Magneto-acoustic waves in an asymmetric magnetic slab

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and
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The University Of Sheffield.

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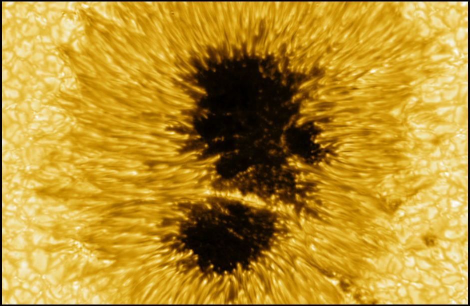
Magnetic slab: a model for solar structures



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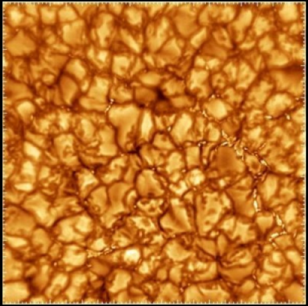

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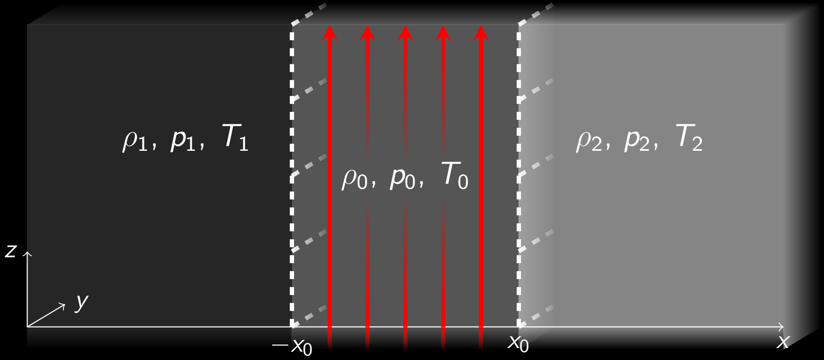
Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- Different density and pressure on each side.

Governing equations

Ideal MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\nabla p - \frac{1}{\mu} \boldsymbol{B} \times (\nabla \times \boldsymbol{B}),$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{p}{\rho^\gamma}\right) = 0,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}),$$

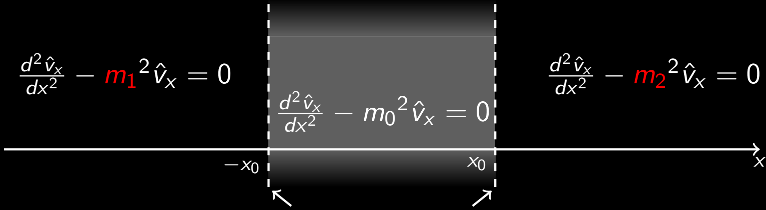
$\boldsymbol{v}$ = plasma velocity, $\boldsymbol{B}$ = magnetic field strength, ρ = density, p = pressure, μ = magnetic permeability, γ = adiabatic index.

Fourier decomposition

Look for *plane wave* solutions of the form:

$$v_x(\mathbf{x}, t) = \hat{v}_x(x)e^{i(kz-\omega t)}, \quad v_y(\mathbf{x}, t) = 0, \quad v_z(\mathbf{x}, t) = \hat{v}_z(x)e^{i(kz-\omega t)},$$

to arrive at the following ordinary differential equations:



$$\frac{d^2\hat{v}_x}{dx^2} - {m_1}^2\hat{v}_x = 0 \qquad \frac{d^2\hat{v}_x}{dx^2} - {m_0}^2\hat{v}_x = 0 \qquad \frac{d^2\hat{v}_x}{dx^2} - {m_2}^2\hat{v}_x = 0$$

Boundary conditions:
pressure and **velocity** continuous across boundaries.

$${m_0}^2 = \frac{(k^2{v_A}^2 - \omega^2)(k^2c_0^2 - \omega^2)}{(c_0^2 + {v_A}^2)(k^2{c_T}^2 - \omega^2)}, \qquad {m_{1,2}}^2 = k^2 - \frac{\omega^2}{{c_{1,2}}^2},$$

$${c_T}^2 = \frac{c_0^2{v_A}^2}{c_0^2 + {v_A}^2}, \qquad v_A = \frac{B_0}{\sqrt{\mu\rho_0}},$$

Mode decomposition

Matching solutions across each interface gives four homogeneous algebraic equations. The determinant of this system must be zero for the existence of non-trivial solutions. To satisfy this, the following relation must hold:

Dispersion relation:

$$\frac{\omega^4 {m_0}^2}{k^2 {v_A}^2 - \omega^2} + \frac{\rho_0}{\rho_1} m_1 \frac{\rho_0}{\rho_2} m_2 (k^2 {v_A}^2 - \omega^2) - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} m_1 + \frac{\rho_0}{\rho_2} m_2 \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

$${m_0}^2 = \frac{(k^2 {v_A}^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + {v_A}^2)(k^2 {c_T}^2 - \omega^2)}, \qquad {m_{1,2}}^2 = k^2 - \frac{\omega^2}{{c_{1,2}}^2},$$

$${c_T}^2 = \frac{c_0^2 {v_A}^2}{c_0^2 + {v_A}^2}, \qquad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

See **Allcock** and Erdélyi, 2017.

Asymmetric slab modes

Slow **quasi-kink** surface mode

Red tubes: magnetic fieldlines,
Red/blue contours: high/low density perturbation,
Direction field: velocity perturbation,

$v_A = 0.9c_0$
$c_2 = 1.2c_0$
$c_1 = c_2\sqrt{\rho_2/\rho_1} = 1.231c_0$

$\rho_2/\rho_0 = 2$
$\rho_1/\rho_0 = 1.9$

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Asymmetric slab modes

Slow **quasi-sausage** surface mode

Red tubes: magnetic fieldlines,
Red/blue contours: high/low density perturbation,
Direction field: velocity perturbation,

$$v_A = 0.9c_0 \qquad \rho_2/\rho_0 = 2$$
$$c_2 = 1.2c_0 \qquad \rho_1/\rho_0 = 1.9$$
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Asymmetric slab modes

Fast **quasi-kink** body mode

Number of nodes: 1
Red tubes: magnetic fieldlines,
Red/blue contours: high/low density perturbation,
Direction field: velocity perturbation,

$v_A = 0.9c_0$ $\rho_2/\rho_0 = 2$
$c_2 = 1.2c_0$ $\rho_1/\rho_0 = 1.9$
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Fast **quasi-kink** body mode

Number of nodes: 2
Red tubes: magnetic fieldlines,
Red/blue contours: high/low density perturbation,
Direction field: velocity perturbation,

$v_A = 0.9c_0$ $\rho_2/\rho_0 = 2$
$c_2 = 1.2c_0$ $\rho_1/\rho_0 = 1.9$
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$v_A = 0.9c_0$ $\rho_2/\rho_0 = 2$
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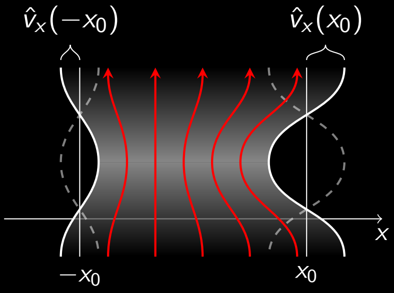
Solar magneto-seismology

Cross-slab amplitude ratio



Cross-slab (signed) amplitude ratio

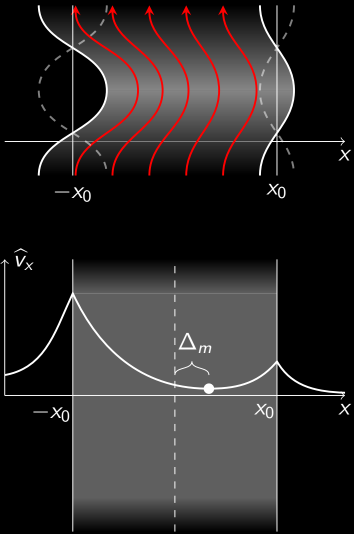
$$R_A := \frac{\hat{v}_x(x_0)}{\hat{v}_x(-x_0)}$$

$\left(\substack{\text{Top} = \text{quasi-kink} \\ \text{Bottom} = \text{quasi-sausage}}\right)$

$$= \left(\substack{+ \\ -}\right) \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 {v_A}^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 \left(\substack{\tanh \\ \coth}\right)(m_0 x_0)}{(k^2 {v_A}^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \left(\substack{\tanh \\ \coth}\right)(m_0 x_0)}$$

Solar magneto-seismology

Minimum/zero amplitude displacement



Solar magneto-seismology

Minimum/zero amplitude displacement

Quasi-kink: displacement of minimum amplitude position

$$\Delta_m = \frac{1}{m_0}\tanh^{-1}(D)$$

Quasi-sausage: displacement of zero amplitude position

$$\Delta_z = \frac{1}{m_0}\coth^{-1}(D)$$

where

$$D = \frac{(k^2{v_A}^2 - \omega^2)m_2\frac{\rho_0}{\rho_2}\tanh(m_0 x_0) - \omega^2 m_0}{(k^2{v_A}^2 - \omega^2)m_2\frac{\rho_0}{\rho_2} - \omega^2 m_0\tanh(m_0 x_0)}$$

Future work

Solar magneto-seismology:

- **Observe**: ρ_i, c_i, ω, k, x_0, and R_A or Δ_m or Δ_z.
- **Solve** to find: v_A and hence B_0.

Generalise the model:

- Add **magnetic field** outside the slab,
- Add **steady flow** - see **Mihai Barbulescu**'s talk at **15:15 tomorrow**.

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