SBA Center

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4th Grade

Sheet (4)

Operations Research

Lecture 1 after Mid - Term

- **Waiting Line Theory**
- **Simulation**

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DR Mona's Part

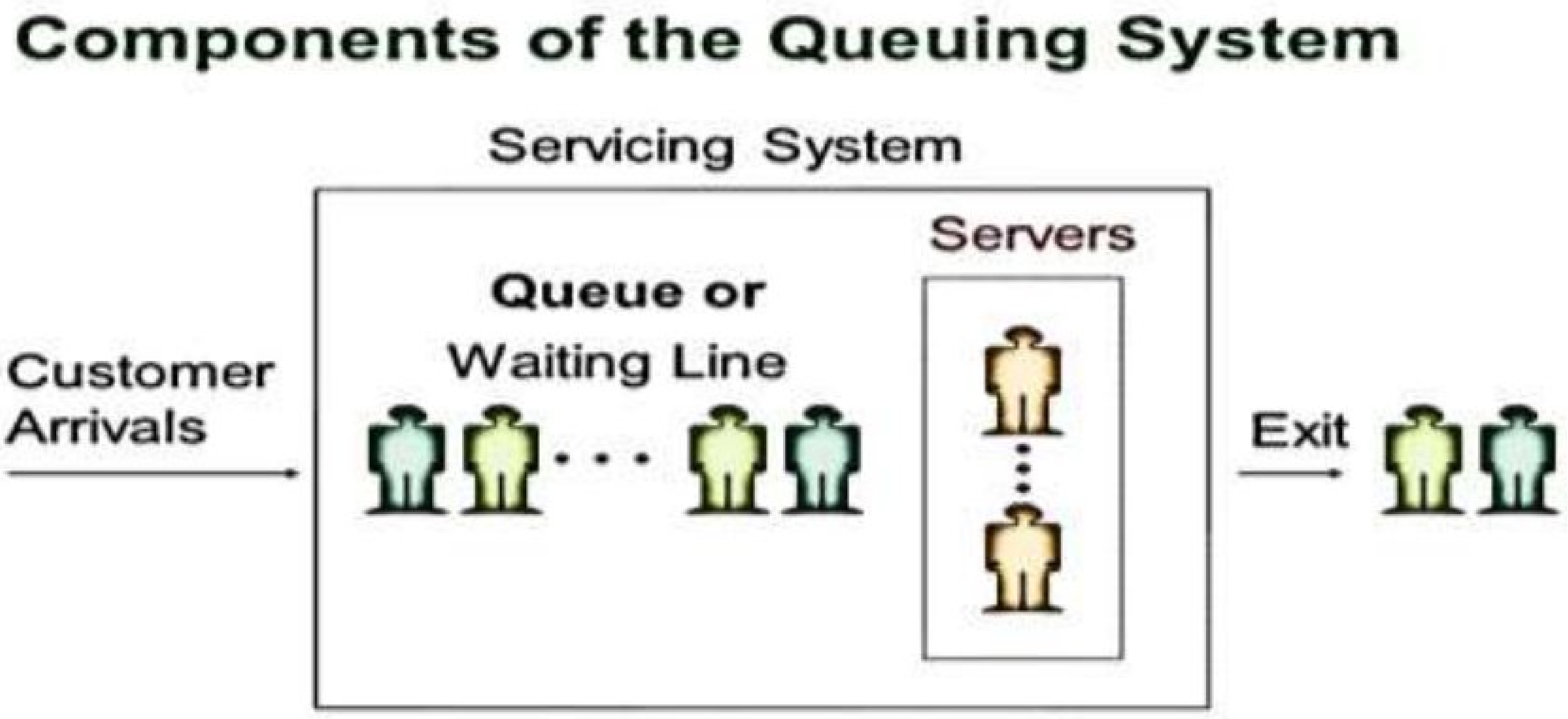
1- Waiting Line Models (Queuing Theory)

Queue: is a line of customers that are waiting their turns to be served.

Examples for Queues or waiting lines:

1- Waiting line to be served in the bank
2- Waiting line at fast food restaurant
3- Waiting line at ATM's
4- Waiting line in the super market at the cashier and so on.

Any waiting line model consists of



ليه بنستخدم ال (Waiting line model) ؟

-بنستخدمه عشان نحسن مدى كفاءة الاماكن او الشركات اللي بتتعامل مع ال (customer) بصورة فردية (عميل واحد فقط في كل دور).

-المقصود بالتحسين هنا هو تقليل وقت الانتظار لكل عميل, و منع حدوث زحمة او تكدس في ال (waiting line).

- و ده بيتم عن طريق انك كمدير شركة لازم تدرس كويس اوي:

1- معدل وصول العملاء (Arrival Rate)
2- معدل حصول كل عميل على الخدمة (Service Rate)
3- ما هي نسبة استغلال الموظفين في الشركة . (Average system utilization)

What queuing Models Tell Us?

- **Average Number of Customers in Line.**
  متوسط عدد العملاء المنتظرين في الصف

- **Average Time in Line for a Customer.**
  متوسط دقائق الانتظار لكل عميل

- Average Number of Customers **in the System** at any time (In Line & Being Served).

متوسط عدد العملاء اللي بيتم خدمتهم دلوقتي

- Average Time in the System for a customer
  متوسط الوقت الذي تستغرقه الخدمة

Note:

(**In the System**) includes customers who are waiting **(+)** the customers being served.

Arrival Rate: (λ)

- The Average number of customers arriving per time period.
- Modeled Using the Poisson distribution.

Service Rate: (μ)

- The Average number of customers that can be served during the period of time.
- Service Times are usually modeled using the exponential distribution.

Average system utilization: (P)

$$\boxed{P = \frac{\lambda}{\mu}}$$

خلي بالك:

لازم ال (Service Rate) تكون اكبر او اسرع من ال (Arrival Rate) عشان ال (waiting line) ميحصلش فيه زحمة و تكدس و ده هيخلي وقت الانتظار لل (customers) يزيد.

In any corporate we want to know the following characteristics

Average number of customers in service (in system) (S)	Average number of customers waiting in the queue (q)
$$Ls = \frac{\lambda}{\mu - \lambda}$$	$$Lq = \frac{\lambda^2}{\mu\,(\mu - \lambda)}$$

AND

Average TIME a customer spends in service (in system) (S)	Average TIME a customer spends in the queue (q)
$$Ws = \frac{1}{\mu - \lambda}$$	$$Wq = \frac{\lambda}{\mu\,(\mu - \lambda)}$$

Example 1:

An Electronics Corporation retains a service crew to repair machine breakdowns that occur (arrive) at an average λ=24. The Crew Can Service an Average of μ = 30 machines Per Hour. What hours is the Corporation Characteristics of queuing system?

Solution

λ=24 **μ = 30**

Average system utilization: (P)

$$P = \frac{\lambda}{\mu} = \frac{24}{30} = 0.80$$

$$\boxed{Po = 1 - 0.80 = 0.20} \rightarrow$$

النسبة الغير مستغلة

Service unit is idle

Average number of machines in service (in system) (S)	Average number of machines in the queue (q)
$Ls = \frac{\lambda}{\mu - \lambda}$ $= \frac{24}{30-24} = \frac{24}{6} = 4$ **Machines**	$Lq = \frac{\lambda^2}{\mu(\mu-\lambda)}$ $= \frac{(24)^2}{30(30-24)} = \frac{(24)^2}{180} = 3.2$ **machines**

Average Time a machine spends in service (in system) (S)	Average Time a machine spends in the queue (q)
$Ws = \frac{1}{\mu - \lambda}$ $= \frac{1}{30-24} = \frac{1 \times 60}{30-24} = 10$ **minutes**	$Wq = \frac{\lambda}{\mu(\mu-\lambda)}$ $= \frac{\lambda}{\mu(\mu-\lambda)} = \frac{24 \times 60}{30(30-24)} = 8$ **minutes**

ضربنا في 60 عشان نحول ال (TIME) من (HOURS) الى (Minutes)

Example 2:

On a certain day, a commercial bank found that an average of **<u>20 customers per hour arrive</u>** at the bank (approximately Poisson in nature). The Teller at the bank can **<u>service an average of 30 customers per hour</u>** with) exponential time distribution) . What are the bank characteristics of queuing system?

Solution

λ=20 **μ = 30**

$P = \frac{\lambda}{\mu} = \frac{20}{30} = 0.67$ Po = 1 – 0.67 = 0.33

<u>Average number</u> of customers in service (in system) (S)	**<u>Average number</u> of customers in the queue (q)**
$Ls = \frac{\lambda}{\mu - \lambda}$ $= \frac{20}{30-20} =$ **2 Customers**	$Lq = \frac{\lambda^2}{\mu\,(\mu-\lambda)}$ $= \frac{(20)^2}{30(30-20)} =$ **1.3 Customers**

<u>Average Time</u> a Customer spends in service (in system) (S)	**<u>Average Time</u> a Customer spends in the queue (q)**
$Ws = \frac{1}{\mu - \lambda}$ $= \frac{1}{30-20} = \frac{1 \times 60}{30-20} =$ **6 minutes**	$Wq = \frac{\lambda}{\mu\,(\mu-\lambda)}$ $= \frac{\lambda}{\mu(\mu-\lambda)} = \frac{20 \times 60}{30\,(30-20)} =$ **4 minutes**

Example 3:

The manager of international airport prepares a study to improve the service. The study showed that the planes **average arrival rate was 32 planes per hour**. The **airport average service rate now is 36 planes per hour**. While after improvements it can **serve on average 39 planes per hour** with. Will the study improve the airport characteristics of queuing system?

Solution

	Before: λ = 32 , μ =36	**After: λ = 32 , μ = 39**
$\mathbf{P = \frac{\lambda}{\mu}}$	$32/36 = 0.89$	$32/39 = 0.82$
$\mathbf{Po = 1 - P}$	$1 - 0.88 = 0.11$	$1 - 0.82 = 0.18$
$\mathbf{Ls = \frac{\lambda}{\mu - \lambda}}$	$\frac{32}{36-32} = 8$ Planes	$\frac{32}{39-32} = 4.57$ Planes
$\mathbf{Lq = \frac{\lambda^2}{\mu(\mu - \lambda)}}$	$\frac{32^2}{36(36-32)} = 7.11$ Planes	$\frac{32^2}{39(39-32)} = 3.75$ Planes
$\mathbf{Wq = \frac{\lambda}{\mu(\mu - \lambda)}}$	$\frac{32x60}{36(36-32)} = 13.3$ minutes	$\frac{32x60}{39(39-32)} = 7$ minutes
$\mathbf{Ws = \frac{1}{\mu - \lambda}}$	$\frac{1x60}{36-32} = 15$ minutes	$\frac{1x60}{39-32} = 8.6$ minutes

It is better to implement the improvement as the waiting in (service &line) decreases after improvements.

2- Simulation محاكاة

Simulation **involves using mathematical models to describe the behavior of real-world systems.**

This can be useful for testing different *scenarios or strategies* in a virtual environment before applying them in the real world, *and using the simulation helps the decision maker to judge and evaluate the model under many conditions or scenarios.*

Simulation depends on uncertainty (probabilities)

The most known simulation technique is > Monte Carlo Simulation Method

Monte Carlo Method:

It is probabilistic method used when the model or the problem contains random variables or random results.

Steps:

- Identify the probability distribution (the outcomes and the percentage or the probability for each outcome)

- make range intervals for the corresponding random results that matches the probability distributions.

- obtain the random numbers.

- Interpret the results.

Theoretical part in simulation (Important):

Monte Carlo method:

- **Solve problems that are difficult to solve mathematically.**
- **Does not produce optimum solution.**
- **Applicable only for random systems.**
- **Allows experimentation without any risk to actual system.**

Example 1:

The manager is concerned about the machine breakdowns. Historical data over the last 100 days are given in the following table.

Number of breakdowns	0	1	2	3	4	5
Frequency	10	30	25	20	10	5

use Monte Carlo random numbers to simulate breakdowns for 10 days periods.
Assume the random numbers are >> 18 25 73 12 54 96 21 31 45 01

Solution

Number of breakdowns	Frequency	Probability	Cumulative probability	Corresponding random number (Range)
0	10	$\frac{10}{100} = 0.10$	0.10	01 to 10
1	30	$\frac{30}{100} = 0.30$	+ 0.40	11 to 40
2	25	$\frac{25}{100} = 0.25$	+ 0.65	41 to 65
3	20	0.20	0.85	66 to 85
4	10	0.10	0.95	86 to 95
5	5	0.05	1.00	96 to 00
Total	**100**	**1**		

نقصد بها 100

Convert random numbers into simulated numbers

Days	Random Numbers	Simulated numbers of break downs
1	18	1
2	25	1
3	73	3
4	12	1
5	54	2
6	96	5
7	21	1
8	31	1
9	45	2
10	01	0
Total		**17**

18 >> found in the range of 11 to 40 >> 1 breakdown

The mean number of break downs = $\frac{17}{10}$

$$= 1.7\ breakdown\ /\ day$$

Example 2:

Use the following data for the past 20 years and the random numbers to simulate breakdowns for the power generator for 10 years periods. What is the mean of expected number of yearly breakdowns.

Number of breakdowns	0	1	2	3	4	5
Frequency	18	8	22	25	17	10

The random numbers are: 20 19 57 48 92 40 55 96 49 78

Solution

Number of breakdowns	Frequency	Probability	Cumulative probability	Corresponding random number (Range)
0	18	$\frac{18}{100} = 0.18$	0.18	01 to 18
1	8	$\frac{8}{100} = 0.08$	+ 0.26	19 to 26
2	22	$\frac{22}{100} = 0.22$	+ 0.48	27 to 48
3	25	0.25	0.73	49 to 73
4	17	0.17	0.90	74 to 90
5	10	0.10	1.00	91 to 00
Total	**100**	**1**		

Convert random numbers into simulated numbers

years	Random Numbers	Simulated numbers of break downs
1	20	1
2	19	1
3	57	3
4	48	2
5	92	5
6	40	2
7	55	3
8	96	5
9	49	3
10	78	4
Total		**29**

The mean number of break downs = $\frac{29}{10}$

$$= 2.9\ breakdown\ /\ year$$

Example 3:

A ware houses manager needs to simulate the demand on a product, the historical records for this product are listed below, random numbers have been generated to simulate the next 10 daily order cycle. What is the average product demand?

Daily demand	0	1	2	3	4	5
Frequency	15	22	35	08	11	9

The random numbers are: 98 94 91 36 06 78 16 80 67 85

Solution

Daily demand	Frequency	Probability	Cumulative probability	Corresponding random number (Range)
0	15	$\frac{15}{100} = 0.15$	0.15	01 to 15
1	22	$\frac{22}{100} = 0.22$	0.37	16 to 37
2	35	$\frac{35}{100} = 0.35$	0.72	38 to 72
3	08	0.08	0.80	73 to 80
4	11	0.11	0.91	81 to 91
5	9	0.09	1.00	92 to 00
Total	100	1		

Convert random numbers into simulated numbers

days	Random Numbers	Simulated numbers of units demanded
1	98	5
2	94	5
3	91	4
4	36	1
5	06	0
6	78	3
7	16	1
8	80	3
9	67	2
10	85	4
Total		28

The mean number of demands = $\frac{28}{10}$

$$= 2.8\ unit\ /\ day$$

TEST BANK waiting line theory

1-) A queue is formed when the demand for a service:

a. Exceeds the capacity to provide that service
b. Is less than the capacity to provide the service
c. a or b
d. None of these

Ans: a

2-) Queuing theory is also termed as............................

a. Game theory
b. Replacement theory
c. Waiting line theory
d. None of these

Ans: c

3-) in queuing theory, ________ refers to those waiting in a queue or receiving service.

a. Service provider
b. Customer
c. Both a and b
d. None of these

Ans: b

4-) in waiting line theory, number of customers waiting in the queue is referred to as................

a. Traffic intensity
b. Queuing system
c. Service pattern
d. Queue length

Ans: d

5-) number of customers in the queue per unit of time is called.....................

a. Queuing system
b. Length of queue
c. Average length of queue
d. None of these

Ans: c

6-) the ration between mean arrival rate and mean service rate is called..................

a. Idle period
b. Average length of queue
c. Average system utilization (Traffic intensity)
d. None of these

Ans: c

7-) commonly assumed probability distribution of arrival pattern is......................

a. Poisson distribution
b. Binomial distribution
c. Normal distribution
d. None of these

Ans: a

8-) commonly assumed probability distribution of service pattern are......................

a. Poisson distribution
b. Exponential distribution
c. Erlang distribution
d. b and c

Ans: b

9-) In queuing theory, ________ stands for mean service rate.

a. μ
b. λ
c. t
d. none of these

Ans: a

10-) .... is a method of analyzing the current movement of the some variable In an effort to predict the future movement of the same variable.

a. Goal programming
b. Queuing theory
c. Markov Analysis
d. Replacement theory

11-) the utilization factor for a system is defined as:

a. The mean number of arrivals per period divided by the mean number of customers served per period.
b. The percent idle time.
c. The average time a customer spends in the system.
d. The average time a customer spends waiting in the queue.

Ans: a

12-) Traffic intensity in Queuing Theory is also called..........................

a. Service factor
b. Arrival factor
c. Utilisation factor
d. None of these

Ans: b

13-) Traffic intensity is computed by using the formula:

A. λ / μ
B. μ / λ
C. 1- (λ / μ)
D. 1- (μ / λ)

14-) the outlet where the services are being provided to the customers is called................

a. Waiting line
b. Service facility
c. Idle facility
d. Traffic intensity

Ans: b

15-) The Operations research technique which helps in minimizing total waiting and service costs is

A. Queuing Theory
B. Decision Theory
C. Both A and B
D. None of the above

Ans: a

16-) arrival rate should be ___ than the service rate.

A. equal
B. bigger
C. smaller
D. None of the above

Ans: c

17-) What are awaiting line's two points of service?

a. Strong and Weak
b. Low and High
c. Fast and Slow
d. Difficult and Easy

Ans: c

18-) Which of the following is not a key operating characteristic for a queuing system

A. utilization factor
B. percent idle time
C. average time spent waiting in the system and queue
D. none of the above

Ans: d

19-) mean average number of items served per time period is.

A. mean arrival rate
B. mean service rate
C. utility percent
D. None of the above

Ans: b