DAGs (contd).

Order on vertices.

- For any edge $(u,v) \in E$, we have $u \leq v$.
- The relation is transitive.

  $u \leq v$ and $v \leq w \Rightarrow u \leq w$.

Want to sort with $\leq$ relation as defined above and $\leq$ is not reflexive, not symmetric but transitive.

Topological sort is a sorting of vertices as per $\leq$.

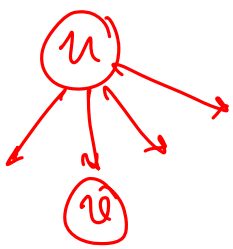
Prereq: Cycle detection:

```
R ← {}
DFS(u):
  Explored[u] ← True
  R ← R ∪ {u}
  For each (u,v) ∈ E :
    If Explored[v] == False:
      DFS(v).
```

DFS(s)
Stack: Last In First Out

Start[u] = current time

End[u] = current time.

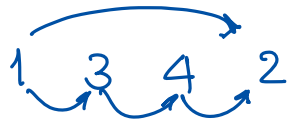


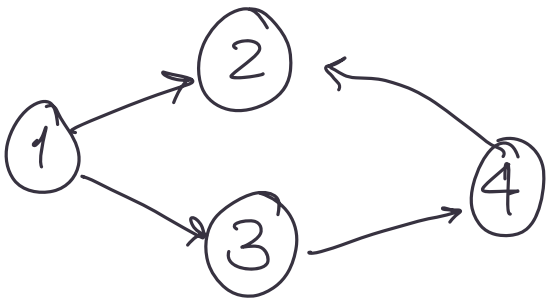
If u was a descendant of v then

- $\text{Start}[v] \leq \text{Start}[u] \leq \text{End}[u] \leq \text{End}[v]$.

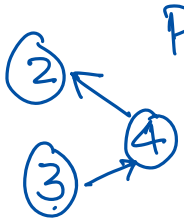
If u and v were unrelated then
$(\text{Start}[u], \text{End}[u])$ and $(\text{Start}[v], \text{End}[v])$
are disjoint.

Remark: Back edges $\Leftrightarrow$ Cycles.





Push 1 into a DS.
Reduce the degree of 2 and 3

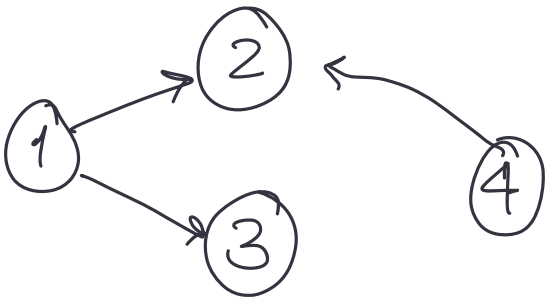


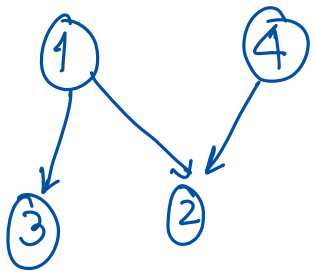
Push 3 into DS
Reduce degree of 4

Topological sort (Attempt 1):

Call this set S

- Start from ~~a~~ vertices of in-degree 0 ← Call this s
- Push all elements of S into a queue.





Topological sort(G): // Do cycle existence check.

- Initialize array InDegree[v] $\forall v \in V(G)$.

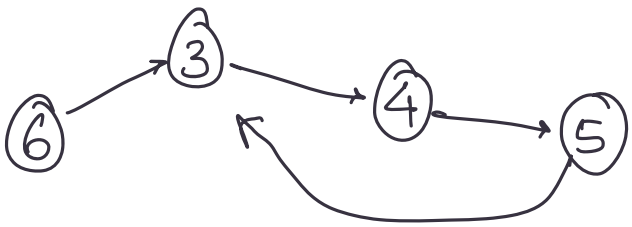
while ∃ a vertex that is not pushed into the DS:

U ← set of vertices w/ indegree 0. // Use another
// If U is empty then say "Not DAG". return.
For all $v \in N(U)$:

InDegree[v] = InDegree[v] − 1. // next indegree zero set $\in N(U)$.

DS.append(U).

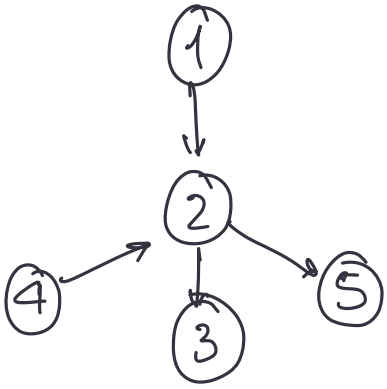
Claim: When done carefully, the complexity is $O(m+n)$.

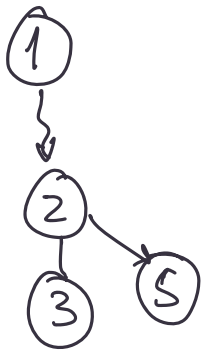


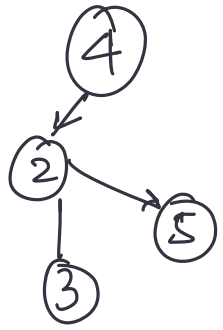
Indeg	6	3	4	5
	0	2	1	1
	X	1	1	1

6

← Can't get any vertex w/ indeg 0
⇒ G is not a DAG.







1 4 2 3 5, 4 1 2 3 5

Claim: Topological sort is given by decreasing order of finish times.

End

1 4 2 5 3, 4 1 2 5 3