Unit 5

Karnaugh Maps

Logic Circuits (Spring 2022)

Minimum Sum-of-Products

- A sum of product terms is a *minimum* SOP expression
  - It has a minimum number of *terms*
  - It has a minimum number of *literals*, for all those expressions with the same minimum number of terms
- Minimum two-level gate circuit
  - It has a minimum number of gates and gate inputs
  - A minimum SOP $\Rightarrow$ a minimum 2-level circuit
- How to find a minimum sum-of-products
  - Combine terms by using the uniting theorem $XY + XY' = X$
  - Eliminate redundant terms by using the consensus theorem or other theorems
  - Eliminate literals by using the theorem $X + X'Y = X + Y$

Minimum Sum-of-Products: Example

- Example: $F(a, b, c) = \Sigma\, m\,(0, 1, 2, 5, 6, 7)$
  - Minimum SOP expression: the first case

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$
$$= a'b' + b'c + bc' + ab$$

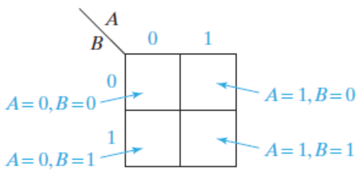
  - Minimum SOP expression: the second case

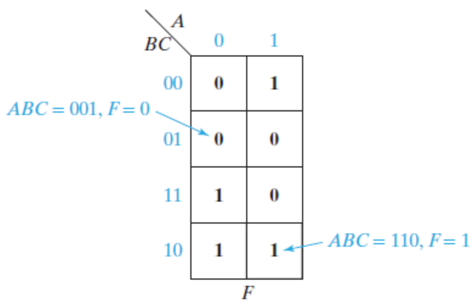
$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$
$$= a'b' + bc' + ac$$

- Combining terms in a different way may lead to a different minimum expressions

Karnaugh Map

- Karnaugh map
  - A systematic way of simplifying switching functions
  - Specifies the value of the function for every combination of values of the independent variables
  - Each 1 on the map corresponds to a minterm

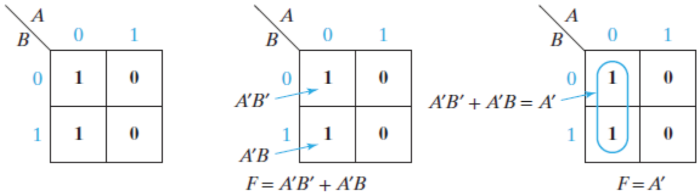




Two-Variable Karnaugh Map

- Variable and value
  - The value of a variable A is listed on the top
  - The value of the other B is listed on the side
- Truth table ⇒ Karnaugh map

A B	F
0 0	1
0 1	1
1 0	0
1 1	0

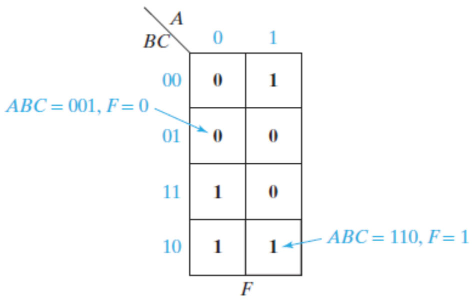


- Properties of Karnaugh map
  - A square represents a minterm
  - Minterms in adjacent squares differ in only one variable
    ⇒ the minterms can be combined
  - Product terms are represented by one or more squares

Three-Variable Karnaugh Map

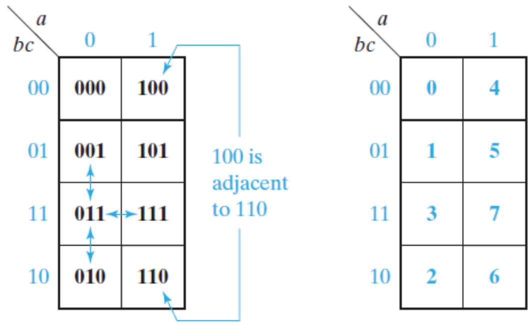
- Variable and value
  - The value of a variable A is listed on the top
  - The values of the other B and C are listed on the side
- Truth table ⇒ Karnaugh map

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0



Drawing a Karnaugh Map

- Locations of minterms

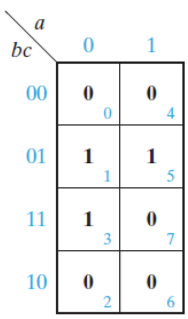


Binary representation Decimal representation

- Minterm $ab'c'$ is adjacent to minterm $a'b'c'$, $ab'c$, and abc'
- Minterm m_4 is adjacent to minter m_0, m_5, and m_6

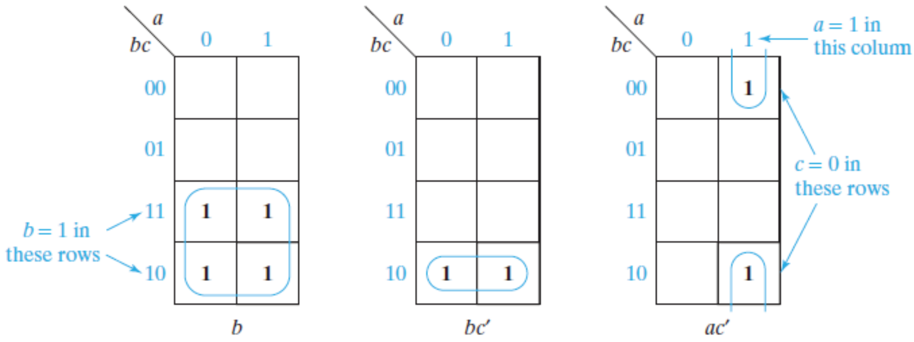
Drawing a Karnaugh Map

- Mapping minterm and maxterm expressions on Karnaugh map
- Example: $F(a, b, c) = \Sigma m(1, 3, 5) = \Pi M(0, 2, 4, 6, 7)$



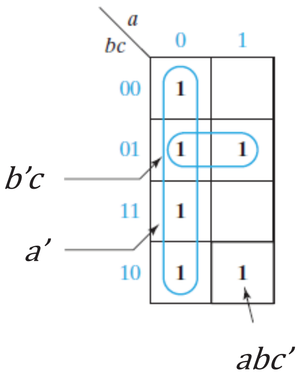
Drawing a Karnaugh Map

- Plotting product terms
- Example: b, bc', ac'



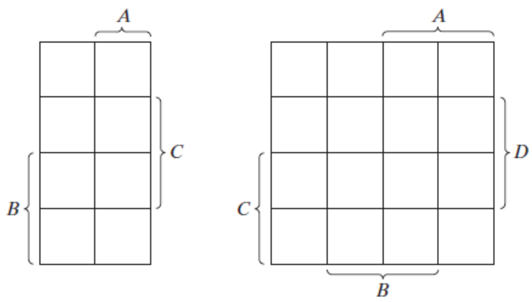
Drawing a Karnaugh Map

- Plotting an expression in algebraic form
- Example: $f(a, b, c) = abc' + b'c + a'$



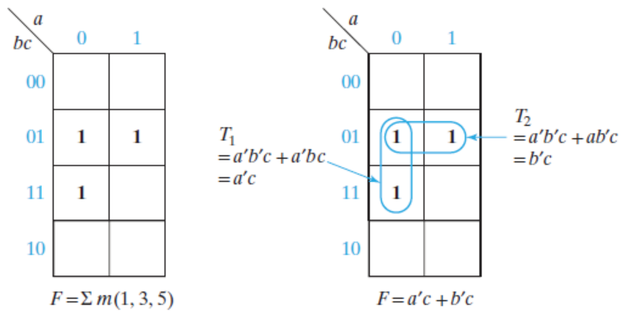
Veitch Diagram

- Other forms of Karnaugh maps
- Using different labeling
  - A = 1 for the half of the map labeled A
  - A = 0 for the other half



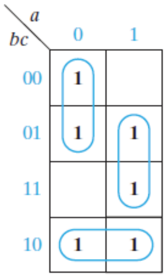
Utilizing a Karnaugh Map

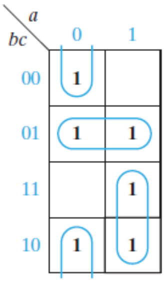
- Simplifying an expression
- Example: $F = \Sigma m(1, 3, 5) = a'c + b'c$



Utilizing a Karnaugh Map

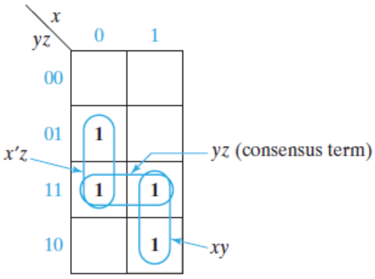
- Simplification based on Karnaugh map
- Example: $F = \Sigma m(0, 1, 2, 5, 6, 7)$

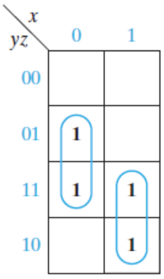




Utilizing a Karnaugh Map

- Consensus theorem
- Example: $xy + x'z + yz = xy + x'z$



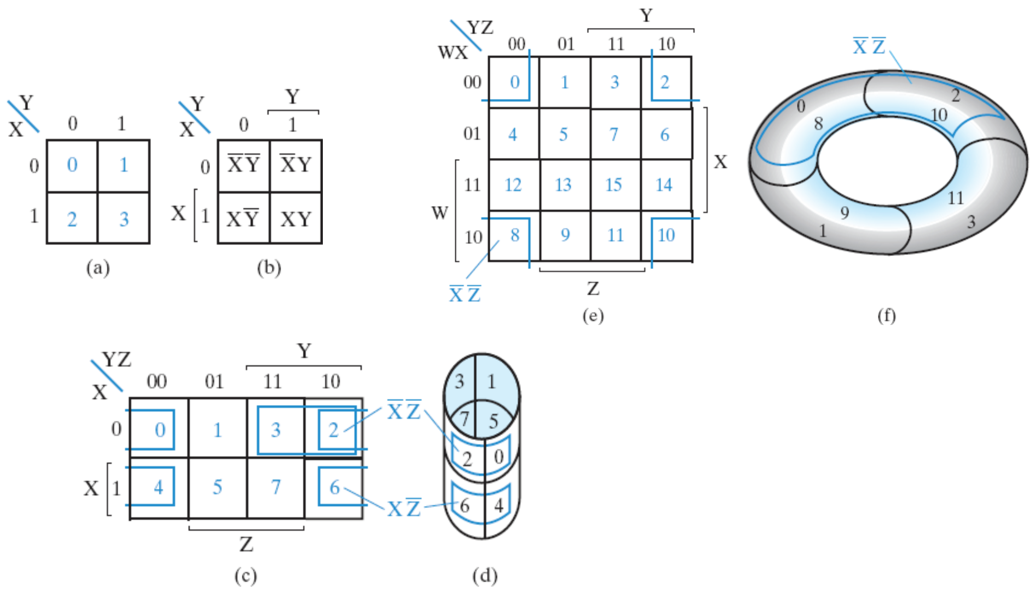


Four-Variable Karnaugh Map

- Location of terms

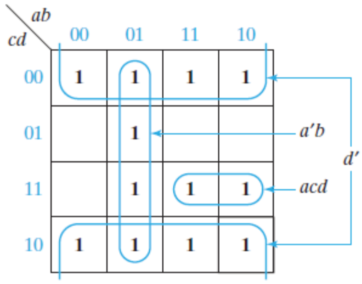
CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Karnaugh Map



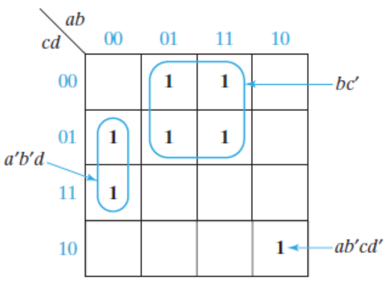
Four-Variable Karnaugh Map

- Plotting a four-variable function
- Example: *acd + a'b + d'*

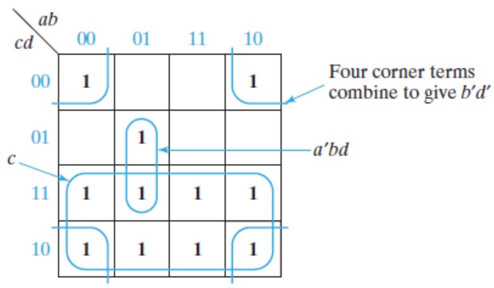


Four-Variable Karnaugh Map

- Simplifying a four-variable expression



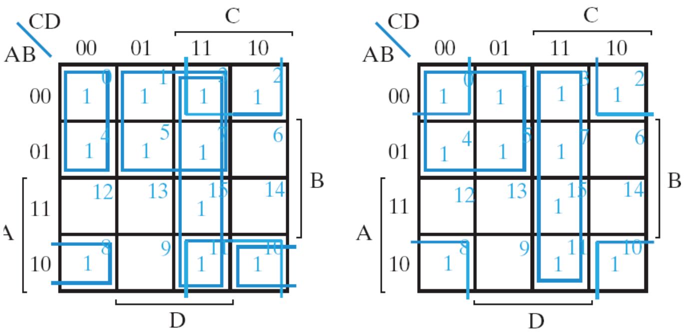
$f_1 = \Sigma\, m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$



$f_2 = \Sigma\, m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

Four-Variable Karnaugh Map

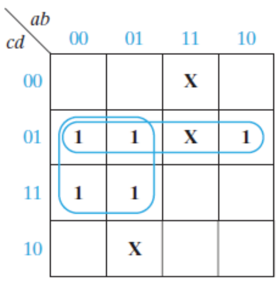
- Simplifying a four-variable expression
- Example: $A'C'D' + A'D + B'C + CD + AB'D'$



- Minimum sum-of-product expression: $A'C' + CD + B'D'$

Four-Variable Karnaugh Map

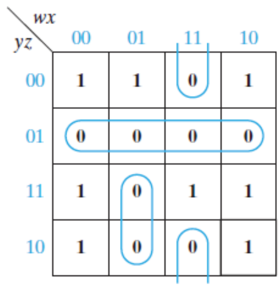
- Expressions with “don’t care”
  - “don’t care” terms are noted as X’s



Four-Variable Karnaugh Map

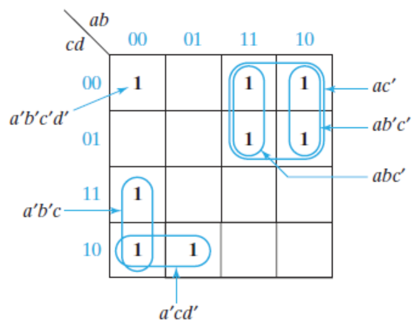
- Application: SOP ⇒ POS form
- Example

$$f = x'z' + wyz + w'y'z' + x'y$$
$$f' = y'z + wxz' + w'xy$$
$$f = (y + z')(w' + x' + z)(w + x' + y')$$



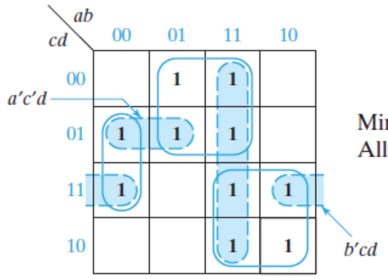
Prime Implicants

- *Implicant*
  - A product term of an expression
  - If the product term is equal to 1, the expression is also equal to 1
- *Prime implicant*
  - An implicant that can not be combined with another implicant
  - The implicant is no longer an implicant if any literal is deleted from it
- Example
  - Prime implicant: *ac', a'b'c, a'cd'*
  - Non prime implicant: *abc', ab'c',* *a'b'c'd'*
- A SOP expression containing a non-prime implicant term cannot be minimum



Prime Implicants: Example

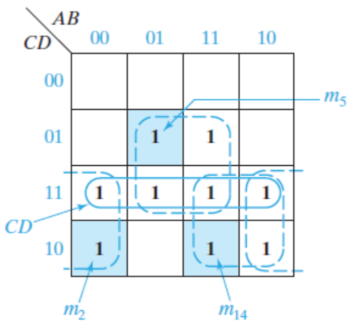
- All of the prime implicants are generally not needed
  ⇒The minimum solution may not include all prime implicants
- Example



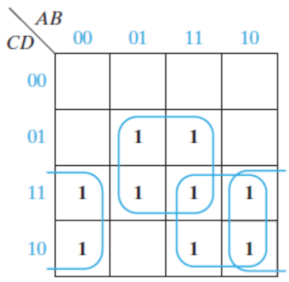
Minimum solution: $F = a'b'd + bc' + ac$
All prime implicants: $a'b'd$, bc', ac, $a'c'd$, ab, $b'cd$

Essential Prime Implicant

- Covering minterms
  - Some minterms can be covered by only a single prime implicant



$f = CD + BD + B'C + AC$

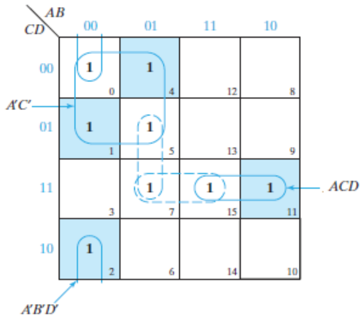


$f = BD + B'C + AC$

- *Essential prime implicant*
  - A prime implicant that covers a minterm that any other prime implicant could not cover
  - Essential prime implicants must be included in the minimum SOP

Essential Prime Implicants: Example

- Example: $F = \Sigma m(0, 1, 2, 4, 5, 7, 11, 15)$
- Prime implicants
  - Essential prime implicants: $A'C', A'B'D', ACD$



- Minimum sum-of-product expression

$$A'C' + A'B'D' + ACD + \begin{Bmatrix} A'BD \\ \text{or} \\ BCD \end{Bmatrix}$$

Procedure to Obtain a Minimum SOP

1. Choose a minterm (a 1) which has not yet been covered.
2. Find all 1's and X's adjacent to that minterm. (Check the n adjacent squares on an n-variable map.)
3. If a single term covers the minterm and all of the adjacent 1's and X's, then that term is an essential prime implicant, so select that term. (Note that "don't-care" terms are treated like 1's in steps 2 and 3 but not in step 1.)
4. Repeat steps 1, 2, and 3 until all essential prime implicants have been chosen.
5. Find a minimum set of prime implicants which cover the remaining 1's on the map. (If there is more than one such set, choose a set with a minimum number of literals.)