Unit 3

Boolean Algebra (Continued)

Logic Circuits (Spring 2022)

SOP Representation

- Sum-of-products (SOP)
  - All products are the products of single variables
  - Examples

    $AB' + CD'E + AC'E'$

    $ABC' + DEFG + H$

- Two distributive laws are used to *multiply out* an expression
  - An expression is fully multiplied out ⇔ it is in sum-of-products form
- *Multiplying out*
  - POS expression ⇒ SOP expression
  - $X(Y+Z) = XY + XZ$

POS Representation

- Product-of-Sums (POS)
  - All sums are the sums of single variables
  - Examples

$$(A + B')(C + D' + E)(A + C' + E')$$

$$(A + B)(C + D + E)F$$

- Two distributive laws are used to *factor* an expression
  - An expression is fully factored ⇔ it is in product-of-sums form
- *Factoring*
  - SOP expression ⇒ POS expression
  - $X + YZ = (X+Y)(X+Z)$

Multiplying Out & Factoring

- Multiplying out
  - $(X+Y)(X'+Z) = XZ + X'Y$
- Factoring
  - $AB + A'C = (A+C)(A'+B)$
- Example

$$(Q + AB')(C'D + Q') = QC'D + Q'AB'$$

$$(Q + AB')(C'D + Q') = QC'D + QQ' + AB'C'D + AB'Q'$$

Multiplying Out Expressions: Example

Example

$$(A + B + C')(A + B + D)(A + B + E)(A + D' + E)(A' + C)$$
$$= (A + B + C'D)(A + B + E)[AC + A'(D' + E)]$$
$$= (A + B + C'DE)(AC + A'D' + A'E)$$
$$= AC + \cancel{ABC} + A'BD' + A'BE + A'C'DE$$

Factoring Expressions: Example

Example of Factoring

$$AC + A'BD' + A'BE + A'C'DE$$
$$= \underbrace{AC}_{XZ} + A'(\underbrace{BD' + BE + C'DE}_{Y})$$
$$= (A + BD' + BE + C'DE)(A' + C)$$
$$= [\underbrace{A + C'DE}_{X} + B(\underbrace{D' + E}_{Z})](A' + C)$$
$$= (A + B + C'DE)(A + \cancel{C'DE} + D' + E)(A' + C)$$
$$= (A + B + C')(A + B + D)(A + B + E)(A + D' + E)(A' + C)$$

Algebraic Simplification: Case 1

- Combining terms
  - $XY + XY' = X$
- Examples

$$abc'd' + abcd' = abd'$$

$$ab'c + abc + a'bc = ab'c + abc + abc + a'bc = ac + bc$$

$$(a + bc)(d + e') + a'(b' + c')(d + e') = d + e'$$

$$[X = d + e', Y = a + bc, Y' = a'(b' + c')]$$

Algebraic Simplification: Case 2

- Eliminating terms
  - $X + XY = X$
  - $XY + X'Z + YZ = XY + X'Z$
- Examples

$$a'b + a'bc = a'b$$

$$a'bc' + bcd + a'bd = a'bc' + bcd$$

Algebraic Simplification: Case 3

- Eliminating literals
  - $X + X'Y = X + Y$
- Examples

$$\begin{aligned} A'B + A'B'C'D' + ABCD' &= A'(B + B'C'D') + ABCD' \\ &= A'(B + C'D') + ABCD' \\ &= B(A' + ACD') + A'C'D' \\ &= B(A' + CD') + A'C'D' \\ &= A'B + BCD' + A'C'D' \end{aligned}$$

Algebraic Simplification: Case 4

- Adding redundant terms
  - XX'
  - $X + X'$
  - Consensus term
  - Don't care term
- Examples

$$\begin{aligned} &WX + XY + X'Z' + WY'Z' \\ &= WX + XY + X'Z' + WY'Z' + WZ' \\ &= WX + XY + X'Z' + WZ' \\ &= WX + XY + X'Z' \end{aligned}$$