Practice problem set

September 16, 2022

Points to note

1. This problem set is **NOT GRADED**
2. These questions are to help you get familiar with graph algorithms and greedy algorithms
3. There is **NO GUARANTEE** that these questions or similar questions will appear in exams

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Questions

1. Consider the following scenario. There are n males and n females in a heterosexual and monogamous society. Every person in the society has an ordered list (increasing order of preference to marry) of members of the opposite sex. A set of tuples of the following kind is said to be a valid matching.

   $$\{(m_1, f_1), (m_2, f_2), \ldots, (m_n, f_n)\}$$

   where $m_i \in$ Males and $f_i \in$ Females $\forall i \in [n]$.
   A valid matching is said to be a stable matching if the following situation does not exist. $\exists (m_i, f_i)$ and (m_j, f_j) s.t pref$_{f_i}(m_i) <$ pref$_{f_i}(m_j)$ and pref$_{m_j}(f_j) <$ pref$_{m_j}(f_i)$ where pref$_{f_i}(m_i)$ is the position at which m_i appears in f_i's ordered list.
   Given below is a greedy algorithm that outputs a valid matching. Argue whether the matching is stable or not.
   Note: pref$_{m_i}$[j] is the j'th entry in m_i's ordered list.

Algorithm 1 Greedy deterministic algorithm for stable matching

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S ← {}
for m_i ∈ {m_1, m_2, ..., m_n} do
    j ← n
    while j ⩾ 1 do
        f_curr ← pref_{m_i}[j]
        if f_curr is not matched yet then
            S ← S ∪ (m_i, f_curr)
            break
        end if
        j ← j − 1
    end while
end for
```

2. Given the MST T of a graph $G(V, E)$ rooted at vertex s, is it guaranteed that every s to t path in T is infact the shortest path from s to t in G?

3. Consider a connected graph $G(V, E)$. Let T, tree rooted at $s \in V$, be the tree obtained by running Dijkstra's on vertex s. Is this tree always the MST of graph G?

4. Let T be the MST of graph $G(V, E)$ and T^* be the tree obtained by running Dijkstra's on some vertex $s \in V$. Assuming all edge weights are unique, answer if T and T^* will change for each of the following change

   (a) Each edge weight is increased by a constant
   (b) Each edge weight is multiplied by a constant
   (c) Each edge weight is squared