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1 (a)

X	Y	Z	X'Y	X'Y'Z	XYZ'	F_1	X'Z	YZ'	F_2
0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	1	0	1
0	1	0	1	0	0	1	0	1	1
0	1	1	1	0	0	1	1	0	1
1	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0
1	1	0	0	0	1	1	0	1	1
1	1	1	0	0	0	0	0	0	0

(b) $X'Y + X'Y'Z + XYZ'$

$= Y(X' + XZ') + X'Y'Z$

$= X'Y + YZ' + X'Y'Z$

$= X'(Y + Y'Z) + YZ'$

$= X'(Y + Z) + YZ'$

$= \underbrace{X'Y}_{\text{Consensus}} + YZ' + X'Z$

$= YZ' + X'Z$ 이다.

(C) $X'Y + X'Y'Z + XYZ' = X'Y(Z+Z') + X'Y'Z + XYZ'$

$= X'Y'Z + X'YZ' + X'Y'Z + XYZ'$ 이다. $\Rightarrow F(X,Y,Z) = \Sigma m(1,2,3,6)$

$X'Z + YZ' = X'Z(Y+Y') + YZ'(X+X')$

$= X'YZ + X'Y'Z + XYZ' + X'YZ'$ 이다. $\Rightarrow F(X,Y,Z) = \Sigma_m(1,2,3,6)$

즉 두 식은 동일 하다.

2. (a) $XY + YZ + X'Z$ ↓ 5번, 1번(D)

$= XY + (X+X')YZ + X'Z$ ↓ 8번

$= XY + XYZ + X'YZ + X'Z$ ↓ 8번

$= XY(1+Z) + X'Z(Y+1)$ ↓ 2번, 1번(D)

$= XY + X'Z$

(b) $XY + X'Z$ ↓ 5번, 1번(D)

$= XY(Z+Z') + X'Z$ ↓ 8번

$= XYZ + XYZ' + X'Z$ ↓ 8번

$= Z(XY + X') + XYZ'$ ↓ 8번(D)

$= Z((X+X')(X'+Y)) + XYZ'$ ↓ 5번

$= Z(1 \cdot (X'+Y)) + XYZ'$ ↓ 1번(D)

$= Z(X'+Y) + XYZ'$ ↓ 8번

$= X'Z + YZ + XYZ'$ ↓ 8번

$= X'Z + Y(Z + XZ')$ ↓ 8번(D)

$= X'Z + Y((Z+X)(Z+Z'))$ ↓ 5번, 1번(D)

$= X'Z + Y(Z+X)$ ↓ 8번

$= X'Z + XY + YZ$

3.

$(X_1 + X_2 + \cdots + X_n)' = X_1' X_2' \cdots X_n'$ 에 대하여

$n=1$ 일 때 $(X_1)' = X_1'$ 이므로 성립한다.

$n=2$ 일 때 $(X_1+X_2)' = X_1'X_2'$ 이므로 성립한다.

$n=k$ 일 때 $(X_1+X_2+\cdots+X_n)' = X_1'X_2'\cdots X_n'$ 이 성립한다고 가정하자.

즉 $(X_1+X_2+\cdots+X_k)' = X_1'X_2'\cdots X_k'$ 일 때,

$n=k+1$ 이라면 $\left[(X_1+X_2+\cdots+X_k)+X_{k+1}\right]' = (X_1+X_2+\cdots+X_k)' \cdot X_{k+1}'$

이므로 $(X_1'X_2'\cdots X_k') \cdot X_{k+1} = X_1'X_2'\cdots X_{k+1}'$ 이다.

따라서 (임의의 양의 정수 n에 대하여) $(X_1 + X_2 + \cdots + X_n)' = X_1' X_2' \cdots X_n'$ 은 참이다.

$(X_1 X_2 \cdots X_n)' = X_1' + X_2' + \cdots + X_n'$ 에 대하여

$n=1$ 일 때 $(X_1)' = X_1'$ 이므로 성립한다.

$n=2$ 일 때 $(X_1X_2)' = X_1' + X_2'$ 이므로 성립한다.

$n=k$ 일 때 $(X_1X_2\cdots X_n)' = X_1' + X_2' + \cdots + X_n'$ 이 성립한다고

가정하면 $(X_1X_2\cdots X_k)' = X_1' + X_2' + \cdots + X_k'$ 이다.

$n=k+1$ 일 때 $\left[(X_1X_2\cdots X_k)\cdot X_{k+1}\right]' = (X_1X_2\cdots X_k)' + X_{k+1}'$

이므로 $X_1' + X_2' + \cdots + X_k' + X_{k+1}'$ 이다.

따라서 (임의의 양의 정수 n에 대하여) $(X_1X_2\cdots X_n)' = X_1' + X_2' + \cdots + X_n'$ 은 참이다.

4. (a) $\left[(WX' + Y)\left[(W+Z)' + (XYZ')'\right]\right]'$

$= (WX' + Y)' + \left[(W+Z)' + (XYZ')\right]'$

$= (WX')' \cdot Y' + \left[((W+Z)')' \cdot (XYZ')'\right]$

$= (W' + X)Y' + \left[(W+Z)(X'+Y'+Z)\right]$

(b) $\left[X + YZ(W + X')\right]'$

$= X' \cdot \left[Y' + Z' + (W+X')'\right]$

$= X'\left(Y' + Z' + XW'\right)$

5. (a) $(A+B)(C+D+E)$

$= AC+AD+AE+BC+BD+BE$

(b) $X(Y+Z)(V+W)$

$= X(YV+YW+ZV+ZW)$

$= XYV+XYW+XZV+XZW$

(c) $(X+Y)ZW+(X+Y)'$ 에서 $X+Y$를 A로 두면

$AZW+A'$

$=(A+A')(ZW+A')$

$=1\cdot(ZW+(X+Y)')$

$=ZW+XY'$

6. (a), $X + YZ + VW$

$= X + (Y+V)(Y+W)(Z+V)(Z+W)$

$= (X+Y+V)(X+Y+W)(X+Z+V)(X+Z+W)$

(b) $X+Y = A$라 두면

$AZW + A'$

$= (A+A')(Z+A')(W+A')$

$= 1\,(Z+(X+Y)')(W+(X+Y)')$

$= (Z+X'Y')(W+X'Y')$

$= (X'+Z)(Y'+Z)(X'+W)(Y'+W)$

7. (a) 전개하면 $XX' + XY = XY$ 이고

$XX' = 0$ 이므로 $0 + XY = XY$ 이다.

(b) $X(X' + Y)$ 의 dual은 $X + (X'Y)$ 이고

XY의 dual은 $X + Y$ 이다.

$X + (X'Y) = (X + X')(X + Y)$ 이고,

$X + X' = 1$ 이므로 위 식은 $1 \cdot (X + Y) = X + Y$ 이다.

8. (a) $AB + (C'+D)(AB)'$ 에서 $AB = Z$ 로 두면

$$Z + Z'(C'+D)$$
$$= (Z+Z')(Z+C'+D)$$
$$= Z + C' + D$$
$$= AB + C' + D$$

(b) $(A'+B'+C)(A'+B+C)'$ 에서 $A'+C = Z$ 로 두면

$$(Z+B')(Z+B)'$$
$$= (Z+B')(Z'B')$$
$$= ZZ'B' + Z'B'B'$$
$$= 0 + Z'B'$$
$$= (A'+C)'B'$$
$$= AB'C'$$

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9. (a) $X'Y' + WX'Z + WX'$

$= X'Y'(Z+Z')\cdot(W+W') + WX'Z(Y+Y') + WX'(Y+Y')(Z+Z')$

$= X'Y'ZW + X'Y'ZW' + X'Y'Z'W + X'Y'Z'W' + X'YZW +$ ~~$X'Y'ZW$~~

$+$ ~~$X'YZW$~~ $+ X'YZ'W +$ ~~$X'Y'ZW$~~ $+$ ~~$X'Y'Z'W$~~

$= F(X,Y,Z,W) = \Sigma m(0,1,2,3,5,7)$

(b) $(X+Y)WZ + (X+Y)'$

$= XWZ + YWZ + X'Y'$

$= XWZ(Y+Y') + YWZ(X+X') + X'Y'(Z+Z')(W+W')$

$= XYZW + XY'ZW +$ ~~$XYWZ$~~ $+ X'YWZ + X'Y'ZW + X'Y'ZW'$

$+ X'Y'Z'W + X'Y'Z'W'$

$= F(X,Y,Z,W) = \Sigma m(0,1,2,3,7,11,15)$

10. (a) $(X'+Y)(W+Z)(X+Y'+Z)$

$= (X'+Y+ZZ'+WW')(XX'+YY'+Z+W)(X+Y'+Z+WW')$

$= (X'+Y+Z+W)(X'+Y+Z+W')(X'+Y+Z'+W)(X'+Y+Z'+W')$

$\cdot (X+Y+Z+W)(X+Y'+Z+W)(X'+Y+Z+W)(X'+Y'+Z+W)$

$\cdot (X+Y'+Z+W)(X+Y'+Z+W')$

$= F(X,Y,Z,W) = \Pi M(3,4,5,6,7,10,11,15)$

(b) $(X+Y)WZ + (X+Y)'$

$= (X+Y+1)((X+Y)'+WZ)$

$= (X'Y'+WZ)$

$= (X'+W)(X'+Z)(Y'+W)(Y'+Z)$

$= (X'+W+YY'+ZZ')(X'+Z+YY'+WW')(Y'+W+XX'+ZZ')(Y'+Z+XX'+WW')$

$= (X'+W+Y+Z)(X'+W+Y+Z')(X'+W+Y'+Z)(X'+W+Y'+Z')$

$\cdot (X'+Z+Y+W)(X'+Z+Y+W')(X'+Z+Y'+W)(X'+Z+Y'+W')$

$\cdot (Y'+W+X+Z)(Y'+W+X+Z')(Y'+W+X'+Z)(Y'+W+X'+Z')$

$\cdot (Y'+Z+X+W)(Y'+Z+X+W')(Y'+Z+X'+W)(Y'+Z+X'+W')$

$= F(X,Y,Z,W) = \Pi M(1,2,3,5,6,7,9,10,11)$

11. (a)

A B C D	Y
0 0 0 0	0
0 0 0 1	1
0 0 1 0	0
0 0 1 1	1
0 1 0 0	0
0 1 0 1	1
0 1 1 0	0
0 1 1 1	1
1 0 0 0	0
1 0 0 1	1
1 0 1 0	X
1 0 1 1	X
1 1 0 0	X
1 1 0 1	X
1 1 1 0	X
1 1 1 1	X

(b)

$$Y = \Sigma m(1,3,5,7,9) + \Sigma d(10,11,12,13,14,15)$$

(c)

$$Y = \Pi M(0,2,4,6,8) + \Pi D(10,11,12,13,14,15)$$

12.

(a) 더해지는 두개의 두 비트 이진수의 각각 A_1A_0, B_1B_0로 두고, 입력으로 들어오는 Carry bit를 C_{in}, 출력되는 두 자리 이진수를 Y_1Y_0, 덧셈을 수행한 후 발생한 Carry bit를 C_{out}이라 하겠습니다.

(a)

A_1	A_0	B_1	B_0	C_{in}	C_{out}	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1
0	0	0	1	0	0	0	1
0	0	0	1	1	0	1	0
0	0	1	0	0	0	1	0
0	0	1	0	1	0	1	1
0	0	1	1	0	0	1	1
0	0	1	1	1	1	0	0
0	1	0	0	0	0	0	1
0	1	0	0	1	0	1	0
0	1	0	1	0	0	1	0
0	1	0	1	1	0	1	1
0	1	1	0	0	0	1	1
0	1	1	0	1	1	0	0
0	1	1	1	0	1	0	0
0	1	1	1	1	1	0	1
1	0	0	0	0	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	0	0	1	1
1	0	0	1	1	1	0	0
1	0	1	0	0	1	0	0
1	0	1	0	1	1	0	1
1	0	1	1	0	1	0	1
1	0	1	1	1	1	1	0
1	1	0	0	0	1	1	0
1	1	0	0	1	1	1	1
1	1	0	1	0	1	1	1
1	1	0	1	1	1	1	1
1	1	1	0	0	1	1	1
1	1	1	0	1	1	1	1
1	1	1	1	0	1	1	1
1	1	1	1	1	1	1	1

(b)

- $C_{out}(A_1, A_0, B_1, B_0, C_{in})$

$= \Sigma m(7, 13, 14, 15, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31)$

- $Y_1(A_1, A_0, B_1, B_0, C_{in})$

$= \Sigma m(3, 4, 5, 6, 9, 10, 11, 12, 16, 17, 18, 23, 24, 29, 30, 31)$

- $Y_0(A_1, A_0, B_1, B_0, C_{in})$

$= \Sigma m(1, 2, 5, 6, 8, 11, 12, 15, 17, 18, 21, 22, 24, 27, 28, 31)$

(c)

- $C_{out}(A_1, A_0, B_1, B_0, C_{in})$

$= \Pi M(0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 16, 17, 18, 24)$

- $Y_1(A_1, A_0, B_1, B_0, C_{in})$

$= \Pi M(0, 1, 2, 7, 8, 13, 14, 15, 19, 20, 21, 22, 25, 26, 27, 28)$

- $Y_0(A_1, A_0, B_1, B_0, C_{in})$

$= \Pi M(0, 3, 4, 7, 9, 10, 13, 14, 16, 19, 20, 23, 25, 26, 29, 30)$

(d)