

Magneto-acoustic waves in asymmetric solar waveguides

Progress in spatial magneto-seismology



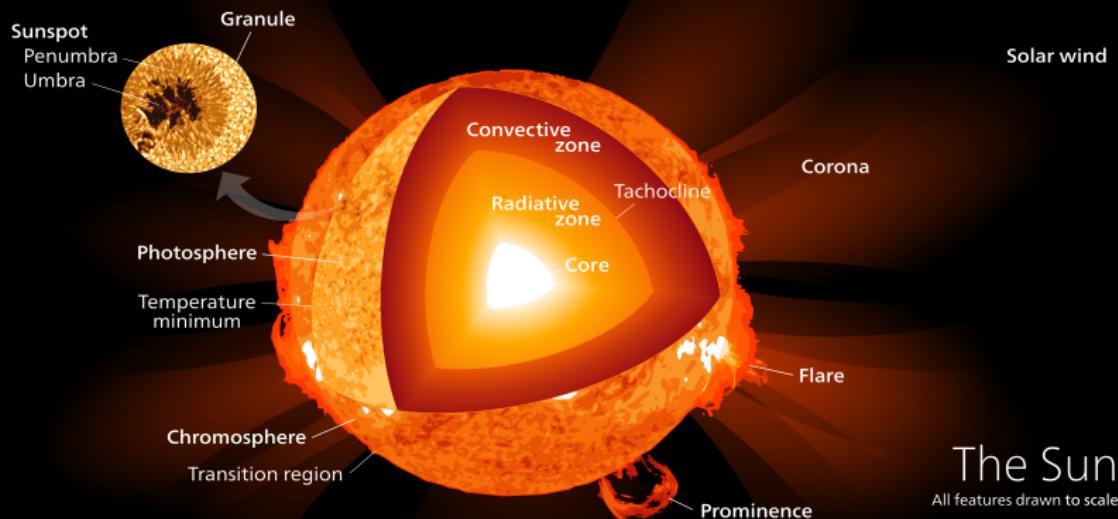
Matthew Allcock
and
Robertus Erdélyi



The
University
Of
Sheffield.



The layers of an onion



Magnetohydrodynamic waves

Ubiquitous in the solar atmosphere

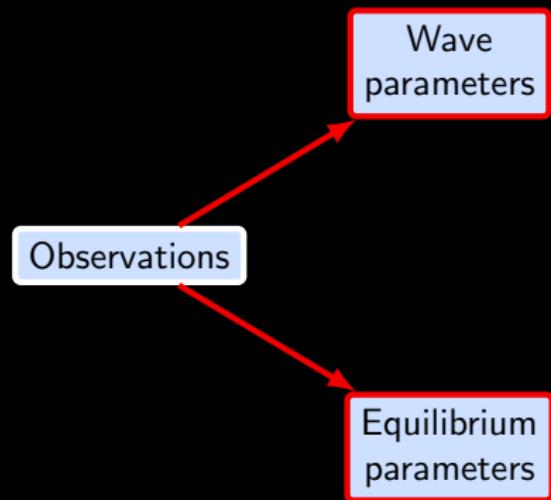
Magnetohydrodynamic waves

Diagnosing information about solar plasma

Observations

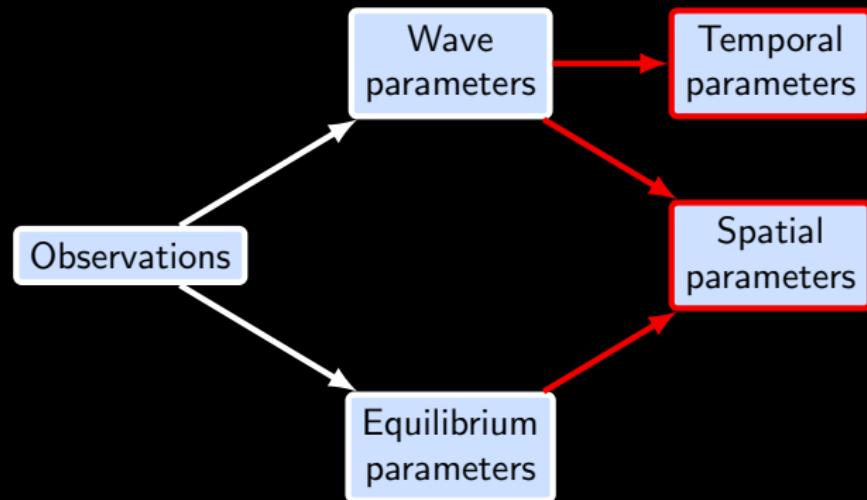
Magnetohydrodynamic waves

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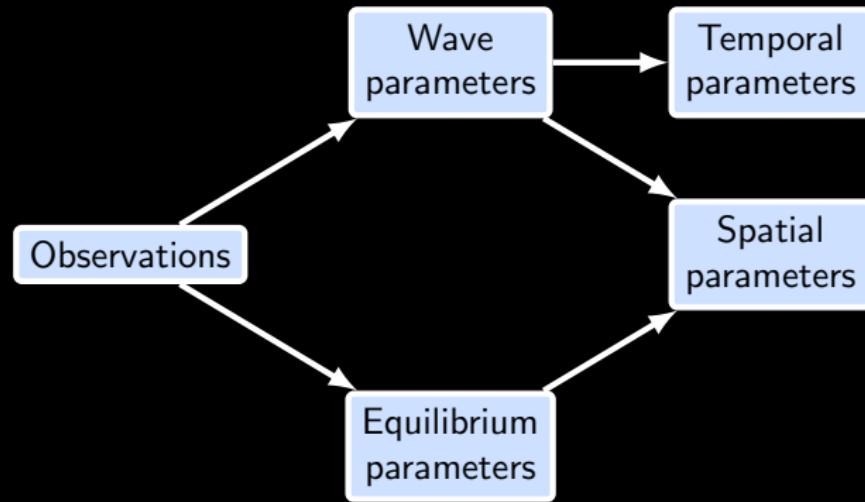
Magnetohydrodynamic waves

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Magnetohydrodynamic waves

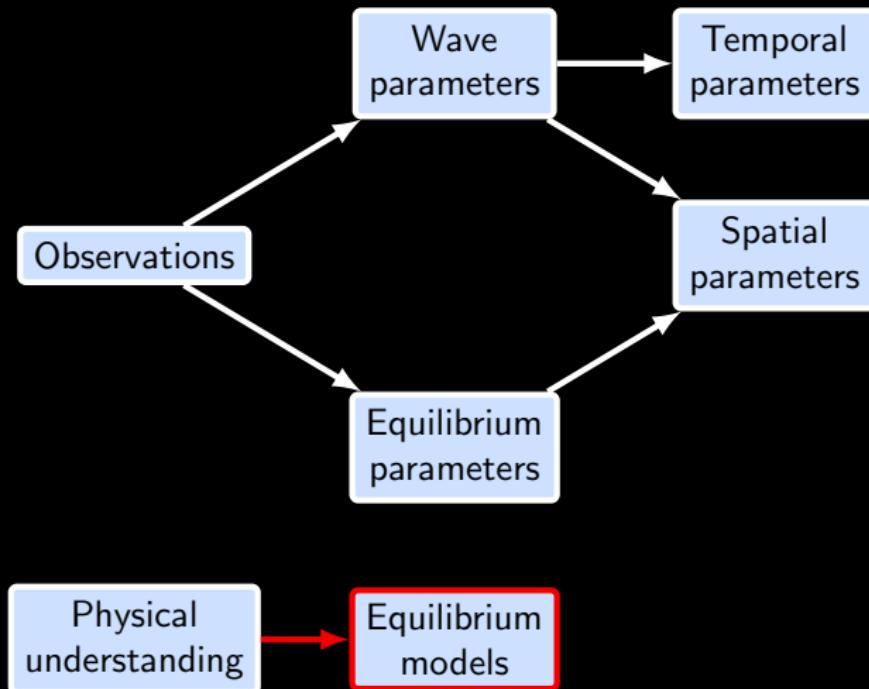
Diagnosing information about solar plasma



Physical
understanding

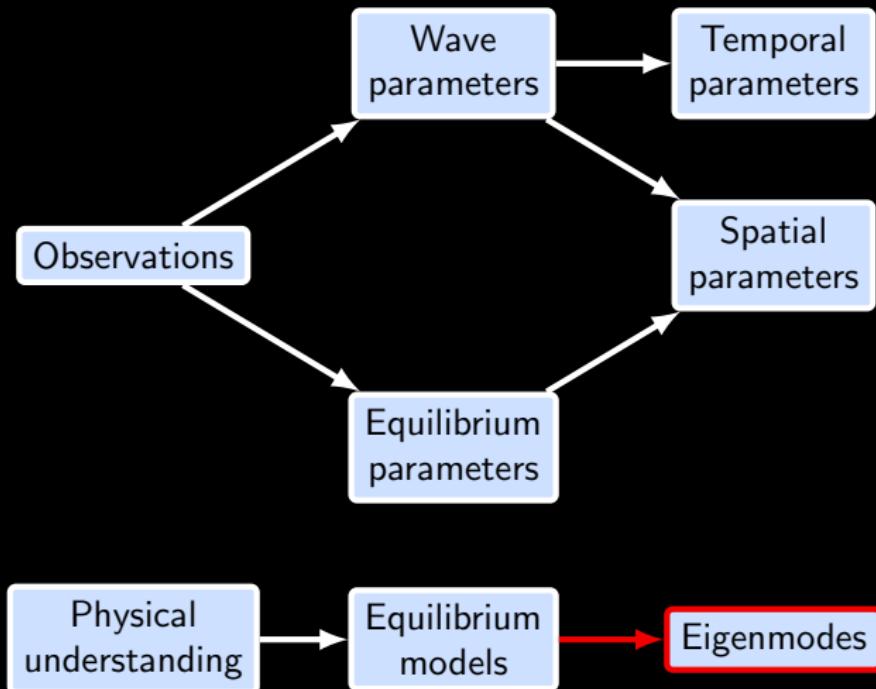
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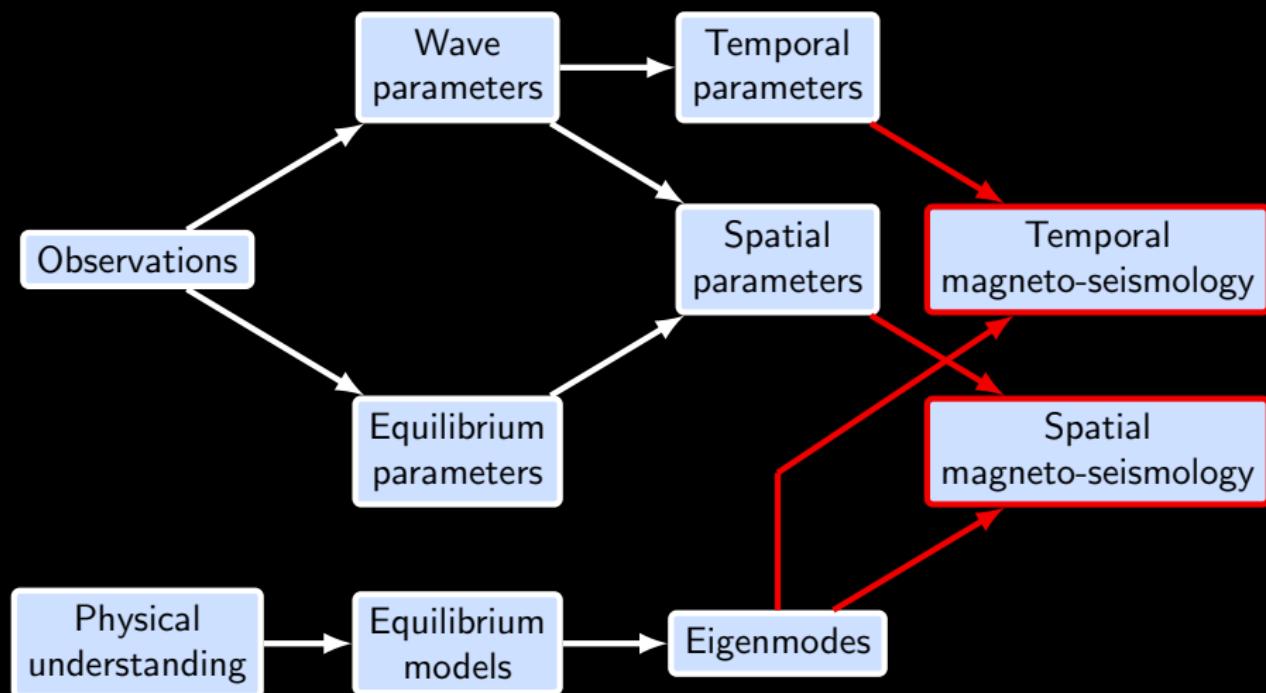
Magnetohydrodynamic waves

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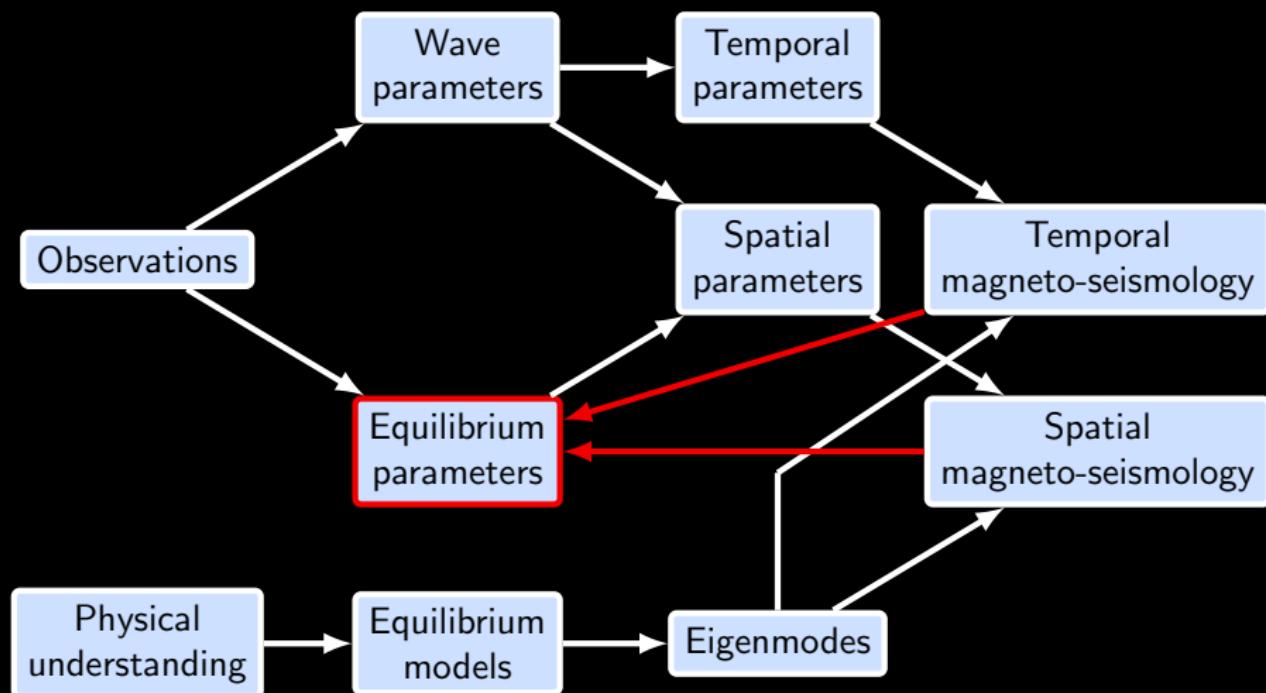
Magnetohydrodynamic waves

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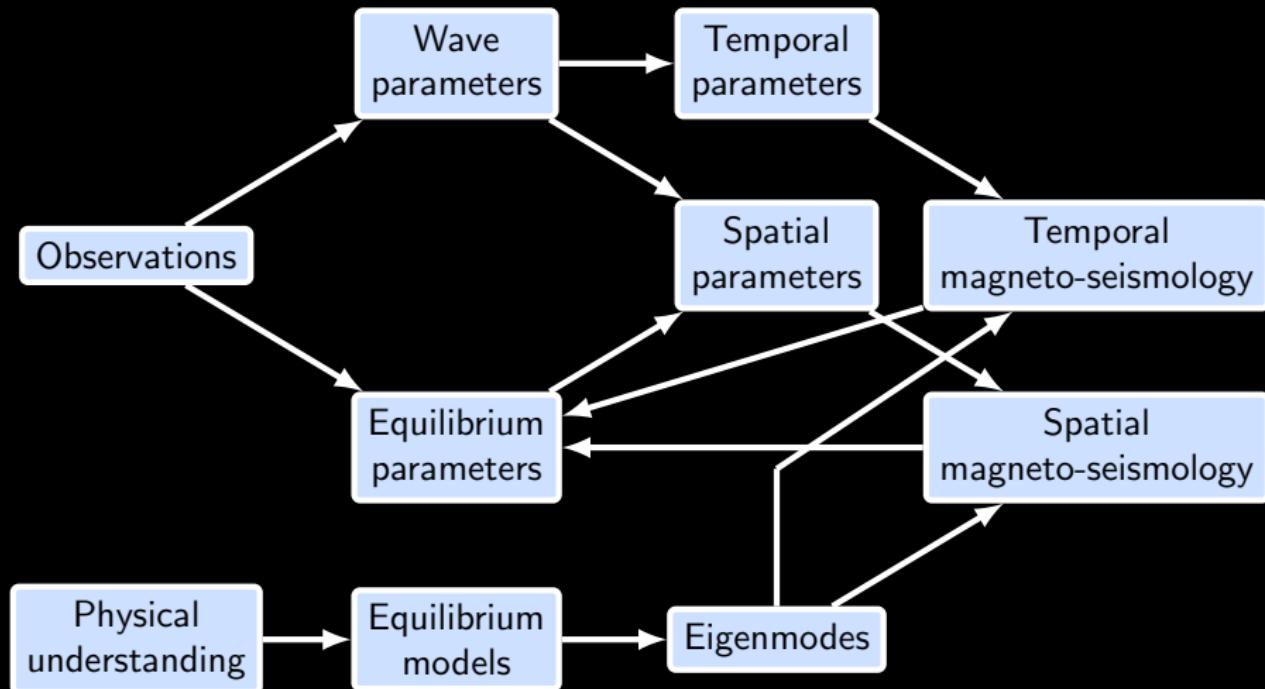
Magnetohydrodynamic waves

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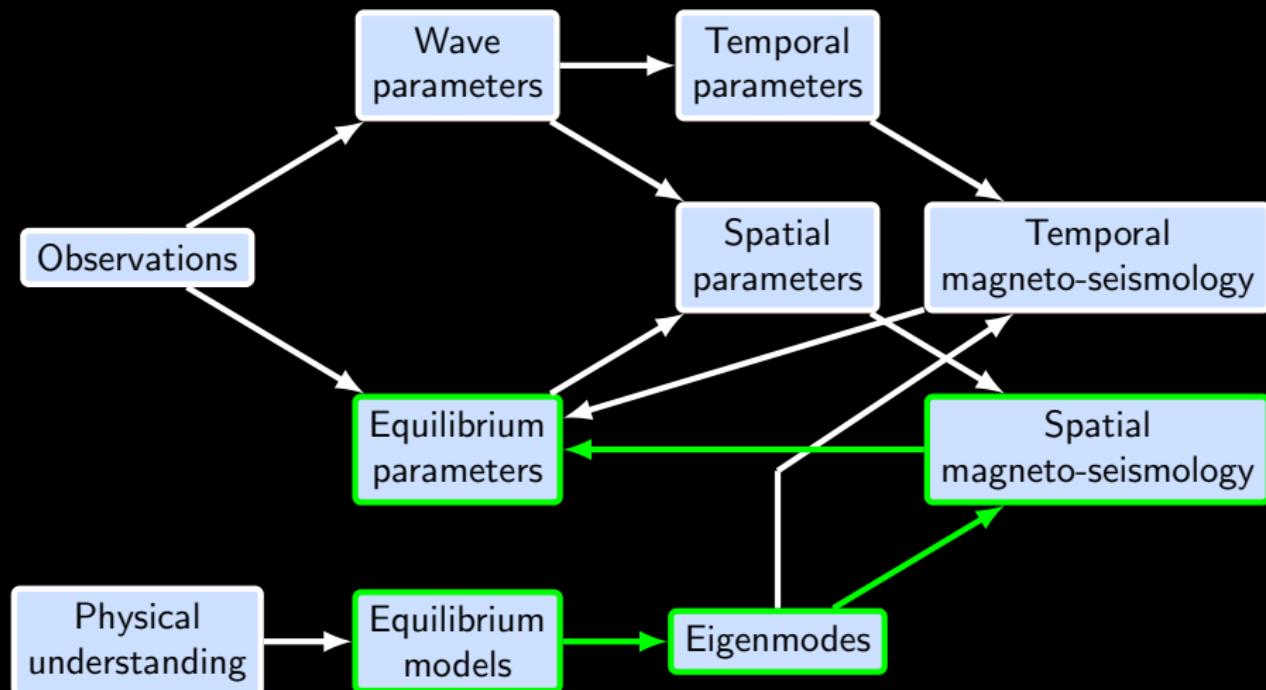
Magnetohydrodynamic waves

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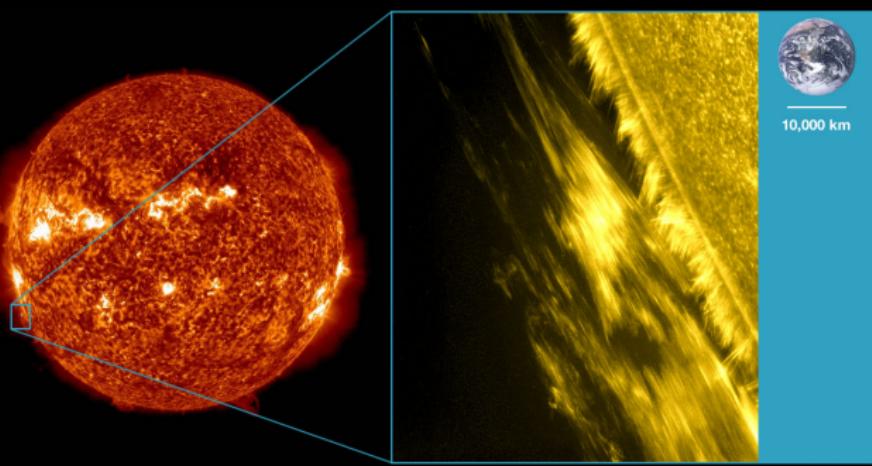


Magnetohydrodynamic waves

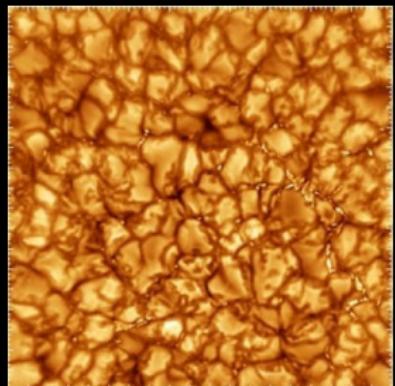
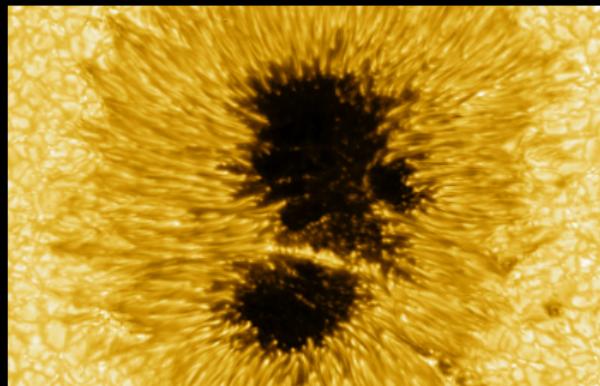
Diagnosing information about solar plasma



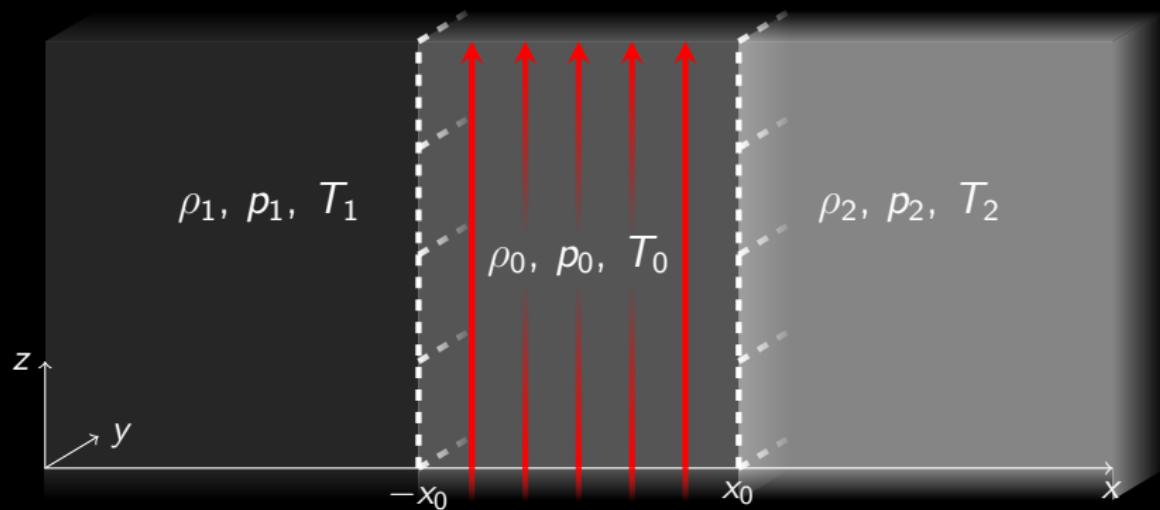
Slab structures on the Sun



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Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- **Different** density and pressure on each side.

Governing equations

Ideal MHD equations:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \frac{1}{\mu} \mathbf{B} \times (\nabla \times \mathbf{B}), \quad \text{momentum}$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \text{mass}$$
$$\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) = 0, \quad \text{energy}$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad \text{magnetic flux}$$

\mathbf{v} = plasma velocity,

ρ = density,

μ = magnetic permeability,

\mathbf{B} = magnetic field strength,

p = pressure,

γ = adiabatic index.

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Asymmetric slab modes

Dispersion relation:

$$\frac{\omega^4 m_0^2}{k^2 v_A^2 - \omega^2} + \frac{\rho_0}{\rho_1} m_1 \frac{\rho_0}{\rho_2} m_2 (k^2 v_A^2 - \omega^2) - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} m_1 + \frac{\rho_0}{\rho_2} m_2 \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad m_{1,2}^2 = k^2 - \frac{\omega^2}{c_{1,2}^2},$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}, \quad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

See **Allcock** and Erdélyi, 2017.

Asymmetric slab modes

Dispersion relation:

$$\frac{\omega^4 m_0^2}{k^2 v_A^2 - \omega^2} + \frac{\rho_0}{\rho_1} \textcolor{red}{m}_1 \frac{\rho_0}{\rho_2} \textcolor{red}{m}_2 (k^2 v_A^2 - \omega^2) - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} \textcolor{red}{m}_1 + \frac{\rho_0}{\rho_2} \textcolor{red}{m}_2 \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

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See **Allcock** and Erdélyi, 2017.

Slab eigenmodes

Symmetric kink surface mode

Slab eigenmodes

Quasi-kink surface mode

Slab eigenmodes

Symmetric sausage surface mode

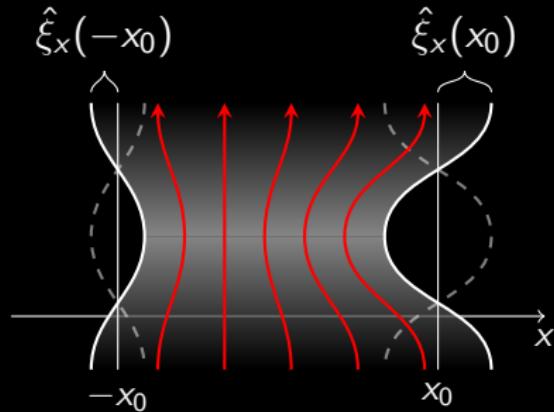
Slab eigenmodes

Quasi-sausage surface mode

Slab eigenmodes

Body modes

Amplitude ratio



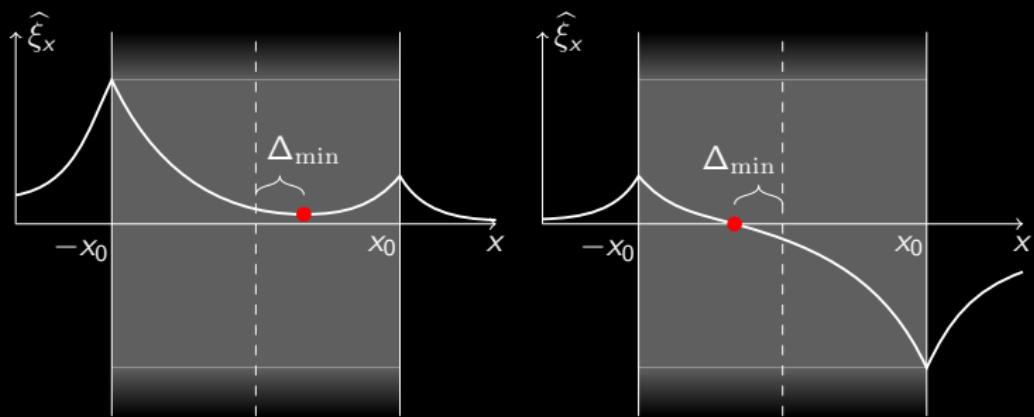
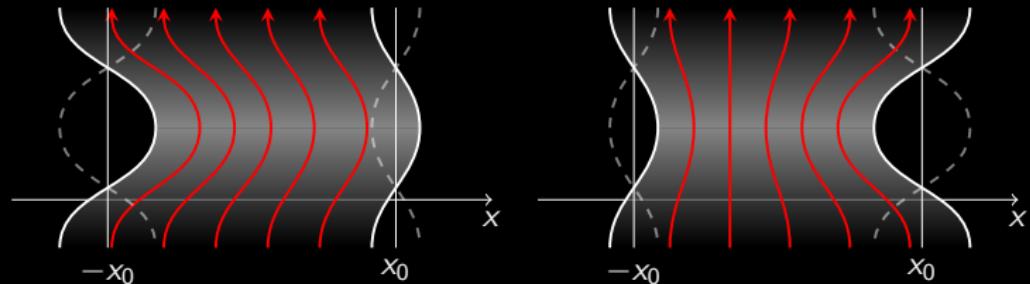
Amplitude ratio

$$R_A := \frac{\hat{\xi}_x(x_0)}{\hat{\xi}_x(-x_0)}$$

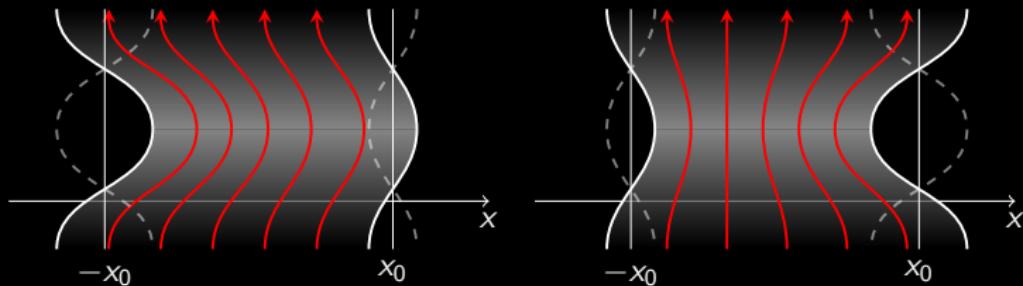
(Top = quasi-kink
Bottom = quasi-sausage)

$$= (+) \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 v_A^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 (\tanh \frac{\rho_0}{\coth}) (m_0 x_0)}{(k^2 v_A^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 (\tanh \frac{\rho_0}{\coth}) (m_0 x_0)}$$

Minimum perturbation shift



Minimum perturbation shift



Quasi-kink:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}(D)$$

Quasi-sausage:

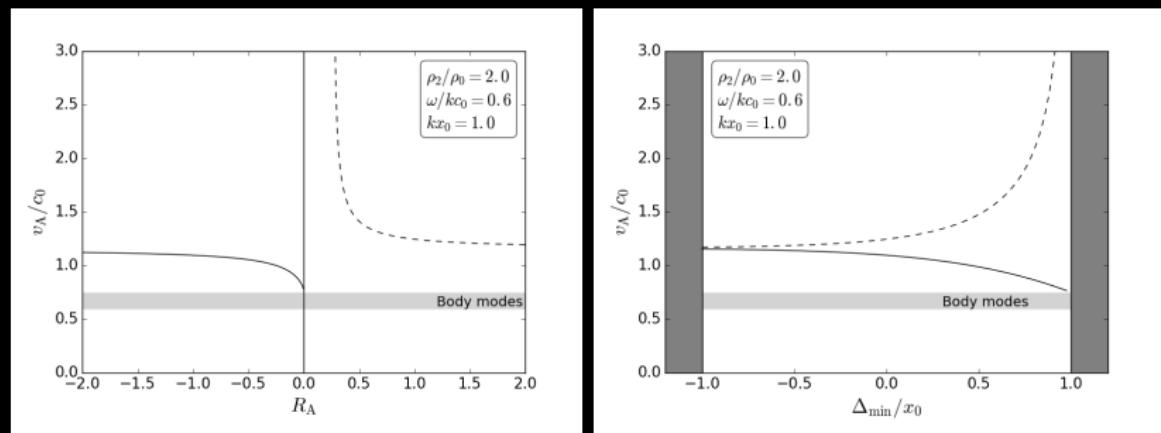
$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1} \left(\frac{1}{D} \right)$$

where $D = \frac{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} \tanh(m_0 x_0) - \omega^2 m_0}{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \tanh(m_0 x_0)}$

Solar magneto-seismology

Parameter inversion

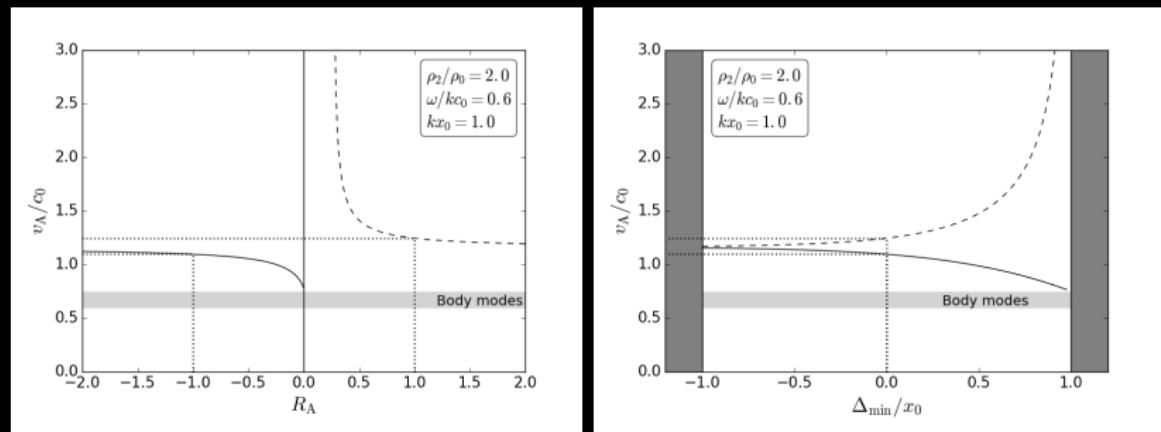
- **Observe:** ω , k , x_0 , T_i , and R_A or Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .



Solar magneto-seismology

Parameter inversion

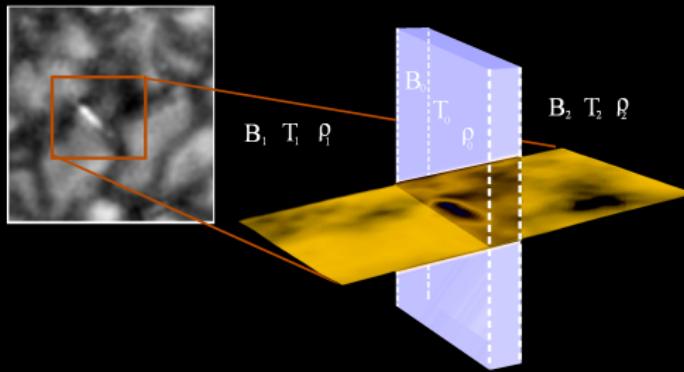
- **Observe:** ω , k , x_0 , T_i , and R_A or Δ_{\min} .
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Future work

Diagnose magnetic field parameters using observations of MHD waves in magnetic structures in the solar atmosphere, for example:

- Elongated **magnetic bright points**,

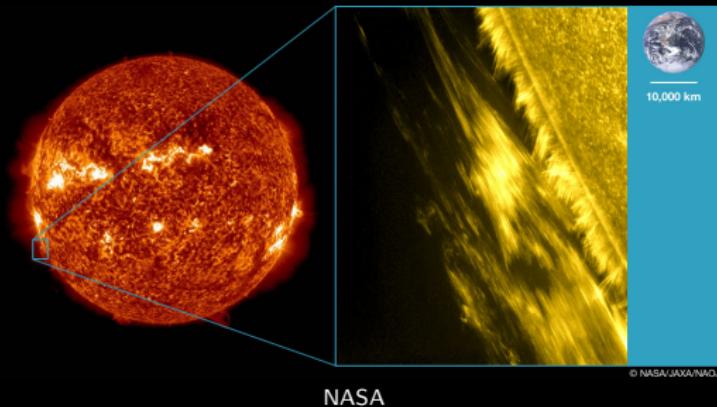


Adaptation of Liu et al., 2017, by N. Zsámberger

Future work

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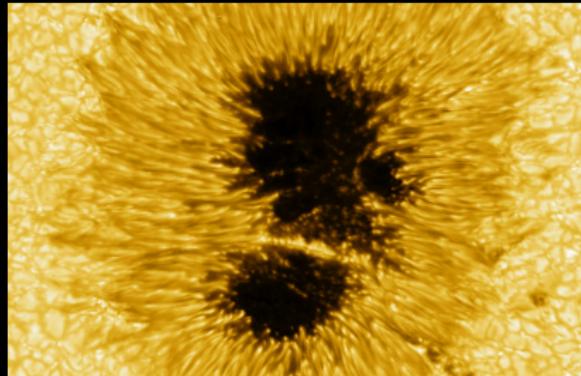
- Elongated **magnetic bright points**,
- **Prominences**,



Future work

Diagnose magnetic field parameters using observations of MHD waves in magnetic structures in the solar atmosphere, for example:

- Elongated **magnetic bright points**,
- **Prominences**,
- Sunspot **light walls**.



Max Planck Institute for Solar System Research



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