

Solar magneto-seismology with asymmetric waveguides



Matthew Allcock
and
Robertus Erdélyi



The
University
Of
Sheffield.



Outline

- 1 Introduction
 - Waves on the Sun
 - Solar magneto-seismology (SMS)
 - A brief history
- 2 SMS with asymmetric wave-guides
 - Motivation
 - Derivation
 - Mode identification
 - Amplitude ratio
 - Minimum perturbation shift
- 3 Looking ahead

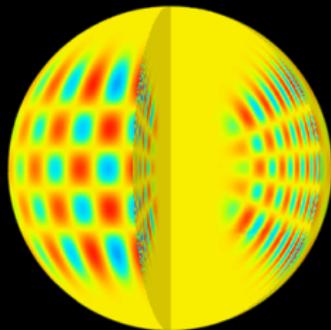
Outline

- 1 Introduction
 - Waves on the Sun
 - Solar magneto-seismology (SMS)
 - A brief history
- 2 SMS with asymmetric wave-guides
 - Motivation
 - Derivation
 - Mode identification
 - Amplitude ratio
 - Minimum perturbation shift
- 3 Looking ahead

Waves on the Sun

Global pressure waves (p -modes):

- Standing modes,
- Spherical harmonics with global Sun as cavity,
- Global and local **helioseismology** for inference of sub-surface flows, density, temperature.



MHD waves:

- Propagating or standing modes,
- Guided by local plasma inhomogeneity,
- Local **magneto-seismology** for inference of background magnetic field strength, heat transport coefficients, density.



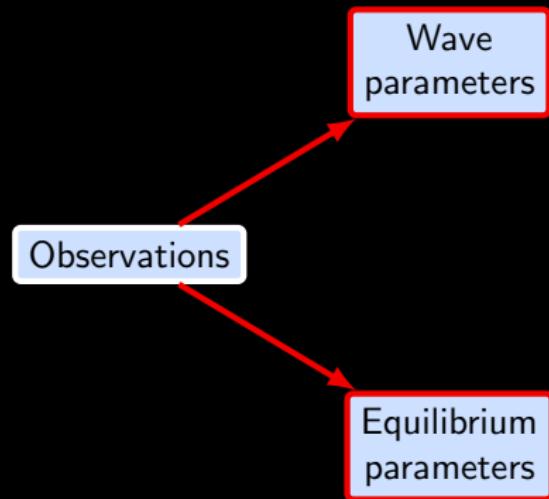
Solar magneto-seismology

An overview

Observations

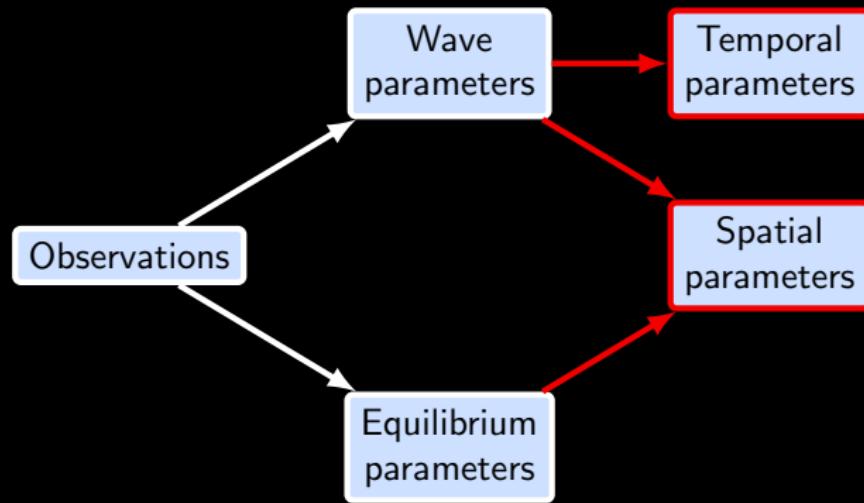
Solar magneto-seismology

An overview



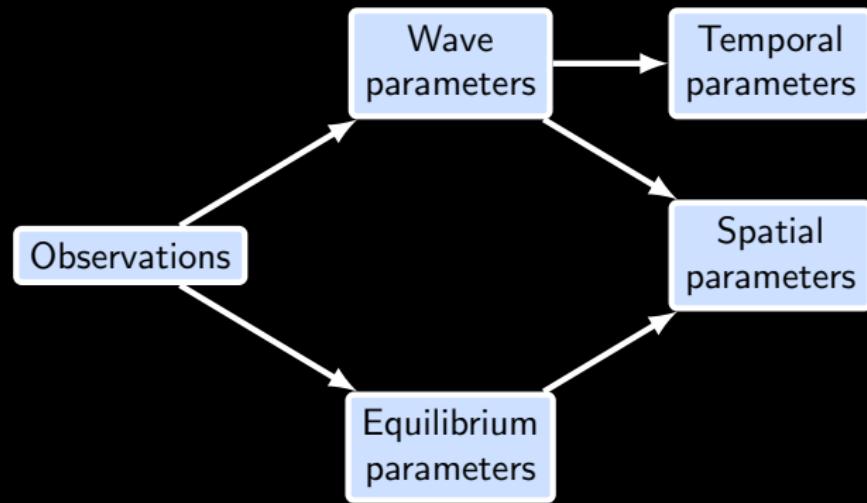
Solar magneto-seismology

An overview



Solar magneto-seismology

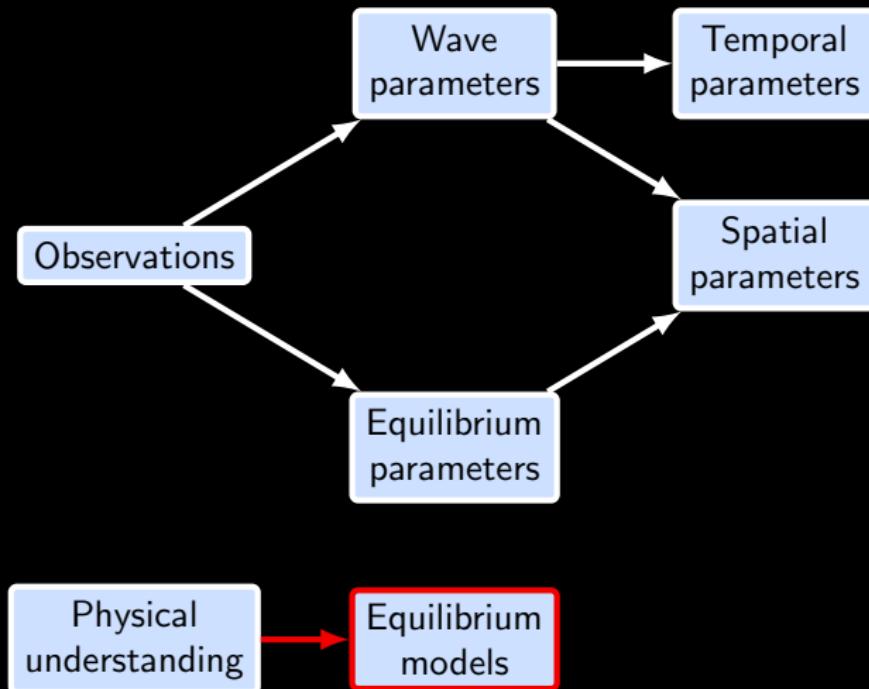
An overview



Physical
understanding

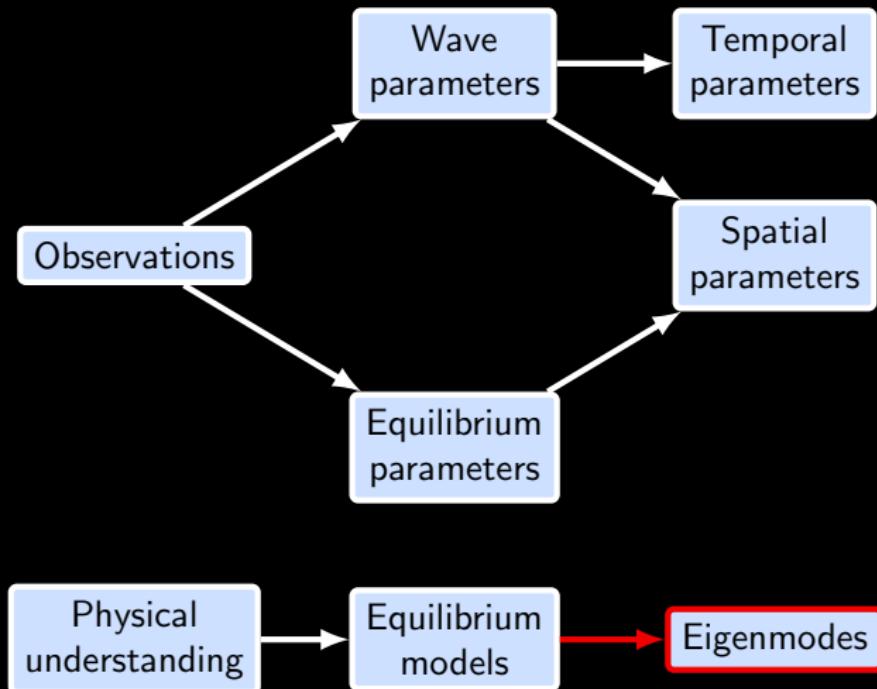
Solar magneto-seismology

An overview



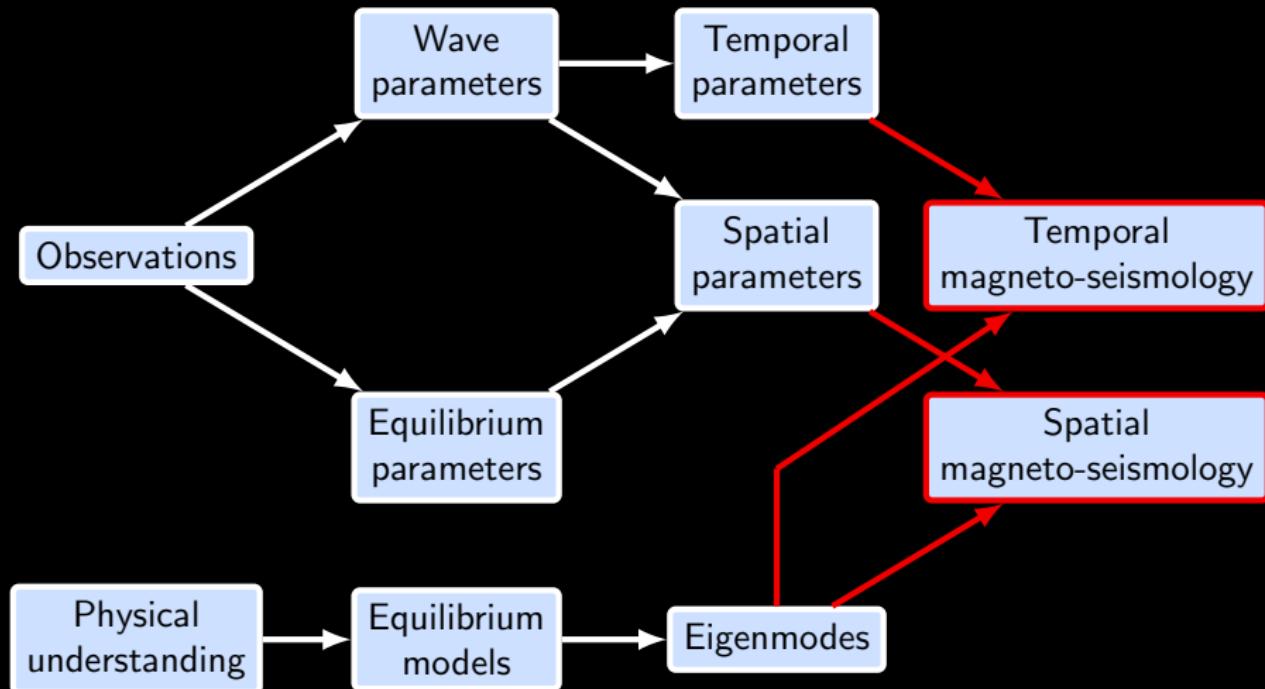
Solar magneto-seismology

An overview



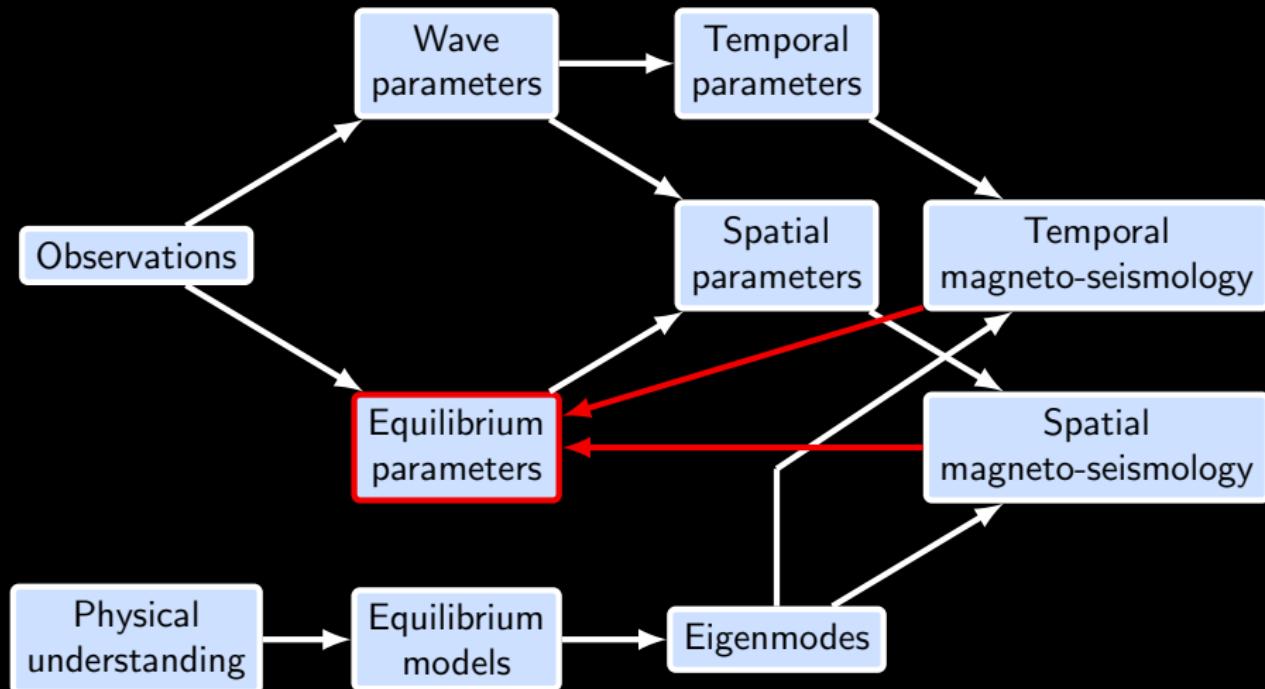
Solar magneto-seismology

An overview



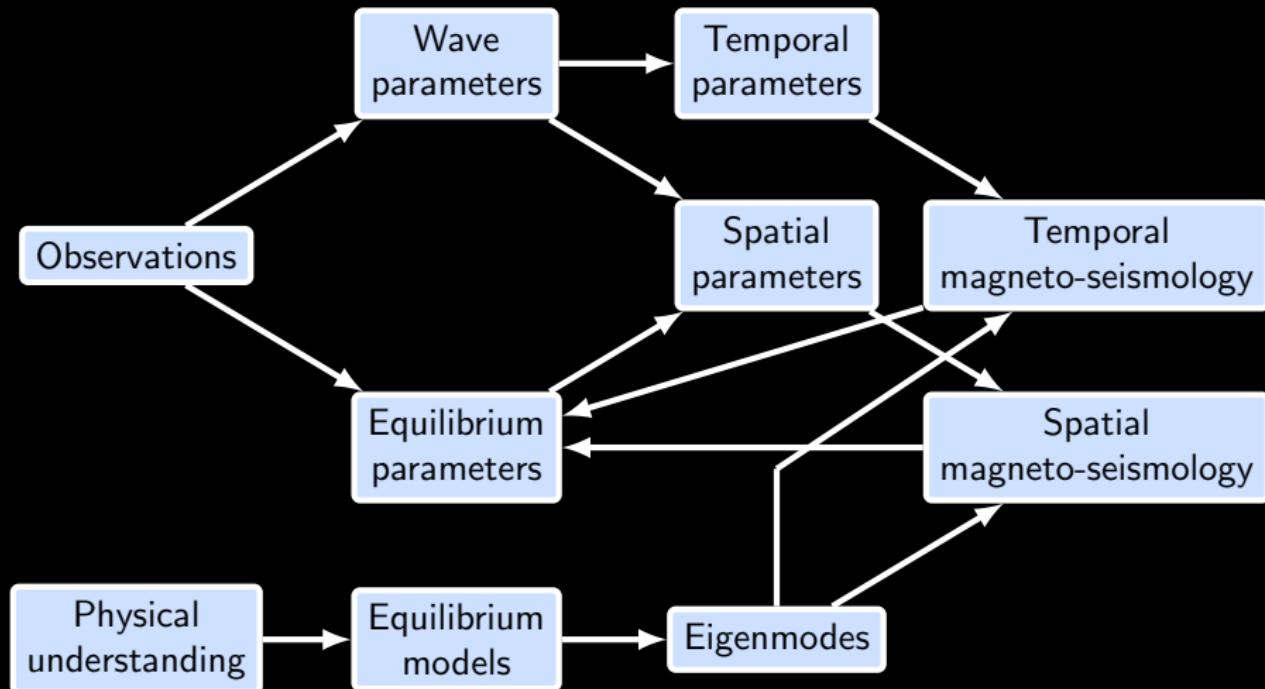
Solar magneto-seismology

An overview



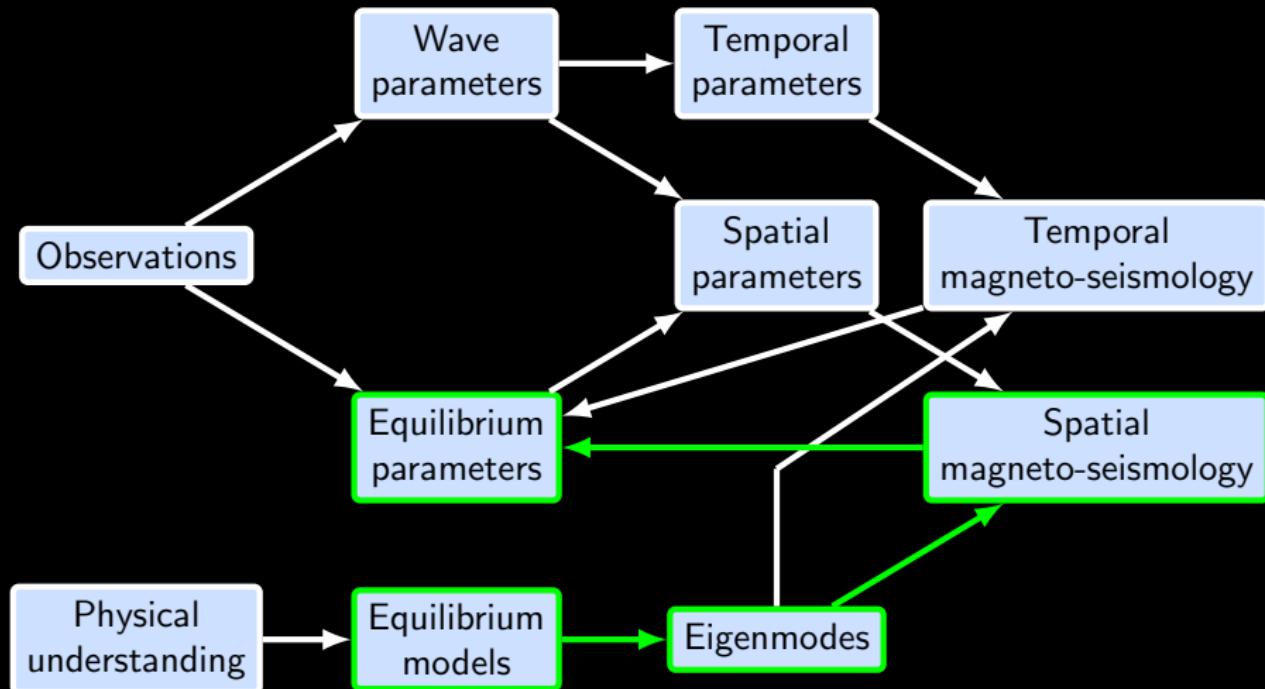
Solar magneto-seismology

An overview



Solar magneto-seismology

An overview



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

1970

1980

1990

2000



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

1970 Uchida, 1970 - **Moreton** wavefronts;

1980

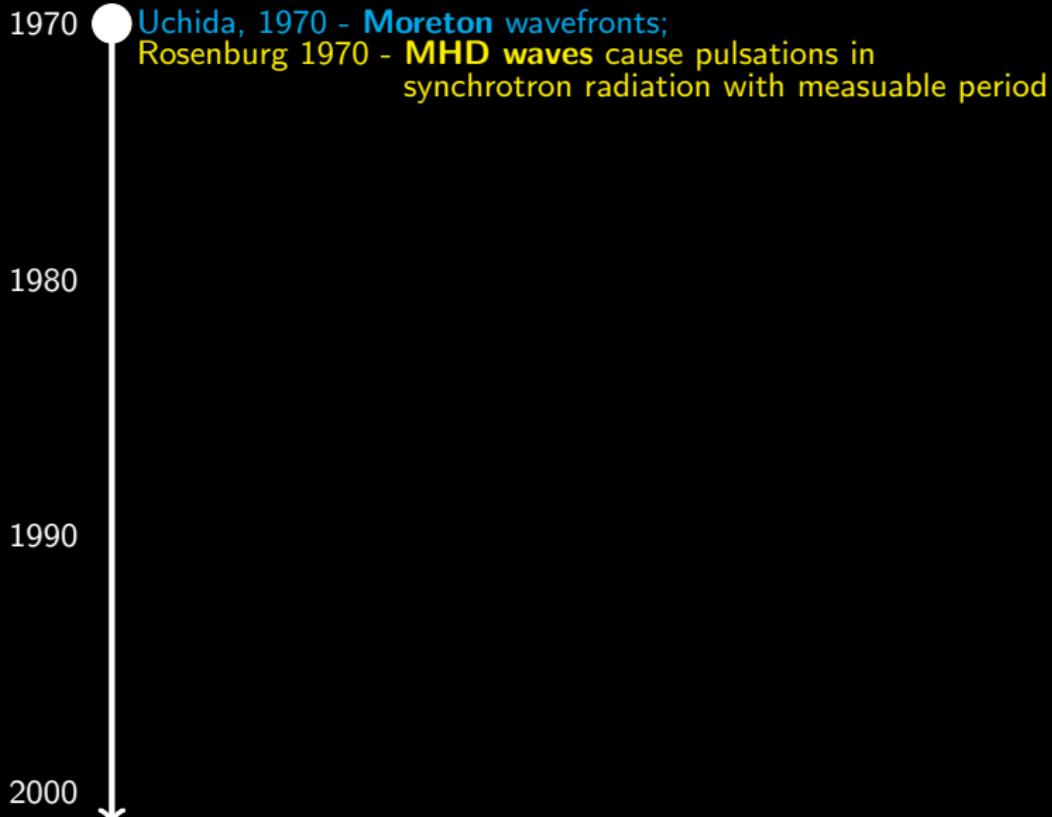
1990

2000



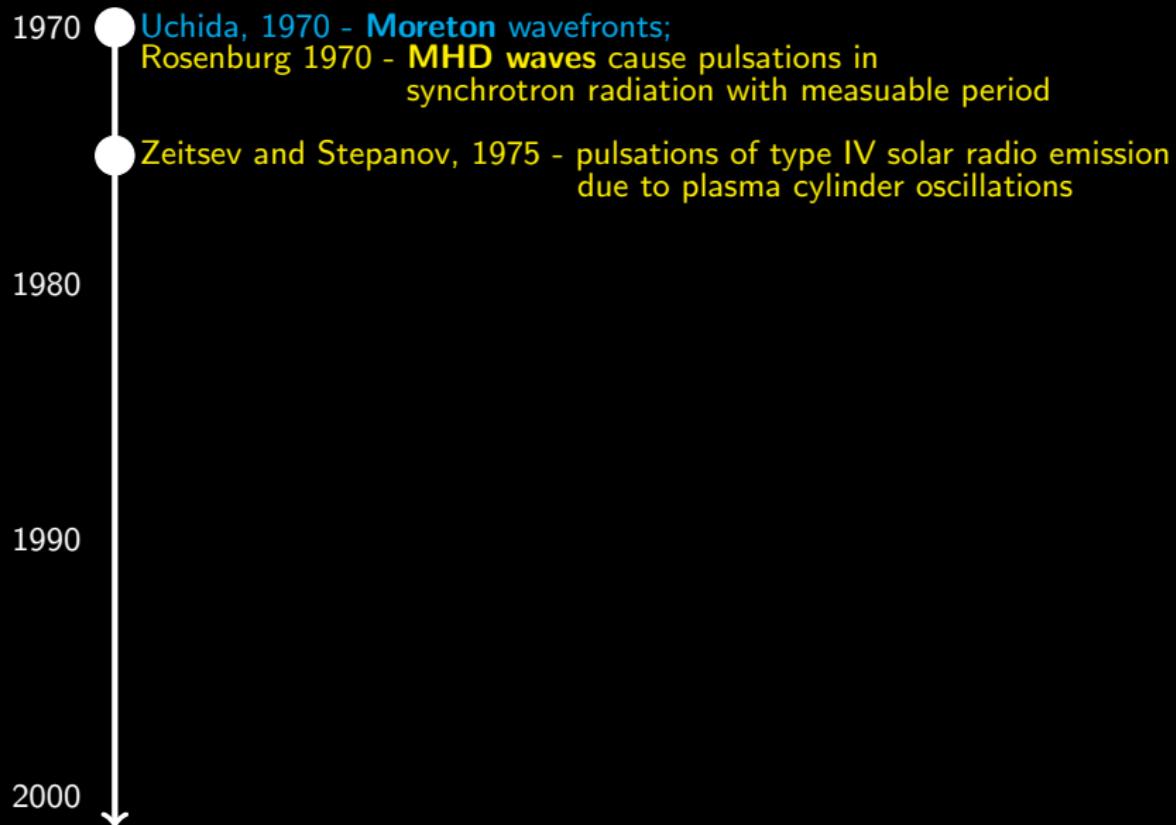
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



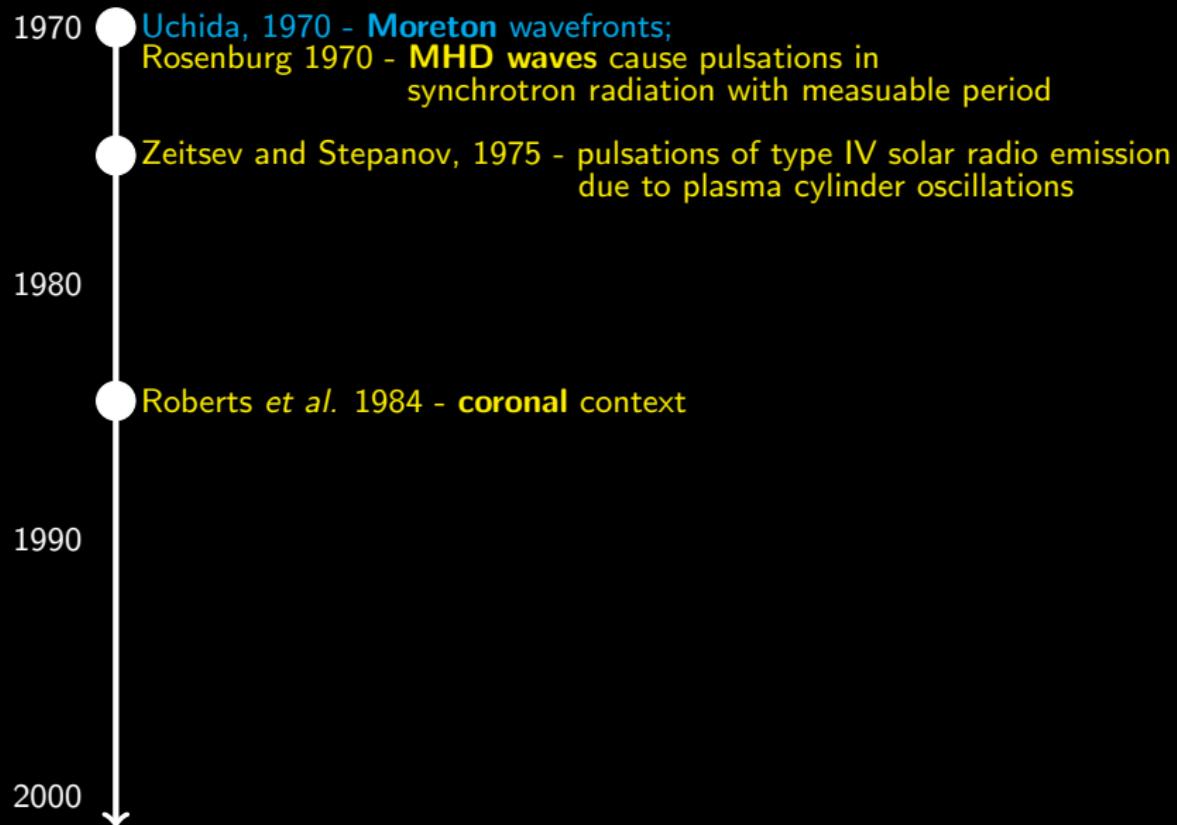
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



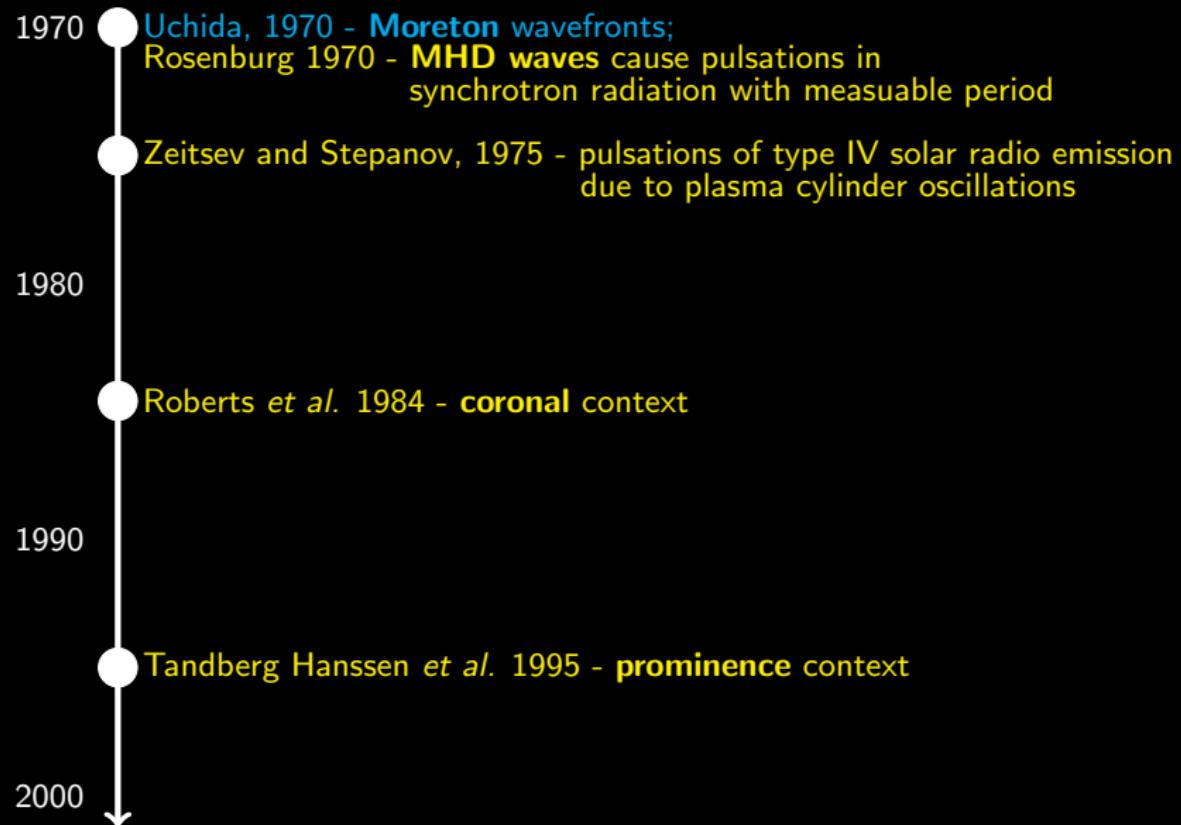
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



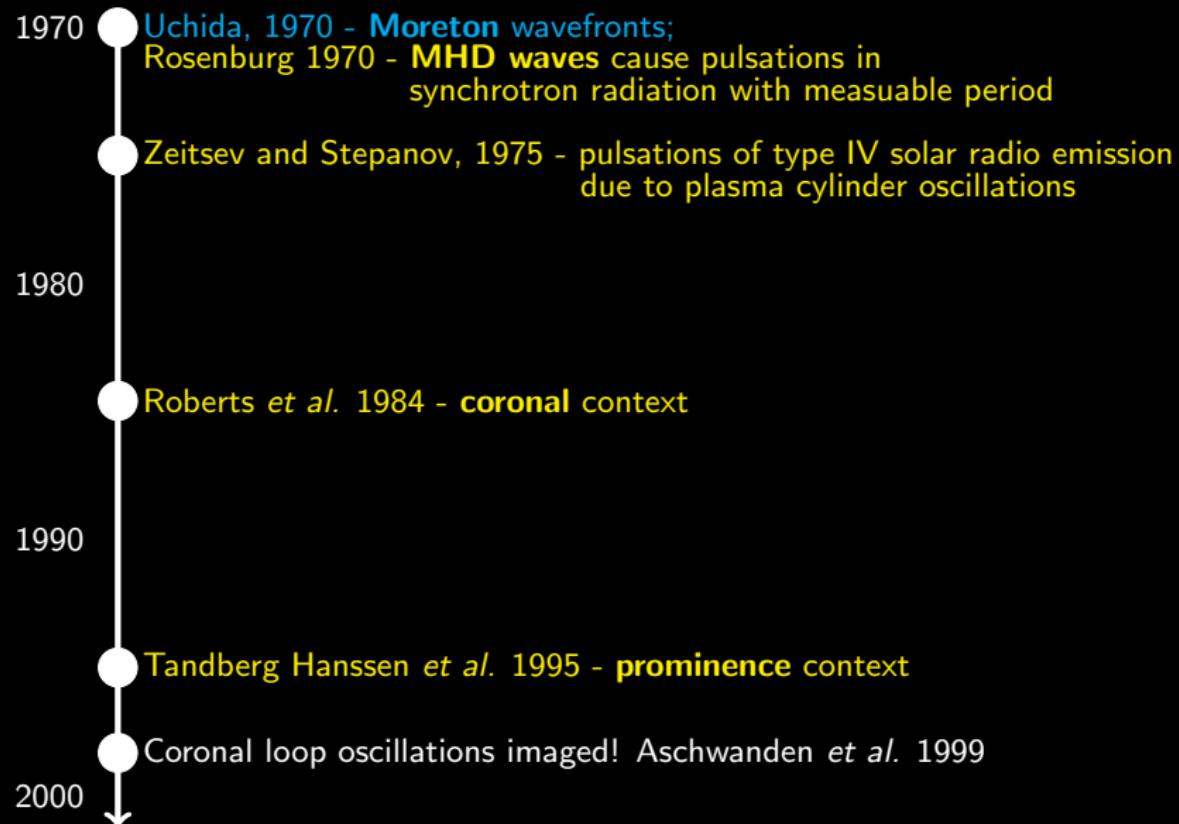
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

2000

2005

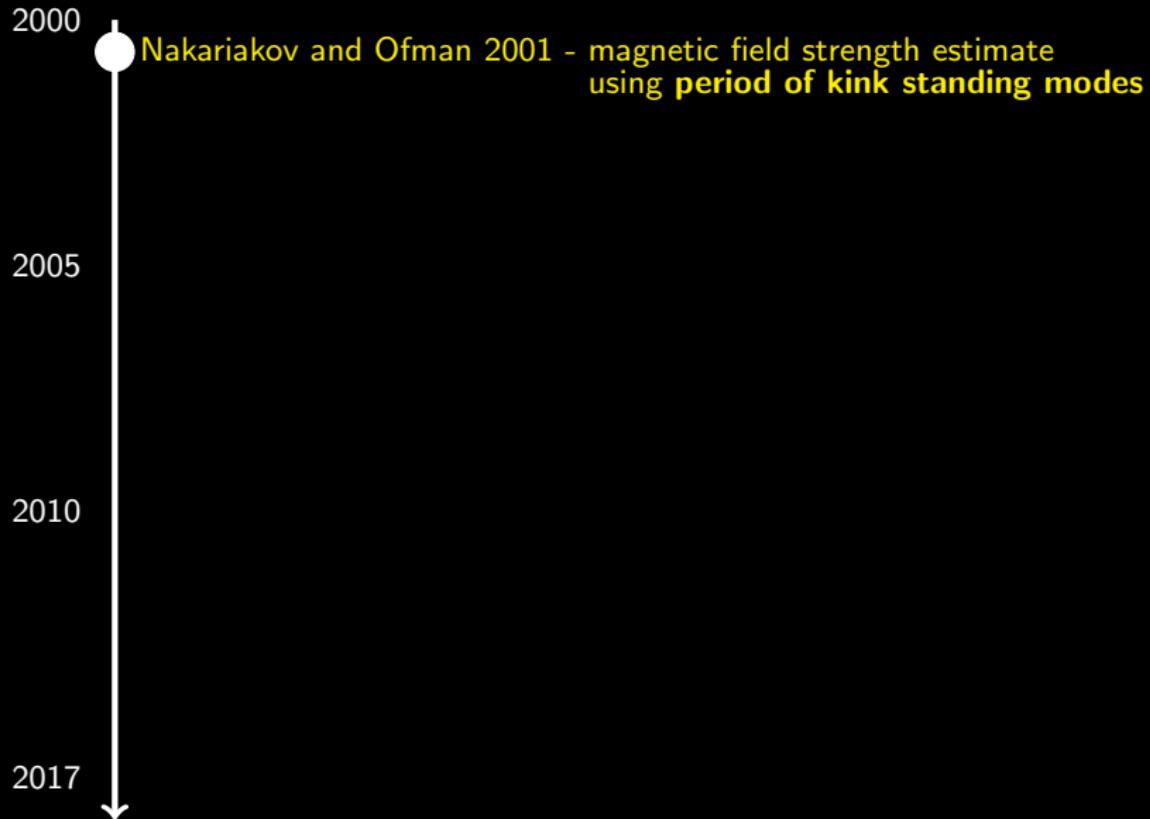
2010

2017



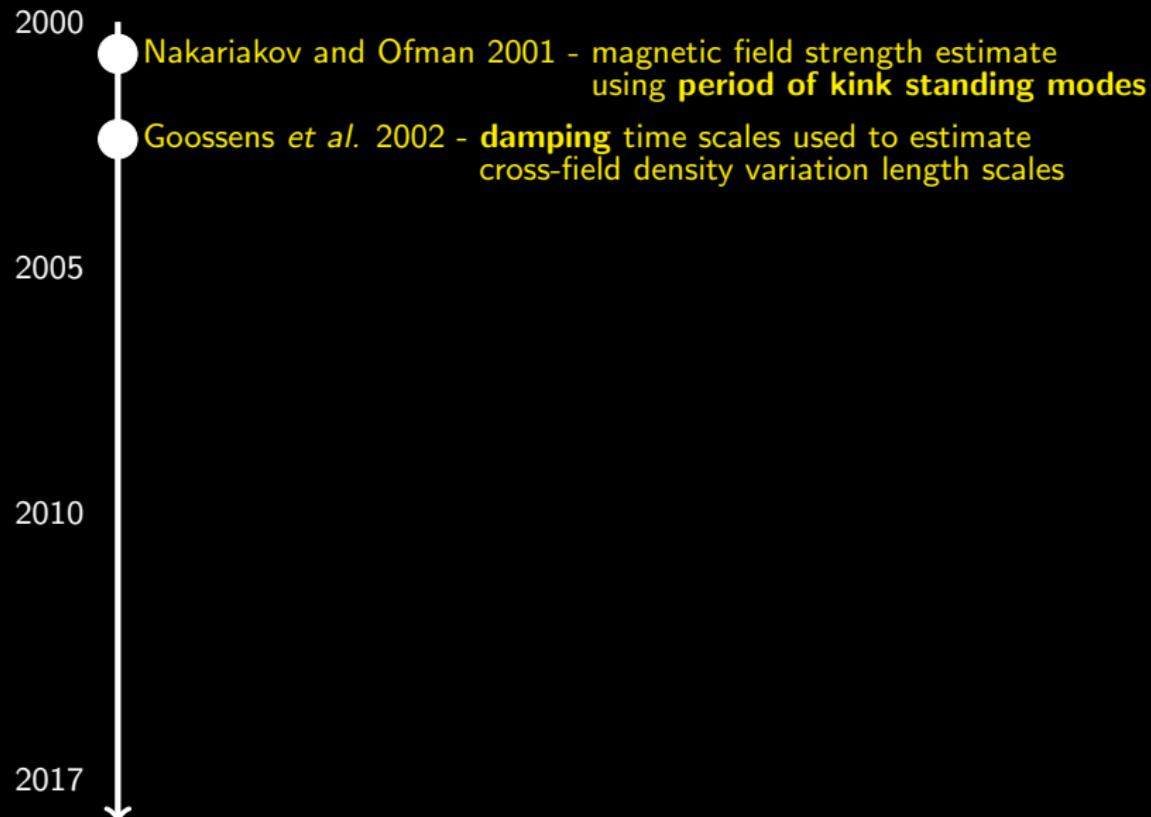
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



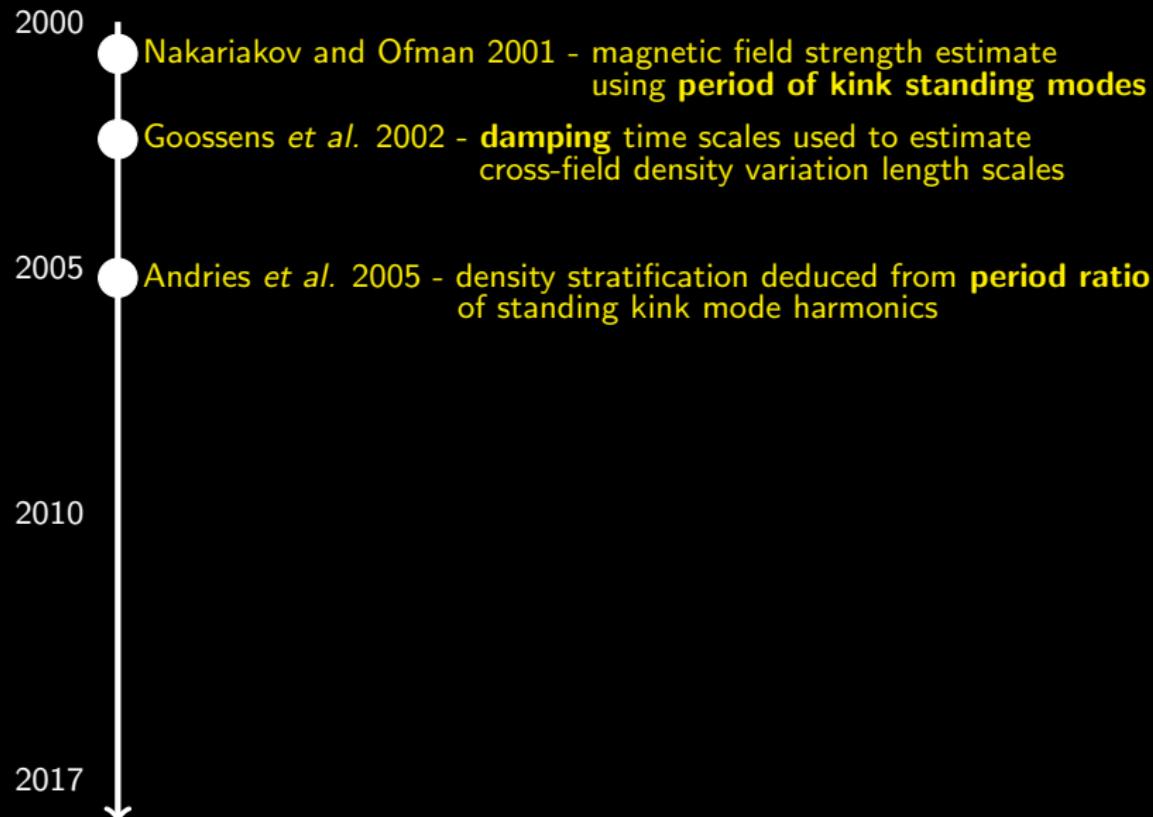
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



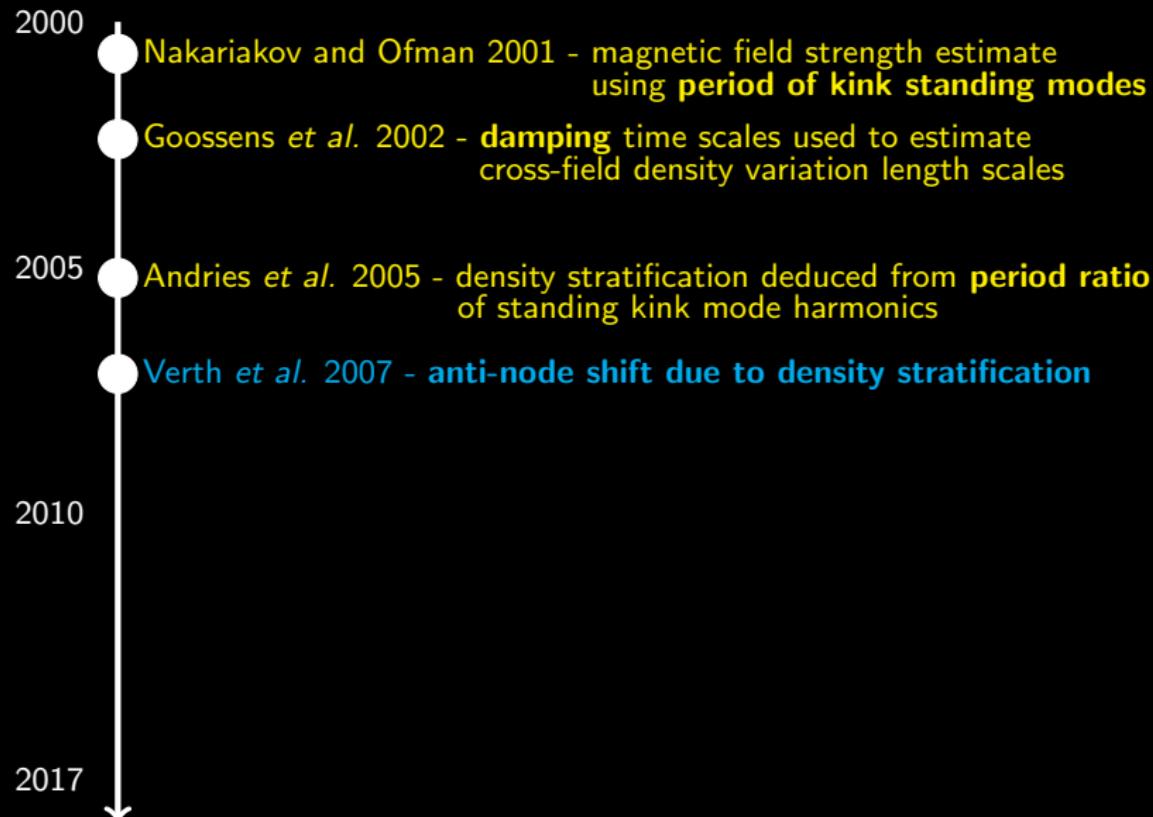
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



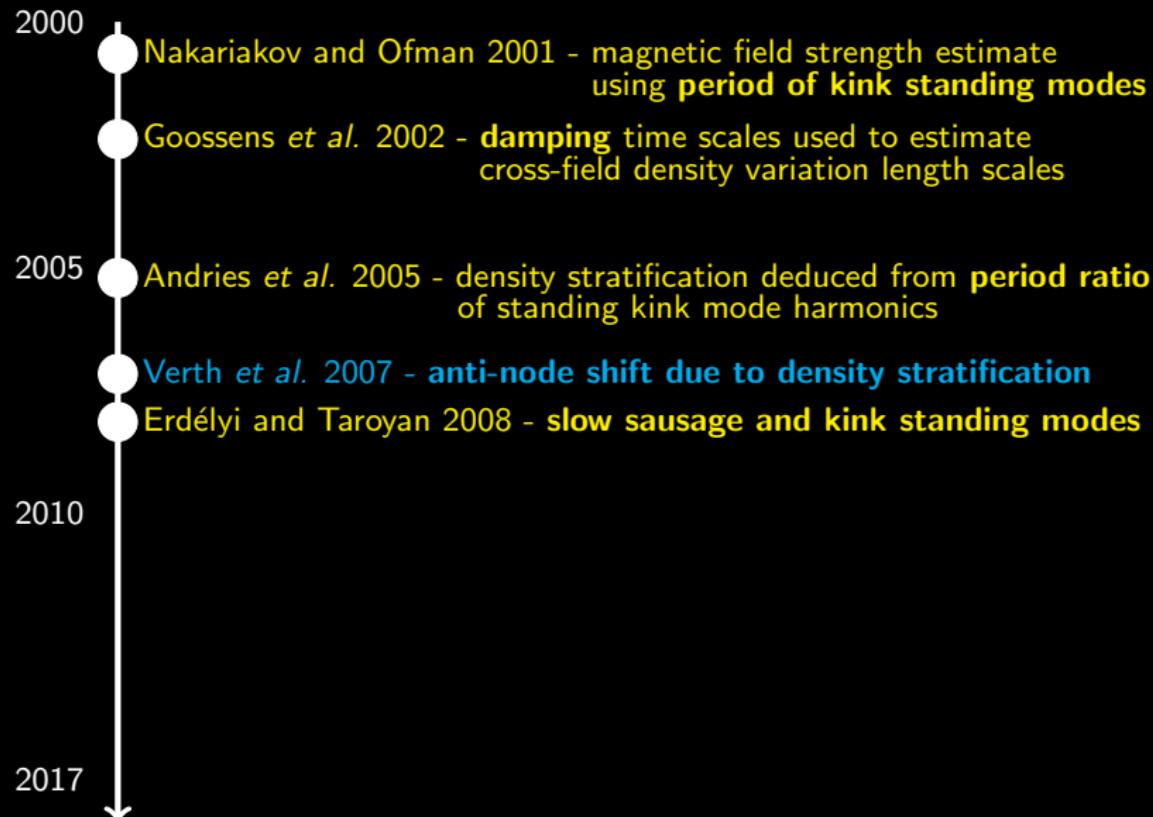
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



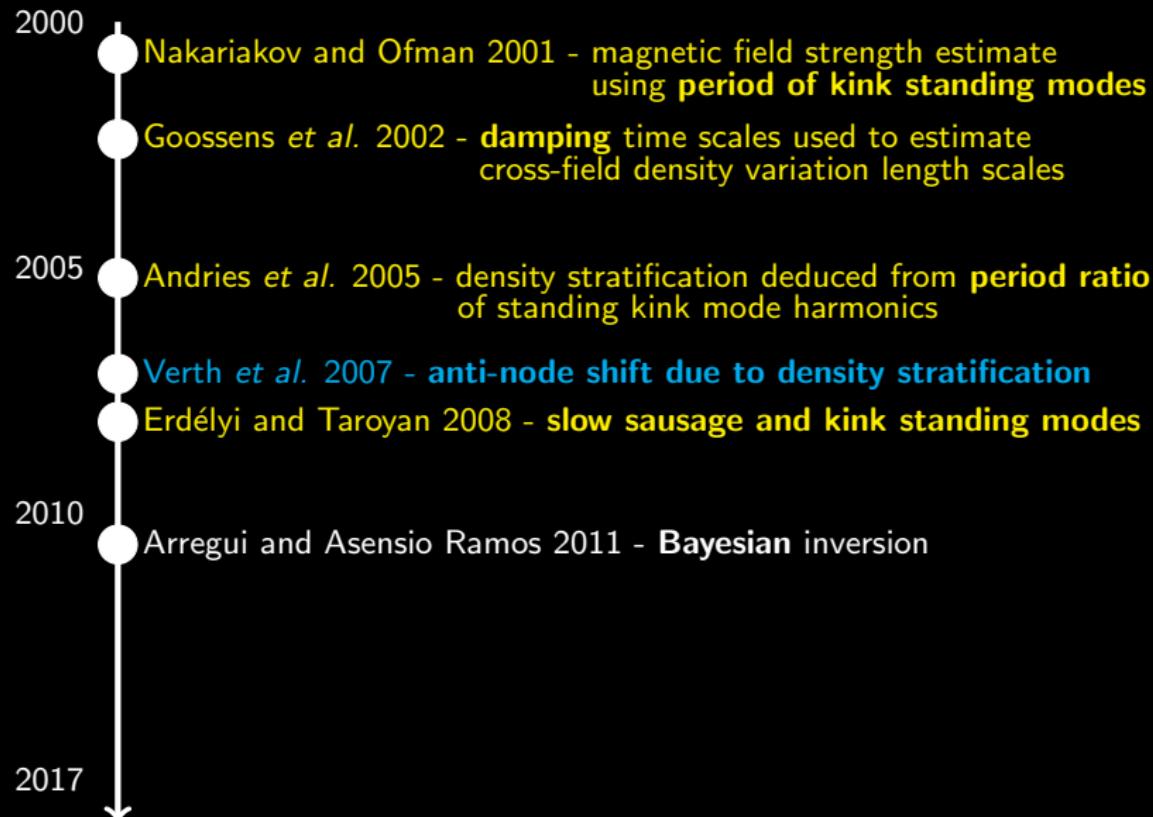
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



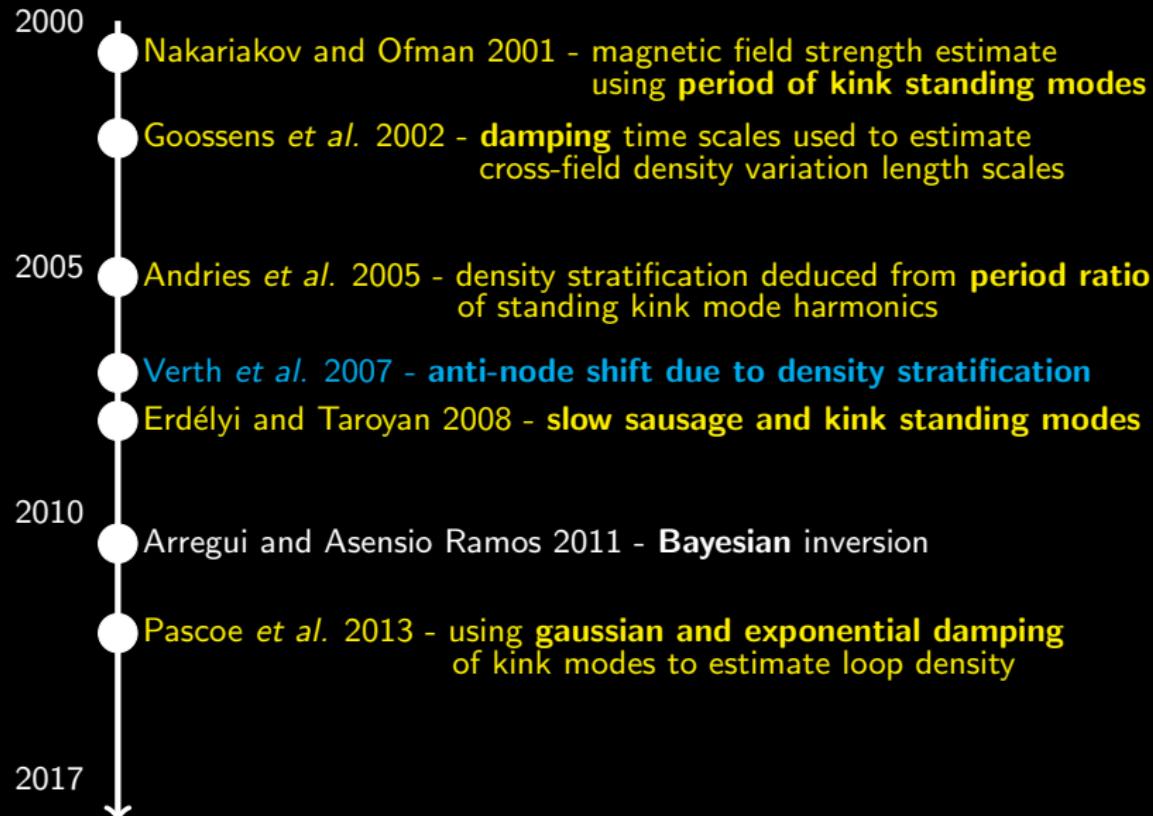
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

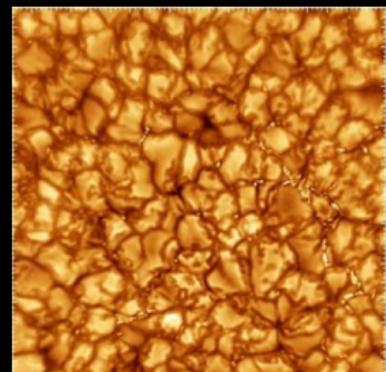
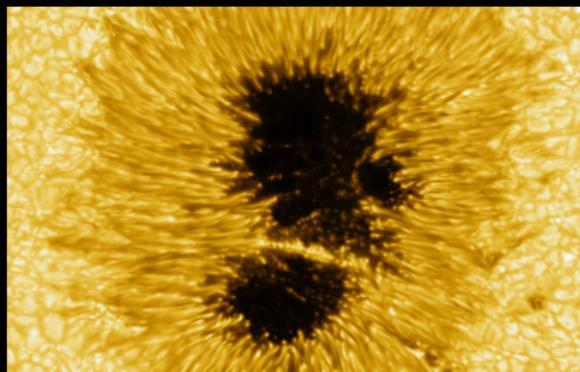
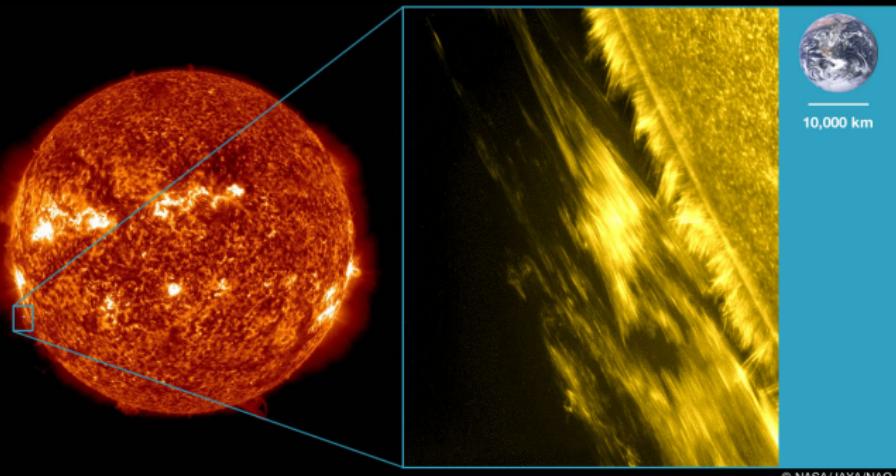
-
- The timeline shows a vertical line with circular markers at five-year intervals from 2000 to 2017. Each marker is followed by a research summary. An arrow at the bottom indicates the progression of time downwards.
- 2000
Nakariakov and Ofman 2001 - magnetic field strength estimate using **period of kink standing modes**
 - 2002
Goossens *et al.* 2002 - **damping** time scales used to estimate cross-field density variation length scales
 - 2005
Andries *et al.* 2005 - density stratification deduced from **period ratio** of standing kink mode harmonics
 - 2007
Verth *et al.* 2007 - **anti-node shift due to density stratification**
 - 2008
Erdélyi and Taroyan 2008 - **slow sausage and kink standing modes**
 - 2010
Arregui and Asensio Ramos 2011 - **Bayesian** inversion
 - 2013
Pascoe *et al.* 2013 - using **gaussian and exponential damping** of kink modes to estimate loop density
 - 2017
Magyar *et al.* - **global** dynamic coronal seismology

Outline

- 1 Introduction
 - Waves on the Sun
 - Solar magneto-seismology (SMS)
 - A brief history
- 2 SMS with asymmetric wave-guides
 - Motivation
 - Derivation
 - Mode identification
 - Amplitude ratio
 - Minimum perturbation shift
- 3 Looking ahead

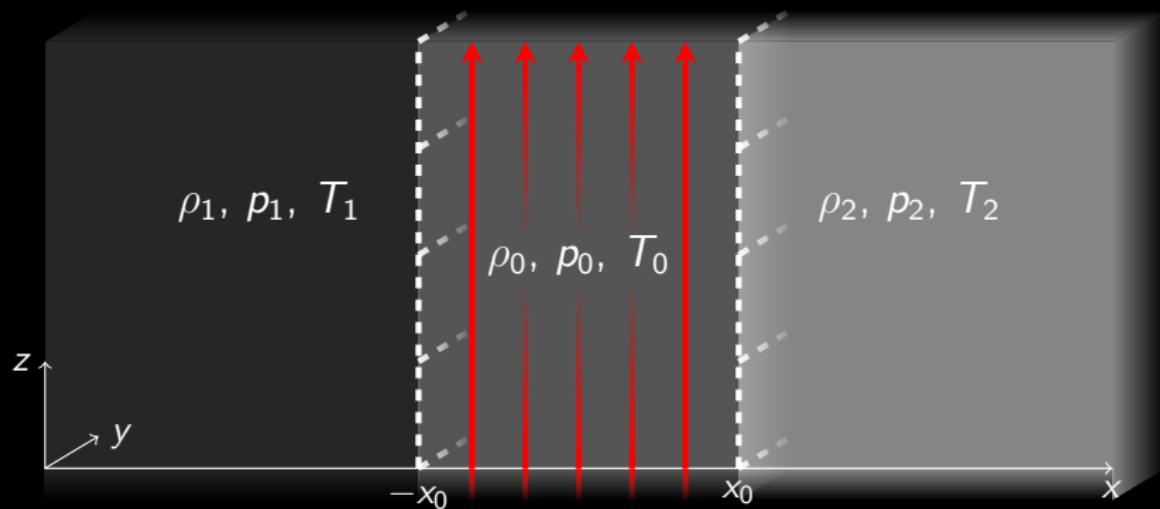
Asymmetric magnetic slab

Motivation



Asymmetric magnetic slab

Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- **Different** density and pressure on each side.

Asymmetric magnetic slab

Governing equations

Ideal MHD equations:

Conservation of:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \frac{1}{\mu} \mathbf{B} \times (\nabla \times \mathbf{B}), \quad \text{momentum}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \text{mass}$$

$$\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) = 0, \quad \text{energy}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad \text{magnetic flux}$$

\mathbf{v} = plasma velocity,

\mathbf{B} = magnetic field strength,

ρ = density,

p = pressure,

μ = magnetic permeability,

γ = adiabatic index.

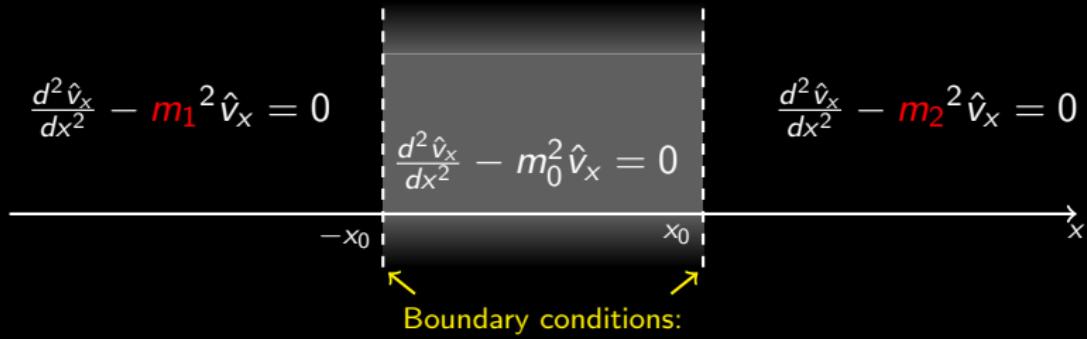
Asymmetric magnetic slab

Fourier decomposition

Look for *plane wave* solutions of the form:

$$v_x(\mathbf{x}, t) = \hat{v}_x(x) e^{i(kz - \omega t)}, \quad v_y(\mathbf{x}, t) = 0, \quad v_z(\mathbf{x}, t) = \hat{v}_z(x) e^{i(kz - \omega t)},$$

to arrive at the following ODEs:



$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad m_{1,2}^2 = k^2 - \frac{\omega^2}{c_{1,2}^2},$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}, \quad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

Asymmetric magnetic slab

Eigenmodes

Dispersion relation:

$$\frac{\omega^4 m_0^2}{k^2 v_A^2 - \omega^2} + \frac{\rho_0}{\rho_1} \textcolor{red}{m_1} \frac{\rho_0}{\rho_2} \textcolor{red}{m_2} (k^2 v_A^2 - \omega^2) \\ - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} \textcolor{red}{m_1} + \frac{\rho_0}{\rho_2} \textcolor{red}{m_2} \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

See **Allcock** and Erdélyi, 2017.

$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad \textcolor{red}{m_{1,2}}^2 = k^2 - \frac{\omega^2}{\textcolor{red}{c_{1,2}}^2},$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}, \quad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

Eigenmodes

Symmetric kink surface mode

Eigenmodes

Quasi-kink surface mode

Eigenmodes

Symmetric sausage surface mode

Eigenmodes

Quasi-sausage surface mode

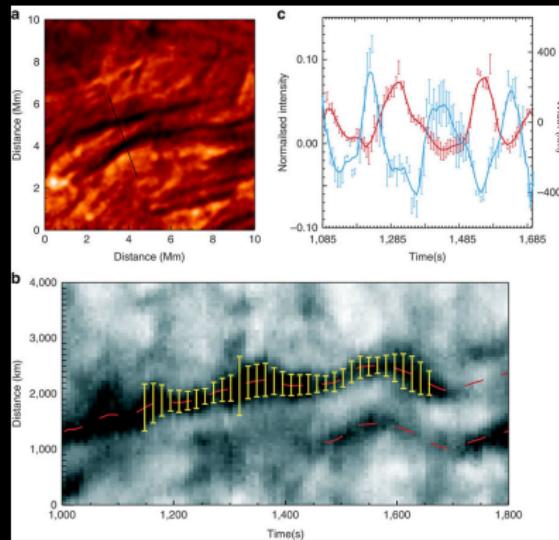
Eigenmodes

Body modes

Asymmetric magnetic slab

Mode identification

Superposition of symmetric kink and sausage modes...

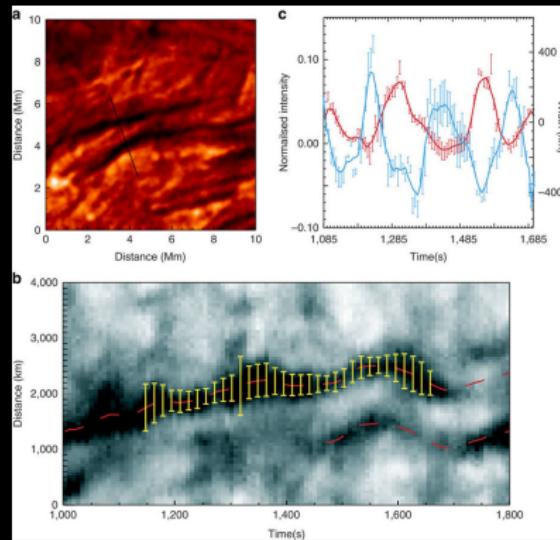


Morton *et al.* 2012

Asymmetric magnetic slab

Mode identification

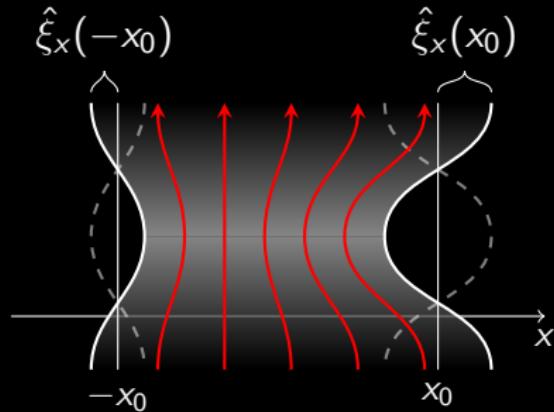
Superposition of symmetric kink and sausage modes...



Morton *et al.* 2012

or asymmetric kink mode?

Amplitude ratio



Amplitude ratio

$$R_A := \frac{\hat{\xi}_x(x_0)}{\hat{\xi}_x(-x_0)}$$

(Top = quasi-kink
Bottom = quasi-sausage)

$$= (+) \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 v_A^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 (\tanh \frac{\rho_0}{\coth}) (m_0 x_0)}{(k^2 v_A^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 (\tanh \frac{\rho_0}{\coth}) (m_0 x_0)}$$

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .

Analytical inversion

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .

| Mode | Approximation of $k^2 v_A^2 / \omega^2$ using amplitude ratio, R_A | | |
|---------|--------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|
| | Thin slab | Incompressible | Low-beta |
| Sausage | $1 + \frac{1}{x_0} \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} + \frac{\rho_1}{\rho_0 m_1}}{R_A + 1} \right)$ | $1 + \left(\frac{R_A \frac{\rho_2 + \rho_1}{\rho_0}}{R_A + 1} \right) \coth kx_0$ | $1 + k \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} + \frac{\rho_1}{\rho_0 m_1}}{R_A + 1} \right) \coth kx_0$ |
| Kink | $1 + k^2 x_0 \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} - \frac{\rho_1}{\rho_0 m_1}}{R_A - 1} \right)$ | $1 + \left(\frac{R_A \frac{\rho_2 - \rho_1}{\rho_0}}{R_A - 1} \right) \tanh kx_0$ | $1 + k \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} - \frac{\rho_1}{\rho_0 m_1}}{R_A - 1} \right) \tanh kx_0$ |

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .

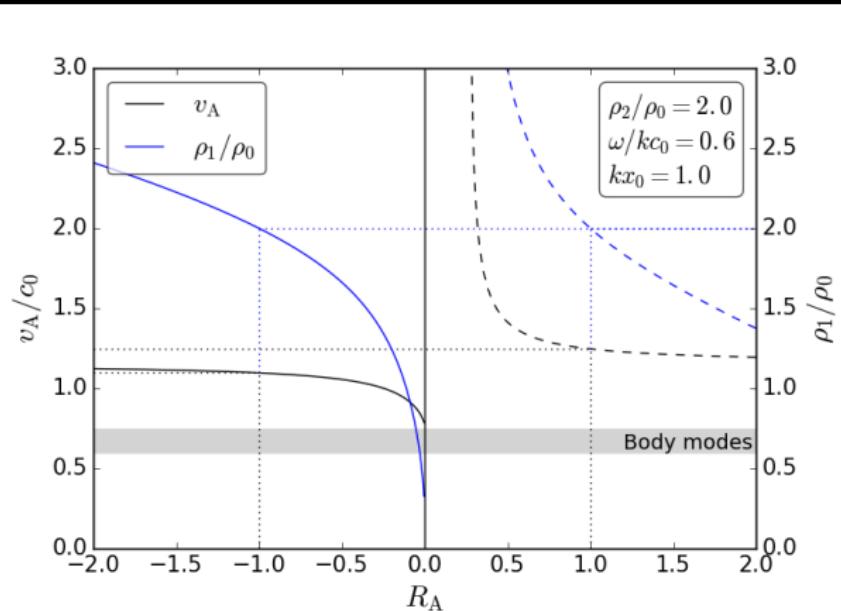
Numerical inversion

Amplitude ratio

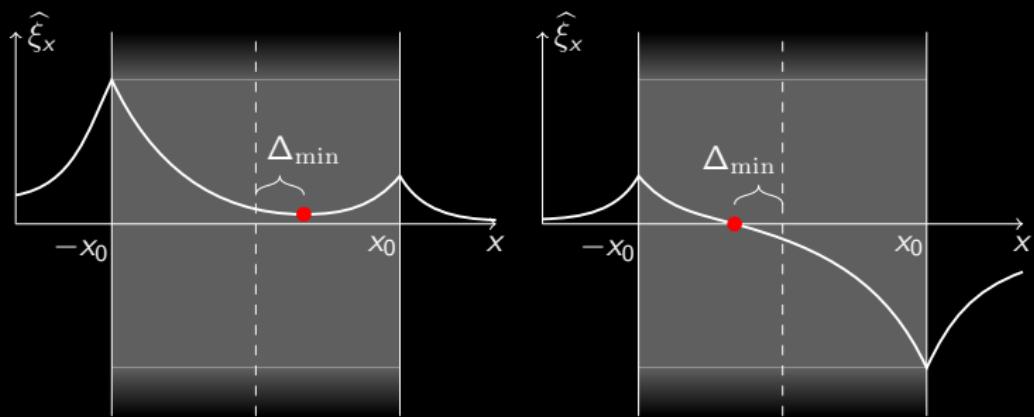
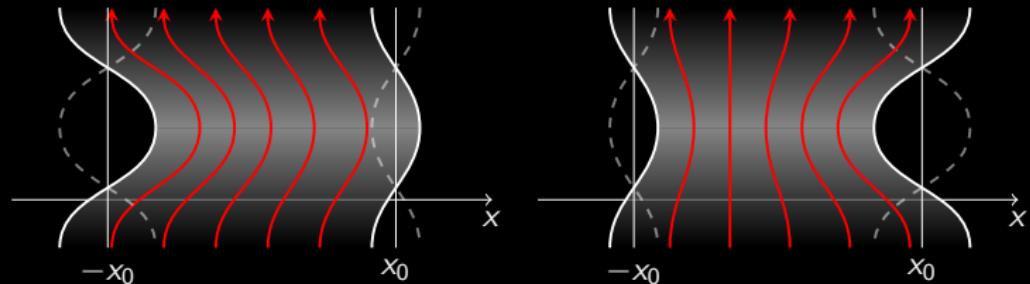
Parameter inversion

Parameter inversion

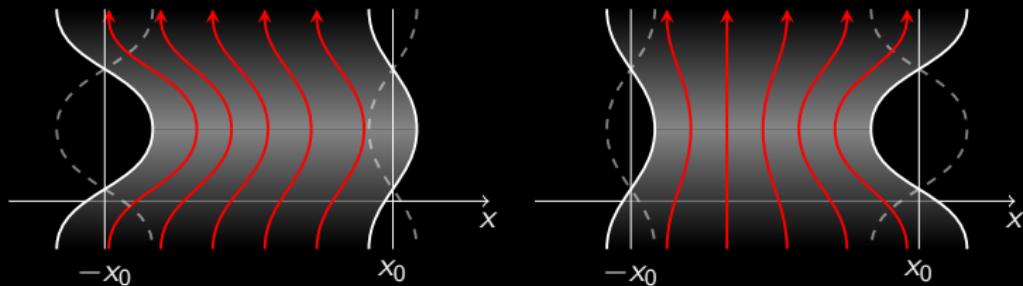
- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .



Minimum perturbation shift



Minimum perturbation shift



Quasi-kink:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}(D)$$

Quasi-sausage:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1} \left(\frac{1}{D} \right)$$

where $D = \frac{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} \tanh(m_0 x_0) - \omega^2 m_0}{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \tanh(m_0 x_0)}$

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .

Analytical inversion

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .

| Mode | Approximation of $k^2 v_A^2 / \omega^2$ using minimum perturbation shift, Δ_{\min} | | |
|---------------|-----------------------------------------------------------------------------------------------------------|----------------------------------------------------------|--------------------------------------------------------------|
| | Thin slab | Incompressible | Low-beta |
| Quasi-sausage | $\frac{\rho_1}{\rho_0 m_1} (x_0 + \Delta_{\min}) + \frac{1}{1 + (\omega/kc_0)^2} + k^2 x_0 \Delta_{\min}$ | $1 + \frac{\rho_1}{\rho_0} \tanh k(x_0 + \Delta_{\min})$ | $1 + \frac{k\rho_1}{m_1\rho_0} \tanh k(x_0 + \Delta_{\min})$ |
| Quasi-kink | $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, defined in text | $1 + \frac{\rho_1}{\rho_0} \coth k(x_0 + \Delta_{\min})$ | $1 + \frac{k\rho_1}{m_1\rho_0} \coth k(x_0 + \Delta_{\min})$ |

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .

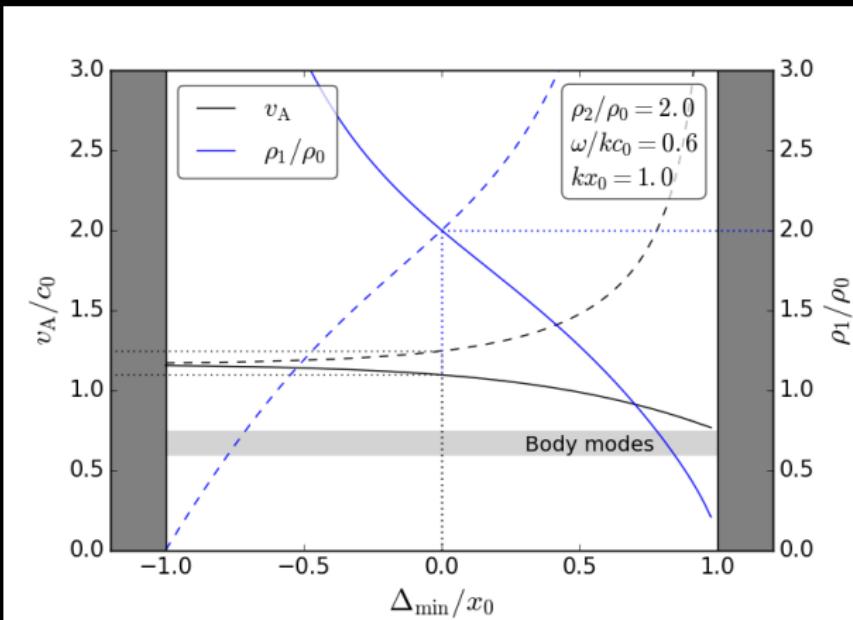
Numerical inversion

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .

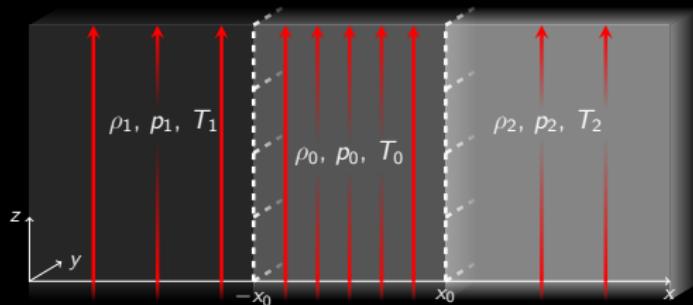


Outline

- 1 Introduction
 - Waves on the Sun
 - Solar magneto-seismology (SMS)
 - A brief history
- 2 SMS with asymmetric wave-guides
 - Motivation
 - Derivation
 - Mode identification
 - Amplitude ratio
 - Minimum perturbation shift
- 3 Looking ahead

Further work

Add **magnetic field** outside the slab to model coronal structures.
See **Zsámberger, Allcock, and Erdélyi**, recommended for publication in ApJ.



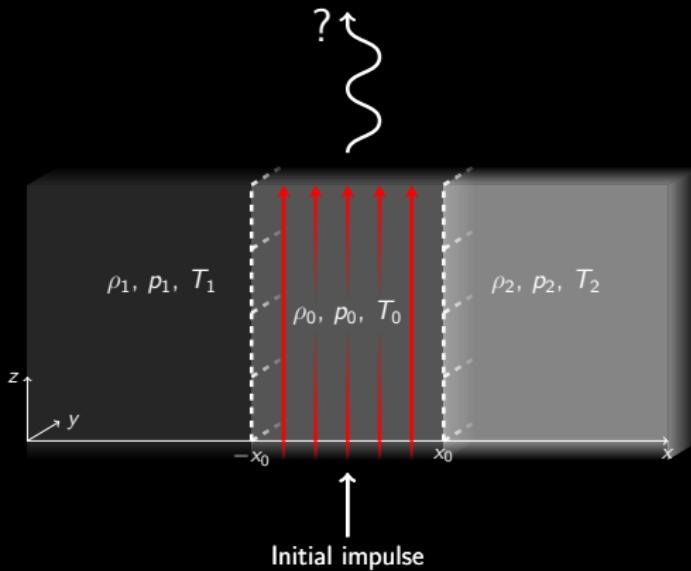
Key result for mode identification:

There can exist **quasi-symmetric modes** - where the oscillations of each interface have equal amplitude - even with asymmetric background.

Symmetric mode $\not\Rightarrow$ symmetric equilibrium

Future work

Initial value problem



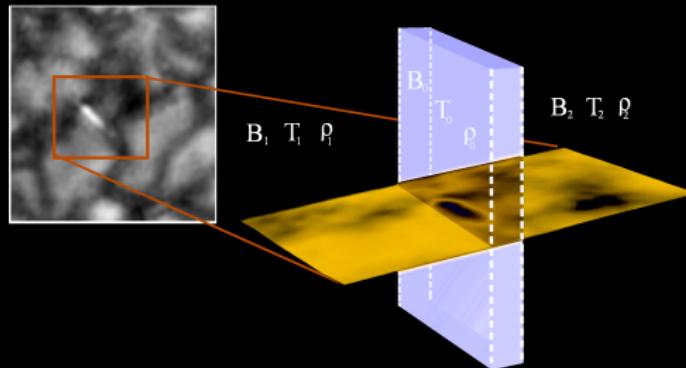
- Determine approximate time for set-up of asymmetric modes.
- Compare this to typical lifetime of solar asymmetric structures.
- Include transition layer to investigate resonant absorption.

Future work

Application

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,



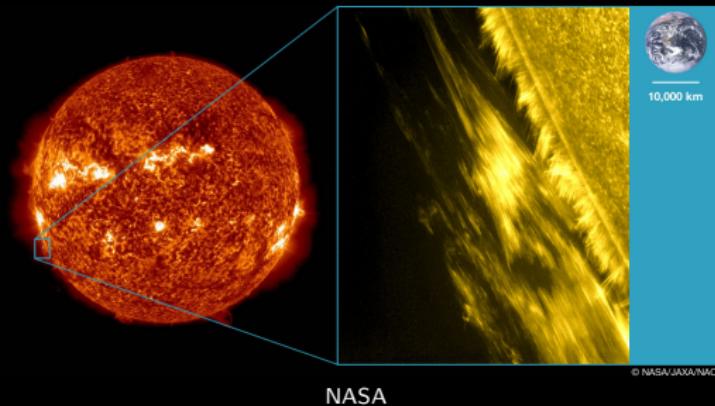
Adaptation of Liu et al., 2017, by Zsámberger et al.

Future work

Application

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,
- **Prominences**,

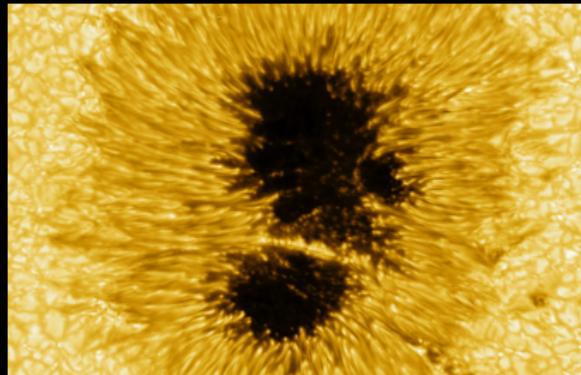


Future work

Application

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,
- **Prominences**,
- Sunspot **light walls**.



Max Planck Institute for Solar System Research

Thank you



matthew_allcock



The
University
Of
Sheffield.

