Magneto-acoustic waves in an asymmetric magnetic slab

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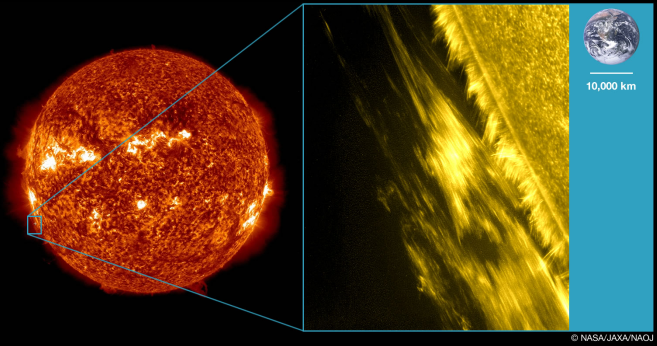
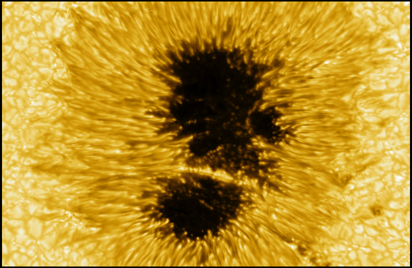
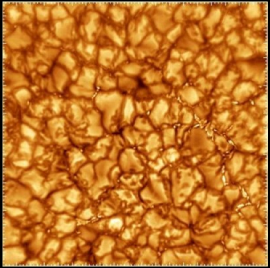
Motivation







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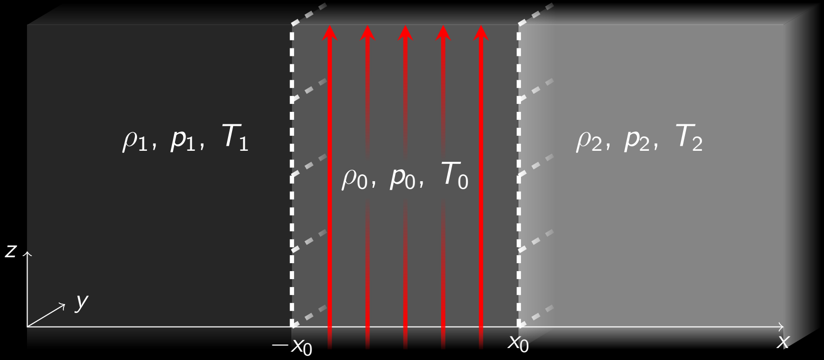
Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- Different density and pressure on each side.

Governing equations

Ideal MHD equations: Conservation of:

$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\nabla p - \frac{1}{\mu}\boldsymbol{B} \times (\nabla \times \boldsymbol{B}), \qquad \text{momentum}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \qquad \text{mass}$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{p}{\rho^\gamma}\right) = 0, \qquad \text{energy}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}), \qquad \text{magnetic flux}$$

$\boldsymbol{v}$ = plasma velocity, $\boldsymbol{B}$ = magnetic field strength,
ρ = density, p = pressure,
μ = magnetic permeability, γ = adiabatic index.

Asymmetric slab modes

Dispersion relation:

$$\frac{\omega^4 {m_0}^2}{k^2 {v_A}^2 - \omega^2} + \frac{\rho_0}{\rho_1} m_1 \frac{\rho_0}{\rho_2} m_2 (k^2 {v_A}^2 - \omega^2)$$
$$- \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} m_1 + \frac{\rho_0}{\rho_2} m_2 \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

$${m_0}^2 = \frac{(k^2 {v_A}^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + {v_A}^2)(k^2 {c_T}^2 - \omega^2)}, \qquad {m_{1,2}}^2 = k^2 - \frac{\omega^2}{{c_{1,2}}^2},$$

$${c_T}^2 = \frac{c_0^2 {v_A}^2}{c_0^2 + {v_A}^2}, \qquad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

See **Allcock** and Erdélyi, 2017.

Asymmetric slab modes

Slow **quasi-kink** surface mode

Red tubes: magnetic fieldlines,
Direction field: velocity perturbation,

$v_A = 0.9c_0$

$c_2 = 1.2c_0$

$c_1 = c_2\sqrt{\rho_2/\rho_1} = 1.231c_0$

$\rho_2/\rho_0 = 2$

$\rho_1/\rho_0 = 1.9$

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Asymmetric slab modes

Slow **quasi-sausage** surface mode

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Asymmetric slab modes

Fast **quasi-kink** body mode

Number of nodes: 1
Red tubes: magnetic fieldlines,
Red/blue contours: high/low density perturbation,
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$v_A = 0.9c_0$
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Fast **quasi-kink** body mode

Number of nodes: 2
Red tubes: magnetic fieldlines,
Red/blue contours: high/low density perturbation,
Direction field: velocity perturbation,

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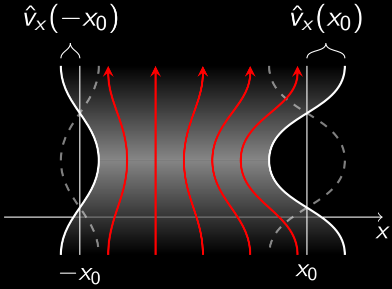
Amplitude ratio

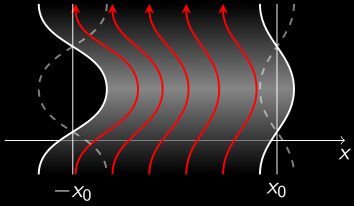


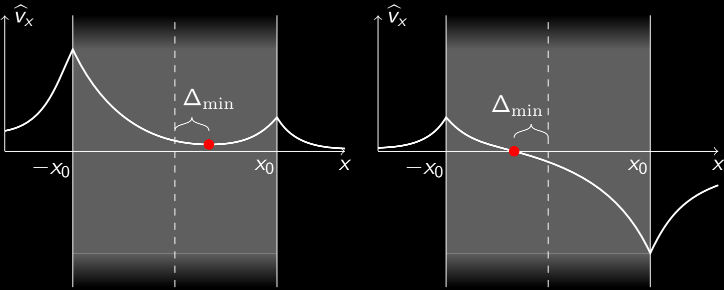
Amplitude ratio

$$R_A := \frac{\hat{v}_x(x_0)}{\hat{v}_x(-x_0)} \qquad \left(\begin{matrix}\text{Top} = \text{quasi-kink}\\ \text{Bottom} = \text{quasi-sausage}\end{matrix}\right)$$

$$= \left(\begin{matrix}+\\-\end{matrix}\right) \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 {v_A}^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 \left(\begin{matrix}\tanh\\ \coth\end{matrix}\right)(m_0 x_0)}{(k^2 {v_A}^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \left(\begin{matrix}\tanh\\ \coth\end{matrix}\right)(m_0 x_0)}$$

Minimum perturbation shift





Minimum perturbation shift

Quasi-sausage:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}\left(\frac{1}{D}\right)$$

Quasi-kink:

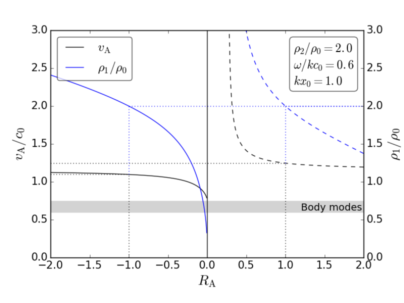
$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}(D)$$

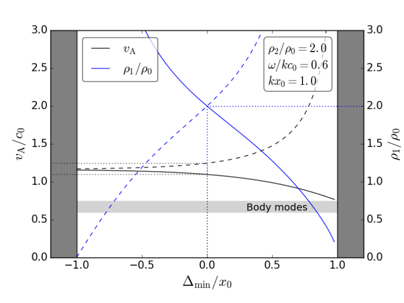
where $$D = \frac{(k^2 {v_A}^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} \tanh(m_0 x_0) - \omega^2 m_0}{(k^2 {v_A}^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \tanh(m_0 x_0)}$$

Solar magneto-seismology

Parameter inversion

- **Observe**: ω, k, x_0, T_i, and R_A or Δ_min.
- **Solve** to find: v_A and hence B_0.





Future work

Apply to observations of, for example:

- **prominences**,
- elongated **magnetic bright points**,
- sunspot **light walls**.

Generalise the model to a more realistic structure:

- Add **magnetic field** outside the slab,
- Add **steady flow**.

"a day without the Sun is, you know, night"

matthew_allcock

The University Of Sheffield.

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