**Question 1a):** By definition of the Poisson Distribution:

Hence:

Recognising that :

Separating a factor of and reindexing such that :

By the definition of :

Similarly, for :

Simplifying :

Letting :

For :

Simplifying :

Letting :

Therefore  **as required.**

**Question 1b):** To calculate the probability that X takes even integers, we can define an indicator function of the form:

Then and we can relate the indicator to . We can observe that:

Taking the expectation:

Hence:

Taking from the Definition of the Poisson distribution:

Factoring out and recognising that :

Therefore, when substituting back into the probability:

**As required.**

**Question 2a):** For and to be independent, we must show by the definition of independence that:

The joint event can be rewritten as follows:

Because on the event , the relation is equivalent to . By the definition of independence of random variables that each take with probability :

Therefore:

To compute , note that and are independent and each combination of is equally likely:

|  |  |  |  |
| --- | --- | --- | --- |
|  | 𝑌 |  | Probability |
|  | 1 |  |  |
|  | −1 |  |  |
|  | 1 |  |  |
|  | −1 |  |  |

From the table:

Hence:

And as this holds for all , and are independent as required.

**Question 2b):** For and to be mutually independent:

For all . Using the counterexample

Because occurs with probability and then automatically. But the product of the marginals is:

Since , the factorisation fails and thus and are not independent. As required.

Question 3a):