계산이론

2022년 1학기

이은주

2장 유한 오토마타
결정적 유한 인식기와 비결정적 유한 인식기의 동치성

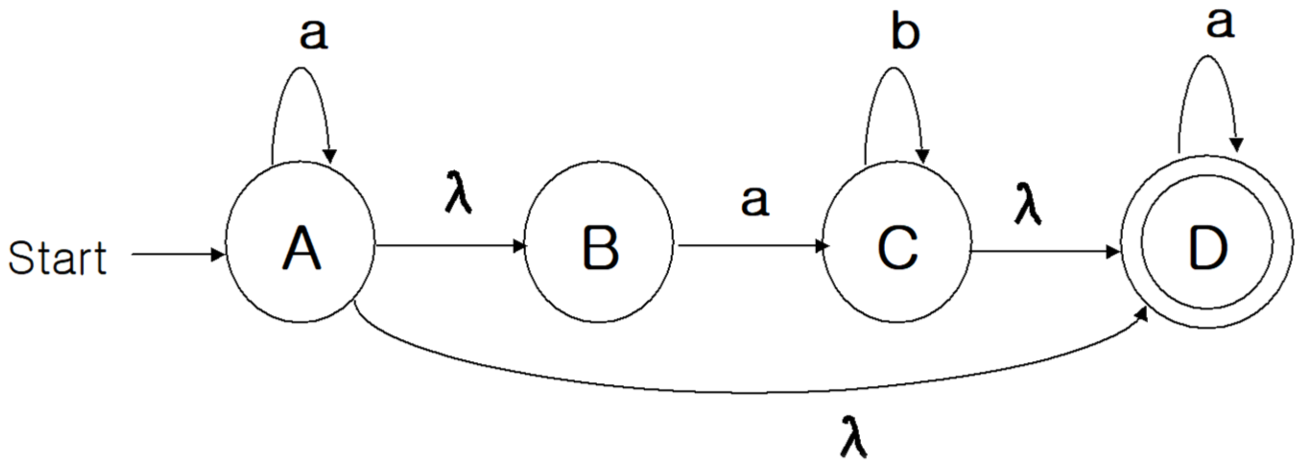
NFA에서 DFA로의 변환

- [정의] **λ-closure(s)** : 상태 s에서 λ-transition 만을 이용하여 도달 가능한 상태들의 집합 (s도 포함)
  - $\lambda\text{-closure}(s) = \{s\} \cup \{ q \mid \delta(p,\lambda)=q,\ p \in \lambda\text{-closure}(s) \}$
  - $\lambda\text{-closure}(T) = \cup_{s \in T}\ \lambda\text{-closure}(s)$

- [정의] **a-successor(s) :** 상태 s에서 입력기호 a에 의해 도달 가능한 상태들의 집합
  - a-successor(s) **=** $\cup_{qi \in T}\ \lambda\text{-closure}(\delta(q_i,a))$ where $T = \lambda\text{-closure}(s)$
  - 즉, $T = \lambda\text{-closure}(s)$
  - $Ta = \delta(T, a)$
  - a-successor(s) **=** $\lambda\text{-closure}(Ta)$

NFA에서 DFA로의 변환

- **[ex]**



  - λ-closure(A) = {A,B,D}
  - λ-closure({A,C}) = λ-closure(A) ∪ λ-closure(C) = {A,B,C,D}

NFA에서 DFA로의 변환

- a-successor(A) :

  $T = \lambda\text{-closure}(A) = \{A,B,D\}$

  $Ta = \delta(T, a) = \delta(A, a) \cup \delta(B, a) \cup \delta(D, a) = \{A,C,D\}$

  $\lambda\text{-closure}(Ta) = \lambda\text{-closure}(\{A,C,D\}) = \{A,B,C,D\}$

  $\therefore$ a-successor(A) = $\{A,B,C,D\}$

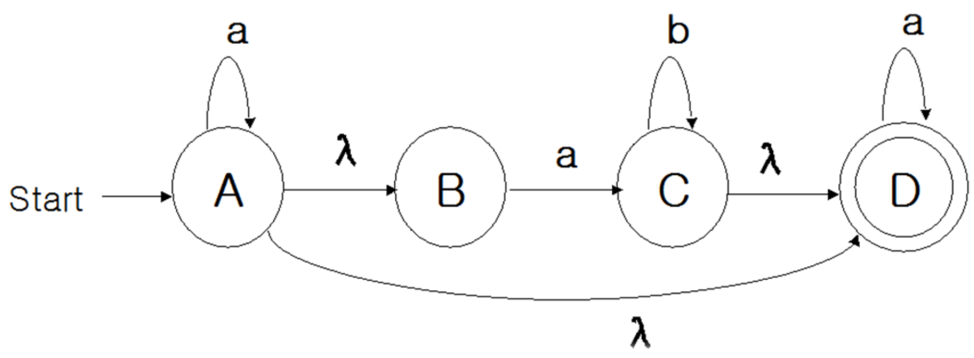
- b-successor(C) :

  $T = \lambda\text{-closure}(C) = \{C,D\}$

  $Tb = \delta(T, b) = \delta(C, b) \cup \delta(D, b) = \{C\}$

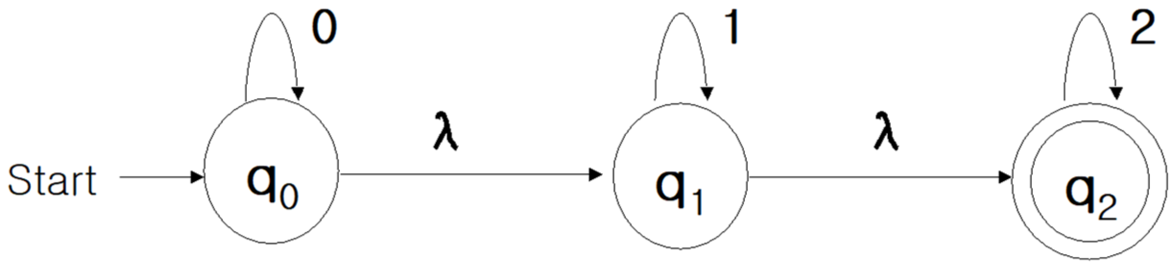
  $\lambda\text{-closure}(Tb) = \lambda\text{-closure}(\{C\}) = \{C,D\}$

  $\therefore$ b-successor(C) = $\{C,D\}$



NFA에서 DFA로의 변환

- **[ex]**



$\lambda\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

$\delta(\{q_0, q_1, q_2\}, 0) = \{q_0\}$

$\lambda\text{-closure}(\{q_0\}) = \{q_0, q_1, q_2\}$

$\therefore$ 0-successor$(q_0) = \{q_0, q_1, q_2\} = A$

$\lambda\text{-closure}(\{q_0, q_1, q_2\}) = \{q_0, q_1, q_2\}$

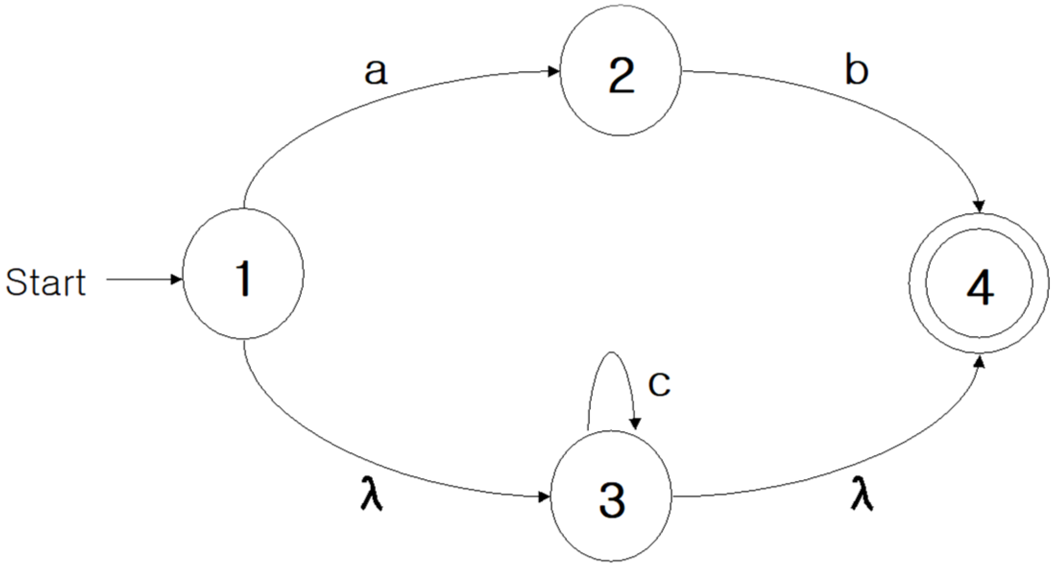
$\delta(\{q_0, q_1, q_2\}, 1)) = \{q_1\}$

$\lambda\text{-closure}(\{q_1\}) = \{q_1, q_2\}$

$\therefore$ 1-successor$(A) = \{q_1, q_2\}$

NFA에서 DFA로의 변환

- **[ex]**



초기화 : λ-closure(1) = **{1,3,4} = A** , Q' = {A}

- marking A :

a-successor(A) = λ-closure(δ({1,3,4},a)) = λ-closure({2}) = **{2} = B** , Q'= {A,B}

b-successor(A) = Φ

c-successor(A) = λ-closure({3}) = **{3,4} = C** , Q' = {A,B,C}

NFA에서 DFA로의 변환

- marking B :

  a-successor(B) = λ-closure(δ({2},a)) = Φ

  b-successor(B) = λ-closure({4}) = **{4} = D** , Q' = {A,B,C,D}

  c-successor(B) = Φ

- marking C :

  a-successor(C) = Φ

  b-successor(C) = Φ

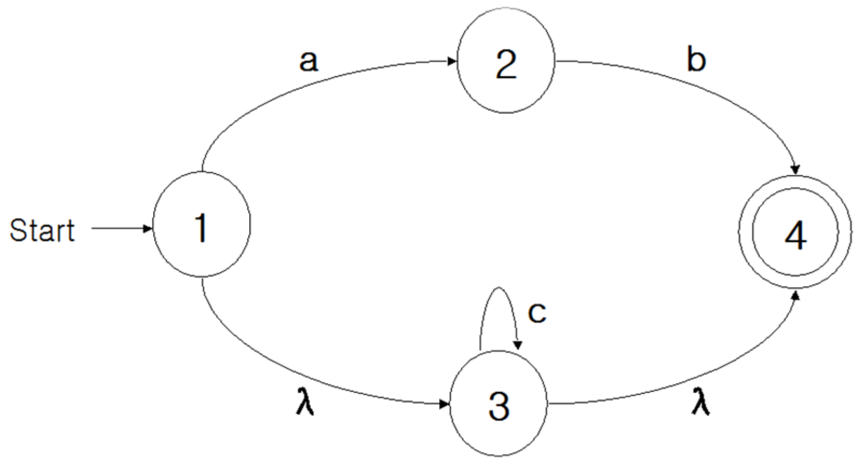
  c-successor(C) = λ-closure({3}) = {3,4} = C

- marking D :

  a-successor(D) = Φ

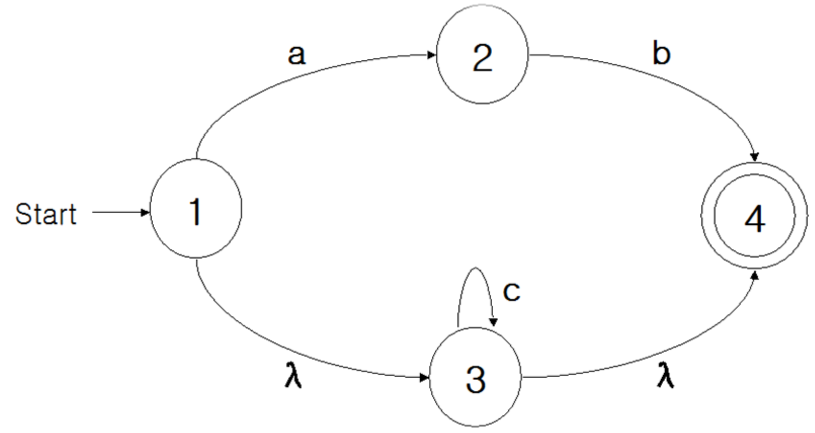
  b-successor(D) = Φ

  c-successor(D) = Φ

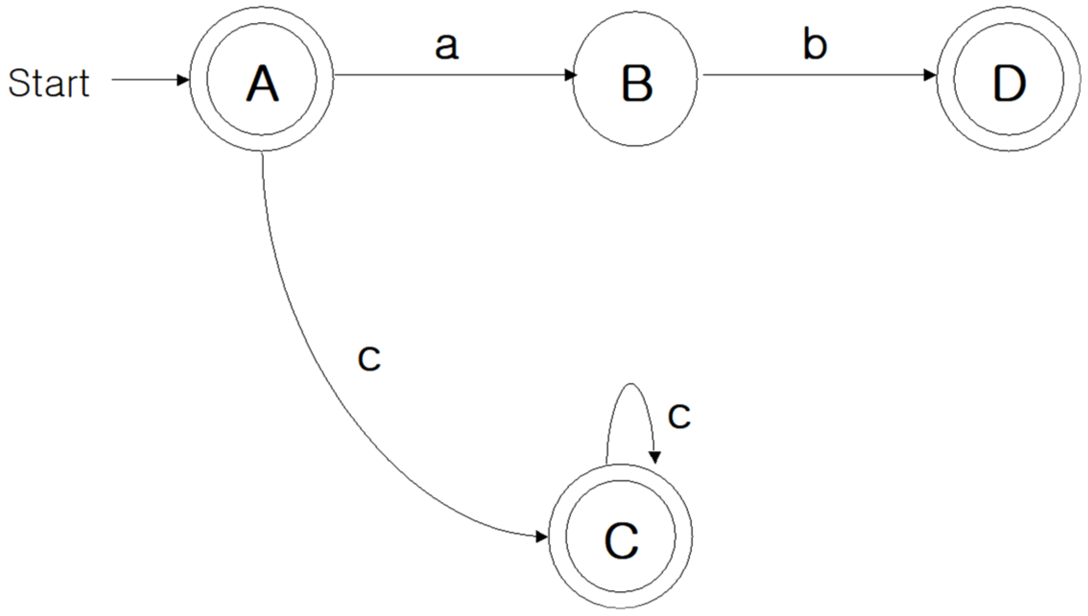


NFA에서 DFA로의 변환

nfa

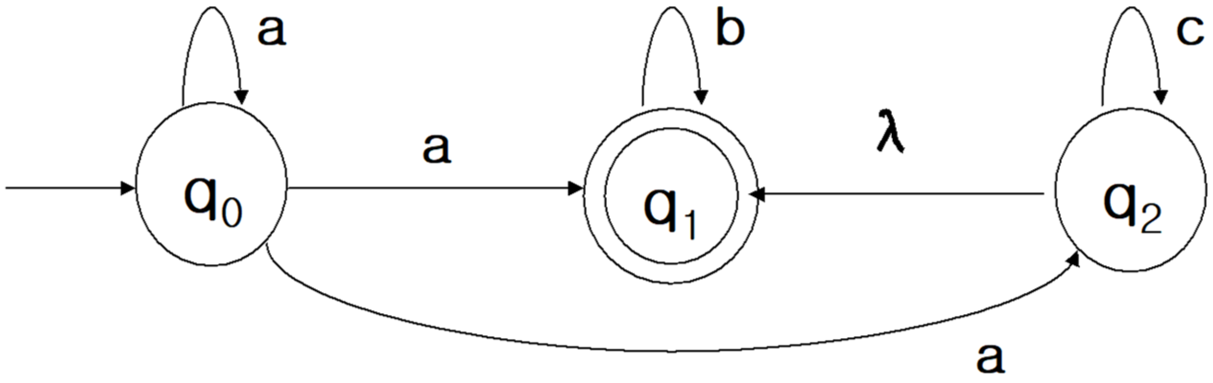


dfa



NFA에서 DFA로의 변환

- **[ex]** nfa ⇨ dfa



초기화 : λ-closure(q_o) = **{q_o} = A** , Q' = {A}

- marking A :

a-successor(A) = λ-closure({q_o,q_1,q_2}) = **{q_o,q_1,q_2} = B** , Q' = {A,B}

b-successor(A) = Φ

c-successor(A) = Φ

NFA에서 DFA로의 변환

- marking B :

  a-successor(B) = $\{q_0,q_1,q_2\}$ = B

  b-successor(B) = **$\{q_1\}$ = C** , Q' = {A,B,C}

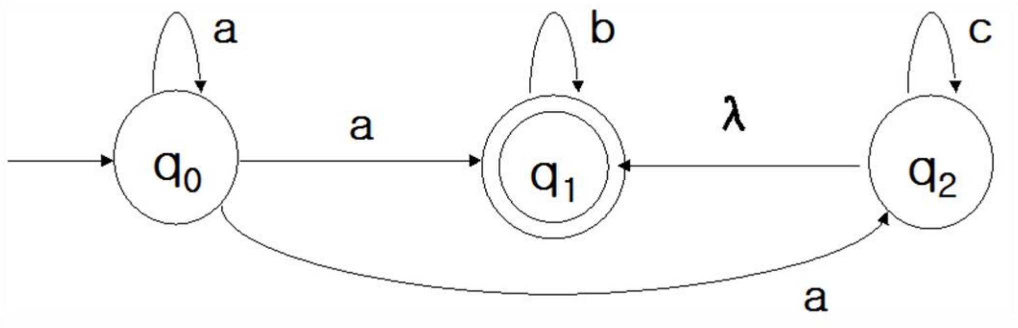
  c-successor(B) = λ-closure($\{q_2\}$) = **$\{q_1,q_2\}$ = D** , Q' = {A,B,C,D}

- marking C :

  a-successor(C) = Φ

  b-successor(C) = $\{q_1\}$ = C

  c-successor(C) = Φ



- marking D :

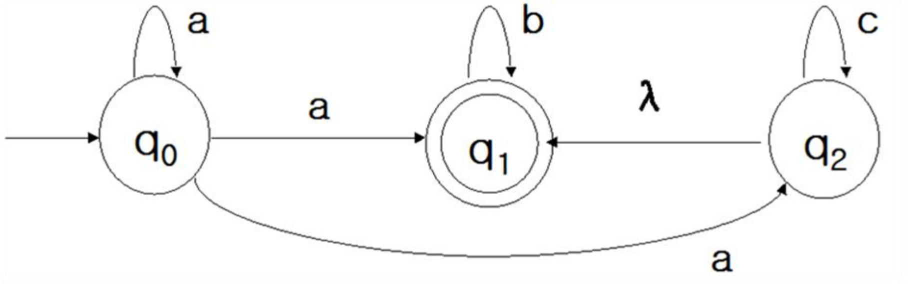
  a-successor(C) = Φ

  b-successor(C) = $\{q_1\}$ = C

  c-successor(C) = λ-closure($\{q_2\}$) = $\{q_1,q_2\}$ = D

NFA에서 DFA로의 변환

nfa



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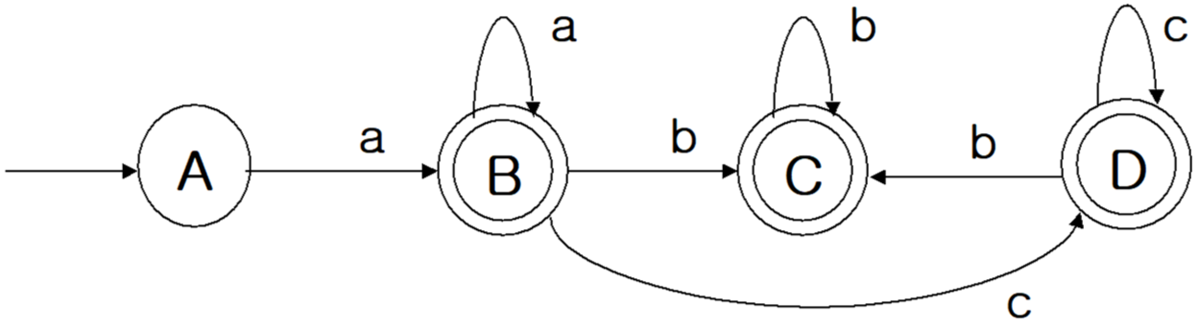
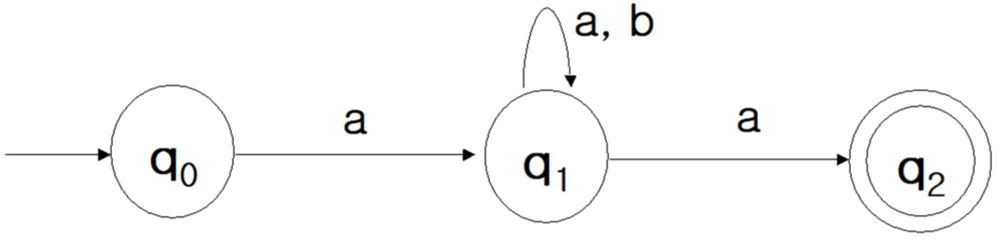


NFA에서 DFA로의 변환

- **[ex]**
  - L(M) = { **awa** | w ∈ {a,b}* }을 인식하는 nfa M ⇨ dfa



  - 초기화 : λ-closure(q_0) = **{q_0} = A**
  - marking A :

    a-successor(A) = **{q_1} = B**

    b-successor(A) = Φ
  - marking B :

    a-successor(B) = **{q_1,q_2} = C**

    b-successor(B) = {q_1} = B

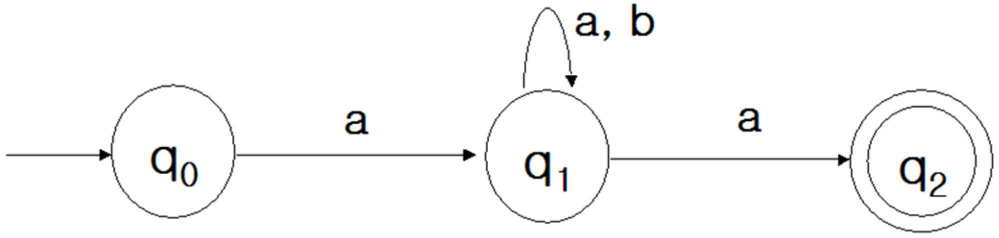
NFA에서 DFA로의 변환

- marking C :

  a-successor(C) = $\{q_1,q_2\}$ = C

  b-successor(C) = $\{q_1\}$ = B

nfa



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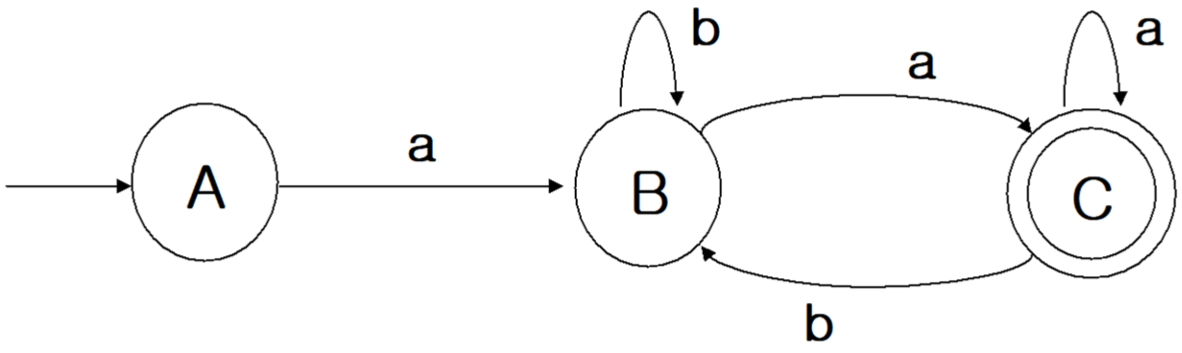
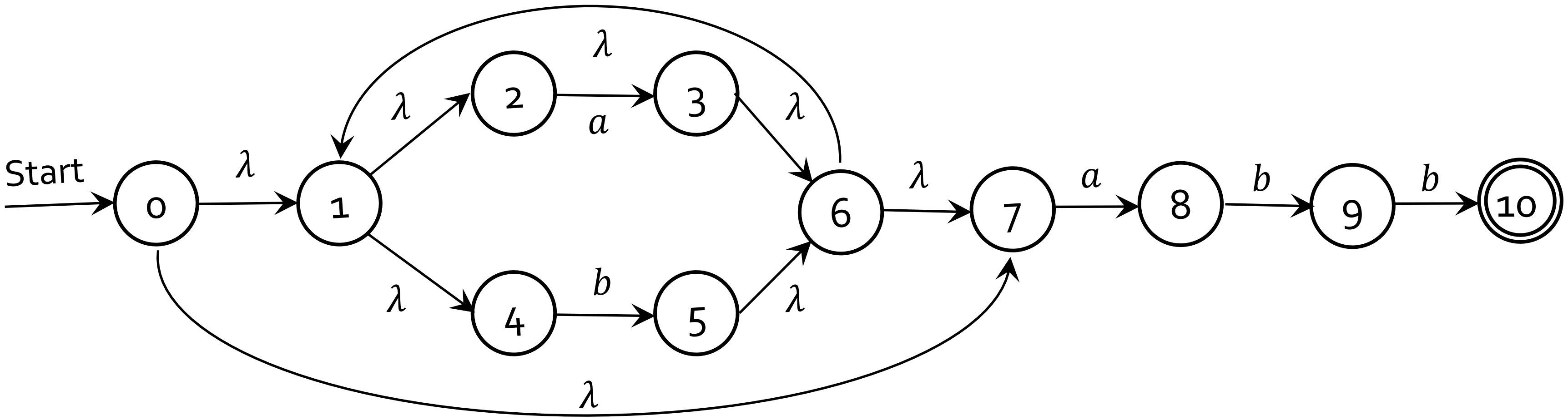


NFA에서 DFA로의 변환

- **[ex]** L(M) = { **wabb** | w∈{a,b}* }을 인식하는 nfa M ⇨ dfa



NFA에서 DFA로의 변환

- 초기화 : λ-closure(0) = **{0,1,2,4,7} = A**

- marking A :

  a-successor(A) = λ-closure(δ({0,1,2,4,7},a)) = λ-closure({3,8})

  = **{1,2,3,4,6,7,8} = B**

  b-successor(A) = λ-closure(δ({0,1,2,4,7},b)) = λ-closure({5})

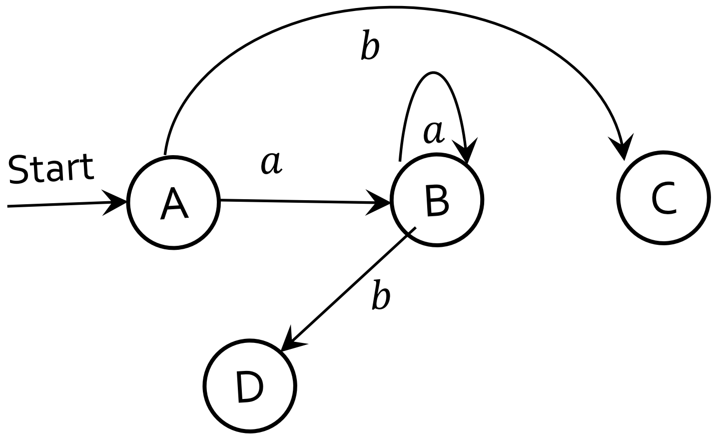
  = **{1,2,4,5,6,7} = C**

- marking B :

  a-successor(B) = λ-closure(δ({1,2,3,4,6,7,8},a)) = λ-closure({3,8}) = B

  b-successor(B) = λ-closure(δ({1,2,3,4,6,7,8},b)) = λ-closure({5,9})

  = **{1,2,4,5,6,7,9} = D**



NFA에서 DFA로의 변환

- marking C :

  a-successor(C) = λ-closure(δ({1,2,4,5,6,7},a)) = λ-closure({3,8}) = B

  b-successor(C) = λ-closure(δ({1,2,4,5,6,7},b)) = λ-closure({5}) = C

- marking D :

  a-successor(D) = λ-closure(δ({1,2,4,5,6,7,9},a)) = λ-closure({3,8}) = B

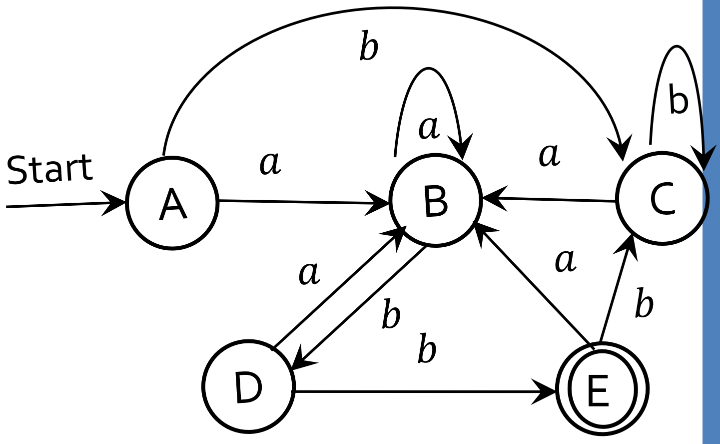
  b-successor(D) = λ-closure(δ({1,2,4,5,6,7,9},b)) = λ-closure({5,10})

  = **{1,2,4,5,6,7,10} = E**

- marking E :

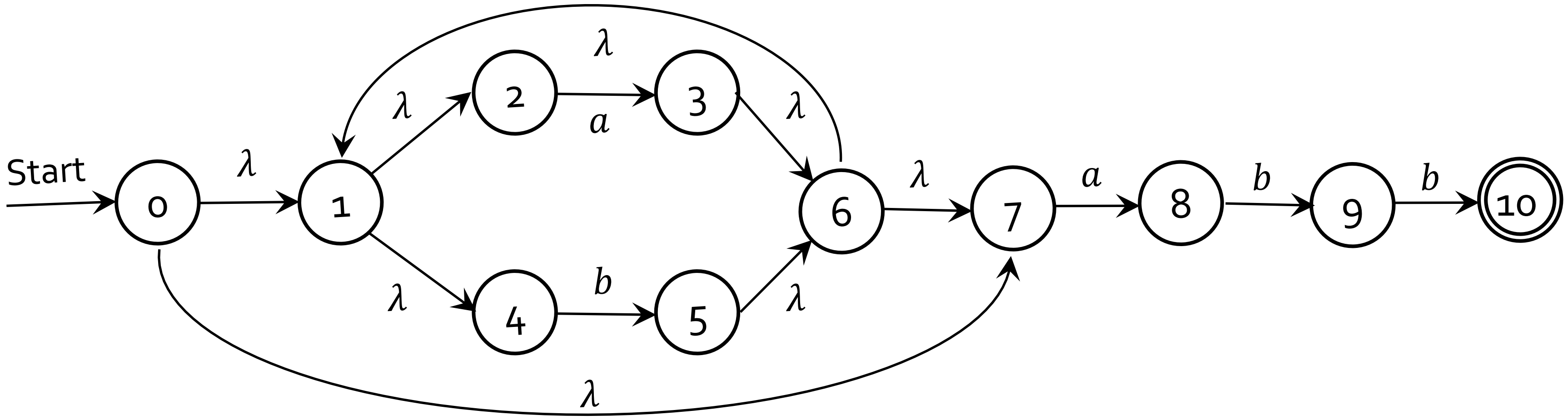
  a-successor(E) = λ-closure(δ({1,2,4,5,6,7,10},a)) = λ-closure({3,8}) = B

  b-successor(E) = λ-closure(δ({1,2,4,5,6,7,10},b)) = λ-closure({5}) = C



NFA에서 DFA로의 변환

nfa



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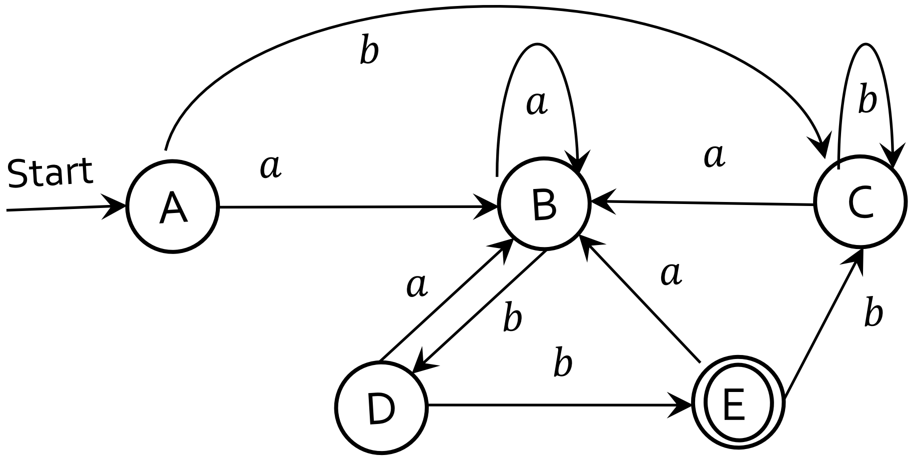
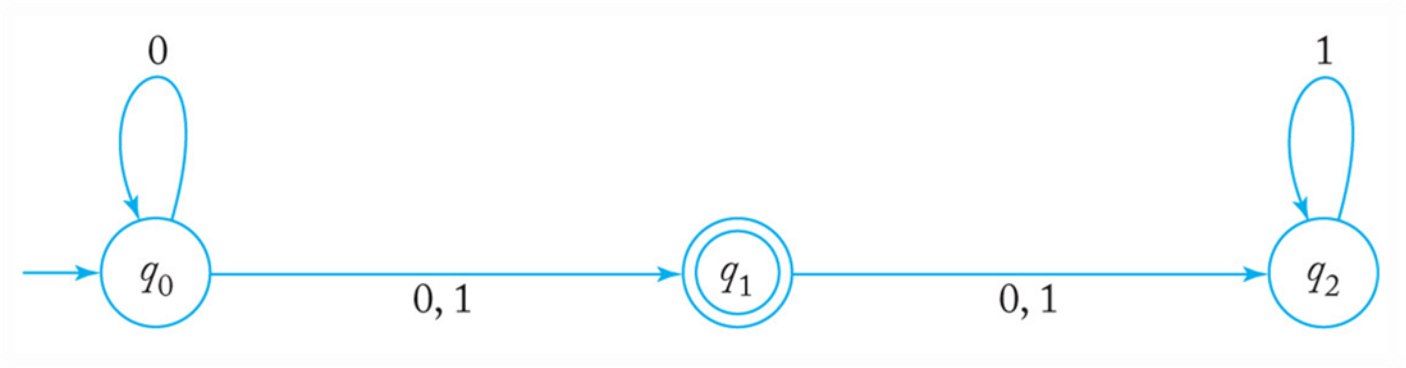


NFA에서 DFA로의 변환

- 예제 2.13(70 page)



$\delta(q_0, 0) = \{ q_0, q_1 \}$

$\delta(q_1, 0) = \{ q_2 \}$

$\delta(q_2, 0) = \varnothing$

$\delta(q_0, 1) = \{ q_1 \}$

$\delta(q_1, 1) = \{ q_2 \}$

$\delta(q_2, 1) = \{ q_2 \}$

NFA에서 DFA로의 변환

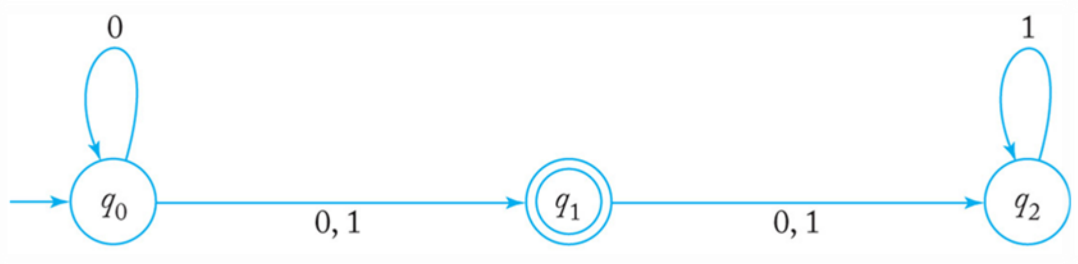
- 예제 2.13(70 page) 계속

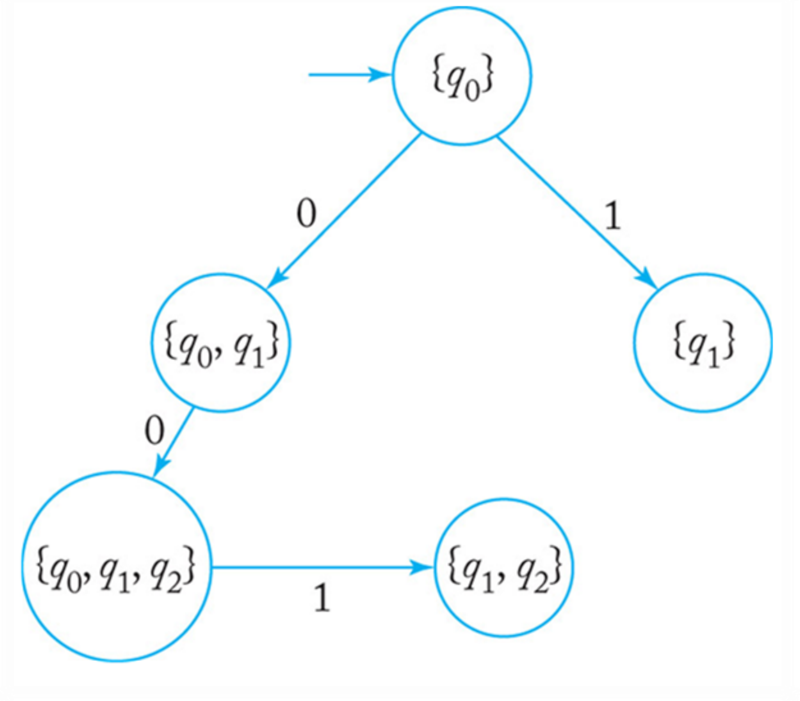
q_0 에서 $\{ q_0, q_1 \}$, $\{ q_1 \}$ 생성

$\delta(q_0, 0) \cup \delta(q_1, 0) = \{ q_0, q_1, q_2 \}$

$\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) = \{ q_1, q_2 \}$

$\{ q_0, q_1, q_2 \}$와 $\{ q_1, q_2 \}$ 추가





NFA에서 DFA로의 변환

- 예제 2.13(70 page) 계속

$\delta(q_1, 0) \cup \delta(q_2, 0) = \{ q_2 \}$

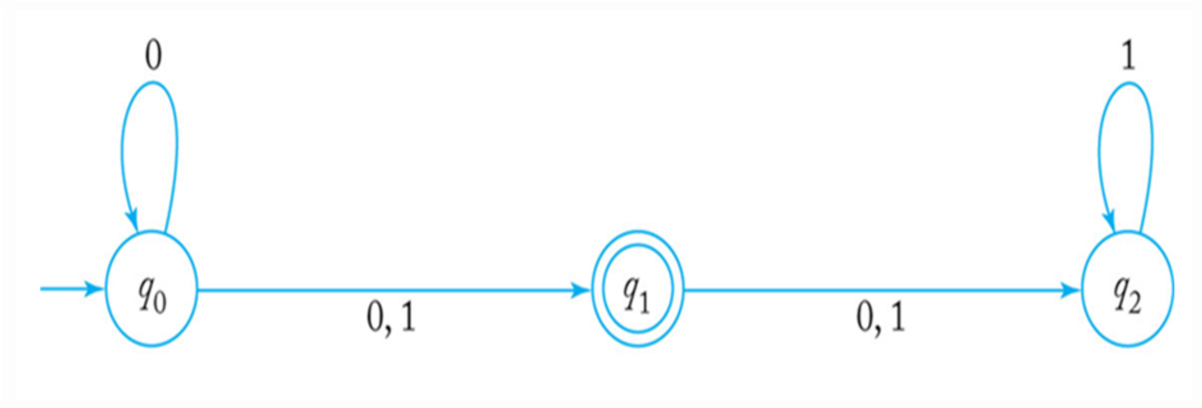
$\delta(q_1, 1) \cup \delta(q_2, 1) = \{ q_2 \}$

$\{ q_1, q_2 \}$ 으로 부터 $\{ q_2 \}$

$\{ q_1 \}$ 으로 부터 $\{ q_2 \}$

$\delta(q_2, 1) = \{ q_2 \}$

$\delta(q_2, 0)$은 없으므로 trap 상태 (labeled $\varnothing$)



NFA에서 DFA로의 변환

- 예제 2.13(70 page) 계속