

# Path Planning

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# Objectives

## Specific Objectives

- Role of path planning in robotics
- Main techniques

## Source

- Stuart Russell & Peter Norvig (2009). Artificial Intelligence: A Modern Approach. (3rd Edition). Ed. Pearson
- P. Muñoz, B. Castaño and M.D. R-Moreno. 3Dana: A Path Planning Algorithm for Surface Robotics (2017). Int. Scientific Journal Engineering Applications of AI. Vol. 60, pp:175-192

# Outline

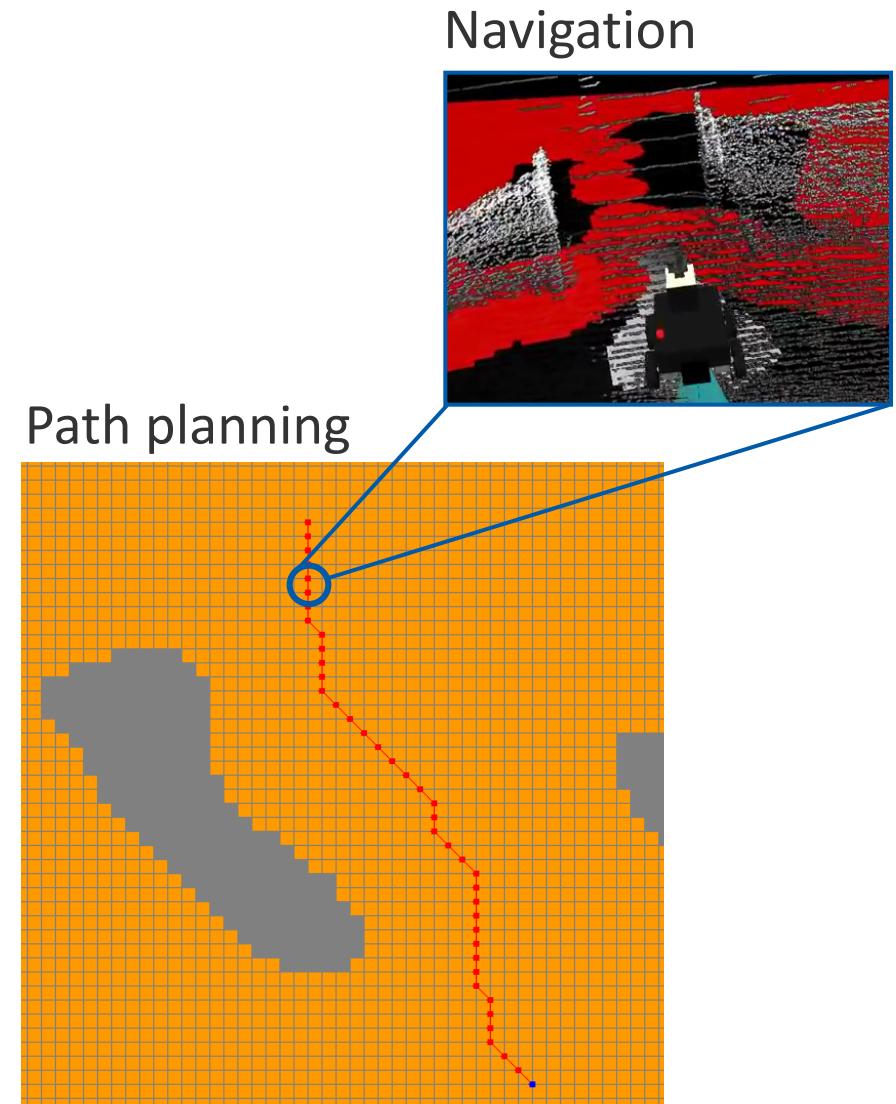
- Introduction
- Representation
- Dijkstra
- A\*
- A\*PS
- Theta\*
- S-Theta\*
- Heuristics
- Conclusions

# Introduction (I)

- Path-planning problem aims to obtain feasible and optimal (or near to it) routes between two or more points
  - Find optimal (or near to it) paths is not trivial
  - Feasible implies not transverse over obstacles or overcome system limitations
  - Usually parameters: path length, run-time, expanded nodes and number of heading changes
- It is a fundamental task in mobile robots and video games

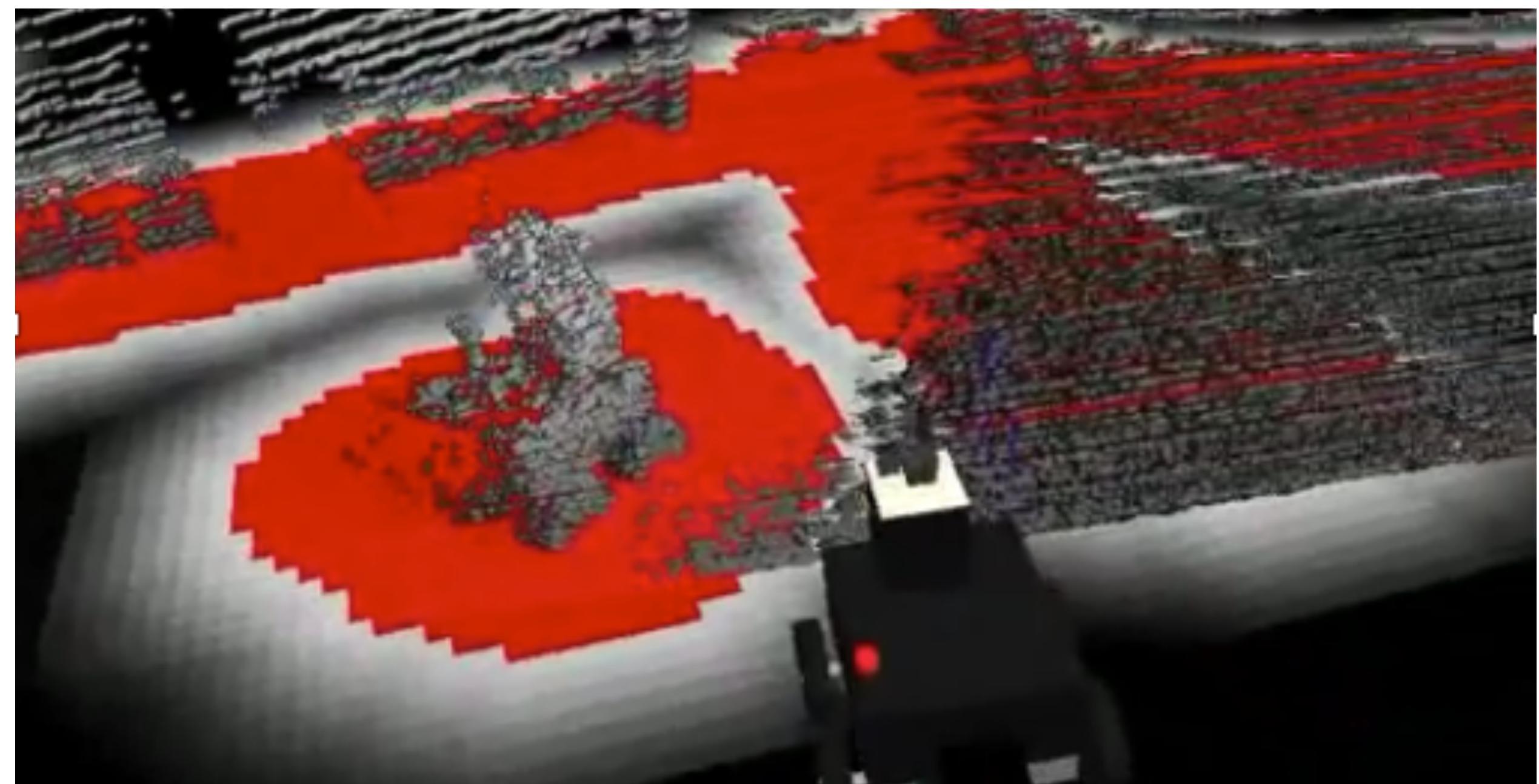
# Introduction (II)

- Navigation
  - Local paths
  - Guided by sensors
- Path Planning
  - Global paths
  - Known terrain
  - Related to AI



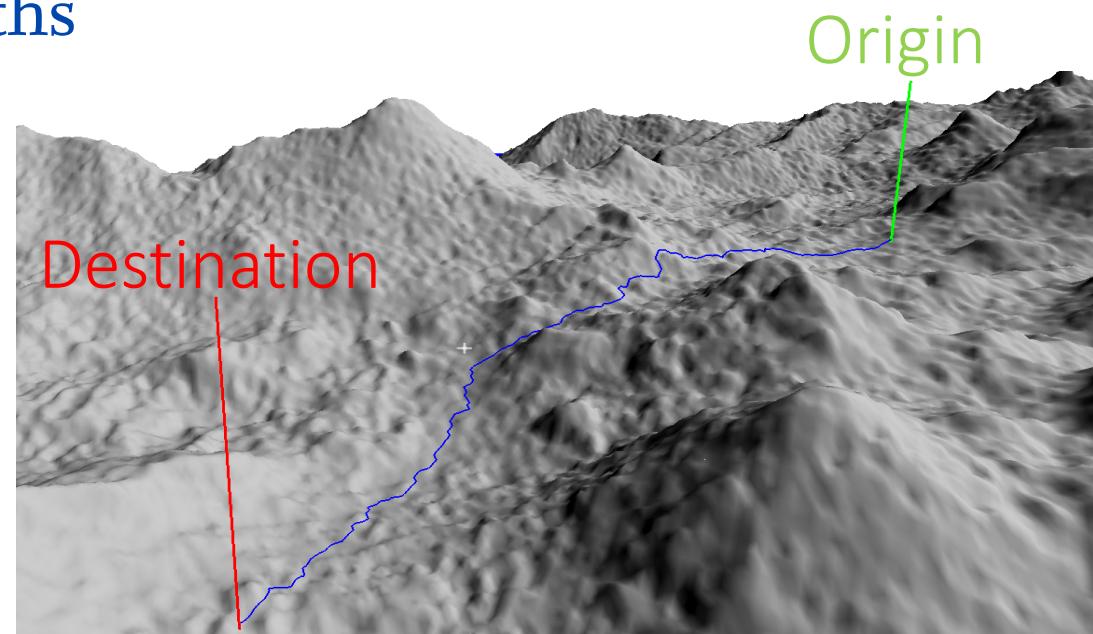
# Introduction (III)

- First point to address: the environment
  - Local planning vs Long term planning
  - Totally observable vs Partially observable
  - Discrete vs Continuous
  - Dynamic environment?    Maybe
  - Extra information on the terrain?    Maybe



# Introduction (IV)

- Long term path-planning is more related to AI
- Easier with fully observable environment
- High effort trying to obtain optimal paths
- In some domains (such as planetary exploration) path-planning and task-planning are highly coupled



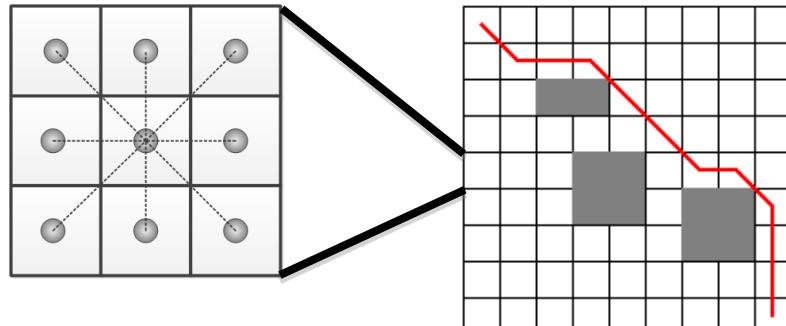
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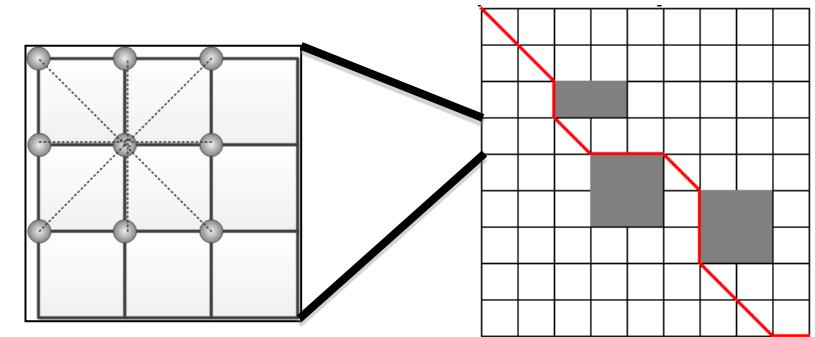
# Representation (I)

- Classical path-planning algorithms are based on A\* algorithm
- Works over 2D grids with blocked and unblocked cells
- Nodes are (*usually*) 8-connected with its neighbors
- Two representations:

Center node

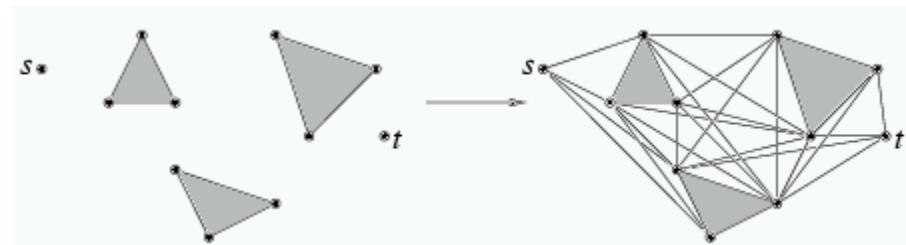
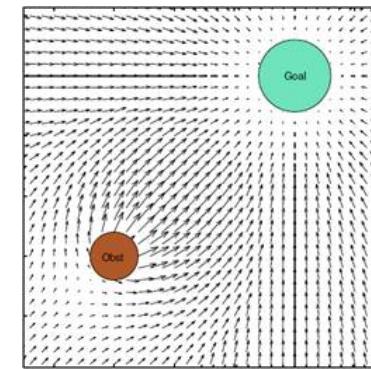
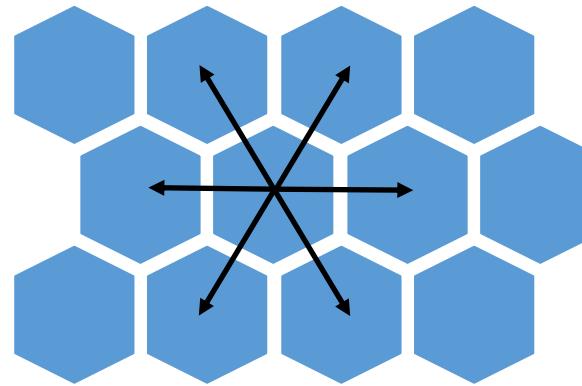


Corner node



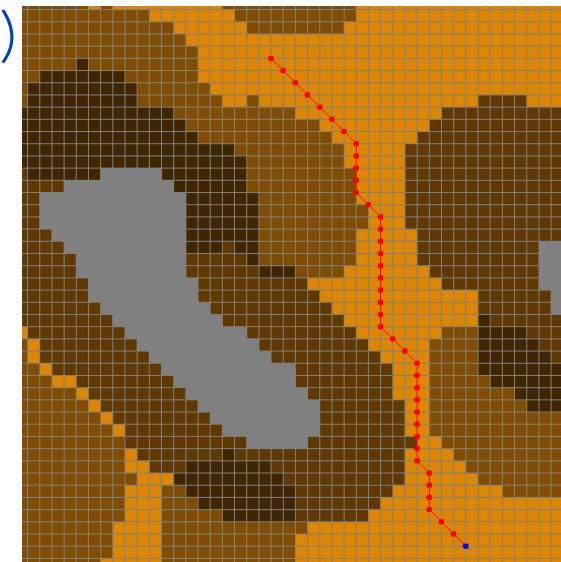
# Representation (II)

- Hexagonal cells
  - 6 neighbours
  - More complex!
- Potential fields
  - Attraction/repulsion
- Visibility graphs
  - Pair of interconnected points
  - Expensive to calculate
  - Fast and optimal solution
  - Video



# Representation (III)

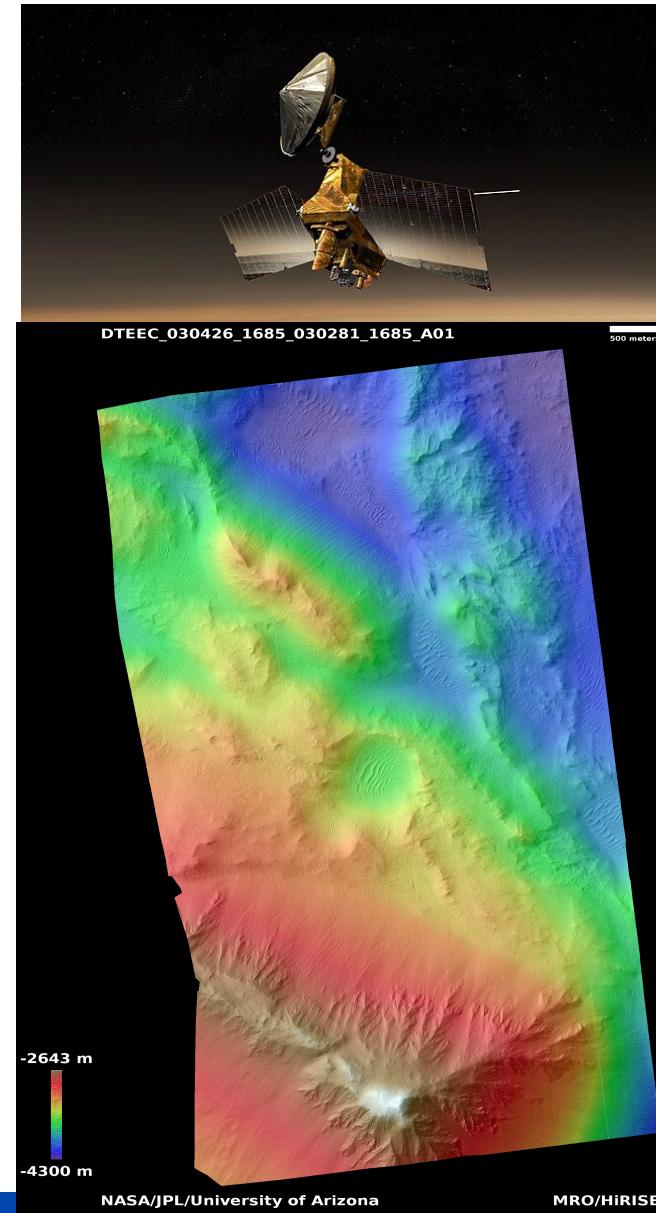
- Costs maps
  - Extension of 2D maps
  - Add information
  - Typically, lineal combination of factors (rocks, hills, etc)
- Goal: avoid hazardous areas



# Representation (IV)

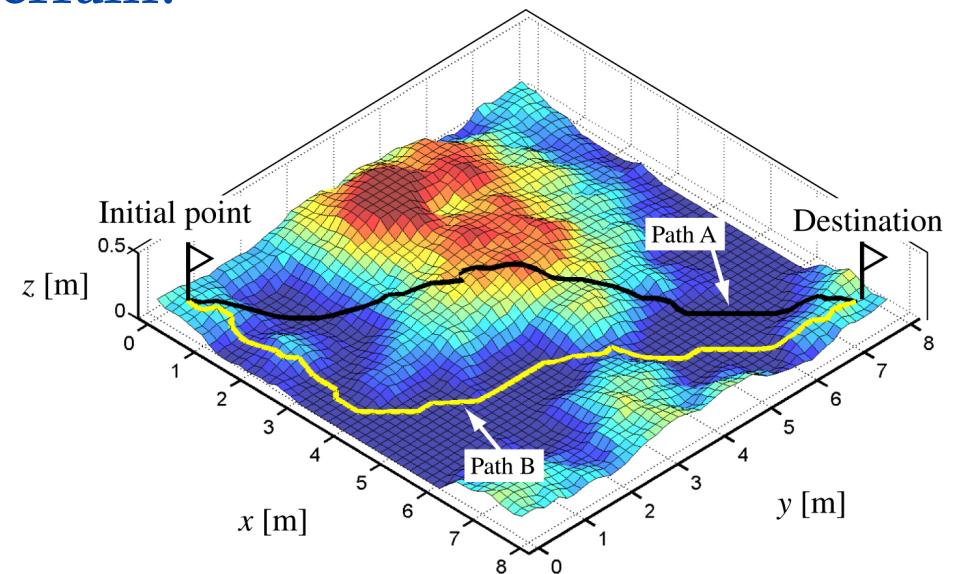
- Digital terrain models (DTM)
- Mars on high resolution
  - From 2m of horizontal resolution
  - Up to 25cm!
  - Vertical resolution in dc
- Used for planning (MER/MSL)
- Free download

[www.uahirise.org/dtm](http://www.uahirise.org/dtm)



# Representation (V)

- For dynamic environments usually a replanning strategy is followed
- Optimize the replanning process?
- What can be taken into consideration into the terrain?
  - Altitude → DEM
  - Hazardous areas



# Representation (VI)

- Deterministic algorithms
  - Non Informed search
  - Heuristic search
- Stochastic algorithms
  - Tree search
  - Genetic algorithms
  - Ant colony



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# Dijkstra

- Non informed search
- Pick the unvisited vertex with the lowest-distance
- Calculate the distance through it to each unvisited neighbor
- Update the neighbor's distance if smaller
- Mark visited when done with neighbors



# Dijkstra

```
1  function Dijkstra(Graph, source):
2      dist[source] := 0                                // Initializations
3      for each vertex v in Graph:
4          if v ≠ source
5              dist[v] := infinity                  // Unknown distance from source to v
6              previous[v] := undefined           // Predecessor of v
7          end if
8          PQ.add_with_priority(v, dist[v])
9      end for
10
11
12     while PQ is not empty:                      // The main loop
13         u := PQ.extract_min()                 // Remove and return best vertex
14         for each neighbor v of u:            // where v has not yet been removed from PQ.
15             alt = dist[u] + length(u, v)
16             if alt < dist[v]                // Relax the edge (u,v)
17                 dist[v] := alt
18                 previous[v] := u
19                 PQ.decrease_priority(v, alt)
20             end if
21         end for
22     end while
23     return previous[]
```

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# A\*

- A\* makes guided search using two values:
  - Accumulate cost ( $G(t)$ ): cost to reach a node
  - Heuristic ( $H(t)$ ): predicted cost to achieve goal from a node (Euclidian distance, Octal distance)
- Node Evaluation:  $F(t) = G(t)+H(t)$
- A\* is simple, fast and guarantee optimal paths in eight-connected grids
- Artificially restricted to  $45^\circ$  headings

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**Algorithm 1** A\* search

---

```
1  $G(s) \leftarrow 0$ 
2  $parent(s) \leftarrow s$ 
3  $open \leftarrow \emptyset$ 
4  $open.insert(s, G(s), H(s))$ 
5  $closed \leftarrow \emptyset$ 
6 while  $open \neq \emptyset$  do
7      $p \leftarrow open.pop()$ 
8     if  $p = g$  then
9         return  $path$ 
10    end if
11     $closed.insert(p)$ 
12    for  $t \in neighbours(p)$  do
13        if  $t \notin closed$  then
14            if  $t \notin open$  then
15                 $G(t) \leftarrow \infty$ 
16                 $parent(t) \leftarrow null$ 
17            end if
18             $UpdateVertex(p, t)$ 
19        end if
20    end for
21 end while
22 return  $fail$ 
```

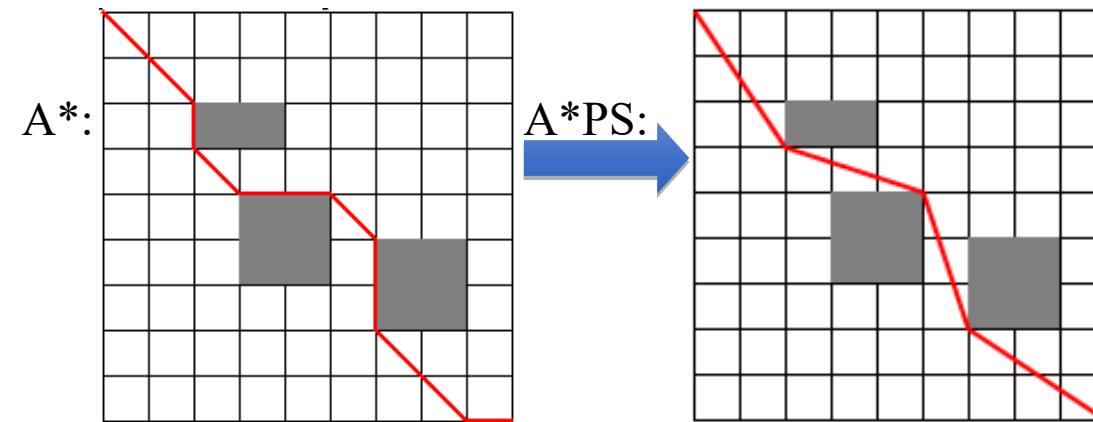
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# A\* Post Processed

- A\* Post Processed (A\*PS) tries to smooth A\* routes by removing intermediate nodes when there is line of sight between 2 no neighbors nodes, that is, there are no obstacles



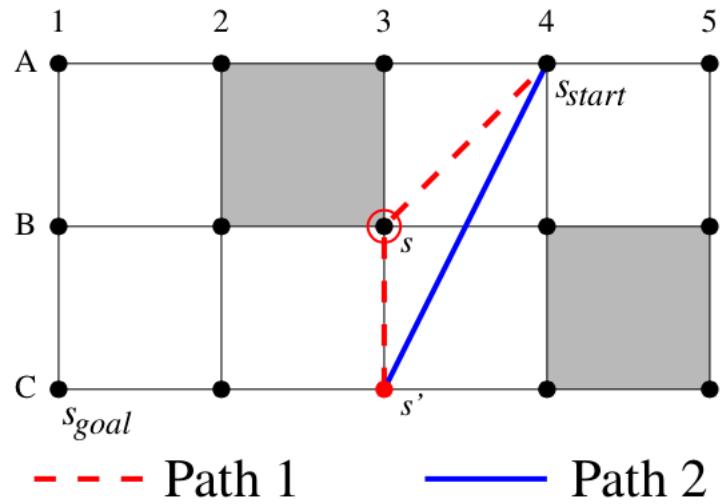
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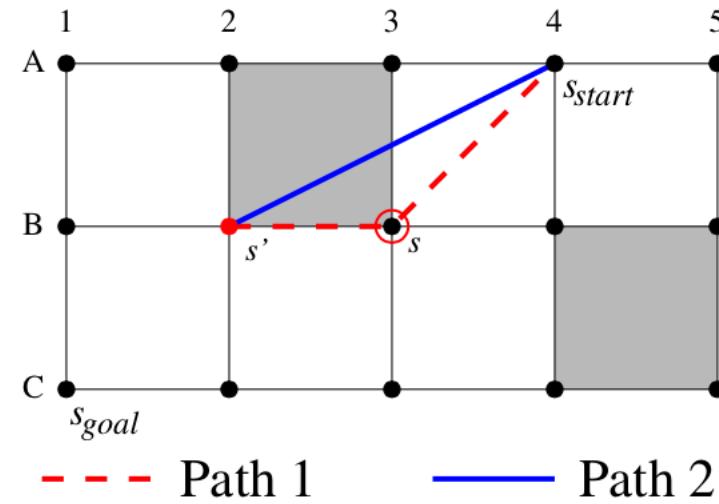
# Theta\*

- Theta\* is a variation of A\* that integrates the line of sight check during search
- For this reason Theta\* is not restricted to  $45^\circ$  headings, and gets more realistic paths than A\* without post processing
- Theta\* is slower than A\* due to line of sight calculation, but paths are shorter and smoothest

# Theta\*



(a) Path 2 is unblocked



(b) Path 2 is blocked

# Theta\*

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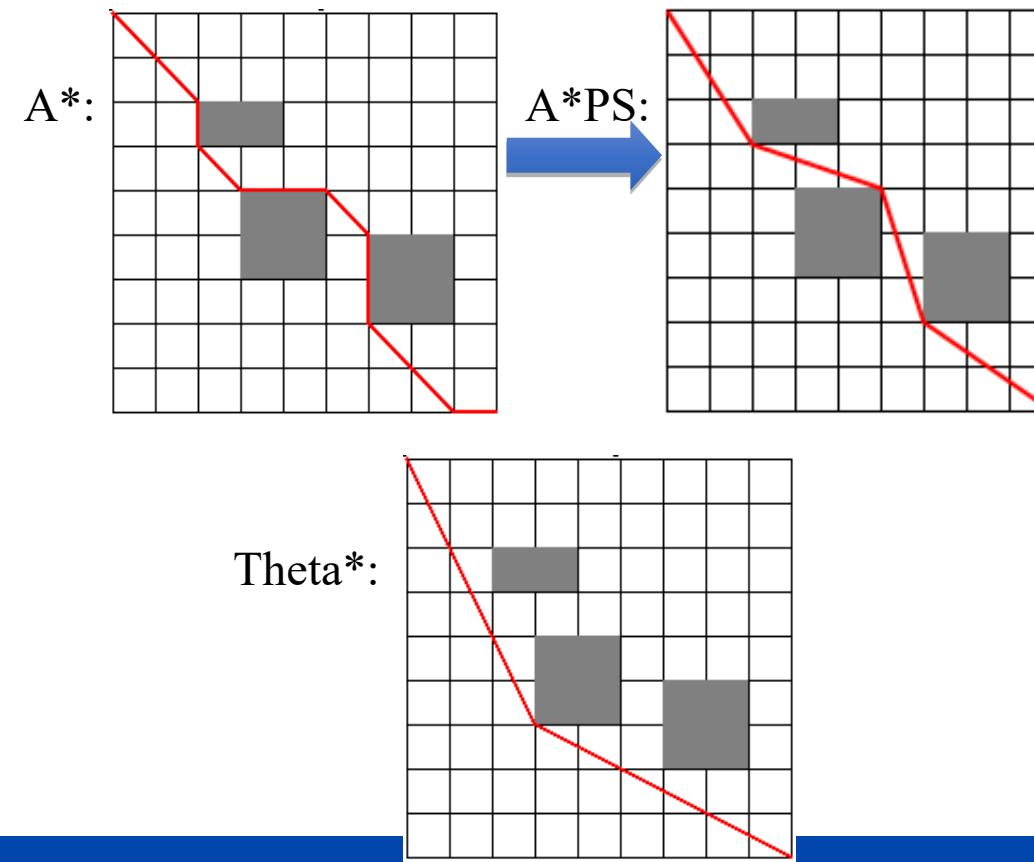
**Algorithm 2** Update vertex function for Basic Theta\*

```
1 UpdateVertex(p, t)
2 if LineOfSight(parent(p), t) then
3     if G(parent(p)) + dist(parent(p), t) < G(t) then
4         G(t)  $\leftarrow$  G(parent(p)) + dist(parent(p), t)
5         parent(t)  $\leftarrow$  parent(p)
6         if t  $\in$  open then
7             open.remove(t)
8         end if
9         open.insert(t, G(t), H(t))
10    end if
11 else
12     if G(p) + dist(p, t) < G(t) then
13         G(t)  $\leftarrow$  G(p) + dist(p, t)
14         parent(t)  $\leftarrow$  p
15         if t  $\in$  open then
16             open.remove(t)
17         end if
18         open.insert(t, G(t), H(t))
19     end if
20 end if
```

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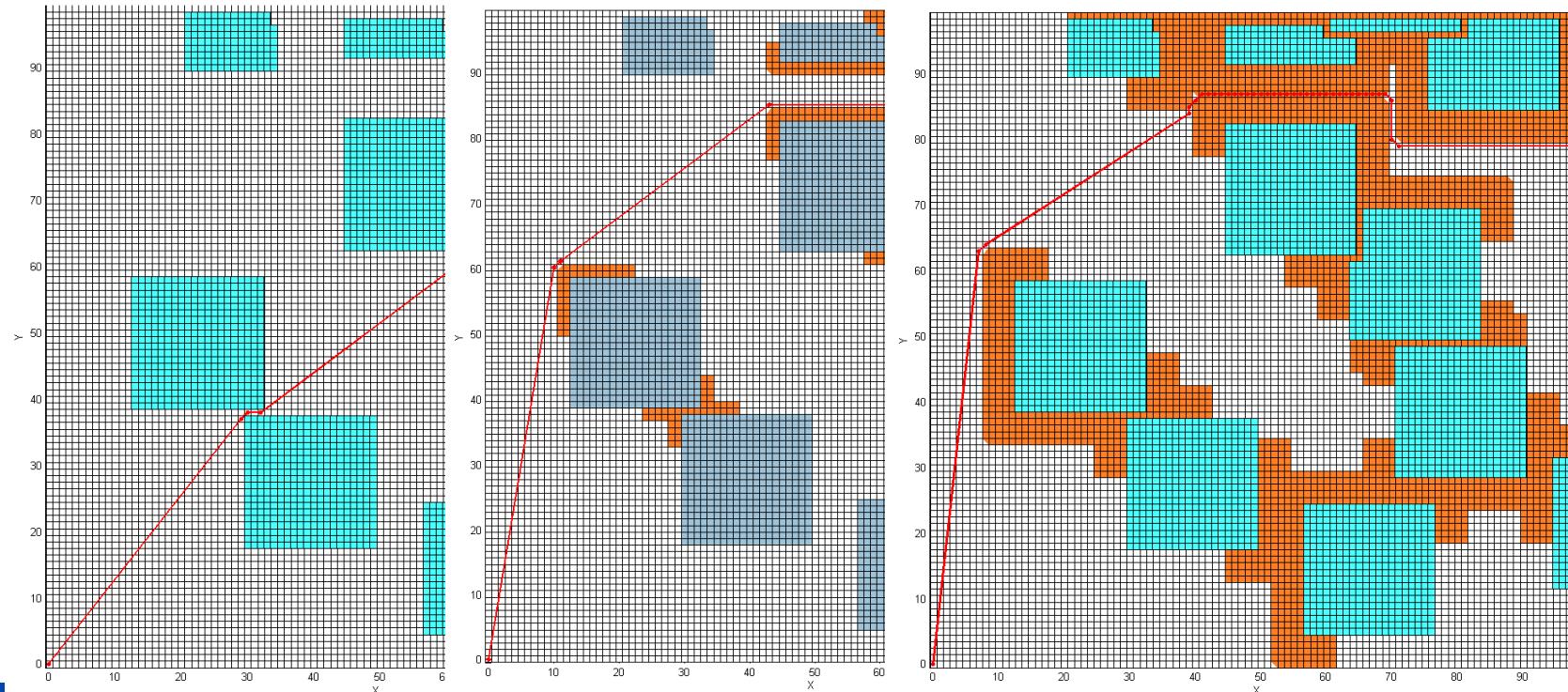
# Theta\*

- A\*PS and Theta\* are any-angle algorithms: not restricted to  $45^\circ$  headings



# Theta\*

- What about crossing the obstacles at the corner?  
→ Safety margin



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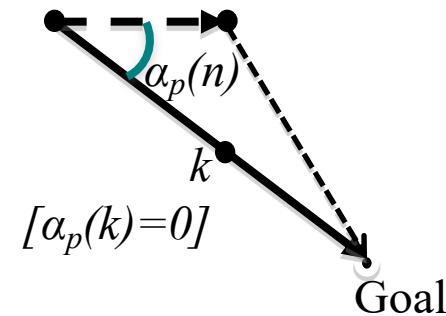
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# S-Theta\*

- Theta\* updates a node depending on the distance to reach it, regardless of its orientation
- Adapt Theta\* to take into consideration heading changes during the search process
  - Robotics hardware is usually very limited
  - Rotation cost is greater than the movement straight
- Best path between two points in a free obstacle grid is straight line
- Achieve less heading changes in exchange of a slight degradation of the path length

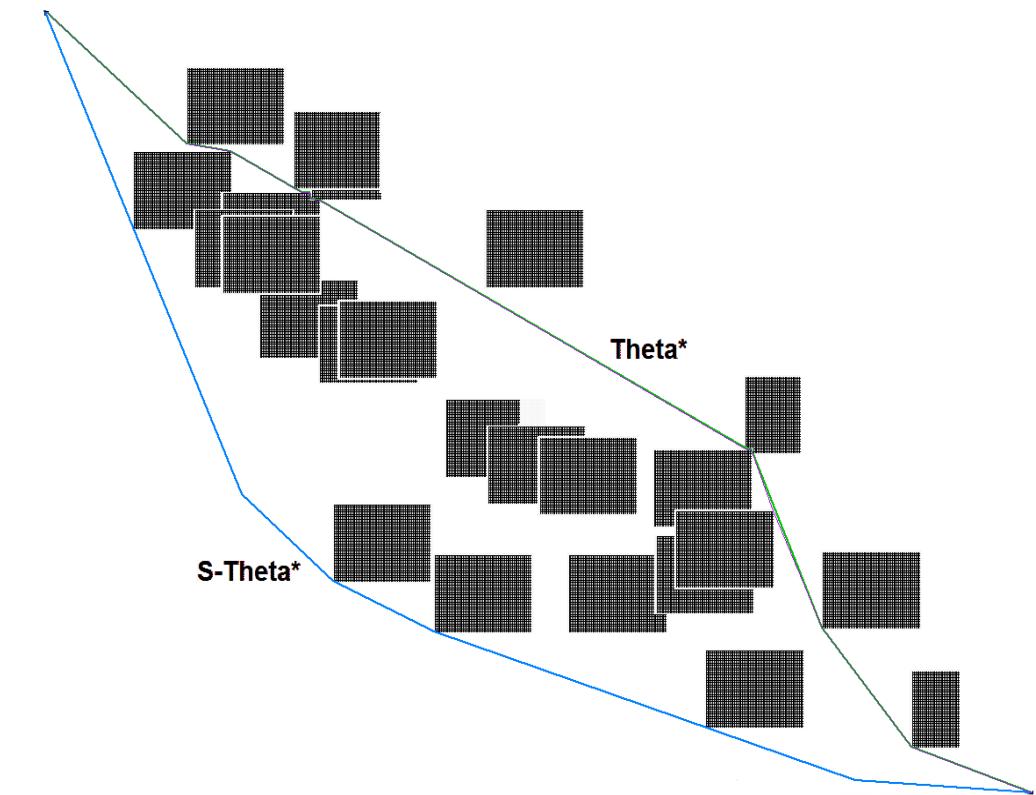
# S-Theta\*

- Include a new term in the cost function:  $\alpha(n)$  that represents the heading change variation to reach a node  $n$  in relationship with the objective and previous nodes
- This term guides the search process to:
  - Smooth heading changes
  - Reduce number of heading changes
- $F(t) = G(t)+H(t)+\alpha(n)$



# S-Theta\*

- $\alpha(n)$  tries to surround obstacles and return the best path
- Does not expand nodes far from the line
- Need to weight
  - $\alpha(n) = \alpha(n) \times N$
  - $\alpha(n)$  is  $[0^\circ, 180^\circ]$



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# Heuristics

- We consider a plane with  $p_1$  at  $(x_1, y_1)$  and  $p_2$  at  $(x_2, y_2)$
- Euclidean
  - $E = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Manhattan
  - $M = |x_1 - x_2| + |y_1 - y_2|$
- Octile
  - $\nabla_x = |x_1 - x_2| \quad \nabla_y = |y_1 - y_2|$
  - $O = \sqrt{r^2 + r^2} \cdot \min(\nabla_x, \nabla_y) + |\nabla_x - \nabla_y|$

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# Conclusions

- Classical path-planning based on informed search methods provides a good approximation for big observable areas
- Heuristics are very relevant
- Representation of the environment is an important point to consider
- Large number of works, but still possibility to improve
  - Field D\*, Block A\*, Lazy Theta\*...