

Temporal Constraints Satisfaction Problems (TCSP)

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Objectives

Specific Objectives

- Main representation in TCSP
- Main techniques

Source

- Dechter, Meiri, and Pearl (1991). Simple Temporal Networks. *Artificial Intelligence* 49:61–95.
- Dana Nau's slides for Automated Planning. Licensed under License <https://creativecommons.org/licenses/by-nc-sa/2.0/>
- Brian William. (2002). Slides. Temporal Planning, Scheduling and Execution

Outline

- Motivation
- Definition
- Simple Temporal Network (STN)
- Properties of STN
- A Completed Plan Forms an STN
- Conclusions
- ToDo Example

Motivation example

- I would like to solve this problem:
 - John goes to work by car, which takes 30–40 minutes
 - Fred goes to work in a carpool, which takes 40–50 minutes
 - Today, John left home between 7:10 and 7:20
 - Fred arrived at work between 8:00 and 8:10
 - We also know that John arrived at work about 10–20 mn after Fred left home
- What are the times that Fred left home?

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Definition

- Involves a set of variables: $X_1 \dots X_n$
- Each variable represents a time point (TP)
- Each constraint represented by a set of intervals: $\{I_1, \dots, I_n\} = \{[a_1, b_1], \dots, [a_n, b_n]\}$
 - $T_i = (a_i \leq X_i \leq b_i) \text{ or } \dots \text{ or } (a_i \leq X_i \leq b_i) \rightarrow \text{unary}$
 - $T_{ij} = (a_i \leq X_i - X_j \leq b_i) \text{ or } \dots \text{ or } (a_n \leq X_i - X_j \leq b_n) \rightarrow \text{binary}$
- a constraint $c_i \rightarrow_j$ between two time points x_i and x_j can be expressed as a union of intervals $x_j - x_i \in I_{ij1} \cup \dots \cup I_{ijn} = [a_{ij1}, b_{ij1}] \cup \dots \cup [a_{ijn}, b_{ijn}]$
- Use Simple Temporal Networks representation

Dechter, Meiri, Pearl, aij89

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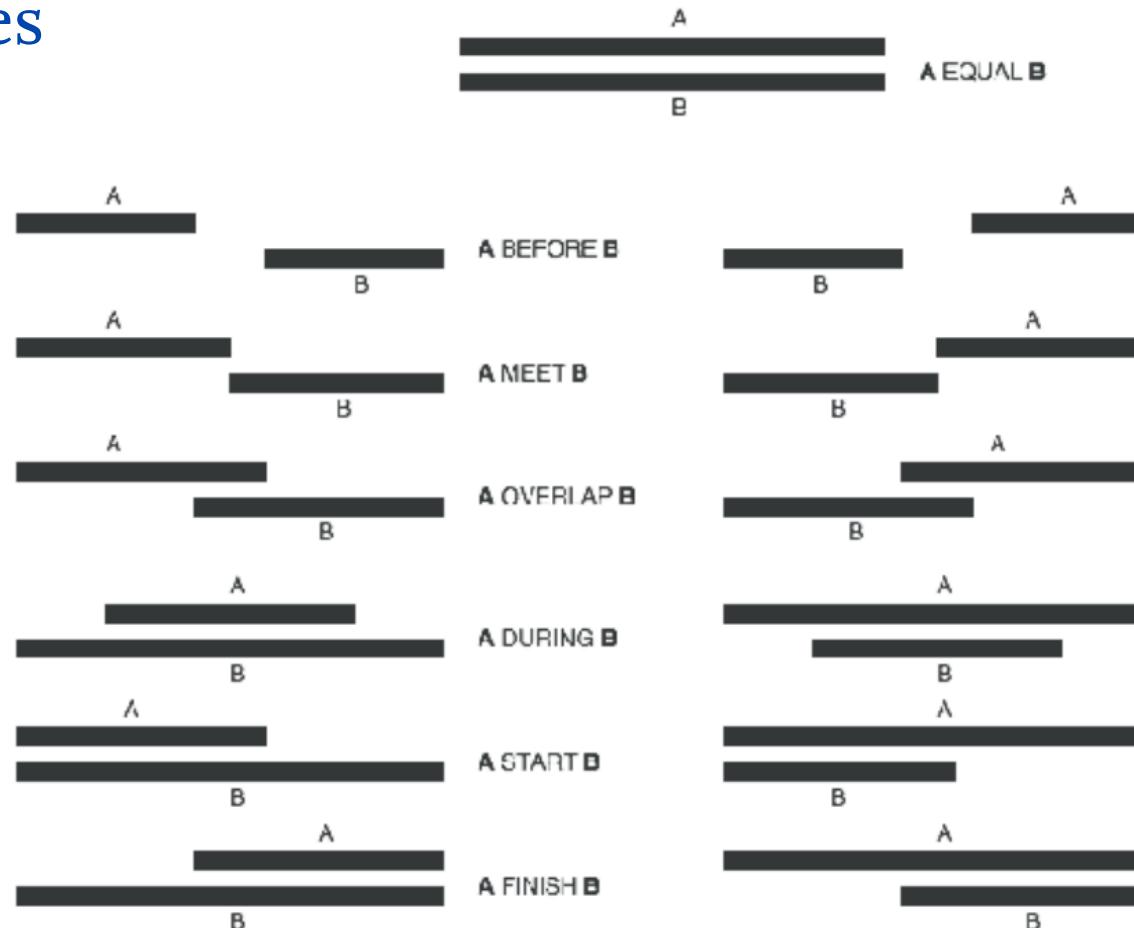
Simple Temporal Network (I)

- Formally:
 - A STN S is a pair (T, C) where T is a set $\{to, t_1, \dots, t_n\}$ of TP variables
 - C is a finite set of binary constraints on those variables, each constraint having the form $t_j - t_i \leq \delta$, for some real number δ
 - The “variable” $to \rightarrow$ represents an arbitrary, fixed reference point on the time-line
 - The constraints in C are called the explicit constraints in S
- Implementation: A directed graph where vertices are TPs and every arc represents a single temporal constraint defined on two TPs

Simple Temporal Network (II)

- A solution to $S=(T, C)$ is a complete set of variable assignments that satisfies all the constraints in C
- Sufficient to represent most Allen relations
 - Simple metric constraints
- Allows to represent a set of TPs X_i at which events occur
 - Unary constraints
 $(a_o \leq X_i \leq b_o) \text{ or } (a_i \leq X_i \leq b_i) \text{ or } \dots$
 - Binary constraints
 $(a_o \leq X_j - X_i \leq b_o) \text{ or } (a_i \leq X_j - X_i \leq b_i) \text{ or } \dots$

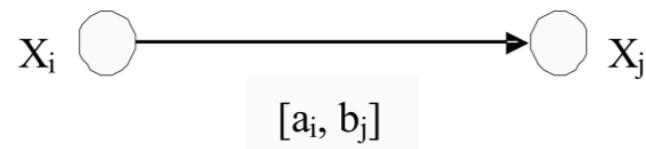
Allen primitives



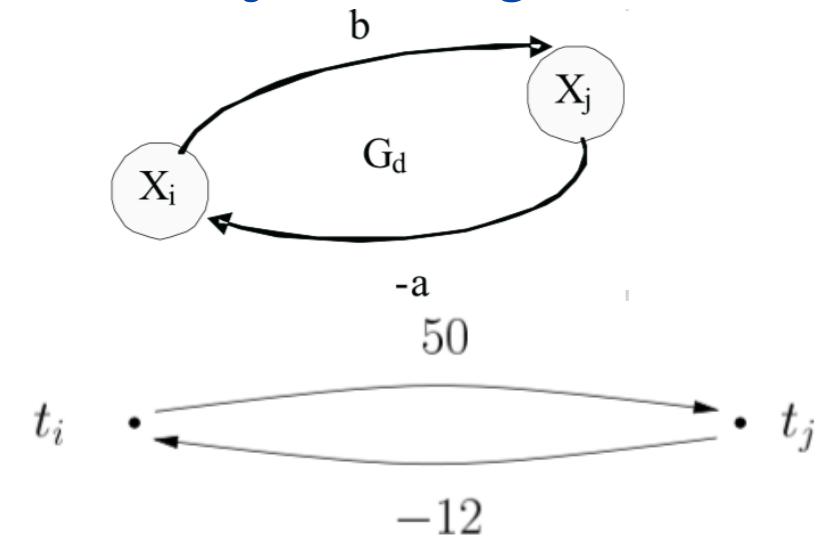
Source: Flávio S. Corrêa da Silva

Simple Temporal Network (III)

- A temporal constraint $X_j - X_i \leq \delta$ in an STN is represented in the corresponding graph by a directed edge from X_i to X_j with weight (or length) δ



- For example: $t_j - t_i \leq 50$ and $t_i - t_j \leq -12$



Simple Temporal Network (IV)

- It is useful to have a TP, called Z/X_0 , whose value is fixed at 0
- Binary constraints involving Z are equivalent to unary constraints:

$$X - Z \leq 7 \iff X \leq 7$$

$$Z - X \leq -3 \iff X \geq 3$$

Motivation example

- John goes to work by car, which takes 30–40 minutes
- Fred goes to work in a carpool, which takes 40–50 minutes
- John left home between 7:10 and 7:20
- Fred arrived at work between 8:00 and 8:10
- John arrived at work after than Fred left home, but not more than 10-20 minutes later than he left

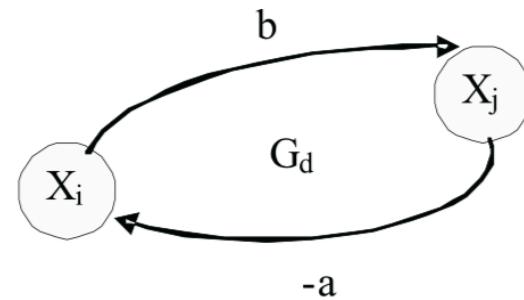
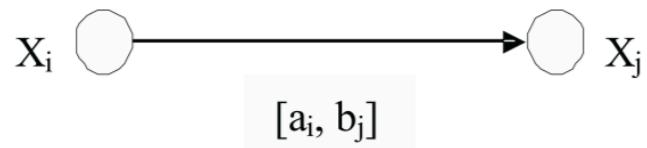
Motivation example: representation

- X_1 John left home
- X_2 John arrived at work
- X_3 Fred left home
- X_4 Fred arrived at work
- We also need a temporal reference point to be able to refer to absolute time points; this is denoted by X_0 will stand for seven o'clock this morning

Example: constraints representation

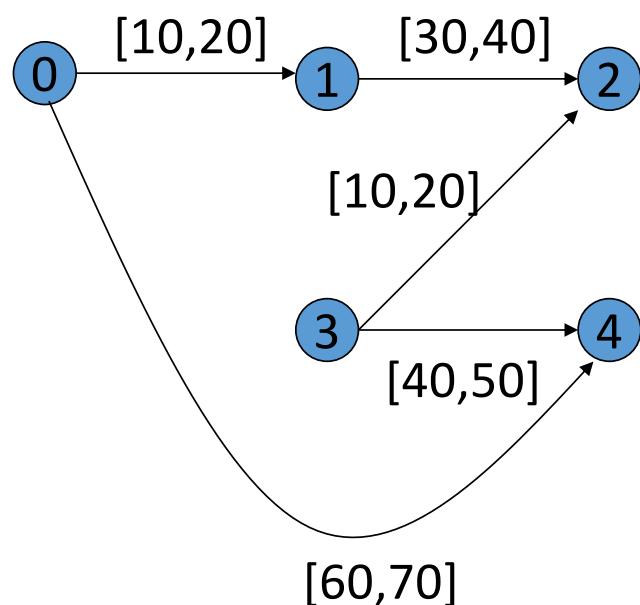
- $T = \{X_0, X_1, X_2, X_3, X_4\}$,
 $X_0/Z = 7\text{am}$
- Time units = minutes
- $C =$
 - $10 \leq X_1 - X_0 \leq 20$ (John leaving between 7:10 and 7:20)
 - $30 \leq X_2 - X_1 \leq 40$ (John duration 30-40mn) $X_2 - X_1 \geq 60$
 - $60 \leq X_4 - X_0 \leq 70$ (Fred arrives between 8:00 and 8:10)
 - $40 \leq X_4 - X_3 \leq 50$ (Fred duration 40-50mn)
 - $10 \leq X_2 - X_3 \leq 20$ (John arrived work about 10-20 mn after Fred left home)

Example: d-graph representation (I)



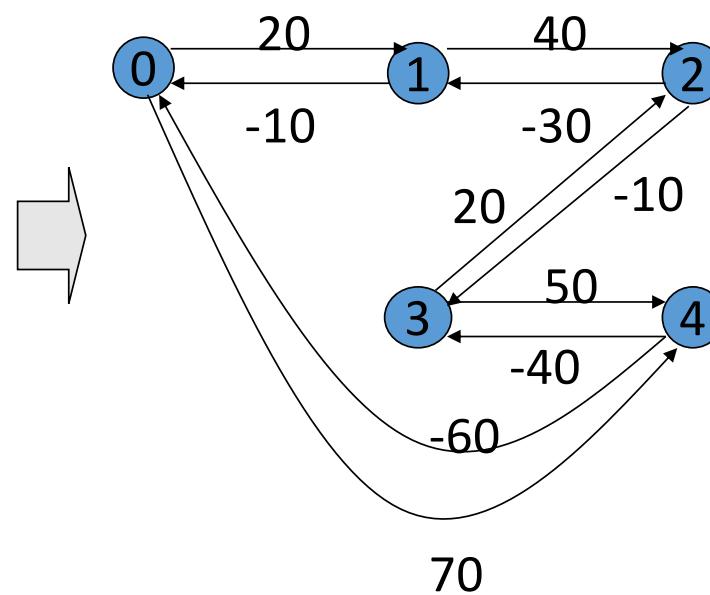
Example: d-graph representation (II)

$$T_{ij} = (a_{ij} \leq X_j - X_i \leq b_{ij})$$



$$X_j - X_i \leq b_{ij}$$

$$X_i - X_j \leq -a_{ij}$$



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Properties of STN (I)

- **Temporal distance** from t_j to t_i in an STN is the length of the shortest path from t_j to t_i in the distance graph G (or negative infinity if no such path exists)
- **Distance Matrix** for an STN $S = (T, C)$ is a matrix D each entry of which equals the temporal distance between the corresponding pair of time-points in T

$$-\mathcal{D}(t_j, t_i) \leq t_j - t_i \leq \mathcal{D}(t_i, t_j)$$

- The distance Matrix for a given STN may be computed from scratch in polynomial time

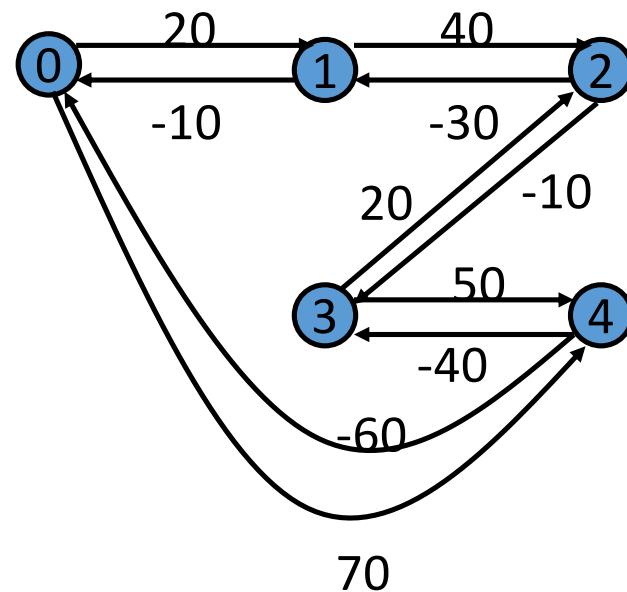
Properties of STN (II)

- An STN S is **consistent** if and only if its corresponding distance graph G has no negative cycles (i.e., if and only if the path length around any loop is non-negative)
- An STN S is **consistent** if and only if all of the diagonal entries in its distance matrix are zero
- Explicit constraints can be combined to form implicit constraints:
 $t_j - t_i \leq 30$ and $t_k - t_j \leq 40 \rightarrow t_k - t_i \leq 70$

Shortest Paths of Gd

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph



STN Minimum Network

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph

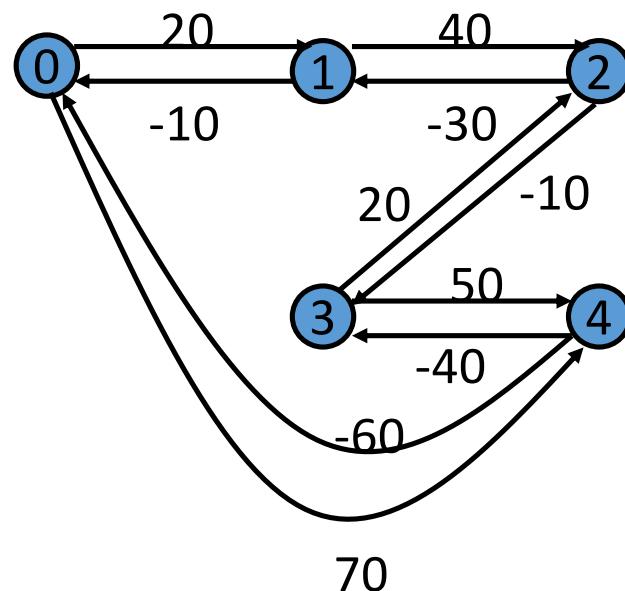
	0	1	2	3	4
0	[0]	[10,20]	[40,50]	[20,30]	[60,70]
1	[-20,-10]	[0]	[30,40]	[10,20]	[50,60]
2	[-50,-40]	[-40,-30]	[0]	[-20,-10]	[20,30]
3	[-30,-20]	[-20,-10]	[10,20]	[0]	[40,50]
4	[-70,-60]	[-60,-50]	[-30,-20]	[-50,-40]	[0]

STN minimum network

Test Consistency: No Negative Cycles

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph

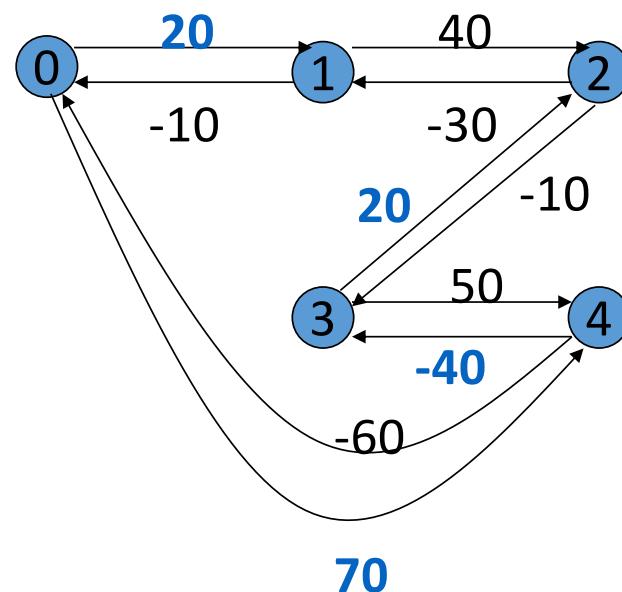


Latest Solution

Node 0 is the reference.

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph

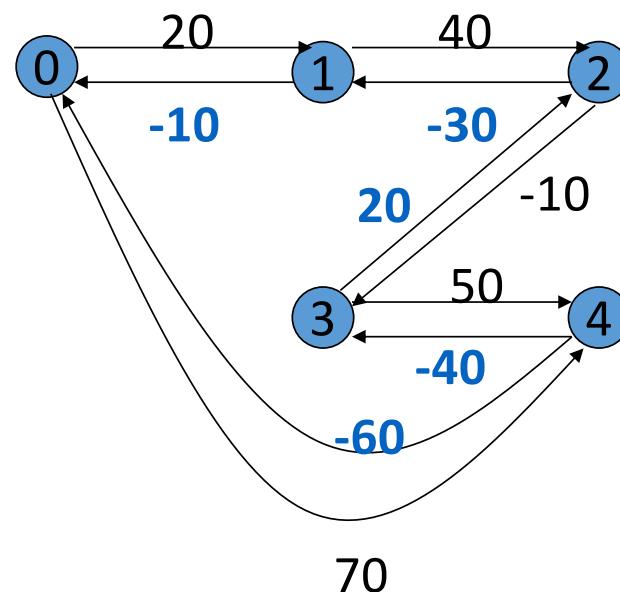


Earliest Solution

Node 0 is the reference.

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph



Feasible Values

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph

- X_1 in $[10, 20]$
- X_2 in $[40, 50]$
- X_3 in $[20, 30]$
- X_4 in $[60, 70]$

Solution by Decomposition

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph

- Select value for 1
→ 15 [10,20]

Solution by Decomposition

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

d-graph

- Select value for 1

→ 15

- Select value for 2,
consistent with 1

→ 45 [40,50], 15+[30,40]

Solution by Decomposition

	0	1	2	3	4	
0	0	20	50	30	70	
1	-10	0	40	20	60	
2	-40	-30	0	-10	30	
3	-20	-10	20	0	50	
4	-60	-50	-20	-40	0	

d-graph

- Select value for 1
→ 15
- Select value for 2,
consistent with 1
→ 45
- Select value for 3,
consistent with 1 & 2
→ 30 [20,30], 15+[10,20],45+[-20,-10]

Solution by Decomposition

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

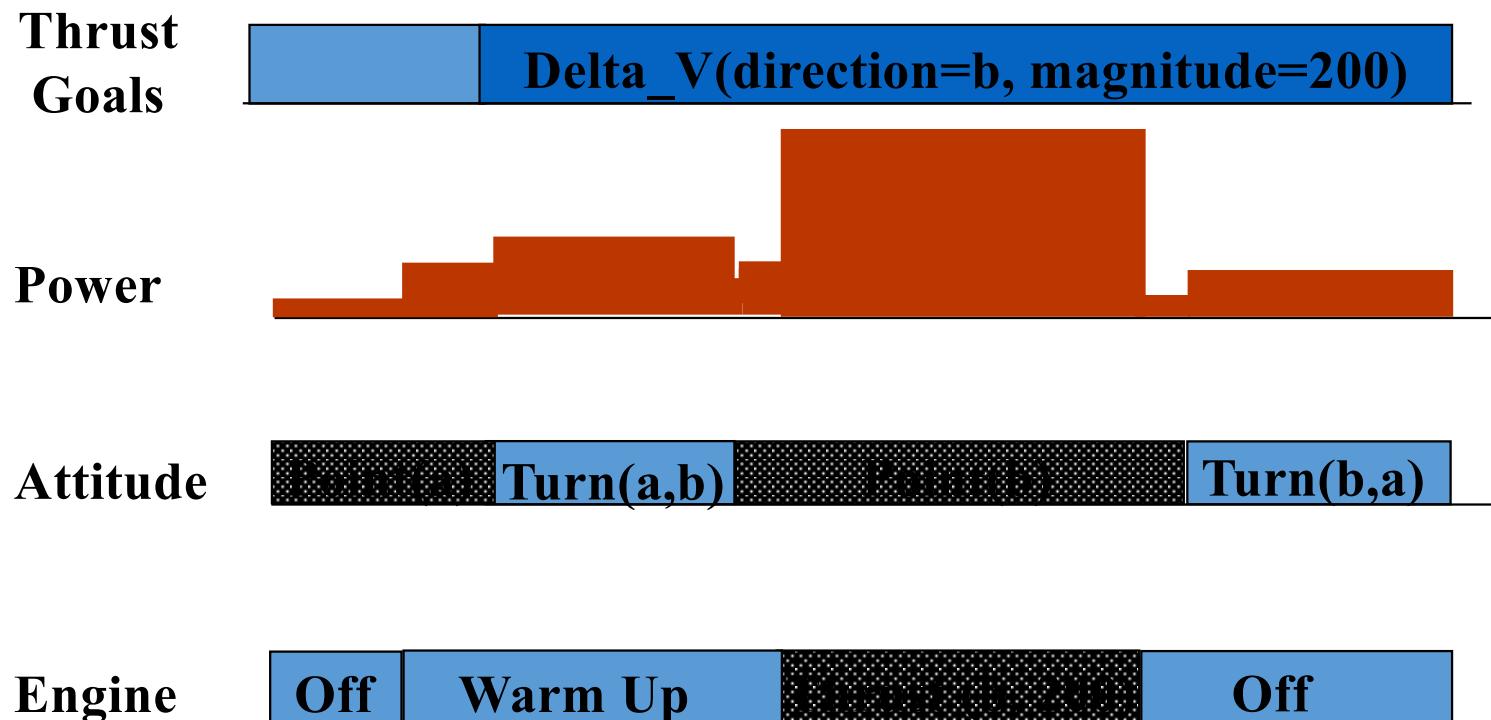
d-graph $\rightarrow O(N^2)$

- Select value for 1
 $\rightarrow 15$
- Select value for 2,
consistent with 1
 $\rightarrow 45$
- Select value for 3,
consistent with 1 & 2
 $\rightarrow 30$
- Select value for 4,
consistent with 1,2 & 3

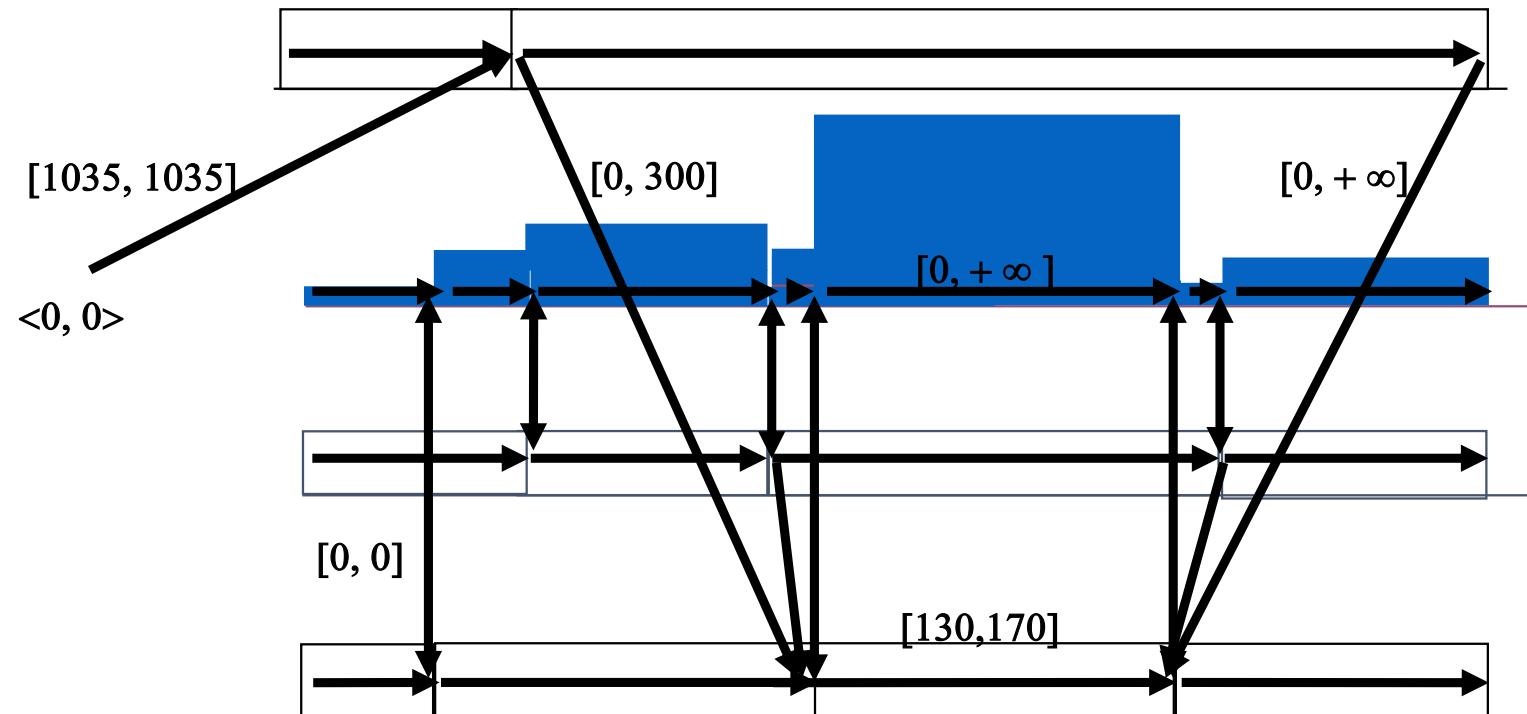
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A Completed Plan Forms an STN



A Completed Plan Forms an STN



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Conclusions

- For time, TCSP → uses STN
- STNs have been used to provide flexible planning and scheduling systems for more than a decade
- Efficient algorithms for checking consistency
- However, STNs cannot represent uncertainty (e.g., actions with uncertain durations) or conditional constraints (e.g., only do X if test result is negative).

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Example

- Goal: Fly from New York to Rome
 - Leave New York after 4p.m., June 8
 - Return to New York before 10p.m., June 18
 - Away from New York no more than 7 days
 - In Rome at least 5 days
 - Return flight lasts no more than 7 hours
- Represent
 - Constraints problem
 - Directed graph
 - Distance matrix