Question Paper

SEP 2024 - UE22EC341A - ISA 1 (set- 1)

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1.a : Marks (4.0)

Two point charges of $1\,mC$ and $-2\,mC$ are located at (3, 2, -1) and (-1, -1, 4) respectively. Determine the electric force on a $10\,nC$ charge located at (0, 3, 1).

Expected Answer

$$
\begin{aligned}
\mathbf{F} &= \sum_{k=1,2} \frac{QQ_k}{4\pi\varepsilon_o R^2}\mathbf{a}_R = \sum_{k=1,2} \frac{QQ_k(\mathbf{r}-\mathbf{r}_k)}{4\pi\varepsilon_o|\mathbf{r}-\mathbf{r}_k|^3} \\
&= \frac{Q}{4\pi\varepsilon_o}\left\{\frac{10^{-3}[(0,3,1)-(3,2,-1)]}{|(0,3,1)-(3,2,-1)|^3} - \frac{2\cdot 10^{-3}[(0,3,1)-(-1,-1,4)]}{|(0,3,1)-(-1,-1,4)|^3}\right\} \\
&= \frac{10^{-3}\cdot 10\cdot 10^{-9}}{4\pi\cdot\dfrac{10^{-9}}{36\pi}}\left[\frac{(-3,1,2)}{(9+1+4)^{3/2}} - \frac{2(1,4,-3)}{(1+16+9)^{3/2}}\right] \\
&= 9\cdot 10^{-2}\left[\frac{(-3,1,2)}{14\sqrt{14}} + \frac{(-2,-8,6)}{26\sqrt{26}}\right] \\
\mathbf{F} &= -6.512\mathbf{a}_x - 3.713\mathbf{a}_y + 7.509\mathbf{a}_z \text{ mN}
\end{aligned}
$$

1.b : Marks (4.0)

Given that $\mathbf{D} = z\rho \cos^2\phi \, \mathbf{a}_z$ C/m^2, calculate the charge density at $(1, \pi/4, 3)$ and the total charge enclosed by the cylinder of radius 1 m with $-2 \leq z \leq 2$ m.

Expected Answer

$$\rho_v = \nabla \cdot \mathbf{D} = \frac{\partial D_z}{\partial z} = \rho \cos^2 \phi$$

At $(1, \pi/4, 3)$, $\rho_v = 1 \cdot \cos^2(\pi/4) = 0.5$ C/m^3. The total charge enclosed by the cylinder can be found in two different ways.

Method 1: This method is based directly on the definition of the total volume charge.

$$\begin{aligned} Q &= \int_v \rho_v \, dv = \int_v \rho \cos^2 \phi \, \rho \, d\phi \, d\rho \, dz \\ &= \int_{z=-2}^{2} dz \int_{\phi=0}^{2\pi} \cos^2 \phi \, d\phi \int_{\rho=0}^{1} \rho^2 \, d\rho = 4(\pi)(1/3) \\ &= \frac{4\pi}{3} \text{ C} \end{aligned}$$

1.c : Marks (4.0)

A wire of diameter 1 mm and conductivity 5×10^7 S/m has 10^{29} free electrons per cubic meter when an electric field of 10 mV/m is applied. Determine

(a) The charge density of free electrons
(b) The current density

(take the electronic charge as $e = -1.6 \times 10^{-19}$ C)

Expected Answer

Solution:
(In this particular problem, convection and conduction currents are the same.)

(a) $\rho_v = ne = (10^{29})(-1.6 \times 10^{-19}) = -1.6 \times 10^{10}$ C/m^3

(b) $J = \sigma E = (5 \times 10^7)(10 \times 10^{-3}) = 500$ kA/m^2

1.d : Marks (4.0)

Two extensive homogeneous isotropic dielectrics meet on plane $z = 0$. For $z > 0$, $\varepsilon_{r1} = 4$ and for $z < 0$, $\varepsilon_{r2} = 3$. A uniform electric field $\mathbf{E}_1 = 5\mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ kV/m exists for $z \geq 0$. Find

(a) $\mathbf{E}_2$ for $z \leq 0$

Expected Answer

(a) Since $\mathbf{a}_z$ is normal to the boundary plane, we obtain the normal components as

$$E_{1n} = \mathbf{E}_1 \cdot \mathbf{a}_n = \mathbf{E}_1 \cdot \mathbf{a}_z = 3$$
$$\mathbf{E}_{1n} = 3\mathbf{a}_z$$
$$\mathbf{E}_{2n} = (\mathbf{E}_2 \cdot \mathbf{a}_z)\mathbf{a}_z$$

Also

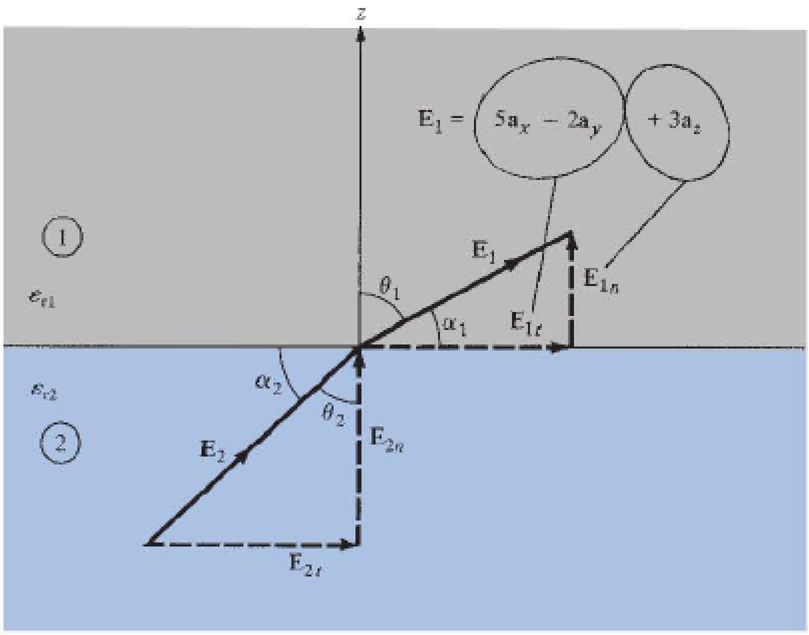
$$\mathbf{E} = \mathbf{E}_n + \mathbf{E}_t$$

Hence,

$$\mathbf{E}_{1t} = \mathbf{E}_1 - \mathbf{E}_{1n} = 5\mathbf{a}_x - 2\mathbf{a}_y$$

Thus

$$\mathbf{E}_{2t} = \mathbf{E}_{1t} = 5\mathbf{a}_x - 2\mathbf{a}_y$$



Similarly,

$$\mathbf{D}_{2n} = \mathbf{D}_{1n} \rightarrow \varepsilon_{r2}\mathbf{E}_{2n} = \varepsilon_{r1}\mathbf{E}_{1n}$$

or

$$\mathbf{E}_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\mathbf{E}_{1n} = \frac{4}{3}(3\mathbf{a}_z) = 4\mathbf{a}_z$$

Thus

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$$
$$= 5\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z \text{ kV/m}$$

5 : Marks (2.0)

A point charge $-4\ nC$ is located at (2, -1, 3). Find the potential at (1, 0, 1).

Expected Answer

$$V(\mathbf{r}) = \frac{Q_1}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_1|}.$$

$$|\mathbf{r} - \mathbf{r}_1| = |(1, 0, 1) - (2, -1, 3)| = |(-1, 1, -2)| = \sqrt{6}$$

Hence potential, $V = -14.697\ kV$

6 : Marks (2.0)

Two point charges $-1\ nC$ and $4\ nC$ are located at (0, 0, 0) and (0, 0, 1). Find the electrostatic energy in the system.

Expected Answer

$$W = W_1 + W_2 \cdot$$
$$= 0 + Q_2 V_{21}$$

$$= Q_2 \cdot \frac{Q_1}{4\pi\varepsilon_0 \left|(0,0,1) - (0,0,0)\right|}$$

$$= \frac{1}{4\pi \cdot \frac{10^{-9}}{36\pi}} \left(-4 \cdot\right.$$

) = -36 nJ

7 : Marks (2.0)

In a dielectric material, $E_x = 5$ V/m and $\mathbf{P} = \frac{1}{10\pi}(3\mathbf{a}_x - \mathbf{a}_y + 4\mathbf{a}_z)$ nC/m^2.

Calculate:

Susceptibility of the material

Expected Answer

$\vec{P} = \chi_e \vec{E}$

$Since\ P_x\ and\ E_x\ are\ known,\ \chi_e = 2.16$

8 : Marks (2.0)

A spherical capacitor with inner conductor radius of 1mm and outer conductor radius of 3mm has a dielectric with $\varepsilon_r = 6$ in between the conductors. Determine the capacitance of the spherical capacitor.

Expected Answer

$$\boxed{C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}}$$

Hence $C = 1\ pF.$

9 : Marks (1.0)

The electric field at a distance r from a point charge is proportional to:

Expected Answer

by definition

10 : Marks (1.0)

The electric field at a distance r from an infinite sheet with uniform surface charge density is:

Expected Answer

by definition

11 : Marks (1.0)

The electric flux density in a medium with permittivity ε is:

Expected Answer

by definition

12 : Marks (1.0)

The electric field $\overline{E}$ at a point is expressed in terms of potential V as:

Expected Answer

by definition

13 : Marks (1.0)

The line integral of a static electric field around a closed path is:

Expected Answer

by definition

14 : Marks (1.0)

The magnitude of moment of an electric dipole is

Expected Answer

by definition

15 : Marks (1.0)

The potential at a distance r from an electric dipole is:

Expected Answer

by definition

16 : Marks (1.0)

The electrostatic energy density is:

Expected Answer

by definition

17 : Marks (1.0)

The susceptibility of a material with a dielectric constant 4 is equal to:

Expected Answer

by definition

18 : Marks (1.0)

The relaxation time for a perfect dielectric material is:

Expected Answer

by definition

19 : Marks (1.0)

The tangential component of electric field at the boundary between a perfect conductor and free space is:

Expected Answer

by definition

20 : Marks (1.0)

A surface charge of density $0.2\ nC/sq.m$ resides at the boundary between a perfect conductor and free space. The magnitude of the electric field at the boundary in the free space is equal to

Expected Answer

$$E = \frac{\rho_s}{\varepsilon_0}$$

21 : Marks (1.0)

In a one dimensional system, where the potential V varies only along x axis, if the initial conditions are: $V=0\ \ at\ \ x=0,\ \ and\ \ V=6\ \ at\ \ x=3$, the expression for the potential is given by:

Expected Answer

$\partial^2 V/\partial x^2 = 0, \quad \partial V/\partial x = c_1, \quad V = c_1\, x + c_2$

$At\ x=0, \quad V=0, \quad hence \quad c_2=0.$

$At\ x=3, \quad V=6, \quad hence \quad c_1=2.$

$So, \quad V=2x$

22 : Marks (1.0)

A metal wire of diameter 2 mm and length 3 meters has a conductivity of 10^6 S/m. The resistance of the wire is:

Expected Answer

$R = \frac{L}{\sigma A} = 0.955\ ohms$

23 : Marks (1.0)

The capacitance of a parallel plate capacitor with free space between the plates is measured as 3 pF, and when the same capacitor is filled with a dielectric material, the capacitance measured is 7.5 pF. The dielectric constant of the matrial is:

Expected Answer

$\varepsilon_r = C/C_0 = 2.5$

24 : Marks (1.0)

An electrostatic system in free space has an energy density of 1 nJ/cubic m. The magnitude of electric field in the system is:

Expected Answer

$$w = \frac{\varepsilon_0 E^2}{2} = 1\ nJ, \quad hence\ \ E = 15.04\ V/m.$$