

Magneto-acoustic waves in asymmetric solar waveguides

Progress in spatial magneto-seismology



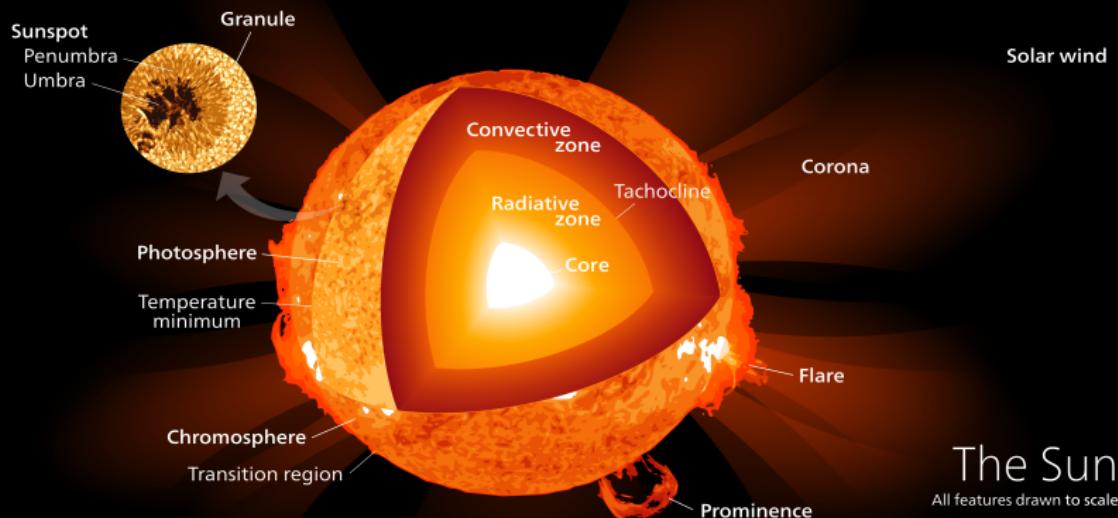
Matthew Allcock
and
Robertus Erdélyi



The
University
Of
Sheffield.



The layers of an onion



The Sun

All features drawn to scale

Magnetohydrodynamic waves

Ubiquitous in the solar atmosphere

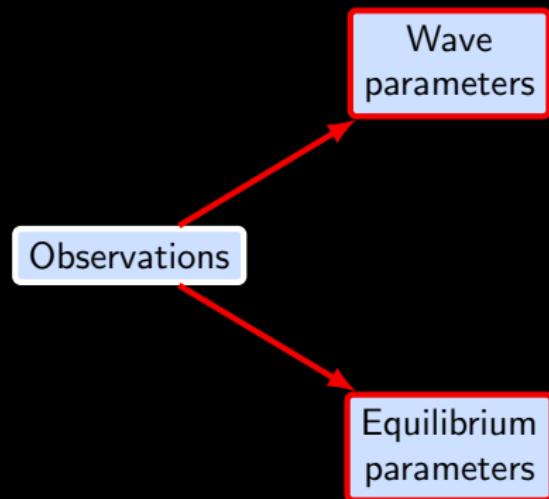
Magnetohydrodynamic waves

Diagnosing information about the background plasma

Observations

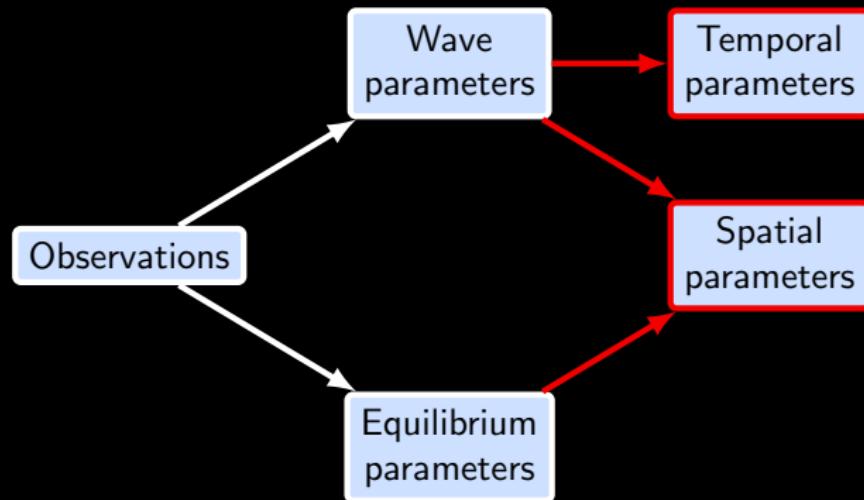
Magnetohydrodynamic waves

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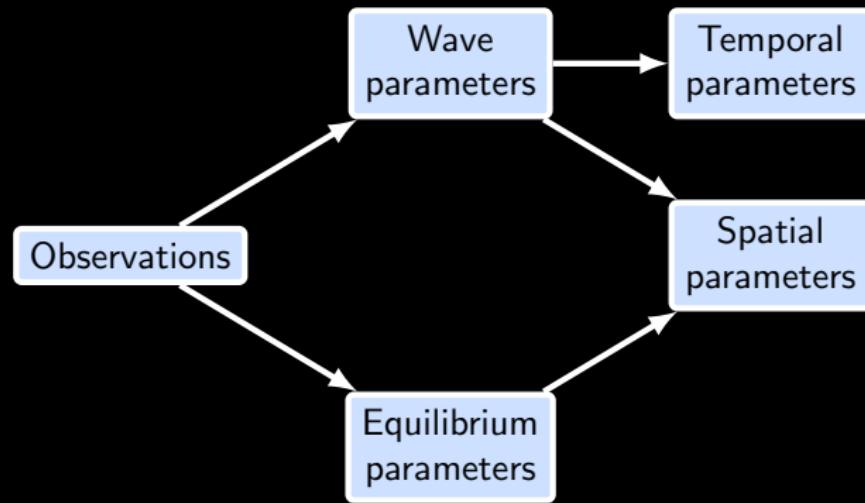
Magnetohydrodynamic waves

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Magnetohydrodynamic waves

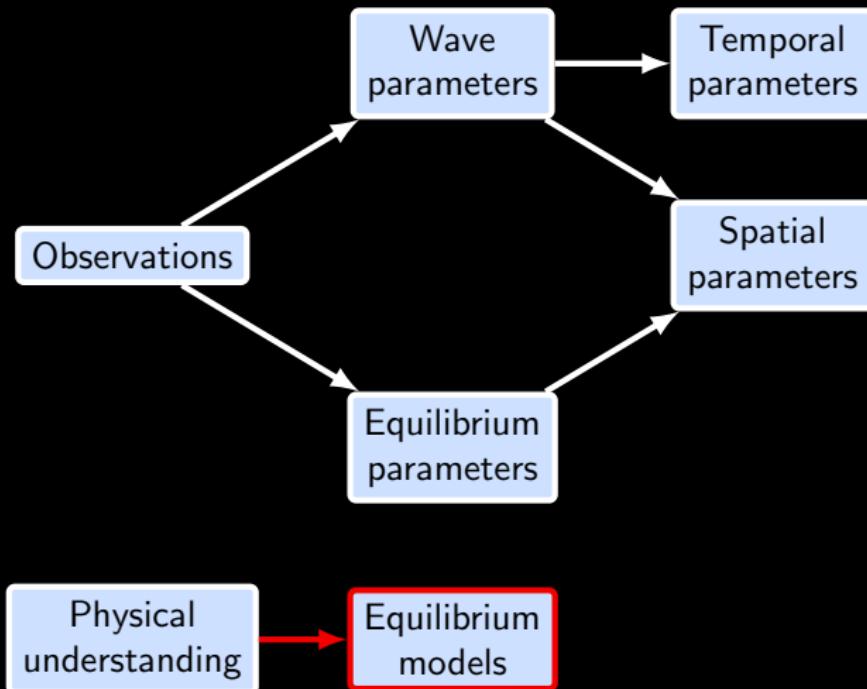
Diagnosing information about the background plasma



Physical
understanding

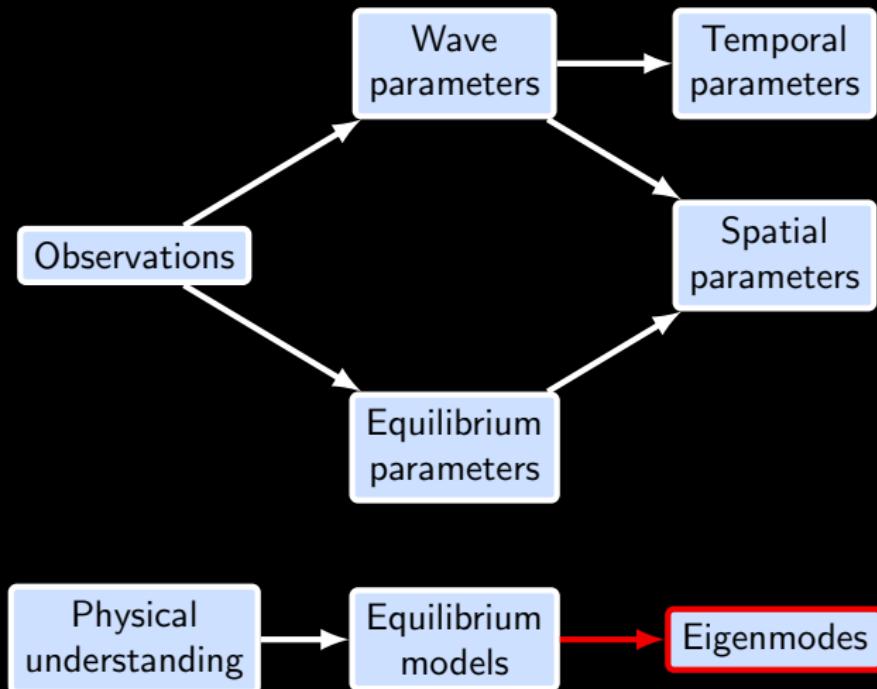
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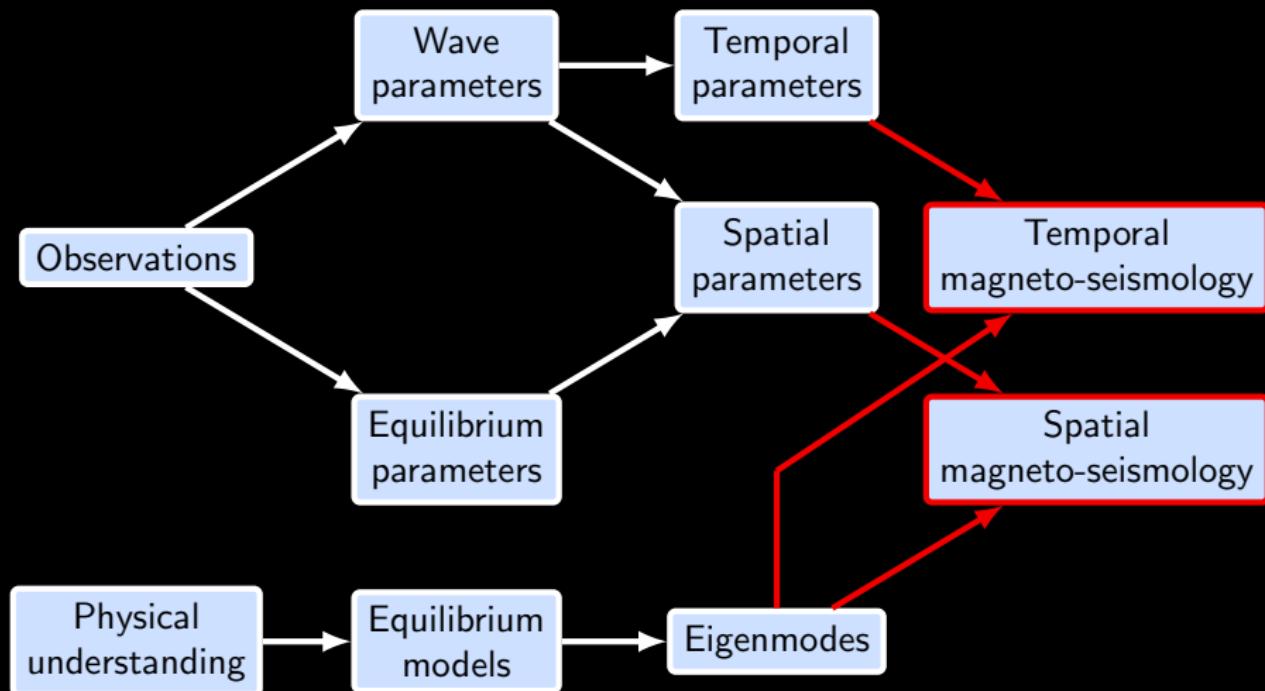
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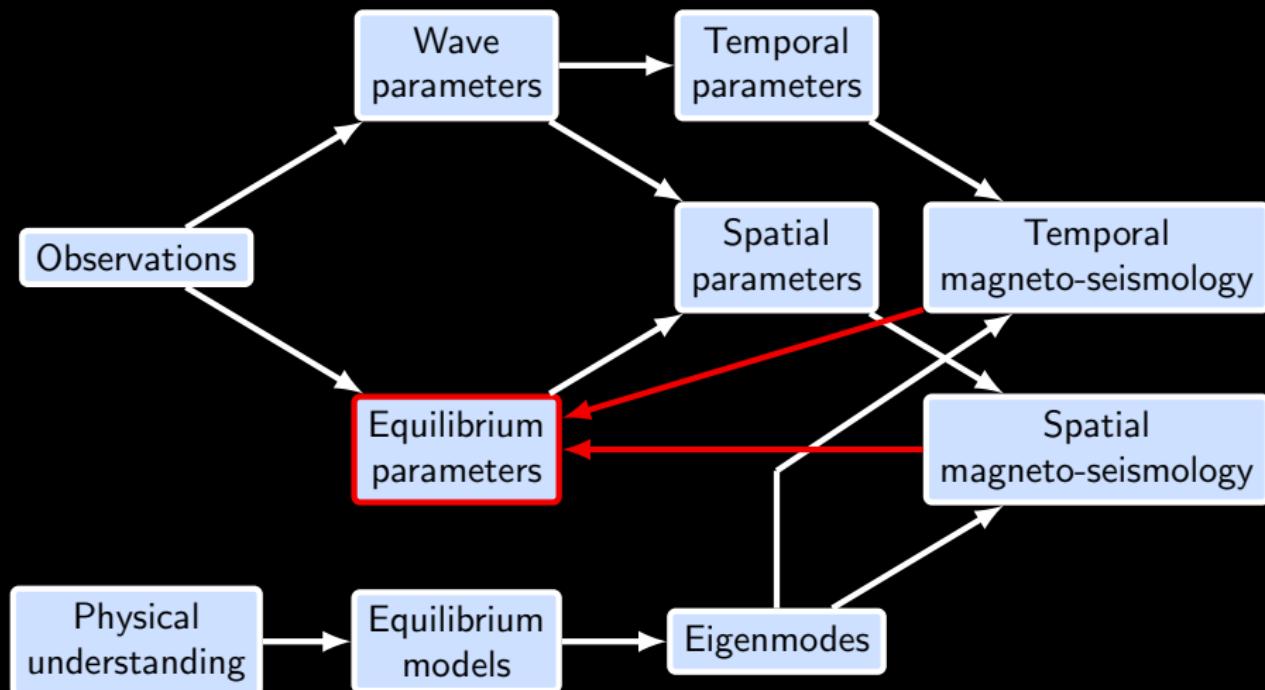
Magnetohydrodynamic waves

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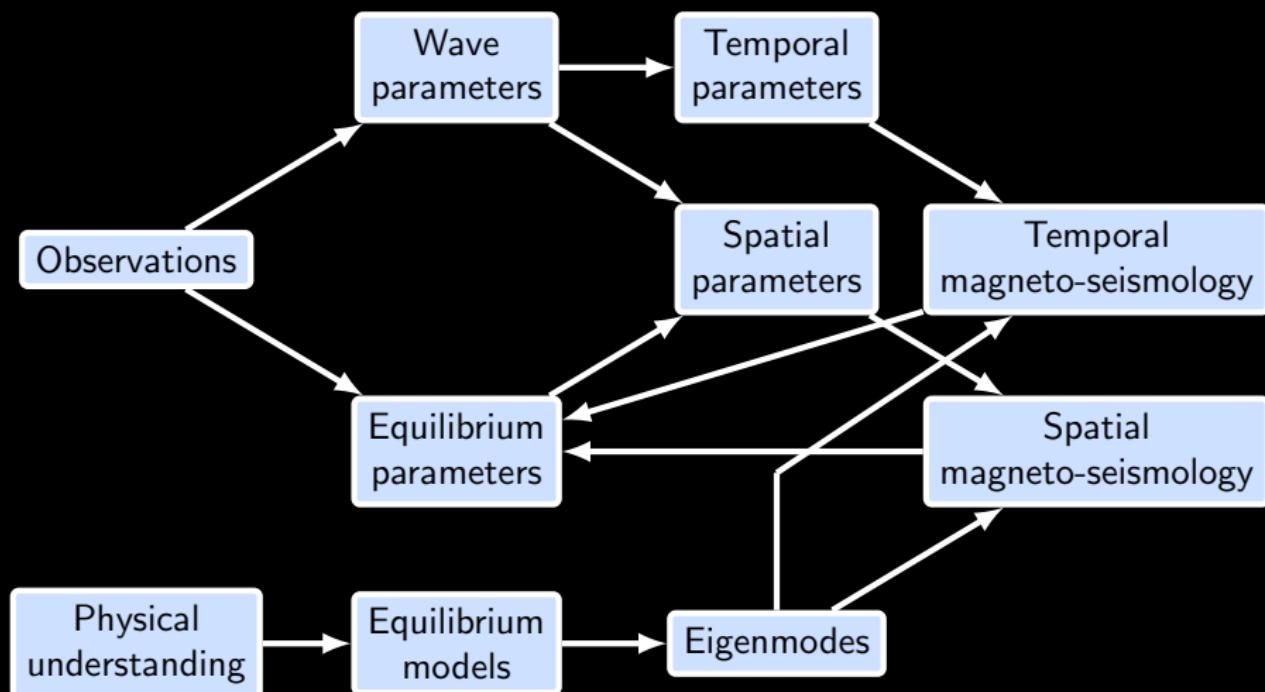
Magnetohydrodynamic waves

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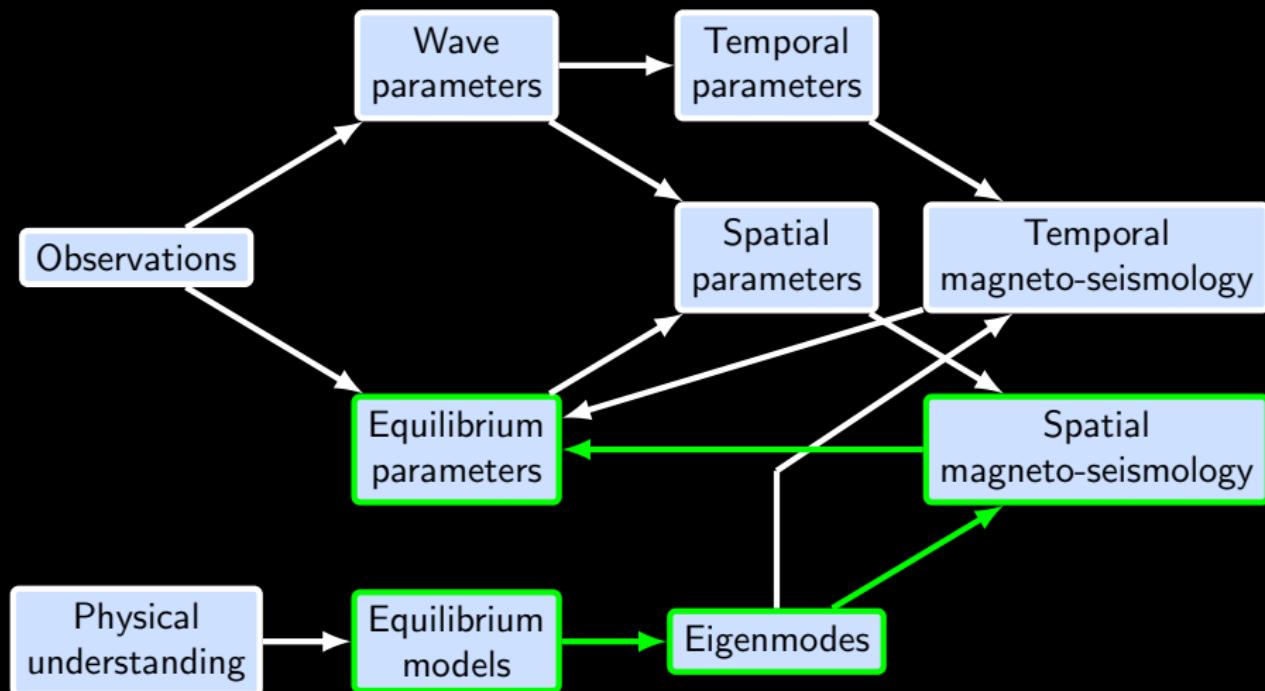
Magnetohydrodynamic waves

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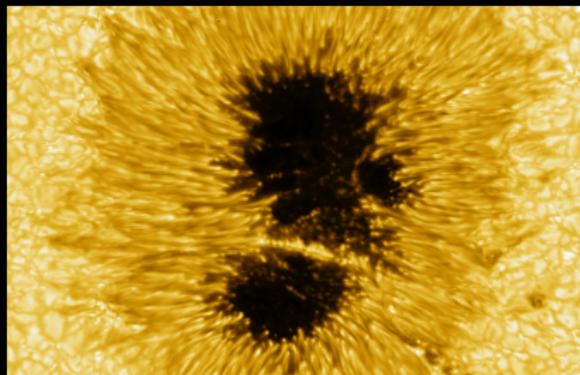
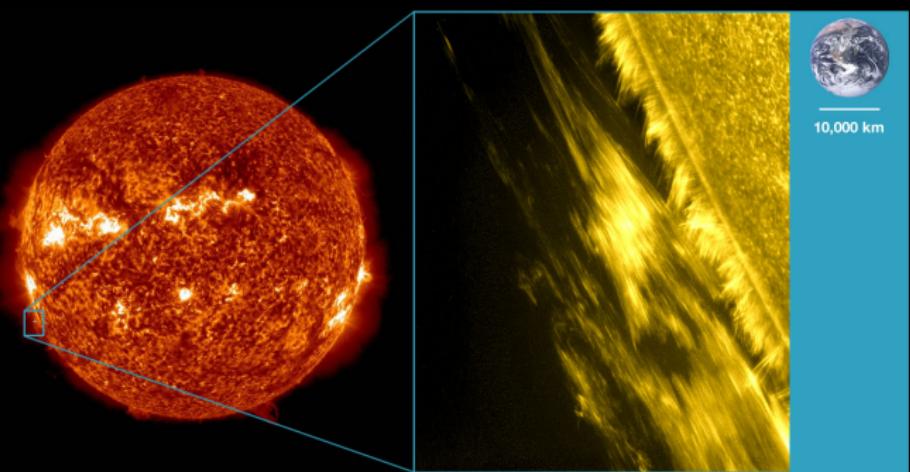


Magnetohydrodynamic waves

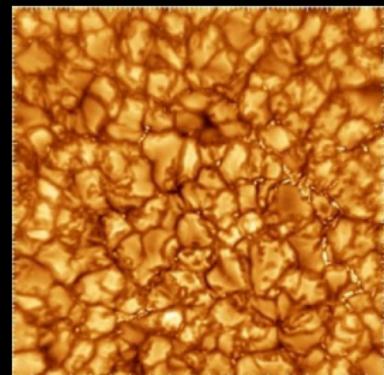
Diagnosing information about the background plasma



Motivation

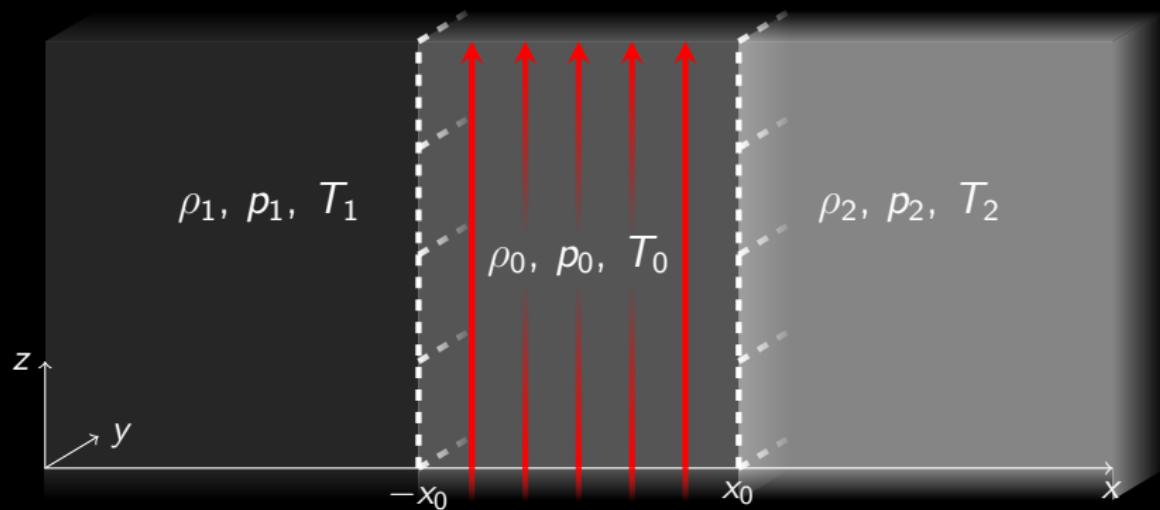


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Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- **Different** density and pressure on each side.

Governing equations

Ideal MHD equations:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \frac{1}{\mu} \mathbf{B} \times (\nabla \times \mathbf{B}), \quad \text{momentum}$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \text{mass}$$
$$\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) = 0, \quad \text{energy}$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad \text{magnetic flux}$$

\mathbf{v} = plasma velocity,

ρ = density,

μ = magnetic permeability,

\mathbf{B} = magnetic field strength,

p = pressure,

γ = adiabatic index.

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Asymmetric slab modes

Dispersion relation:

$$\frac{\omega^4 m_0^2}{k^2 v_A^2 - \omega^2} + \frac{\rho_0}{\rho_1} m_1 \frac{\rho_0}{\rho_2} m_2 (k^2 v_A^2 - \omega^2) - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} m_1 + \frac{\rho_0}{\rho_2} m_2 \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad m_{1,2}^2 = k^2 - \frac{\omega^2}{c_{1,2}^2},$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}, \quad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

See **Allcock** and Erdélyi, 2017.

Asymmetric slab modes

Dispersion relation:

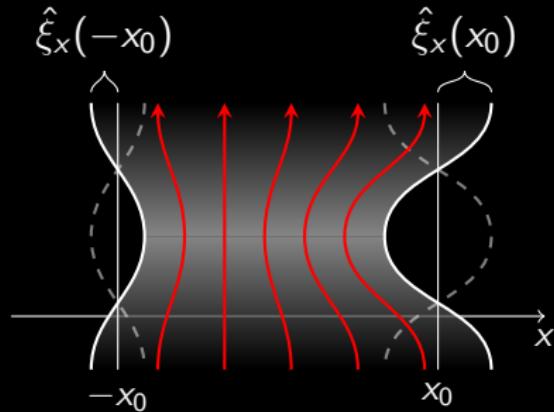
$$\frac{\omega^4 m_0^2}{k^2 v_A^2 - \omega^2} + \frac{\rho_0}{\rho_1} \textcolor{red}{m}_1 \frac{\rho_0}{\rho_2} \textcolor{red}{m}_2 (k^2 v_A^2 - \omega^2) - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} \textcolor{red}{m}_1 + \frac{\rho_0}{\rho_2} \textcolor{red}{m}_2 \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad \textcolor{red}{m}_{1,2}^2 = k^2 - \frac{\omega^2}{\textcolor{red}{c}_{1,2}^2},$$

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See **Allcock** and Erdélyi, 2017.

Amplitude ratio



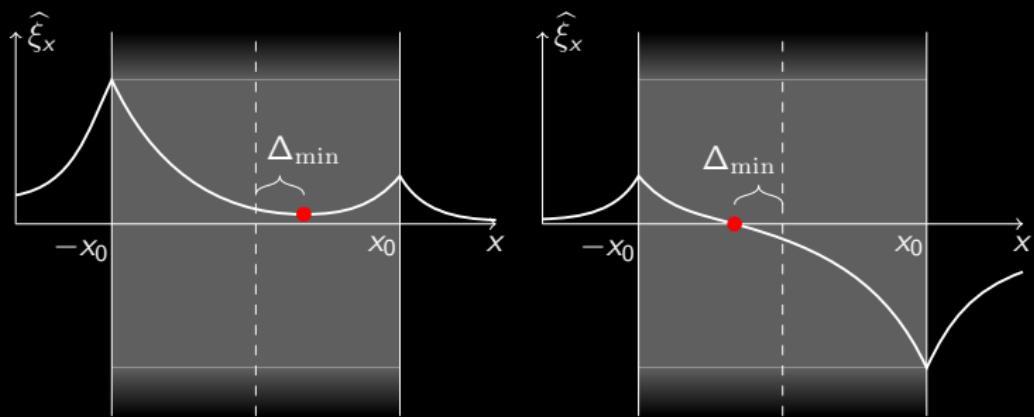
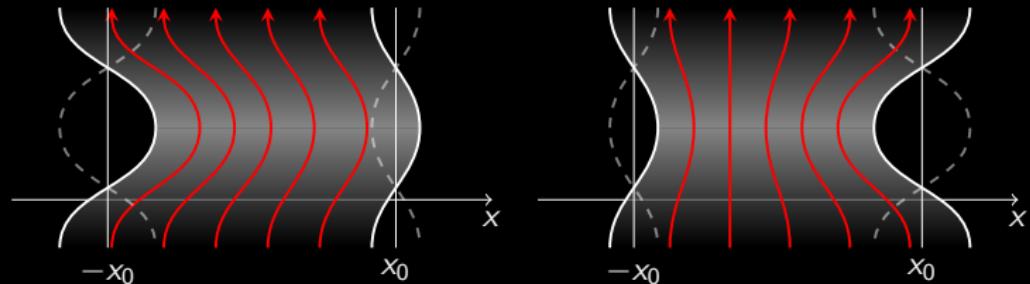
Amplitude ratio

$$R_A := \frac{\hat{\xi}_x(x_0)}{\hat{\xi}_x(-x_0)}$$

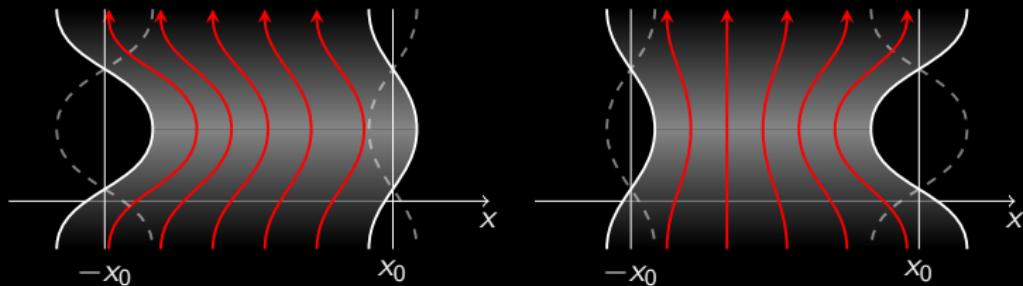
(Top = quasi-kink
Bottom = quasi-sausage)

$$= (+) \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 v_A^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 (\tanh \frac{\rho_0}{\coth}) (m_0 x_0)}{(k^2 v_A^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 (\tanh \frac{\rho_0}{\coth}) (m_0 x_0)}$$

Minimum perturbation shift



Minimum perturbation shift



Quasi-kink:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}(D)$$

Quasi-sausage:

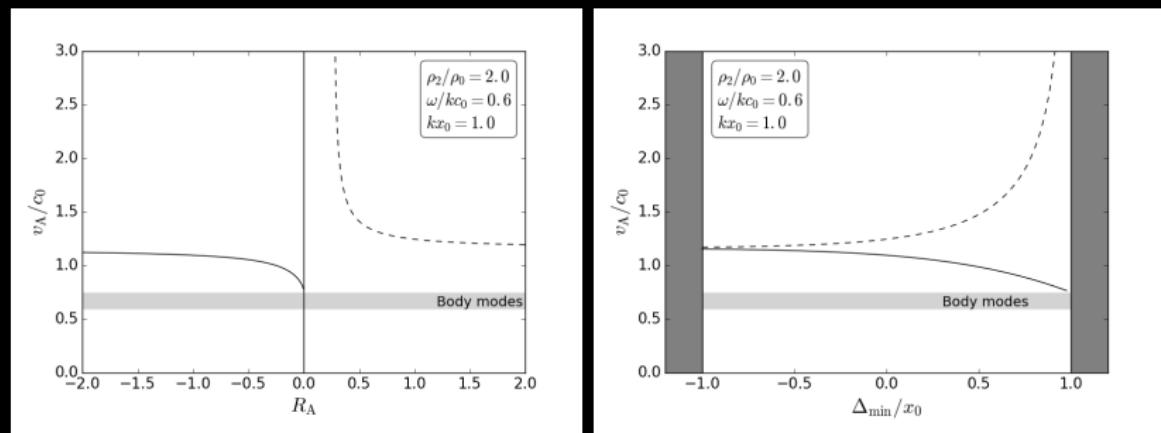
$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1} \left(\frac{1}{D} \right)$$

where $D = \frac{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} \tanh(m_0 x_0) - \omega^2 m_0}{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \tanh(m_0 x_0)}$

Solar magneto-seismology

Parameter inversion

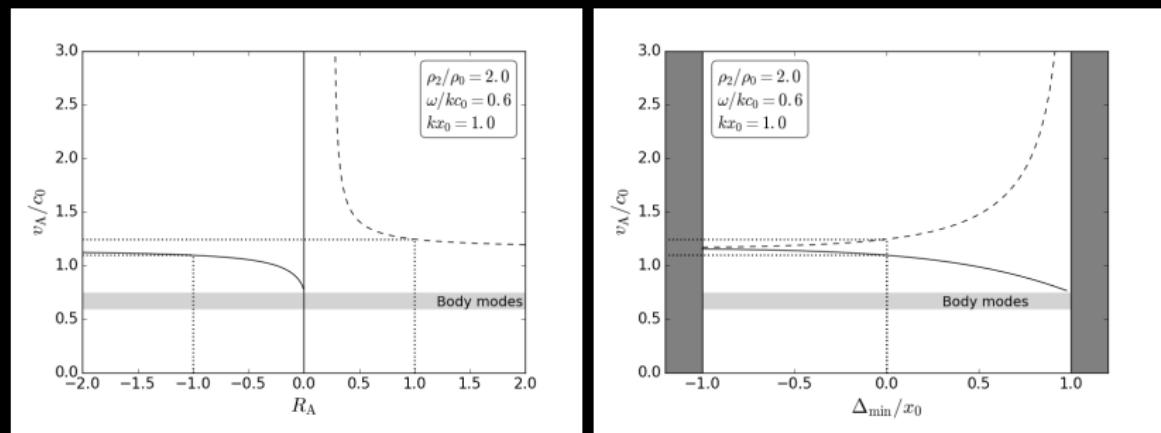
- **Observe:** ω , k , x_0 , T_i , and R_A or Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .



Solar magneto-seismology

Parameter inversion

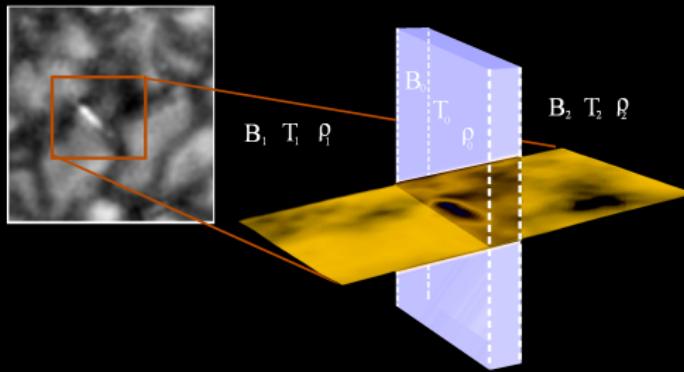
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Future work

Diagnose magnetic field parameters using observations of MHD waves in magnetic structures in the solar atmosphere, for example:

- Elongated **magnetic bright points**,

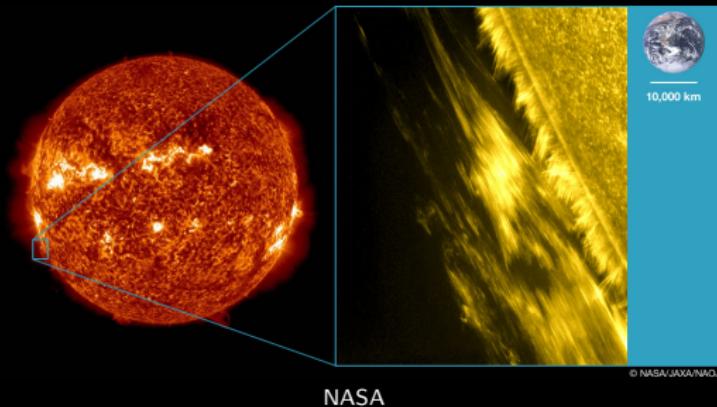


Adaptation of Liu et al., 2017, by N. Zsámberger

Future work

Diagnose magnetic field parameters using observations of MHD waves in magnetic structures in the solar atmosphere, for example:

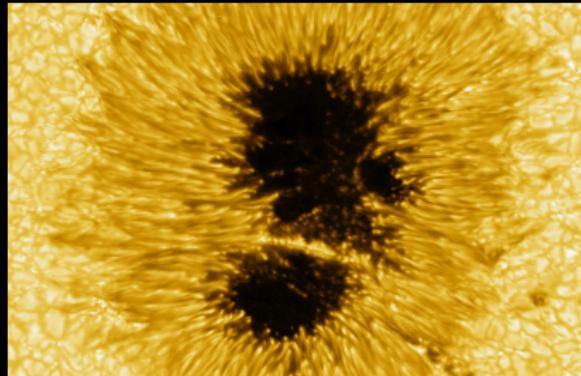
- Elongated **magnetic bright points**,
- **Prominences**,



Future work

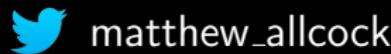
Diagnose magnetic field parameters using observations of MHD waves in magnetic structures in the solar atmosphere, for example:

- Elongated **magnetic bright points**,
- **Prominences**,
- Sunspot **light walls**.



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"a day without the Sun is, you know, night"



The
University
Of
Sheffield.

