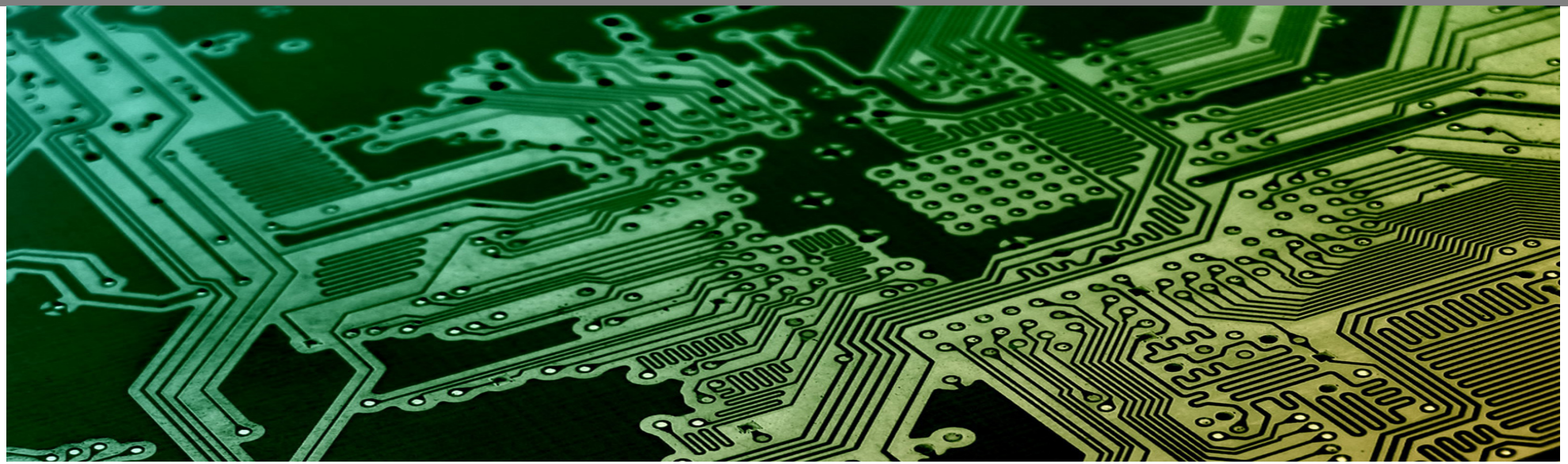
Karlsruhe Institute of Technology

Evolutionary Hypergraph Partitioning

Presentation · December 12, 2017

Robin Andre

INSTITUTE OF THEORETICAL INFORMATICS ·

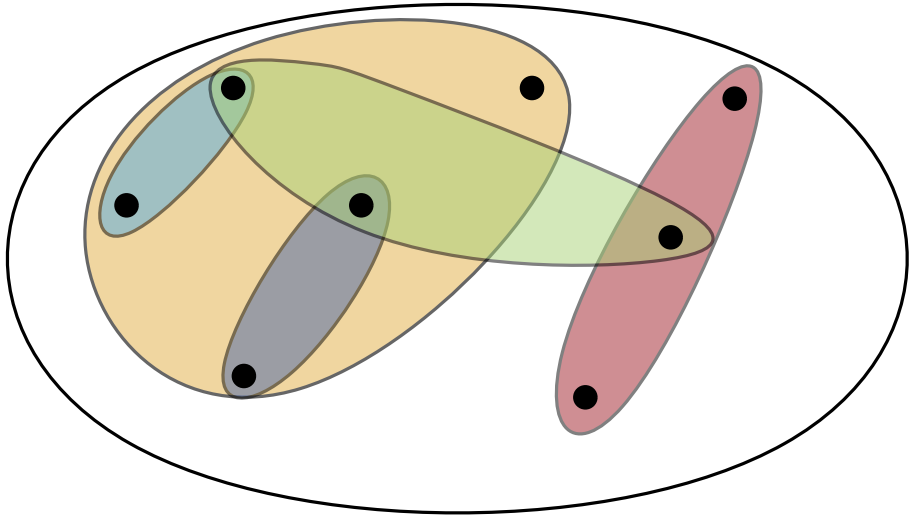


Hypergraph Partitioning

Hypergraph Partitioning

A k partition of a hypergraph is $H = V_1 \cup V_2 \cup ... \cup V_k$

A partition is balanced if $\forall\ 1 \leq i \leq k\ :\ c(V_i) \leq (1 + \epsilon)\lceil \frac{c(V)}{k} \rceil$



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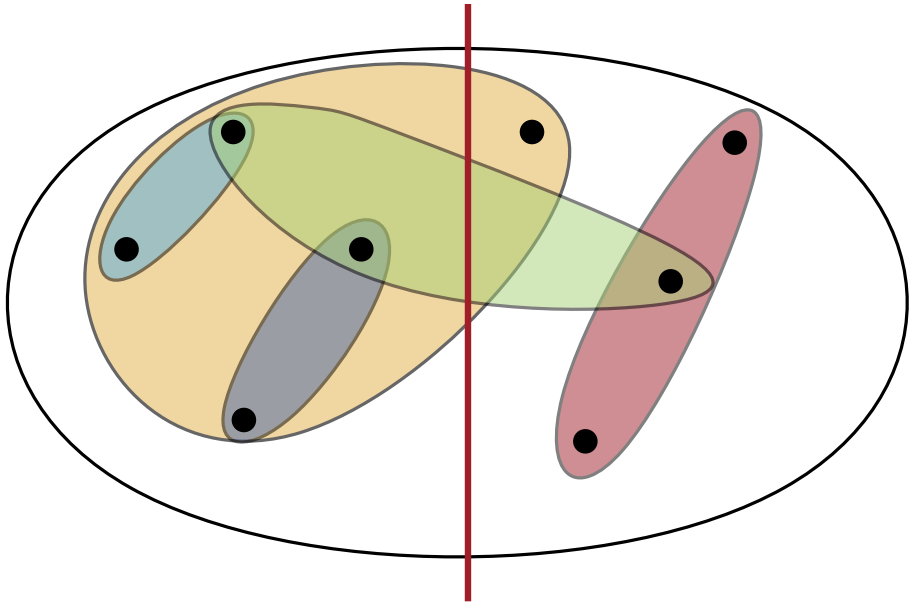
$k = 2; cut = 2; (\lambda - 1) = 2$



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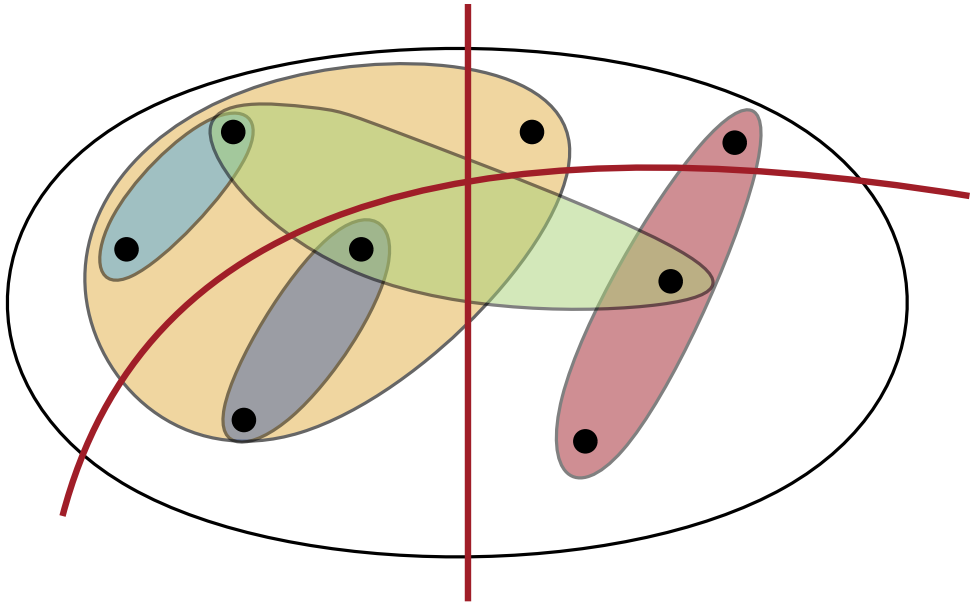
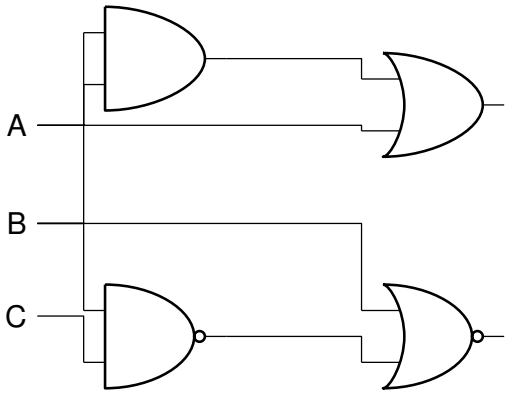
$k = 4;\ cut = 3;\ (\lambda - 1) = 5$



Motivation

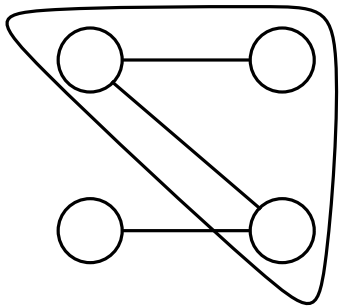
- Hypergraph partitioning is NP-hard
- Many applications benefit from the best possible solution
- Evolutionary Algorithms are generating high quality solutions

VLSI Design

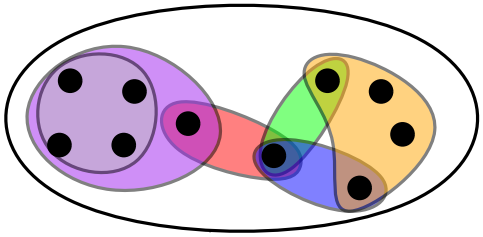


Scientific Computing

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$



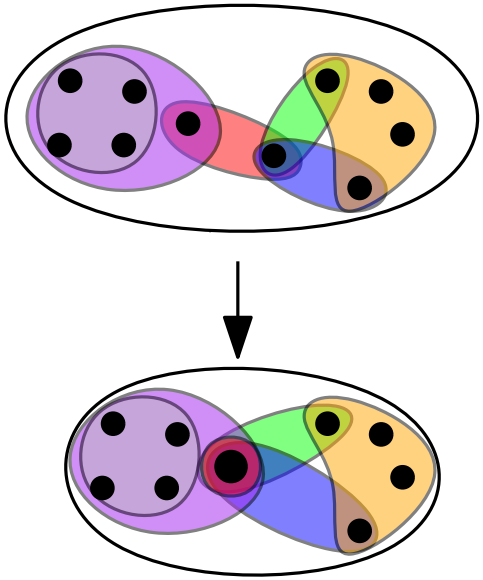
KaHyPar - Multilevel Partitioner



Coarsening:

- H is reduced to a smaller problem H_C
- n-level coarsening
- H_C should be sufficiently small

KaHyPar - Multilevel Partitioner



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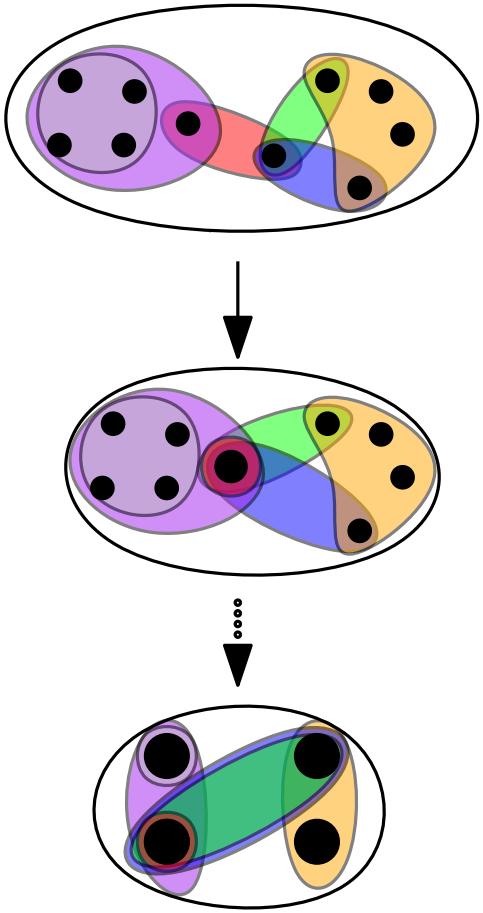

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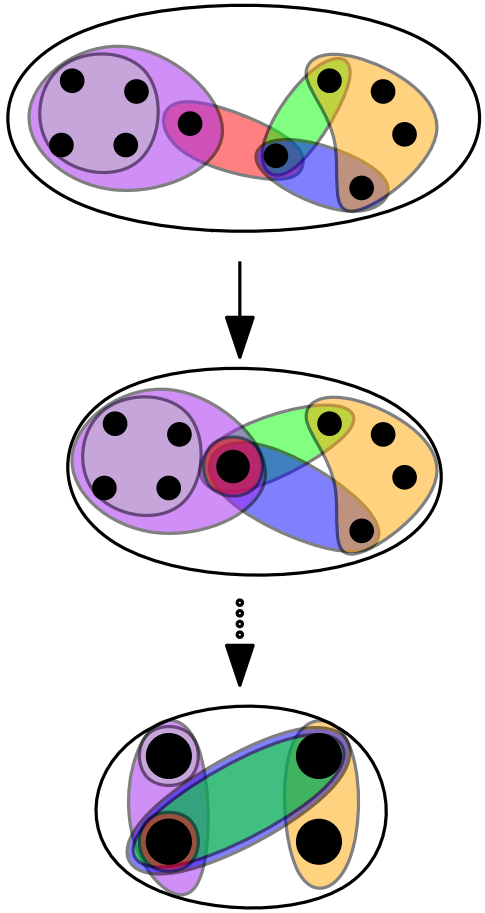
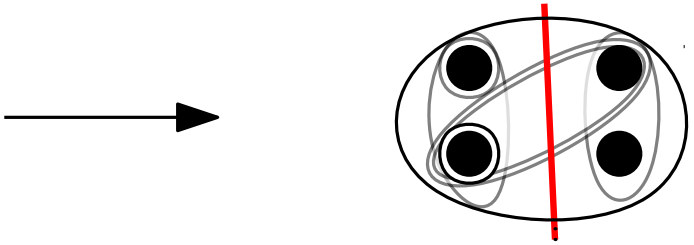
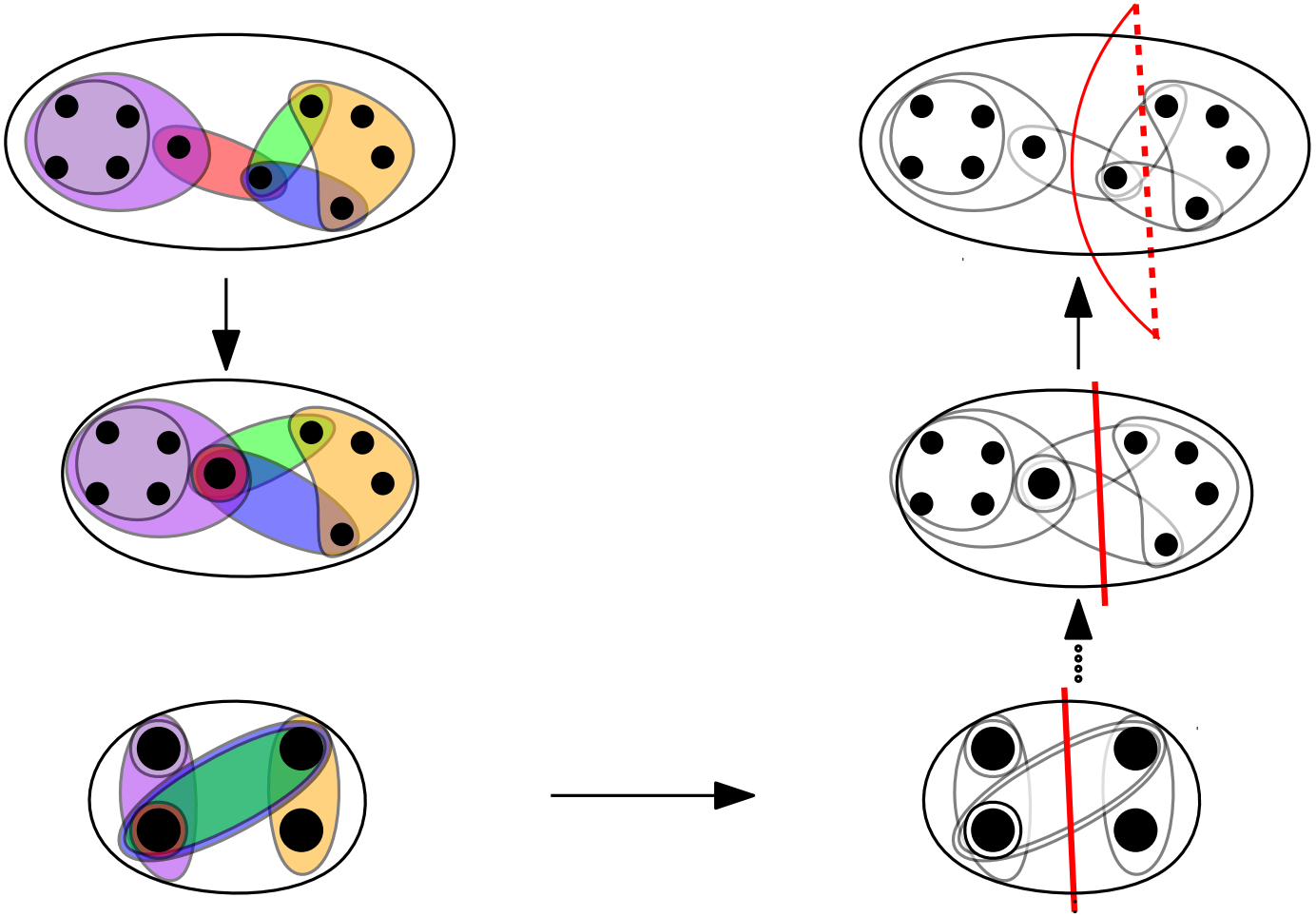




Initial Partitioning:

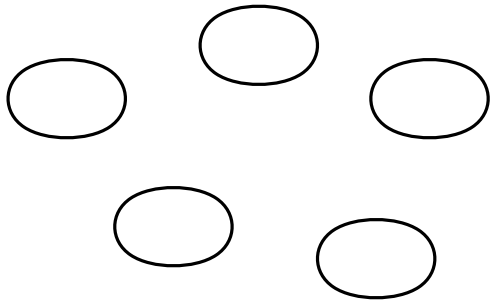
- An algorithm generates an Initial Partitioning for H_C

KaHyPar - Multilevel Partitioner



Evolutionary Process

- Choose individuals for recombination



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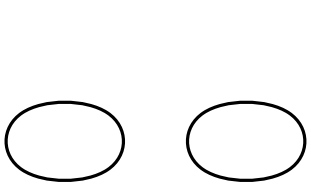
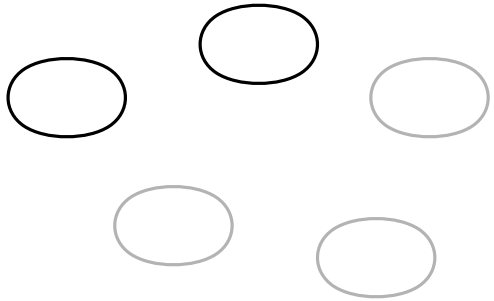
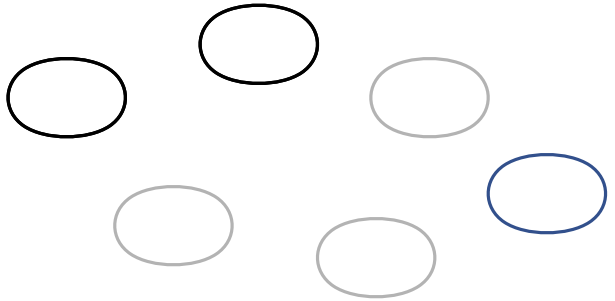




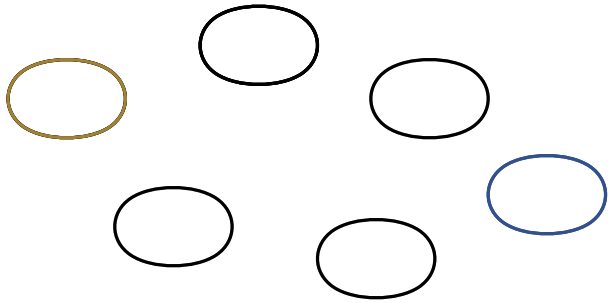
Evolutionary Process

- Choose individuals for recombination
- Generate offspring O



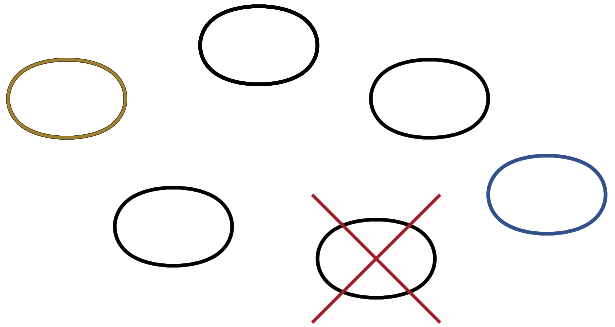
Evolutionary Process

- Choose individuals for recombination
- Generate offspring O
- Perform mutations M



Evolutionary Process

- Choose individuals for recombination
- Generate offspring O
- Perform mutations M
- Select survivors



Evolutionary Algorithm

- *KaHyPar* generates multiple partitions
- dynamic allocation $\delta = 15\%$
- balances time/hypergraph size

Evolutionary Algorithm

3.1s $time = 100s$ ⟶ ~ 5 iterations

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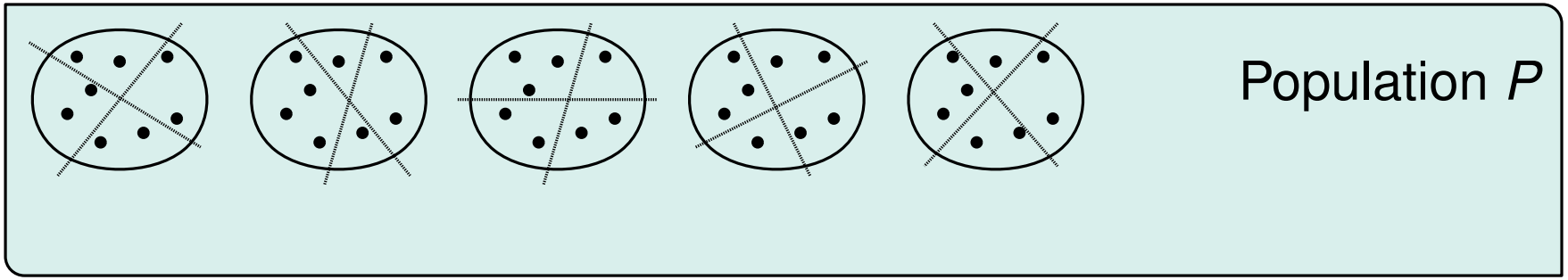

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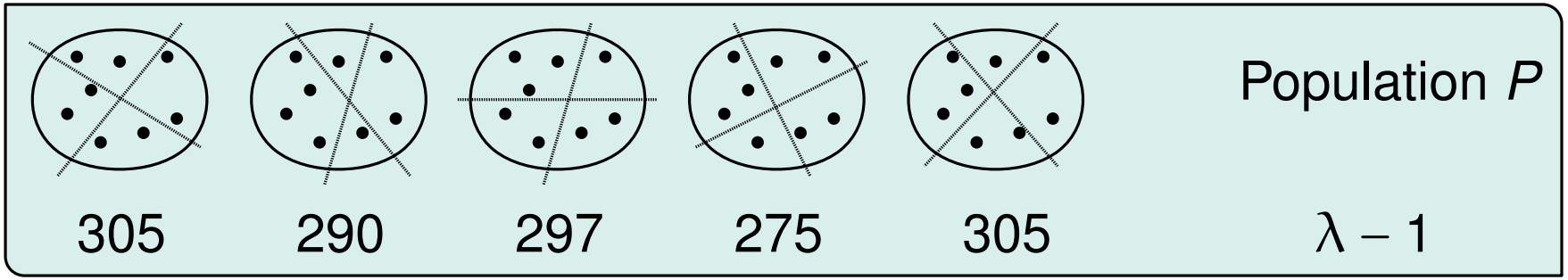

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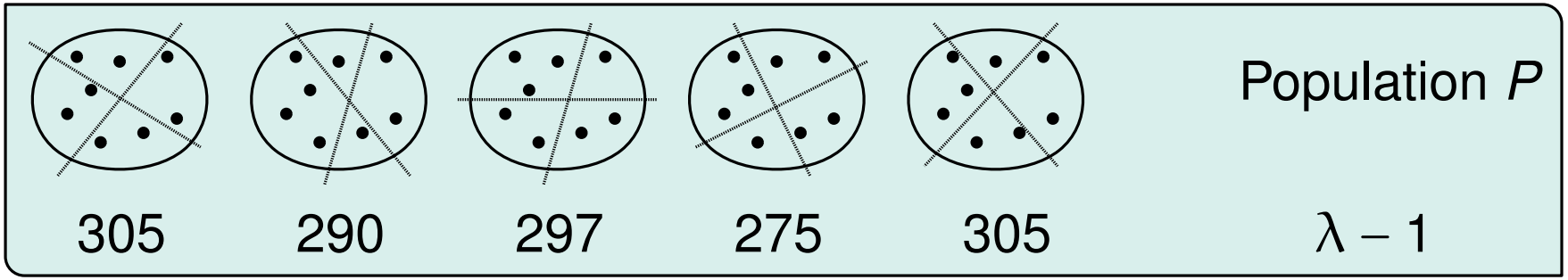

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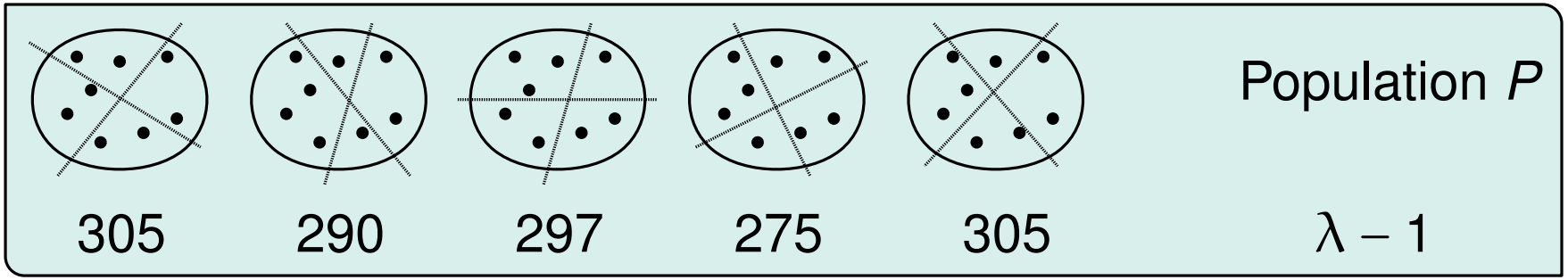
high quality solutions

Tournament Selection



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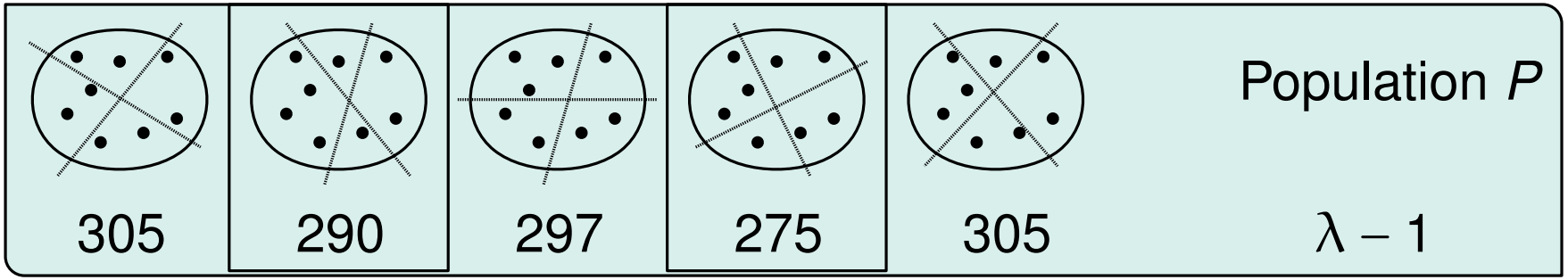
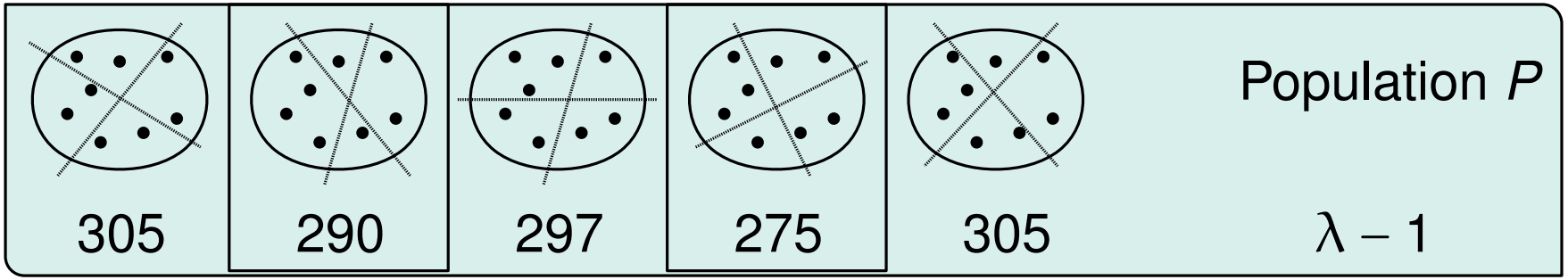


- Select 2 random individuals

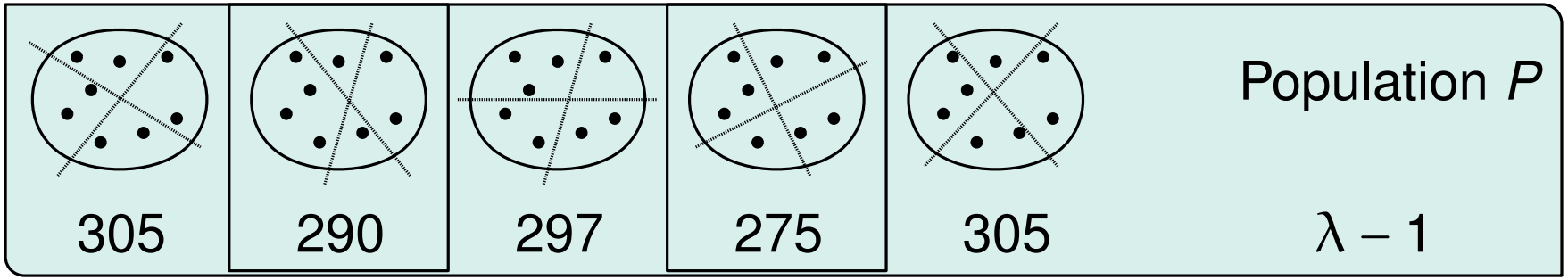
Tournament Selection



290 > 275

- Select 2 random individuals
- Compare their fitness

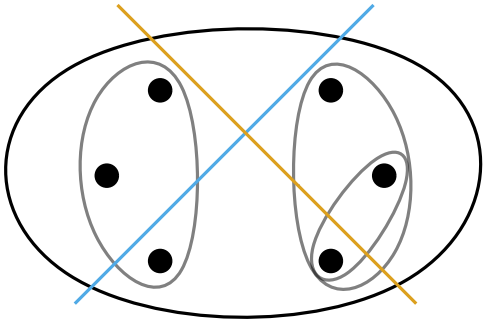
Tournament Selection

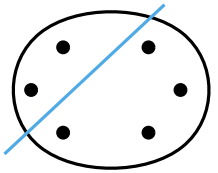


290 > 275

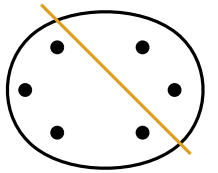
- Select 2 random individuals
- Compare their fitness
- Choose the better individual

Combine Operator





P_1

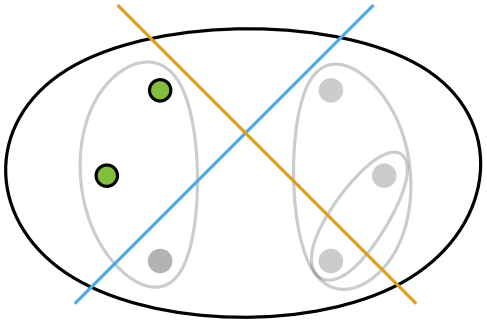


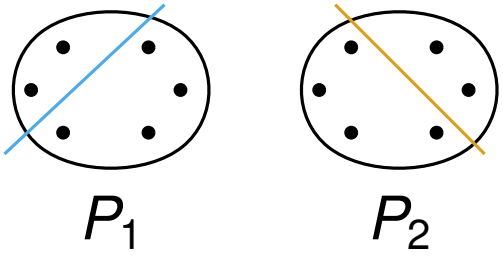
P_2

Coarsening:

- contractions must respect P_1 & P_2
- does not change solution quality

Combine Operator





Valid Contraction

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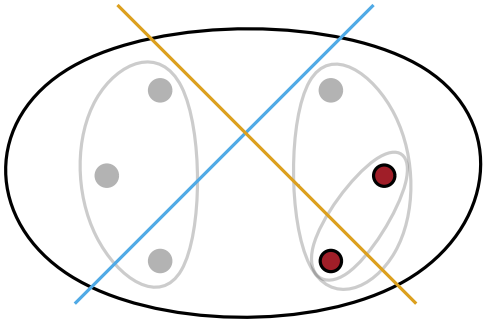

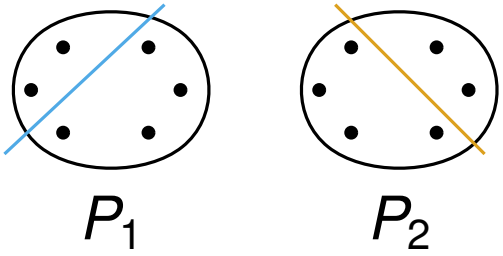

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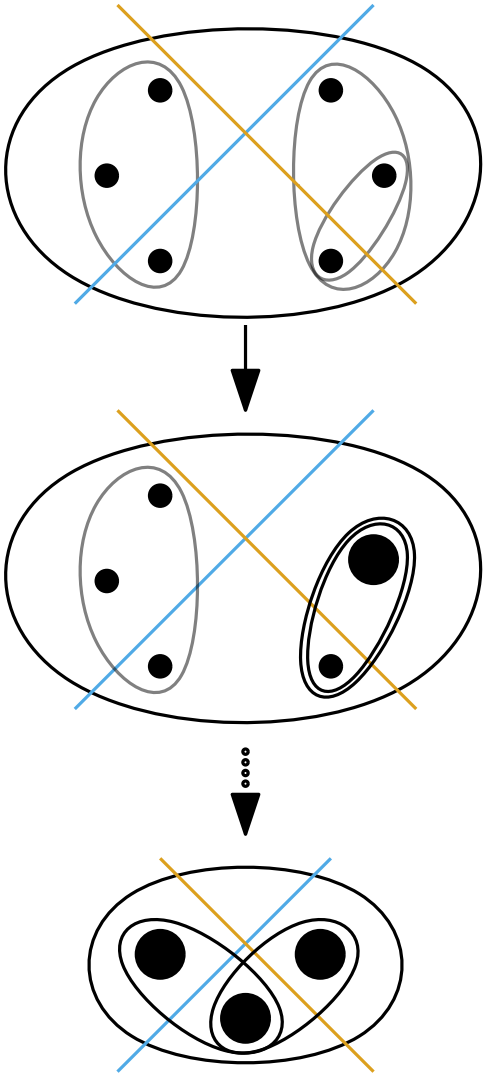

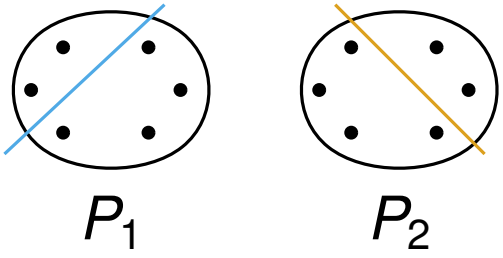

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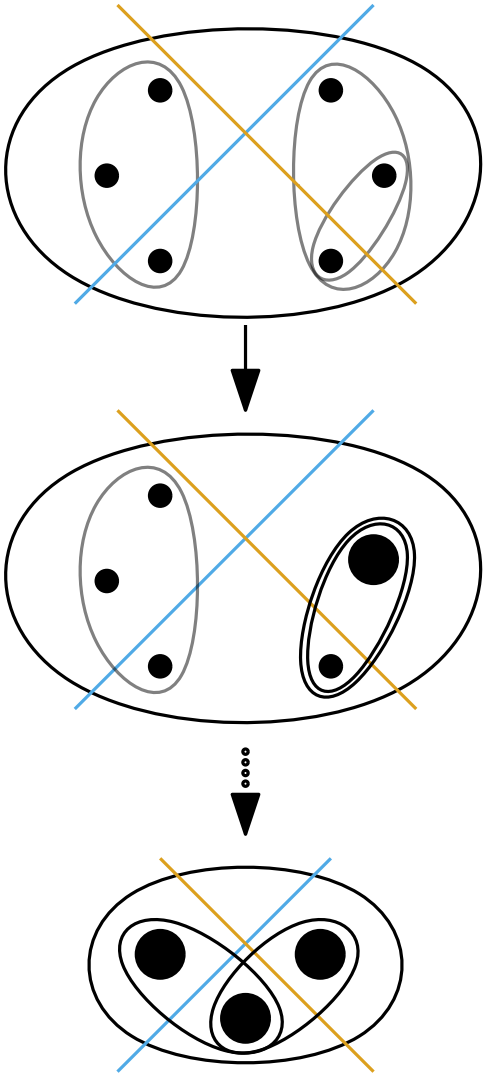

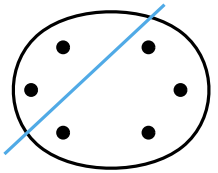

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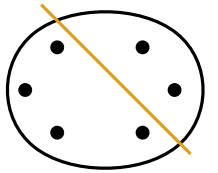
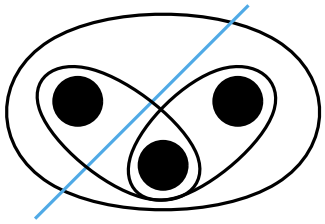


P_2

Initial Partitioning:

- Use the better parent partition (P_1)
- Maintains solution quality



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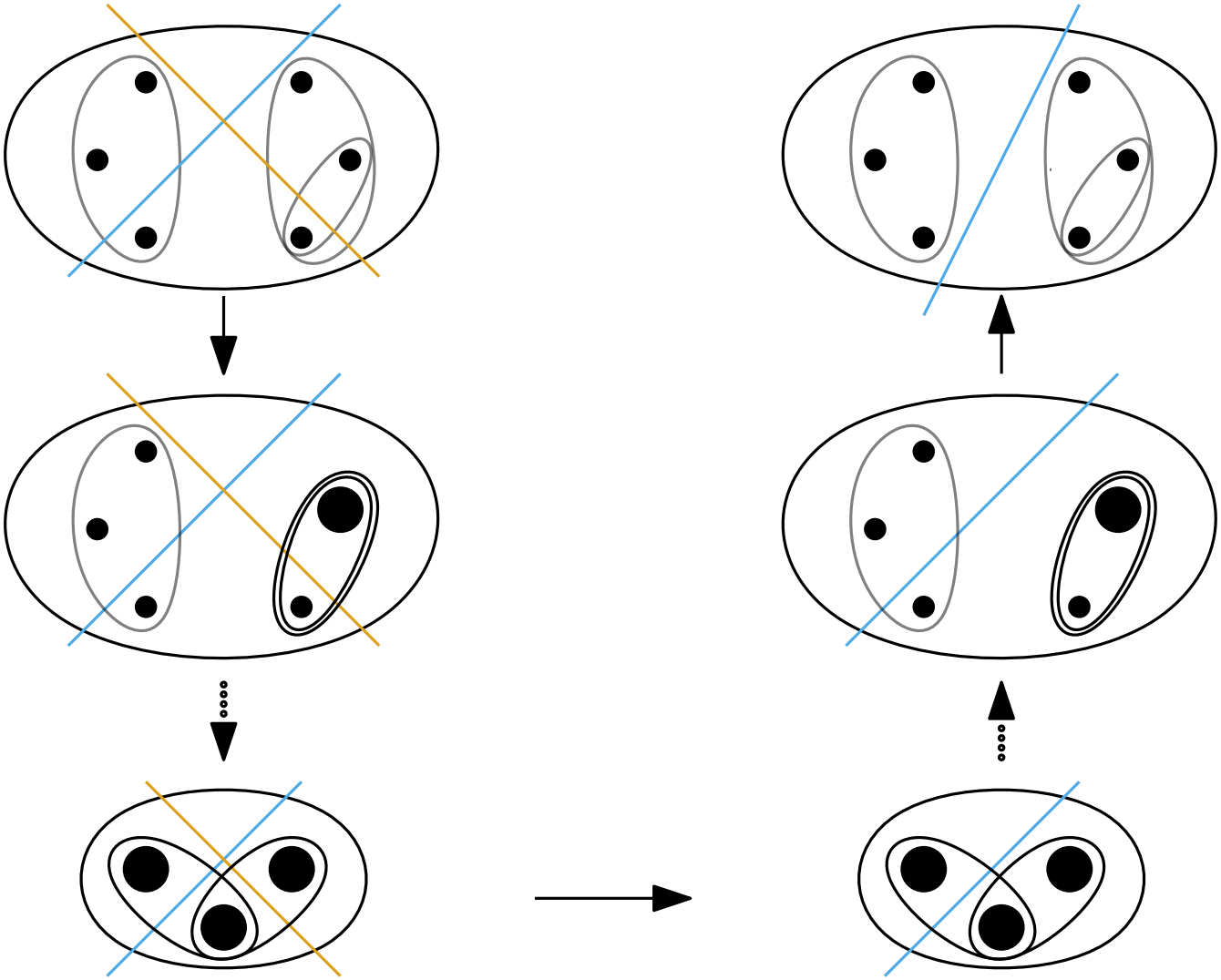
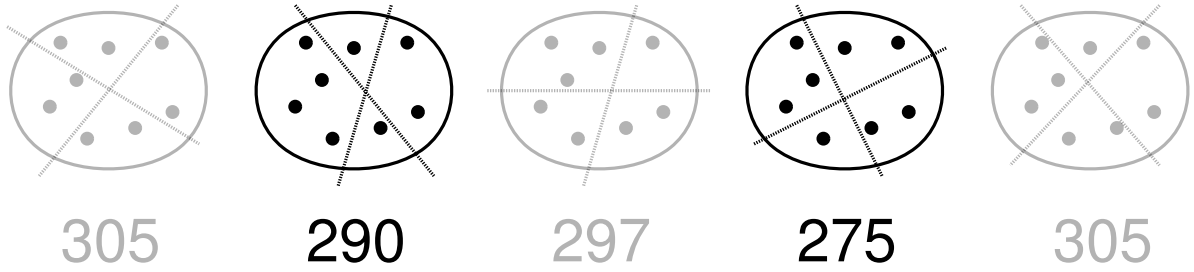
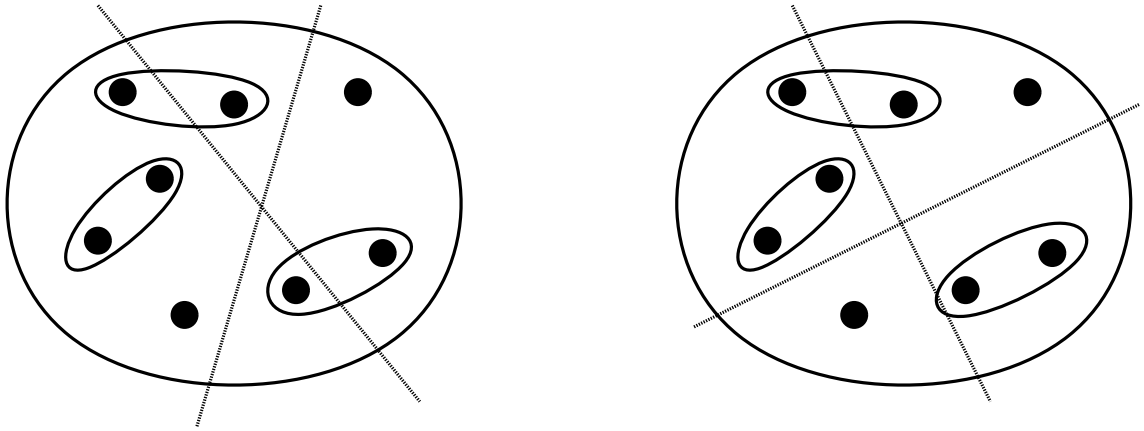


Edge Frequency Multicombine



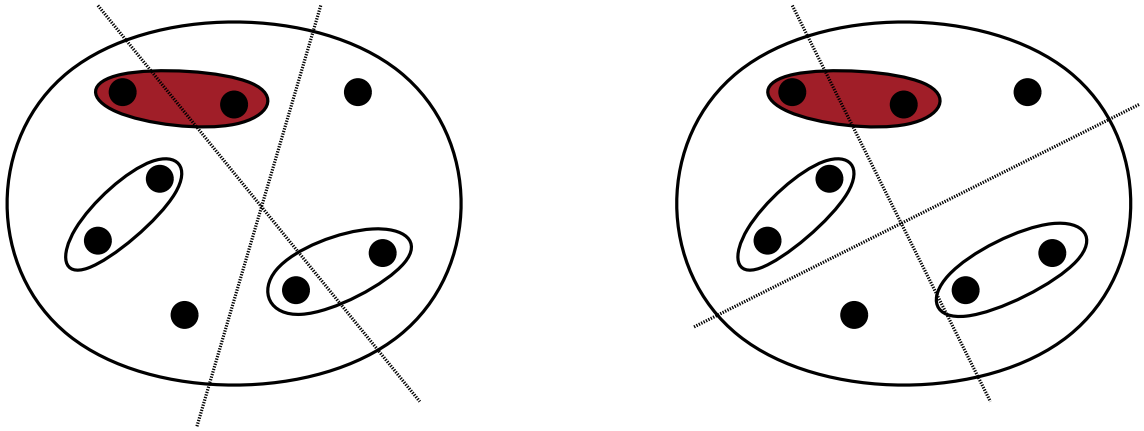
- We inspect the $\sqrt{|P|}$ best individuals of P
- Each hyperedge e is analyzed on its frequency in the selected elements

Edge Frequency Multicombine



- Frequent edges are beneficial to the solution quality
- Contracting frequent edges may be detrimental to solution quality
- Additionally it may limit other contractions

Edge Frequency Multicombine



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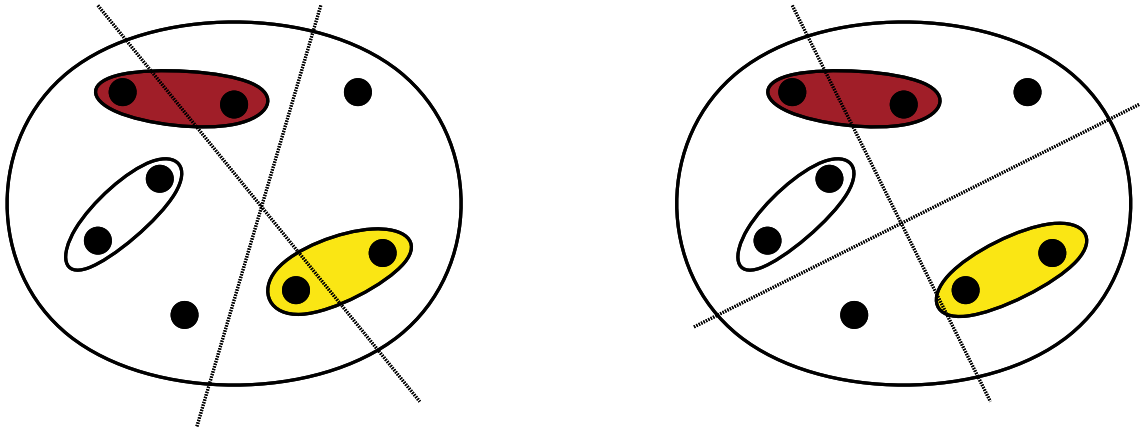

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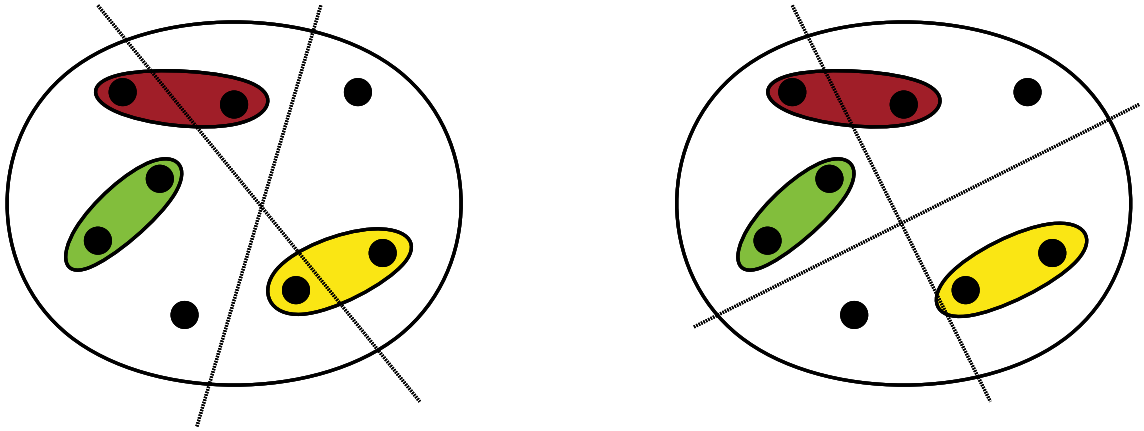

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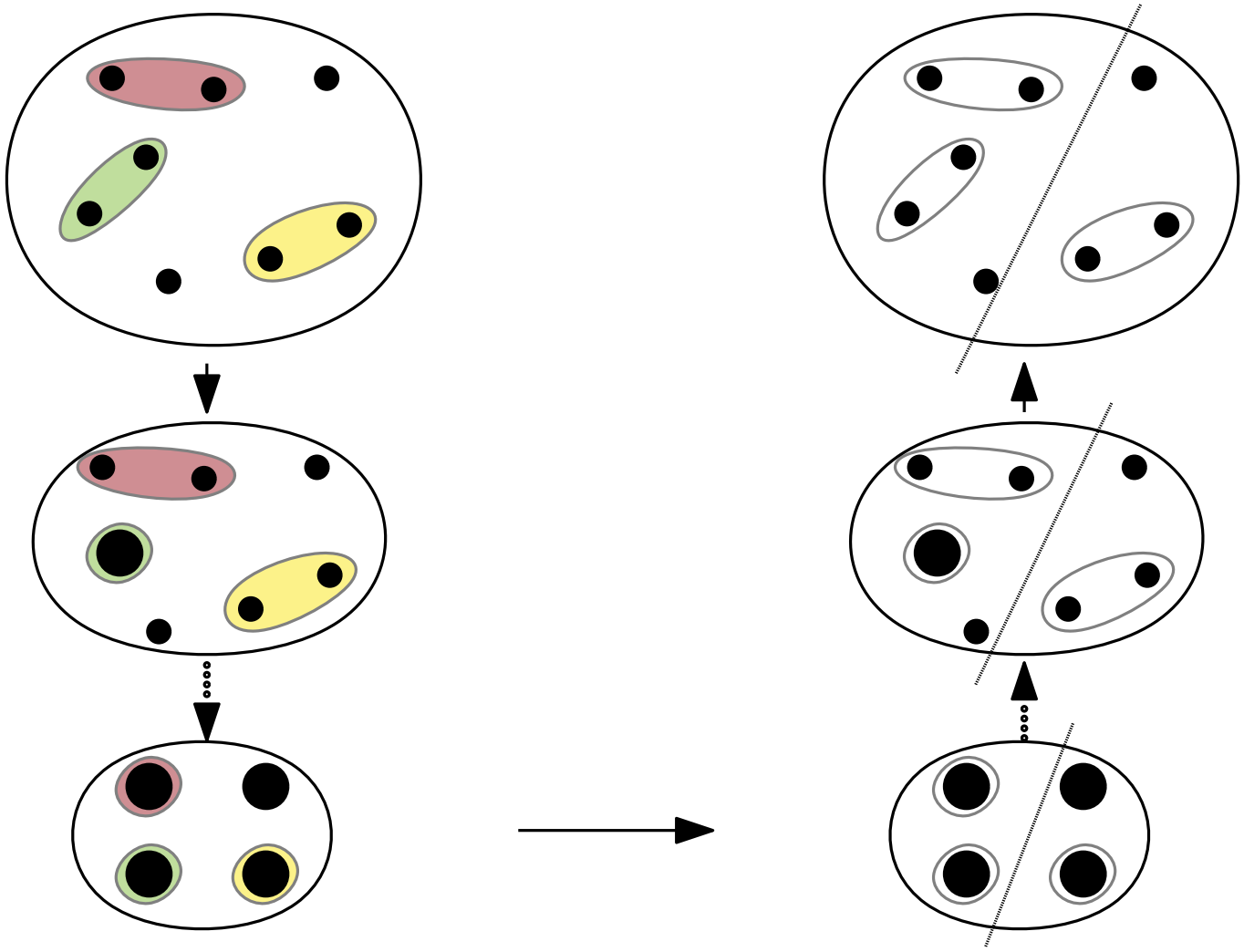
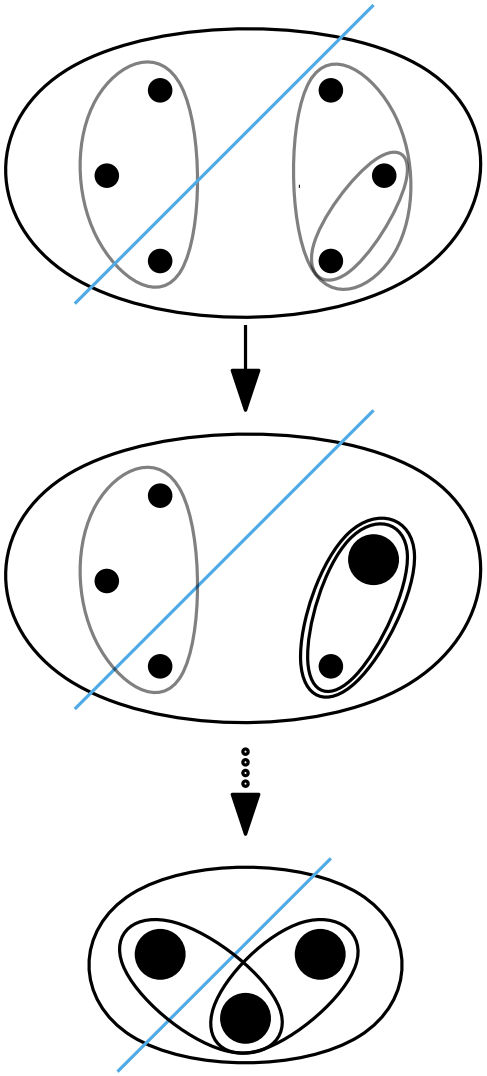
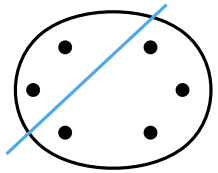


V-Cycle (+ New Initial Partitioning)





P

Coarsening:

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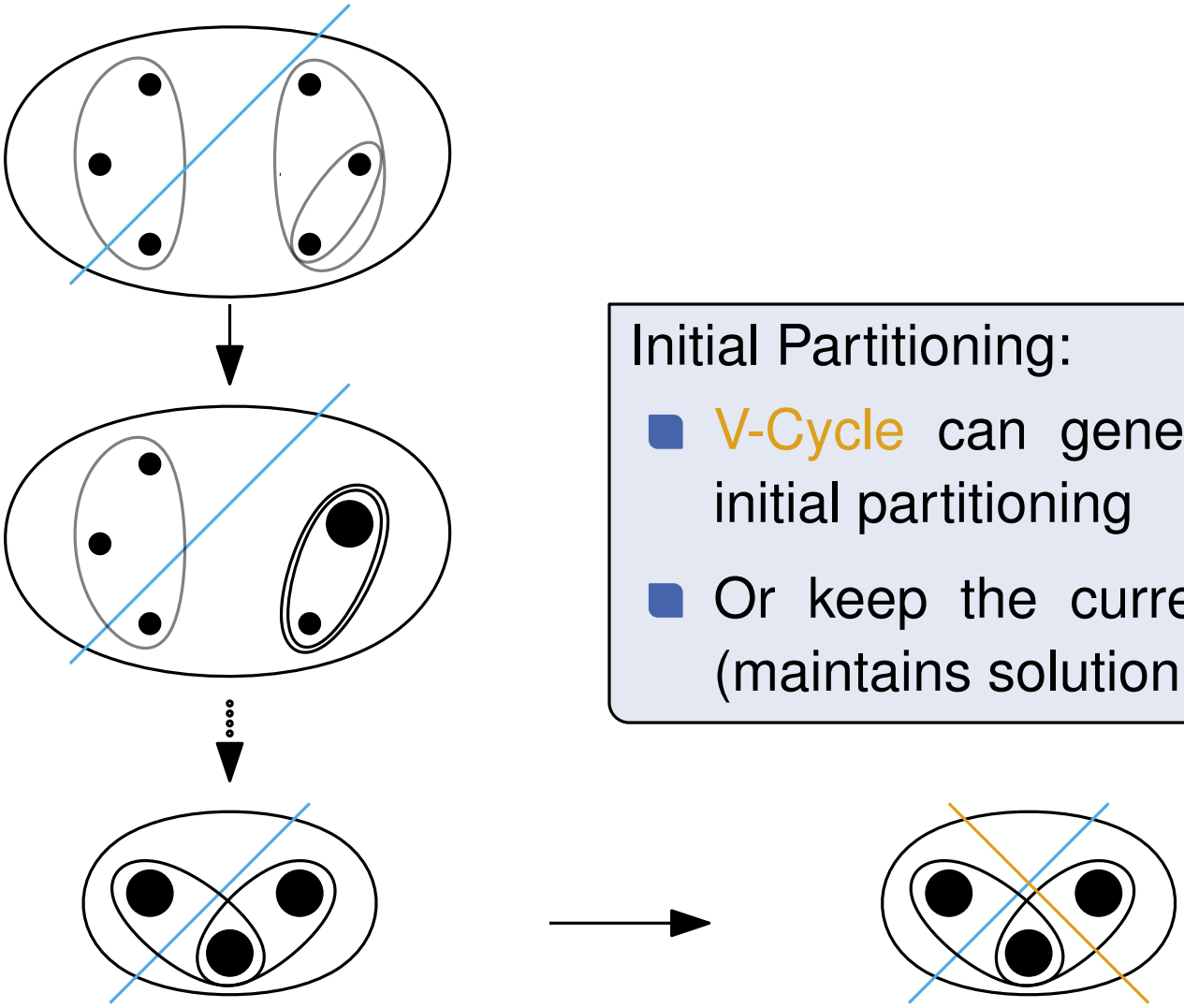
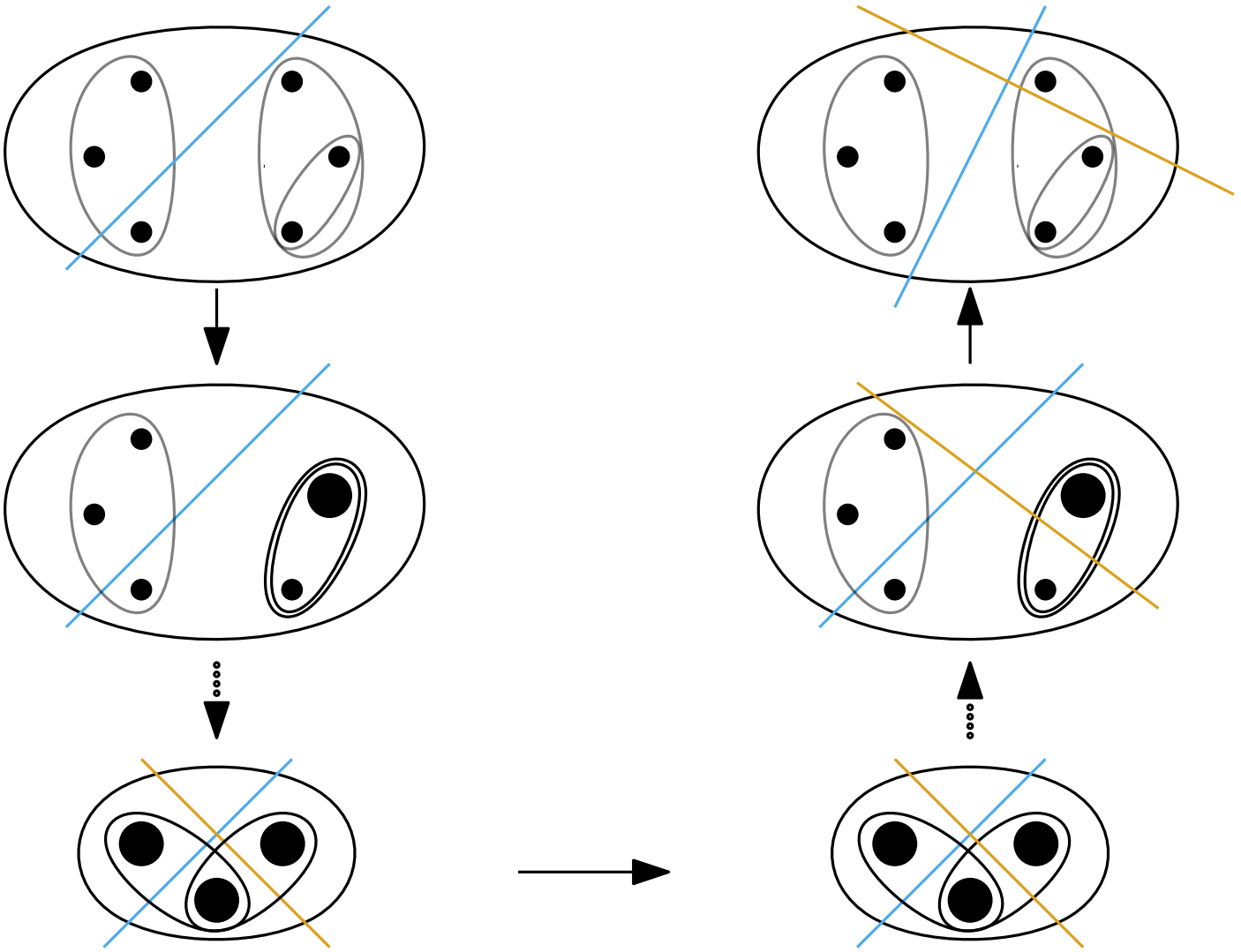
V-Cycle (+ New Initial Partitioning)



Initial Partitioning:

- V-Cycle can generate a new initial partitioning
- Or keep the current partition (maintains solution quality)

V-Cycle (+ New Initial Partitioning)

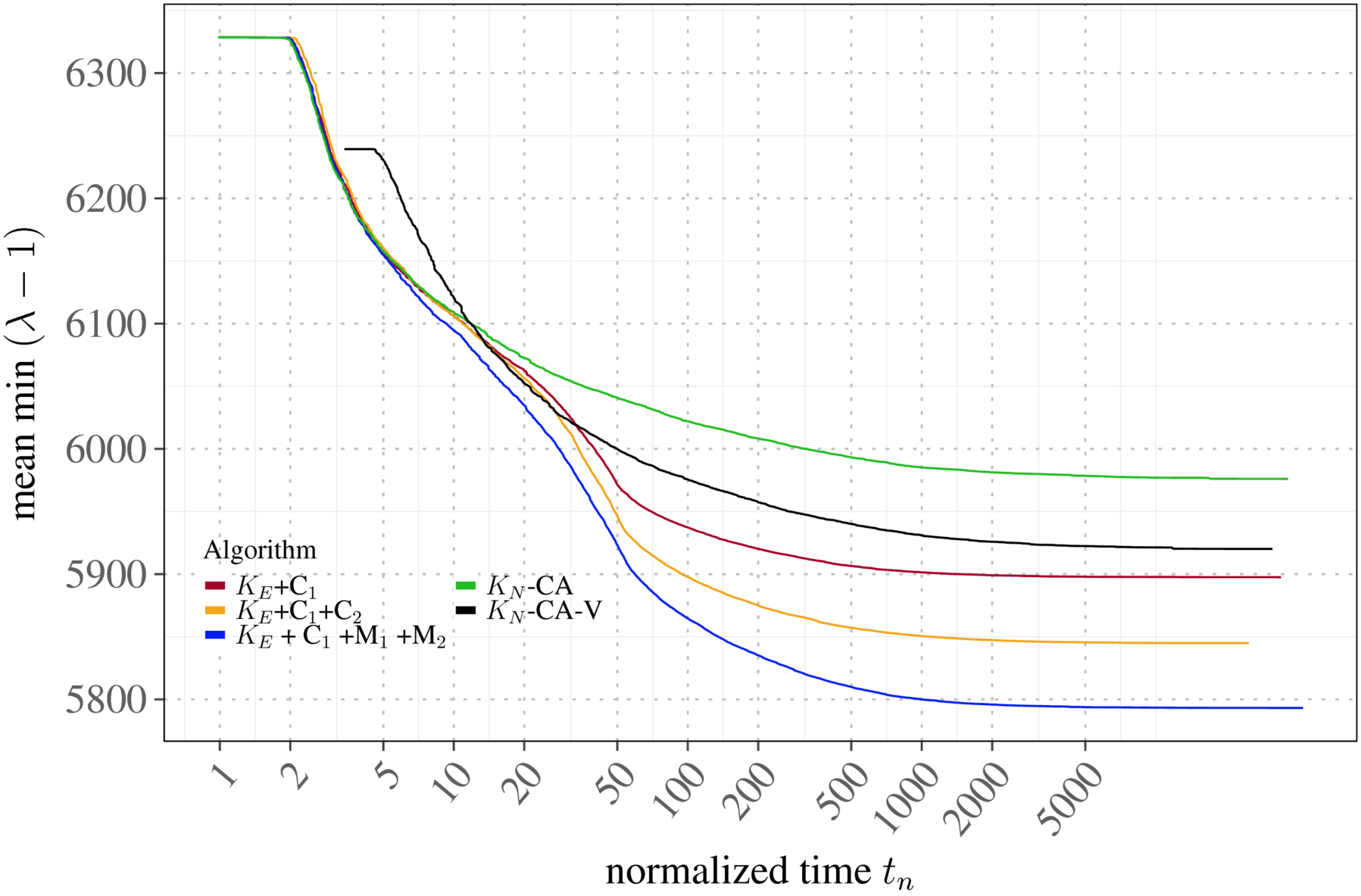


Experimental Setup

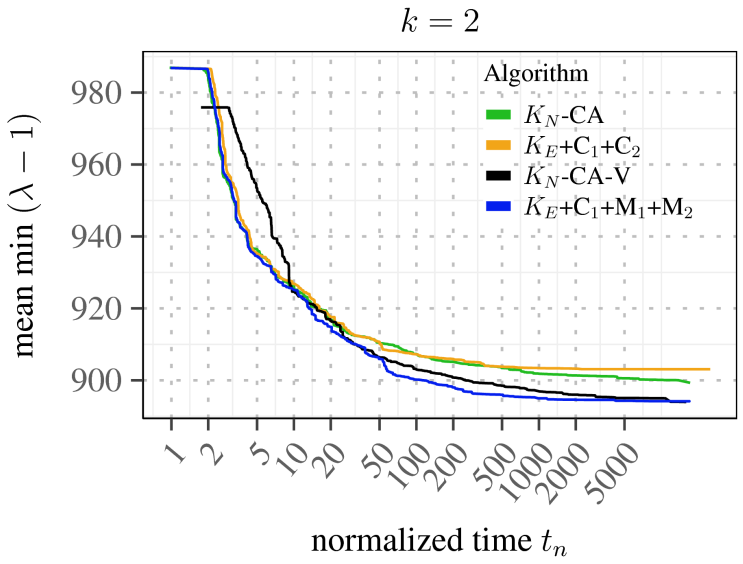
- $k = \{2, 4, 8, 16, 32, 64, 128\}; \epsilon = 0.03$
- 90 hypergraphs
- Comparison against repeated *KaHyPar*-CA

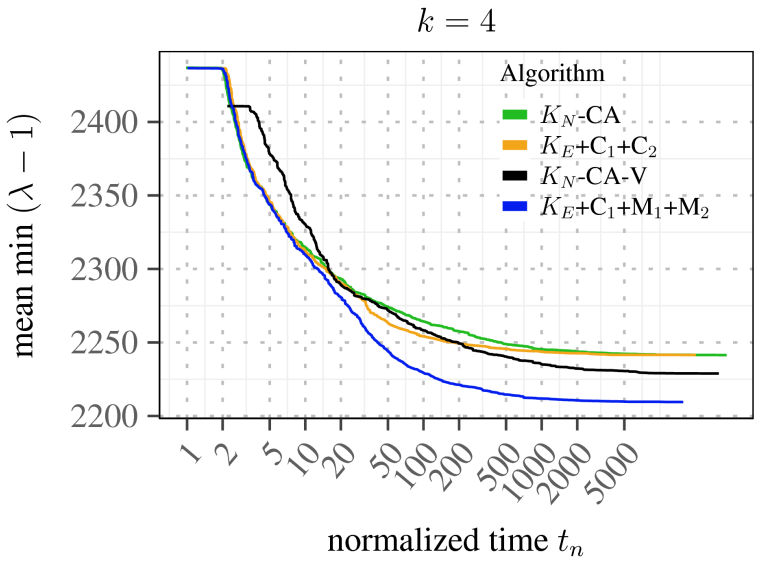
- The run time is normalized $t_n = \frac{time}{t_1}$; t_1 := duration of first iteration.
  - Allows comparing differently sized hypergraphs
  - Algorithmic components can be analyzed on run time

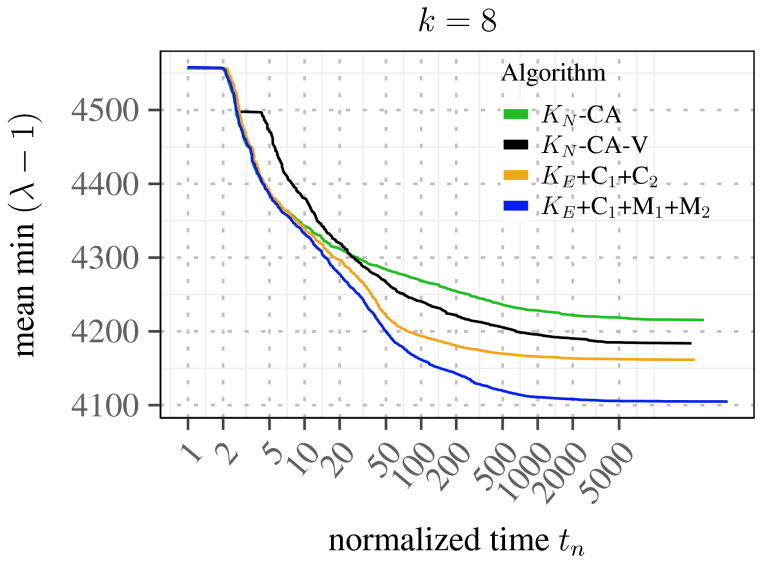
Results

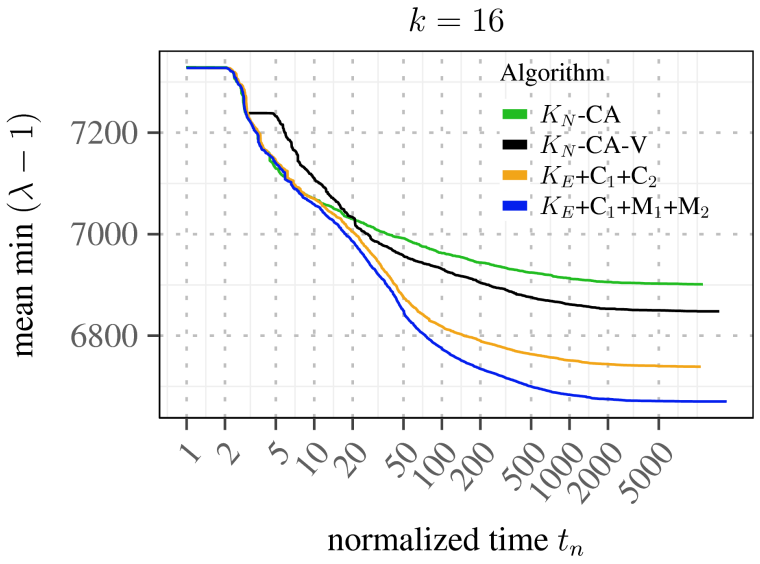

mean min ($\lambda - 1$)
6300
6200
6100
6000
5900
5800
1
2
5
10
20
50
100
200
500
1000
2000
5000
normalized time t_n
Algorithm
K_E+C$_1$
K_E+C$_1$+C$_2$
K_E + C$_1$ +M$_1$ +M$_2$
K_N-CA
K_N-CA-V

Results

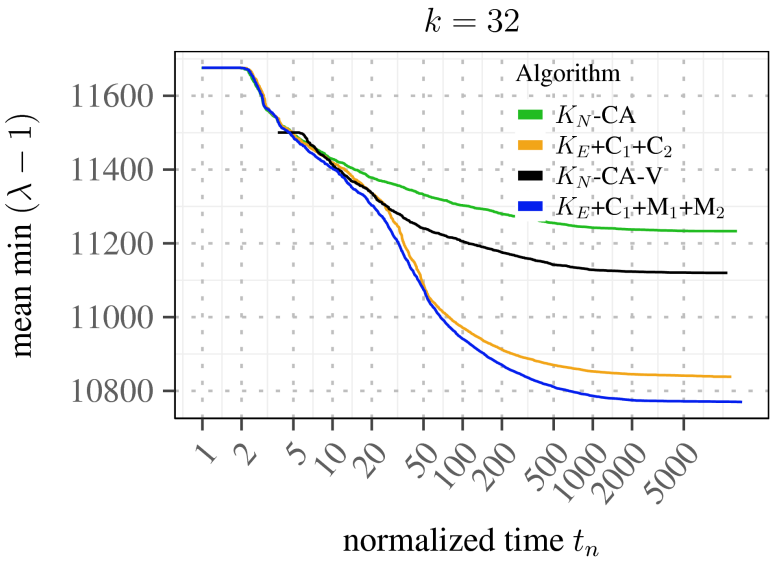

$k = 2$
mean min $(\lambda - 1)$
980
960
940
920
900
Algorithm
K_N-CA
K_E+C$_1$+C$_2$
K_N-CA-V
K_E+C$_1$+M$_1$+M$_2$
1 2 5 10 20 50 100 200 500 1000 2000 5000
normalized time t_n

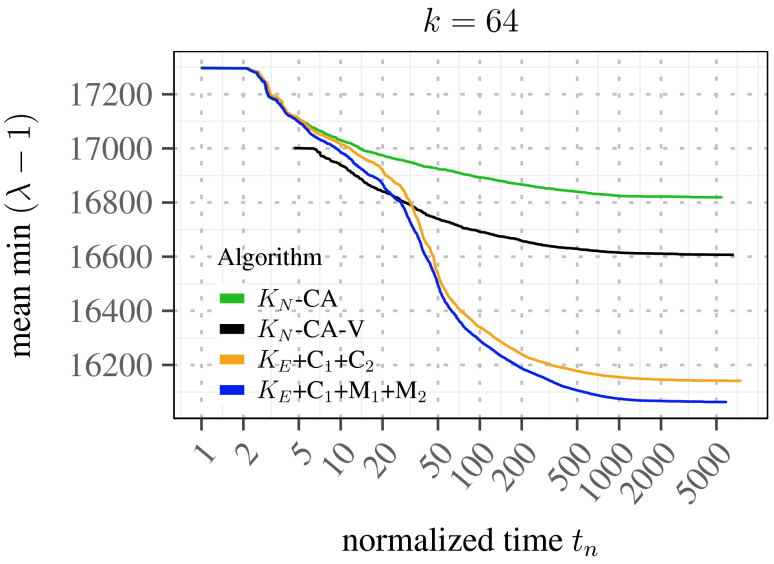

$k = 4$
mean min $(\lambda - 1)$
2400
2350
2300
2250
2200
Algorithm
K_N-CA
K_E+C$_1$+C$_2$
K_N-CA-V
K_E+C$_1$+M$_1$+M$_2$
1 2 5 10 20 50 100 200 500 1000 2000 5000
normalized time t_n


$k = 8$
mean min $(\lambda - 1)$
4500
4400
4300
4200
4100
Algorithm
K_N-CA
K_N-CA-V
K_E+C$_1$+C$_2$
K_E+C$_1$+M$_1$+M$_2$
1 2 5 10 20 50 100 200 500 1000 2000 5000
normalized time t_n


$k = 16$
mean min $(\lambda - 1)$
7200
7000
6800
Algorithm
K_N-CA
K_N-CA-V
K_E+C$_1$+C$_2$
K_E+C$_1$+M$_1$+M$_2$
1 2 5 10 20 50 100 200 500 1000 2000 5000
normalized time t_n

Results


$k = 32$
mean min $(\lambda - 1)$
11600
11400
11200
11000
10800
Algorithm
K_N-CA
K_E+C$_1$+C$_2$
K_N-CA-V
K_E+C$_1$+M$_1$+M$_2$
1 2 5 10 20 50 100 200 500 1000 2000 5000
normalized time t_n


$k = 64$
mean min $(\lambda - 1)$
17200
17000
16800
16600
16400
16200
Algorithm
K_N-CA
K_N-CA-V
K_E+C$_1$+C$_2$
K_E+C$_1$+M$_1$+M$_2$
1 2 5 10 20 50 100 200 500 1000 2000 5000
normalized time t_n

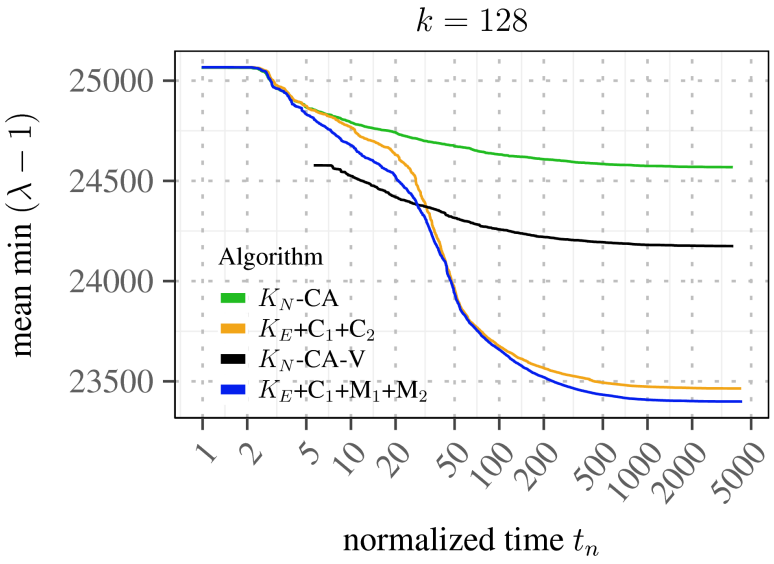

$k = 128$
mean min $(\lambda - 1)$
25000
24500
24000
23500
Algorithm
K_N-CA
K_E+C$_1$+C$_2$
K_N-CA-V
K_E+C$_1$+M$_1$+M$_2$
1 2 5 10 20 50 100 200 500 1000 2000 5000
normalized time t_n

Table of Improvements

k	$K_E + C_1 + C_2$		$K_E + C_1 + M_1 + M_2$	
	K_N-CA-V	K_N-CA	K_N-CA-V	K_N-CA
all k	1.7%	2.7%	2.2%	3.2%
2	-0.2%	0.4%	0.2%	0.8%
4	-0.2%	0.3%	0.9%	1.3%
8	0.7%	1.6%	1.9%	2.7%
16	1.9%	2.8%	2.6%	3.5%
32	2.9%	3.9%	3.2%	4.2%
64	3.2%	4.7%	3.4%	4.8%
128	3.3%	5.0%	3.3%	5.0%

Conclusion

Conclusion

- $(\lambda - 1)$ improvement of up to 5%
- High integration of evolutionary aspects in the multilevel approach

Future Work

- Added parallelization for faster partitioning
- Different approach for generating the initial population
- Time cost analysis for evolutionary operators