A blue and white logo

Description automatically generated

University of Hertfordshire

School of Engineering and Computer Science

MSc Computer Science (Data science and analytics)

Module: 7COM1039-0509-2022 – Advanced Computer Science Masters Project

Date:18-09-2023

**Final Project Report**

Project title – A study on the performance of GARCH and Stochastic Volatility models for Forecasting Stock Market Volatility: A Time Series Analysis

Name: Mallesh Dharmaraj

Student ID: 20070455

Supervisor: Gani Nashi

**Declaration**

This report is submitted in partial fulfilment of the requirement for the degree of Master of  
Science in **7COM1039-0509-2022 – Advanced Computer Science Masters Project** at the University of Hertfordshire (UH). It is my own work except where indicated in the report. I did not use human participants in my MSc Project.

**Mallesh Dharmaraj**.  
I hereby give permission for the report to be made available on the university  
website provided the source is acknowledged.

**Abstract**

This research project delves into the intricate world of financial market volatility forecasting, employing two prominent models: the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and the Stochastic Volatility (SV) model. The primary objective is to rigorously assess the effectiveness of these models in predicting stock market volatility across diverse time periods and forecasting horizons. The project meticulously follows a well-defined methodology, encompassing data collection, model implementation, ethical considerations, and extensive performance evaluations, including metrics such as Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE).

Key findings reveal the nuanced performance of the GARCH model, incorporating Maximum Likelihood techniques and a student’s t-distribution for parameter estimation, and the SV model, incorporating Markov Chain Monte Carlo (MCMC) techniques and a student’s t-distribution for parameter estimation, under distinct market conditions and forecasting scenarios. The SV consistently outperforms GARCH (1,1) in next-day volatility predictions for considered stock market data, such as S&P 500 and FTSE 100 stock markets, making it an ideal choice for daily trading and short-term decision-making.

However, for longer-term forecasts often required in real-world applications, GARCH (1,1) emerges as the preferred option. It consistently exhibits superior accuracy for weekly and monthly forecasting periods, particularly during high volatility periods, underscoring the critical role of forecasting horizons in model selection.

The study concludes by offering practical recommendations for model selection, data preparation, and forecasting horizons, addressing the evolving nature of volatility forecasting. Future research directions, including hybrid models and machine learning integration, are also discussed, shedding light on the ever-evolving landscape of financial modelling and risk management.

In summary, this project contributes vital insights into the behaviour of GARCH and SV models during periods of extreme market volatility. Its findings guide practitioners and researchers in selecting the most suitable models for their specific financial applications, emphasizing the importance of considering forecast horizons, market conditions, and data characteristics in this critical decision-making process. Ultimately, it advances the field of volatility forecasting, aiding investors, and decision-makers in navigating the complexities of financial markets.

**Acknowledgment**

I would like to express my special thanks to my supervisor, **Mr. Gani Nashi**, for their invaluable guidance, expertise, and unwavering support. Their mentorship and insightful feedback were instrumental in shaping this project and enhancing my research skills.

I am also indebted to the faculty and staff of University of Hertfordshire, whose dedication to education and research provided me with a stimulating academic environment.

I extend my heartfelt thanks to my family and friends for their patience, encouragement, and belief in my abilities. Your support has been a constant source of motivation.

This project would not have been possible without the collective effort of these individuals, and for that, I am truly grateful.

**Name: Mallesh Dharmaraj**

**Date: 18/09/2023**

Table of Contents

[1.0 Introduction 9](#_Toc145946787)

[1.0.1 Volatility 9](#_Toc145946788)

[1.0.2 Importance of Volatility Forecasting 9](#_Toc145946789)

[1.0.3 Time Series Analysis 9](#_Toc145946790)

[1.0.4 Is it right to forecast volatility using historical data? 9](#_Toc145946791)

[1.0.5 Volatility models 10](#_Toc145946792)

[1.0.6 Why GARCH and SV volatility models? 10](#_Toc145946793)

[1.0.7 GARCH and SV models 10](#_Toc145946794)

[1.2 Aim of the Project 10](#_Toc145946795)

[1.3 Research Questions 11](#_Toc145946796)

[1.4 Objectives 11](#_Toc145946797)

[2.0 Literature review 11](#_Toc145946798)

[2.1 Comparative Studies 11](#_Toc145946799)

[2.2 Research Gap 12](#_Toc145946800)

[2.3 Project planning according to literature review 12](#_Toc145946801)

[2.4 Key Concepts and Definitions of literature review 13](#_Toc145946802)

[2.4 Conclusion: 15](#_Toc145946803)

[3.0 Methodology 15](#_Toc145946804)

[3.0.1 Introduction 15](#_Toc145946805)

[3.0.2. Application and technology: 16](#_Toc145946806)

[3.1 Data Collection and Pre-processing 17](#_Toc145946807)

[3.1.1 Data Sources 17](#_Toc145946808)

[3.1.2 Data Collection 18](#_Toc145946809)

[3.1.3 Data pre-processing 18](#_Toc145946810)

[3.1.4 Data Exploration 18](#_Toc145946811)

[3.2 Data Split 22](#_Toc145946812)

[3.3 Modelling and Forecasting 23](#_Toc145946813)

[3.3.1 GARCH Model 23](#_Toc145946814)

[3.3.2 Stochastic Volatility (SV) Model 26](#_Toc145946815)

[3.4 Model Performance Evaluation 27](#_Toc145946816)

[3.5 Consideration of ethical/legal/professional and social issues 27](#_Toc145946817)

[3.5.1 Ethical Considerations 27](#_Toc145946818)

[3.5.2 Legal Considerations 27](#_Toc145946819)

[3.5.3 Professional Considerations 28](#_Toc145946820)

[3.5.4 Social Considerations 28](#_Toc145946821)

[3.6 Limitations 28](#_Toc145946822)

[3.7 Conclusion 28](#_Toc145946823)

[4.0 Results and Analysis: 29](#_Toc145946824)

[4.1 Conclusion: 37](#_Toc145946825)

[5.0 Evaluation and Discussion 38](#_Toc145946826)

[5.1 Achievements 39](#_Toc145946827)

[5.1.1 Aim of the Project Achievement 39](#_Toc145946828)

[5.1.2 Research Questions Achievement 40](#_Toc145946829)

[5.1.3 Project objective achievements 40](#_Toc145946830)

[5.2 Project Plan 41](#_Toc145946831)

[6.0 Recommendations 42](#_Toc145946832)

[7.0 Conclusion 42](#_Toc145946833)

[8.0 Bibliography: 43](#_Toc145946834)

[9.0 Appendix 44](#_Toc145946835)

[9.1. Appendix A: complete code 44](#_Toc145946836)

[9.2 Appendix B: all GARCH models summary: 49](#_Toc145946837)

**Equation List:**

[Equation 1: GARCH Model Parameters 13](#_Toc145945952)

[Equation 2: SV Model Parameters 14](#_Toc145945953)

**Tables List:**

[Table 1: Languages, IDE, Libraries and Visualization tools 16](#_Toc145953257)

[Table 2: Dataset 18](#_Toc145953258)

[Table 3: Model summary for Time period: 01-01-2001 to 29-12-2006 24](#_Toc145953259)

[Table 4: Model summary for Time period: 01-01-2014 to 29-03-2020 25](#_Toc145953260)

[Table 5: FTSE 100 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006. 30](#_Toc145953261)

[Table 6: FTSE 100 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020. 31](#_Toc145953262)

[Table 7: FTSE 100 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006. 32](#_Toc145953263)

[Table 8: FTSE 100 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020. 33](#_Toc145953264)

[Table 9: S&P 500 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006. 34](#_Toc145953265)

[Table 10: S&P 500 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020. 35](#_Toc145953266)

[Table 11: S&P 500 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006. 36](#_Toc145953267)

[Table 12: S&P 500 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020. 37](#_Toc145953268)

[Table 13: Results of Ding & Meade (2010) 38](#_Toc145953269)

[Table 14: Results of Avilés Ochoa & Flores Sosa (2021) 39](#_Toc145953270)

[Table 15: Compare 20 days volatility forecasting results with results of Ding & Meade (2010) studies. 39](#_Toc145953271)

[Table 16: Project plan 41](#_Toc145953272)

**Figure List:**

[Figure 1: Forecasting sample Figure 9](#_Toc145949217)

[Figure 2: Environment setup 16](#_Toc145949218)

[Figure 3: Required Libraries 17](#_Toc145949219)

[Figure 4: Return calculation 18](#_Toc145949220)

[Figure 5: Volatility calculation 18](#_Toc145949221)

[Figure 6: S&P 500 and FTSE 100 data description for time period:01-01-2001 to 29-12-2006 19](#_Toc145949222)

[Figure 7: S&P 500 and FTSE 100 data description for time period:01-01-2014 to 29-03-20020 19](#_Toc145949223)

[Figure 8: Close price of S&P 500 for Time period: 01-01-2001 to 29-12-2006. 20](#_Toc145949224)

[Figure 9: Close price of FTSE 100 for Time period: 01-01-2001 to 29-12-2006. 20](#_Toc145949225)

[Figure 10: Returns of S&P 500 for Time period: 01-01-2001 to 29-12-2006. 20](#_Toc145949226)

[Figure 11: Returns of FTSE 100 for Time period: 01-01-2001 to 29-12-2006. 20](#_Toc145949227)

[Figure 12: Volatility of S&P 500 for Time period: 01-01-2001 to 29-12-2006. 21](#_Toc145949228)

[Figure 13: Volatility of FTSE 100 for Time period: 01-01-2001 to 29-12-2006. 21](#_Toc145949229)

[Figure 14: Close price of S&P 500 for Time period: 01-01-2014 to 29-03-2020 21](#_Toc145949230)

[Figure 15: Close price of FTSE 100 for Time period: 01-01-2014 to 29-03-2020 21](#_Toc145949231)

[Figure 16: Returns of S&P 500 for Time period: 01-01-2014 to 29-03-2020. 22](#_Toc145949232)

[Figure 17: Returns for FTSE 100 for Time period: 01-01-2014 to 29-03-2020. 22](#_Toc145949233)

[Figure 18: Volatility of S&P 500 for Time period: 01-01-2014 to 29-03-2020. 22](#_Toc145949234)

[Figure 19: Volatility of FTSE 100 for Time period: 01-01-2014 to 29-03-2020. 22](#_Toc145949235)

[Figure 20: GARCH modelling 23](#_Toc145949236)

[Figure 21: SV modelling 26](#_Toc145949237)

[Figure 22: FTSE 100 Volatility forecasting with actual returns Results for Time Period: 01-01-2001 to 29-12-2006. 29](#_Toc145949238)

[Figure 23: FTSE 100 Volatility forecasting with actual returns Results for Time Period: 01-01-2014 to 29-03-2020. 30](#_Toc145949239)

[Figure 24: FTSE 100 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2014 to 29-03-2020. 31](#_Toc145949240)

[Figure 25: FTSE 100 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2014 to 29-03-2020. 32](#_Toc145949241)

[Figure 26: S&P 500 Volatility forecasting with actual returns Results for Time Period: 01-01-2001 to 29-12-2006. 33](#_Toc145949242)

[Figure 27: S&P 500 Volatility forecasting with actual returns Results for Time Period: 01-01-2014 to 29-03-2020. 34](#_Toc145949243)

[Figure 28: S&P 500 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2001 to 29-12-2006. 35](#_Toc145949244)

[Figure 29: S&P 500 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2014 to 29-03-2020. 36](#_Toc145949245)

[Figure 30: Gantt chart 41](#_Toc145949246)

# 1.0 Introduction

Unpredictable Markets, Unsteady Economies, and the Urgent Need to Decode the Future. In a world of ever-shifting uncertainties, one force stands out as the ultimate puzzle it is “Volatility”. As inflation rates oscillate, stock markets take unpredictable leaps, and property prices fluctuate wildly, the need to understand and predict volatility becomes paramount.

### 1.0.1 Volatility

The degree of change or fluctuation in the price or value of a financial asset over a certain period is referred to as volatility. It expresses the degree of uncertainty and risk connected with a certain investment. Volatility is often measured using standard deviation or variance. Volatility is influenced by a variety of factors such as market circumstances, economic developments, investor sentiment, and external shocks. In other contexts, "volatility" can refer to the general concept of unpredictability or variability.

### 1.0.2 Importance of Volatility Forecasting

Forecasting volatility is critical in financial markets. Accurate volatility predictions allow market players to understand the possible risks and returns of their investment portfolios. It aids in identifying optimal asset allocation, developing hedging strategies, and controlling portfolio risk. Volatility forecasts also aid in the price of derivative products like options and futures, which rely heavily on future volatility estimates. Furthermore, volatility estimates are employed in risk management practises such as value-at-risk (VaR) computations to quantify prospective losses in adverse market situations.

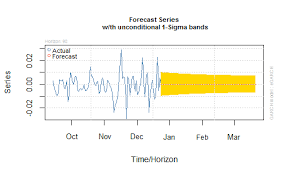


Figure : Forecasting sample Figure

### 1.0.3 Time Series Analysis

Volatility forecasting often involves analysing time-ordered data points using a statistical technique known as time series analysis. This methodology aims to uncover patterns within the data, such as trends or cycles, and leverages these patterns to make future predictions. Time series analysis is particularly relevant in finance and economics, where historical data patterns can offer valuable insights into market behaviour.

### 1.0.4 Is it right to forecast volatility using historical data?

In the financial community, it is common practise to predict volatility using past data, and this practise has empirical studies to support it and many volatility models which are constructed using historical data. It is not without difficulties and restrictions, though. Is it truly "right" in the strictest sense? That is dependent on several variables, such as the accuracy of the data, the suitability of the model, the time horizon for predicting, and the volatility of the financial markets. The key is to be aware of this approach's limitations and to use it carefully, frequently along with other data and methodologies. As market conditions change, it's also crucial to continuously validate and update the volatility models. Utilising historical data might be entirely suitable and useful in some situations and for some objectives.

### 1.0.5 Volatility models

Volatility models are indispensable tools in financial econometrics, serving crucial roles in forecasting, risk management, and derivatives pricing. Over the years, several notable models have emerged to address the dynamic nature of volatility. These include historical volatility models like the Simple Moving Average (SMA) and Exponentially Weighted Moving Average (EWMA), time-series models such as Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH), as well as Stochastic Volatility models like Taylor's Model, Heston Model, and the SABR Model. Jump Diffusion Models, exemplified by Merton's Jump Diffusion Model, and Local Volatility Models, like Dupire's Model, have also contributed to this toolkit. In the era of data science, machine learning techniques such as Long Short-Term Memory (LSTM) networks are sometimes integrated with traditional models for enhanced volatility forecasting. Analysts choose from this array of models based on factors like data characteristics, forecasting horizons, and specific applications. Among these models, GARCH and Stochastic Volatility (SV) models stand out as widely used methods in the field of financial econometrics for volatility forecasting.

### 1.0.6 Why GARCH and SV volatility models?

We chose to compare GARCH and SV models for our project due to their well-established theoretical foundation, interpretability, and practical relevance in financial risk management. These models offer a deeper understanding of financial data dynamics and provide interpretable parameters, which is crucial in finance. Additionally, by focusing on these models, I aim to build a strong foundation in traditional statistical and econometric methods and serve as a foundation for further research into more advanced models, including neural networks and machine learning techniques within the context of financial volatility forecasting.

### 1.0.7 GARCH and SV models

**GARCH Model:** It was introduced by Robert F. Engle in 1982 (with the ARCH model) The ARCH model allowed for time-varying volatility but had limitations in capturing the dynamics of volatility, later generalized by Tim Bollerslev in 1986 (hence the name GARCH) The GARCH model allowed for more flexibility in modelling volatility patterns, making it a significant improvement over its predecessors. It accounts for the autoregressive and conditional heteroskedasticity of financial time series data. To estimate the conditional variance of asset returns, the GARCH model integrates lagged squared residuals and lagged conditional variance.

**Stochastic Volatility (SV) Model:** Taylor's (1986) Stochastic Volatility model adopts a new approach by treating volatility as a stochastic process. In contrast to the Black-Scholes model, which assumes time-invariant volatility, the SV model allows for time-varying volatility. It describes the instantaneous variance as an autoregressive process, capturing volatility's persistence and mean-reverting tendency. The SV model has gained popularity due to its ability to capture complex volatility patterns and dynamics.

Both the models can be used for widespread applications in finance, including risk management, option pricing, portfolio optimization, and volatility forecasting.

## 1.2 Aim of the Project

The goal of this study is to implement, investigate and evaluate the GARCH and SV models performance in forecasting stock market volatility. The study aims to assess the model accuracy, efficiency, and ability to capture volatility features. The study seeks to provide insights into the chosen models strengths, limits, and practical implementation possibilities by undertaking an in-depth investigation of their performance on stock market volatility data. The outcomes of the study will add to the body of knowledge on volatility forecasting and provide recommendations for model selection and design.

## 1.3 Research Questions

* How accurate are the GARCH and SV models in forecasting stock market volatility?
* How do the GARCH models perform in comparison to SV models?
* Can the chosen models effectively represent the volatility characteristics of stock market data?

## 1.4 Objectives

* Conduct a comprehensive literature review to understand the fundamentals, theoretical foundations and empirical evidence related to GARCH and SV models for volatility forecasting.
* Acquire and pre-process key financial data, such as price and volatility history and market indicators.
* Implement and estimate GARCH and SV models using relevant programming language and statistical methodologies.
* Evaluate and compare the chosen models' accuracy and efficiency in capturing stock market volatility.
* Based on the research findings, make model selection and configuration recommendations.
* Improve data collecting, preparation, and analysis skills, as well as programming language expertise for quantitative finance modelling.
* Apply research insights to real-world trading scenarios and investment decision-making.

# 2.0 Literature review

Volatility forecasting and pricing in finance have evolved significantly since the mid-20th century, with notable contributions from researchers. In the 1960s, Fisher Black, Myron Scholes, and Robert Merton laid the foundation with the Black-Scholes-Merton (BSM) model for options pricing. However, the BSM model's reliance on constant volatility, continuous trading, and risk-free interest rates limits its applicability in real-world market conditions. In the 1980s, Robert Engle and Clive Granger introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model, later extended by Tim Bollerslev to the Generalized ARCH (GARCH) model, which better captured time-varying volatility. Concurrently, researchers like Taylor and Heston developed Stochastic Volatility (SV) models, treating volatility as a stochastic process. This research focuses on GARCH and SV models, recognizing the ongoing evolution of volatility modelling.

## 2.1 Comparative Studies

Few studies have evaluated the forecasting and pricing performance of the GARCH and SV models in the stock market. Lehar et al. (2002) conducted a comprehensive study encompassing option pricing, where they compared the effectiveness of the Black–Scholes, GARCH, and SV models. Analysing FTSE 100 transaction data spanning 1210 trading days, they revealed that the GARCH model exhibited superior pricing accuracy when contrasted with the SV model and constant volatility models. Notably, this study hinted at the potential of student’s t-distribution distributions for future research in modelling financial market dynamics.

Later years, advancements in computational power have paved the way for more extensive comparative research. Chan & Grant (2015) ventured into the domain of energy commodities pricing, exploring oil, gas, and petroleum prices over an extended period. Their findings diverged from the traditional financial context, as SV models consistently outperformed GARCH models in this specialized domain. Particularly noteworthy was the success of an SV variant incorporating moving average innovations. Although this study deviates from our dataset, it underscores the prowess of SV models, particularly in asset-specific analyses, broadening the horizon of potential applications.

Ding & Meade (2010) embarked on a comprehensive study spanning diverse financial markets, encompassing Euro/US$, FTSE 100, S&P 500, HSBC, Electricity, Gold, and Oil. Their analysis covered various forecasting horizons, examining 1564 data points over six years. Their research revealed that the SV model excelled in high volatility scenarios with evident stochastic volatility mechanisms. Conversely, the GARCH model outperformed SV(MCL) in situations where the precise volatility process remained uncertain. This real-world data-driven focus aligns with our objective of assessing model performance across varying conditions.

Kim et al. (2021) ventured into the realm of cryptocurrencies, expanding the horizon of application for GARCH and SV models. Their investigation covered nine well-known cryptocurrencies and assessed volatility using both model types. Their findings emphasized the superior predictive accuracy of the SV model, especially for longer forecasting horizons. The GARCH models relied on maximum likelihood estimates, while the SV model adopted a Bayesian approach, incorporating Markov Chain Monte Carlo (MCMC) techniques and a student’s t-distribution for parameter estimation. This study, spanning different cryptocurrencies and horizons, underpins the adaptability of these models.

In the domain of exchange rates, Avilés Ochoa & Flores Sosa (2021) conducted a comparative study contrasting GARCH and SV models. Their research delved into the Mexican peso-US dollar volatility, revealing the superior predictive accuracy of SV models, particularly when coupled with Bayesian estimating techniques like Monte Carlo Markov Chains (MCMC). Exploring the sensitivity of current volatility to recent data points, their study provided insights into the nuances of volatility dynamics. This real-world application, spanning an extensive time range and utilizing various evaluation metrics, enriches our understanding of the models potential in different financial contexts.

In summary, research on GARCH and SV models in various financial contexts offers valuable insights. Lehar et al. (2002) highlight GARCH's pricing accuracy in stock markets. Chan & Grant (2015) show SV models' dominance in energy commodities. Ding & Meade's (2010) analysis across markets showcases the situational excellence of SV and GARCH models. Kim et al. (2021) and Avilés Ochoa & Flores Sosa expand (2021) the models applications in cryptocurrencies and exchange rates, emphasizing SV models versatility and predictive accuracy, particularly with Bayesian techniques. These studies stress the need to consider specific contexts and model dynamics when choosing between GARCH and SV models for volatility forecasting.

## 2.2 Research Gap

The existing research has extensively compared GARCH and SV models in various financial contexts. However, there is a research gap in assessing how these models perform in highly volatile situations, such as the COVID-19 pandemic. This gap is crucial because extreme volatility poses unique challenges, and understanding how these models behave during such events is essential for risk management and policy decisions. This research project aims to fill this gap by evaluating GARCH and SV models performance during extreme volatility, providing valuable insights for academia and industry.

## 2.3 Project planning according to literature review

The collective insights from previous studies contribute significantly to our understanding of the performance of GARCH and SV models in diverse financial contexts. To embark on our research journey, we need to carefully consider several key aspects, starting with the selection of relevant data. Drawing inspiration from studies conducted by Lehar et al. (2002) and Chan & Grant (2015), we opt for datasets derived from prominent stock market indices such as the FTSE and the S&P 500, ensuring the representativeness of our data.

In the realm of model selection, it's crucial to employ effective criteria. Following the precedent set by Namugaya et al. (2014), we choose to use the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to determine the most suitable GARCH model (GARCH(P, Q)). These criteria strike a balance between model goodness of fit and complexity.

Parameter estimation is a critical step in modelling. For the GARCH model, we adopt the Maximum Likelihood method with a student’s t-distribution, consistent with methodologies used by Lehar et al. (2002) and Ding & Meade (2010). Leveraging a student’s t-distribution, as suggested by Lehar et al. (2002), proves to be essential for capturing the nuances of volatility.

In contrast, for parameter estimation in the SV model, we employ a Bayesian analysis approach paired with Markov Chain Monte Carlo (MCMC) techniques and a student’s t-distribution. This approach, as demonstrated by Kim, Jun, & Lee (2021) and Avilés Ochoa and Flores Sosa (2021), has shown its effectiveness in modelling SV in financial data.

To assess the accuracy of volatility forecasts, we employ well-established evaluation metrics like Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). These metrics, consistently used in previous studies, provide a robust framework for gauging the precision of volatility forecasts, following the footsteps of Ding & Meade (2010), Kim et al. (2021), and Avilés Ochoa & Flores Sosa (2021).

Considering different time horizons is essential for comprehensive model evaluation. Our approach aligns with studies like Ding & Meade (2010) and Kim et al. (2021), who analysed various forecasting horizons to capture short-term and long-term performance nuances effectively.

In our research, we will conduct a comparative analysis with previous studies, such as Ding & Meade (2010) and Avilés Ochoa & Flores Sosa (2021). This analysis will provide valuable insights into how GARCH and SV models perform relative to our chosen datasets and evaluation metrics. By building on the knowledge and methodologies established in these prior studies, our research aims to deepen our understanding of model dynamics and their applicability in diverse financial contexts.

In conclusion, this research project leverages the insights and methodologies gleaned from previous studies to undertake a comprehensive analysis of GARCH and SV models in diverse financial contexts. Our careful data selection, model criteria, parameter estimation techniques, and evaluation metrics draw inspiration from established research practices. By comparing our findings with prior studies, we aim to contribute valuable insights into the performance of these models across varying time horizons and datasets. Ultimately, this research endeavours to enhance our understanding of the nuanced dynamics of GARCH and SV models, shedding light on their adaptability and effectiveness in forecasting and pricing in the ever-evolving world of finance.

## 2.4 Key Concepts and Definitions of literature review

**GARCH(p,q) Model Parameters:**

Equation : GARCH Model Parameters

σt2 = γ + ∑(i=1 to p) αi\*rt-12 + ∑(i=1 to q) βi\*σt-12

* Conditional Variance(σt2): Represents the expected variance of the returns at time t
* ARCH Coefficient (α): Represents the weight or impact of past squared residuals on the current volatility (rt-12).
* GARCH Coefficient (β): Represents the weight or impact of lagged conditional variances on the current volatility(σt-12).
* Constant(γ): Is the constant term or intercept that represents the long-term average conditional variance.

**Stochastic Volatility (SV) Model Parameters:**

Equation : SV Model Parameters

Observation equation: yt = ϵt \* eht​​/2 ( ϵt ∼ N(0,1) )

State Equation: σt2 = μ + ϕ \* (ht-1 - μ) + ηt ( ηt ∼ N(0,σ2) )

* Observed Return (yt​): Represents the observed return of the financial asset at time t, which is modelled as a function of the underlying stochastic volatility process.
* Volatility of Log-Volatility (σ2): Represents the volatility of the underlying log-volatility process, controlling how dramatically the log-volatility can change from one period to the next.
* Latent Log-Volatility (ht): Represents the unobserved log-volatility process at time t, driving the volatility of the observed returns.
* Standardized Innovation (ϵt​): Represents the standardized shock term in the return process, with mean zero and unit variance, reflecting unexpected changes in returns.
* Mean Log-Volatility (μ): Represents the long-term average of the log-volatility process, serving as the equilibrium level to which the log-volatility tends to revert.
* Persistence Parameter (ϕ): Represents the persistence of the log-volatility process, indicating how quickly deviations from the mean log-volatility decay over time.

The SV model goes into the stochastic aspect of volatility, capturing its unobserved properties, while the GARCH model emphasises historical residuals and variances. Making a choice between these models and interpreting their parameters can give analysts and investors important knowledge about market behaviour, assisting in making well-informed choices and managing risk.

**Markov Chain Monte Carlo:**

Markov Chain Monte Carlo is a computational technique used in statistics, particularly Bayesian statistics, to approximate complex probability distributions and solve a wide range of statistical and computational problems. It relies on Markov chains, random sampling (Monte Carlo), and Bayesian inference to approximate complex probability distributions. Key components include the Metropolis-Hastings algorithm for generating samples, a burn-in period to discard initial samples, and convergence assessment to ensure representative results. MCMC is widely used in Bayesian modelling, posterior inference, model calibration, and many other statistical applications. It allows researchers to handle complex models, estimate parameters, and quantify uncertainty in a wide range of fields, including physics, biology, finance, and machine learning.

**Student’s t- distribution:**

The student’s t-distribution is a probability bell shaped distribution used in statistics to model the variability of sample statistics, especially the sample mean, when dealing with small sample sizes and situations where the population standard deviation is unknown. Key features include its shape, which resembles the normal distribution but with thicker tails. The degrees of freedom (df) determine its shape, with smaller ‘df’ values resulting in fatter tails. It's commonly used in hypothesis testing and constructing confidence intervals for population parameters in small samples. Critical values aid in hypothesis testing, and confidence intervals are wider due to increased uncertainty. In essence, the t-distribution is a statistical tool that helps handle uncertainty in situations where we have limited data and don't know the population standard deviation, allowing for more accurate inference about population parameters.

**AIC and BIC:**

The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are methods used for selecting the best model from a group of candidate models in statistics and machine learning.

AIC assesses a model's goodness of fit while penalizing complexity. It balances how well a model fits the data with the number of parameters used. Lower AIC values indicate a better trade-off between model fit and simplicity.

BIC, on the other hand, is more stringent in penalizing model complexity than AIC. It encourages simpler models by imposing a heavier penalty on additional parameters. Like AIC, lower BIC values indicate better-fitting models.

In summary, AIC and BIC help researchers choose the best model by considering how well it fits data while penalizing complex models. AIC is less strict, while BIC strongly prefers simpler models with larger datasets. Your choice depends on your specific analysis objectives and dataset characteristics.

## 2.4 Conclusion:

In conclusion, this literature review has provided a comprehensive overview of the landscape surrounding GARCH and SV models in the realm of financial forecasting and volatility modelling. It highlights the historical evolution of volatility modelling, starting with the Black-Scholes-Merton model and progressing through the development of ARCH and GARCH models, as well as the introduction of SV models. The comparative studies discussed in this review have shed light on the strengths and weaknesses of these models across various financial contexts, emphasizing the importance of considering specific datasets, contexts, and evaluation criteria when choosing between them.

A notable research gap identified in the literature is the lack of analysis regarding how GARCH and SV models perform during periods of extreme volatility, such as the COVID-19 pandemic. This research project seeks to address this gap and contribute valuable insights into the behaviour of these models under high-volatility conditions, which is essential for risk management and policy decision-making.

The project's planning, guided by insights from previous studies, involves careful data selection, model criteria, parameter estimation techniques, and evaluation metrics. By following established research practices and conducting a comparative analysis with prior studies, this research aims to deepen our understanding of the dynamics of GARCH and SV models. Ultimately, it strives to enhance our knowledge of their adaptability and effectiveness in the dynamic landscape of financial forecasting and pricing.

Future research can continue to study and modify these models, such as integration with machine learning and deep learning techniques, considering new datasets and using advanced estimating techniques to improve volatility forecasting accuracy.

# 3.0 Methodology

### 3.0.1 Introduction

This comprehensive methodology aims to analyse and forecast market volatility in two distinct time periods. The first period corresponds to the COVID-19 market crash, spanning from January 1, 2014, to March 29, 2020, this time period is chosen to identify the research gap. The second period aligns with Ding & Meade's (2010) research timeframe, ranging from January 1, 2001, to December 29, 2006, this time period is taken into consideration to have comparable results with previous studies to identify credibility of our results. The primary focus is on comparing the performance of GARCH and SV models for volatility forecasting. Additionally, the methodology considers two different approaches for forecasting volatility: one using actual returns and the other using forecasted returns, both calculated using rolling windows of various horizons (5, 10, 15, and 20 days).

The foundation of this methodology draws upon previous research in financial time series analysis, highlighting the effectiveness of SV and GARCH models in capturing and forecasting market volatility. These models provide insights into time-varying and autoregressive components of volatility.

### 3.0.2. Application and technology:

A new python environment “GARCHandSV” was successfully created in Anaconda-Navigator, ensuring a clean and isolated environment for the project.

A screenshot of a computer

Description automatically generated

Figure : Environment setup

The Languages that we are using and the libraries and visualization tools libraries for the project.

Table : Languages, IDE, Libraries and Visualization tools

|  |  |
| --- | --- |
| Languages used for developing | Python 3.9 |
| IDE | Anaconda-Navigator Jupiter notebook |
| Libraries | NumPy, Pandas, Sklearn, datetime, yfinance, arch and pymc3 |
| Visualization Tool | Matplotlib, and statsmodels |

Below are the necessary libraries for conducting time series analysis and volatility modelling. Following breakdown what each line of code does:

A screenshot of a computer program

Description automatically generated

Figure : Required Libraries

* **from datetime import datetime, timedelta:** This line imports the datetime module, enabling date and time manipulation in Python, and specifically imports the datetime and timedelta classes for working with dates and time intervals.
* **import pandas as pd:** This line imports the powerful Pandas library, commonly used for structured data handling and analysis, and assigns it the alias pd.
* **import matplotlib.pyplot as plt:** This line imports the pyplot module from the Matplotlib library, a popular choice for creating various types of visualizations in Python, and aliases it as plt.
* **from statsmodels.graphics.tsaplots import plot\_acf, plot\_pacf:** This line imports specific functions, plot\_acf and plot\_pacf, from the tsaplots module of the statsmodels library, which are essential for visualizing autocorrelation and partial autocorrelation functions in time series data analysis.
* **import numpy as np:** This line imports the fundamental NumPy library, vital for numerical computations involving arrays and matrices, and assigns it the alias np.
* **import yfinance as yf:** This line imports the yfinance library, allowing you to fetch financial data from Yahoo Finance in Python, with the alias yf.
* **from arch import arch\_model:** This line imports the arch\_model class from the arch library, which is used for estimating and predicting volatility models, including GARCH models.
* **from arch.\_\_future\_\_ import reindexing:** This line imports a module from the arch library related to future functionality, indicating potential enhancements or changes in the library.
* **import pymc3 as pm:** This line imports the pymc3 library, a tool for Bayesian statistical modeling and probabilistic programming, and assigns it the alias pm.
* **import warnings:** This line imports the warnings module, facilitating the handling and control of warning messages during Python code execution.

In summary, this code sets up the environment for conducting time series analysis, volatility modelling, and financial data retrieval by importing the necessary Python libraries and modules. These libraries are essential for performing various tasks, such as data manipulation, visualization, statistical analysis, and modelling in the context of time series data and financial markets.

## 3.1 Data Collection and Pre-processing

### 3.1.1 Data Sources

**Stock Indices:** Historical data for two major stock indices S&P 500 and FTSE 100 will be obtained. Data is collected using the Yahoo Finance API ‘yfinance’. It simplifies the retrieval and analysis of financial data for various purposes, such as financial modelling and analysis. You can use it to retrieve data for specific years, making it a valuable resource for financial research and analysis. (Reference: <https://pypi.org/project/yfinance/>)

**Stock indices:**

* S&P 500 (^GSPC): S&P 500 represents the performance of the 500 largest publicly traded companies in the United States and is a key indicator of the U.S. stock market.
* FTSE 100 (^FTSE): FTSE 100 represents the 100 largest companies listed on the London Stock Exchange (LSE) and is a major benchmark for the UK stock market.

### 3.1.2 Data Collection

Historical data for two distinct time periods will be collected for the specified time periods and stock indices. In which we would concentrate on daily close price and calculate the daily returns and Volatility.

**Time periods:**

* COVID-19 Market Crash Period (January 1, 2014, to March 29, 2020)
* Ding & Meade's (2010) Research Period (January 1, 2001, to December 29, 2006)

The data retrieved includes daily adjusted closing prices for both S&P 500 and FTSE 100

### 3.1.3 Data pre-processing

* The retrieved data is saved as CSV files for future use.
* The column 'Adj Close' in the CSV files is renamed to 'Close' for clarity.
* Daily returns will be computed as the percentage change in closing prices. This calculation will be performed for both S&P 500 and FTSE 100.

Daily Return=Today’s Close Price−Yesterday’s Close Price/Yesterday’s Close Price.

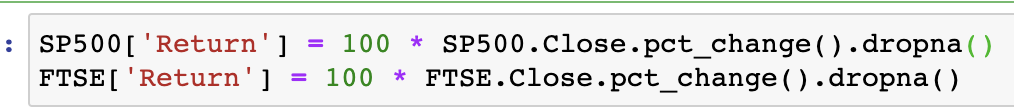


Figure : Return calculation

* Rolling volatility will be computed for each time series using a 20-day rolling window. The standard deviation of returns within the window will represent the volatility. following the same approach used by Ding & Meade's (2010).

Rolling Volatility=Standard Deviation of Daily Returns over 20 DaysRolling.

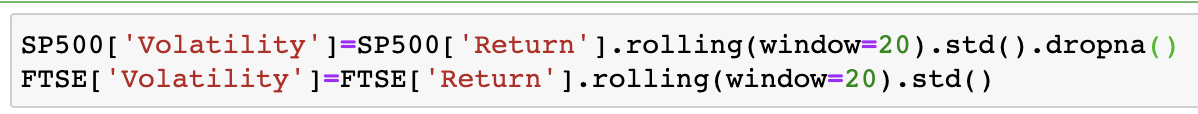


Figure : Volatility calculation

* Any rows with missing values (usually resulting from the rolling window) are dropped from the dataset.

This data collection and pre-processing methodology prepares the datasets for subsequent analysis and forecasting of volatility using GARCH and SV models.

### 3.1.4 Data Exploration

The resulting datasets contain Date, Close, Return and Volatility columns:

Table : Dataset

|  |  |
| --- | --- |
| **Column Name** | **Description** |
| Date | The date of the data point. |
| Close | The adjusted closing price of the index. |
| Return | The daily returns of the index. |
| Volatility | The rolling volatility of the index. |

In the data description below, we can see that there is a difference in the count of data points even for the same period. For example, for the time period January 1, 2001, to December 29, 2006, the count for S&P 500 is 1486, and for FTSE 100 is 1495. Similarly, for the time period January 1, 2014, to March 29, 2020, the count for S&P 500 is 1550, and for FTSE 100 is 1559. This difference is due to the number of bank holidays in the US and UK, but this variation is consistent with Ding & Meade's (2010), Kim et al. (2021), and many other previous research papers. All data points within these periods will be considered for analysis, as this variation does not impact the forecasting comparison.

A screenshot of a computer

Description automatically generatedA screenshot of a computer screen

Description automatically generated

Figure : S&P 500 and FTSE 100 data description for time period:01-01-2001 to 29-12-2006

A screenshot of a computer

Description automatically generatedA screenshot of a calculator

Description automatically generated

Figure : S&P 500 and FTSE 100 data description for time period:01-01-2014 to 29-03-20020

While the historical data of the S&P 500 and FTSE 100 may appear similar, it is essential to forecast volatility for both stock market indices. This approach allows us to assess how well our models perform in relation to each dataset. Even though the underlying data sources share some similarities, variations in their volatility patterns may exist.

We conduct volatility forecasts for the most recent days of two specific time periods: January 1, 2001, to December 29, 2006, and January 1, 2014, to March 29, 2020. This distinction is crucial because it showcases how the models operate in contrasting market conditions.

* Time Period: January 1, 2001, to December 29, 2006: In this period, we anticipate observing medium variance levels towards the end of the dataset.

A graph showing the price of a stock market

Description automatically generated

Figure : Close price of S&P 500 for Time period: 01-01-2001 to 29-12-2006.

A graph showing the price of a stock market

Description automatically generated

Figure : Close price of FTSE 100 for Time period: 01-01-2001 to 29-12-2006.

A blue line graph with numbers

Description automatically generated

Figure : Returns of S&P 500 for Time period: 01-01-2001 to 29-12-2006.

A blue line graph with numbers

Description automatically generated

Figure : Returns of FTSE 100 for Time period: 01-01-2001 to 29-12-2006.

A graph showing the value of a stock market

Description automatically generated

Figure : Volatility of S&P 500 for Time period: 01-01-2001 to 29-12-2006.

A graph showing a number of volatility

Description automatically generated

Figure : Volatility of FTSE 100 for Time period: 01-01-2001 to 29-12-2006.

* Time Period: January 1, 2014, to March 29, 2020: Conversely, for this timeframe, we expect to witness significantly higher variance levels as we approach the end of the data set.

A graph with blue lines

Description automatically generated

Figure : Close price of S&P 500 for Time period: 01-01-2014 to 29-03-2020

A graph with blue lines

Description automatically generated

Figure : Close price of FTSE 100 for Time period: 01-01-2014 to 29-03-2020

A graph with blue lines

Description automatically generated

Figure : Returns of S&P 500 for Time period: 01-01-2014 to 29-03-2020.

A graph showing a wave of data

Description automatically generated with medium confidence

Figure : Returns for FTSE 100 for Time period: 01-01-2014 to 29-03-2020.

A graph with blue lines

Description automatically generated

Figure : Volatility of S&P 500 for Time period: 01-01-2014 to 29-03-2020.

A graph with blue lines

Description automatically generated

Figure : Volatility of FTSE 100 for Time period: 01-01-2014 to 29-03-2020.

The comparative analysis of these two distinct time periods enables us to gauge how well our GARCH and SV models perform under varying market scenarios. It provides valuable insights into the models effectiveness in forecasting volatility during periods of differing market dynamics.

## 3.2 Data Split

In this research project, our primary goal is to conduct volatility forecasts across a range of distinct time horizons, spanning 5 days, 10 days, 15 days, and 20 days. This strategic selection of forecasting windows allows us to comprehensively evaluate the efficacy of our forecasting models under varying scenarios. This methodological approach is consistent with established research practices, as exemplified by prior studies like Ding & Meade (2010) and Kim et al. (2021).

It's crucial to acknowledge that our choice of relatively short forecasting horizons is influenced by the computational demands associated with employing SV models. These models entail intricate computations, necessitating extensive computational resources and time. Consequently, we partition our data into training and test sets, where each forecasting period (5 days, 10 days, 15 days, and 20 days) serves as the test data. The remaining data is allocated for model training.

To illustrate, if our goal is to forecast volatility for a 20-day horizon within the time period of January 1, 2001, to December 29, 2006, our test data encompasses the final 20 days of this period, specifically from November 29, 2006, to December 28, 2006. Meanwhile, the training data spans from January 1, 2001, to November 28, 2006, with the exclusion of the initial month during data pre-processing. It's worth noting that accounting for holidays, the 20-day data interval effectively corresponds to one month of trading data. This meticulous approach ensures a rigorous evaluation of our models' forecasting capabilities across diverse temporal contexts.

## 3.3 Modelling and Forecasting

It's important to note that our GARCH modelling and forecasting code was initially sourced from the ritvikmath GitHub repository (<https://github.com/ritvikmath/Time-Series-Analysis/blob/master/GARCH%20Stock%20Modeling.ipynb>). Subsequently, we meticulously adapted and customized the code to suit the specific requirements and objectives of our project. Similarly, SV modelling code was initially sourced from the Raj Jaishwal kaggle code (<https://www.kaggle.com/code/rajjais3003/stochastic-volatility-model-using-nifty50-data-set>). Subsequently, we meticulously adapted and customized the code to suit the specific requirements and objectives of our project. The complete code is available in appendix A.

### 3.3.1 GARCH Model

The GARCH model encompasses both autoregressive and moving average components. We have implemented the GARCH(P, Q) model within the 'arch' library, specifically configuring GARCH(1,1), GARCH(1,2), GARCH(2,2), and GARCH(2,1) models, with parameter estimation using Maximum Likelihood Estimation (MLE) and a student’s t-distribution.

This methodology is closely aligned with the research studies by Avilés Ochoa & Flores Sosa (2021) and Kim et al. (2021), particularly in the context of modelling residuals. To elaborate further on our approach, we meticulously configure the GARCH models, systematically updating the P and Q values, where P is ARCH coefficient and Q is GARCH coefficient. This iterative approach is motivated by our analysis, which suggests that models with P and Q values beyond 2 do not yield statistically significant improvements in modelling performance.

Here's a breakdown of the key steps involved in specifying and configuring the GARCH model:



Figure : GARCH modelling

**Model Specification:** We define the GARCH model using the 'arch' library's arch\_model function. This function takes several arguments:

* train2[1:]: Financial return data, typically presented as a time series of returns, is used as input. The [1:] indexing is used to exclude the first data point, often treated as an initial value.
* vol="GARCH": We specify that the model aims to estimate volatility using the GARCH model.
* p and q: These parameters determine the orders of the GARCH and ARCH terms in our model. p represents the autoregressive term order (alpha), while q signifies the moving average term order (beta).
* dist='t': We indicate the use of a student’s t-distribution to model the error terms, a common choice in financial applications due to its ability to capture fat tails in the data.

**Model Fitting:** We apply the fit method to the FTSE\_model\_garch object. This step estimates the model parameters using Maximum Likelihood Estimation (MLE). The method iteratively optimizes the parameters to maximize the log-likelihood function.

**Summary of Model Results:** Finally, we utilize the summary() method to generate a comprehensive summary of the model results. This summary provides detailed information about the estimated parameters, goodness-of-fit statistics (such as AIC and BIC), and other relevant details, assisting in the assessment of the model's performance.

In summary, the parameters of the GARCH model with a student’s t-distribution are estimated using MLE. The arch library takes care of the optimization process, and you can examine the results and statistics by calling summary on the fitted model object (FTSE\_results\_garch). The estimated parameters will be displayed in the summary output, including values, standard errors, t-statistics, and p-values, among other information. These estimated parameters capture important characteristics of the volatility dynamics in your financial return data.

**GARCH Model Selection:**

Selecting the appropriate GARCH model involves considering multiple factors, such as goodness of fit, the statistical significance of parameters, and the project's context according to model summary. For the sake of our analysis, we have meticulously compiled the essential results from all model summaries into tables (all the model summary results are available in appendix B). In our analysis of S&P 500 data across two distinct time periods, we have opted for the GARCH(1,1) model over other GARCH variations. Here's a detailed rationale for this choice:

Table : Model summary for Time period: 01-01-2001 to 29-12-2006

S&P 500 FTSE100



Table : Model summary for Time period: 01-01-2014 to 29-03-2020

S&P 500 FTSE 100

AIC and BIC Values: AIC and BIC serve as essential criteria for model selection, with lower values indicating a better fit. This is aligned with previous studies of Namugaya et al. (2014). In all datasets, the AIC and BIC values for the GARCH(1,1) model consistently rank the lowest. An exception is the S&P 500 data from January 1, 2001, to December 29, 2006, where the GARCH(2,1) model achieves the lowest AIC value. Nevertheless, the GARCH(1,1) model maintains the second-lowest AIC and is nearly on par with the lowest BIC. This suggests that GARCH(1,1) strikes a favourable balance between capturing data patterns and model complexity in all four cases.

Statistical Significance: The p-values (p>|t|) associated with model parameters indicate their statistical significance. Lower p-values (usually below 0.05) imply significant parameters. Across all datasets, the p>|t| values for the GARCH(1,1) model reveal that estimated parameters, especially Alpha[1] and Beta[1], are statistically significant, boasting low p-values close to zero. This implies that the model parameters are accurately estimated and play a crucial role in capturing volatility dynamics.

In summary, the GARCH(1,1) model was selected as the optimal choice for our analysis based on several key factors: lower AIC and BIC values, statistical significance of parameters and also because of model simplicity, interpretability, and enhanced stability. This choice aligns with our commitment to robust and interpretable results across different datasets and time periods, emphasizing the GARCH(1,1) models versatility and effectiveness in capturing volatility dynamics.

**GARCH(1,1) Model Volatility Forecasting**

Forecasting Approach 1 (Actual Returns): The GARCH(1,1) models will be employed to forecast volatility for various forecast horizons (5, 10, 15, and 20 days) using actual returns in a rolling manner. Typically, we start by training our model on the entire training dataset to predict the volatility for the next day. Then, in a rolling fashion, we gradually incorporate the actual returns from the test data, which represent the following day, to predict the volatility for the day after that. This process continues iteratively until we have forecasted the volatility for all the days in our test dataset.

Forecasting Approach 2 (Forecasted Returns): The GARCH(1,1) models will forecast volatility same forecast horizons as first approach while simultaneously returns will be calculated from forecasted volatility in a rolling manner. Similar to the first approach, we initiate the process by training our model on the complete training dataset to predict the volatility for the next day. Then, in a rolling fashion, we gradually incorporate the calculated returns from forecasted volatility of that day, which represent the following day, to predict the volatility for the day after that. This process continues iteratively until we have forecasted the volatility for all the days in our test dataset.

In summary, the first approach, despite technically calculating the volatility for the next day by considering the fitness of the data up to that point, provides an accurate measure of how forecasting works for the next day when we evaluate it by averaging the forecasted errors. In contrast, the second approach represents the actual scenario where we obtain forecasted volatility for various horizons. This approach, distinct from the first, doesn't rely on actual returns but rather employs calculated returns derived from forecasted volatility. Essentially, the first approach can be viewed as an in-sample analysis since it involves fitting the model to almost the entire dataset to forecast the next day's volatility, while the second approach is an out-of-sample analysis where we forecast volatility across different time horizons.

### 3.3.2 Stochastic Volatility (SV) Model

The SV model used to model and forecast time-varying volatility in financial data. In this project, we employ the SV model within the PyMC3 framework, following methodologies observed in recent studies (e.g., Avilés Ochoa & Flores Sosa, 2021).

A computer screen shot of a program

Description automatically generated

Figure : SV modelling

* Model Initialization: We initialize a PyMC3 model using pm.Model().
* Model Definition: Within the PyMC3 context, we define the SV model:
  + sigma and nu are parameters of the SV model, representing volatility and degrees of freedom, respectively.
  + s represents the stochastic process for volatility, defined as a Gaussian random walk with a standard deviation of sigma.
  + r defines the observation model using Student's t-distribution, with nu controlling the heavy-tailedness of the distribution.
* Sampling: We sample from the SV model using pm.sample (n\_samples) to estimate the posterior distribution of the model parameters.
* Estimating Current Volatility: We calculate the estimated current volatility state (s\_current) by taking the mean of the trace samples of the s variable at the final time step.

This modelling approach enables us to effectively capture and analyse the ever-changing volatility in financial data. It's important to note that Bayesian models like the SV model provide a parameter distribution instead of a fixed-point estimate. Consequently, estimated parameters may exhibit slight variations across iterations, but they remain pertinent for reliable forecasting.

**SV Model Volatility Forecasting**

In SV modelling, the primary approach to forecasting involves predicting a set of returns, and subsequently, we calculate volatility by computing the standard deviation of these return values.

Similar to GARCH (1,1) model there were exactly same kind of two forecasting approach were done for SV as well, where first approach using actual returns in a rolling manner and the second approach using forecasted return in a rolling manner. However, in the second approach for GARCH (1,1), returns are calculated using forecasted volatility. In contrast, with the SV model, our initial step is to forecast a set of returns, and we determine the forecasted return by taking the mean of these return values. This distinction sets the SV model apart from the GARCH (1,1) model.

In summary, much like the GARCH (1,1) model for volatility forecasting, the first approach can be considered an in-sample analysis. This approach entails fitting the model to nearly the entire dataset to forecast the next day's volatility. In contrast, the second approach represents an out-of-sample analysis, involving the forecasting of volatility across various time horizons.

## 3.4 Model Performance Evaluation

To assess the accuracy and efficiency of the GARCH and SV models in forecasting stock market volatility, we employ the following evaluation metrics:

* Mean Squared Error (MSE): MSE measures the average squared differences between predicted and actual volatility values.
* Root Mean Squared Error (RMSE): RMSE is the square root of the MSE, providing a measure of the model's prediction error in the original units.
* Mean Absolute Error (MAE): MAE measures the average absolute differences between predicted and actual volatility values.

Our performance evaluation encompasses various forecasting horizons, including 5 days, 10 days, 15 days, and 20 days, for both the first and second forecasting approaches of GARCH and SV models. This comprehensive analysis allows for meaningful comparisons between the models.

## 3.5 Consideration of ethical/legal/professional and social issues

In conducting this research project on volatility forecasting using GARCH and SV models, it is crucial to consider various ethical, legal, professional, and social issues that may arise throughout the process. The following are the key considerations in each of these areas for this project:

### 3.5.1 Ethical Considerations

* Data Privacy: All financial data used in the research was obtained through legal means and in compliance with data privacy regulations. Sensitive information was treated with utmost confidentiality, and personally identifiable data was not disclosed. Data handling and storage were done securely to protect the privacy and anonymity of individuals and entities represented in the data.
* Avoidance of Biased Reporting: The research findings were presented objectively, and there was no selective reporting to promote any particular agenda or viewpoint.

### 3.5.2 Legal Considerations

* Copyright and Intellectual Property: Proper citation and permission were sought for copyrighted content used beyond fair use.
* Data Usage Rights: Data sources used in the research were verified to ensure compliance with usage rights.
* Compliance with Regulations: All relevant financial regulations and securities laws were followed during the use of financial data and investment analysis.

### 3.5.3 Professional Considerations

* Integrity and Honesty: The research was conducted with utmost integrity and honesty, avoiding any fabrication, falsification, or plagiarism.
* Peer Review: Consideration was given to submitting the research to reputable peer-reviewed journals to ensure the quality and validity of findings.
* Transparency: The research methodology, data sources, and modelling techniques were clearly documented to enable reproducibility and verification by others.

### 3.5.4 Social Considerations

* Implications of Research Findings: The potential impact of the research on financial markets, investors, and society was considered, and any implications or consequences of the results were discussed.
* Responsible Use of Models: Limitations of the GARCH and SV models were acknowledged, and reliance on model forecasts for critical decision-making was avoided.
* Promotion of Diversity and Inclusivity: The research process and presentation promoted diversity and inclusivity in the financial industry and academic community.

## 3.6 Limitations

While this research project on evaluating and SV models in extreme volatility situations offers valuable insights, it's essential to acknowledge its limitations:

**Model Dynamics are Complex:** GARCH and SV models are powerful tools, but their effectiveness depends on factors like model assumptions, parameter estimation, and the choice of evaluation metrics. These models are not one-size-fits-all solutions.

**External Factors:** The performance of financial models can also be affected by external factors like trading costs, liquidity constraints, and transaction delays. These factors were not explicitly addressed in the research but can significantly impact real-world trading strategies.

**Time Horizon:** This project evaluates model performance across various time horizons, but the choice of these horizons may influence the results. Different models may excel in short-term or long-term forecasting, and their relative performance may vary accordingly.

Despite these limitations, this research project contributes valuable insights into the behaviour of GARCH and SV models during periods of extreme volatility. It serves as a foundation for further exploration and refinement of these models in different financial contexts and highlights the need for careful consideration of model selection and evaluation methods in real-world applications.

## 3.7 Conclusion

In summary, our methodology for analysing and forecasting market volatility with GARCH and SV models encompasses model selection, data collection, and ethical considerations. We've chosen the GARCH(1,1) model due to its balanced performance, simplicity, and interpretability. Our approach includes two forecasting methods, one using actual returns and the other using forecasted returns, allowing us to assess model performance under different scenarios.

We've also prioritized ethical and legal considerations, ensuring data privacy and copyright compliance. Our research maintains integrity, transparency, and responsible model use acknowledging the limitations. By applying this methodology to study market behaviour during the COVID-19 crash and a medium volatility timeframe, we aim to provide valuable insights for investors and researchers while advancing financial modelling and risk management practices.

# 4.0 Results and Analysis:

In the following sections, we will present the results and analysis of our study, providing a comprehensive overview the performance of GARCH(1,1) and SV models in the context of volatility forecasting. Specifically, we assess their effectiveness in forecasting volatility for two prominent stock market indices, the FTSE 100, and the S&P 500, across distinct time periods. Additionally, we compare the models performance under different scenarios, using both forecasted returns and actual returns while forecasting volatility.

**Below are the graphs and tables with results and analysis for the GARCH(1,1) and SV models for volatility forecasting for FTSE 100 across different time periods:**

**With Actual Returns for FTSE 100:**

**Results and Analysis for Time Period: January 1, 2001, to December 29, 2006:**

5 days forecast. 10 days forecast.

A graph showing the price of a stock market

Description automatically generatedA graph of a graph with numbers and lines

Description automatically generated with medium confidence

15 days forecast. 20 days forecast.

A graph of a graph showing the price of a stock market

Description automatically generated with medium confidenceA graph of a graph showing the price of a stock market

Description automatically generated with medium confidence

Figure : FTSE 100 Volatility forecasting with actual returns Results for Time Period: 01-01-2001 to 29-12-2006.

Table : FTSE 100 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006.



In terms of MSE, RMSE, MAE, the SV outperforms GARCH (1,1) in all volatility forecasting. Additionally. Also, we can’t see any difference as the forecasting range increases because technically it is just forecasting the next day volatility not for whole forecasting period.

**With Actual Returns for FTSE 100:**

**Results and Analysis for Time Period: January 1, 2014, to March 29, 2020:**

5 days forecast. 10 days forecast.

A graph with lines and numbers

Description automatically generatedA graph of a graph showing the price of a stock market

Description automatically generated with medium confidence

15 days forecast . 20 days forecast.

A graph of a graph with numbers and lines

Description automatically generated with medium confidenceA graph of a graph with numbers and lines

Description automatically generated with medium confidence

Figure : FTSE 100 Volatility forecasting with actual returns Results for Time Period: 01-01-2014 to 29-03-2020.

Table : FTSE 100 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020.



In terms of MSE, RMSE, MAE, the SV outperforms GARCH (1,1) in all volatility forecasting. Additionally. Also, we can’t see any difference as the forecasting range increases because technically it is just forecasting the next day volatility not for whole forecasting period.

**With Forecasted Returns for FTSE 100:**

**Results and Analysis for Time Period: January 1, 2001, to December 29, 2006:**

5 days forecast. 10 days forecast.

A graph of a graph showing the price of a stock market

Description automatically generated with medium confidence A graph of a graph showing the number of the same graph

Description automatically generated with medium confidence

15 days forecast. 20 days forecast.

A graph of a graph showing the price of a stock market

Description automatically generated with medium confidence A graph of a graph showing the price of a stock market

Description automatically generated with medium confidence

Figure : FTSE 100 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2014 to 29-03-2020.

Table : FTSE 100 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006.



In terms of MSE, RMSE, MAE, the SV outperforms GARCH (1,1) in the 5-day volatility forecast. However, when it comes to the 10-day, 15-day, and 20-day volatility forecasts, GARCH (1,1) exhibits superior performance compared to SV. Additionally, it is noteworthy that SV's forecasting errors tend to increase as the forecasting horizon extends, while GARCH (1,1) does not display a clear pattern in this regard.

**With Forecasted Returns for FTSE 100:**

**Results and Analysis for Time Period: January 1, 2014, to March 29, 2020:**

5 days forecast. 10 days forecast.

A graph of a graph with numbers and lines

Description automatically generated with medium confidence A graph of a graph showing the price of a stock market

Description automatically generated with medium confidence

15 days forecast . 20 days forecast.

A graph with numbers and lines

Description automatically generated A graph showing the price of a stock market

Description automatically generated

Figure : FTSE 100 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2014 to 29-03-2020.

Table : FTSE 100 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020.



In terms of MSE, RMSE, MAE, the GARCH (1,1) outperforms SV in all volatility forecasting. Additionally, it is noteworthy that SV's forecasting errors tend to increase as the forecasting horizon extends, while GARCH (1,1) does not display a clear pattern in this regard.

**Below are the graphs and tables with results and analysis for the GARCH (1,1) and SV models for volatility forecasting for S&P 500 across different time periods:**

**With Actual Returns for S&P 500:**

**Results and Analysis for Time Period: January 1, 2001, to December 29, 2006:**

5 days forecast 10 days forecast

A graph with lines and numbers

Description automatically generatedA graph of a graph showing the price of a stock market

Description automatically generated with medium confidence

15 days forecast 20 days forecast

A graph of a graph with numbers and lines

Description automatically generated with medium confidenceA graph of a graph showing the price of a stock market

Description automatically generated with medium confidence

Figure : S&P 500 Volatility forecasting with actual returns Results for Time Period: 01-01-2001 to 29-12-2006.

Table : S&P 500 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006.



In terms of MSE, RMSE, MAE, the SV outperforms GARCH (1,1) in most volatility forecasting. Additionally. Also, we can’t see any difference as the forecasting range increases because technically it is just forecasting the next day volatility not for whole forecasting period.

**With Actual Returns for S&P 500:**

**Results and Analysis for Time Period: January 1, 2014, to March 29, 2020:**

5 days forecast 10 days forecast

A graph of a graph with lines and numbers

Description automatically generated with medium confidence A graph of a graph with numbers and lines

Description automatically generated with medium confidence

15 days forecast 20 days forecast

A graph of a graph showing the price of a stock market

Description automatically generated with medium confidence A graph with numbers and lines

Description automatically generated

Figure : S&P 500 Volatility forecasting with actual returns Results for Time Period: 01-01-2014 to 29-03-2020.

Table : S&P 500 Volatility forecasting with actual returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020.



In terms of MSE, RMSE, MAE, the SV outperforms GARCH (1,1) in almost all volatility forecasting. Additionally. Also, we can’t see any difference as the forecasting range increases because technically it is just forecasting the next day volatility not for whole forecasting period.

**With Forecasted Returns for S&P 500:**

**Results and Analysis for Time Period: January 1, 2001, to December 29, 2006:**

5 days forecast 10 days forecast

A graph showing the growth of a stock market

Description automatically generated with medium confidenceA graph with numbers and lines

Description automatically generated

15 days forecast 20 days forecast

A graph of a graph showing the price of a stock market

Description automatically generated with medium confidenceA graph of a graph showing the price of a stock market

Description automatically generated with medium confidence

Figure : S&P 500 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2001 to 29-12-2006.

Table : S&P 500 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2001 to 29-12-2006.



In terms of MSE, RMSE, MAE, the SV outperforms GARCH (1,1) in the 5-day and 20-day volatility forecast. However, when it comes to the 10-day and 15-day volatility forecasts, GARCH (1,1) exhibits superior performance compared to SV. Additionally, it is noteworthy that SV's forecasting errors tend to increase as the forecasting horizon extends, while GARCH (1,1) does not display a clear pattern in this regard.

**With Forecasted Returns for S&P 500:**

**Analysis for Time Period: January 1, 2014, to March 29, 2020:**

5 days forecast. 10 days forecast.

A graph of a graph showing the difference between sv forecast and sv forecast

Description automatically generatedA graph showing the price of a stock market

Description automatically generated

15 days forecast. 20 days forecast.

A graph of a graph with numbers and lines

Description automatically generated with medium confidenceA graph showing the price of a stock market

Description automatically generated

Figure : S&P 500 Volatility forecasting with forecasted returns Results for Time Period: 01-01-2014 to 29-03-2020.

Table : S&P 500 Volatility forecasting with forecasted returns MSE, RMSE, MAE values for Time Period: 01-01-2014 to 29-03-2020.



In terms of MSE, RMSE, MAE, the GARCH (1,1) outperforms SV in all volatility forecasting. Additionally, it is noteworthy that SV's forecasting errors tend to increase as the forecasting horizon extends, while GARCH (1,1) does not display a clear pattern in this regard.

## 4.1 Conclusion:

When the actual return considered volatility forecasting, whether using SV or GARCH (1,1) models, essentially involves a rolling prediction that assesses how effectively these models perform in predicting the next day's volatility. Our analysis reveals a notable trend where SV consistently outperforms GARCH (1,1) in next-day volatility forecasting for both the S&P 500 and FTSE 100 stock markets this is a better option for daily traders. However, real-world scenarios like portfolio management often require forecasting weekly, monthly, or even quarterly volatility forecast, where future return values are not readily available for model fitting. This underscores the significance of the forecast horizon, an aspect that significantly impacts model performance.

When considering different time periods, we observed nuances in model performance. During the moderate volatility period from January 1, 2001, to December 29, 2006, both SV and GARCH (1,1) exhibited comparable accuracy, with a slight edge in favour of GARCH (1,1) when assessing overall performance. However, in the high volatility period from January 1, 2014, to March 29, 2020, GARCH (1,1) consistently outperformed SV in all volatility forecasting scenarios. This highlights the sensitivity of model performance to the prevailing market conditions.

It is evident that forecasting errors tend to escalate during high volatility periods compared to moderate volatility periods, underscoring the challenges of predicting market behaviour during turbulent times. Additionally, as the forecasting horizon lengthened, SV exhibited a tendency for increasing forecasting errors, albeit without a precise pattern, while GARCH (1,1) displayed similar trends, indicating the importance of considering forecast horizon in model selection.

In conclusion, the choice between the GARCH (1,1) and SV models for volatility forecasting should be made judiciously, considering the specific context, data availability, and the desired forecasting horizon. These models offer distinct advantages and disadvantages, rendering them suitable for different applications. For instance, SV excels when provided with a wealth of historical return data, making it a valuable choice for daily trading, options pricing, etc., in the stock market. On the other hand, GARCH (1,1) is well-suited for forecasting overall market volatility in stock market like portfolio management, risk management, etc., when compared to SV model.

Ultimately, our analysis underscores the critical role that volatility forecasting plays in financial decision-making, urging practitioners to select models carefully based on their specific needs and market conditions.

# 5.0 Evaluation and Discussion

When comparing our research findings to previous studies conducted by researchers such as Ding & Meade (2010) and Avilés Ochoa & Flores Sosa (2021), we identified an interesting alignment. Specifically, one of our selected time periods and forecasting horizons (Time period: January 1, 2001, to December 29, 2006, Horizon: 20 days volatility forecast) matches that of Ding & Meade (2010) for both the FTSE 100 and S&P 500. However, it's crucial to note that our models for estimating parameters and conducting forecasting differ from those employed by Ding & Meade (2010), as we drew upon the methodologies outlined in Avilés Ochoa & Flores Sosa (2021), which focused on volatility forecasting for exchange rates rather than the stock market. Our approach leveraged advanced techniques, including Bayesian parameter estimation, the incorporation of Markov Chain Monte Carlo (MCMC) methods, and the utilization of a student’s t-distribution for SV model and using maximum likelihood utilization of a student’s t-distribution for GARCH model. Additionally, we benefited from improved computational capabilities, notably the use of ARCH and PYMC3 libraries, which contributed to more accurate forecasts. As a result, our research yielded slightly lower Root Mean Squared Error (RMSE) values compared to Ding & Meade (2010).

Here are the research findings of Ding & Meade (2010), with a note that in this study, a "one-month forecasting" interval is equivalent to a 20-day forecast when adjusted for holidays according to this study.

Table : Results of Ding & Meade (2010)

A table with numbers and letters

Description automatically generated

It's worth acknowledging that while our results exhibited improved forecasting accuracy compared to Ding & Meade (2010), they may not surpass the findings of Avilés Ochoa & Flores Sosa (2021), primarily due to the differing nature of our data (stock market vs. exchange rates). Although our modelling of GARCH and SV models aligns with Avilés Ochoa & Flores Sosa (2021), the comparison may not be entirely valid, given the distinct characteristics of financial markets. Nevertheless, our results remain robust and fall within the range of outcomes observed in other research studies, validating their credibility within the scope of this project.

below are the research findings from Avilés Ochoa & Flores Sosa (2021). It's important to highlight that their study focused on exchange rate data for forecasting log volatility, while our study focus on stock market data to forecast percentage volatility.

Table : Results of Avilés Ochoa & Flores Sosa (2021)

A table of numbers and symbols

Description automatically generated

Here we compare 20 days volatility forecasting results with results of Ding & Meade (2010) studies.

Table : Compare 20 days volatility forecasting results with results of Ding & Meade (2010) studies.



In conclusion, our findings indicate that the GARCH model outperforms the SV model in out-of-sample forecasting, consistent with the results obtained by Ding & Meade (2010). Specifically, GARCH(1,1) demonstrated superior overall accuracy in both moderately volatile and highly volatile scenarios. This alignment with Ding & Meade (2010) further strengthens the reliability of our results, which are underpinned by enhanced forecasting methodologies.

Overall, our research results hold their own against established studies, lending credibility to our approach and findings.

## 5.1 Achievements

### 5.1.1 Aim of the Project Achievement

The project's aim was to implement, investigate, and assess the performance of GARCH and SV models in forecasting stock market volatility, which was achieved through several key steps. Firstly, the project successfully implemented both GARCH and SV models, as demonstrated in sections 3.0 and 4.0, where comprehensive analyses, results, and evaluations for both model types across varying timeframes are presented. Secondly, a thorough investigation and evaluation process was conducted using diverse metrics, including Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE), to gauge accuracy and efficiency. These evaluations are extensively detailed in section 4.0, offering insights into model performance under different conditions.

Furthermore, the project effectively examined the models capability to capture volatility features by analysing their performance in various market scenarios, encompassing periods of both moderate and high volatility, as illustrated in section 4.0. Lastly, the study not only evaluated model performance but also delved into their respective strengths and limitations. It provided practical guidance on when to consider each model, considering factors such as forecasting horizons and market conditions. These valuable insights are discussed comprehensively in section 4.1.

### 5.1.2 Research Questions Achievement

The research questions have also been addressed effectively:

* Accuracy of Models: The project assessed the accuracy of both GARCH and SV models in forecasting stock market volatility. This assessment is evident throughout the analysis presented in section 4.0, which includes metrics like MSE, RMSE, and MAE to measure accuracy.
* Comparison of GARCH and SV Models: The study compared the performance of GARCH models to SV models, answering the question of how these models perform relative to each other. This comparison is evident in the detailed analysis in section 4.0, which provides insights into which model performs better under various conditions.
* Representation of Volatility Characteristics: The project sought to determine if the chosen models effectively represent the volatility characteristics of stock market data. This question is addressed through the comprehensive evaluation of model performance, which includes capturing volatility features and discussing the suitability of each model for different scenarios. These findings are presented in section 4.0 and section 4.1.

### 5.1.3 Project objective achievements

* The project incorporates theoretical foundations and empirical evidence related to GARCH and SV models, suggesting a comprehensive understanding of literature review. These findings are presented in all sections.
* The project mentions acquiring financial data and using it for analysis. It discusses data sources, implying that data acquisition and pre-processing have been conducted. These findings are presented in section 2.0.
* The project goes into great detail regarding the implementation of both GARCH and SV models. It discusses model selection, parameter estimation, and programming libraries used, indicating successful implementation. These findings are presented in section 3.0.
* The core of the project is the evaluation and comparison of GARCH and SV models. It includes extensive analysis using metrics like MSE, RMSE, and MAE to assess accuracy and efficiency, clearly achieving this objective. These findings are presented in section 4.0.
* The project provides recommendations for model selection and discusses the strengths and limitations of each model. This directly fulfils the objective of making recommendations based on research findings. These findings are presented in Section 4.1
* While not explicitly mentioned, it's reasonable to assume that the project contributes improved skills in data-related tasks, Financial Modelling, Model Selection, Data Handling, Statistical Modelling, Data Analysis skills and as well as programming language expertise, through the application of quantitative finance modelling techniques.
* The project does not explicitly mention real-world application, but it has the potential to inform practical trading and investment decisions. This objective is partially achieved Due to time constraints within the scope of this project, but Future steps could involve applying research insights to real-world scenarios.

## 5.2 Project Plan

Table : Project plan

A blue and white list with black text

Description automatically generated

A graph with orange rectangles

Description automatically generated

Figure : Gantt chart

# 6.0 Recommendations

* **Model Selection and Configuration:** Based on the project's findings, consider using the SV model for daily volatility forecasting, particularly for short-term traders. For medium-term forecasting horizons the GARCH model be more suitable. Experiment with different model configurations, including varying lag orders or alternative distributions (e.g., Student's t-distribution, ML, MCMC), to fine-tune model performance based on the specific requirements of your forecasting task.
* **Data Handling and Preparation:** Ensure the financial data used is of high quality, free from outliers, and properly pre-processed. Employ robust data cleaning techniques and validation methods. Explore the possibility of incorporating additional market indicators for improved model accuracy.
* **Forecasting Horizon Considerations:** Recognize the importance of choosing the right forecasting horizon. Tailor the selection between GARCH and SV models based on the need for daily, short-term, or long-term volatility forecasts.
* **Practical Application:** Apply insights gained from the research to real-world scenarios in the financial industry. Implement recommended models thoughtfully, considering specific contexts and risk tolerance in trading or investment.
* **Further Exploration:** Extend the analysis to other financial assets, such as individual stocks or different stock indices, to understand how models perform in various market segments. Investigate the development of trading strategies based on volatility forecasts and assess their effectiveness in risk management and returns generation.

These concise recommendations aim to guide future applications of volatility forecasting models, considering the achievements and outcomes of our project.

# 7.0 Conclusion

In this project, we investigated volatility forecasting in financial markets using two frequently used models: the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model and the Stochastic Volatility (SV) model. Our major goal was to evaluate the effectiveness of these models in predicting stock market volatility over various time periods and forecasting horizons. We arrived at many major conclusions after intensive data analysis and model evaluation:

Our investigation revealed nuanced insights into the performance of GARCH(1,1) and SV models under distinct market conditions and forecasting scenarios. When employing actual returns for forecasting, SV consistently outperformed GARCH(1,1) in next-day volatility forecasting for both the S&P 500 and FTSE 100 stock markets. This finding underscores the suitability of SV for daily trading and short-term decision-making.

However, the real-world applications of volatility forecasting often demand predictions over longer horizons. In these cases, where future return values are not readily available for model fitting, GARCH(1,1) emerged as the preferred choice. GARCH(1,1) emerged as the favoured choice in these instances, where future return values are not easily available for model fitting. GARCH(1,1) regularly outperformed SV for forecasting periods of 10 days, 15 days, and 20 days during high volatility periods, according to our research.

Furthermore, the sensitivity of model performance to prevailing market conditions became evident. During moderate volatility periods, both SV and GARCH(1,1) displayed comparable accuracy, with a slight edge in favour of GARCH(1,1) when assessing overall performance. However, during high volatility periods, GARCH(1,1) consistently outperformed SV across all forecasting scenarios.

Comparing our findings with previous studies by Ding & Meade (2010) and Avilés Ochoa & Flores Sosa (2021), we noted valuable alignments and distinctions. While we shared a time period with Ding & Meade (2010) for comparison, our research employed advanced methodologies inspired by Avilés Ochoa & Flores Sosa (2021). As a result, we achieved slightly improved Root Mean Squared Error (RMSE) values compared to Ding & Meade (2010). However, the comparison with Avilés Ochoa & Flores Sosa (2021) was limited by differences in data types (stock market vs. exchange rates).

**Implications and Future Directions**

Our findings emphasise the importance of volatility forecasting in financial decision-making. The GARCH and SV models should be chosen with caution, taking into account individual needs, data availability, and the desired forecasting horizon. These models have various advantages and disadvantages, making them appropriate for a variety of applications.

In the future, researchers can explore hybrid GARCH-SV models, assess external factors impact, integrate machine learning, study volatility clustering, and develop real-time forecasting. These avenues aim to advance volatility forecasting and its applications.

**Final Remarks**

In conclusion, this project contributes valuable insights into the behaviour of GARCH and SV models during periods of extreme market volatility. Our findings guide practitioners in selecting the most appropriate model for their specific financial applications. We emphasize the importance of considering the forecast horizon, prevailing market conditions, and data characteristics when making this critical choice. Ultimately, our research advances the field of volatility forecasting, aiding investors, and decision-makers in navigating the complexities of financial markets.

This project serves as a foundation for future research in financial modelling and risk management, promoting evidence-based decision-making and enhancing our understanding of market dynamics.

# 8.0 Bibliography:

* Avilés Ochoa, E., & Flores Sosa, M. M. (2021) ‘Comparison of the GARCH and stochastic models: An application to the Mexican peso-us dollar exchange rate’. *Contaduría y Administración*. 66 (2) pp.1-14.
* Bollerslev, T. (1986) ‘Generalized Autoregressive Conditional Heteroskedasticity’. *Journal of Econometrics*. 31 (3) pp.307-327.
* Chan, J. C. C., & Grant, A. L. (2015) ‘Modeling Energy Price Dynamics: GARCH versus Stochastic Volatility’. Research School of Economics, Australian National University. November 2015.
* Ding, J., & Meade, N. (2010) ‘Forecasting accuracy of stochastic volatility, GARCH and EWMA models under different volatility scenarios’. *Applied Financial Economics*. 20 (10) pp.771-783.
* Fischer, T. and Krauss, C., (2018) ‘Deep learning with long short-term memory networks for financial market predictions’. European Journal of Operational Research, 270(2), pp.654-669.
* Franses, P. H., van der Leij, M., & Paap, R. (2005)’ A Simple Test for GARCH against a Stochastic Volatility Model (Econometric Institute Report EI 2005-41)’. Erasmus University Rotterdam.
* Jaishwal(NA), R. Stochastic Volatility Model using Nifty50 Data Set. Kaggle. <https://www.kaggle.com/code/rajjais3003/stochastic-volatility-model-using-nifty50-data-set>
* Kim, J.-M., Jun, C., & Lee, J. (2021)’ Forecasting the Volatility of the Cryptocurrency Market by GARCH and Stochastic Volatility’. *Mathematics*, 9, 1614. DOI: <https://doi.org/10.3390/math9141614>
* Lehar, A., Scheicher, M., & Schittenkopf, C. (2002) ‘GARCH vs. Stochastic Volatility: Option Pricing and Risk Management’. *Journal of Banking & Finance*. 26 pp.323–345.
* Marti, G. (2023). *Decoding the Quant Market: A Guide to Machine Learning in Trading*. Palaiseau: Ecole Polytechnique.
* Namugaya, J., Weke, P. G. O., & Charles, W. M. (2014) ‘Modelling Volatility of Stock Returns: Is GARCH(1,1) Enough?’. *International Journal of Sciences: Basic and Applied Research (IJSBAR)*. 16(2) pp.216-223.
* Poon, S-H. (2005). A Practical Guide to Forecasting Financial Market Volatility. John Wiley & Sons Ltd.
* ritvikmath. (NA). GARCH Stock Modeling. GitHub. <https://github.com/ritvikmath/Time-Series-Analysis/blob/master/GARCH%20Stock%20Modeling.ipynb>
* Salvatier, J., Wiecki, T.V., & Fonnesbeck, C. (2016) ‘Probabilistic programming in Python using PyMC3’. *PeerJ Computer Science*, 2, e55. DOI: 10.7717/peerj-cs.55.
* Sheppard, K. (2021, March 3). bashtage/arch: Release 4.18 (Version v4.18). Zenodo. <https://doi.org/10.5281/zenodo.593254>.
* Taylor, S.J., 1986. *Modeling Financial Time Series*. Chichester: Wiley.

# 9.0 Appendix

## 9.2 Appendix B: all GARCH models summary:

**Time range 01-01-2001 to 29-12-2006:**

**GARCH(1,1):**

A screenshot of a computer

Description automatically generated

**GARCH(1,2):**

A screenshot of a computer screen

Description automatically generated

**GARCH(2,2):**

**A screenshot of a computer

Description automatically generated**

**GARCH(2,1):**

**A screenshot of a computer

Description automatically generated**

**Time range 01-01-2014 to 29-03-2020:**

**GARCH(1,1):**

**A screenshot of a computer screen

Description automatically generated**

**GARCH(1,2):**

**A screenshot of a computer screen

Description automatically generated**

**GARCH(2,2):**

**A screenshot of a computer

Description automatically generated**

**GARCH(2,1):**

**A screenshot of a computer screen

Description automatically generated**