

Magneto-acoustic waves in an asymmetric magnetic slab

Progress in spatial magneto-seismology



Matthew Allcock
and
Robertus Erdélyi



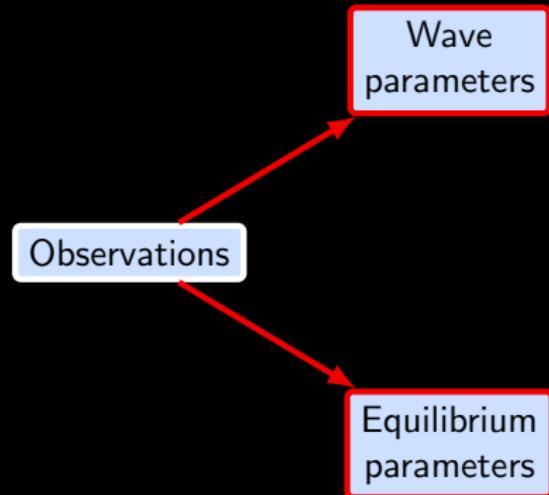
The
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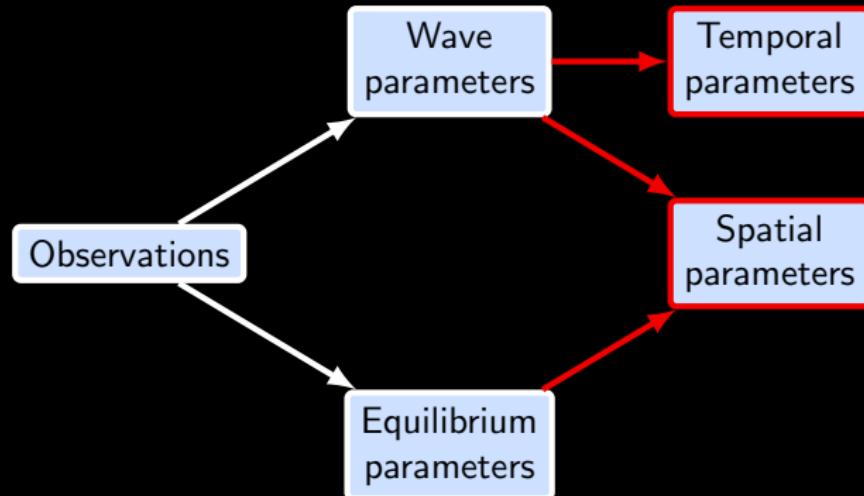
Solar magneto-seismology

Observations

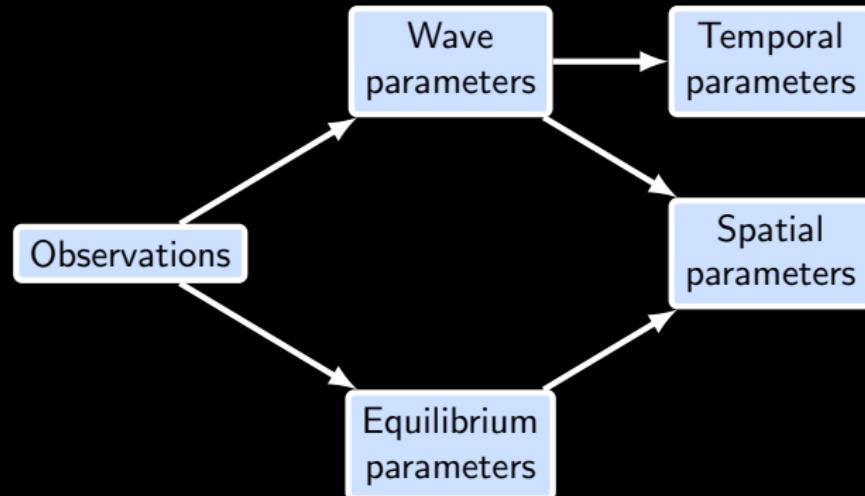
Solar magneto-seismology



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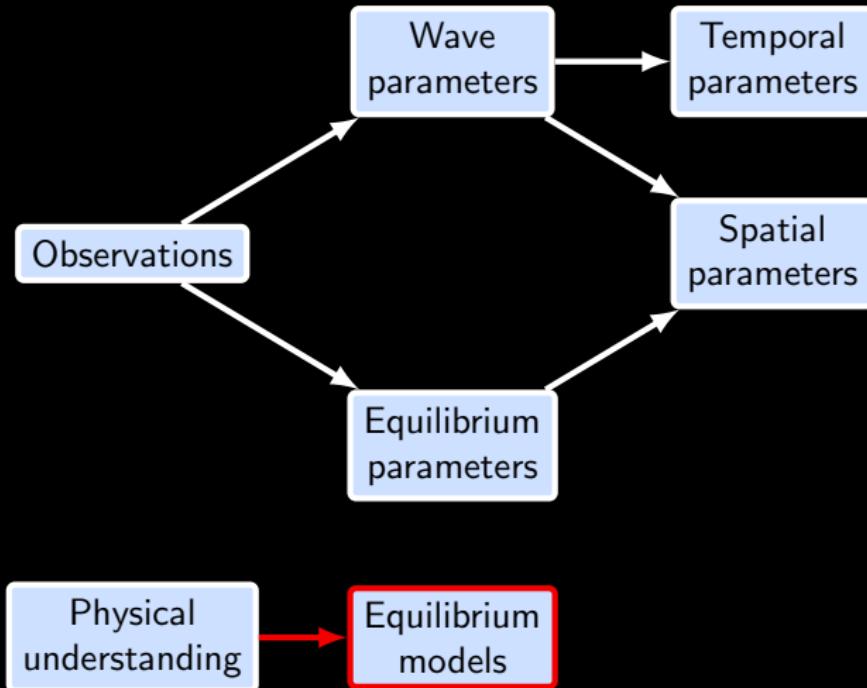


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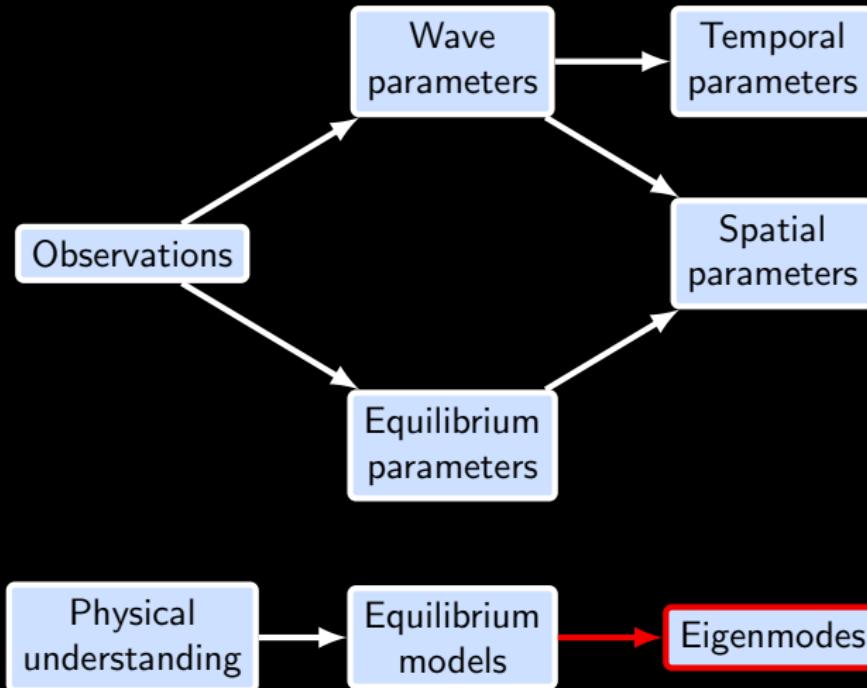


Physical
understanding

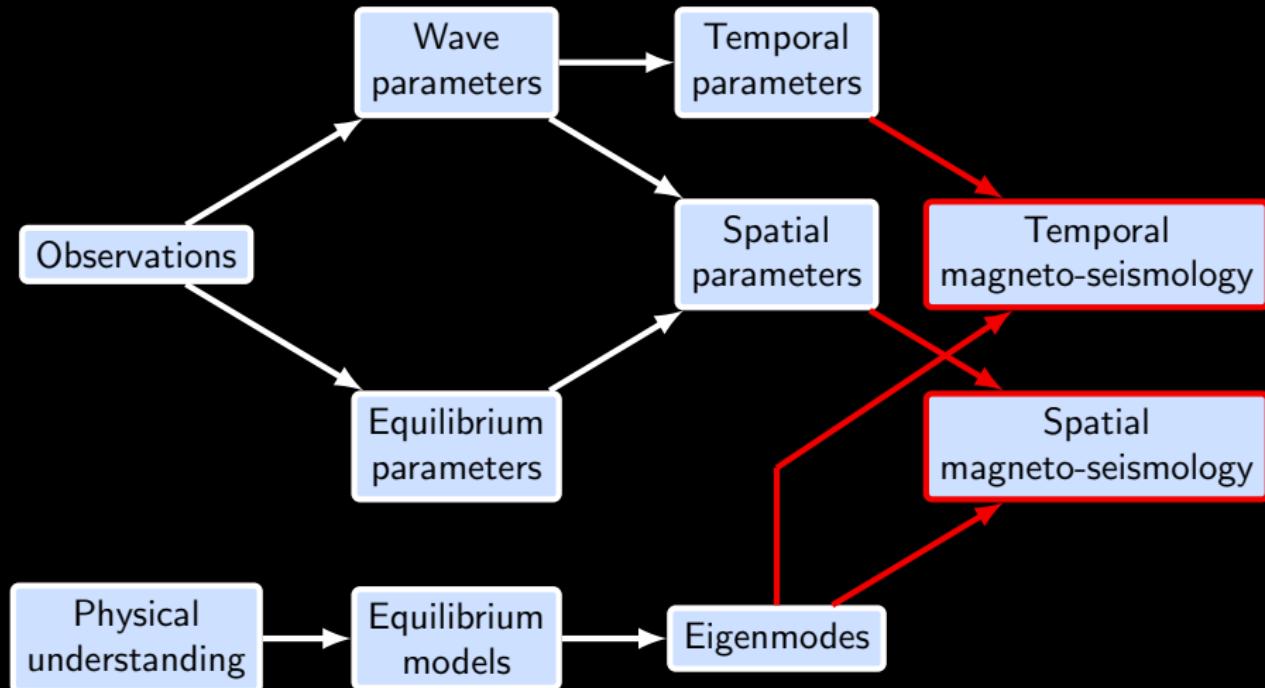
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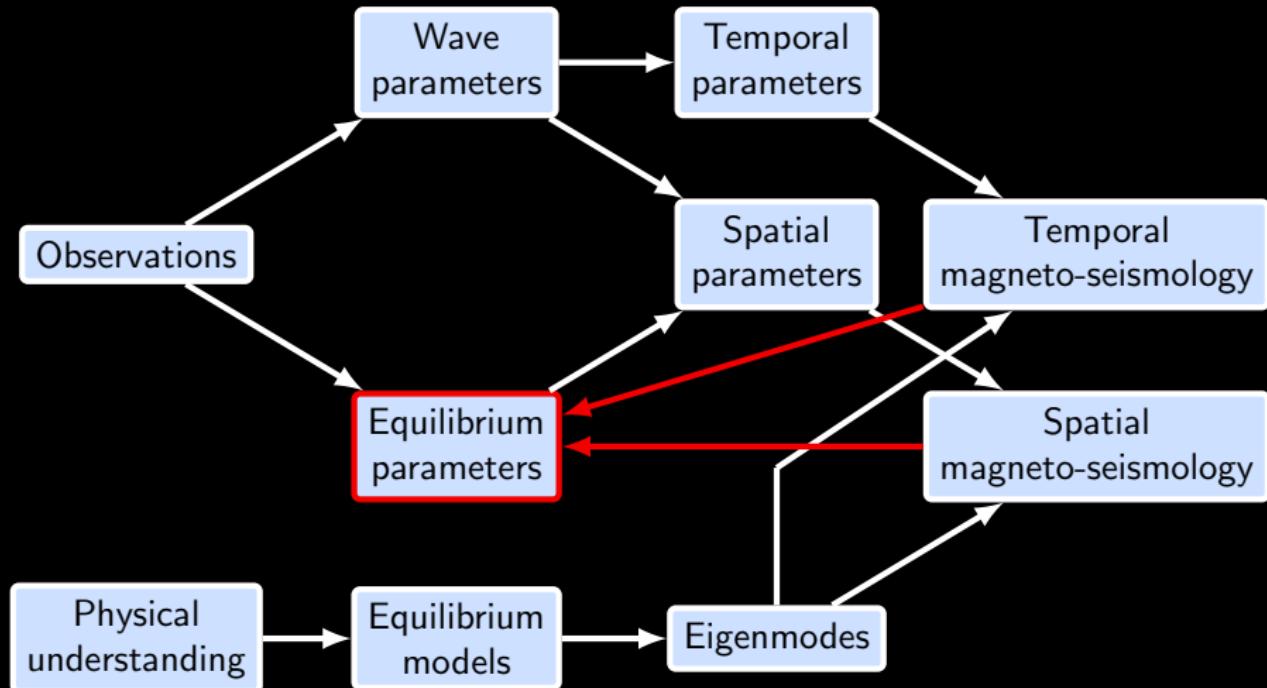
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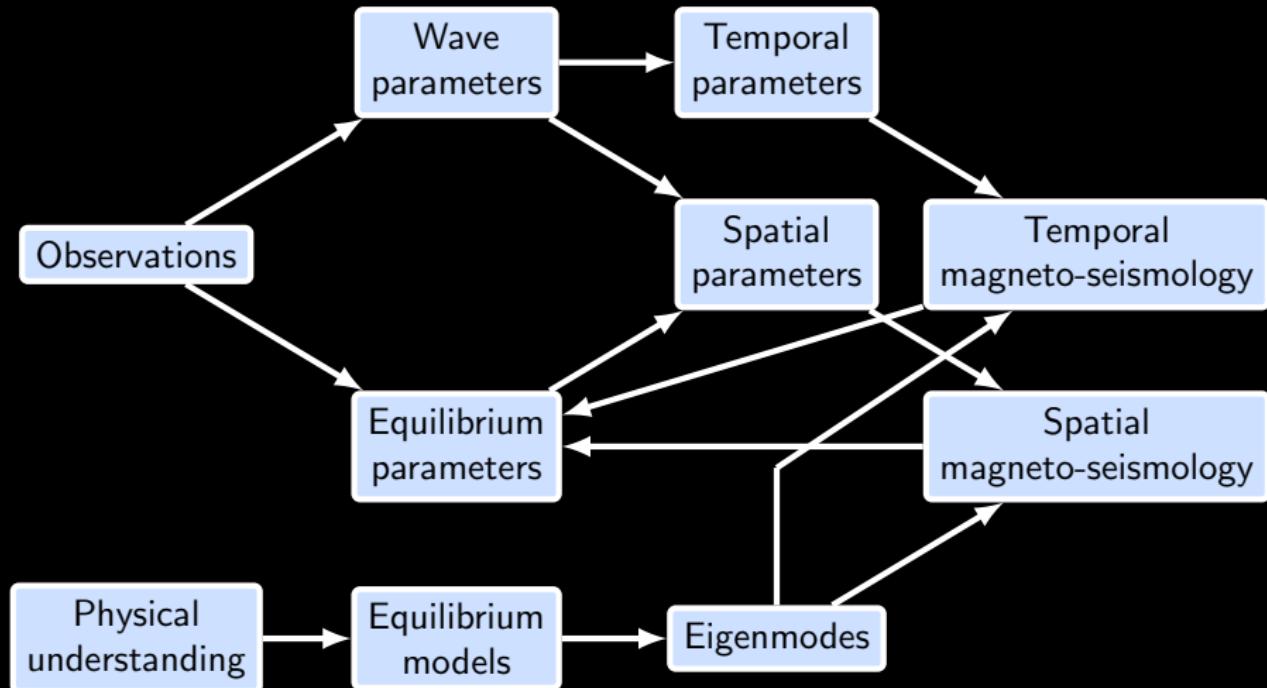
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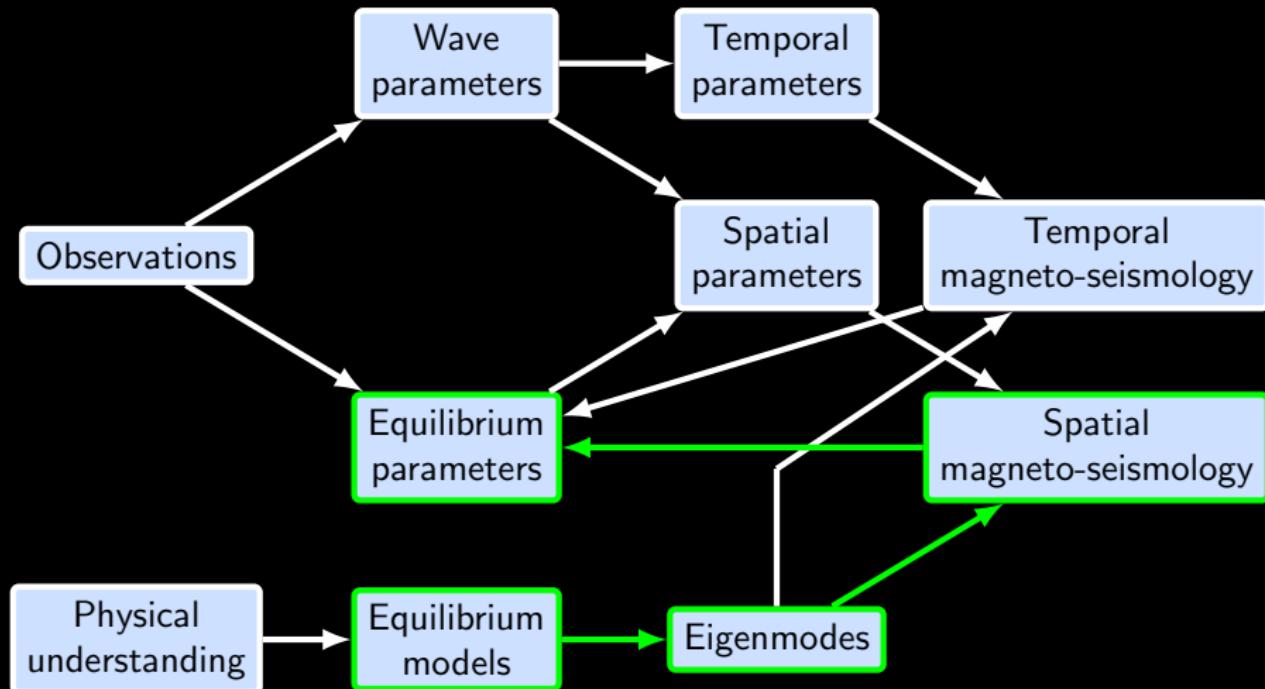
Solar magneto-seismology



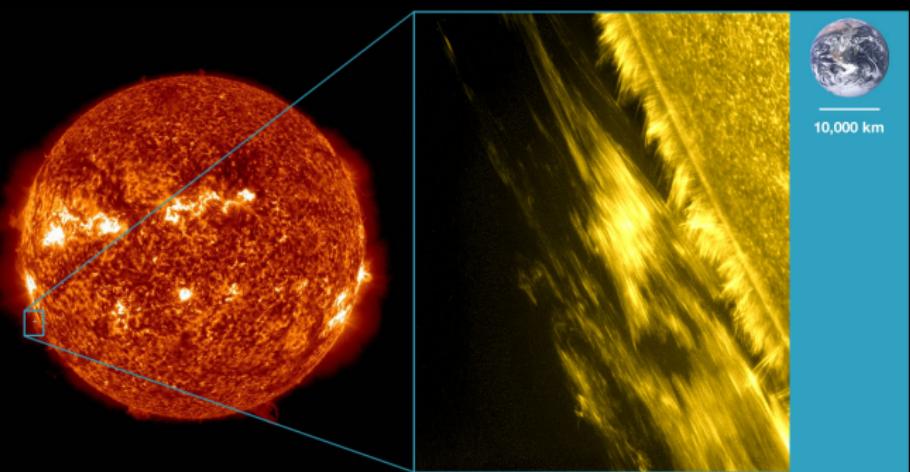
Solar magneto-seismology



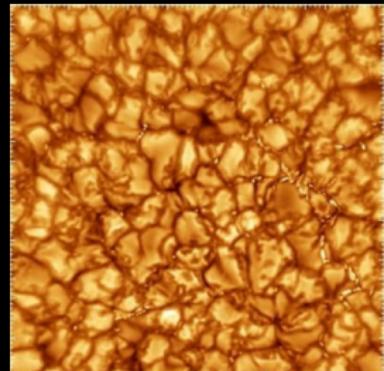
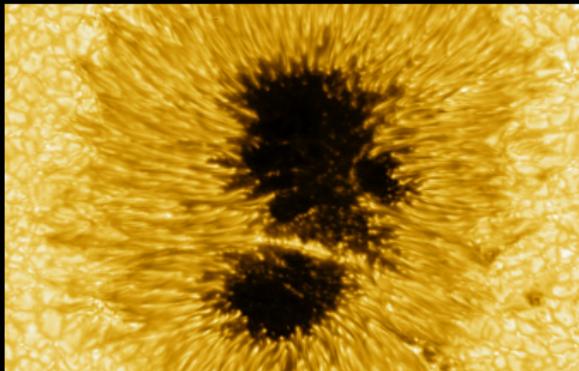
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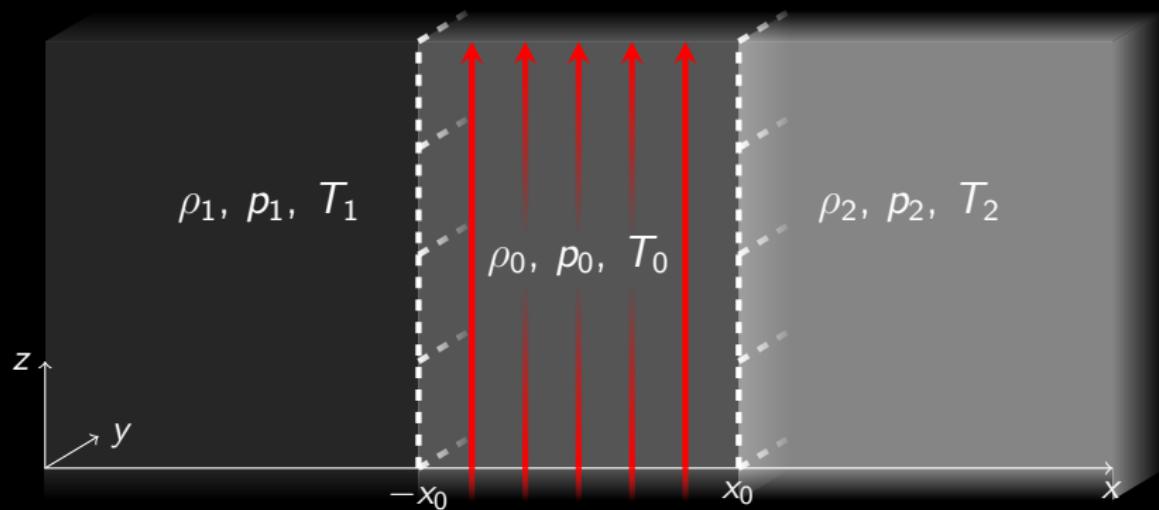
Motivation



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Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- **Different** density and pressure on each side.

Governing equations

Ideal MHD equations:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \frac{1}{\mu} \mathbf{B} \times (\nabla \times \mathbf{B}), \quad \text{momentum}$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \text{mass}$$
$$\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) = 0, \quad \text{energy}$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad \text{magnetic flux}$$

\mathbf{v} = plasma velocity,

ρ = density,

μ = magnetic permeability,

\mathbf{B} = magnetic field strength,

p = pressure,

γ = adiabatic index.

Asymmetric slab modes

Dispersion relation:

$$\frac{\omega^4 m_0^2}{k^2 v_A^2 - \omega^2} + \frac{\rho_0}{\rho_1} \textcolor{red}{m_1} \frac{\rho_0}{\rho_2} \textcolor{red}{m_2} (k^2 v_A^2 - \omega^2) \\ - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} \textcolor{red}{m_1} + \frac{\rho_0}{\rho_2} \textcolor{red}{m_2} \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad \textcolor{red}{m_{1,2}}^2 = k^2 - \frac{\omega^2}{\textcolor{red}{c_{1,2}}^2},$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}, \quad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

See **Allcock** and Erdélyi, 2017.

Asymmetric slab modes

Slow **quasi-kink** surface mode

Red tubes: magnetic fieldlines,
Direction field: velocity perturbation,

$$v_A = 0.9c_0$$

$$c_2 = 1.2c_0$$

$$c_1 = c_2\sqrt{\rho_2/\rho_1} = 1.231c_0$$

$$\rho_2/\rho_0 = 2$$

$$\rho_1/\rho_0 = 1.9$$



Asymmetric slab modes

Slow **quasi-sausage** surface mode

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Asymmetric slab modes

Fast **quasi-kink** body mode

Number of nodes: 1

Red tubes: magnetic fieldlines,

Red/blue contours: **high/low** density perturbation,

Direction field: velocity perturbation,

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Fast **quasi-kink** body mode

Number of nodes: 2

Red tubes: magnetic fieldlines,

Red/blue contours: **high/low** density perturbation,

Direction field: velocity perturbation,

$$v_A = 0.9c_0$$

$$c_2 = 1.2c_0$$

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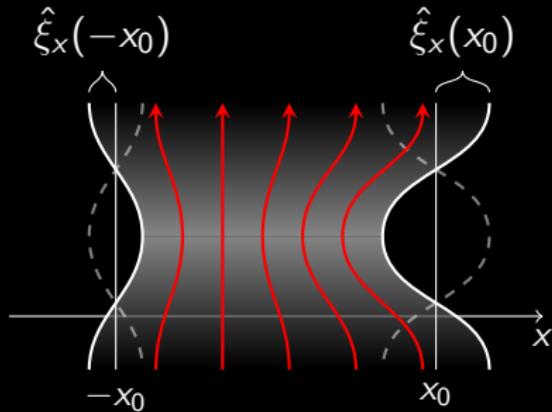
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Amplitude ratio



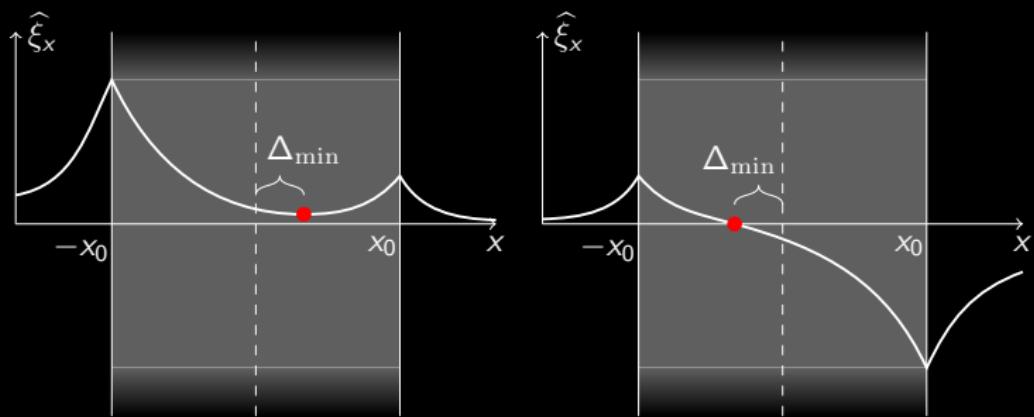
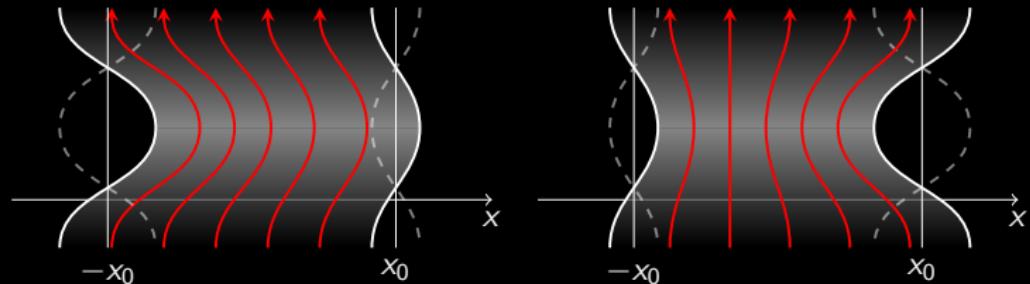
Amplitude ratio

$$R_A := \frac{\hat{\xi}_x(x_0)}{\hat{\xi}_x(-x_0)}$$

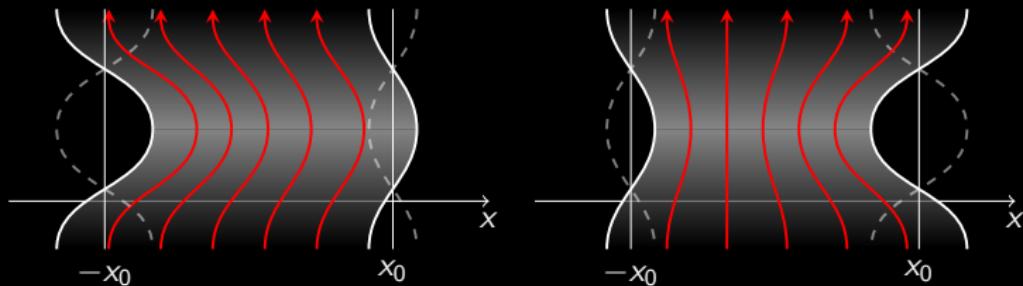
(Top = quasi-kink
Bottom = quasi-sausage)

$$= \left(\frac{+}{-}\right) \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 v_A^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 \left(\frac{\tanh}{\coth}\right) (m_0 x_0)}{(k^2 v_A^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \left(\frac{\tanh}{\coth}\right) (m_0 x_0)}$$

Minimum perturbation shift



Minimum perturbation shift



Quasi-kink:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}(D)$$

Quasi-sausage:

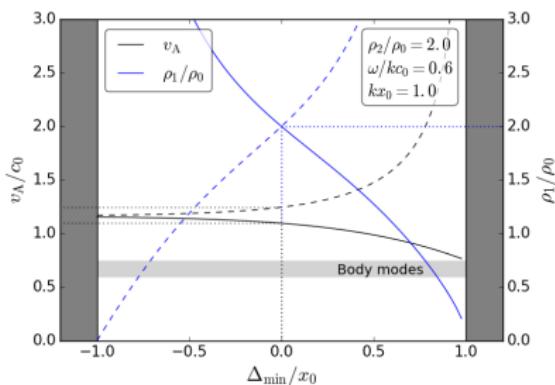
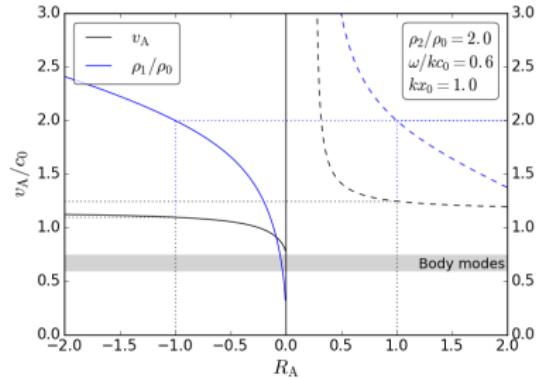
$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1} \left(\frac{1}{D} \right)$$

where $D = \frac{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} \tanh(m_0 x_0) - \omega^2 m_0}{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \tanh(m_0 x_0)}$

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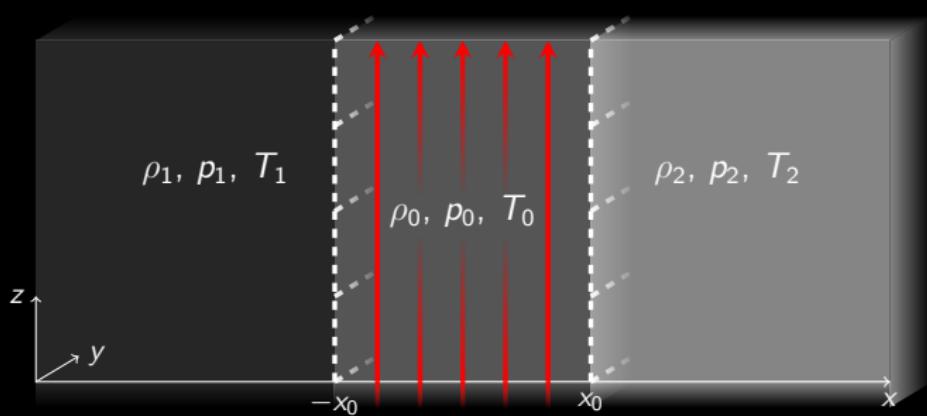
Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A or Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .



Further work

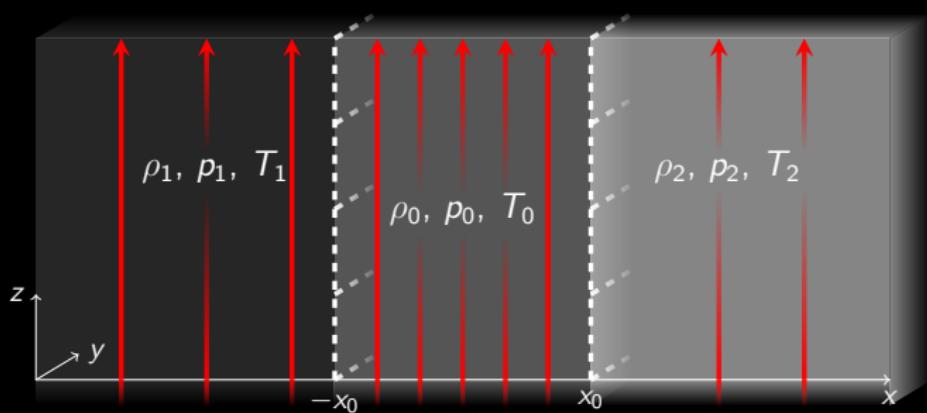
Generalise the model to a variety of structures:



Further work

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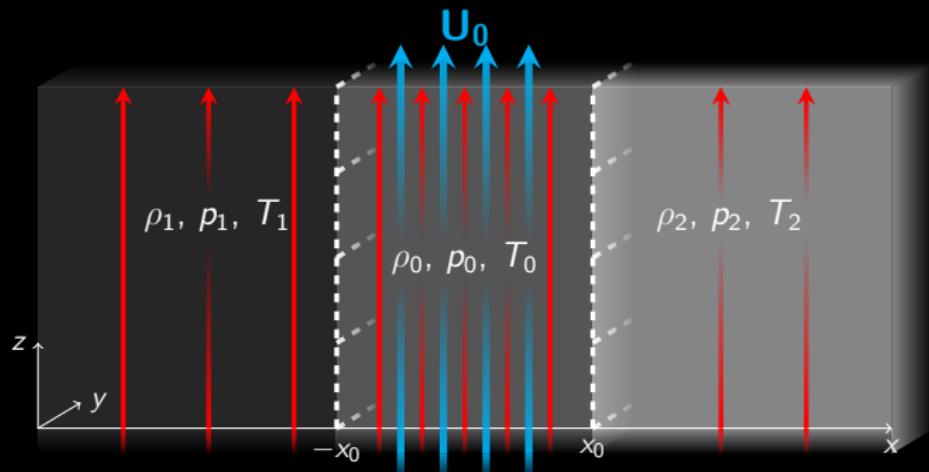
- Add **magnetic field** outside the slab - coronal structures. See **Zsámberger and Erdélyi**, published soon.



Further work

Generalise the model to a variety of structures:

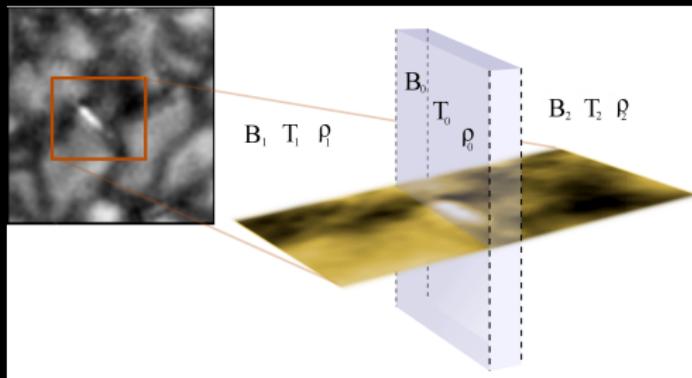
- Add **magnetic field** outside the slab - coronal structures. See **Zsmberger and Erdlyi**, published soon.
- Add **steady flow** - dynamic structures e.g. solar wind. See **Mihai Barbulescu's poster**.



Future work

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,

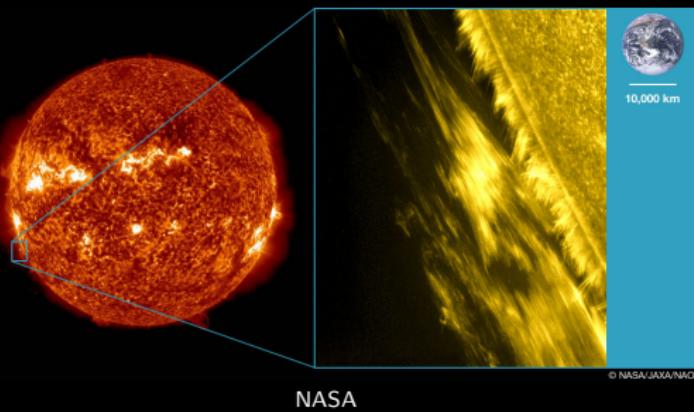


Adaptation of Liu et al., 2017, by N. Zsámberger

Future work

Apply to observations of MHD waves in, for example:

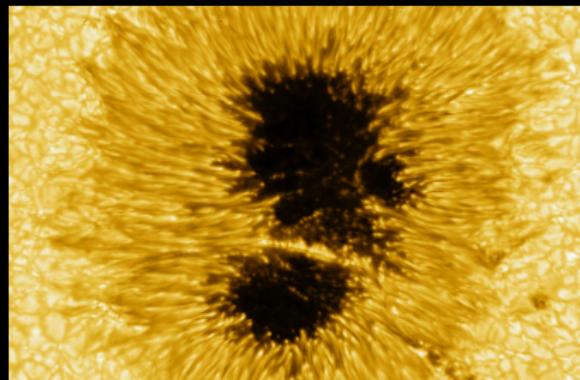
- Elongated **magnetic bright points**,
- **Prominences**,



Future work

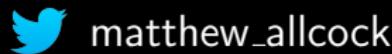
Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,
- **Prominences**,
- Sunspot **light walls**.



Max Planck Institute for Solar System Research

"a day without the Sun is, you know, night"



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Sheffield.

