Lecture 2 – Binary numbers and representations

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Chapter 1 (second half)

Addition

- Octal number system
  - $(73)_8 + (157)_8$
  - $(57)_8 + (23)_8$

- Hexadecimal number system
  - $(AA)_{16} + (BB)_{16}$
  - $(BAD)_{16} + (DAD)_{16}$

- Binary number system
  - $(1101)_2 + (111)_2$
  - $(10101)_2 + (100)_2$

Subtraction

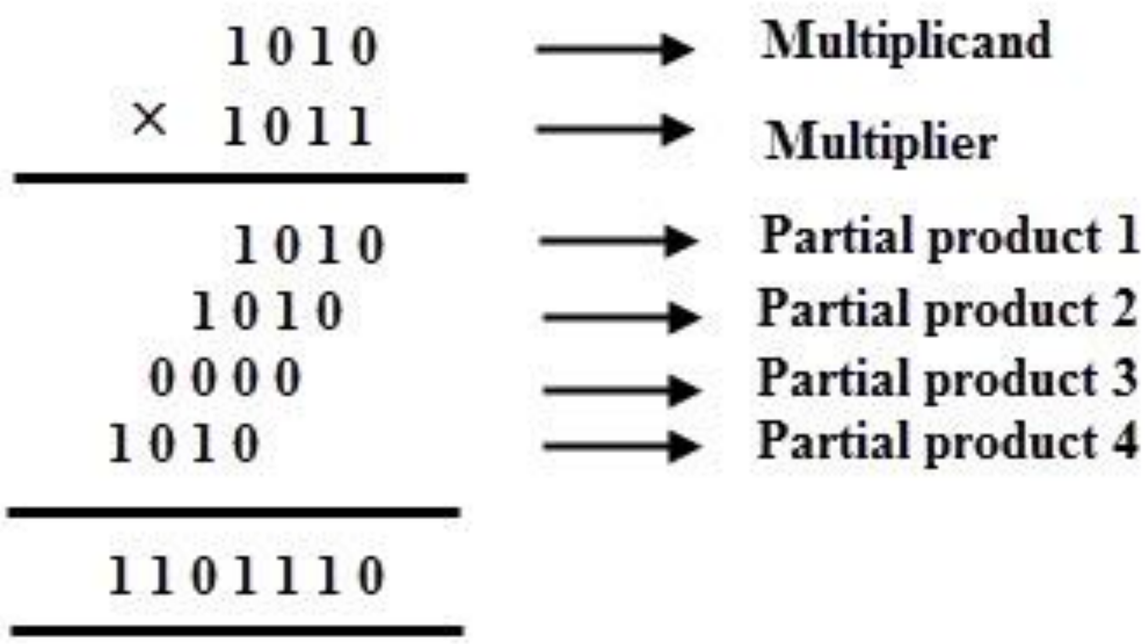
- Octal number system
  - $(172)_8$ - $(167)_8$
  - $(32)_8$ - $(21)_8$

- Hexadecimal number system
  - $(BB)_{16}$ - $(AA)_{16}$
  - $(DAD)_{16}$ - $(BAD)_{16}$

- Binary number system
  - $(1101)_2$ - $(111)_2$
  - $(10101)_2$ - $(100)_2$

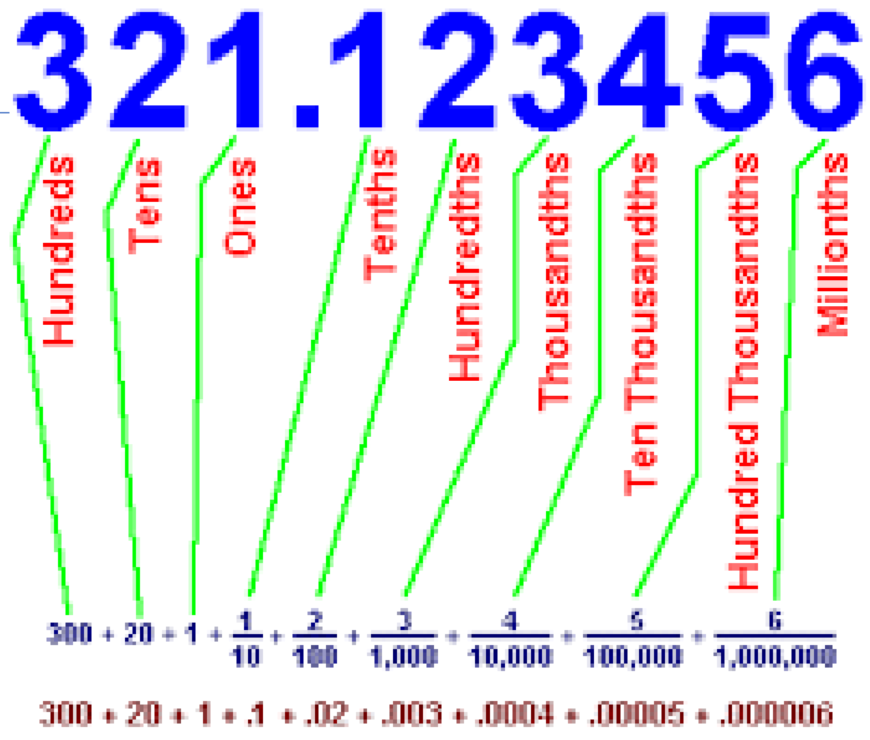
Multiplication

- Binary number system



- Examples:
  - $(111)_2*(110)_2$
  - $(1011)_2*(1010)_2$

The “decimal” point



- The powers of radix decrease after the decimal point
- Binary to decimal:
  - $(1.011)_2 = 1*2^0+0*2^{-1}+1*2^{-2}+1*2^{-3}$
    $= 1+0.25+0.125$
    $= 1.375$
  - $(0.1101)_2$
- Decimal to binary:
  - $(0.75)_{10} = 0.75*2 = 1.50$ 1
    $0.5*2 = 1.00$ 1
    $=(11)_2$
  - $(0.625)_{10}$

Complements of numbers

- Complement operations are run on a single number in any given base
- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation
- Simplifying operations leads to simpler, less expensive circuits to implement the operations
- There are two types of complements for each base-*r* system:

1. The radix complement [r’s complement] – called the 10’s complement in decimal, 2’s complement in binary and so on
2. The diminished radix complement [(r-1)’s complement] – called the 9’s complement in decimal, 1’s complement in binary and so on

Diminished radix complement

- Given a number N in base r having n digits, the (r - 1)'s complement of N , i.e., its diminished radix complement, is defined as $(r^n - 1) - N$
- For decimal numbers, the 9's complement of N is $(10^n - 1) - N$
- In this case, $10^n \ - \ 1$ is a number represented by n 9s
- For example, if n = 4, we have 10^4 = 10,000 and 10^4 - 1 = 9999
- It follows that the 9's complement of a decimal number is obtained by subtracting each digit from 9
- Examples:
  - 1242
  - 9981

Diminished radix complement

- For binary numbers, the 1's complement of *N* is $(2^n-1) - N$.
- Again, $(2^n – 1)$ is a binary number represented by *n* 1s
- For example, if $n = 4$, we have $2^4 = (10000)_2$ and $2^4 - 1 = (1111)_2$. Thus, the 1's complement of a binary number is obtained by subtracting each digit from 1
- However, when subtracting binary digits from 1, we can have either 1 - 0 = 1 or 1 - 1 = 0, which causes the bit to change from 0 to 1 or from 1 to 0, respectively
- Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.**
- Examples:
  - 11100101
  - 10000