

# Tutorial 2 - Probability and Statistics

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## Question 1

If  $E$  and  $F$  are two independent events such that  $P(E \cap F) = 1/6$ ,  $P(E' \cap F') = 1/6$  and  $(P(E) - P(F))(1 - P(F)) > 0$ , then derive the relation between  $P(E)$  and  $P(F)$ .

## Question 2

Suppose we roll two 6-sided die. Consider the events:

A = 'odd on die 1'

B = 'odd on die 2'

C = 'odd sum'.

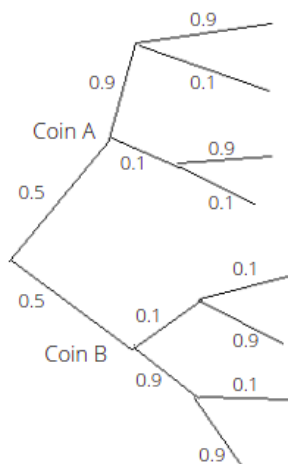
1. Are A, B, and C pairwise independent?
2. Are they mutually independent?

## Question 3

A person's birthday occurs on a day  $i$  with probability  $P_i$ , where  $i = 1, \dots, n$ . Assume independent assignment of birthdays among different people. In a room with  $k$  people, let  $P_k$  be the probability that no two people share a birthday. When is this probability maximized?

## Question 4

There are two biased coins – A and B. The probability of choosing either coin is 0.5. Once the coins are chosen, we perform the experiment of tossing the coins as shown in the figure above. The probabilities for heads and respective coins are given below:



Coin A –

Probability of Head: 0.9

Probability of Tail: 0.1

Coin B –

Probability of Head: 0.1

Probability of Tail: 0.9

Event A: Coin A is chosen

Event B: first 10 tosses are heads

What is the probability of  $(A|B)$  i.e. if we get 10 successive heads, then what is the probability that coin A was chosen?

## Question 5

A coin is tossed twice. Alice claims that the event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses is a head. Is she right? Does it make a difference if the coin is fair or unfair? How can we generalize Alice's reasoning?

## Question 6

Suppose 30% of the women in a class received an A on the test and 25% of the men received an A. The class is 60% women. Given that a person chosen at random received an A, what is the probability this person is a woman?

## Question 7

Can an event  $A$  independent of itself?

## Question 8

Harish has an MCQ quiz (4 options) this week. In the quiz, Harish either knows the answer, guesses it or copies from a friend. Probability of making a guess is  $1/3$  and copying the answer is  $1/6$ . The probability that his answer is correct given that he copied it is  $1/8$ . Find the probability that he knew the answer to that question given that he correctly answered it

## Question 9

A total of 500 married couples are polled about their salaries with the following results:-

	husband makes less than \$25K	husband makes more than \$25K
wife makes less than \$25K	212	198
wife makes more than \$25K	36	54

1. Find the probability that a husband earns less than \$25K.
2. Find the probability that a wife earns more than \$25K, given that the husband earns as that much as well.
3. Find the probability that a wife earns more than \$25K, given that the husband makes less than \$25K.

## Question 10

We roll a fair four-sided die. If the result is 1 or 2, we roll once more but otherwise, we stop. What is the probability that the sum total of our rolls is at least 4?

## Question 11

Let  $C_1, C_2, \dots, C_M$  be disjoint partitions of the sample space  $S$ . Let  $A$  and  $B$  be two events. Suppose we know that:

1.  $A$  and  $B$  are conditionally independent given  $C_i$   $i \in 1, 2, \dots, M$
2.  $B$  is independent of all  $C_i$ s

Prove that  $A$  and  $B$  are independent.

## Question 12

A professor thinks students who attend offline classes are more likely to get  $A$ s in the probability course. To check this theory, the professor combines the data from the past few years:

1. 600 students have taken the course
2. 120 students have gotten  $A$ s
3. 200 students lived on campus
4. 80 students lived off campus and got  $A$ s

Does this data suggest that "getting an  $A$ " and "living on campus" are dependent or independent?

## Question 13

In an entrance examination with multiple choice questions, with each question having four options and a single correct answer, suppose that only 20% candidates think they know the answer to one difficult question and only half of them know it correctly and the other half get it wrong. The remaining candidates pick one option out of the four randomly and tick the same. If a candidate has correctly answered the question, what is the (conditional) probability that she knew the answer?

## Question 14

There are 3 coins. One is two headed coin, the other one comes heads 75 percent of the time and the third is a fair coin. A coin is selected at random and flipped.

1. What is the probability that the flipped coin will come up head?
2. Given that the coin that was flipped comes up head, what is the probability that it was fair coin?

## Question 15

Sarah and Bob draw 13 cards each from a standard deck of 52. Given that Sarah has exactly two aces, what is the probability that Bob has exactly one ace?

## Question 16

A biased coin (with probability of obtaining head equal to  $p > 0$ ) is tossed repeatedly and independently until the first head is observed. Compute the probability that the first head appears at an even numbered toss.

## Question 17

An urn has 5 blue balls and 8 red balls. Each ball that is selected is returned to the urn along with an additional ball of the same color. Suppose that 3 balls are drawn in this way.

1. What is the probability that the three balls are blue?
2. What is the probability that only 1 ball is blue?

## Question 18

You enter a chess tournament where your probability of winning a game is .3 against half of the players (call them type 1), 0.4 against a quarter of the players (call them type 2) and 0.5 against the remaining quarter of the players (call them type 3). You play a game against a randomly chosen opponent. Suppose you won the game. What is the probability that you had an opponent of type 1?

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