CHAPTER 2

Asymmetric waveguides - eigenvalue problem

2.1 Chapter introduction

2.2 Asymmetric slab in a non-magnetic environment

2.2.1 Model description

The simplest model of an asymmetric waveguide is an asymmetric magnetic slab in a non-magnetic environment[1]. Figure 2.1 illustrates the construction of this mathematical model, where a three-dimensional, unbounded, inviscid plasma is separated into three regions by two parallel planar interfaces at $x = \pm x_0$. The magnetic field is in the z-direction and has magnitude

$$B(x) = \begin{cases} B_1 & \text{if } x < -x_0, \\ B_0 & \text{if } |x| \leq x_0, \\ B_2 & \text{if } x > x_0, \end{cases} \tag{2.1}$$

where B_j, for $j = 0, 1, 2$ are constant. In the present case, we let $B_1 = B_2 = 0$ so that the plasma in the environment is non-magnetic. Within each region, denoted by subscripts 0, 1, and 2, the plasma is uniform and the equilibrium plasma pressure, density, and temperature are denoted by p_j, ρ_j, and T_j, respectively, for $j = 0, 1, 2$.

The effects of gravity are ignored throughout; it is important to note, however, that equilibrium density stratification itself in the solar atmosphere can be a consequence of gravity, but gravity can be ignored if the gravity scale height is large compared to the wavelength and the thickness of the magnetic slab, which is safe to assume for many small-scale solar atmospheric structures.

[1]More precisely, the simplest model of an asymmetric MHD waveguide is an interface between different plasmas; the asymmetric slab is the simplest asymmetric waveguide that can oscillate in a collective body mode (see Section ??).

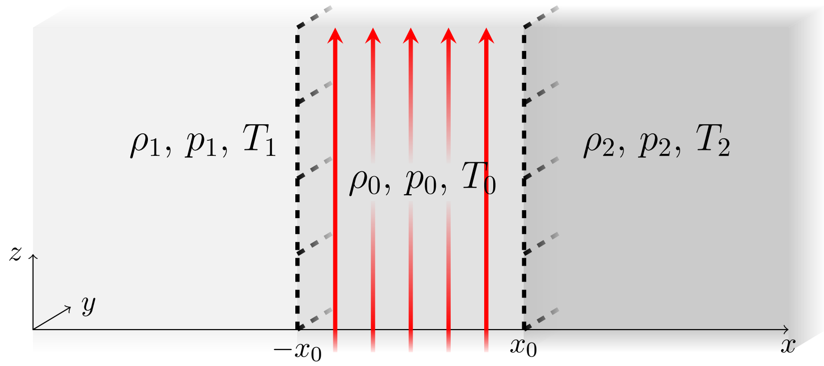

Figure 2.1: Equilibrium state inside the slab, ($|x| \leq x_0$) and outside the slab, ($x < -x_0$ and $x > x_0$). The red arrows illustrate the vertical magnetic field, $B(x)\widehat{\mathbf{z}}$, and the dashed black lines indicate the boundaries of the slab.

2.2.2 The dispersion relation

2.2.2.1 Derivation

2.2.2.2 First-order symmetric slab

2.2.3 Analytical solutions

2.2.3.1 Spurious solutions

2.2.3.2 Limiting case - incompressible plasma

2.2.3.3 Limiting case - zero-beta

2.2.3.4 Limiting case - thin slab

2.2.3.5 Limiting case - wide slab

2.2.4 Numerical solutions

2.2.4.1 Varying the degree of asymmetry

2.2.5 Eigenfunctions

2.2.5.1 Analogy to coupled spring and mass oscillator

Actually do the maths for this.

2.3 Asymmetric slab in a magnetic environment

2.3.1 Model description

2.3.2 The dispersion relation

2.3.3 Implications for observations

2.3.3.1 Quasi-symmetric modes

2.3.3.2 Asymmetric mode or superposition of symmetric modes?

Table of observable indicators of each case.

2.3.3.3 Possible alternative causes of observed asymmetry

Possibilities:

- Asymmetric ICs
- Non-collective oscillations
- Observational artefact

Include discussion about how to differentiate between these.

Bibliography