

Magneto-acoustic waves in an asymmetric magnetic slab

Progress in spatial magneto-seismology



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and
Robertus Erdélyi



The
University
Of
Sheffield.



Outline

- 1 Introduction
 - Waves on the Sun
 - Solar magneto-seismology (SMS)
 - A brief history
- 2 SMS with asymmetric wave-guides
 - Motivation
 - Mode identification
 - Amplitude ratio
 - Minimum perturbation shift
- 3 Looking ahead

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1 Introduction

- Waves on the Sun
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2 SMS with asymmetric wave-guides

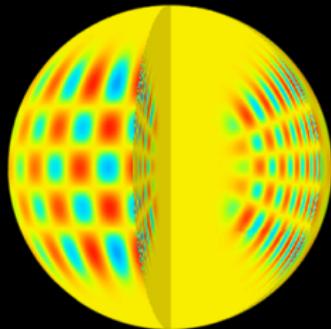
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3 Looking ahead

Waves on the Sun

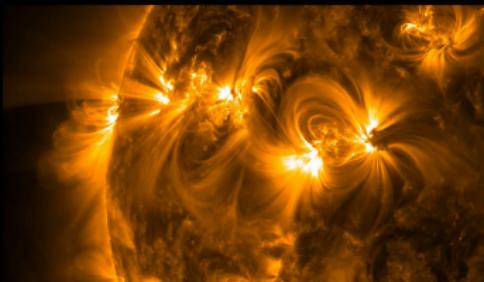
Global pressure modes
(p-modes):

- Standing modes,
- Spherical harmonics with global Sun as cavity,
- Global and local **helioseismology** for inference of sub-surface flows, density, temperature.



MHD waves:

- Propagating or standing modes,
- Guided by local plasma inhomogeneity,
- Local **magneto-seismology** for inference of background magnetic field strength, heat transport coefficients, density.



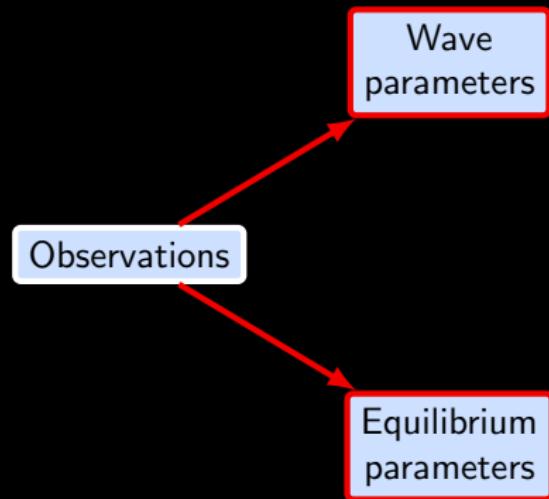
Solar magneto-seismology

An overview

Observations

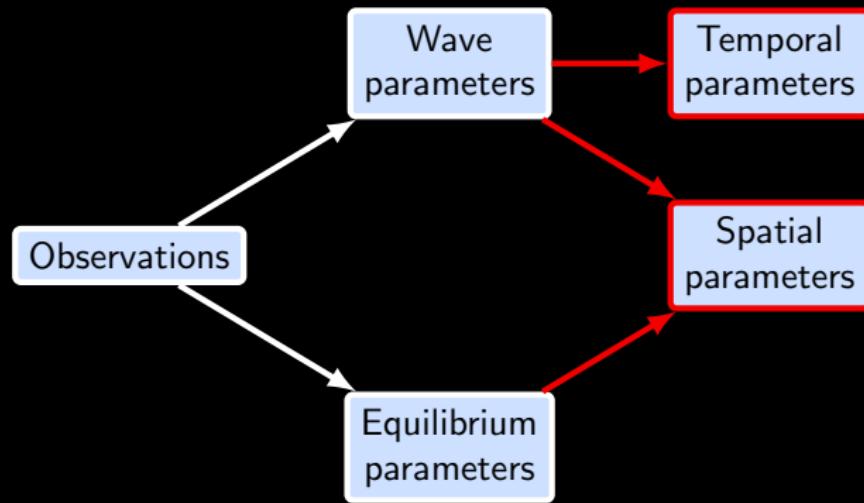
Solar magneto-seismology

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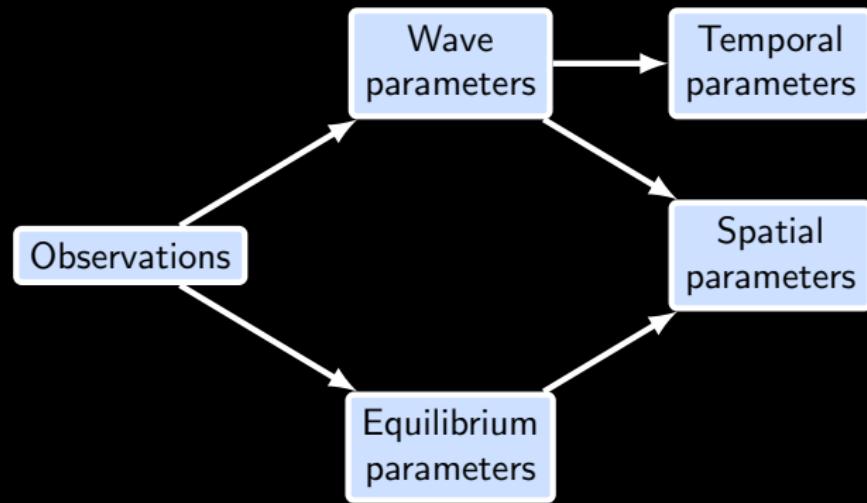
Solar magneto-seismology

An overview



Solar magneto-seismology

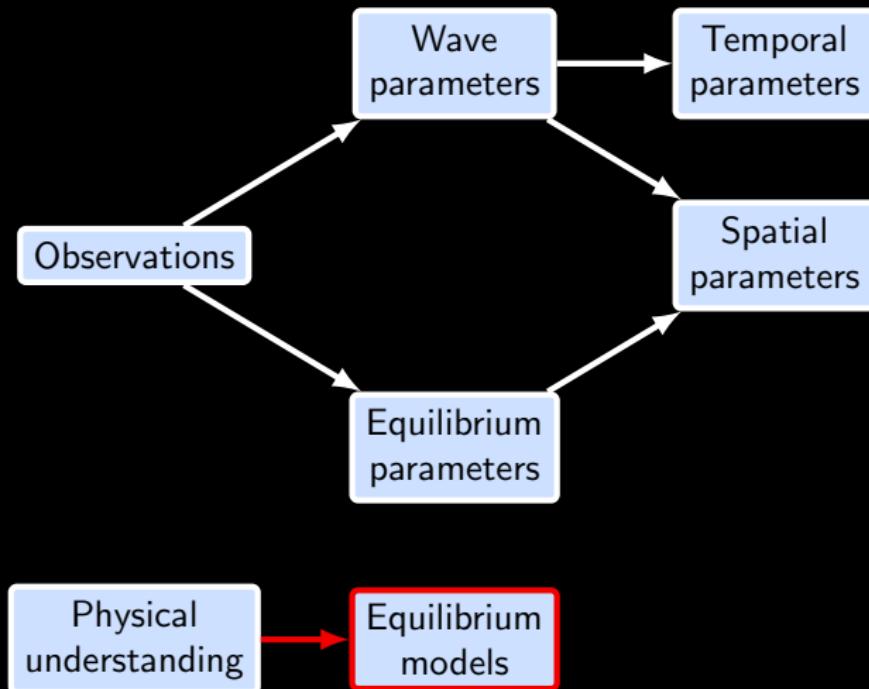
An overview



Physical
understanding

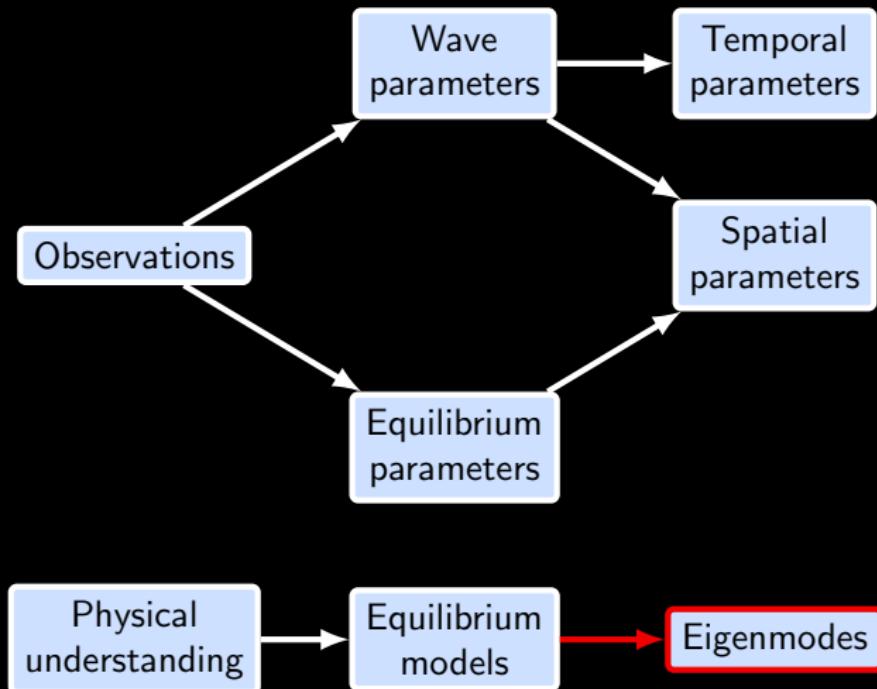
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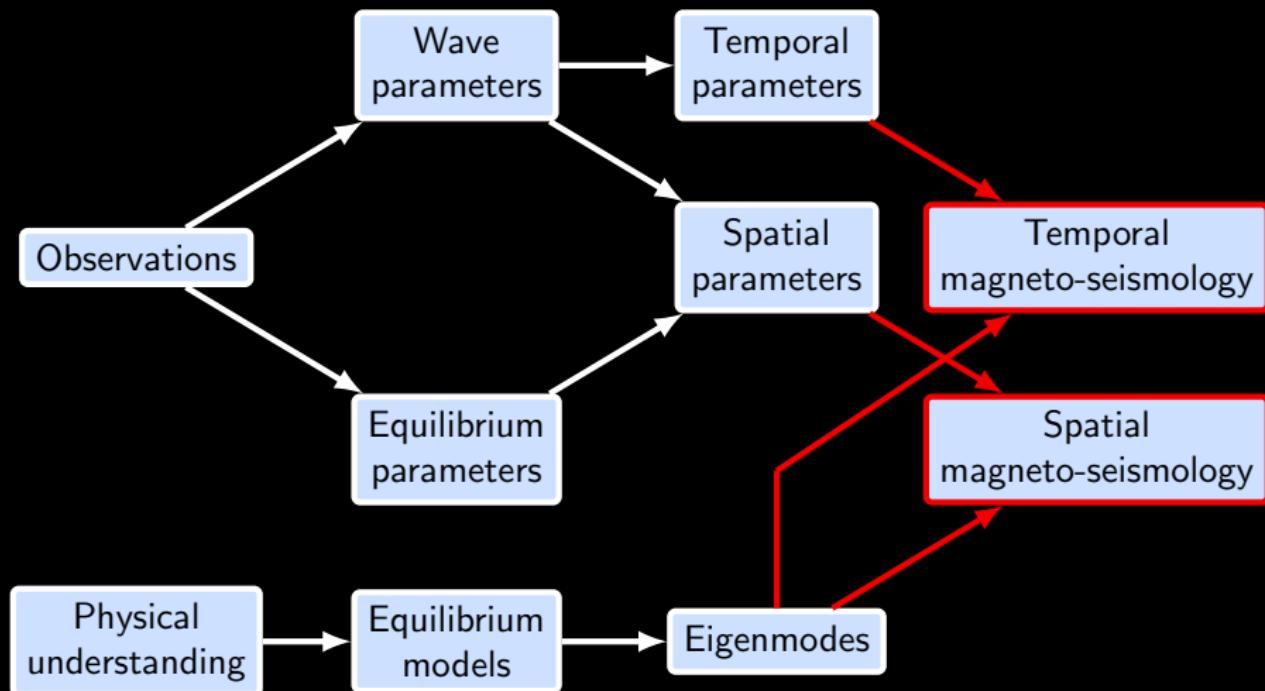
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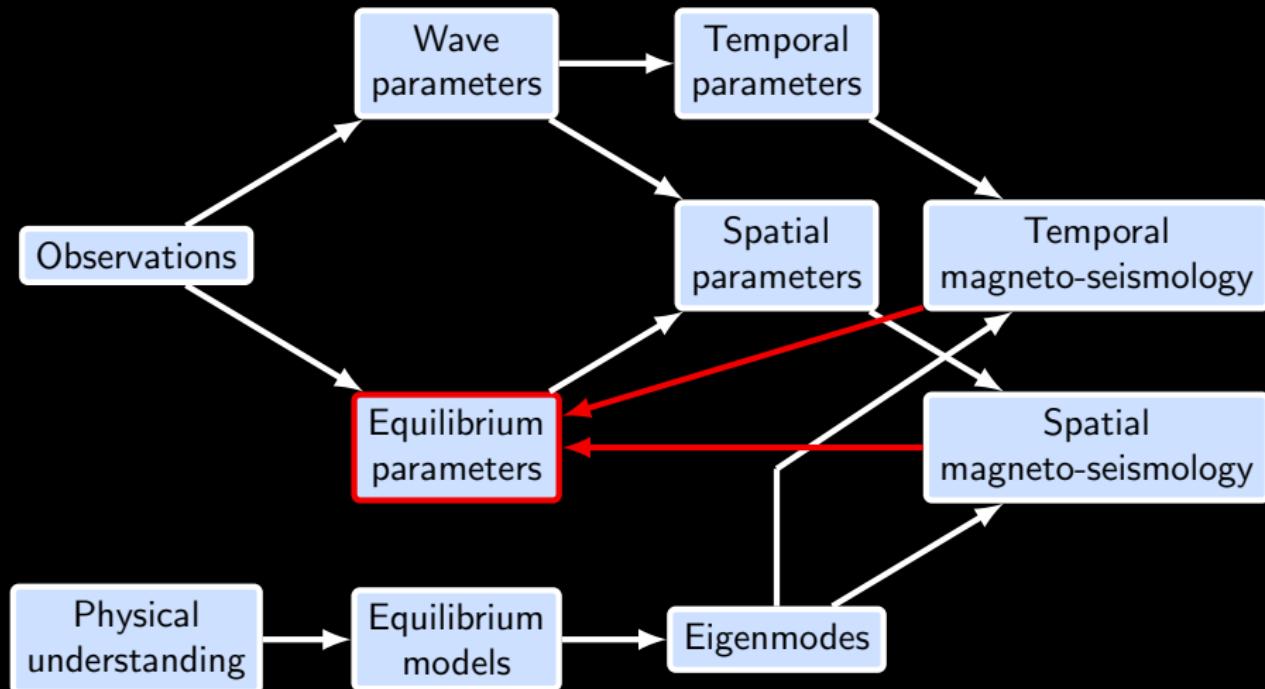
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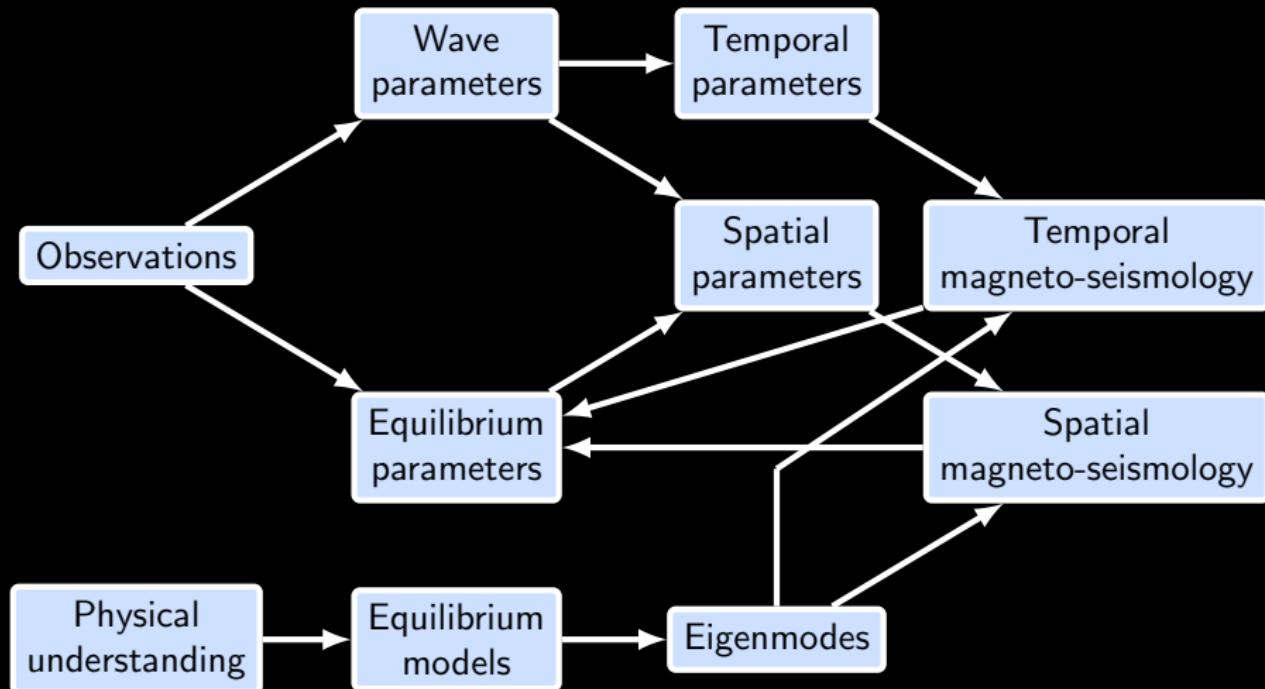
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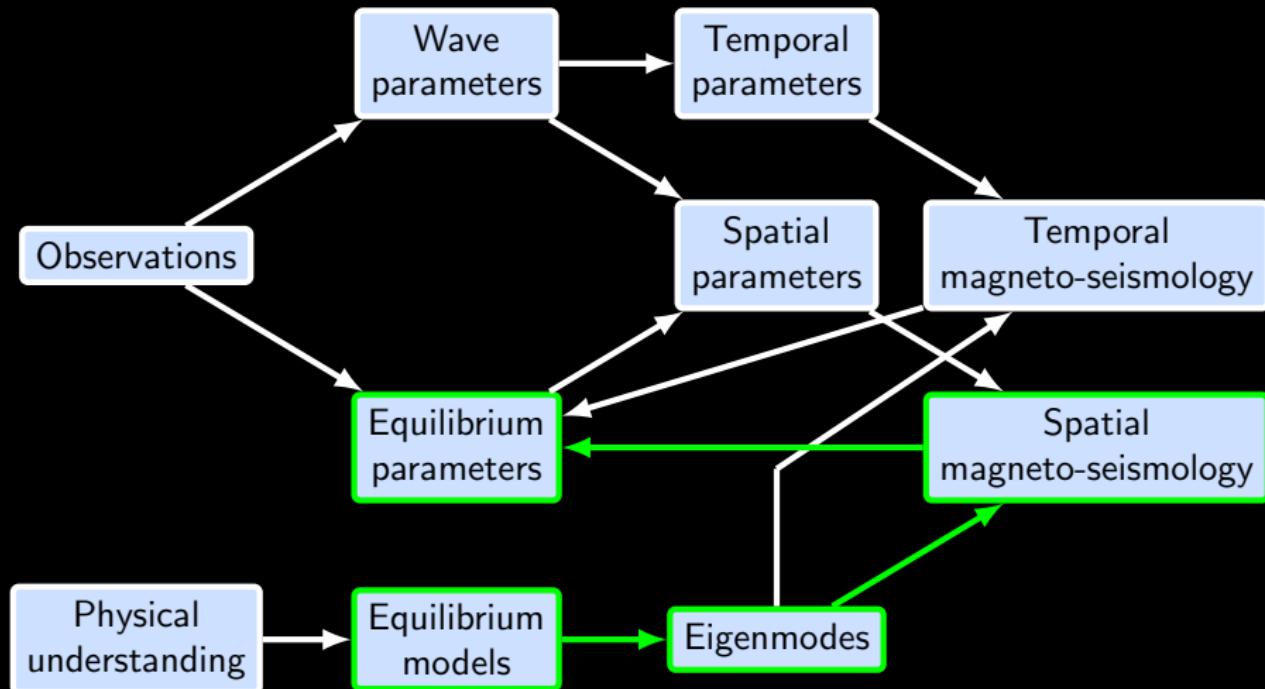
Solar magneto-seismology

An overview



Solar magneto-seismology

An overview



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

1970

1980

1990

2000



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

1970 Uchida, 1970 - **Moreton** wavefronts;

1980

1990

2000



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

1970 Uchida, 1970 - **Moreton wavefronts**;
Rosenburg 1970 - **MHD waves** cause pulsations in
synchrotron radiation with measurable period

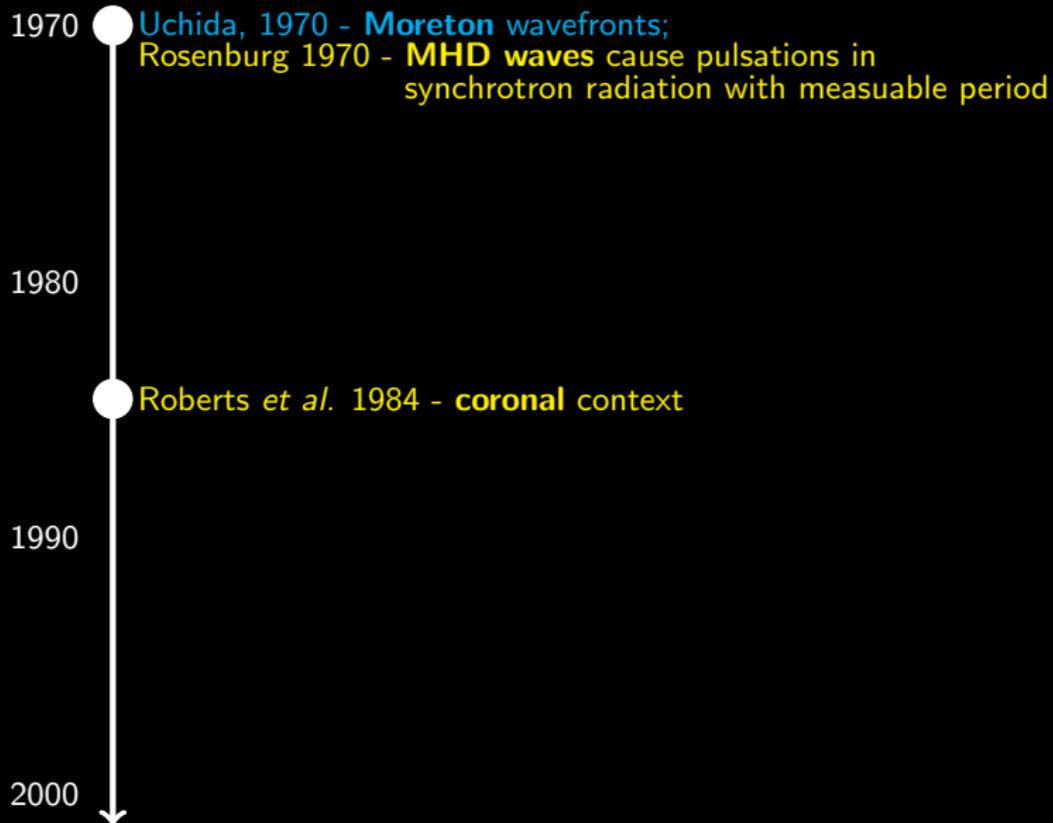
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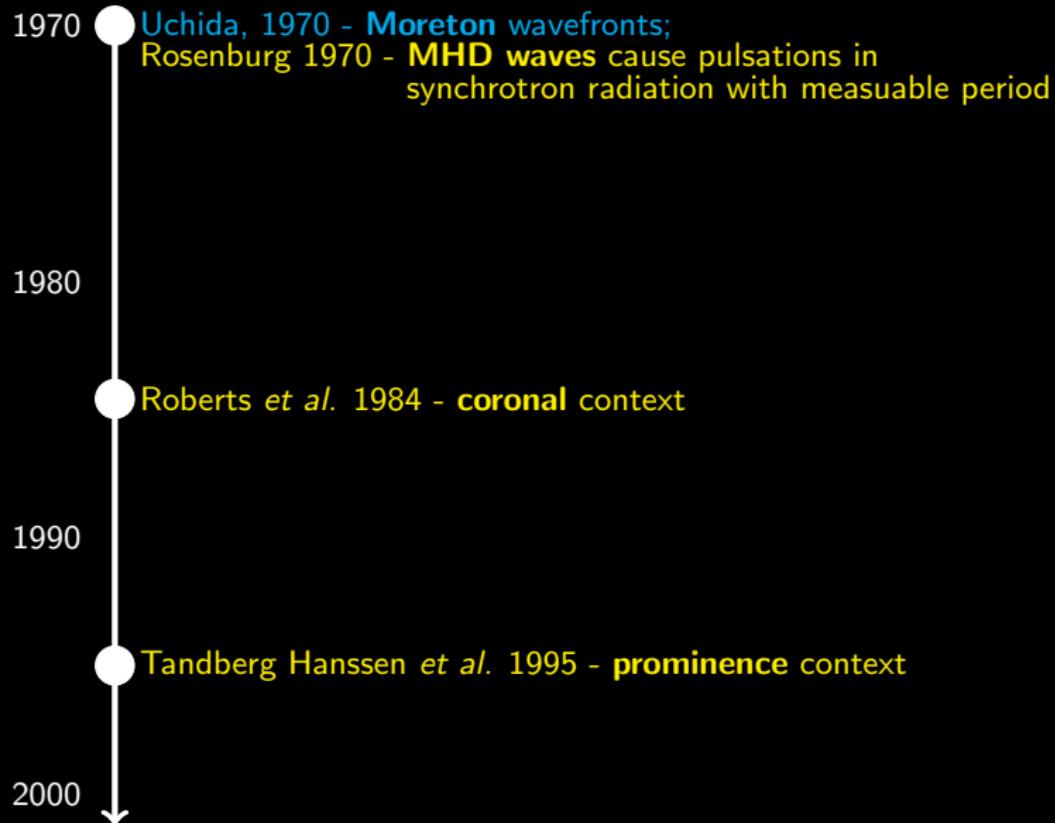
Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



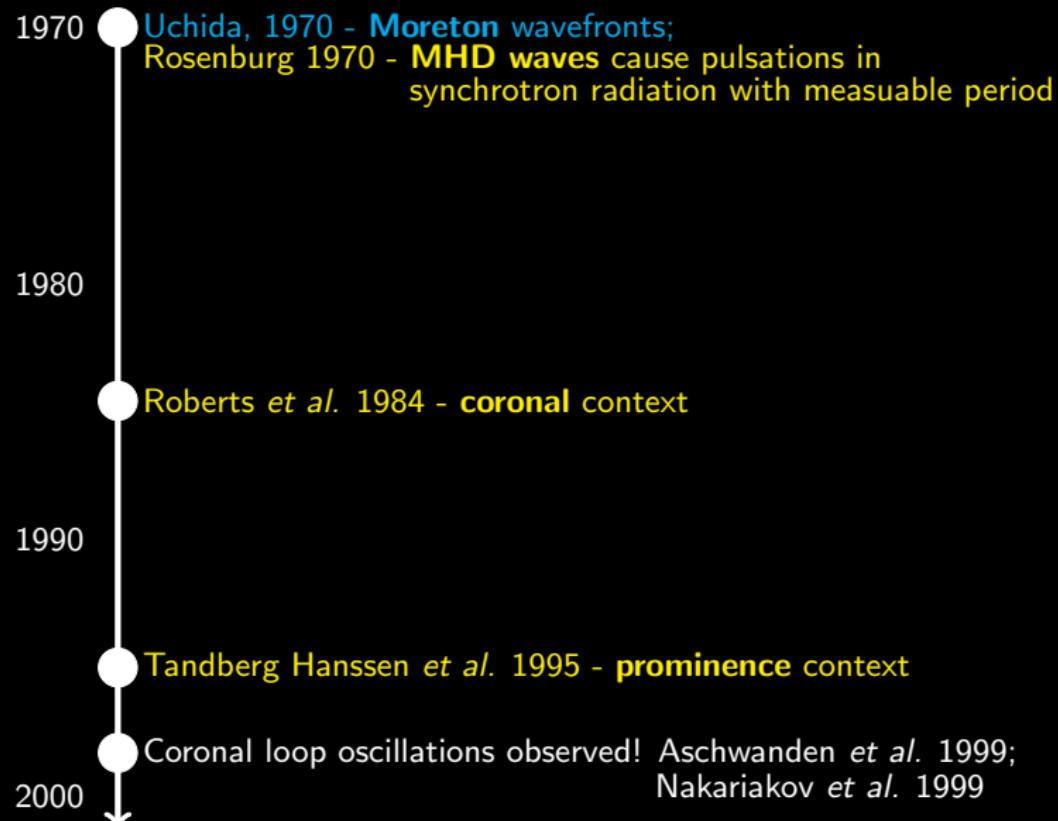
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A brief history - **temporal** and **spatial** seismology



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology



Solar magneto-seismology

A brief history - **temporal** and **spatial** seismology

2000

2005

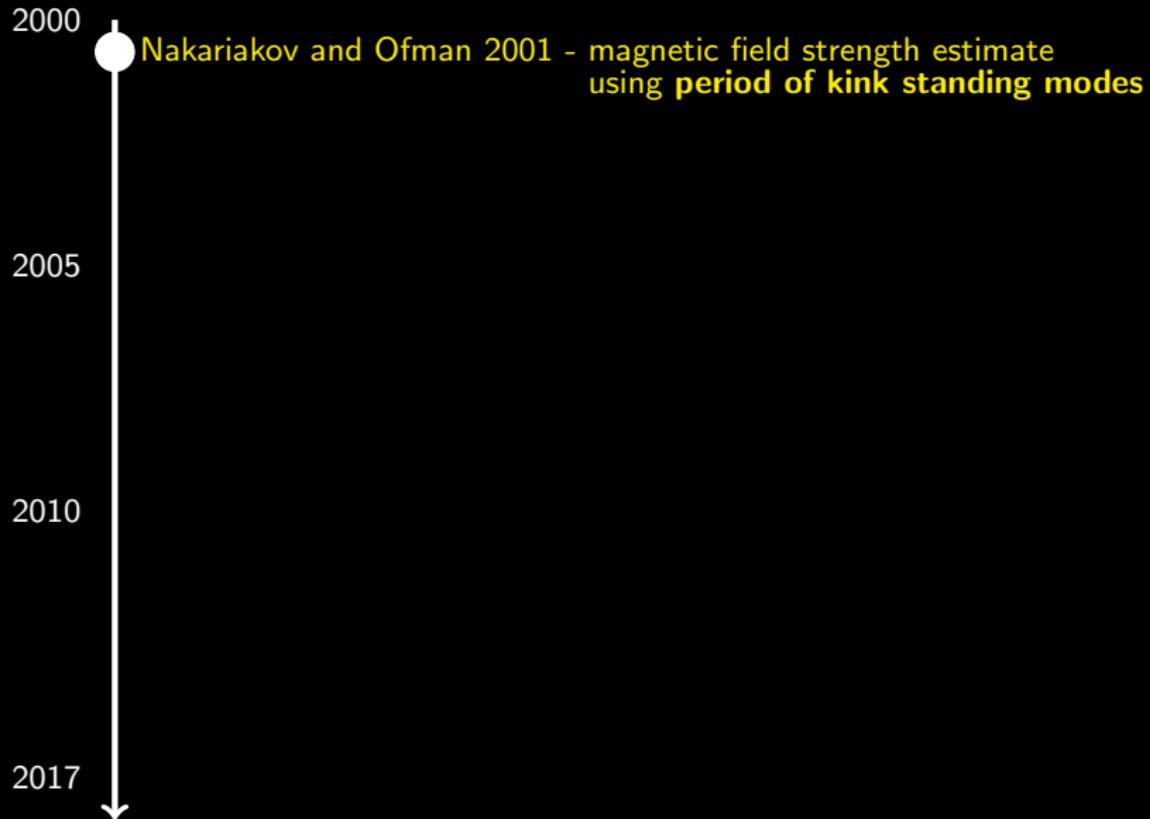
2010

2017



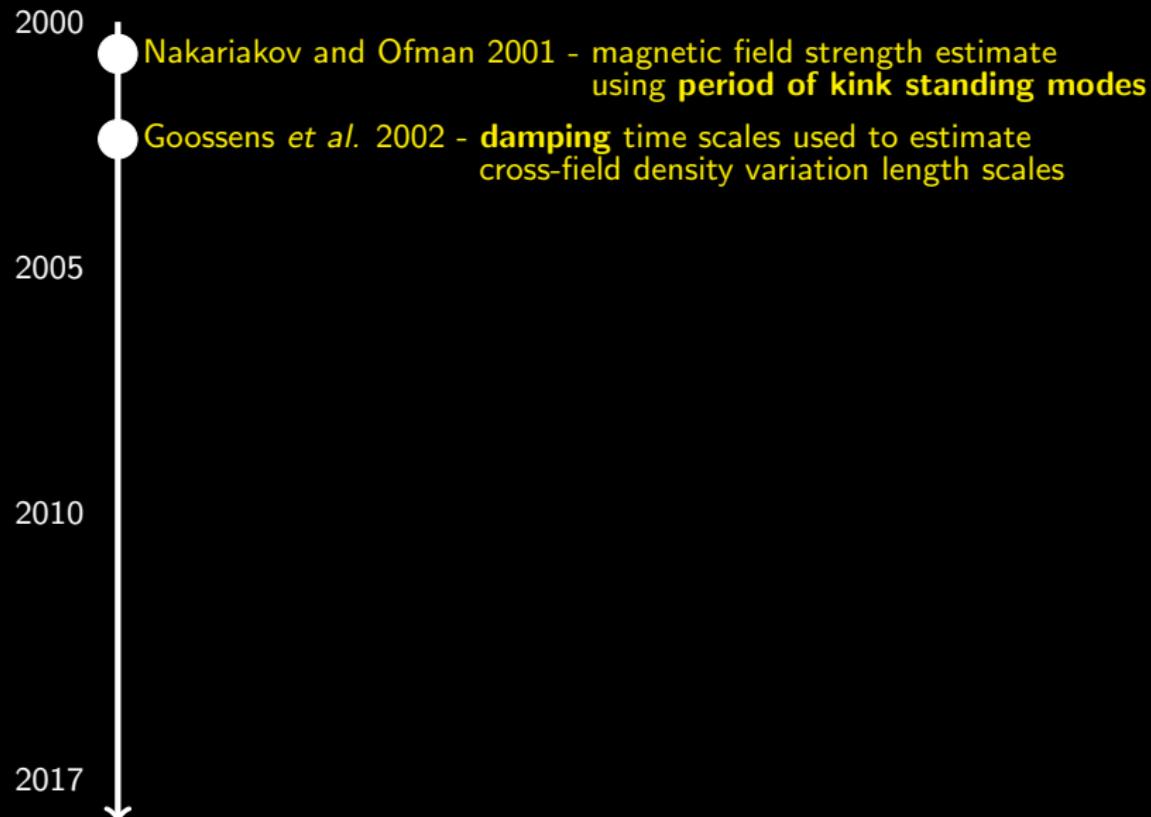
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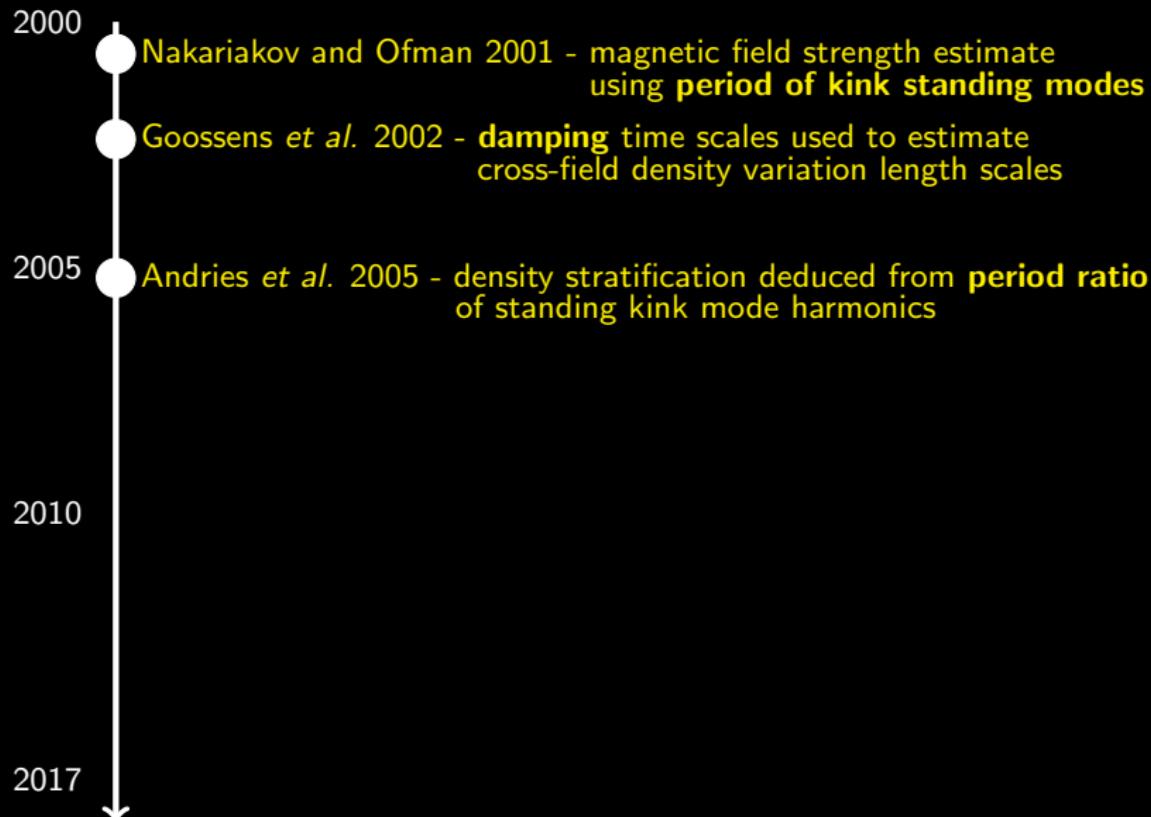
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A brief history - **temporal** and **spatial** seismology



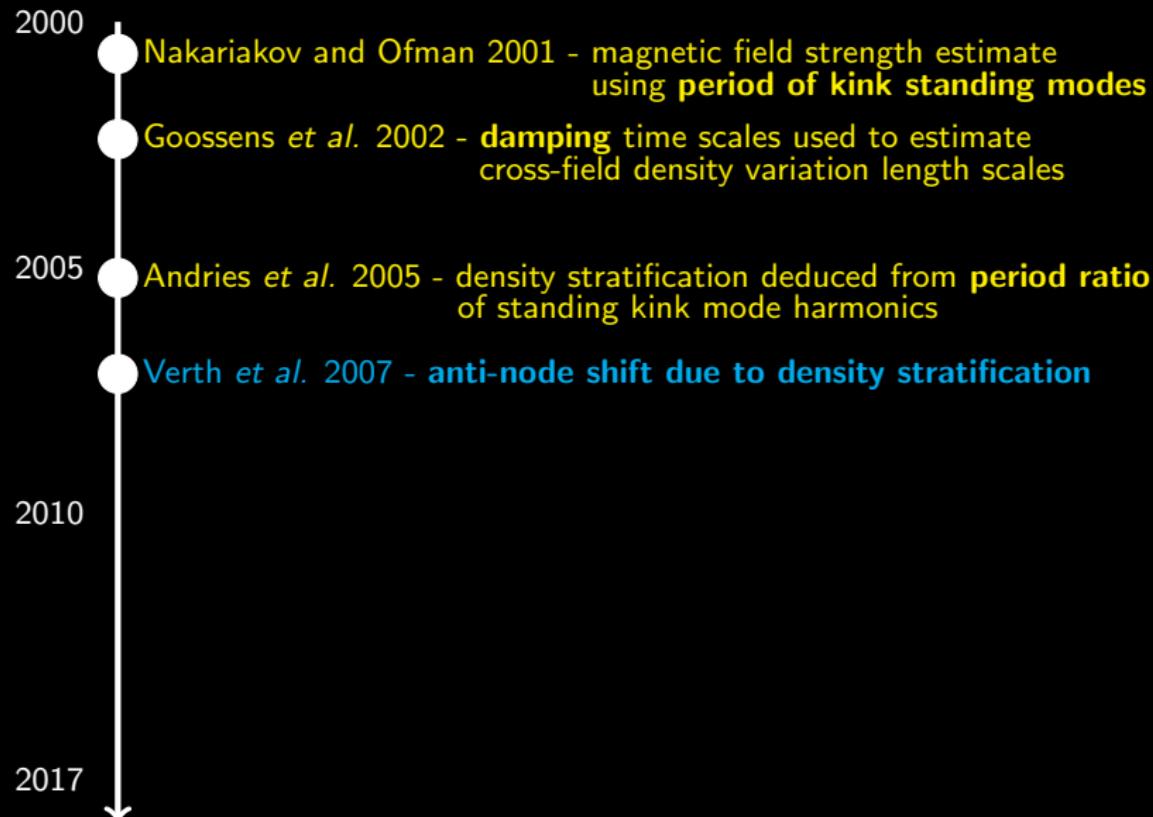
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A brief history - **temporal** and **spatial** seismology



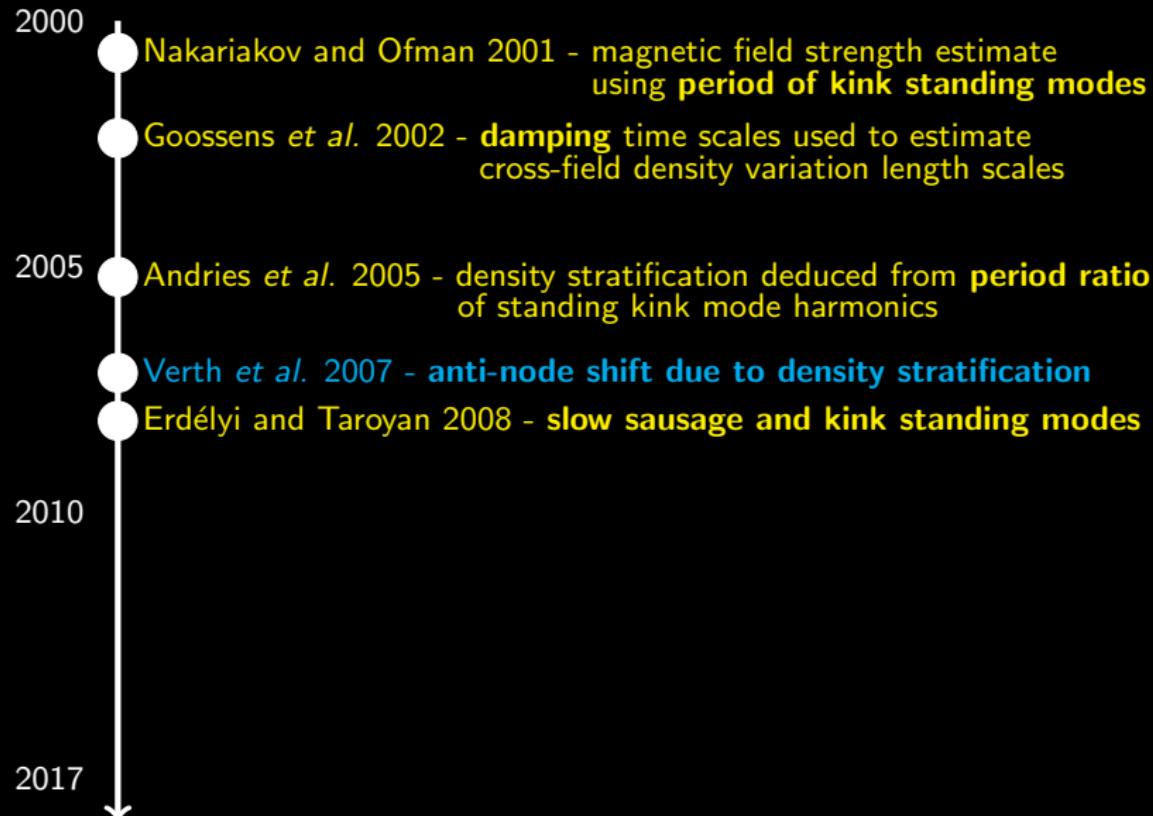
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A brief history - **temporal** and **spatial** seismology



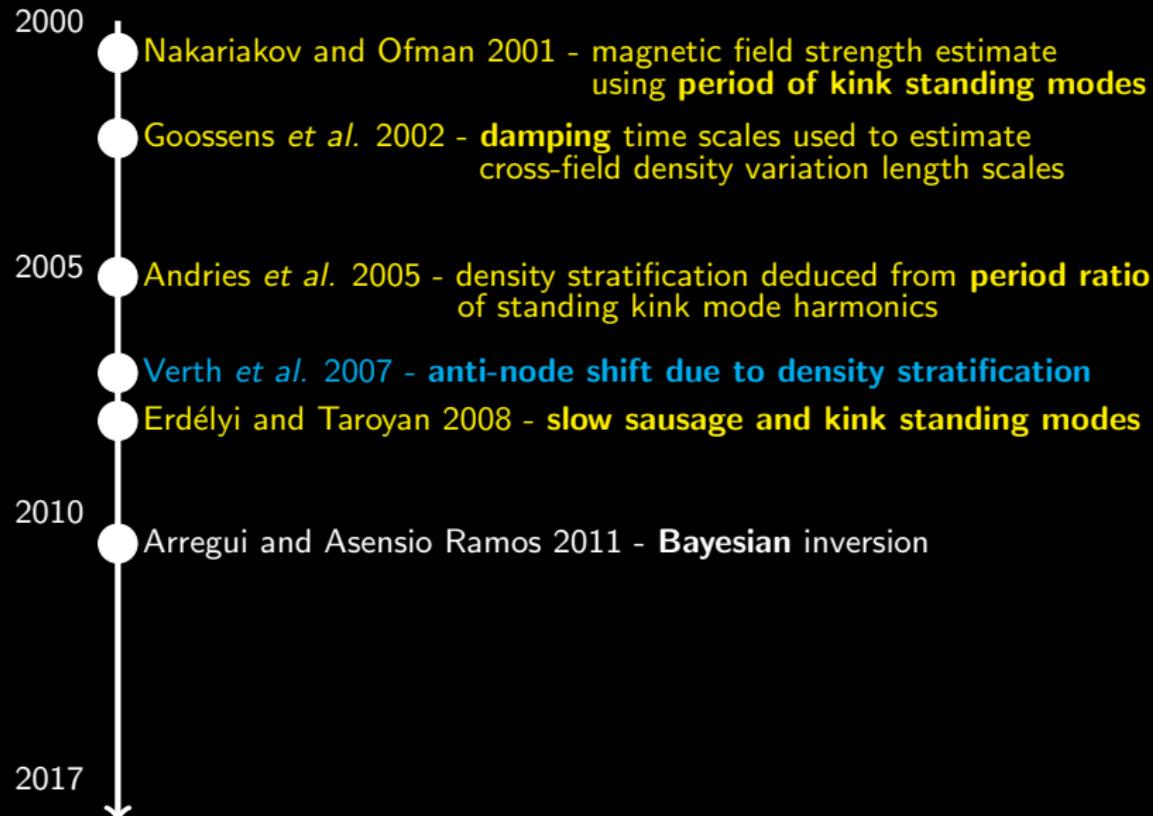
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A brief history - **temporal** and **spatial** seismology



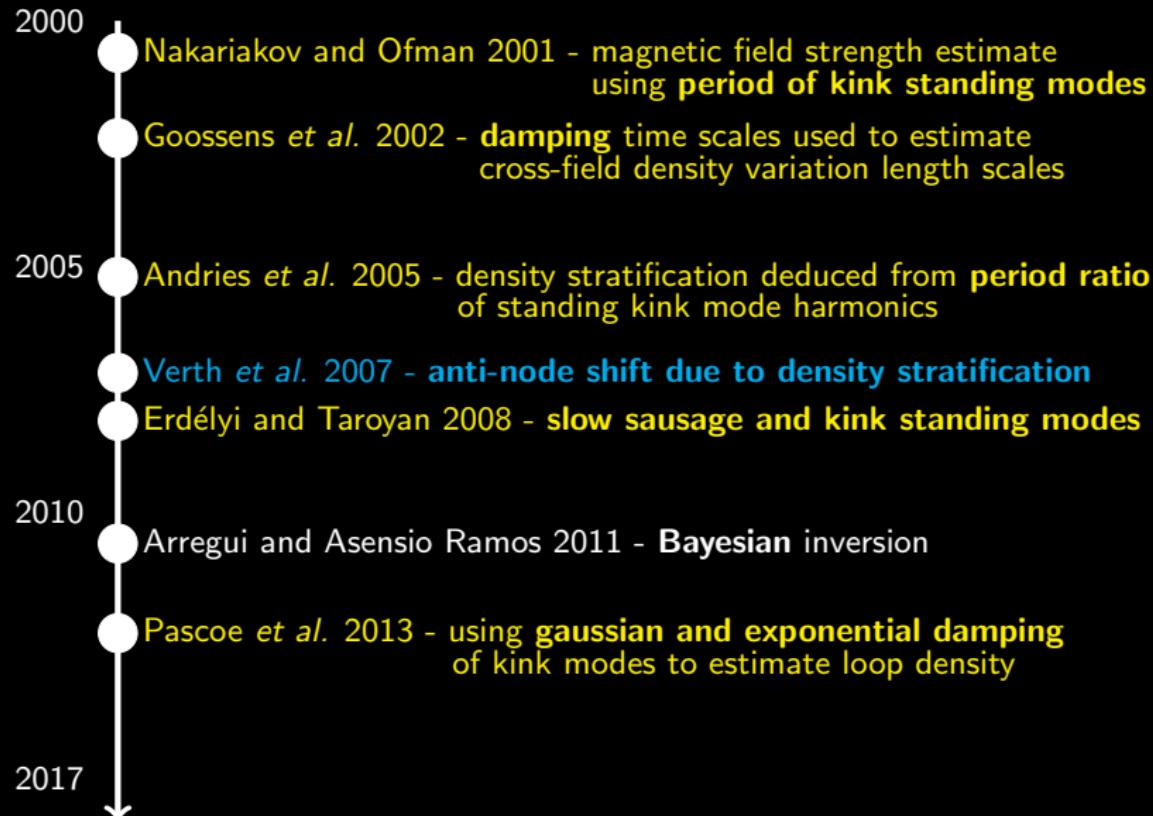
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Solar magneto-seismology

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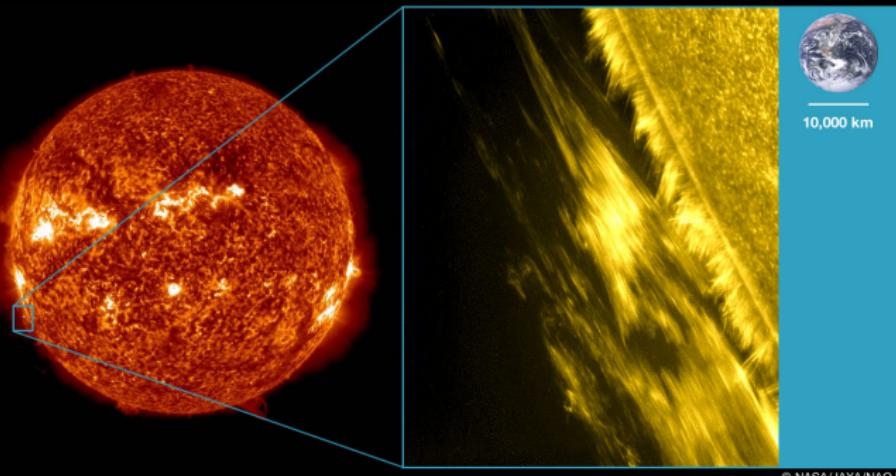
-
- The timeline diagram illustrates the progression of solar magneto-seismology research from 2000 to 2017. A vertical white line with circular markers at each year mark serves as the timeline. To the left of the line, the years are listed: 2000, 2005, 2010, and 2017. To the right of the line, the research findings are detailed:
- 2000: Nakariakov and Ofman 2001 - magnetic field strength estimate using **period of kink standing modes**
 - 2002: Goossens *et al.* 2002 - **damping** time scales used to estimate cross-field density variation length scales
 - 2005: Andries *et al.* 2005 - density stratification deduced from **period ratio** of standing kink mode harmonics
 - 2007: Verth *et al.* 2007 - **anti-node shift due to density stratification**
 - 2008: Erdélyi and Taroyan 2008 - **slow sausage and kink standing modes**
 - 2010: Arregui and Asensio Ramos 2011 - **Bayesian** inversion
 - 2013: Pascoe *et al.* 2013 - using **gaussian and exponential damping** of kink modes to estimate loop density
 - 2017: Magyar *et al.* - **global dynamic coronal seismology**

Outline

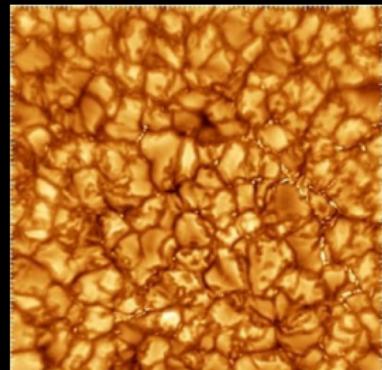
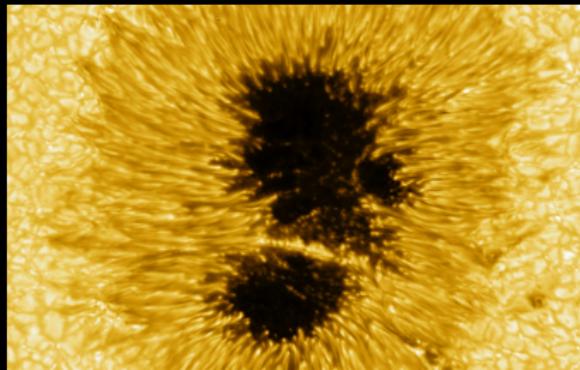
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Asymmetric magnetic slab

Motivation

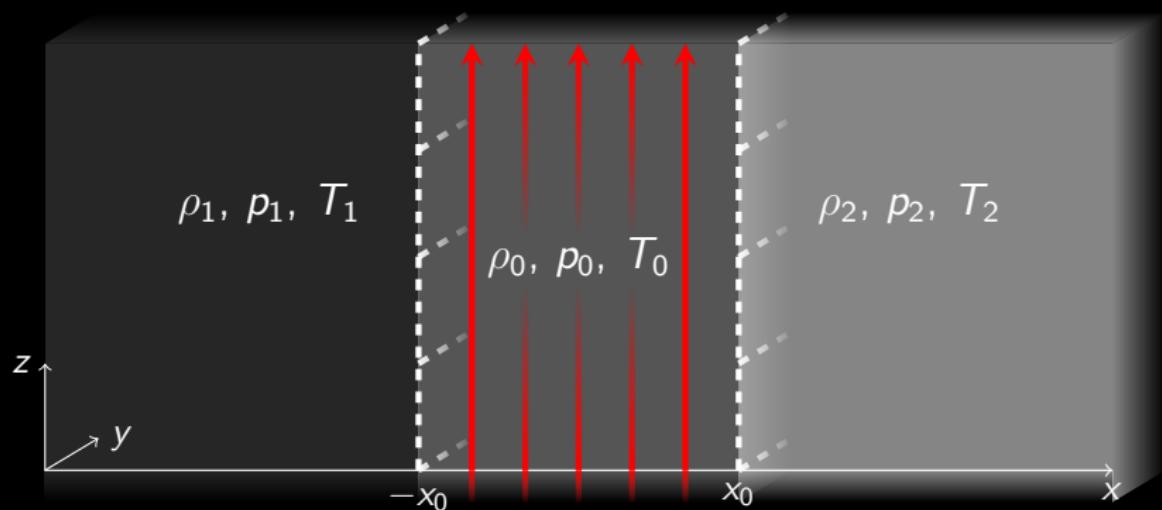


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Asymmetric magnetic slab

Equilibrium conditions



- Uniform magnetic field in the slab.
- Field-free plasma outside.
- **Different** density and pressure on each side.

Asymmetric magnetic slab

Governing equations

Ideal MHD equations:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \frac{1}{\mu} \mathbf{B} \times (\nabla \times \mathbf{B}), \quad \text{momentum}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \text{mass}$$

$$\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) = 0, \quad \text{energy}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad \text{magnetic flux}$$

\mathbf{v} = plasma velocity,

ρ = density,

μ = magnetic permeability,

\mathbf{B} = magnetic field strength,

p = pressure,

γ = adiabatic index.

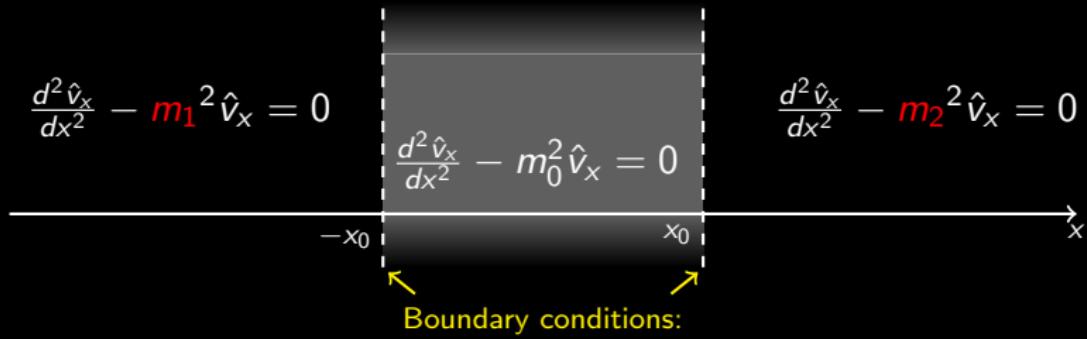
Asymmetric magnetic slab

Fourier decomposition

Look for *plane wave* solutions of the form:

$$v_x(\mathbf{x}, t) = \hat{v}_x(x) e^{i(kz - \omega t)}, \quad v_y(\mathbf{x}, t) = 0, \quad v_z(\mathbf{x}, t) = \hat{v}_z(x) e^{i(kz - \omega t)},$$

to arrive at the following ODEs:



$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad m_{1,2}^2 = k^2 - \frac{\omega^2}{c_{1,2}^2},$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}, \quad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

Asymmetric magnetic slab

Eigenmodes

Dispersion relation:

$$\frac{\omega^4 m_0^2}{k^2 v_A^2 - \omega^2} + \frac{\rho_0}{\rho_1} \textcolor{red}{m_1} \frac{\rho_0}{\rho_2} \textcolor{red}{m_2} (k^2 v_A^2 - \omega^2) \\ - \frac{1}{2} m_0 \omega^2 \left(\frac{\rho_0}{\rho_1} \textcolor{red}{m_1} + \frac{\rho_0}{\rho_2} \textcolor{red}{m_2} \right) (\tanh m_0 x_0 + \coth m_0 x_0) = 0,$$

See **Allcock** and Erdélyi, 2017.

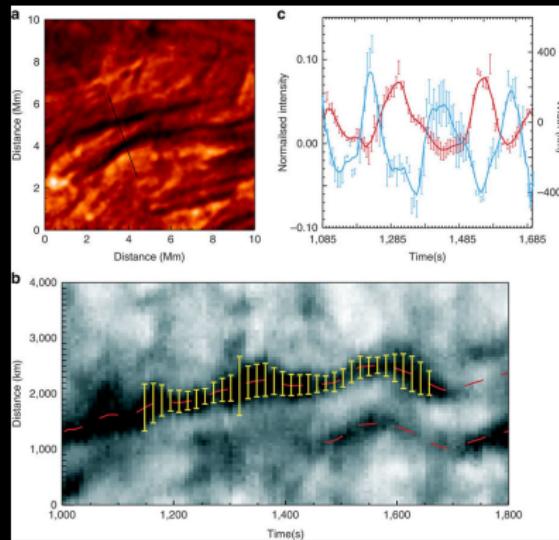
$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad \textcolor{red}{m_{1,2}}^2 = k^2 - \frac{\omega^2}{\textcolor{red}{c_{1,2}}^2},$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}, \quad v_A = \frac{B_0}{\sqrt{\mu \rho_0}},$$

Asymmetric magnetic slab

Mode identification

Superposition of symmetric kink and sausage modes...

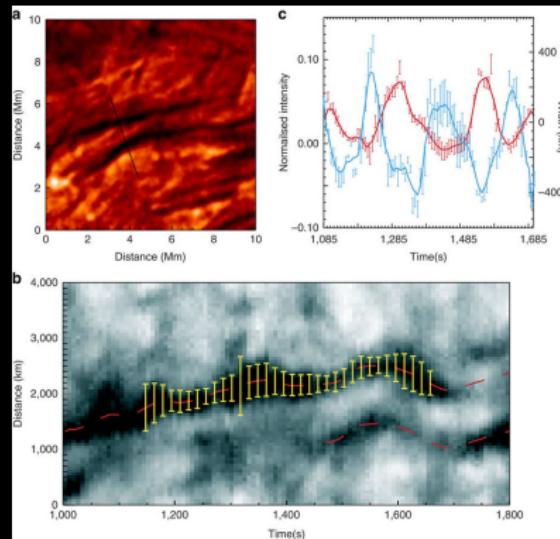


Morton *et al.* 2012

Asymmetric magnetic slab

Mode identification

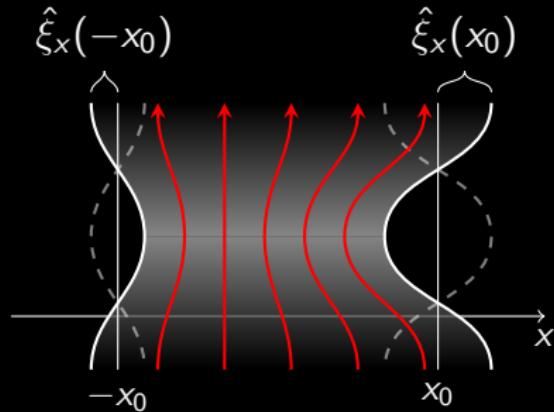
Superposition of symmetric kink and sausage modes...



Morton *et al.* 2012

or asymmetric kink mode?

Amplitude ratio



Amplitude ratio

$$R_A := \frac{\hat{\xi}_x(x_0)}{\hat{\xi}_x(-x_0)}$$

(Top = quasi-kink
Bottom = quasi-sausage)

$$= \left(\frac{+}{-}\right) \frac{\rho_1 m_2}{\rho_2 m_1} \frac{(k^2 v_A^2 - \omega^2) m_1 \frac{\rho_0}{\rho_1} - \omega^2 m_0 \left(\frac{\tanh}{\coth}\right) (m_0 x_0)}{(k^2 v_A^2 - \omega^2) m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \left(\frac{\tanh}{\coth}\right) (m_0 x_0)}$$

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
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Analytical inversion

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .

Mode	Approximation of $k^2 v_A^2 / \omega^2$ using amplitude ratio, R_A		
	Thin slab	Incompressible	Low-beta
Sausage	$1 + \frac{1}{x_0} \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} + \frac{\rho_1}{\rho_0 m_1}}{R_A + 1} \right)$	$1 + \left(\frac{R_A \frac{\rho_2 + \rho_1}{\rho_0}}{R_A + 1} \right) \coth kx_0$	$1 + k \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} + \frac{\rho_1}{\rho_0 m_1}}{R_A + 1} \right) \coth kx_0$
Kink	$1 + k^2 x_0 \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} - \frac{\rho_1}{\rho_0 m_1}}{R_A - 1} \right)$	$1 + \left(\frac{R_A \frac{\rho_2 - \rho_1}{\rho_0}}{R_A - 1} \right) \tanh kx_0$	$1 + k \left(\frac{R_A \frac{\rho_2}{\rho_0 m_2} - \frac{\rho_1}{\rho_0 m_1}}{R_A - 1} \right) \tanh kx_0$

Amplitude ratio

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and R_A .
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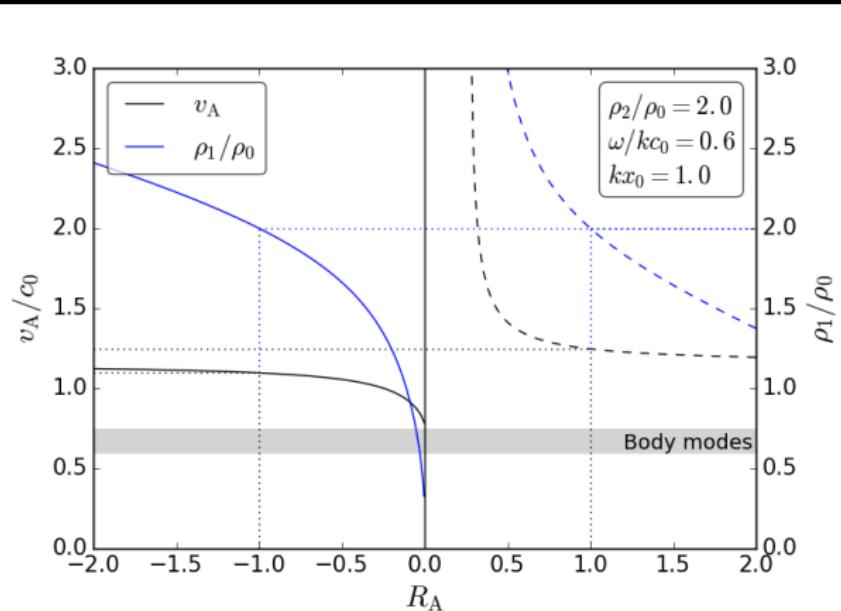
Numerical inversion

Amplitude ratio

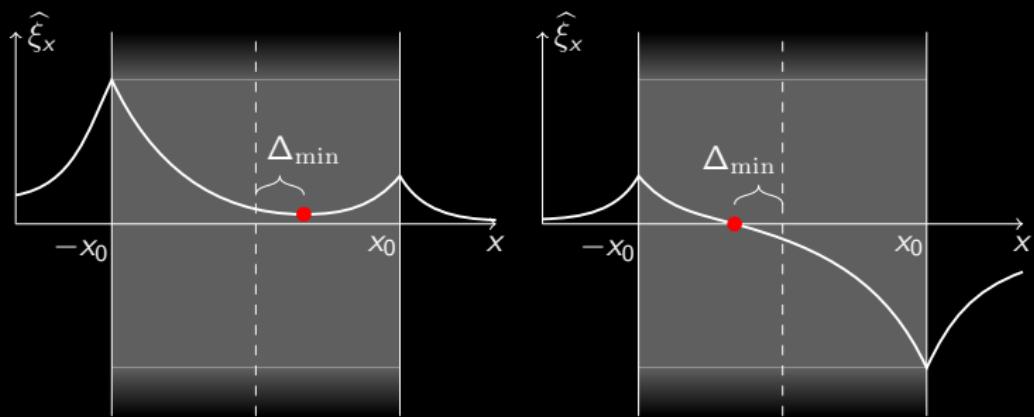
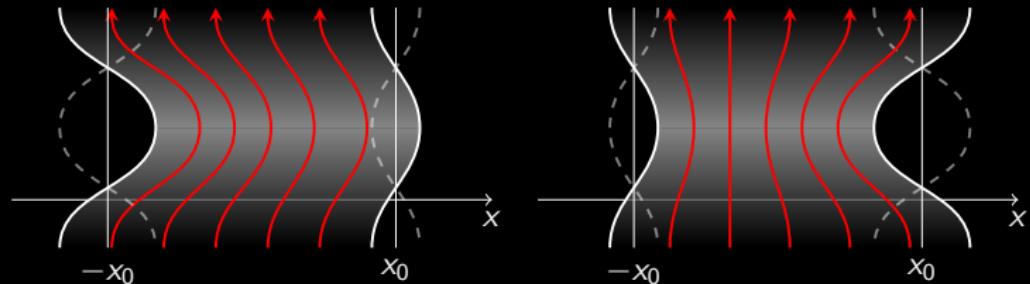
Parameter inversion

Parameter inversion

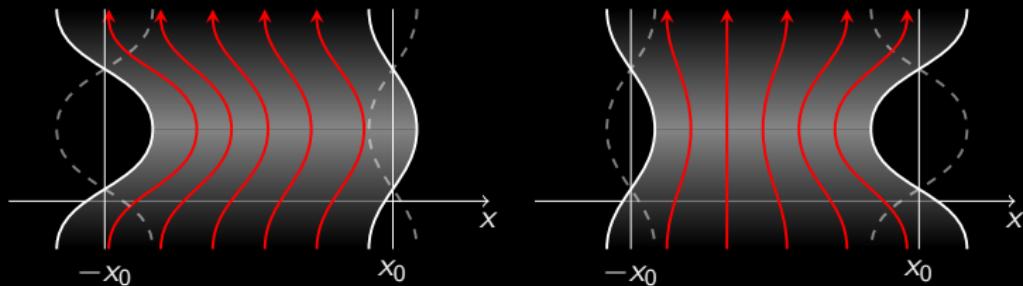
- **Observe:** ω , k , x_0 , T_i , and R_A .
- **Solve** to find: v_A and hence B_0 .



Minimum perturbation shift



Minimum perturbation shift



Quasi-kink:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1}(D)$$

Quasi-sausage:

$$\Delta_{\min} = \frac{1}{m_0} \tanh^{-1} \left(\frac{1}{D} \right)$$

where $D = \frac{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} \tanh(m_0 x_0) - \omega^2 m_0}{(k^2 v_A^2 - \omega^2)m_2 \frac{\rho_0}{\rho_2} - \omega^2 m_0 \tanh(m_0 x_0)}$

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .

Minimum perturbation shift

Parameter inversion

Parameter inversion

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Analytical inversion

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
- **Solve** to find: v_A and hence B_0 .

Mode	Approximation of $k^2 v_A^2 / \omega^2$ using minimum perturbation shift, Δ_{\min}		
	Thin slab	Incompressible	Low-beta
Quasi-sausage	$\frac{\rho_1}{\rho_0 m_1} (x_0 + \Delta_{\min}) + \frac{1}{1 + (\omega/kc_0)^2} + k^2 x_0 \Delta_{\min}$	$1 + \frac{\rho_1}{\rho_0} \tanh k(x_0 + \Delta_{\min})$	$1 + \frac{k\rho_1}{m_1\rho_0} \tanh k(x_0 + \Delta_{\min})$
Quasi-kink	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, defined in text	$1 + \frac{\rho_1}{\rho_0} \coth k(x_0 + \Delta_{\min})$	$1 + \frac{k\rho_1}{m_1\rho_0} \coth k(x_0 + \Delta_{\min})$

Minimum perturbation shift

Parameter inversion

Parameter inversion

- **Observe:** ω , k , x_0 , T_i , and Δ_{\min} .
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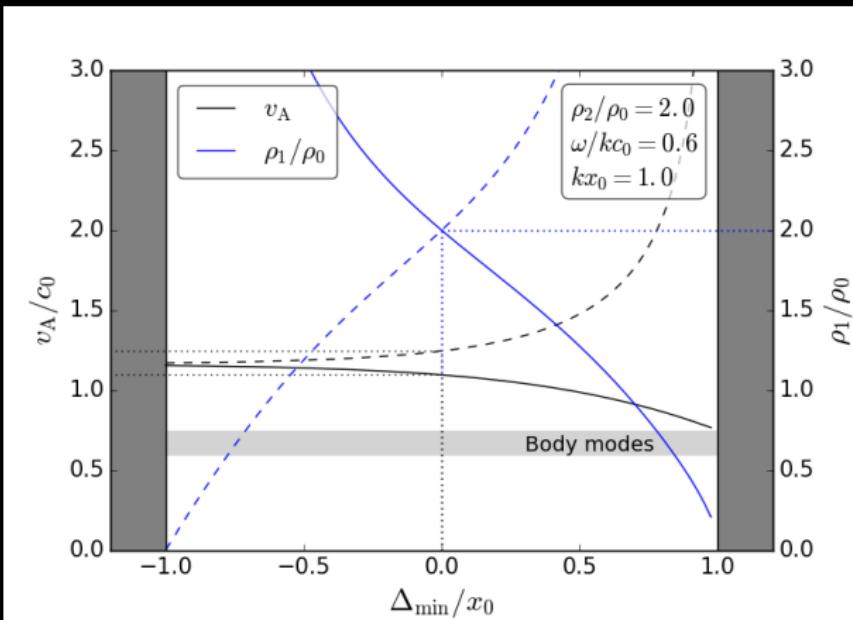
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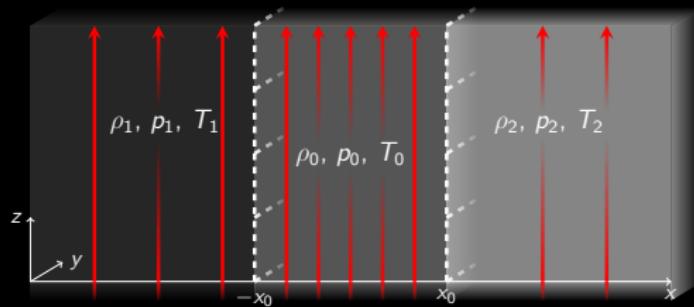
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3 Looking ahead

Further work

External magnetic field

Add **magnetic field** outside the slab - coronal structures. See **Zsamberger, Allcock and Erdelyi**, submitted.



Key result for mode identification:

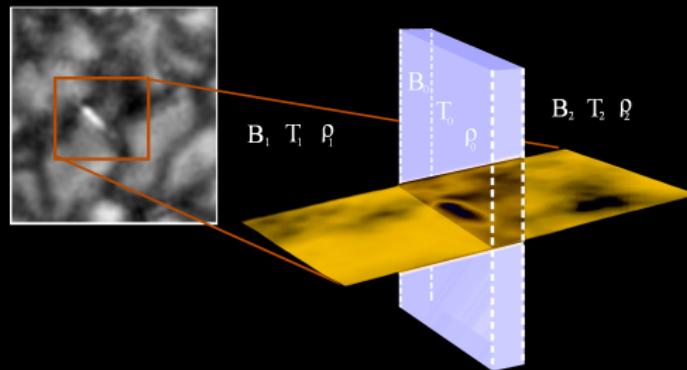
There can exist **quasi-symmetric modes** - where the oscillations of each interface have equal amplitude - even with asymmetric background.

Symmetric mode $\not\Rightarrow$ symmetric equilibrium

Future work

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,

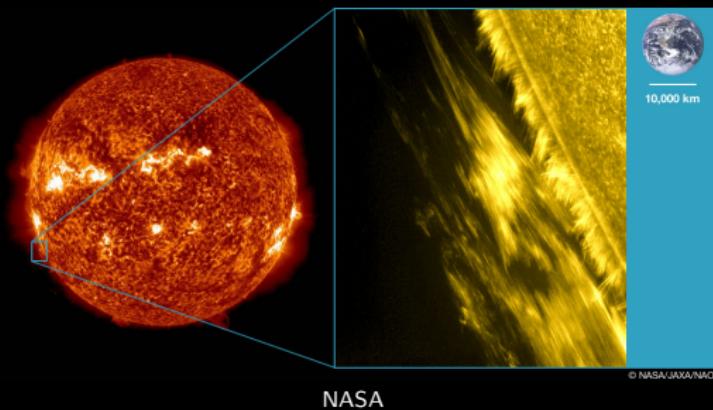


Adaptation of Liu et al., 2017, by N. Zsámberger

Future work

Apply to observations of MHD waves in, for example:

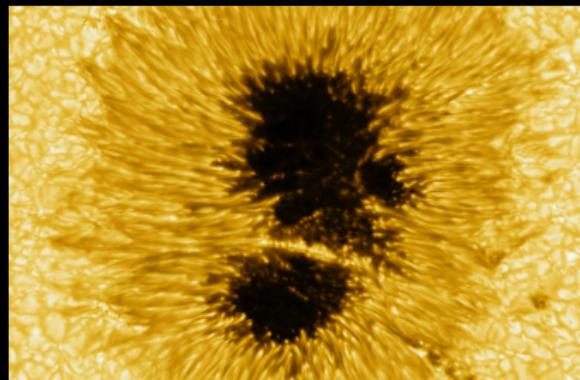
- Elongated **magnetic bright points**,
- **Prominences**,



Future work

Apply to observations of MHD waves in, for example:

- Elongated **magnetic bright points**,
- **Prominences**,
- Sunspot **light walls**.



Max Planck Institute for Solar System Research

Thank you



matthew_allcock



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