Magneto-acoustic waves in an asymmetric magnetic slab

Progress in spatial magneto-seismology

Matthew Allcock
and
Robertus Erdélyi

The University Of Sheffield.

Outline

1. Introduction
   - Waves on the Sun
   - Solar magneto-seismology (SMS)
   - A brief history
2. SMS with asymmetric wave-guides
   - Motivation
   - Mode identification
   - Amplitude ratio
   - Minimum perturbation shift
3. Looking ahead

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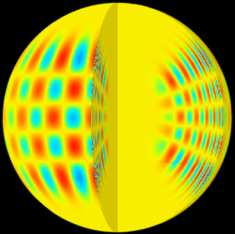
Waves on the Sun

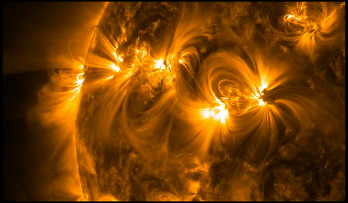
Global pressure modes (p-modes):

- Standing modes,
- Spherical harmonics with global Sun as cavity,
- Global and local **helioseismology** for inference of sub-surface flows, density, temperature.



MHD waves:

- Propagating or standing modes,
- Guided by local plasma inhomogeneity,
- Local **magneto-seismology** for inference of background magnetic field strength, heat transport coefficients, density.



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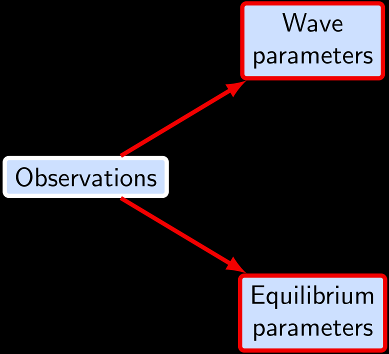
Solar magneto-seismology

An overview

Observations

Solar magneto-seismology

An overview



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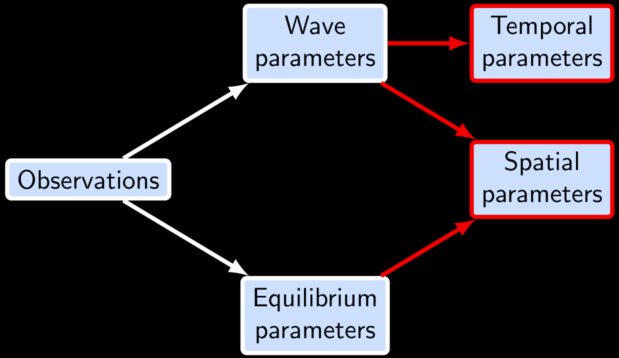

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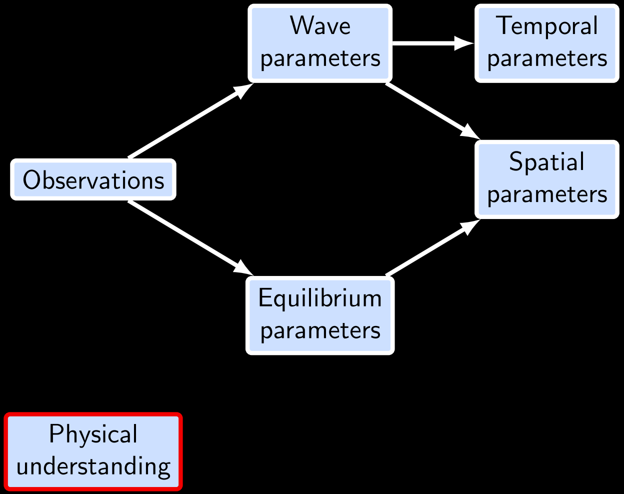

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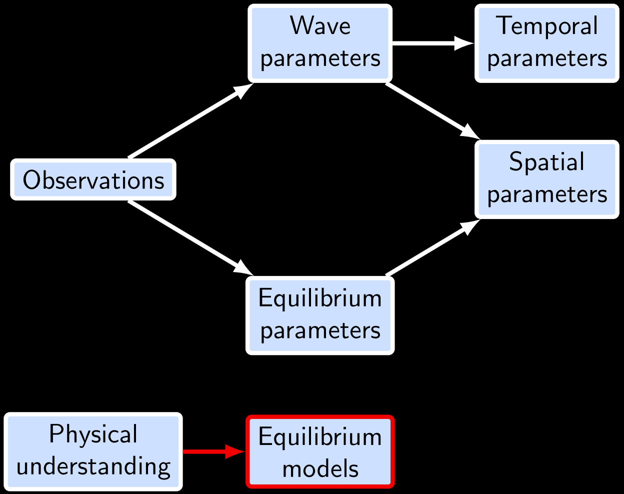

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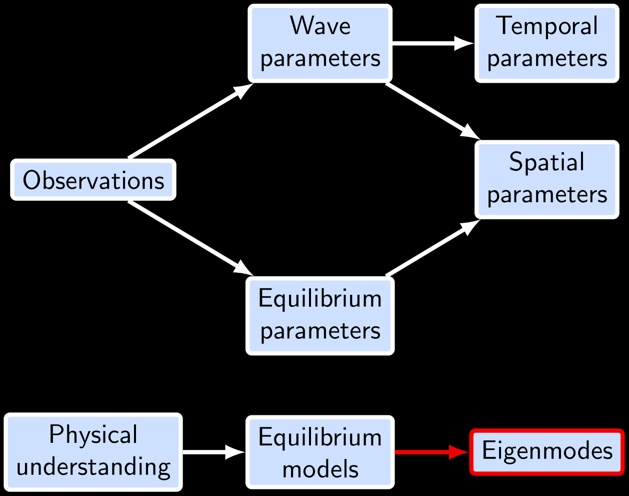

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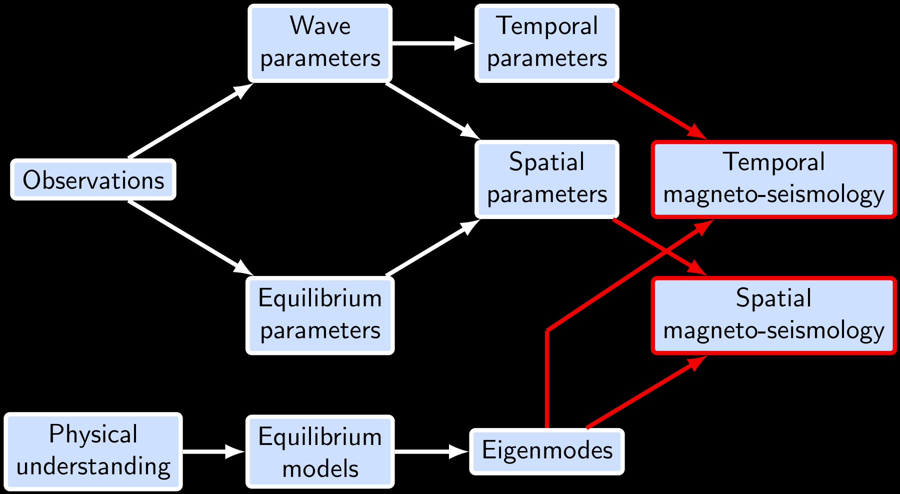

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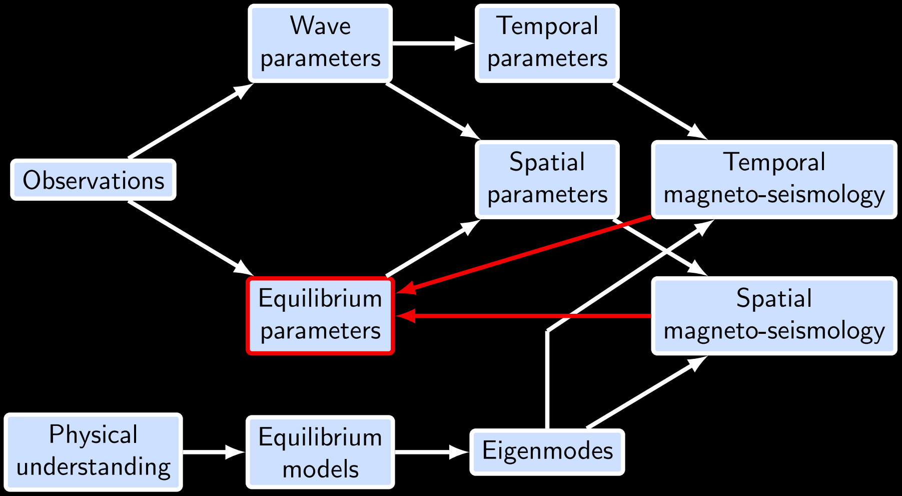

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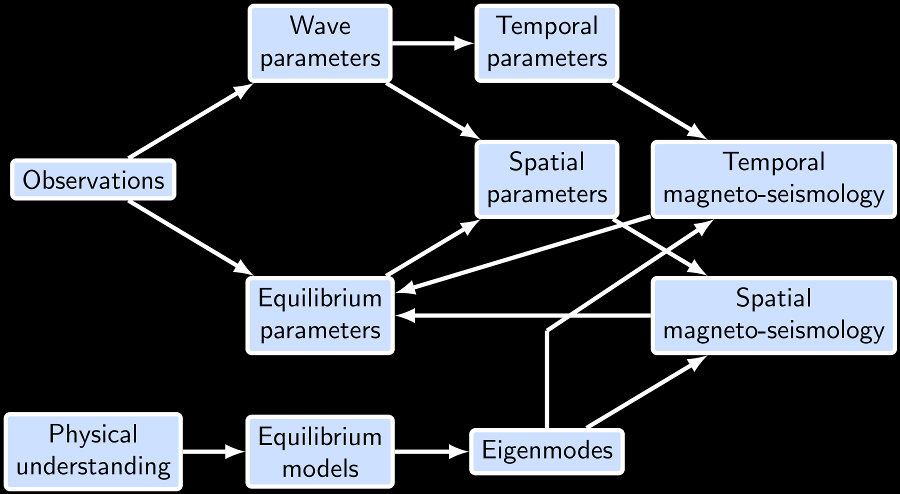

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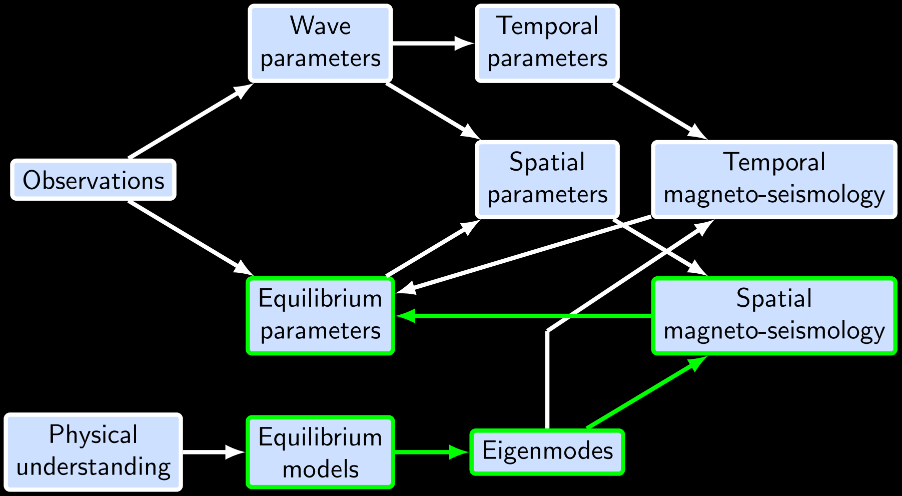

Solar magneto-seismology

A brief history - temporal and spatial seismology

1970

1980

1990

2000

Solar magneto-seismology

A brief history - temporal and spatial seismology

1970 Uchida, 1970 - **Moreton** wavefronts;

1980

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Solar magneto-seismology

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1970 Uchida, 1970 - **Moreton** wavefronts;
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Tandberg Hanssen *et al.* 1995 - **prominence** context

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Coronal loop oscillations observed! Aschwanden *et al.* 1999; Nakariakov *et al.* 1999

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Solar magneto-seismology

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2017

- Magyar *et al.* - **global** dynamic coronal seismology

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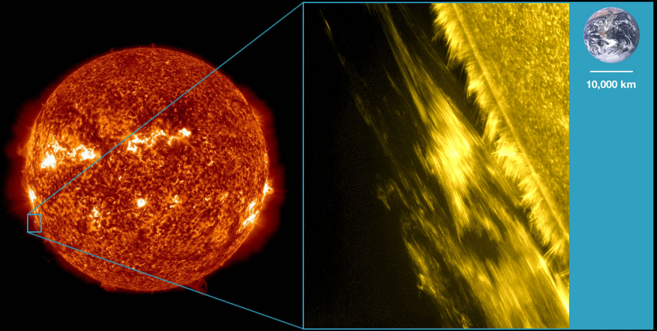
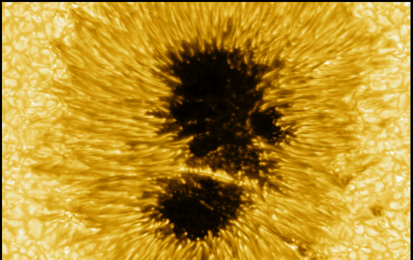
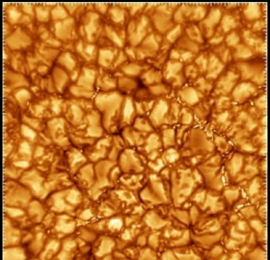
Asymmetric magnetic slab

Motivation

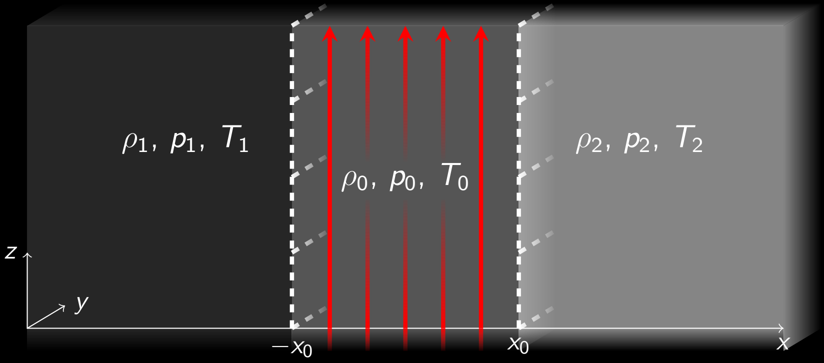






Asymmetric magnetic slab

Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- Different density and pressure on each side.

Ideal MHD equations: Conservation of:

$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\nabla p - \frac{1}{\mu}\boldsymbol{B} \times (\nabla \times \boldsymbol{B}),$$ momentum

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$ mass

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{p}{\rho^\gamma}\right) = 0,$$ energy

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}),$$ magnetic flux

$\boldsymbol{v} =$ plasma velocity, $\boldsymbol{B} =$ magnetic field strength,
$\rho =$ density, $p =$ pressure,
$\mu =$ magnetic permeability, $\gamma =$ adiabatic index.

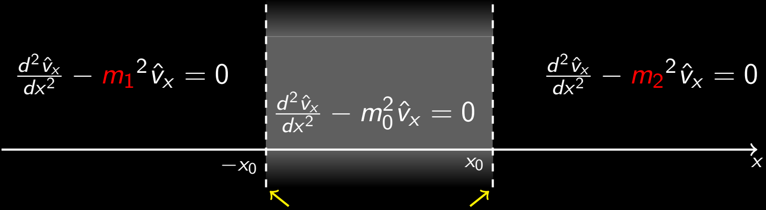
Asymmetric magnetic slab

Fourier decomposition

Look for *plane wave* solutions of the form:

$$v_x(\mathbf{x}, t) = \hat{v}_x(x)e^{i(kz-\omega t)}, \quad v_y(\mathbf{x}, t) = 0, \quad v_z(\mathbf{x}, t) = \hat{v}_z(x)e^{i(kz-\omega t)},$$

to arrive at the following ODEs:



Boundary conditions:
pressure and **velocity** continuous across boundaries.

$${m_0}^2 = \frac{(k^2{v_A}^2 - \omega^2)(k^2c_0^2 - \omega^2)}{(c_0^2 + {v_A}^2)(k^2{c_T}^2 - \omega^2)}, \qquad {m_{1,2}}^2 = k^2 - \frac{\omega^2}{{c_{1,2}}^2},$$

$${c_T}^2 = \frac{c_0^2{v_A}^2}{c_0^2 + {v_A}^2}, \qquad v_A = \frac{B_0}{\sqrt{\mu\rho_0}},$$

Asymmetric magnetic slab

Eigenmodes

Dispersion relation:

$$\frac{\omega^4 {m_0}^2}{k^2 {v_A}^2 - \omega^2} + \frac{\rho_0}{\rho_1} m_1 \frac{\rho_0}{\rho_2} m_2 (k^2 {v_A}^2 - \omega^2) - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} m_1 + \frac{\rho_0}{\rho_2} m_2 \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

See **Allcock** and Erdélyi, 2017.

$${m_0}^2 = \frac{(k^2 {v_A}^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + {v_A}^2)(k^2 {c_T}^2 - \omega^2)}, \qquad {m_{1,2}}^2 = k^2 - \frac{\omega^2}{{c_{1,2}}^2},$$

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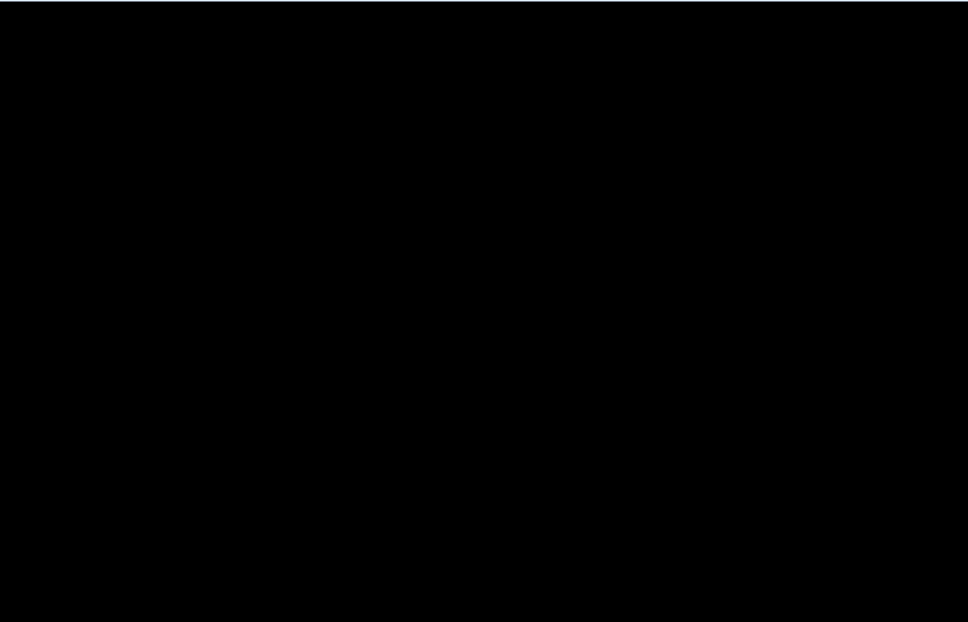
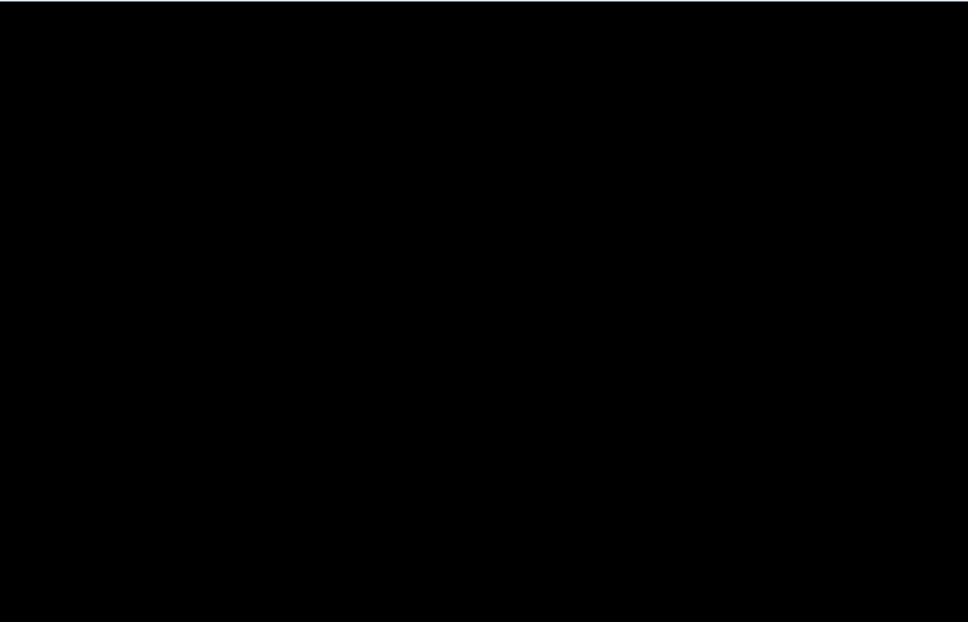
Eigenmodes

Symmetric kink surface mode



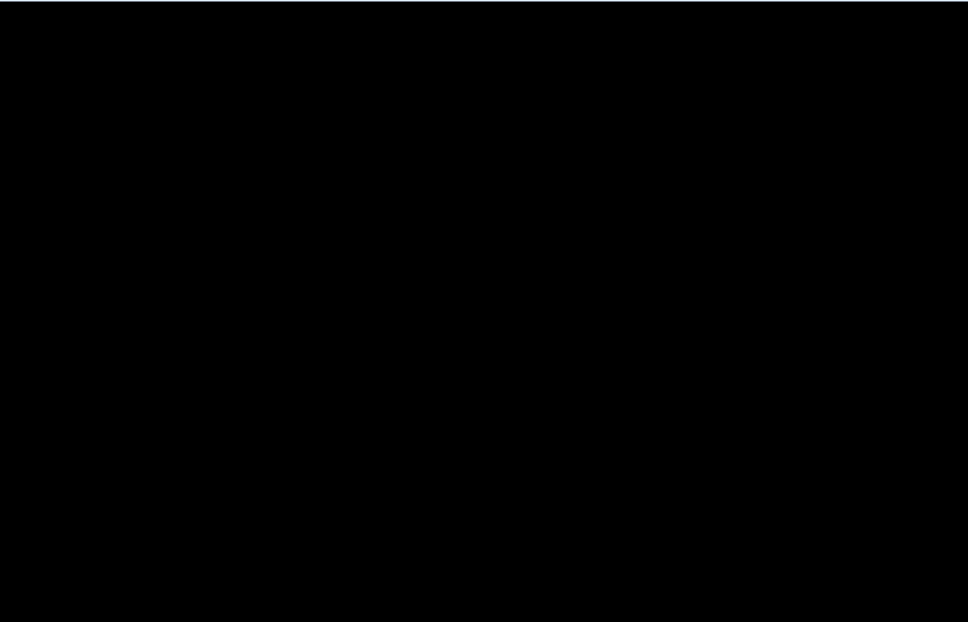
Eigenmodes

Quasi-kink surface mode



Eigenmodes

Symmetric sausage surface mode



Eigenmodes

Quasi-sausage surface mode

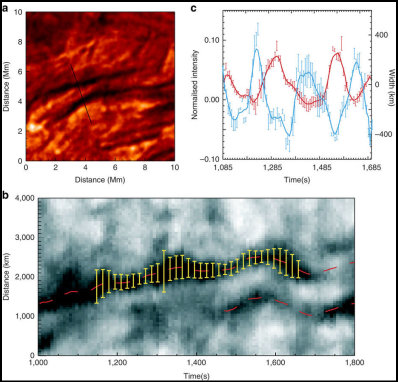
Eigenmodes

Body modes

Asymmetric magnetic slab

Mode identification

Superposition of symmetric kink and sausage modes...

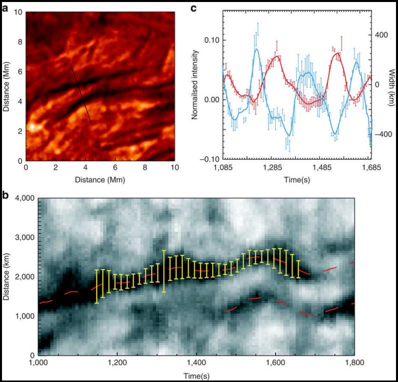


Morton *et al.* 2012

Asymmetric magnetic slab

Mode identification

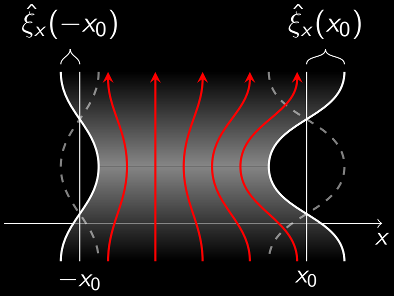
Superposition of symmetric kink and sausage modes...



Morton *et al.* 2012

or asymmetric kink mode?

Amplitude ratio



Amplitude ratio

$$R_A := \frac{\hat{\xi}_x(x_0)}{\hat{\xi}_x(-x_0)} \qquad \left(\begin{smallmatrix}\text{Top = quasi-kink}\\ \text{Bottom = quasi-sausage}\end{smallmatrix}\right)$$

$$= \binom{+}{-} \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 {v_A}^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 \binom{\tanh}{\coth} (m_0 x_0)}{(k^2 {v_A}^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \binom{\tanh}{\coth} (m_0 x_0)}$$

Parameter inversion

- **Observe**: ω, k, x_0, T_i, and R_{A}.
- **Solve** to find: v_A and hence B_0.

Parameter inversion

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Analytical inversion

Amplitude ratio

Parameter inversion

Parameter inversion

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Mode	Approximation of $k^2 v_{\mathrm{A}}{}^2/\omega^2$ using amplitude ratio, R_{A}		
	Thin slab	Incompressible	Low-beta
Sausage	$1+\frac{1}{x_0}\left(\frac{R_{\mathrm{A}}\frac{\rho_2}{\rho_0 m_2}+\frac{\rho_1}{\rho_0 m_1}}{R_{\mathrm{A}}+1}\right)$	$1+\left(\frac{R_{\mathrm{A}}\frac{\rho_2}{\rho_0}+\frac{\rho_1}{\rho_0}}{R_{\mathrm{A}}+1}\right)\coth kx_0$	$1+k\left(\frac{R_{\mathrm{A}}\frac{\rho_2}{\rho_0 m_2}+\frac{\rho_1}{\rho_0 m_1}}{R_{\mathrm{A}}+1}\right)\coth kx_0$
Kink	$1+k^2 x_0\left(\frac{R_{\mathrm{A}}\frac{\rho_2}{\rho_0 m_2}-\frac{\rho_1}{\rho_0 m_1}}{R_{\mathrm{A}}-1}\right)$	$1+\left(\frac{R_{\mathrm{A}}\frac{\rho_2}{\rho_0}-\frac{\rho_1}{\rho_0}}{R_{\mathrm{A}}-1}\right)\tanh kx_0$	$1+k\left(\frac{R_{\mathrm{A}}\frac{\rho_2}{\rho_0 m_2}-\frac{\rho_1}{\rho_0 m_1}}{R_{\mathrm{A}}-1}\right)\tanh kx_0$

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Numerical inversion

Amplitude ratio

Parameter inversion

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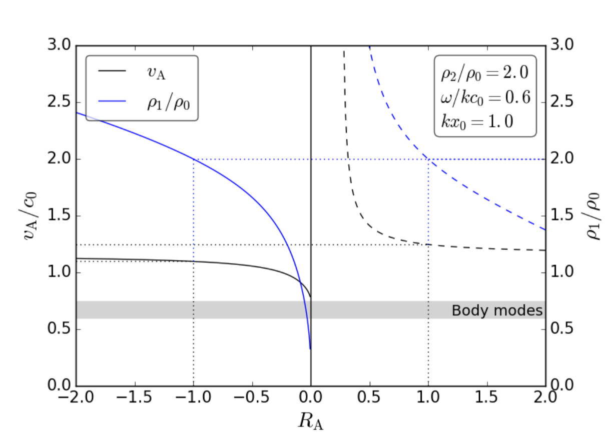
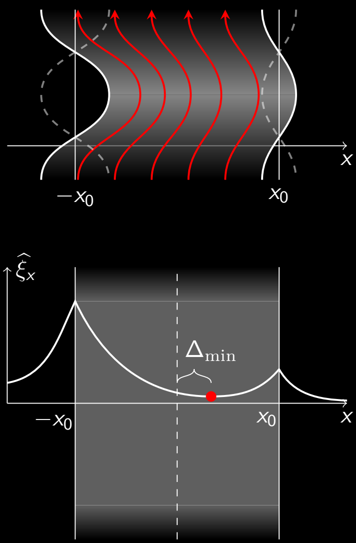


Minimum perturbation shift



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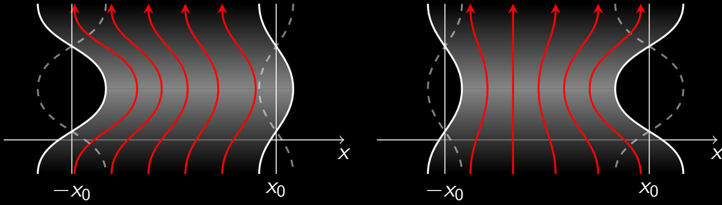

Quasi-kink:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}(D)$$

Quasi-sausage:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}\left(\frac{1}{D}\right)$$

where $$D = \frac{(k^2 {v_A}^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} \tanh(m_0 x_0) - \omega^2 m_0}{(k^2 {v_A}^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \tanh(m_0 x_0)}$$

Minimum perturbation shift

Parameter inversion

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Analytical inversion

Minimum perturbation shift

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Mode	Approximation of $k^2 v_{\mathrm{A}}^2/\omega^2$ using minimum perturbation shift, Δ_{min}		
	Thin slab	Incompressible	Low-beta
Quasi-sausage	$\frac{\rho_1}{\rho_0 m_1}(x_0+\Delta_{\mathrm{min}}) + \frac{1}{1+(\omega/kc_0)^2} + k^2 x_0 \Delta_{\mathrm{min}}$	$1+\frac{\rho_1}{\rho_0}\tanh k(x_0+\Delta_{\mathrm{min}})$	$1+\frac{k\rho_1}{m_1\rho_0}\tanh k(x_0+\Delta_{\mathrm{min}})$
Quasi-kink	$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, defined in text	$1+\frac{\rho_1}{\rho_0}\coth k(x_0+\Delta_{\mathrm{min}})$	$1+\frac{k\rho_1}{m_1\rho_0}\coth k(x_0+\Delta_{\mathrm{min}})$

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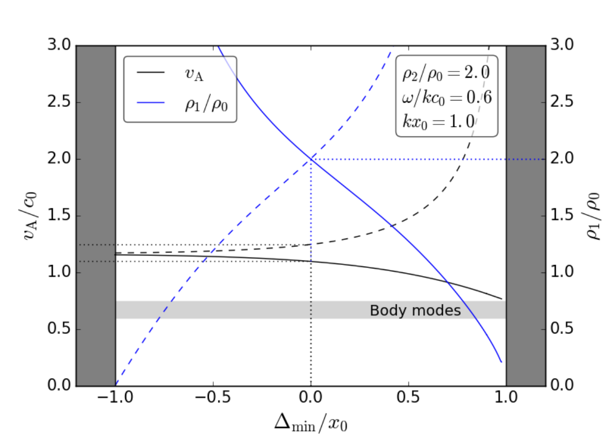
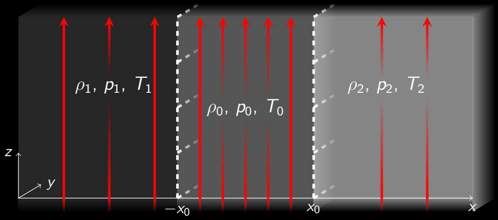


Outline

Further work

Add **magnetic field** outside the slab to model coronal structures. See **Zsámberger**, **Allcock** and **Erdélyi**, submitted.



Key result for mode identification:

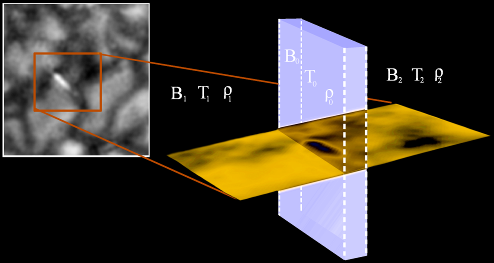
There can exist **quasi-symmetric modes** - where the oscillations of each interface have equal amplitude - even with asymmetric background.

Symmetric mode $\not\Rightarrow$ symmetric equilibrium

Future work

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,



Adaptation of Liu et al., 2017, by N. Zsámberger

Future work

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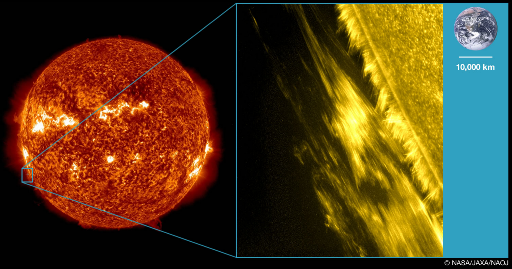
- **Prominences**,

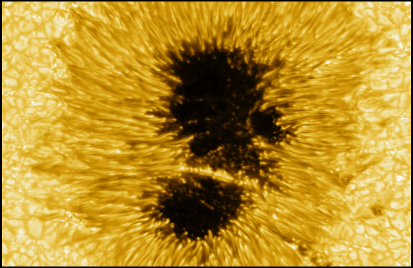


NASA

Future work

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,
- **Prominences**,
- Sunspot **light walls**.



Max Planck Institute for Solar System Research

Thank you

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