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	School Home	Home
☀	5	-5
☁	3	1

	☀	☁
☀	0.8	0.2
☁	0.4	0.6

	☀	☁
☀	0.9	0.1
☁	0.3	0.7

$$R_{picnic} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \quad R_{stay} = \begin{bmatrix} -5 \\ 1 \end{bmatrix} \quad P_{picnic} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix} \quad P_{stay} = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

Step 1:
For Sunny:

$r_\pi = 0.5 \times (5) + 0.5 \times (-5) = 2.5 + (-2.5) = 0$

For Cloudy:

$r_\pi = 0.5 \times (3) + 0.5 \times (1) = 1.5 + 0.5 = 2$

$\left.\right\} r_\pi = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Step 2:
Row 1 (Sunny):
- $P_\pi(1,1) = 0.5 \times 0.8 + 0.5 \times 0.9 = 0.4 + 0.45 = 0.85$
- $P_\pi(1,2) = 0.5 \times 0.2 + 0.5 \times 0.1 = 0.1 + 0.05 = 0.15$

Row 2 (Cloudy):
- $P_\pi(2,1) = 0.5 \times 0.4 + 0.5 \times 0.3 = 0.2 + 0.15 = 0.35$
- $P_\pi(2,2) = 0.5 \times 0.6 + 0.5 \times 0.7 = 0.3 + 0.35 = 0.65$

$\left.\right\} P_\pi = \begin{bmatrix} 0.85 & 0.15 \\ 0.35 & 0.65 \end{bmatrix}$

Step 3:
General form: $v_\pi(s) = r_\pi(s) + \gamma \sum P_\pi(s,s') v_\pi(s')$

Sunny:

$v_1 = 0 + 0.9(0.85v_1 + 0.15v_2) = 0 + 0.765v_1 + 0.135v_2 \rightarrow 0.235v_1 - 0.135v_2 = 0$

Cloudy:

$v_2 = 2 + 0.9(0.35v_1 + 0.65v_2) = 2 + 0.315v_1 + 0.585v_2 \rightarrow -0.315v_1 + 0.415v_2 = 2$

Step 4:
Cloudy:

$0.235v_1 - 0.135v_2 = 0$

$\dfrac{0.235v_1}{0.235} = \dfrac{0 + 0.135v_2}{0.235}$

$v_1 = \dfrac{0.135v_2}{0.235}$

$-0.315v_1 + 0.415v_2 = 2$

$-0.315\left(\dfrac{0.135v_2}{0.235}\right) + 0.415v_2 = 2$

$(-0.315 \times 0.574v_2) + 0.415v_2 = 2$

$-0.180v_2 + 0.415v_2 = 2$

$0.235v_2 = 2$

$v_\pi(cloudy) = \dfrac{2}{0.2352}$

$= 8.54$

Sunny:

$$v_1 = \frac{0.135\, v_2}{0.235}$$

$$v_n(\text{sunny}) = 4.906$$

$$v_1 = \frac{0.135\,(8.54)}{0.235} = \frac{1.1529}{0.235}$$

Step 5:

General Form: $v_*(s) = \max \{ R(s,a) + \gamma_s' \sum P(s'|s,a) v*(s') \}$

For Sunny (v_1) using Picnic:

$$v_*(\text{sunny}) = 5 + 0.9\,(0.8\,v_1 + 0.2\,v_2)$$

For Cloudy (v_2) using Picnic:

$$v_*(\text{cloudy}) = 3 + 0.9\,(0.4\,v_1 + 0.6\,v_2)$$

Sunny:

$$v_1 = 5 + 0.72\,v_1 + 0.18\,v_2$$

$$v_1 - 0.72\,v_1 - 0.18\,v_2 = 5$$

$$0.28\,v_1 - 0.18\,v_2 = 5$$

Cloudy:

$$v_2 = 3 + 0.36\,v_1 + 0.54\,v_2$$

$$v_2 - 0.36\,v_1 - 0.54\,v_2 = 3$$

$$-0.36\,v_1 + 0.46\,v_2 = 3$$

Step 6:

From the sunny equation:

$$0.28\,v_1 = 5 + 0.18\,v_2$$

$$v_1 = \frac{5 + 0.18\,v_2}{0.28}$$

Using the equation for cloudy:

$$-0.36\,v_1 + 0.46\,v_2 = 3$$

$$-0.36\,v_1 \left(\frac{5 + 0.18\,v_2}{0.28} \right) + 0.46\,v_2 = 3$$

$v_*(\text{cloudy})$

$$-6.36\,v_1, -0.28\,v_1 \; -0.36 \times 0.18\,v_2 = 0.36 \times 5$$

$$0.1008_n - 0.0648\,v_2 = 1.8$$

$$0.28 \times (-0.36)\,v_1 + 0.28 \times 0.46\,v_2 = 0.28 \times 3$$

$$-0.1008\,v_1 + 0.1288\,v_2 = 0.84$$

$$(-0.0648 + 0.1288)\,v_2 = 1.8 + 0.84$$

$$0.0640\,v_2 = 2.64$$

$$v_2 = \frac{2.64}{0.0640} = 41.25$$

$$v*(\text{cloudy}) = v_2 = 41.25$$

$v_*(\text{sunny})$

$$0.28\,v_1 - 0.18\,v_2 = 5$$

$$0.28\,v_1 - 0.18\,(41.25) = 5$$

$$0.28\,v_1 - 7.425 = 5$$

$$0.28\,v_1 = 5 + 7.425 = 12.925$$

$$v_1 = \frac{12.425}{0.28} = 44.375$$

$$v*(\text{sunny}) = v_1 = 44.375$$

Step 7:

$q(1, School) = 5 + 0.9 (0.8 * 44.375 + 0.2 * 41.25) = 44.375$

$q(1, Home) = -5 + 0.9 (0.9 * 44.375 + 0.1 * 41.25) = 34.657$

$q(2, School) = 3 + 0.9 (0.9 * 44.375 + 0.6 * 41.25) = 41.25$

$q(2, Home) = 1 + 0.9 (0.3 * 44.375 + 0.7 * 41.25) = 38.969$