CS 302.1 - Automata Theory

Lecture 06

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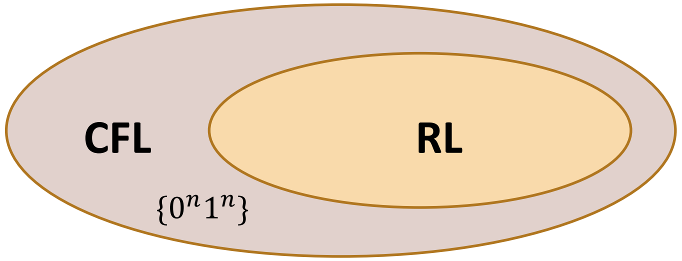
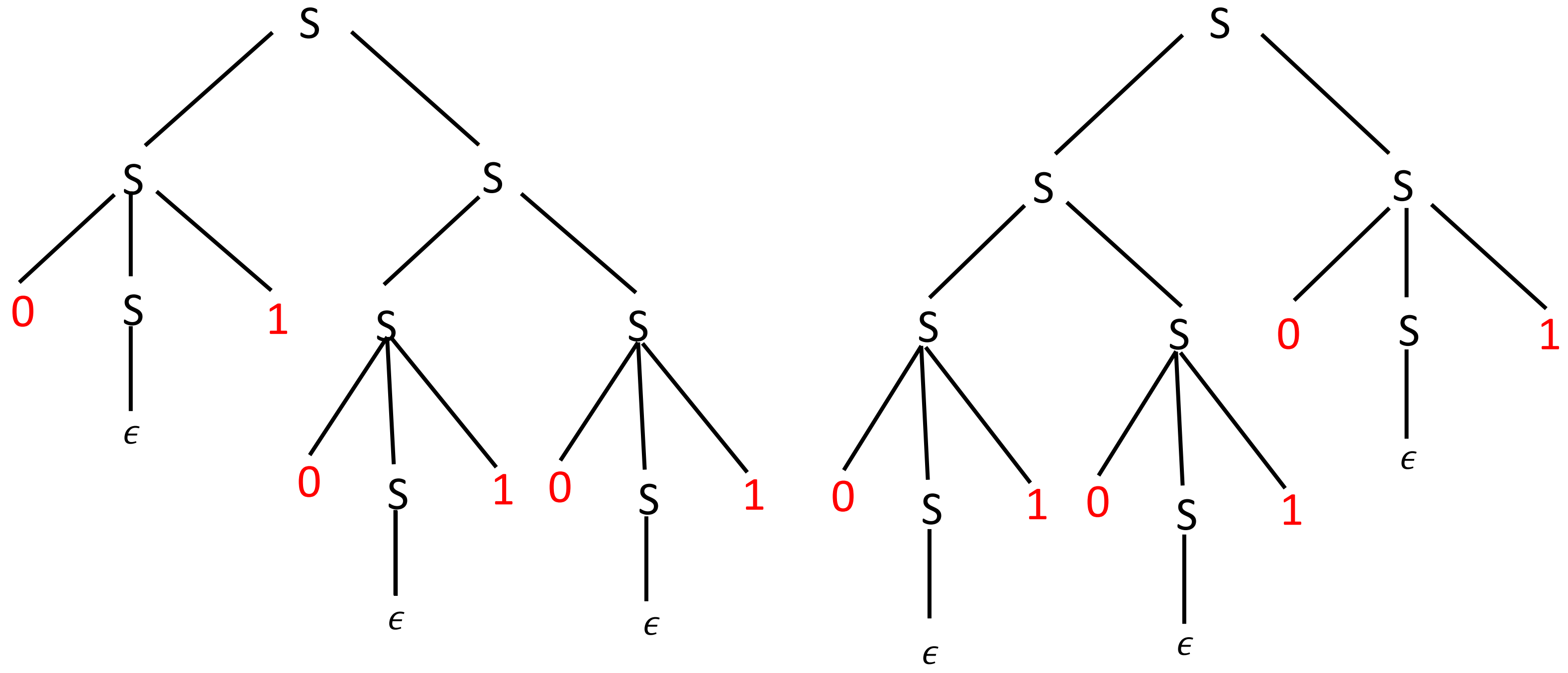
Quick Recap

Context-Free Grammars: If the *rules* of the underlying grammar G are of the form

$$\boldsymbol{V} \to (\boldsymbol{V} \cup \boldsymbol{T})^*$$

then such a grammar is called **Context-Free.**





Parse trees: These are ordered trees that provide alternative representations of the derivation of a grammar.

Ambiguous grammars: There exists $\boldsymbol{\omega} \in \boldsymbol{L}(\boldsymbol{G})$, such that there are **two or more leftmost derivations for $\boldsymbol{\omega}$** (or equivalently two or more rightmost derivations) or equivalently **two or more parse trees for $\boldsymbol{\omega}$. Ambiguity** may not be desirable

Quick Recap

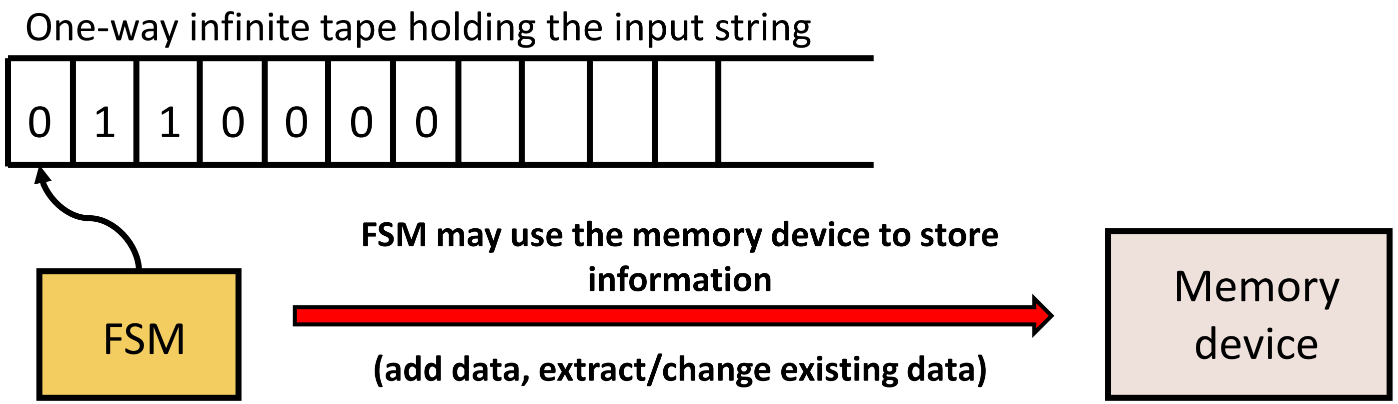
Chomsky Normal Form: If every *rule* of the CFG is of the form

$$\boldsymbol{A \to BC} \qquad [B, C \text{ are not start variables}]$$
$$\boldsymbol{A \to a} \qquad [a \text{ is a terminal}]$$
$$\boldsymbol{S \to \epsilon} \qquad [S \text{ is the Start Variable}]$$

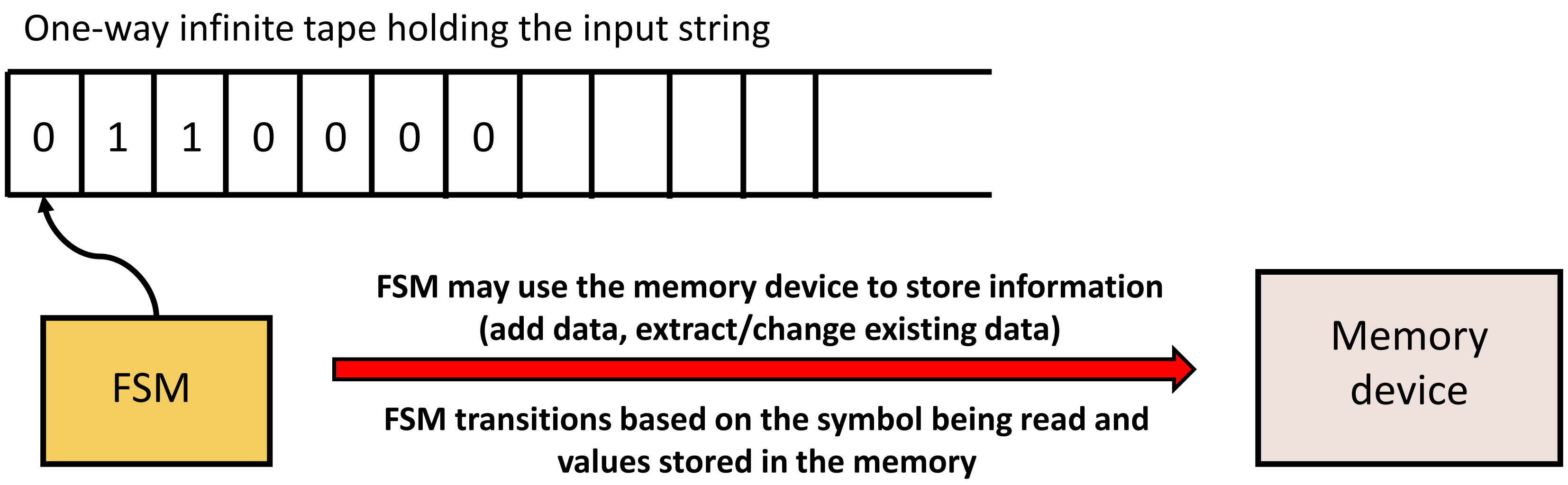
- Any CFG can be converted to a grammar in CNF that generates the same language.
- The number of steps required to derive a string $w = 2|w| - 1$.
- Is crucial in deciding whether w is generated by a CFG G.

Pushdown Automata

- Automata that recognizes CFLs
- FSM + memory device
- FSM transitions by reading an input symbol and by interacting with the device



Pushdown Automata



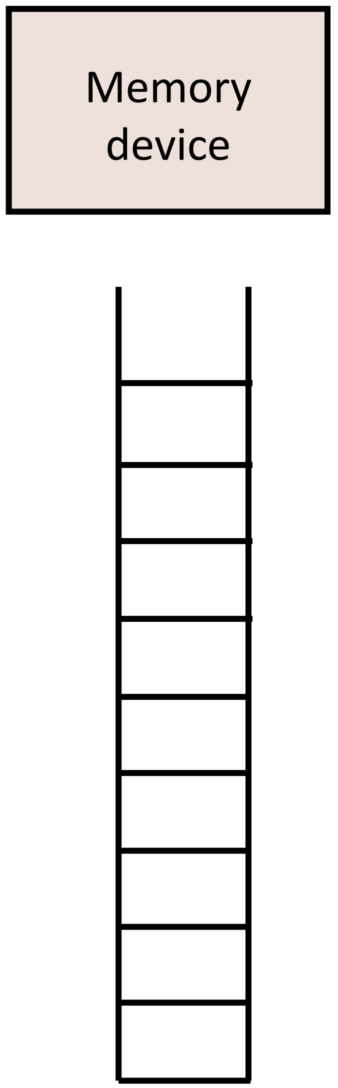
The memory device

- Simple memory device with unbounded memory.

Pushdown Automata

The memory device

- Simple memory device with unbounded memory.
- Consider a **STACK**
- At any stage, new elements can be added to the Stack (**PUSH**).
- At any stage, the element at the **top** of the STACK can be read by removing it from the stack (**POP**).



Pushdown Automata

The memory device

- Simple memory device with unbounded memory.
- Consider a **STACK**
- At any stage, elements can be pushed or popped.

PUSH

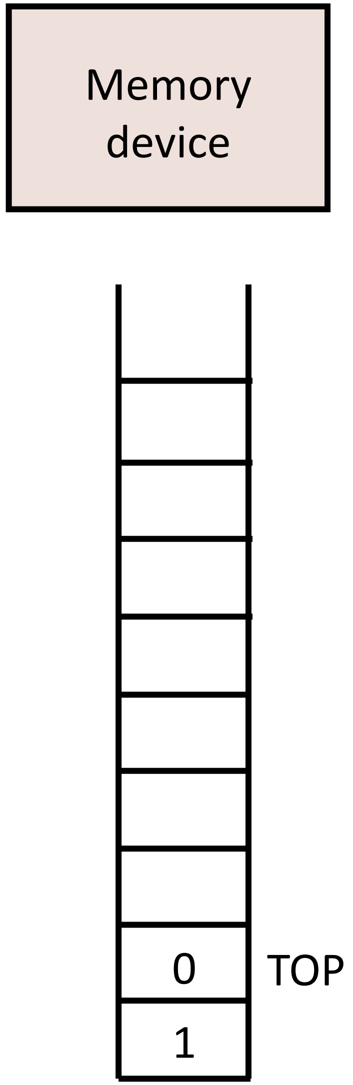
- New symbols can be **pushed** in to the STACK.

  E.g: PUSH 1

- The Top of the STACK now covers the old stack top, i.e.

  TOP = TOP + 1

- The size of the stack keeps growing.



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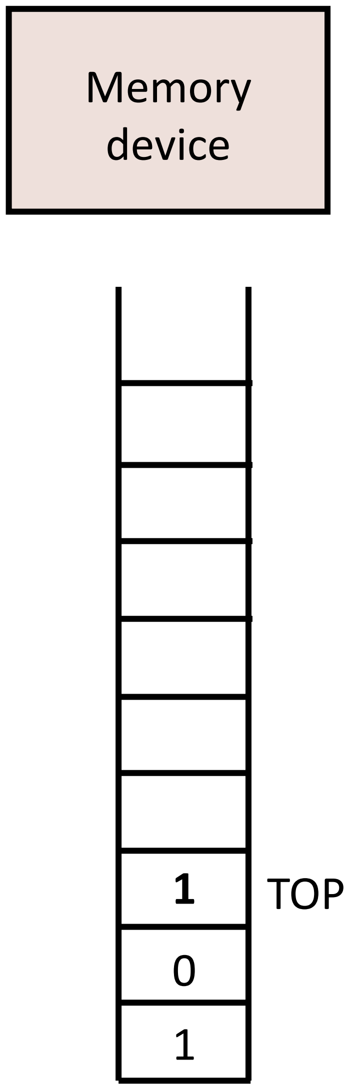

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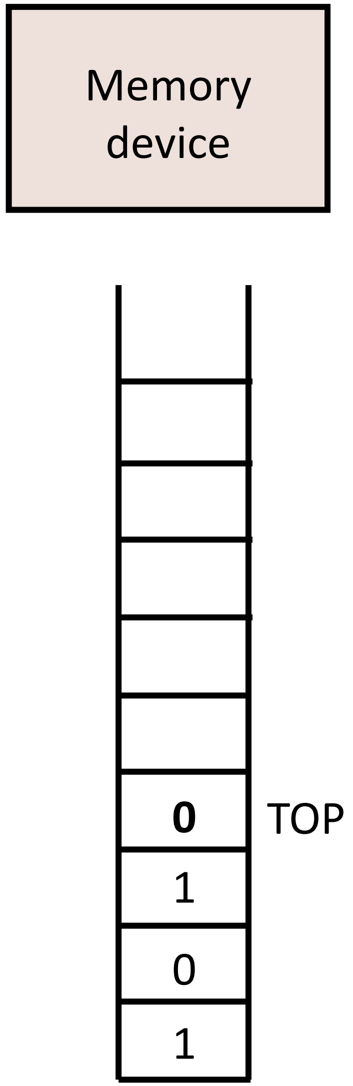

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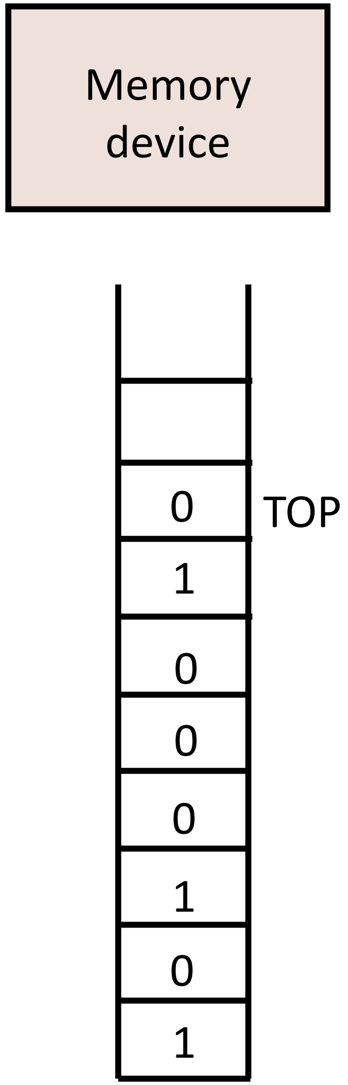

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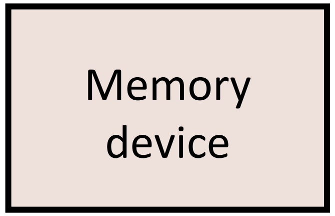


POP

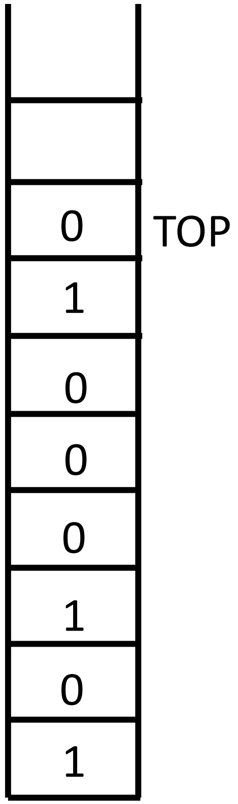
- The element from the TOP of the stack can be **popped** out

E.g.: **POP 0**

- The Top of the STACK moves to the element below.

TOP = TOP - 1

- Successive **POP** operations shrink the stack size. Elements can be popped until EMPTY.
- **Last In First Out (LIFO)**: The last element that was pushed is the first to be popped out



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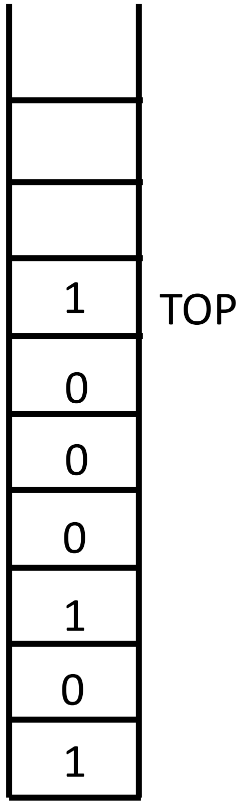

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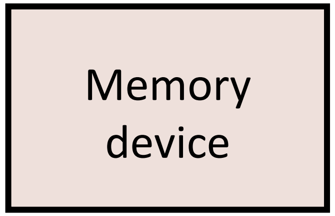

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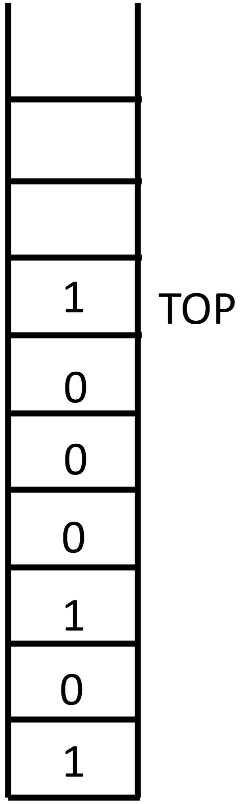

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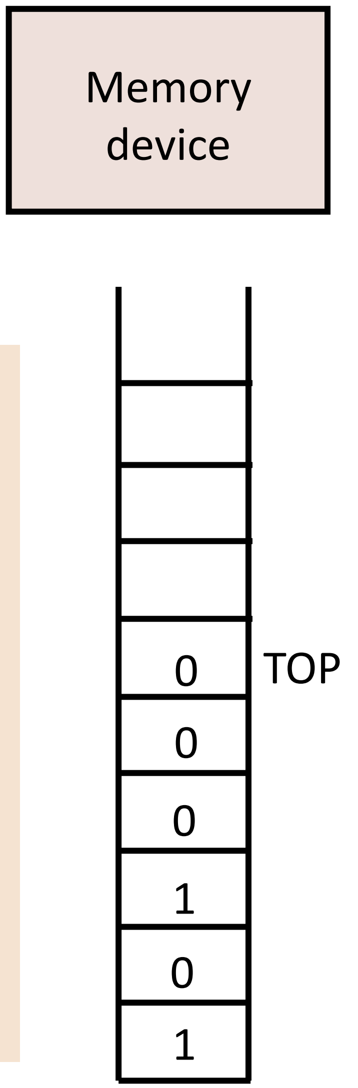

Pushdown Automata

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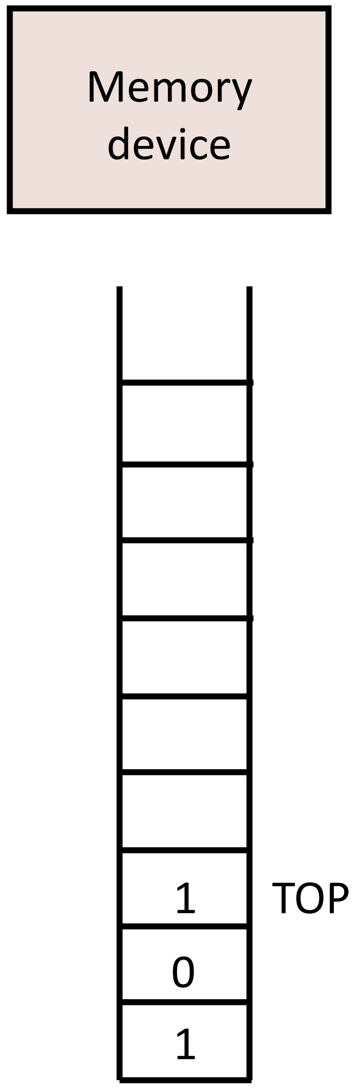


POP

- The element from the TOP of the stack can be **popped** out.
- TOP = TOP - 1
- Elements can be popped until STACK is EMPTY.

- How would you know that the STACK is EMPTY?

Pushdown Automata

The memory device

- Simple memory device with unbounded memory.
- Consider a **STACK**
- Last In First Out (**LIFO**)

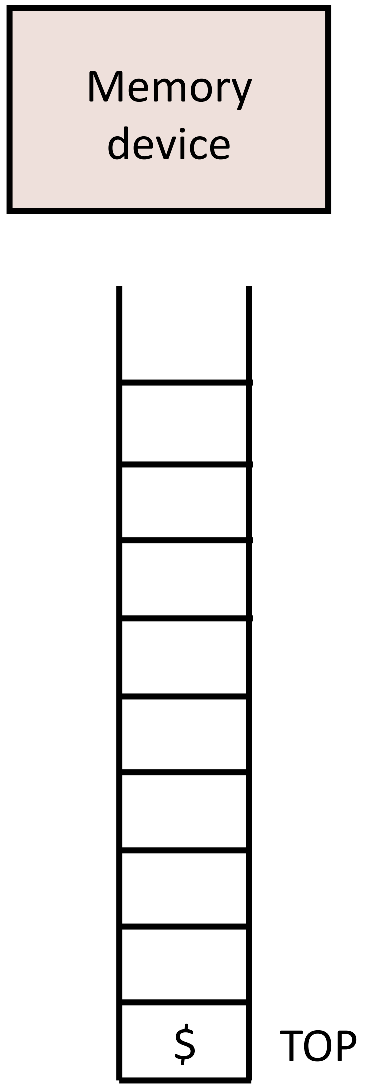
POP

- The element from the TOP of the stack can be **popped** out.
- TOP = TOP - 1
- Elements can be popped until STACK is EMPTY.

- How would you know that the STACK is EMPTY?

- There is generally some special symbol (say $) that demarcates the bottom of the STACK.

- This element is Pushed at the very beginning. Whenever TOP = $, the STACK is EMPTY.



Pushdown Automata

Memory device of PDA: STACK

- STACK is a **LIFO** data structure of unbounded memory
- Only the TOP element can be read from the STACK.
- The bottom of the STACK contains a special symbol ($)
- Characterized by two operations:

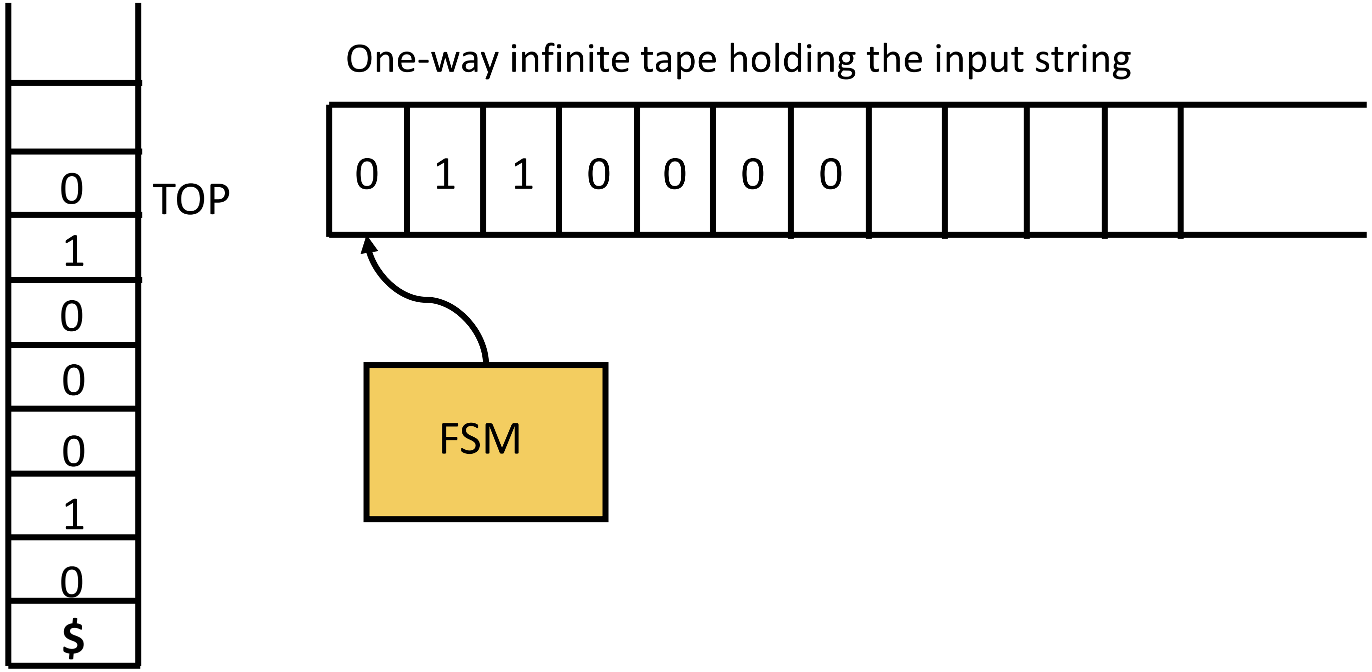
PUSH

- New symbols can be **pushed** in to the STACK.
- TOP = TOP + 1

POP

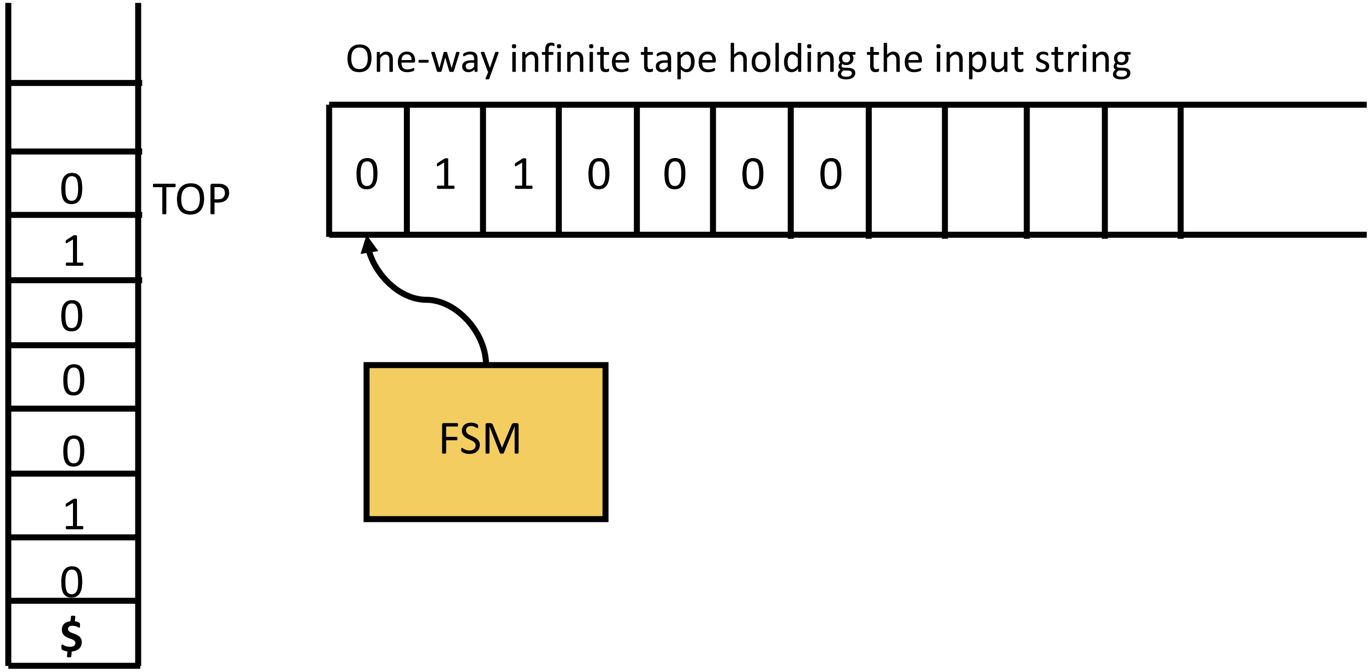
- The element from the TOP of the stack can be **popped** out.
- TOP = TOP - 1
- Elements can be popped until STACK is EMPTY.

Pushdown Automata



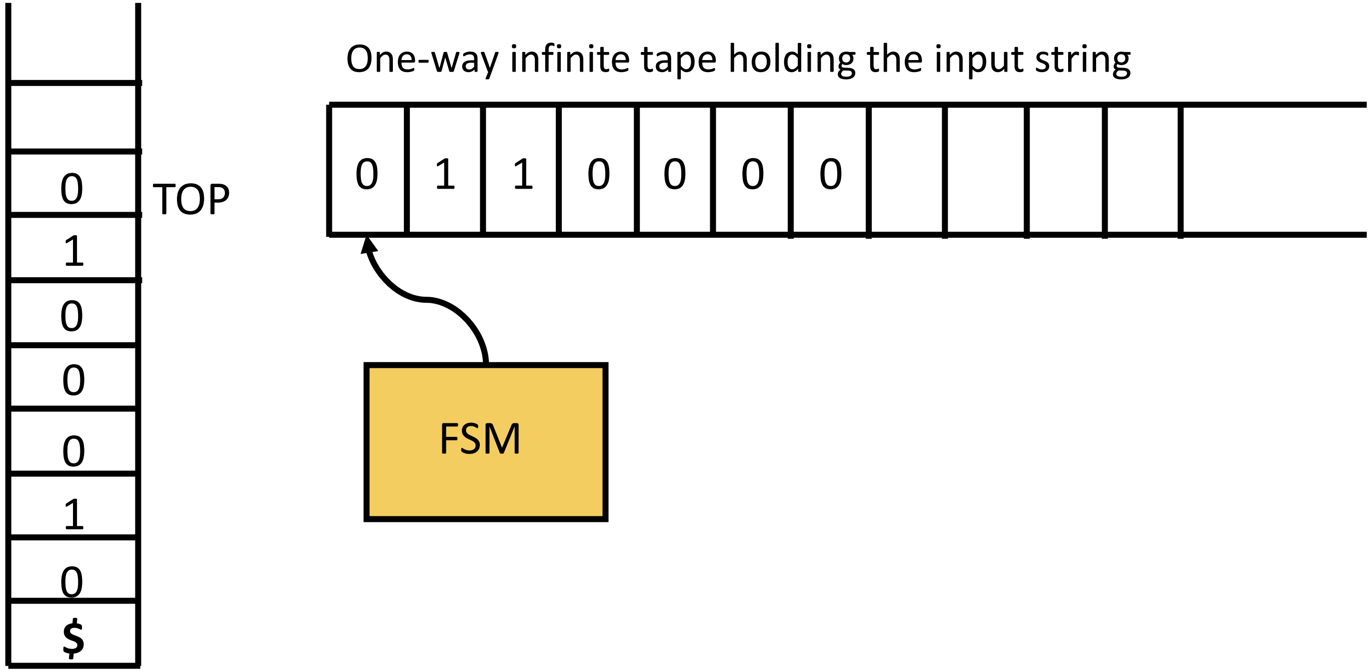
- A Pushdown Automata (PDA) is a finite automaton that has access to a stack.
- The FSM:

Pushdown Automata



- A Pushdown Automata (PDA) is a finite automaton that has access to a stack.
- The FSM:
  - Transitions based on the Input symbol and the element at the top of the stack (e.g.: If I/P symbol = 0 & TOP = 0, transition from i to j)

Pushdown Automata



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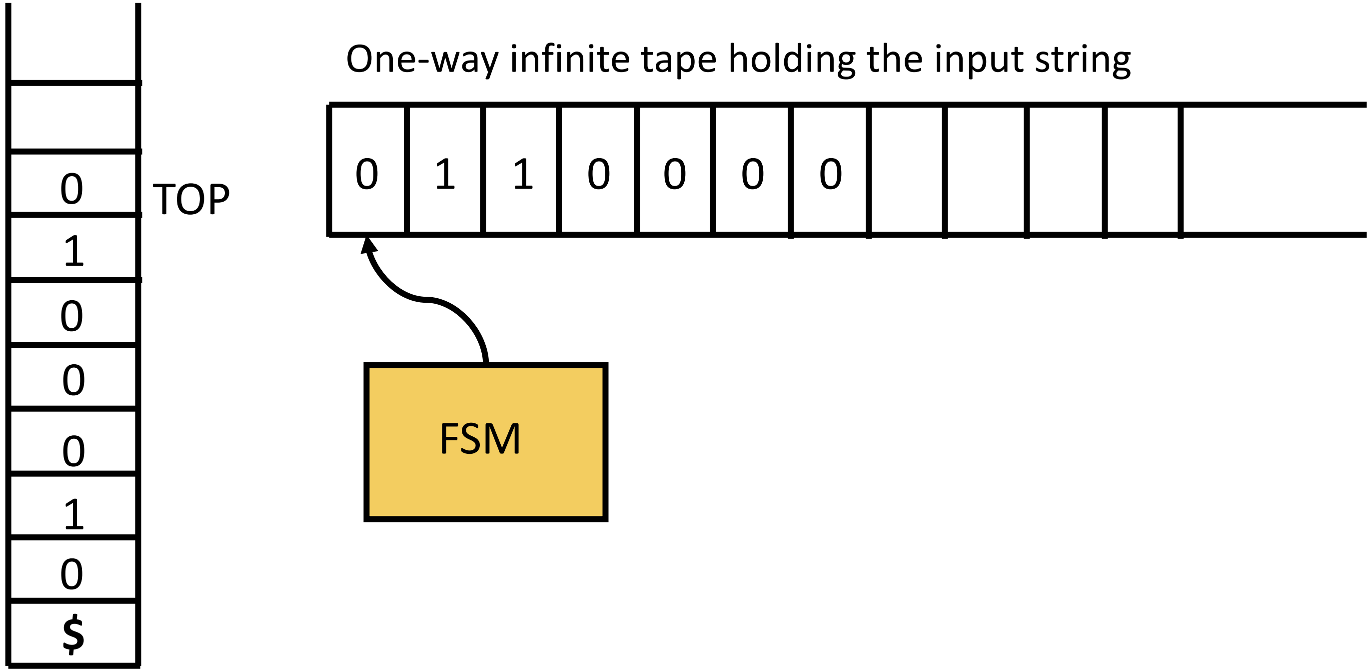
- The FSM:
  - Transitions based on the Input symbol and the element at the top of the stack (e.g.: If I/P symbol = 0 & TOP = 0, transition from i to j).
  - **How can we read the TOP? By popping**

Pushdown Automata



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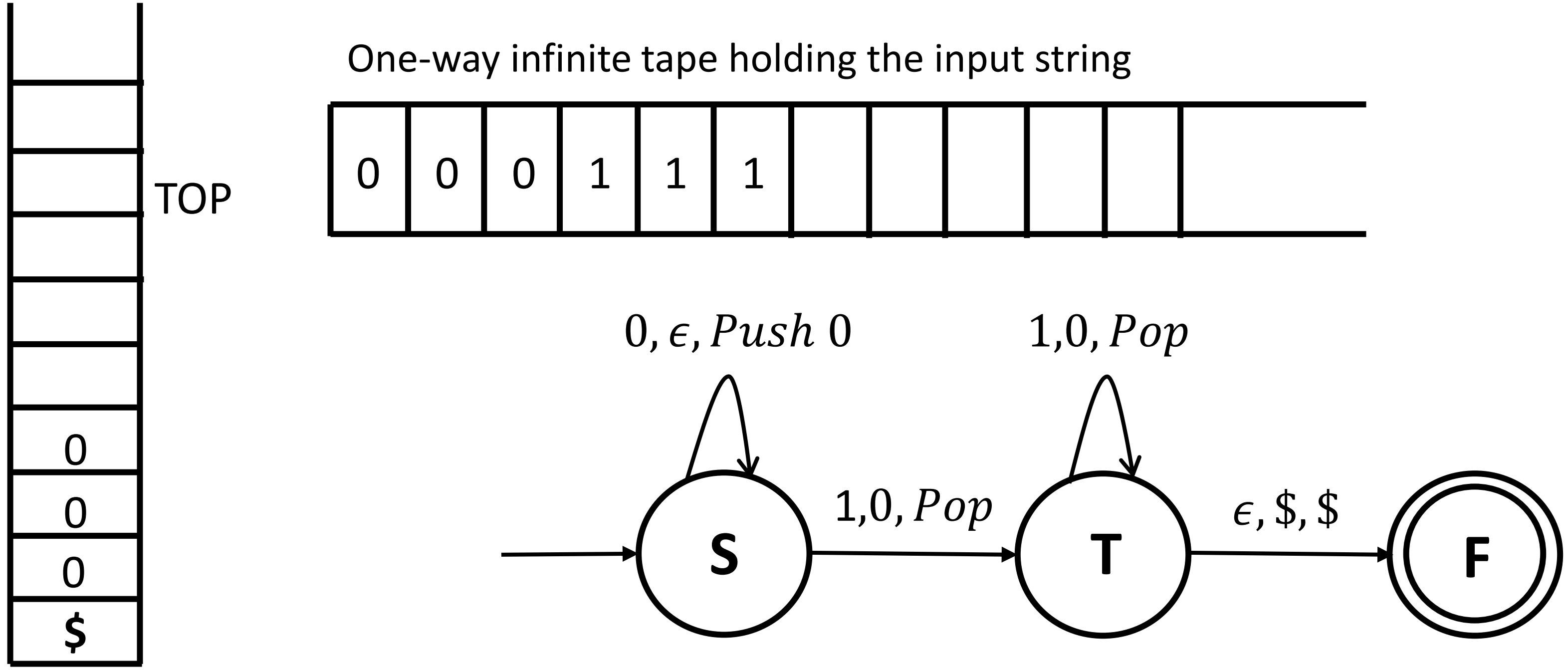
- The FSM:
  - Transitions based on the Input symbol and the element at the top of the stack (e.g.: If I/P symbol = 0 & TOP = 0, transition from i to j) – **Pop 0**

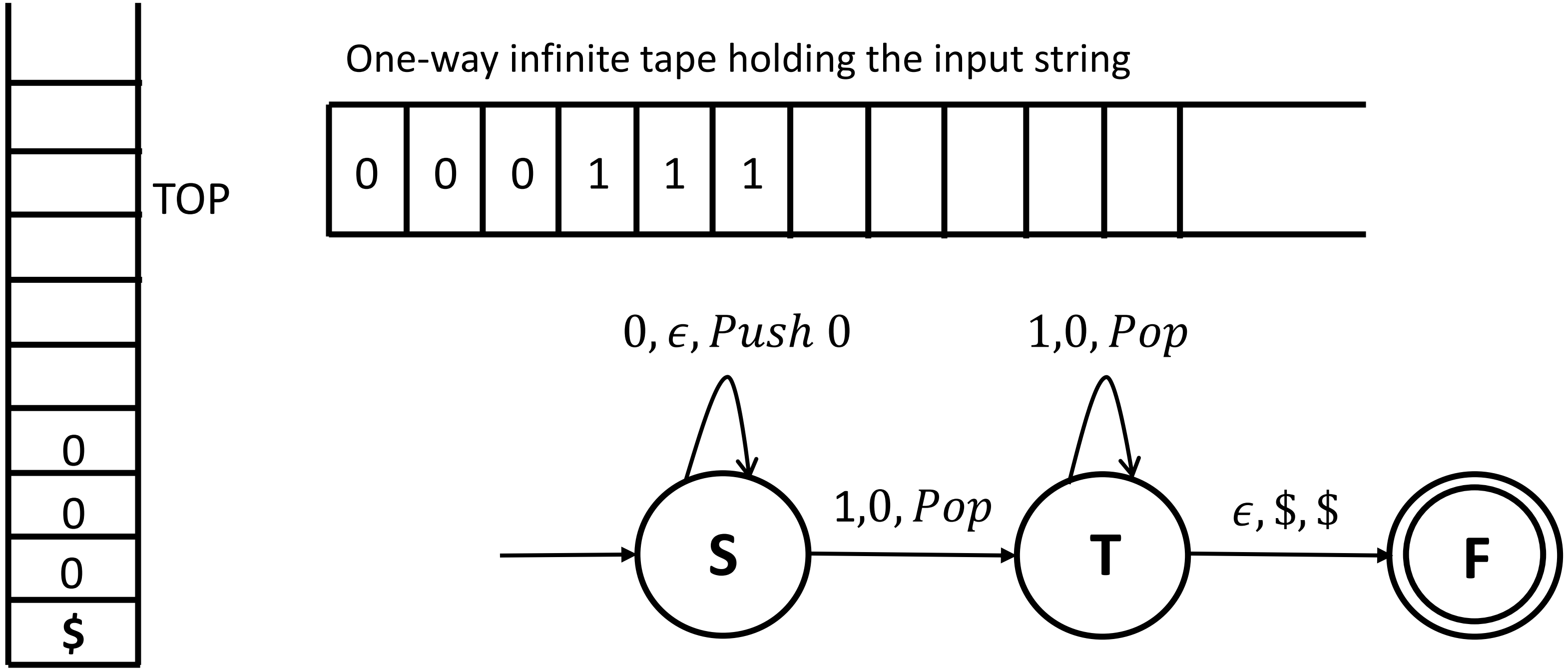
  - Pushes new elements into the Stack (e.g.: If I/P symbol = 0, PUSH 0, transition from i to j).

Pushdown Automata



- A Pushdown Automata (PDA) is a finite automaton that has access to a stack.
- The FSM:
  - Transitions based on the Input symbol and the element at the top of the stack
  - Pops the element at the top of the Stack.
  - Pushes new elements into the Stack.

Pushdown Automata

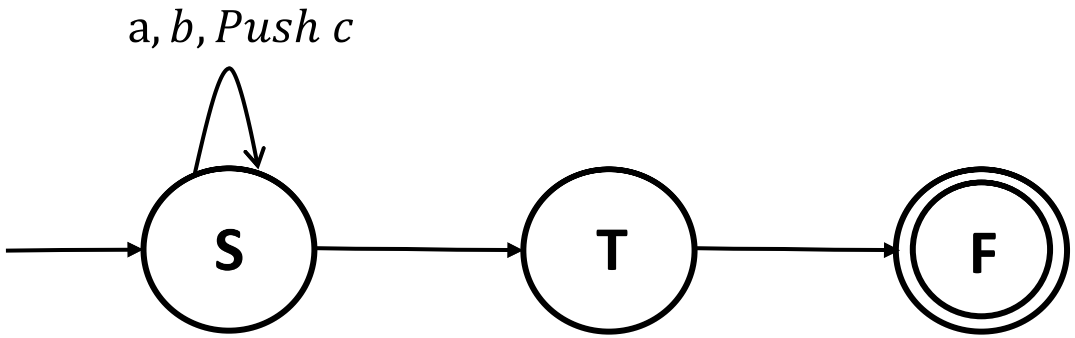


PDAs are **non-deterministic**.

- ϵ-transitions
- Multiple transitions/input symbol possible

Pushdown Automata

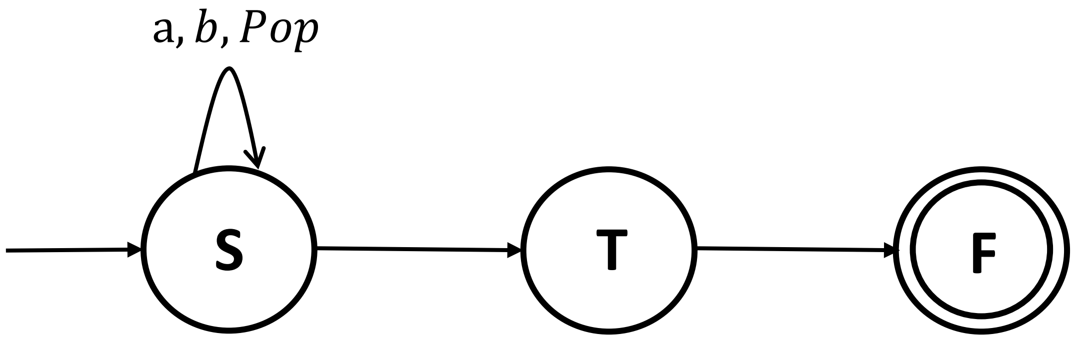
- How to represent a transition in a PDA?



If input symbol = a, Stack top = b (if b is popped) and Push c onto the Stack

Pushdown Automata

- How to represent a transition in a PDA?

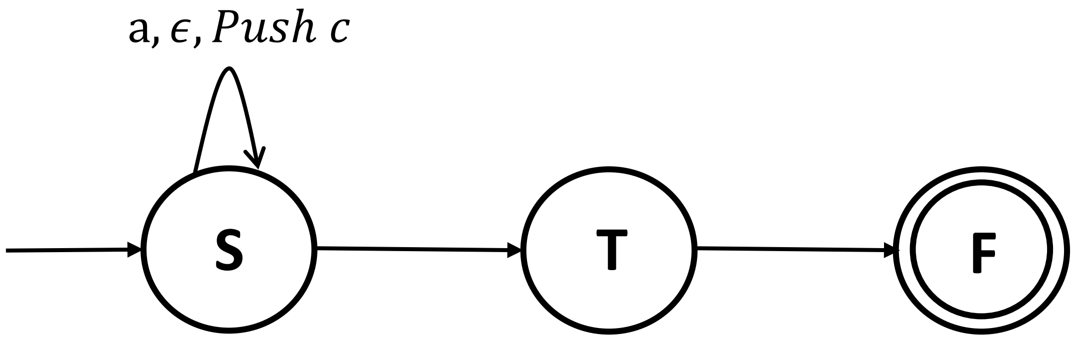


If input symbol = a, then Pop b

(If the symbol read is a and the stack TOP $= b$, then remain in S)

Pushdown Automata

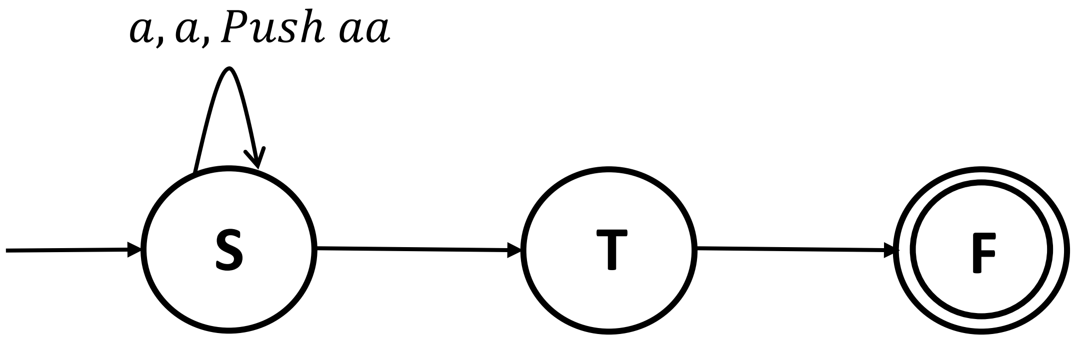
- How to represent a transition in a PDA?



If input symbol = a, then Push c

Pushdown Automata

- How to represent a transition in a PDA?



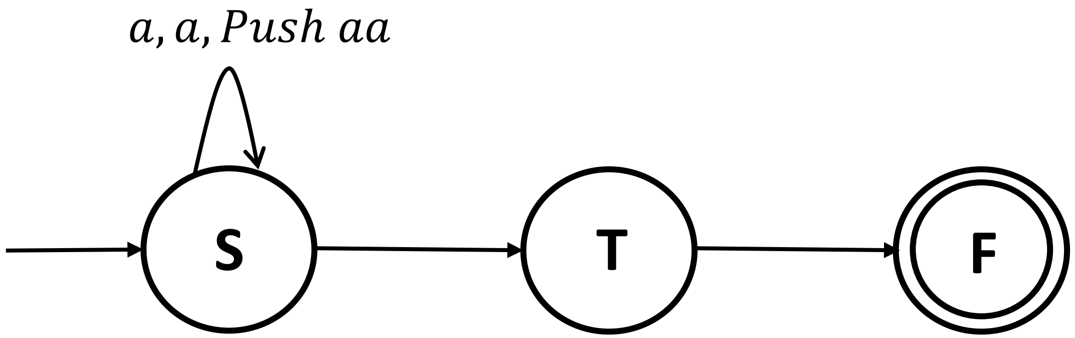
If input symbol = a, then Pop a and Push aa.

So effectively, the PDA pushes a onto the stack if it reads a on the input tape and the stack top = a.

This is a “shorthand” for describing two operations:

Pushdown Automata

- How to represent a transition in a PDA?



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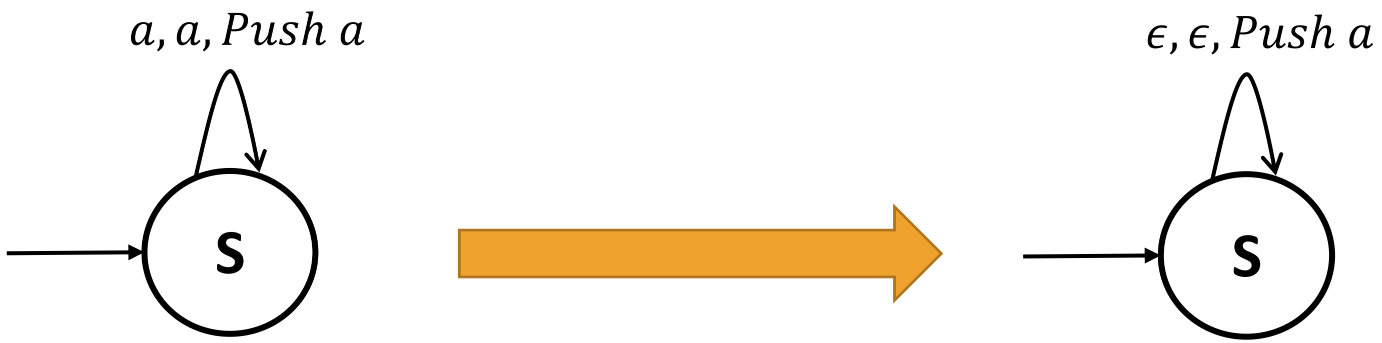

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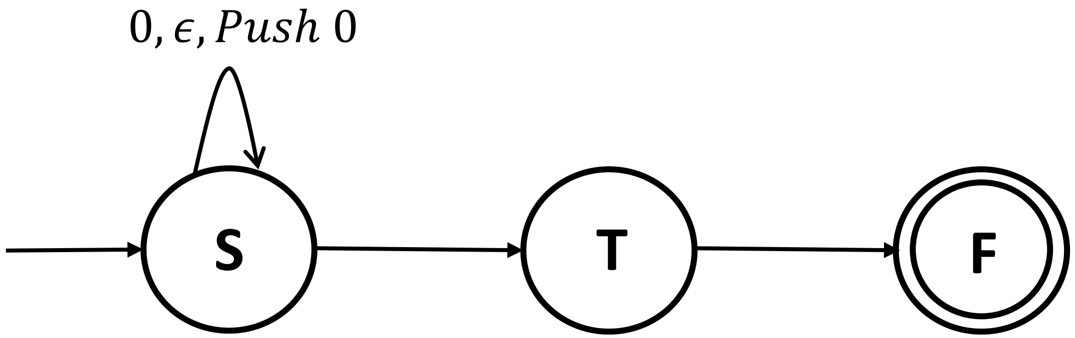
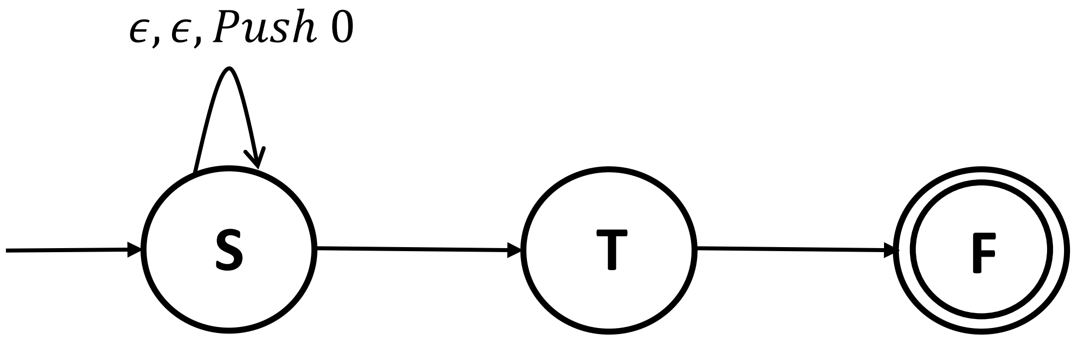


If input symbol = 0, Push 0 onto the Stack irrespective of the element at the top of the stack

Pushdown Automata

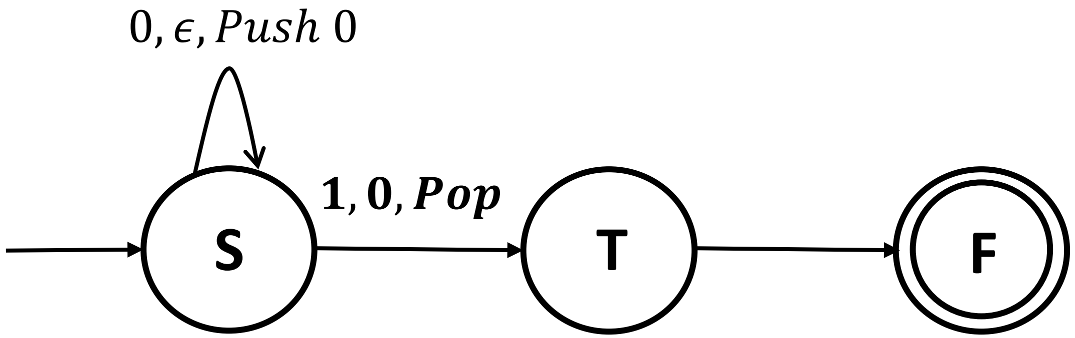
- How to represent a transition in a PDA?



Without reading the input symbol and the Stack top, Push 0 onto the Stack

Pushdown Automata

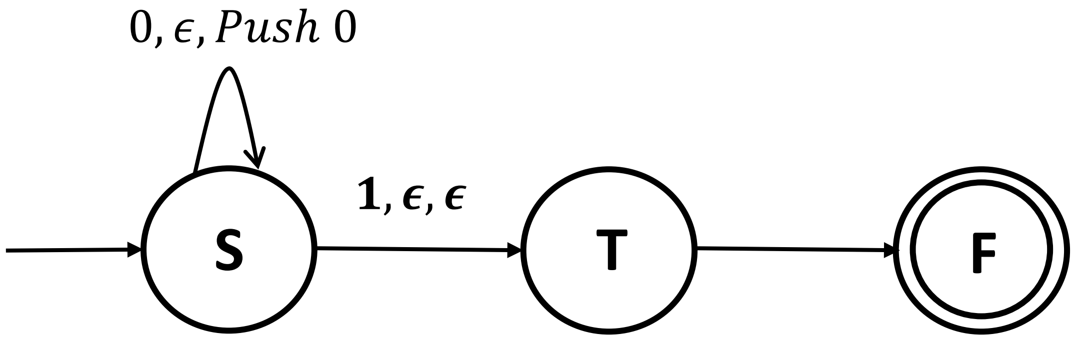
- How to represent a transition in a PDA?



If the input symbol is 1, and the element at the top of the stack is 0 **(Pop 0)**, then transition from S to T

Pushdown Automata

- How to represent a transition in a PDA?

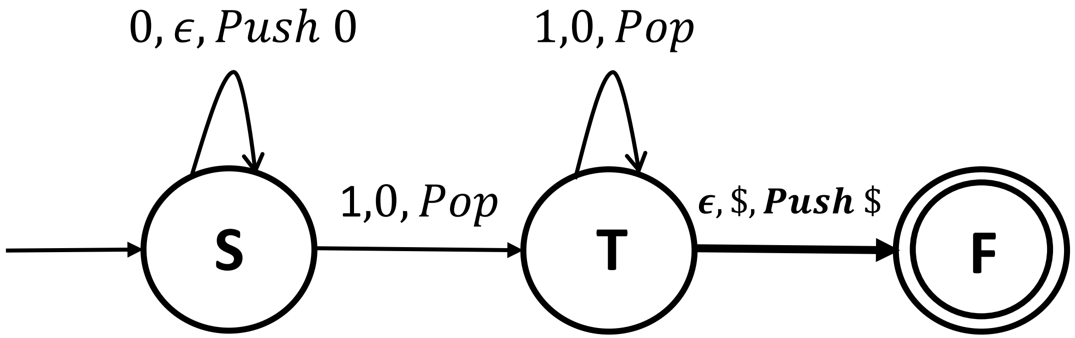


If the input symbol is 1, transition to T by ignoring the stack top completely.

If this happens at every step of the execution of the PDA, then it is as powerful as an NFA.

Pushdown Automata

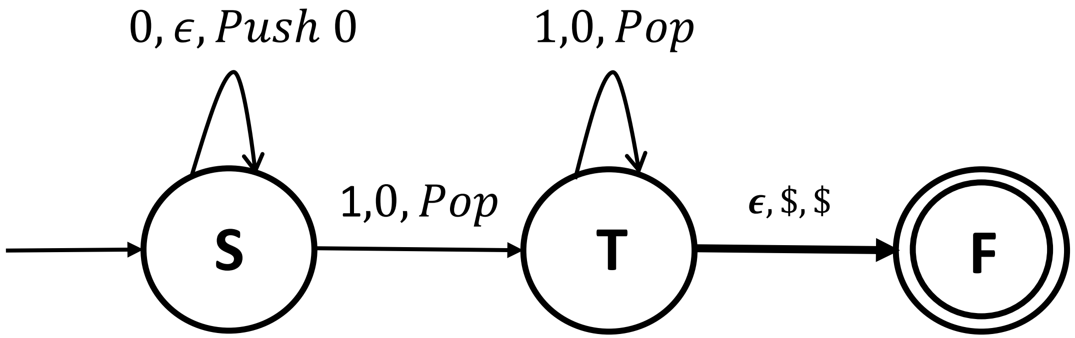
- How to represent a transition in a PDA?



If the Stack is empty, i.e. TOP = \$, transition to F from T, without reading the input

Pushdown Automata

- How to represent a transition in a PDA?



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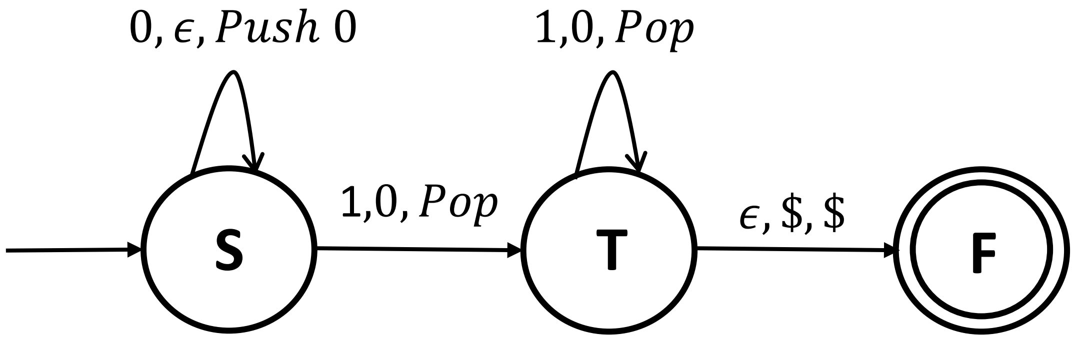
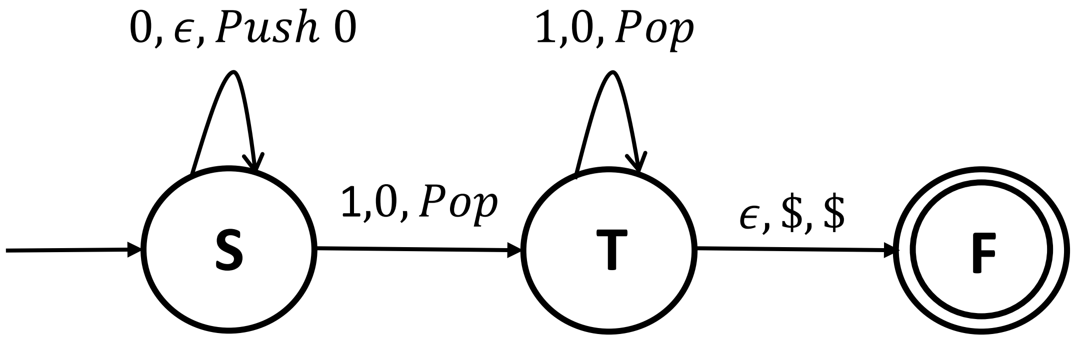


What is the language accepted by this PDA?

Pushdown Automata

- How to represent a transition in a PDA?



What is the language recognized by this PDA?

Verify that it is $\boldsymbol{L = \{0^n 1^n, n \geq 1\}}$

Pushdown Automata

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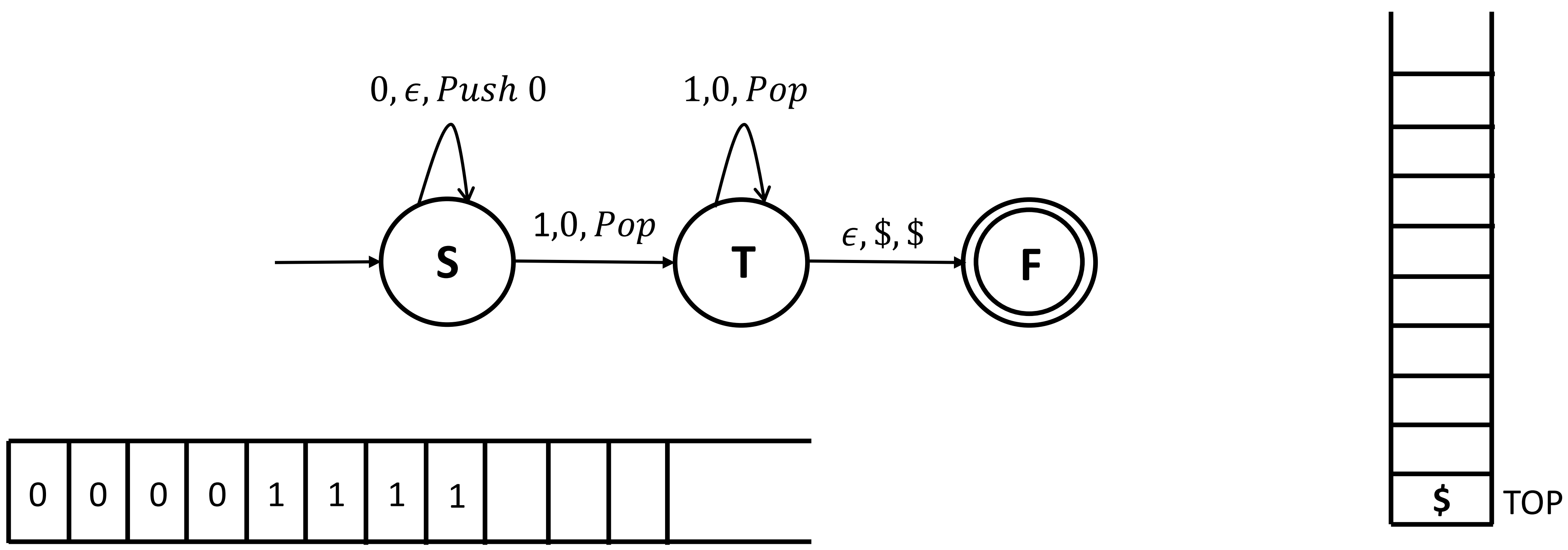

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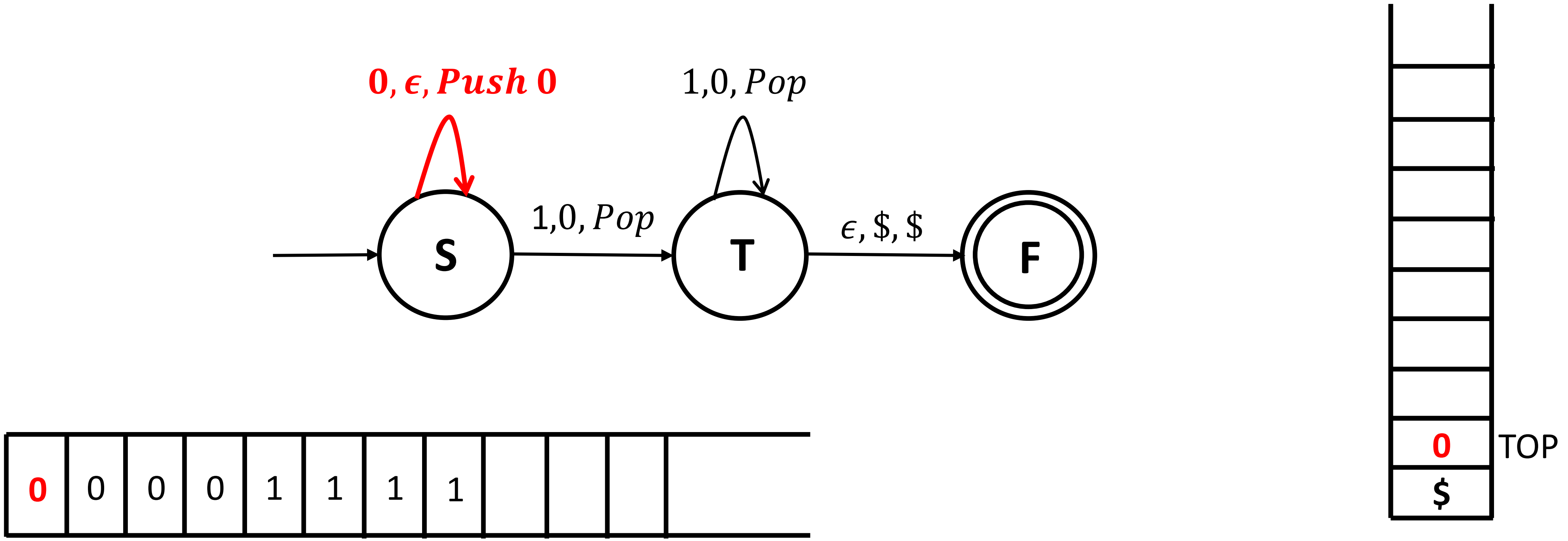

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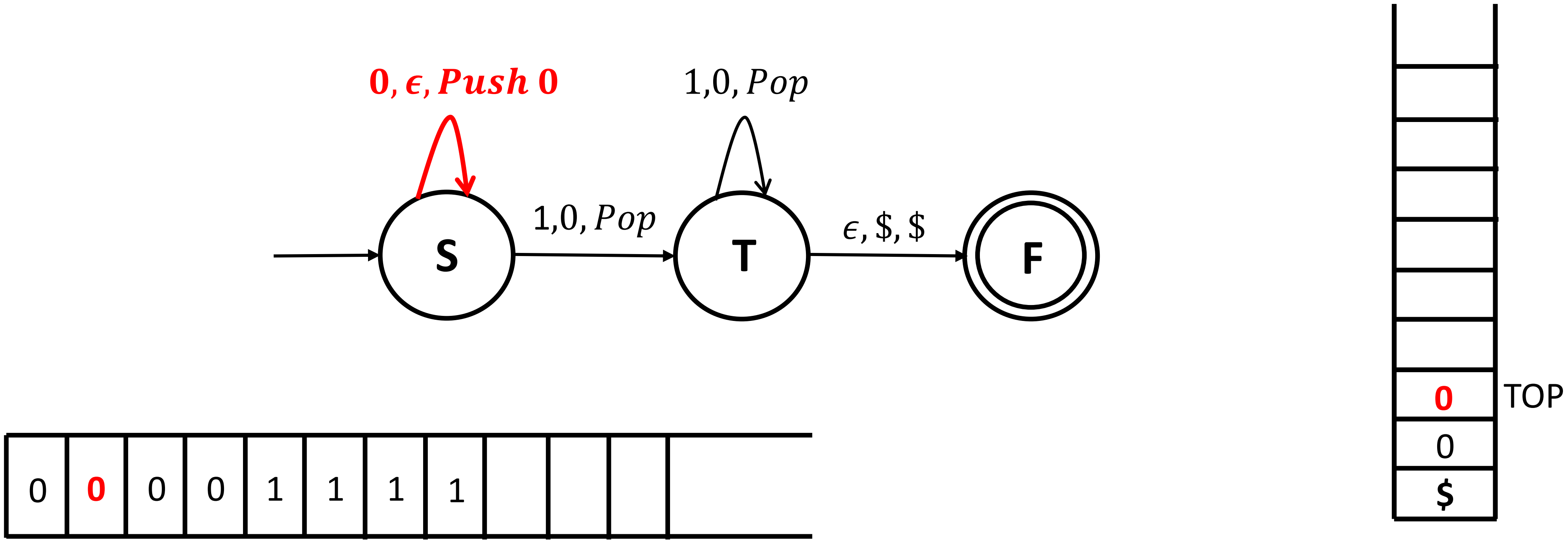

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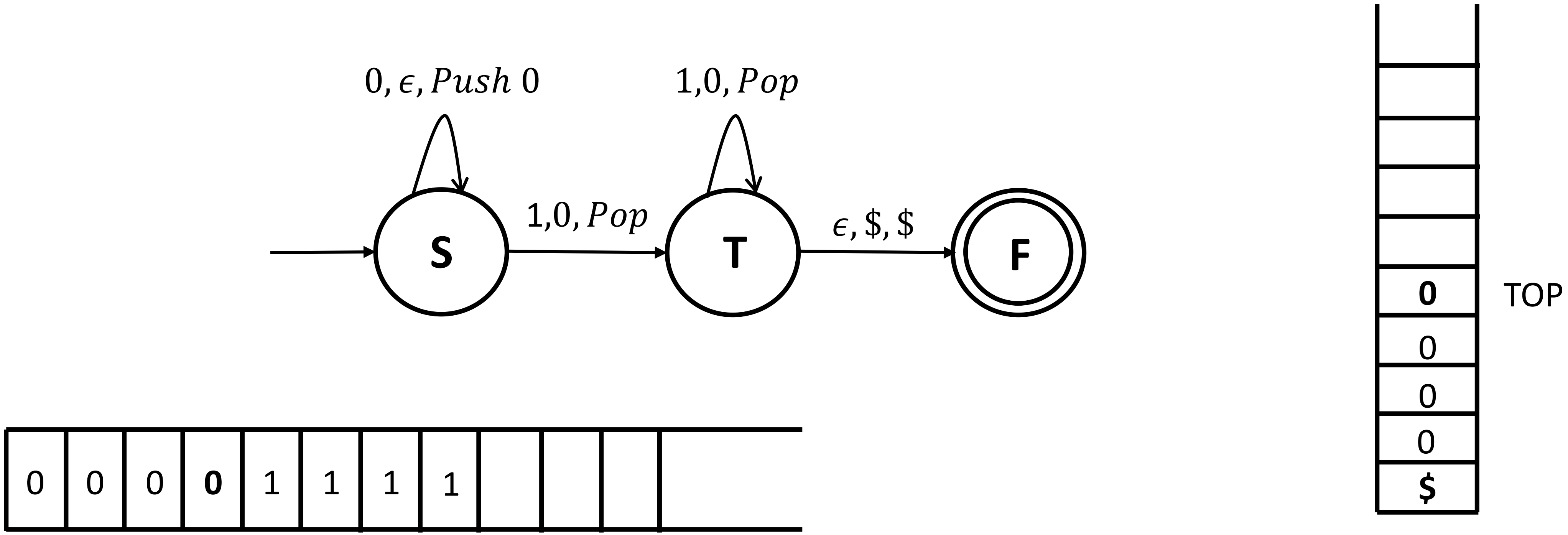

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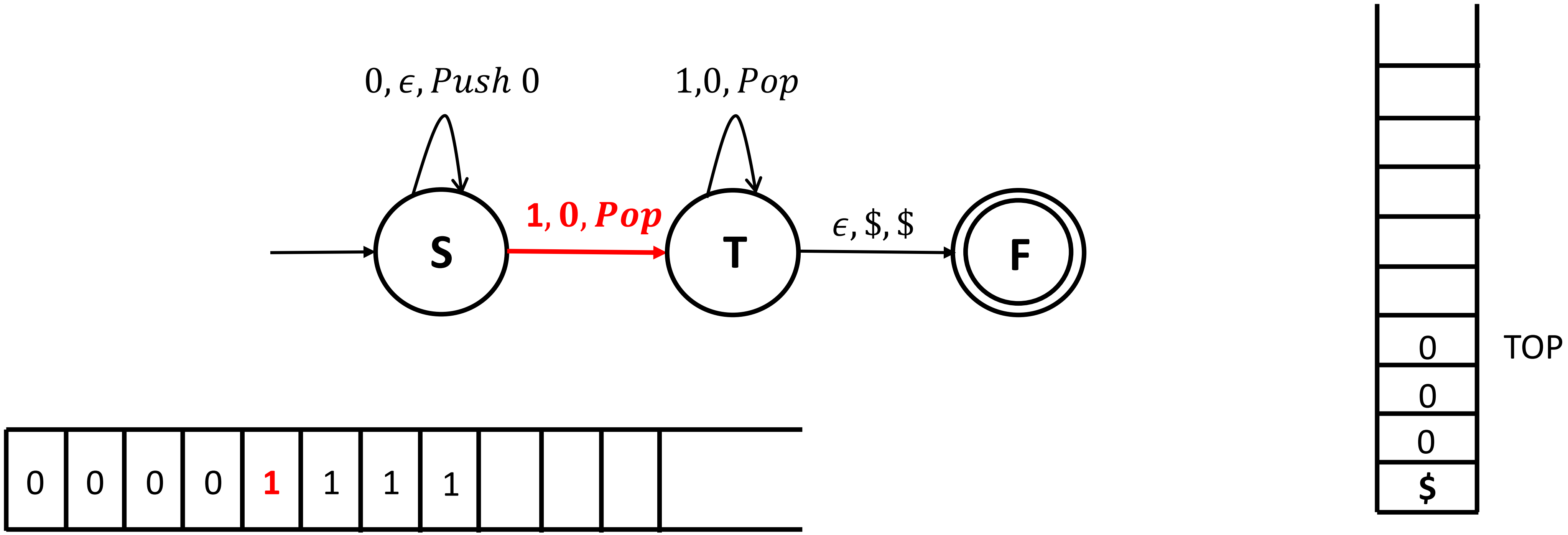

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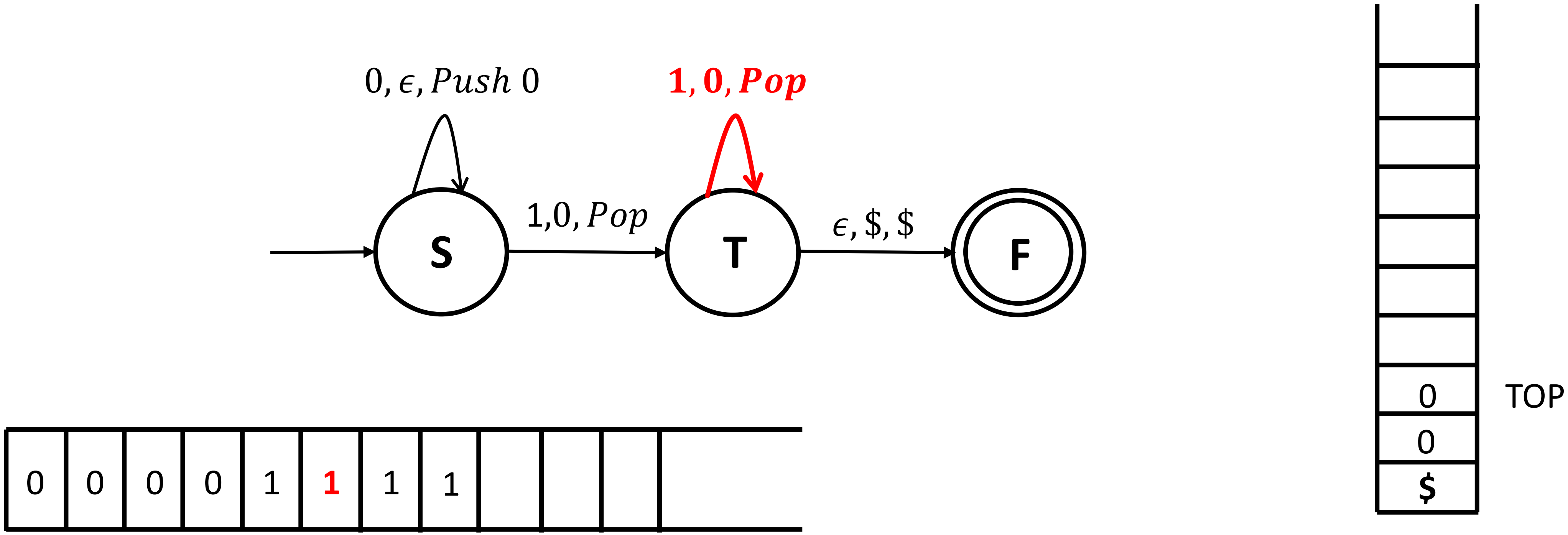

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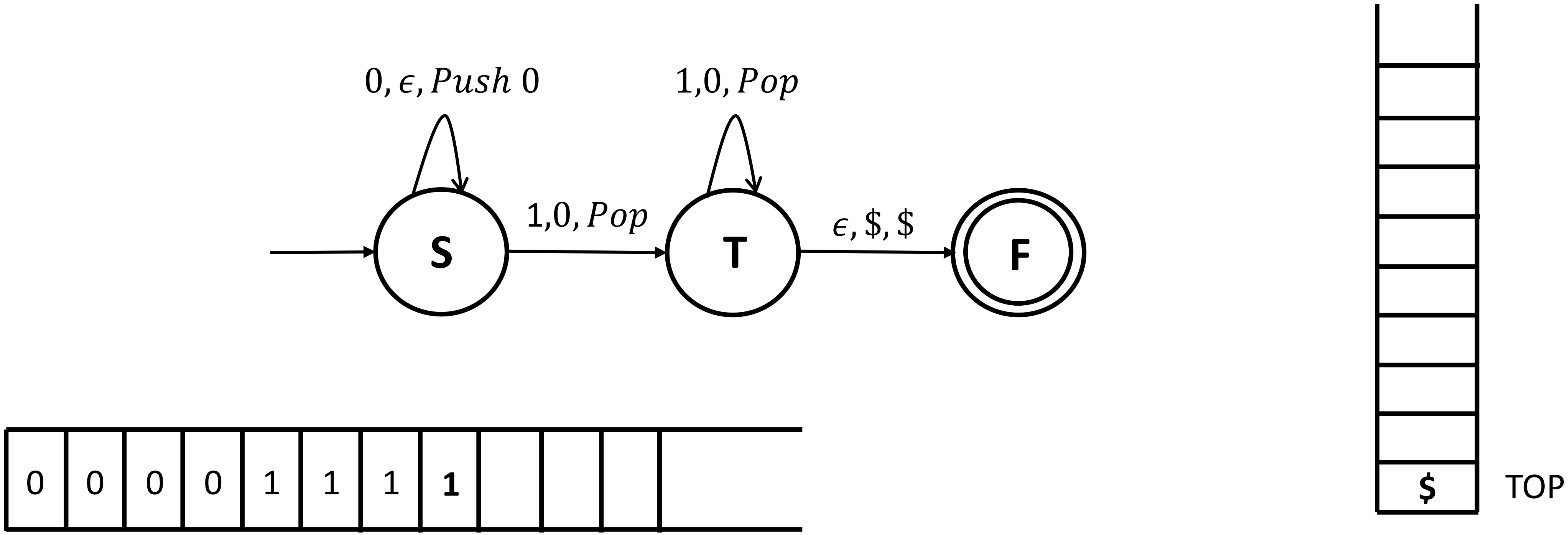

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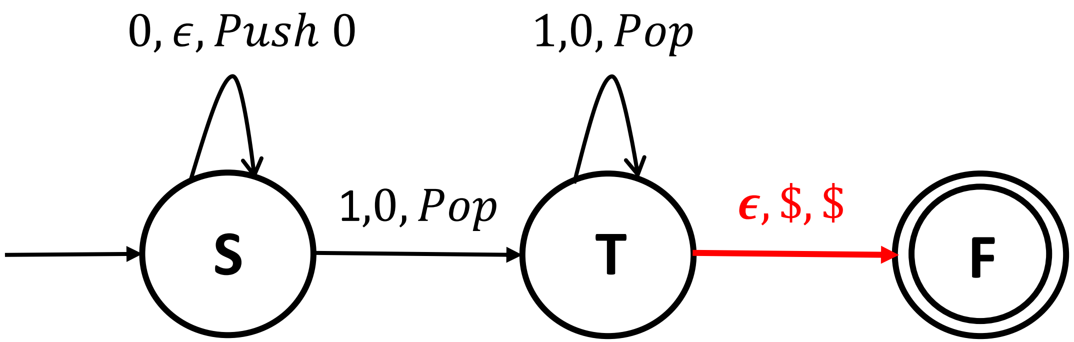
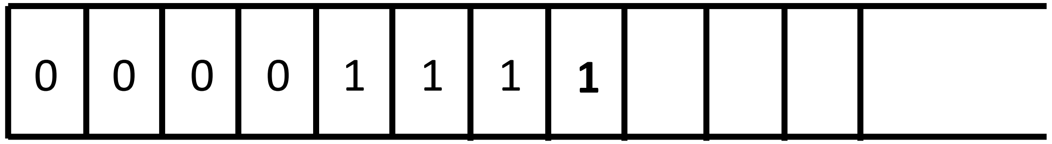
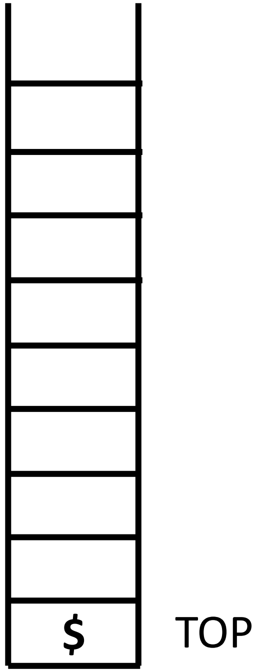


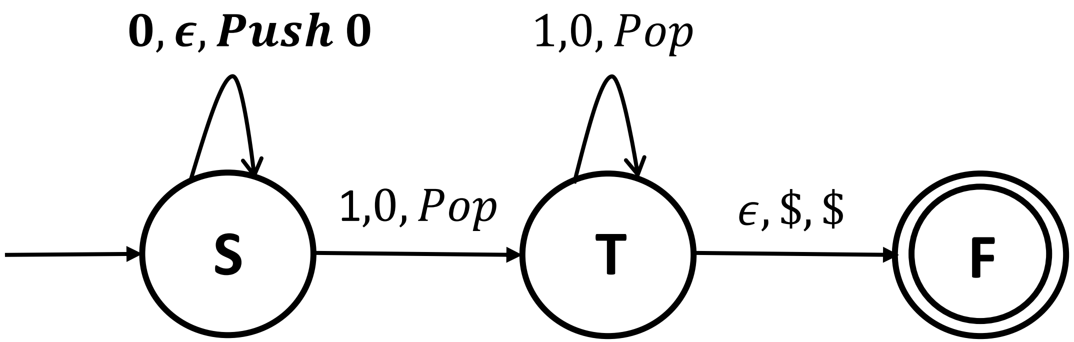




The language recognized by the PDA: $\boldsymbol{L = \{0^n 1^n, n \geq 1\}}$

Pushdown Automata

What is the language recognized by this PDA?



In some references (such as Sipser):

- The transitions of the PDA are labelled as "$\boldsymbol{a}, \boldsymbol{b} \rightarrow \boldsymbol{c}$", implying: If the input symbol read is $\boldsymbol{a}$, the element at the top of the stack is $\boldsymbol{b}$ (pop $\boldsymbol{b}$) and push $\boldsymbol{c}$ on to the Stack.

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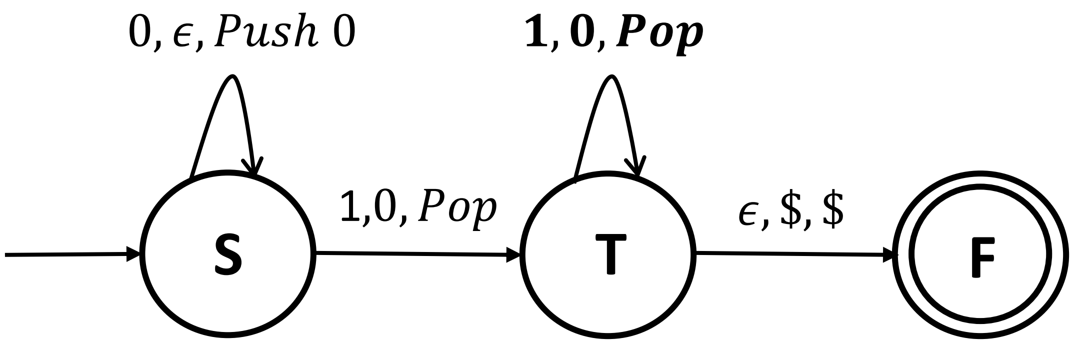

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- The label "$\boldsymbol{a}, \boldsymbol{b} \rightarrow \boldsymbol{\epsilon}$" implies that if the input symbol is $\boldsymbol{a}$ then **pop b**.

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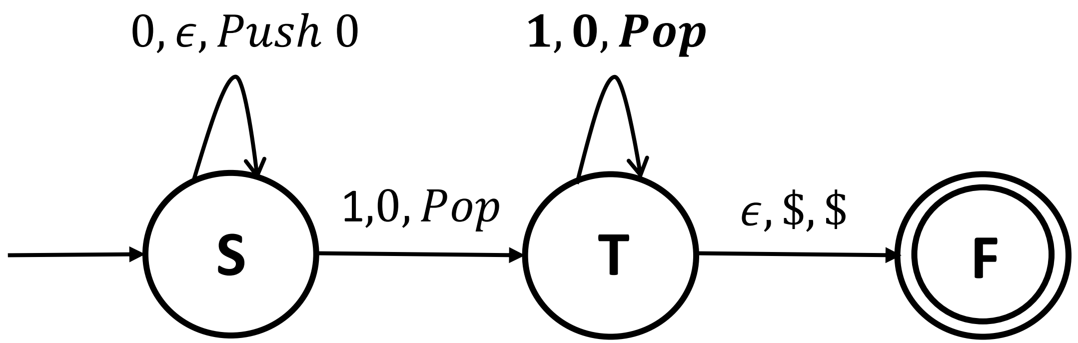

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- The label "$\boldsymbol{a, b \rightarrow \epsilon}$" implies that if the input symbol is $\boldsymbol{a}$ and $\boldsymbol{b}$ is **popped**.

- The symbol signifying the bottom of the Stack \$ is pushed at the very beginning.

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Formally, a PDA M is a 6-tuple $(\mathrm{Q}, \Sigma, \Gamma, \delta, q_0, F\ )$ where

- Q is a finite set called the ***states.***
- Σ is the set of input ***alphabets.***
- Γ is the set of ***Stack alphabets***
- $\delta: Q \times \Sigma_\epsilon \times \Gamma_\epsilon \mapsto \mathcal{P}(Q \times \Gamma_\epsilon)$ is the ***transition function*** [$\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$ and $\Gamma_\epsilon\ = \Gamma \cup \{\epsilon\}$]
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- It **reaches a final state** when the entire string is read.

OR

- The **stack is empty** when the entire string is read.

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These two notions of acceptance are equivalent

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- $\boldsymbol{\delta(q_i, a, b) = (q_j, \epsilon)}$: If the input symbol read is a, and the stack top = b (b is popped) then transition from q_i to q_j

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- $\boldsymbol{\delta(q_i, \epsilon, \$) = (q_j, \$)}$: Transition from q_i to q_j if the stack is empty.

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- If the input symbol read is a and a is popped, then Push a and remain at q_i : ?

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- The Language of the PDA P is the set of strings the PDA accepts, i.e.

$$L = \{w | P \text{ accepts } w\}$$

There exists an accepting run for $\boldsymbol{w}$ on $\boldsymbol{P}$

- If $\mathcal{L}(P) = L$, then the PDA P recognizes L

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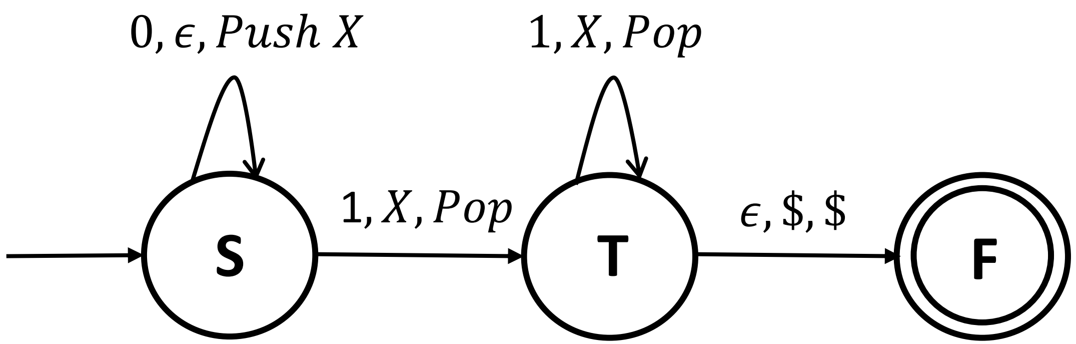
- If $\mathcal{L}(P) = L$, then the PDA P recognizes L
- Stack alphabet **can be different** from the input alphabet



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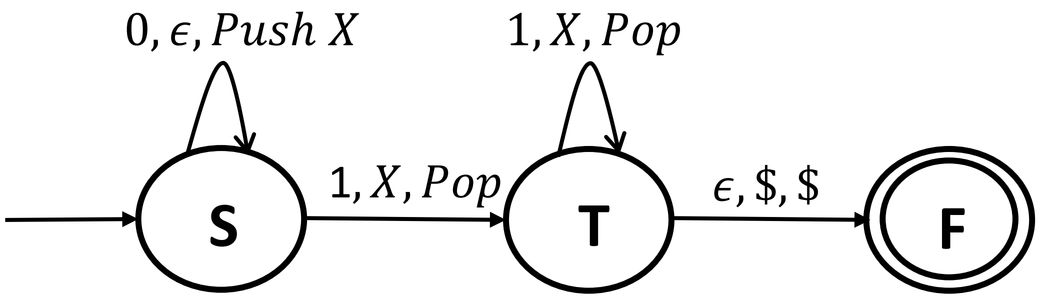

$\delta(S, 0, \epsilon) = (S, X)$

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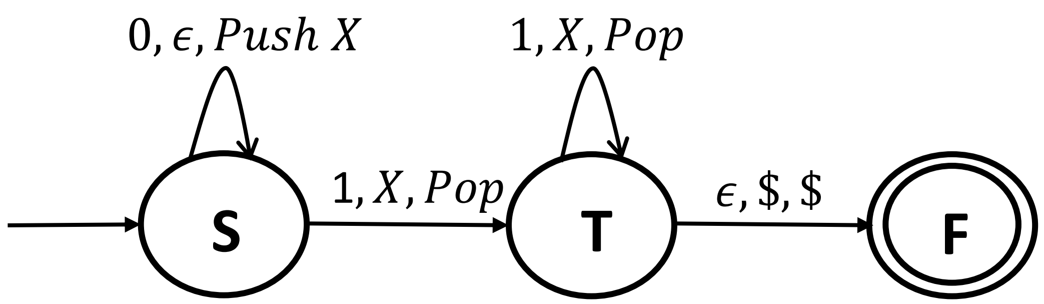

$$\delta(S, 0, \epsilon) = (S, X)$$
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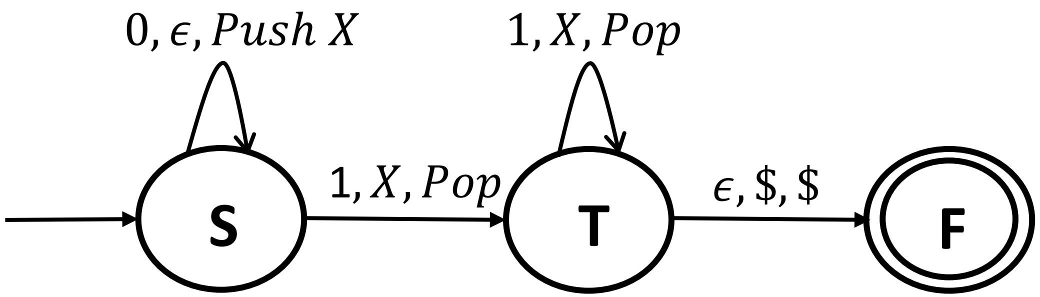

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Let $\Sigma = \{0,1\}$ consider the language $\boldsymbol{L} = \{\boldsymbol{w} \in \boldsymbol{\Sigma}^* \ |\boldsymbol{w}$ **is a Palindrome**$\}$. Design a PDA P that recognizes L.

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- Push first half of the input string onto the stack.
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- The above intuition is applicable for even length palindromes of the form ww^R.

- What about odd length palindromes?
  - Non-determinism to the rescue once again

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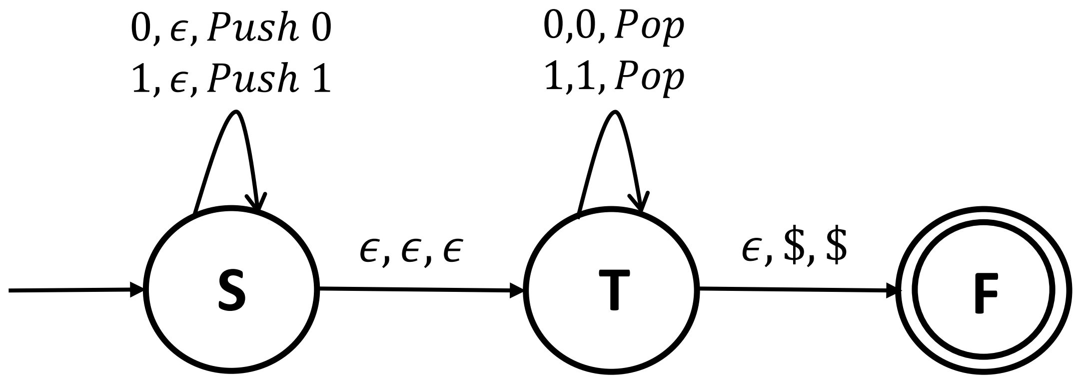

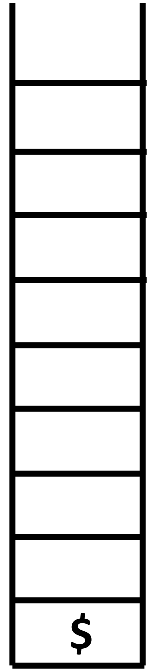

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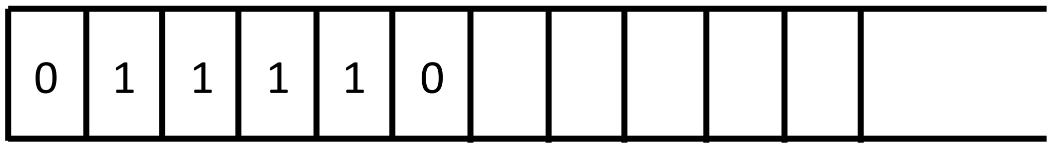

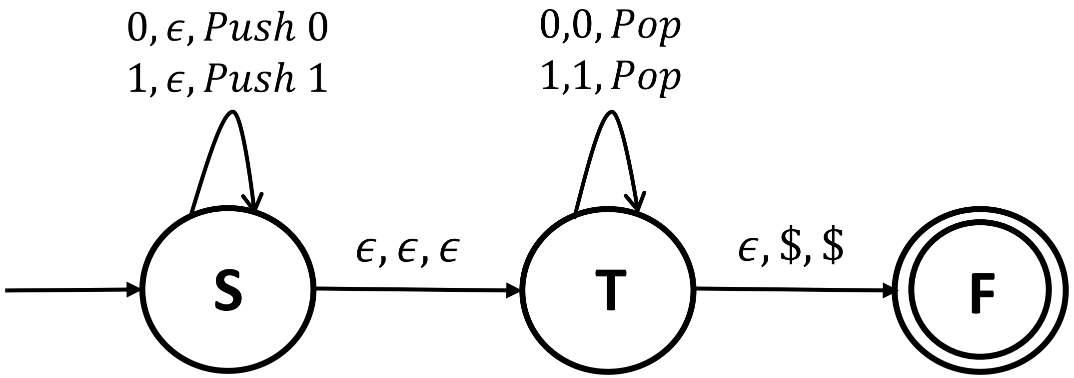

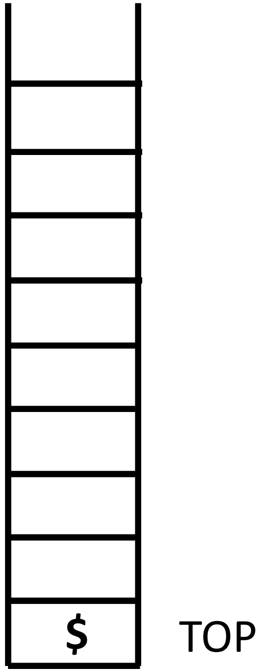

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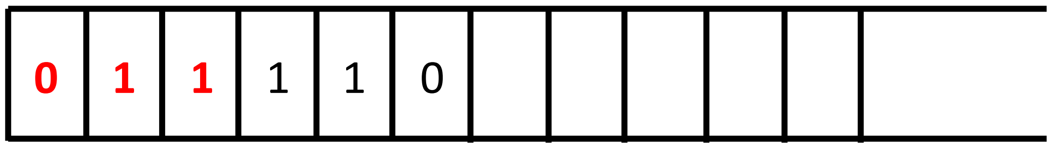

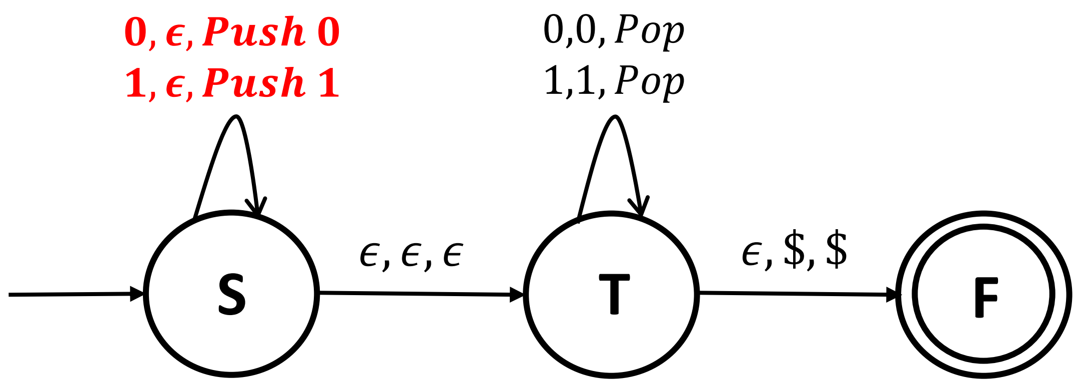

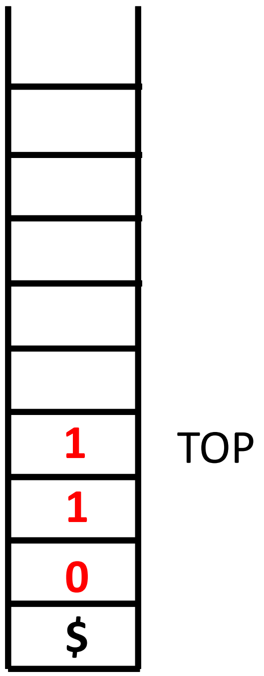

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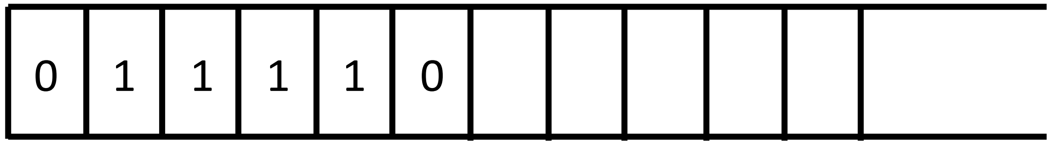

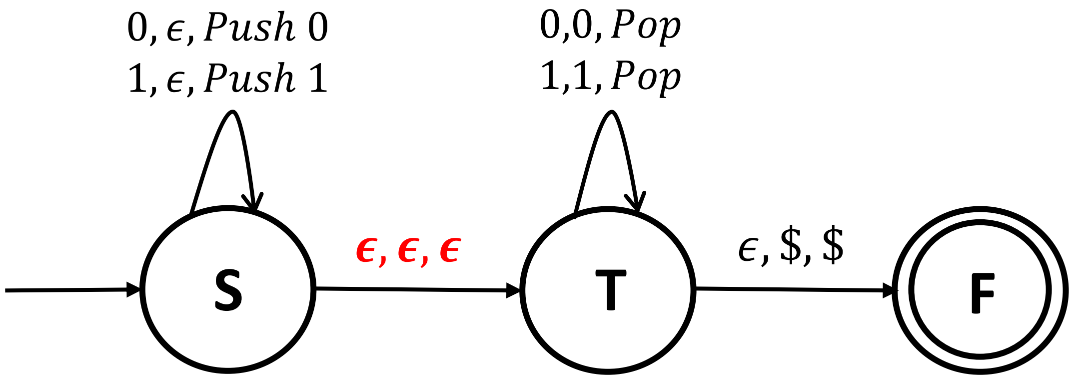

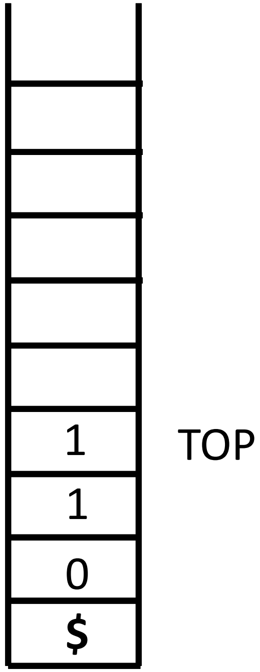

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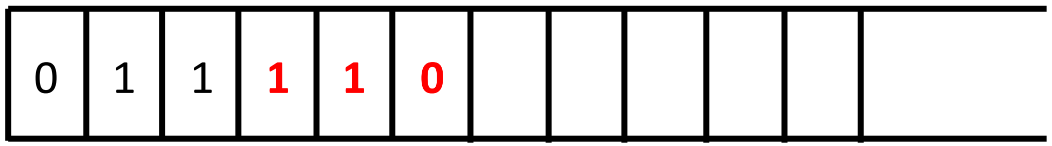

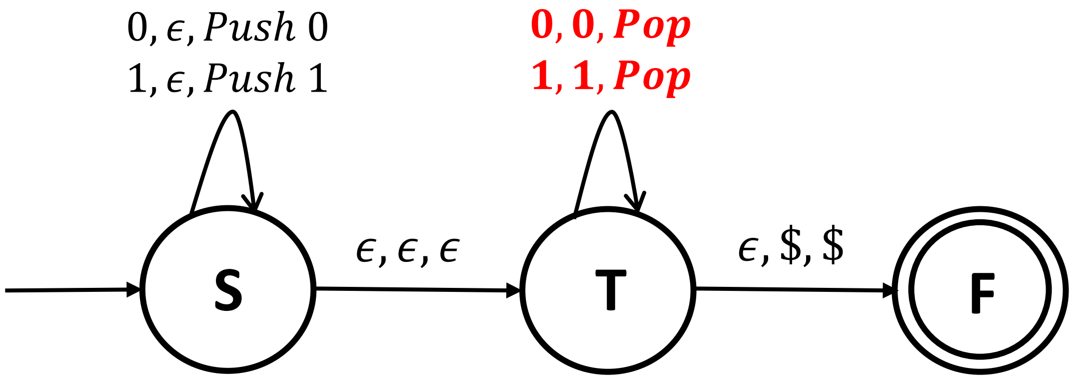

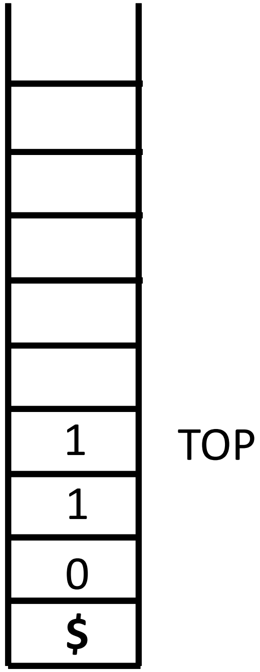

Pushdown Automata

Let $\Sigma = \{0,1\}$ consider the language $\boldsymbol{L} = \{\boldsymbol{w} \in \boldsymbol{\Sigma}^* \mid \boldsymbol{w}$ **is a Palindrome**$\}$. Design a PDA P that recognizes L.

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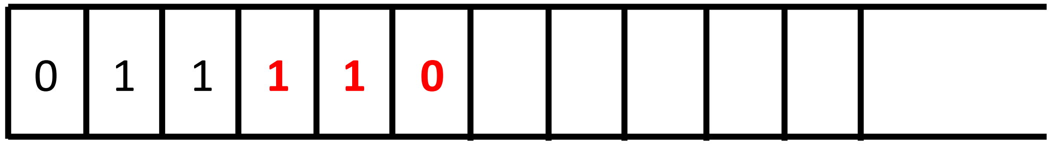

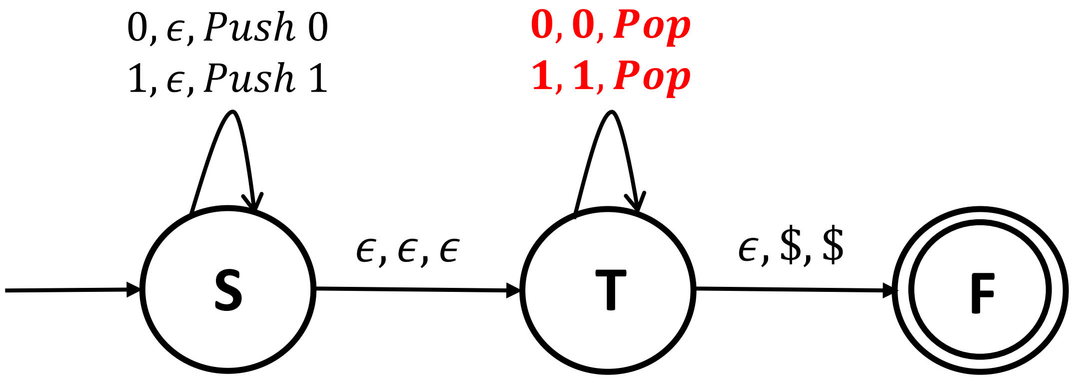

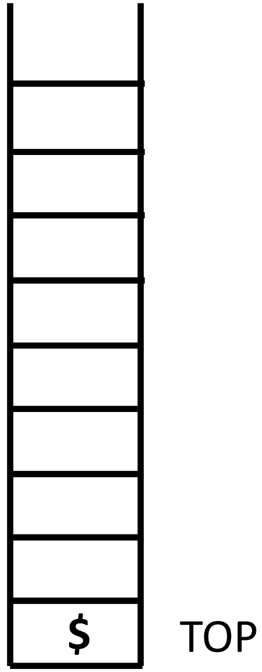

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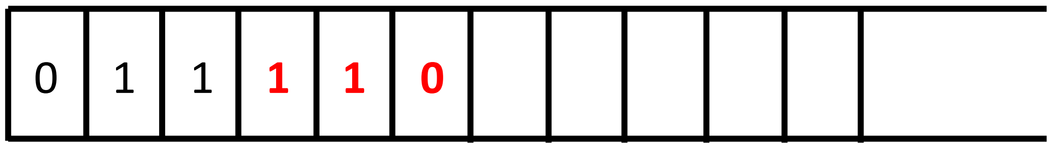

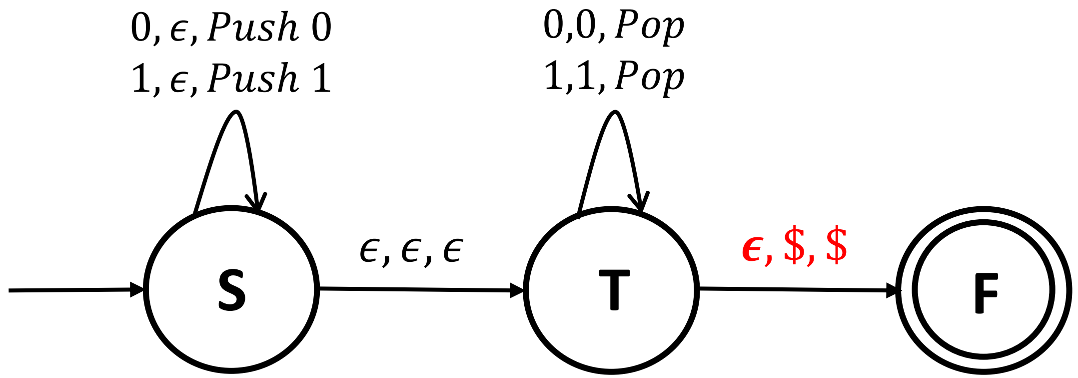

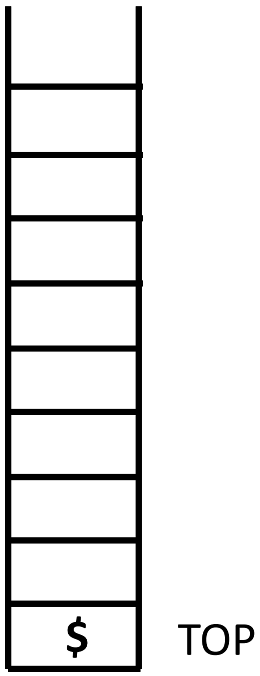

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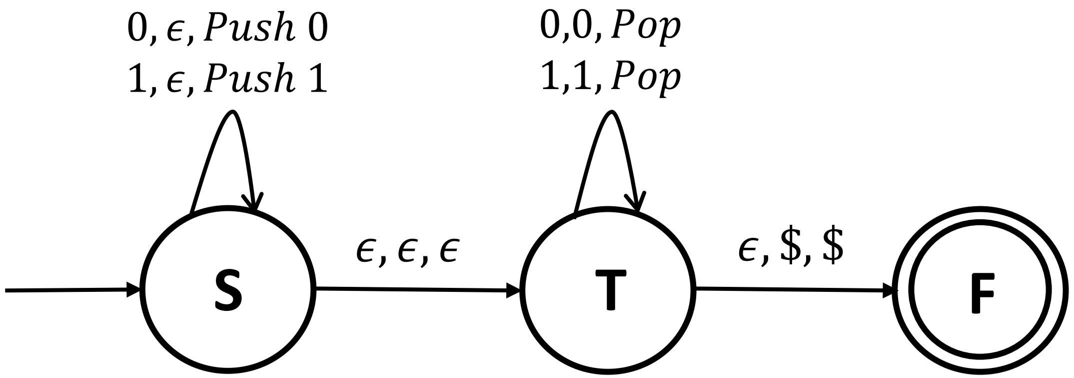
- What about odd length palindromes?



Recognizes even length palindromes of the form: $\boldsymbol{ww^R}$

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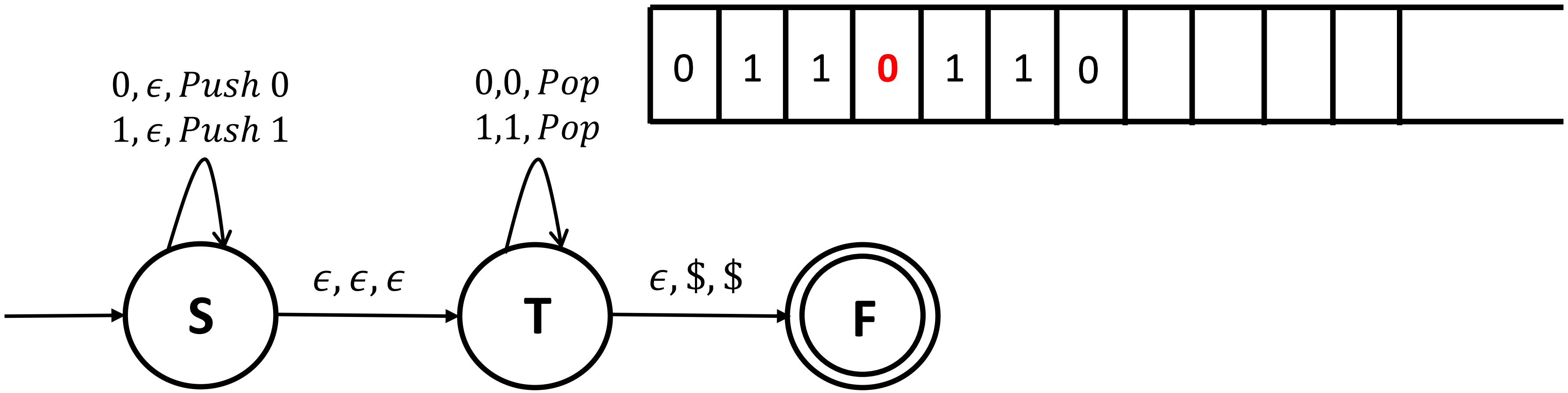
Odd length palindromes are of the form $\boldsymbol{wcw^R}$, such that $\boldsymbol{c \in \Sigma}$



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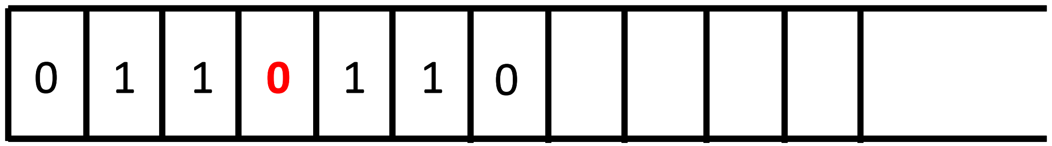

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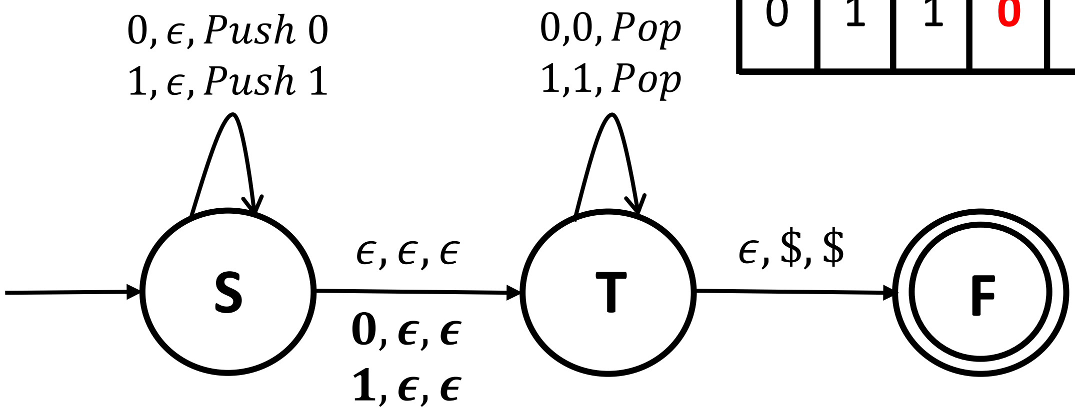

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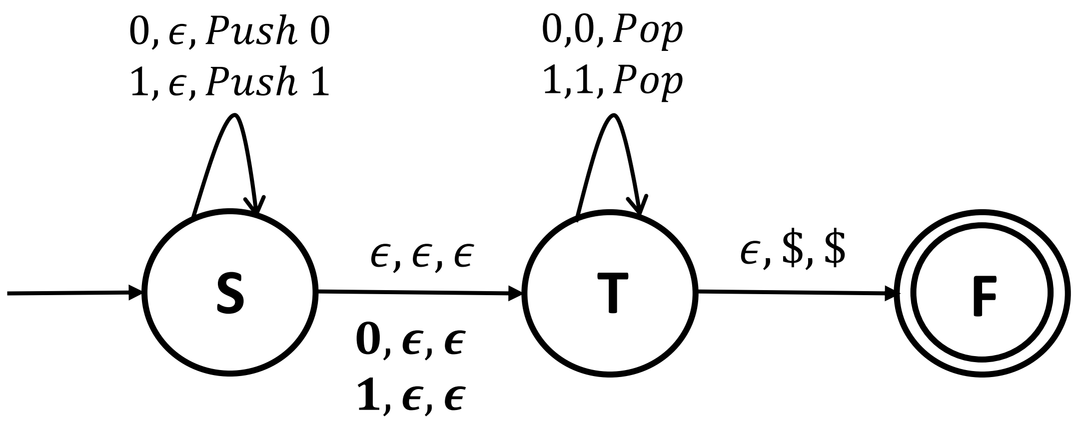

The transitions $\boldsymbol{0, \epsilon, \epsilon}$ **and** $\boldsymbol{1, \epsilon, \epsilon}$ **allow the PDA to consume one symbol** and then begin matching what it has encountered thus far.

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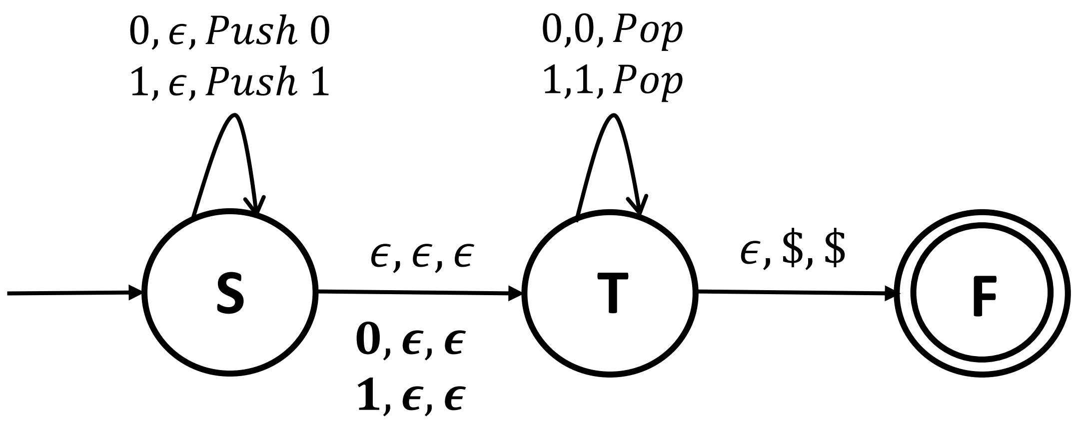

The transitions $\mathbf{0}, \boldsymbol{\epsilon}, \boldsymbol{\epsilon}$ **and** $\mathbf{1}, \boldsymbol{\epsilon}, \boldsymbol{\epsilon}$ **allow the PDA to consume one symbol** and then begin matching what it has encountered thus far.

This allows the PDA to **recognize strings of the form:** $\boldsymbol{\omega c w^R}$, where the aforementioned transitions non-deterministically guessed $c \in \{0,1\}$

Thank You!