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Force/Moment Contribution Sims

We can use the Barrowman method as a low-fidelity estimate of the force/moment contributions of each part of the vehicle. Refer to the following papers to come up with equations describing the force/moment contributions of each component of the vehicle (specifically nosecone, body, and fins)

https://ntrs.nasa.gov/api/citations/20010047838/downloads/20010047838.pdf

http://argoshpr.ch/joomla1/articles/pdf/sentinel39-galejs.pdf

https://mae-nas.eng.usu.edu/MAE_5900_Web/5900/USLI_2010/Flight_Mechanics/Barrowman.html

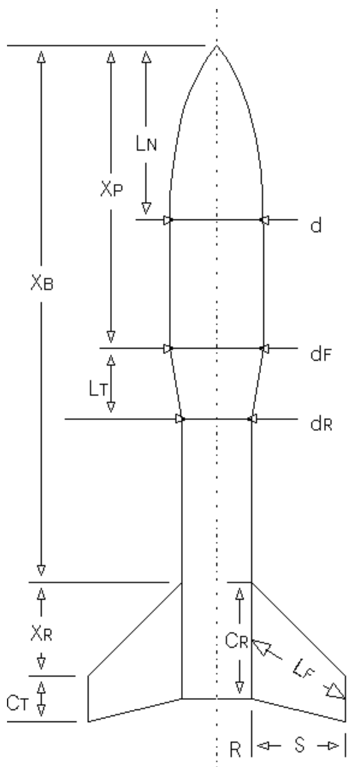
https://arc.aiaa.org/doi/abs/10.2514/6.2021-3267

https://arc.aiaa.org/doi/epdf/10.2514/6.1979-504

https://www.youtube.com/watch?app=desktop&v=Phwm-Tc7EdM

http://ftp.demec.ufpr.br/foguete/bibliografia/Barrowman_report.pdf

https://dept.aem.umn.edu/~./faculty/flaten/Rocketry_Remote_Lessons_Fall_2017/Barrowman_CP_Technical_Report_33.pdf



What equations do we need? Can all this information computed in the 6-DOF sim -aerodynamics functions.

Simplified Model

The Barrowman equations

List of Parameters: -

LN➔ length of nose
d➔ diameter at base of nose
dF➔ diameter at front of transition
dR➔ diameter at rear of transition
LT➔ length of transition
XP➔ distance from tip of nose to front of transition.
CR➔ fin root chord
CT➔ fin tip chord
S➔ fin semispan
LF➔ length of fin mid-chord line
R➔ radius of body at aft end
XR➔ distance between fin root leading edge and fin tip leading edge parallel to body
XB➔ distance from nose tip to fin root chord leading edge.
N➔ number of fins

Goals:

Nose Cone Terms

- $(C_N)_N = 2$
- For Cone: $X_N = \frac{2}{3}L_N$
- For Ogive: $X_N = 0.466L_N$
- For Parabolic: $X_N = \frac{1}{2}L_N$

Conical Transition Terms

$$(C_N)_T = 2\left[\left(\frac{d_R}{d}\right)^2 - \left(\frac{d_F}{d}\right)^2\right] => X_T = X_P + \frac{L_T}{3}[1 + \frac{1-\left(\frac{d_F}{d_R}\right)}{1-\left(\frac{d_F}{d_R}\right)^2}]$$

Fin Terms

$$(C_N)_F = \left[1+\frac{R}{S+R}\right][\frac{4N\left(\frac{S}{d}\right)^2}{1+\sqrt{1+\left(\frac{2L_F}{C_R+C_T}\right)^2}}] => X_F = X_B + \frac{X_R}{3}\frac{(C_R+2C_T)}{(C_R+C_T)} + \frac{1}{6}[(C_R+C_T) - \frac{(C_RC_T)}{(C_R+C_T)}]$$

Finding CP

$$(C_N)_R = (C_N)_N + (C_N)_T + (C_N)_F$$

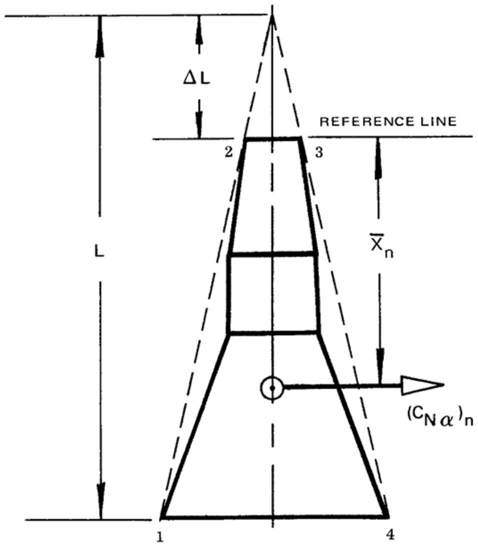
$$\overline{X} = \frac{(C_N)_N X_N + (C_N)_T X_T + (C_N)_F X_F}{(C_N)_R}$$

Becomes

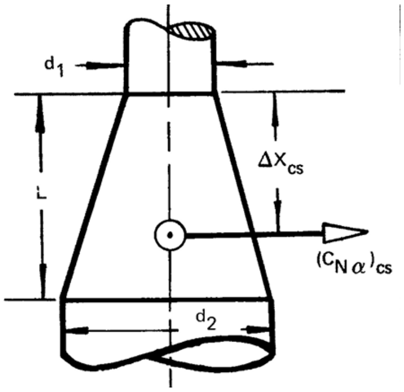
$$\overline{X} = \frac{(C_N)_N X_N + 2\left[\left(\frac{d_R}{d}\right)^2 - \left(\frac{d_F}{d}\right)^2\right]\left[X_P + \frac{L_T}{3}\left[1+\frac{1-\left(\frac{d_F}{d_R}\right)}{1-\left(\frac{d_F}{d_R}\right)^2}\right]\right] + \left[1+\frac{R}{S+R}\right]\left[\frac{4N\left(\frac{S}{d}\right)^2}{1+\sqrt{1+\left(\frac{2L_F}{C_R+C_T}\right)^2}}\right]\left[X_B + \frac{X_R}{3}\frac{(C_R+2C_T)}{(C_R+C_T)} + \right.}{(C_N)_N + 2\left[\left(\frac{d_R}{d}\right)^2 - \left(\frac{d_F}{d}\right)^2\right] + \left[1+\frac{R}{S+R}\right][\frac{4N\left(\frac{S}{d}\right)^2}{1+\sqrt{1+\left(\frac{2L_F}{C_R+C_T}\right)^2}}]}$$

Extension to the above: -

Simplified method to get the X_n, subtract the values derived from L with delta L



Conical Shoulder

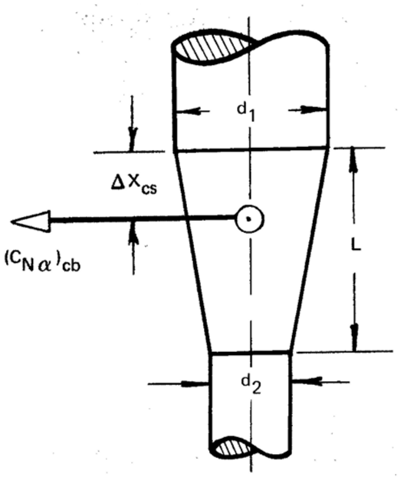


$$C_{N\alpha} = 2\,[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2]$$

$$X_{CS_{total}} = X_{CS} + \Delta X_{CS} = X_{CS} + \frac{L}{3}[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}]$$

d: diameter of nose base, X_CS: the distance from the tip of the nose to the front of the conical shoulder.

Conical Boattail

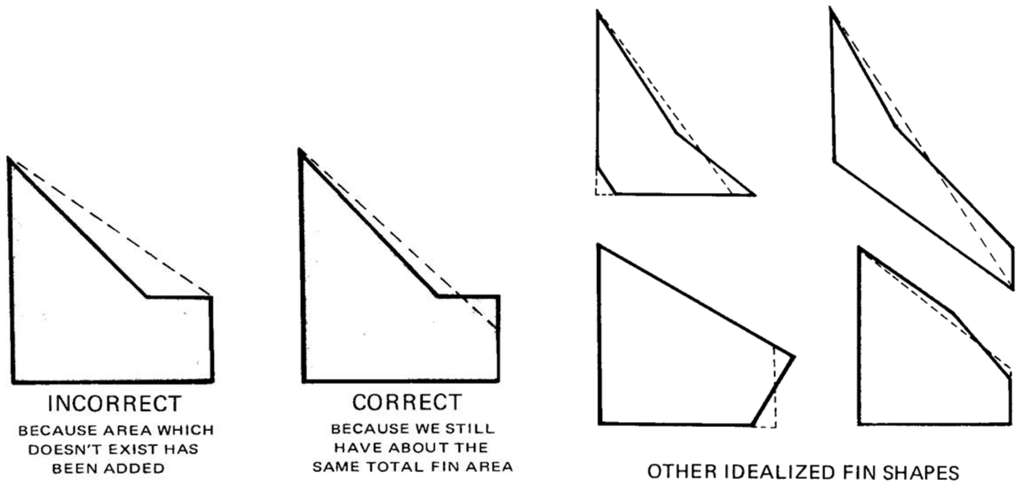


$$C_{N\alpha} = 2\,[\left(\frac{d_2}{d}\right)^2 - \left(\frac{d_1}{d}\right)^2]$$

$$X_{CB_{total}} = X_{CB} + \Delta X_{CB} = X_{CB} + \frac{L}{3}[1 + \frac{1 - \frac{d_1}{d_2}}{1 - \left(\frac{d_1}{d_2}\right)^2}]$$

d: diameter of nose base, X_CB: the distance from the tip of the nose to the front of the conical boattail.

Fin



Approximation of fins to idealized fins shapes: -

$$(C_N)_F = \left[1+\frac{R}{S+R}\right]\left[\frac{4N\left(\frac{S}{d}\right)^2}{1+\sqrt{1+\left(\frac{2L_F}{C_R+C_T}\right)^2}}\right]$$

Fin Interference Factor: -

$$K_{fb} = 1+\frac{R}{S+R} \quad for\ n = 3\ or\ 4$$
$$K_{fb} = 1+\frac{0.5R}{S+R} \quad for\ n = 5$$
$$(C_{N\,\alpha})_{fb} = K_{bf}(C_{N\,\alpha})_f$$
$$X_{f\ total} = X_f + \Delta X_f = X_f + \frac{m(a+2b)}{3(a+b)} + \frac{1}{6}(a+b-\frac{ab}{a+b})$$

The center of pressure of tail does not depend on number of fins.

Sanity-check on appropriate CG placement

$$X_{CG} = X_{center\ of\ pressure} - D$$

Where D: *the largest diameter*

$$(X_{CG})_{body} = \frac{W_1(X_{CG})_1 + W_2(X_{CG})_2 + W_3(X_{CG})_3 + \cdots}{W_B}$$

Each N is measured through the following equation,

$$N = \frac{C_{N\,\alpha}}{2}\rho V^2 \alpha A_r$$

$$\overline{X} = \frac{(N_N)_N\overline{X_N} + (N_T)_T\overline{X_T} + (N_F)_F\overline{X_F}}{(N)_{total}} = \frac{(C_N)_1\overline{X}_1 + (C_N)_T\overline{X}_T + (C_N)_F\overline{X}_F}{(C_N)_{total}}$$

Normal Force Aerodynamics

Based off barrowman's original paper.

Tail Normal Force Coefficient Derivatives

$$\left(C_{N_a}\right)_1 = \frac{2\pi}{2+\sqrt{4+\left(\frac{\beta AR}{\cos\Gamma_C}\right)^2}}$$ Subsonic Conditions for one-fin

$$\left(C_{N_a}\right)_T = \frac{N\pi AR\left(\frac{A_f}{A_r}\right)}{2+\sqrt{4+\left(\frac{\beta AR}{\cos\Gamma_C}\right)^2}}$$ Subsonic Conditions for multi-fin

W. F. Hilton's Highspeed Aerodynamics Supersonic Conditions

- 3 Fin configuration (2 effective panels at 30 degrees dihedral angle)

$$(C_N)_T = \frac{3}{2}\left(C_{N_a}\right)_1$$

- 4 fin configuration (2 effective panels at 0 degrees dihedral angle)

$$(C_N)_T = \frac{4}{2}\left(C_{N_a}\right)_1$$

Generalized formula:

$$(C_N)_T = \frac{N}{2}\left(C_{N_a}\right)_1 \textit{where N is the number of fins (3-4)}$$

Tail Center of Pressure

Geometric Property

$$\overline{X_T} = 1_T + \frac{X_t}{3}\frac{(C_r + 2C_t)}{(C_r + C_t)} + \frac{1}{6}\left[(C_r + C_t) - \frac{(C_r C_t)}{(C_r + C_t)}\right]$$

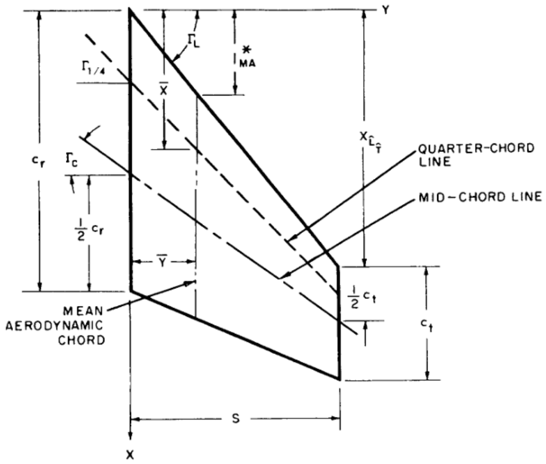
Where,

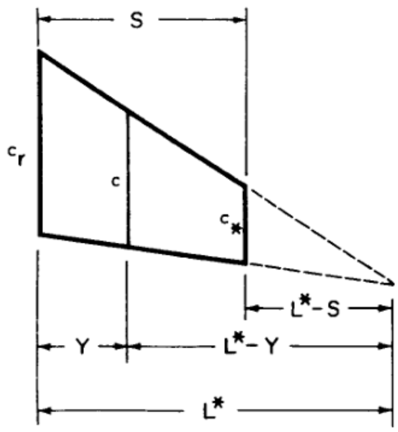
1_T: distance from nose tip to fin root chord leading edge

X_t: *distance between fin root leading edge and fin tip leading edge parallel to body*

C_r: *fin root chord*

C_t: *fin tip chord*





Tail Roll Forcing Moment Coefficient

$C_{\ell_\delta} = N\left(C_{N_a}\right)_1 \frac{Y_T}{L_r}$ Subsonic Roll forcing Moment

Where,

N: *number of fins*

Y_T: *Spanwise Center of Pressure Location Measured from the Longitudinal Axis*

L_r: *Reference Length*

Tail Roll Damping Moment Coefficient

$C_{\ell_p} = -\frac{N c_r S}{6L_r^2}[(1+3\lambda)s^2 + 4(1+2\lambda)s r_t + 6(1+\lambda)r_t^2](C_N)_1$

Subsonic Roll Damping Moment

$C_{\ell_p} = 1000NC_\ell$

Supersonic Roll Damping Moment

Where,

$\lambda: Fin\ Tampter\ Ratio \frac{C_t}{C_r}$

$r_t: Body\ Radius\ at\ Tail$

Body Normal Force

Ignoring compressibility in subsonic flow

Steady state normal load

$n(x) = \rho V \frac{\partial}{\partial x}[A(x)w(x)]$

$n(x) = \rho V^2 \alpha \frac{\partial}{\partial x}[A(x)]$ for small angle of attack

Where $A(x)$ Local cross-sectional area of the body.

$$(C_{N_\alpha})_B = 2\frac{A_{BN}}{A_r}$$

Frustums pressure distribution

$$P = -P_o + Ce^{(S_o s + S_1\frac{s^2}{2} + S_2\frac{s^3}{3} + \cdots)}$$
$$P = P_e - (P_c - P_2)e^{-\eta}$$

Where,

$$\eta = \left(\frac{\partial p}{\partial s}\right)_2 \frac{x - x_2}{(P_c - P_2)cos\delta_2}$$

$s: surface\ coordinate$

$P_c: pressure\ on\ a\ cone\ heaving\ the\ same\ slope\ as\ the\ frustrum$

$\delta: angle\ between\ surface\ and\ longitudial\ axis$

$(\quad)_2: the\ value\ taken\ at\ the\ front\ edge\ of\ the\ frustrum$

$$\frac{\partial P}{\partial s} - \lambda\frac{\partial \delta}{\partial s} = \frac{1}{\cos\mu}\frac{\partial P}{\partial C_1} = -\frac{\lambda}{\cos\mu}\left(\frac{\partial \delta}{\partial C_1} + \frac{\sin\mu\, sin\delta}{r}\right)$$

Where,

$$\lambda = \frac{2\gamma_P}{\sin 2\mu}$$

$C_1: First\ Characteristic\ Coordinate$

$\mu: Mach\ Angle$

$r: Local\ Radius$

$\gamma: ratio\ of\ specific\ heats$

$$(C_{n_\alpha})_B = \frac{2\pi}{A_B}\int_0^l A\, r\, dx$$

where,

$A_B: Body\ Base\ Area, l: body\ length$

Body Center of Pressure

$$\bar{X}_B = \frac{\left(C_{m_\alpha}\right)_B}{\left(C_{N_\alpha}\right)_B} L_r$$

$\bar{X}_B = 1_0 - \frac{V_B}{A_B}$ for subsonic flow

$$\bar{X}_B = \left(\frac{C_{m_\alpha}}{C_{N_\alpha}}\right) d$$

Tail-Body Interference Effects

$$\left(C_{N_\alpha}\right)_{T(B)} = \left(C_{N_\alpha}\right)_T K_{T(B)}$$
$$\left(C_{N_\alpha}\right)_{B(T)} = \left(C_{N_\alpha}\right)_T K_{B(T)}$$
$$\left(C_{l_\delta}\right)_{T(B)} = \left(C_{N_\alpha}\right)_T K_{T(B)}$$

Tail in the Presence of the Body

$$K_{T(B)} = \frac{2}{\pi\left(1-\frac{1}{\tau}\right)^2}\left\{\left(1+\frac{1}{\tau^4}\right)\left[\frac{1}{2}\tan^{-1}\frac{1}{2}\left(\tau-\frac{1}{\tau}\right)\right]-\frac{1}{\tau^2}\left[\left(\tau-\frac{1}{\tau}\right)+2\tan^{-1}\left(\frac{1}{\tau}\right)\right]\right\}$$

Longitudinal tail center-of-pressure is independent of the interference flow.

$$\bar{X}_{T(B)} = \bar{X}_T$$

Body in the Presence of the Tail

Subsonic variation of center of pressure

$$\bar{X}_{B(T)} = 1_T - \frac{C_r}{4} + \left[\frac{\sqrt{s^2-r_t^2}\cosh^{-1}\left(\frac{s}{r_1}\right) - s + \frac{\pi}{2r_t}}{\frac{r_t}{\sqrt{s^2-r_1^2}}\cosh^{-1}\left(\frac{s}{r_t}\right)+\frac{s}{r_t}-\frac{\pi}{2}} - r_t\right] tan\Gamma_{\frac{1}{4}}$$

Body interference pitch moment derivative

$$\left(C_{m_\alpha}\right)_{B(T)} = -\frac{8(\beta m)^{\frac{3}{2}}}{A_r\pi\beta(\beta_m+1)}\int_0^d dy \int_{\beta_y}^{C_r} xcos^{-1}\frac{\frac{x}{\beta}-\beta my}{mx-y}\, dx$$

Fins with supersonic leading edges, $m\beta > 1$. For subsonic leading edges, $m\beta < 1$

$$\left(C_{m_\alpha}\right)_{B(T)} = -\frac{4\beta m}{A_r\pi\beta\sqrt{\beta^2m^2-1}}\int_0^d dy \int_{\beta_y}^{C_r} \frac{x\sqrt{\frac{x}{\beta}+y}}{\sqrt{mx-y}}dx$$

Canted Tail in the Presence of the Body

Effect of fin cant at AoA,

$$K_{T(B)} = \frac{1}{\pi^2}\left[\frac{\pi^2(\tau+1)^2}{4\,\tau^2}+\frac{\pi(\tau^2+1)^2}{\tau^2(\tau-1)}\sin^{-1}\left(\frac{\tau^2-1}{\tau^2+1}\right)-\frac{2\pi(\tau+1)}{\tau(\tau-1)}\right.$$
$$\left.+\frac{(\tau^2+1)^2}{\tau^{2(\tau-1)^2}}\left(\sin^{-1}\frac{\tau^2-1}{\tau^2+1}\right)^2-\frac{8}{(\tau-1)^2}\ln\left(\frac{\tau^2+1}{2\tau}\right)\right]$$

Total Normal Force

The aggregate normal force coefficient derivative.

Pitch Damping Moment

$$C_m = \left(C_{N_\alpha}\right)_{T(B)} \frac{(\Delta x)^2}{L_r} \left(\frac{q}{V}\right)$$

$$C_{mq} = \frac{\partial C_m}{\partial\left(\frac{qL_r}{2V}\right)} \Big|_{\frac{qL_r}{2V}=0}$$

Roll Damping in the Presence of the Body

$$\left(C_{l_P}\right)_{T(B)} = \left(C_{l_P}\right) K_{R(B)}$$

Where the chord variation $K_{R(B)}$

$$K_{R(B)} = 1 + \frac{\frac{\tau - \lambda}{\tau} - \frac{1-\lambda}{\tau - 1}\ln\tau}{\frac{(\tau+1)(\tau-\lambda)}{2} - \frac{(1-\lambda)(\tau^3-1)}{3(\tau-1)}}$$

Axial Force Aerodynamics

Skin Friction

$C_f = \frac{D_{friction}}{\bar{q}A_w}$ $\quad$ $C_f: incomp\ skin\ friction\ coefficient$

Incompressible Flow Skin Friction

$C_f = \frac{1.328}{\sqrt{R_e}}$ $\quad$ $R_e: laminar\ flow$

For $R_e = 5 \times 10^6$ boundary layer becomes transitional to turbulent

Based of experimental data (not theoretical solution is applicable)

$$\log(R_e C_f) = \frac{0.242}{\sqrt{C_f}}$$

Simper formula with $\pm 2\%$ tolerance,

$$C_f = \frac{1}{(3.46 \log R_e - 5.6)^2}$$

Transitional curve form approximated by subtracting an increment from above.

$\Delta C_f = \frac{K}{R_e}$ $\quad$ $K = 1700\ experimental\ Prandtl$

That lead $C_f = \frac{1}{(3.46 \log R_e - 5.6)^2} - \frac{1700}{R_e}$

$$(R_e)_{critical} = 51 \left(\frac{R_s}{L_r}\right)^{-1.039}$$

For R_e higher than the critical value, the skin friction coefficient can be considered independent of R_e

$$C_f = 0.032 \left(\frac{R_e}{L_r}\right)^{0.2}$$

Compressible Subsonic Variation

$C_{f_c} = C_f(1 - 0.09\, M^2)$ laminar skin friction

$C_{f_c} = C_f(1 - 0.12 M^2)$ turbulent flow with roughness

Supersonic Variation

Laminar skin friction coefficient variation

$$C_{f_c} = \frac{C_f}{(1+0.045M^2)^{0.25}}$$

$$C_{f_c} = \frac{C_f}{(1+kM^2)^{0.58}}$$
$$where\ K = 0.15\ for\ heat\ transfer, K = 0.0512\ for\ cooled\ wall$$

Turbulent skin friction with roughness drop

$$C_{f_c} = \frac{C_f}{1+0.18M^2}$$

Drag Coefficient

Skin Friction drag coefficient.

Pressure Drag

Tail

$$\Delta C_D = (1-M^2)^{-0.417} - 1 \quad (M < 0.9)$$
$$\Delta C_D = 1 - 1.5\,(M - 0.9) \quad (0.9 \le M \le 1)$$
$$\Delta C_D = 1.214 - \frac{0.502}{M^2} + \frac{0.1095}{M^4} + \frac{0.0231}{M^6} \quad (1 < M)$$

Cross flow theory

$$\frac{C_{D\,angle}}{C_{\perp}} = \cos^3\Gamma$$

Total leading edge drag of n fins.

$$C_{D_{L_T}} = 2n\left(\frac{Sr_L}{A_r}\right)\cos^2\Gamma_{\mathrm{L}}\,(\Delta C_D)$$

Tailing edge drag.

$$C_{D_{B_T}} = \frac{0.135}{\sqrt[3]{C\_f\_B}}$$

Body

Total

$$C_D = \left(C_{D_T}\right)_T + \left(C_{D_T}\right)_B$$