Chap1. Introduction to probability

1-3 Experiments & Events

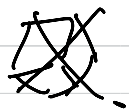
* Experiment (실험)

→ 어떤 과정 (real or hypothesis)

→ 과정 끝에 나올 수 있는 결과물이 예측 가능해야 함

ex) 주사위 던지기.

1 나옴
2 "
3 "
4 "
5 "
6 "



Def) (Sample space) : A set of all possible outcomes of an experiment

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Def) (Event): Well-defined set of possible outputs of an experiment.

→ Sample space의 subset.

E : 짝수가 나옴

$= \{2, 4, 6\} \subseteq S$.

Cor. Sample space는 event이다

1.4. set theory.

i) $\subset$: subset

$A \subset B$: A는 B의 subset이다.

$\Rightarrow \underline{\forall x \in A, \ x \in B}$.

Thm 1-4-1) A, B, C = set (event) , S : sample space.

i) $A \subset S$ ii) if $A \subset B$ and $B \subset C$ then $A \subset C$

★ iii) if $A \subset B$ and $B \subset A$ then $A = B$.

ex) $A = \{$짝수가 나옴$\}$ $A = \{2, 4, 6\}$

$B = \{$1 보다 큰수가 나옴$\}$ $B = \{2, 3, 4, 5, 6\}$

$A \subset B$.

ii). ϕ : empty set (공집합)

event $= \phi \longrightarrow$ 해당 event는 일어나지 않는다.

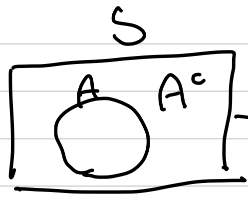
Thm for any event A. $\phi \subset A$

ex) $C = \{$7 이상 숫자가 나옴$\}$ $C = \phi$.

iii) complement. → 여집합

A = event

A^c : A가 일어나지 않는 event



→ Venn diagram

Thm 1-4-3 A: event

i) $(A^c)^c = A$ ii) $\phi^c = S$ iii) $S^c = \phi$.

iv) Union. A, B = event

→ or.

$A \cup B$: A 혹은 B가 일어나는 event

↳ (A와 B가 모두 일어나는 event를 포함)

* n개의 event $A_1, A_2, \ldots A_n$ 의 union

⇒ $\bigcup_{i=1}^{n} A_i$ $n \to \infty$ 도 가능.

→ cup

Thm 1.4.4. A, B = event

i) $A \cup B = B \cup A$ ii) $A \cup A = A$ iii) $A \cup A^c = S$

iv) $A \cup \phi = A$ v) $A \cup S = S$ vi) if $A \subset B$

$A \cup B = B$.

결합법칙

Thm Associative property (A, B, C = event)

$A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C)$

v) Intersection (A, B = event)

$A \cap B$: A와 B가 모두 일어나는 event.

ex) A: 짝수가 나옴 $A = \{2, 4, 6\}$

B: 4보다 작은수가 나옴 $B = \{1, 2, 3\}$

$A \cup B = \{1, 2, 3, 4, 6\}$

$A \cap B = \{2\}$.

* A와 B가 disjoint → $A \cap B = \emptyset$.

* n개의 event $A_1, A_2, \cdots, A_n$ 이 있을때.

intersection of $A_1 \cdots A_n \Rightarrow \bigcap_{i=1}^{n} A_i$ 로 표기.

$n \to \infty$ cap.

Thm. A, B = event.

i) $A \cap B = B \cap A$ ii) $A \cap A = A$ iii) $A \cap A^c = \emptyset$.

iv) $A \cap \emptyset = \emptyset$ v) $A \cap S = A$

ex) 동전던지기: 동전을 3번 던져서 나오는 결과물.

각 trY 결과 = 앞(H) 뒤(T), 순서대로 표기.

$S = \{ \underset{s_1}{HHH}, \underset{s_2}{THH}, \underset{s_3}{HTH}, \underset{s_4}{HHT}, \underset{s_5}{HTT}, \underset{s_6}{THT}, \underset{s_7}{TTH}, \underset{s_8}{TTT} \}$.

event A = 앞이 적어도 한번 나옴

$= \{ s_1, s_2, s_3, s_4, s_5, s_6, s_7 \}$

B = 두번째 시도에 앞면이 나옴

$= \{ s_1, s_2, s_4, s_6 \}$

C = 세번 " 뒷면 "

$= \{ s_4, s_5, s_6, s_8 \}$.

i) $B \subset A$, ii) $A^c = \{s_8\}$ iii) $B \cap C = \{s_4, s_6\}$

iv) $B \cup C = \{s_1, s_2, s_4, s_5, s_6, s_8\}$

v) $A \cap (B \cup C) = \{s_1, s_2, s_4, s_5, s_6\}$

* Additional properties of sets.

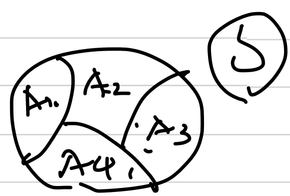
i) De Morgan's law.

- $(A \cup B)^c = A^c \cap B^c$
- $(A \cap B)^c = A^c \cup B^c$

ii) Distributive property. (분배법칙)

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

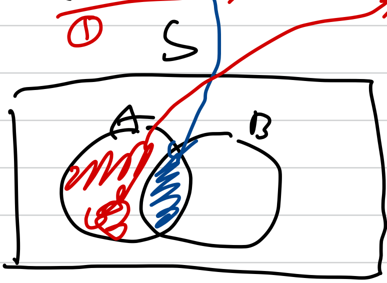
iii) Partitioning a set



$$A = \underbrace{(A \cap B)}_{①} \cup \underbrace{(A \cap B^c)}_{②}$$

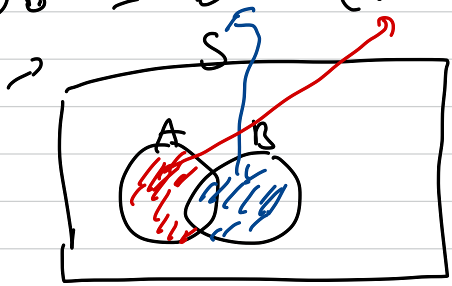
i) $A_1 \cup \cdots A_4 = S$

ii) mutually disjoint



①, ② → A의 partition

$$A \cup B = \overset{①}{B} \cup \overset{②}{(A \cap B^c)}$$



①, ② → $A \cup B$의 partition.

1.5 The definition of probability

S: sample space E: event

P(E) : Event E에 대해 어떤 property 를 만족하는 값.

↳ 길이, 부피, (measure)

↳ axiom(공리) 3개.
↳ 정해진다.

Axiom.

i) $\forall A \subseteq S$, $P(A) \geq 0$.

ii) $P(S) = 1$.

iii) For every countable infinite sequence of (mutually) disjoint events $A_1, A_2, \ldots$

1-1 corresponds to $\mathbb{Z}$.
$\mathbb{Z}$, $\mathbb{N}$, $\mathbb{Q}$ (유리수)
~~$\mathbb{R}$~~ ~~실수~~

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

↓

iii) 은 finite한 sequence $A_1 \cdots A_n$ 에 대해서도 성립 ⇒ For any mutually disjoint $A_1 \cdots A_n$.

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

<u>Def</u>) S : sample space

probability (probability measure) on S

⇒ <u>P</u>(A) for all event A, which satisfies the axiom i), ii), and iii)

Thm 1-5-1 $P(\emptyset) = 0$.

pf) countable infinite sequence of events

$A_1, A_2 \cdots$ 에 대해 $\forall i$, $A_i = \underline{\emptyset}$

이라면 $\forall i, j$, $A_i \cap A_j = \emptyset$

⇒ $A_1, A_2 \cdots$ are <u>mutually disjoint</u>

by axiom iii)

$$\underline{P(\emptyset)} = P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(\underline{A_i}) = \sum_{i=1}^{\infty} \underline{P(\emptyset)}$$

⇒ 자기 자신을 무한번 더해서 자기 자신이 나오는 유일한 수. ⇒ 0.

* other properties (직접 해보기)

A = event

i) $P(A^c) = 1 - P(A)$

ii) if $A \subset B$ then $P(A) \le P(B)$,

iii) $0 \le P(A) \le 1$

iv) $P(A \cap B^c) = P(A) - P(A \cap B)$

Thm 1.5.7. A, B = event

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

pf) $A \cup B = B \cup (A \cap B^c)$

B 와 $A \cap B^c$ 는 disjoint 하므로

by axiom iii), $P(A \cup B) = P(B) + P(A \cap B^c)$

$= P(B) + P(A) - P(A \cap B)$

ex) 학생기) 사탕이나 초콜렛을 좋아함

$P(\underbrace{\text{사탕 좋아함}}_{A}) = 0.3$

$P(\underbrace{\text{초콜렛 〃}}_{B}) = 0.8$

$P(\text{사탕과 초콜렛 모두 좋아함}) = ?$

$$\underbrace{P(A \cup B)}_{1} = \underbrace{P(A)}_{0.3} + \underbrace{P(B)}_{0.8} - P(A \cap B)$$

$$P(A \cap B) = 0.1$$

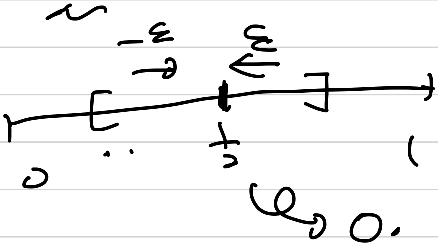
Thm 1.5.8 (Bonferroni $\underbrace{\text{inequality}}_{\text{부등식}}$)

$\forall n$ events $A_1 \cdots A_n$, (disjoint 조건 X)

$$P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$$

pf) Induction on n. //

* $\boxed{\text{불가능}}$ ($A = \emptyset$) $\xrightarrow{\text{O}}$ $\boxed{\text{확률이 } 0}$; $\xleftarrow{\text{X}}$



$\downarrow$

$P(\text{뽑은 값이 } \frac{1}{2}) = 0$