CLL110 Transport Phenomena
Department of Chemical Engineering IIT Delhi
Semester 1 2024-25, B slot

Tutorial 3

Exercise

Following pages and problem number refers to Transport Phenomena by BSL, 2nd Edition.

Reading assignment: Chapter 1, page 11-21, 34-37
Chapter 3: page 75-78
Chapter 2, page 40-56

Problem: Write the shell momentum balance for flow between two parallel plates, top of which is moving at constant velocity "V" while the bottom plate is stationary. The distance between two plates is h.

Problems: 2A1, 2A3 (on Page number 62), Problems screen shots are given below

$\beta = 0$

2A.1 **Thickness of a falling film.** Water at 20°C is flowing down a vertical wall with Re = 10. Calculate **(a)** the flow rate, in gallons per hour per foot of wall width, and **(b)** the film thickness in inches.
Answers: **(a)** 0.727 gal/hr · ft; **(b)** 0.00361 in.

$Re = \frac{4\delta\langle v_z\rangle\rho}{\mu}$ $\quad \frac{Q}{W}? \to a$

2A.3 **Volume flow rate through an annulus.** A horizontal annulus, 27 ft in length, has an inner radius of 0.495 in. and an outer radius of 1.1 in. A 60% aqueous solution of sucrose ($C_{12}H_{22}O_{11}$) is to be pumped through the annulus at 20°C. At this temperature the solution density is 80.3 lb/ft^3 and the viscosity is 136.8 $lb_m/ft \cdot hr$. What is the volume flow rate when the impressed pressure difference is 5.39 psi?
Answer: 0.110 ft^3/s

Problems: 2B1, 2B2, (Page 63), Problems screen shots are given below

2B.1 **Different choice of coordinates for the falling film problem.** Rederive the velocity profile and the average velocity in §2.2, by replacing x by a coordinate $\bar{x}$ measured away from the wall; that is, $\bar{x} = 0$ is the wall surface, and $\bar{x} = \delta$ is the liquid–gas interface. Show that the velocity distribution is then given by

$$v_z = (\rho g \delta^2/\mu)[(\bar{x}/\delta) - \tfrac{1}{2}(\bar{x}/\delta)^2] \cos\beta \qquad (2B.1\text{-}1)$$

and then use this to get the average velocity. Show how one can get Eq. 2B.1-1 from Eq. 2.2-18 by making a change of variable.

2B.2 Alternate procedure for solving flow problems. In this chapter we have used the following procedure: (i) derive an equation for the momentum flux, (ii) integrate this equation, (iii) insert Newton's law to get a first-order differential equation for the velocity, (iv) integrate the latter to get the velocity distribution. Another method is: (i) derive an equation for the momentum flux, (ii) insert Newton's law to get a second-order differential equation for the velocity profile, (iii) integrate the latter to get the velocity distribution. Apply this second method to the falling film problem by substituting Eq. 2.2-14 into Eq. 2.2-10 and continuing as directed until the velocity distribution has been obtained and the integration constants evaluated.

Problems: 2B3 (Page 63), 2B7 (page 64) , Problems screen shots are given below

2B.3 Laminar flow in a narrow slit (see Fig. 2B.3).

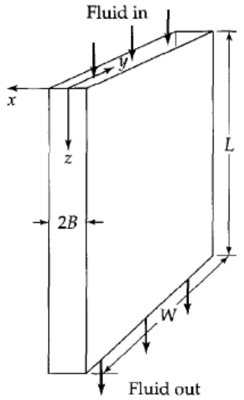


Fig. 2B.3 Flow through a slit, with $B << W << L$.

(a) A Newtonian fluid is in laminar flow in a narrow slit formed by two parallel walls a distance $2B$ apart. It is understood that $B << W$, so that "edge effects" are unimportant. Make a differential momentum balance, and obtain the following expressions for the momentum-flux and velocity distributions:

$$\tau_{xz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right)x \tag{2B.3-1}$$

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L}\left[1 - \left(\frac{x}{B}\right)^2\right] \tag{2B.3-2}$$

In these expressions $\mathcal{P} = p + \rho g h = p - \rho g z$.

(b) What is the ratio of the average velocity to the maximum velocity for this flow?

(c) Obtain the slit analog of the Hagen–Poiseuille equation.

(d) Draw a meaningful sketch to show why the above analysis is inapplicable if $B = W$.

(e) How can the result in (b) be obtained from the results of §2.5?

Answers: **(b)** $\langle v_z \rangle / v_{z,\max} = \frac{2}{3}$

(c) $w = \dfrac{2}{3}\dfrac{(\mathcal{P}_0 - \mathcal{P}_L)B^3 W\rho}{\mu L}$

2B.7 Annular flow with inner cylinder moving axially (see Fig. 2B.7). A cylindrical rod of radius κR moves axially with velocity $v_z = v_0$ along the axis of a cylindrical cavity of radius R as seen in the figure. The pressure at both ends of the cavity is the same, so that the fluid moves through the annular region solely because of the rod motion.

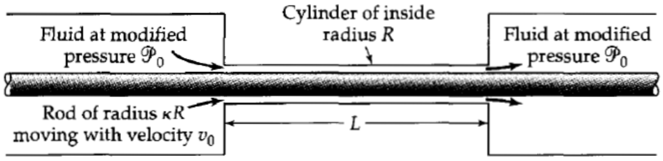


Fig. 2B.7 Annular flow with the inner cylinder moving axially.

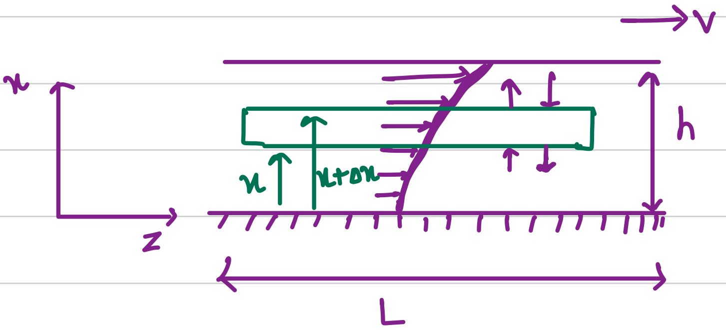
(a) Find the velocity distribution in the narrow annular region.

(b) Find the mass rate of flow through the annular region.

(c) Obtain the viscous force acting on the rod over the length L.

Ans ① :

$\underset{\sim}{v} = v(v_x, v_y, v_z)$



$v_x = 0$
$v_y = 0$ } laminar flow & incompressible flow

$v_z = v_z(x, \cancel{y}^{0}, z, \cancel{t}^{0})$ — Steady state + fully developed

$v_z = v_z(x, z)$

- mass conservation in Control volume :

↳ eq of Continuity : $\underset{\sim}{\nabla} \cdot v = 0$

$$\frac{\partial \cancel{v_x}^{0}}{\partial x} + \frac{\partial \cancel{v_y}^{0}}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \implies \frac{\partial v_z}{\partial z} = 0 \quad \text{(independent of z)}$$

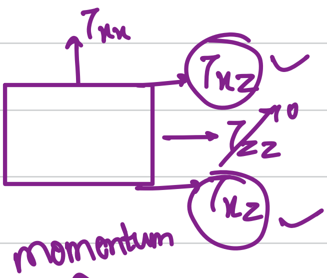
$$\boxed{v_z = v_z(x)}$$

⇒ Momentum Balance : $\cancel{\text{Rate of accumulation of momentum in C.V}}^{0} = \cancel{\text{Rate of momentum due to convection}}^{0} + \Sigma F_i$

Bulk Flow

$$\rho v_z v_z \Delta x W\Big|_{in} - \rho v_z v_z \Delta x W\Big|_{out} = 0$$

Rate of acc. of mom. C.V ⇒ Rate of momentum due to convection + Rate of mom. due to shear force + forces



$\tau_{yz} = 0$

$$\boxed{\tau_{xz} = \mu \frac{dv_z}{dx}}$$

$$\lim_{\Delta x \to 0} \frac{\tau_{xz} \cancel{LW}\Big|_x - \tau_{xz} \cancel{LW}\Big|_{x+\Delta x}}{\cancel{LW} \Delta x} = 0 \implies -\frac{\partial(\tau_{xz})}{\partial x} = 0 \implies \tau_{xz} = -\mu \frac{dv_z}{dx}$$

$$= \mu \frac{\partial^2 v_z}{\partial x^2} = 0$$

force

$$\lim_{\Delta x \to 0} \frac{\Rightarrow -\tau_{xz} wL\big|_{x} + \tau_{xz} wL\big|_{x+\Delta x}}{wL\,\Delta x} = \frac{\partial(\tau_{xz})}{\partial x} = 0$$

$$\Rightarrow \tau_{xz} = \mu \frac{dv_z}{dx}$$

$$\Rightarrow \mu \frac{\partial^2 v_z}{\partial x^2} = 0$$

$$\boxed{\mu \frac{\partial^2 v_z}{\partial x^2} = 0} \Rightarrow \frac{\partial v_z}{\partial x} = C_1$$

$$\Rightarrow v_z = C_1 x + C_2$$

At $x=0 \Rightarrow v_z = 0$ $\quad (C_2 = 0)$

At $x=h \Rightarrow v_z = v$ $\quad (C_1 = v/h)$

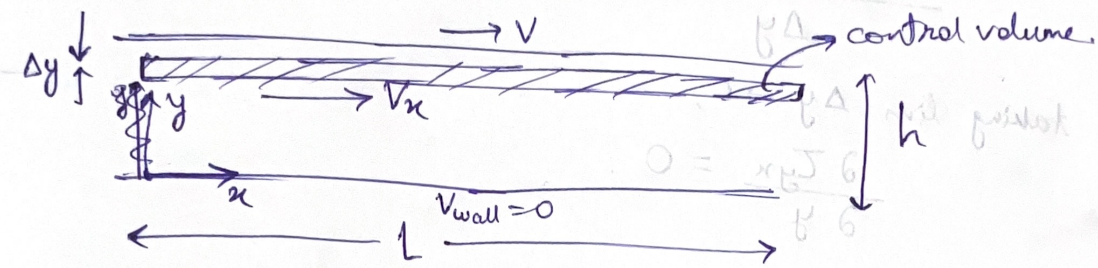
$$\boxed{v_z = \frac{v}{h} x}$$

Ans②:

Sarang N. Mendhe 2023CH70465 Tut-3

Q.1

width = W.



Assumptions : ① steady state flow
② No slip condition
③ Irrotational fluid.
④ Constant ρ
⑤ laminar flow

$V_y = 0$

$V_z = 0$

$V_x = V_x(x, y, z, t)$

Since there is no boundary at z, V_x can't depend on z.

$\therefore V_x = V_x(x, y)$

By eq^n of continuity,

$$\frac{\partial V_x}{\partial x} = 0 \Rightarrow V_x = V_x(y)$$

since velocity is only in x dir^n,
only $\tau_{xx}, \tau_{yx}, \tau_{zx}$ are non zero.

since momentum is transported only in y dir^n,
$\therefore \tau_{yx}$ is only non zero component.
↪ momentum flux

Momentum Shell Balance :

$$\text{Rt. of Acc.} = \cancel{V_x \cdot \rho \Delta y W V_x}\Big|_{x=0} - \cancel{V_x \cdot \rho \Delta y W V_x}\Big|_{x=L}$$

$$+ \tau_{yx} L W\Big|_y - \tau_{yx} L W\Big|_{y+\Delta y}$$

$$\frac{\tau_{yx}|_{y+\Delta y} - \tau_{yx}|_{y}}{\Delta y} = 0$$

taking lim $\Delta y \to 0$,

$$\frac{\partial \tau_{yx}}{\partial y} = 0.$$

$$\tau_{yx} = c_1$$

$$\tau_{yx} = -\mu \frac{dV_x}{dy} \quad \text{(Momentum flux).}$$

$$-\mu \frac{dV_x}{dy} = c_1$$

$$V_x = -\frac{c_1}{\mu} y + c_2$$

at $y = 0$, $V_x = 0$, $\therefore c_2 = 0$

$$\therefore V_x = -\frac{c_1}{\mu} y$$

at $y = h$, $V_x = V$

$$\therefore V = -\frac{c_1}{\mu} h$$

$$c_1 = -\frac{\mu V}{h}.$$

$$\therefore V_x = \frac{Vy}{h}$$

7.1) $Re = \dfrac{4\delta \langle v_z \rangle \rho}{\mu}$ for falling film.

$\dfrac{Q}{W} = ?$ $\qquad Q = \delta W \cdot \langle v_z \rangle$

$\cancel{\dfrac{Q}{W} = \delta \langle v_z \rangle} = \cancel{\dfrac{\mu Re}{4\rho}} = \cancel{\dfrac{10^{-2} \times 10}{4 \times 10^{3}}}$

$\cancel{= \dfrac{\mu Re}{4\rho}} \qquad \cancel{= 0.25 \times 10^{-4}}$

$\cancel{= 2.5 \times 10^{-5}\ cm^2/s}$

$Re = \dfrac{4\, W \cdot \delta \langle v_z \rangle \rho}{W \mu} = \dfrac{4 Q \rho}{W \mu}$

$\dfrac{Q}{W} = \dfrac{\mu Re}{4\rho} = \dfrac{\cancel{\mu}}{\cancel{4\rho}} \times \dfrac{\cancel{4}\, \delta \cancel{\rho}}{\cancel{\mu}} \times \dfrac{\cancel{3}}{\cancel{3}}\, \dfrac{\rho g \delta^2}{3} = \dfrac{\rho g \delta^3}{3}$

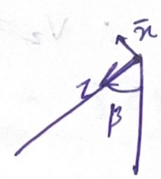
$\therefore \quad \delta = \left(\dfrac{3\mu Re}{4\rho^2 g}\right)^{1/3}$

$\therefore$ a) 0.727 gal/hr·ft.

b) 0.00361

(2B 1) By setting up a shell momentum balance and taking $\lim \Delta\bar{x} \to 0$,

$$\frac{d\tau_{\bar{x}z}}{d\bar{x}} = \rho g \cos\beta$$



But the new variable

x is given as $-\bar{x} + \delta$

$\therefore \bar{x} = \delta - x$

$$\therefore \frac{d\tau_{xz}}{dx} = +\rho g \cos\beta$$

$$\therefore \tau_{xz} = +\rho g \cos\beta\, x + C_1$$

Since there is no shear force at air-liquid interface, at $x = 0$, $\tau_{xz} = 0$

$\boxed{\therefore C_1 = 0}$

$$\tau_{xz} = -\mu \frac{dv_z}{dx}$$

$$\therefore \frac{dv_z}{dx} = -\frac{\rho g \cos\beta}{\mu} x + C_1$$

$$v_z = -\frac{\rho g \cos\beta\, x^2}{2\mu} \cancel{+ c_1 x} + c_2$$

at : $x = \delta$, $v_z = 0$,

$$\therefore v_z = \frac{\rho g \cos\beta}{2\mu} \delta^2 \left(1 - \left(\frac{x}{\delta}\right)^2\right).$$

Q.5 (2B-2)

$$\frac{d\tau_{xz}}{dx} = \rho g \cos\beta$$

put $\tau_{xz} = -\mu \frac{dv_z}{dx}$

$$\therefore \frac{d^2 v_z}{dx^2} = \frac{-\rho g \cos\beta}{\mu}$$

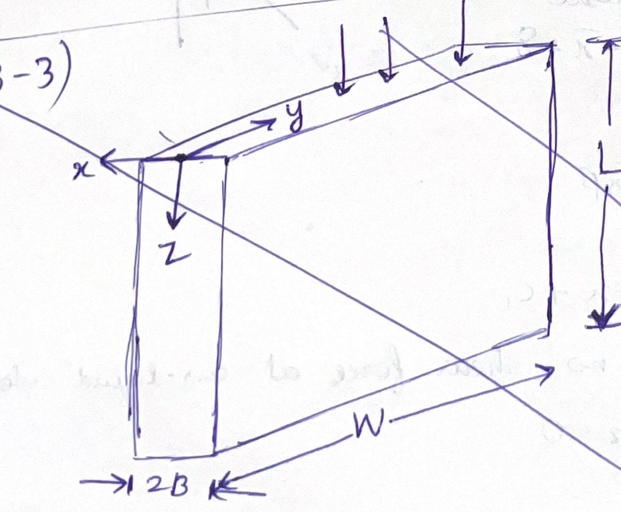
$$v_z = -\frac{\rho g \cos\beta}{2\mu} x^2 + c_1 x + c_2$$

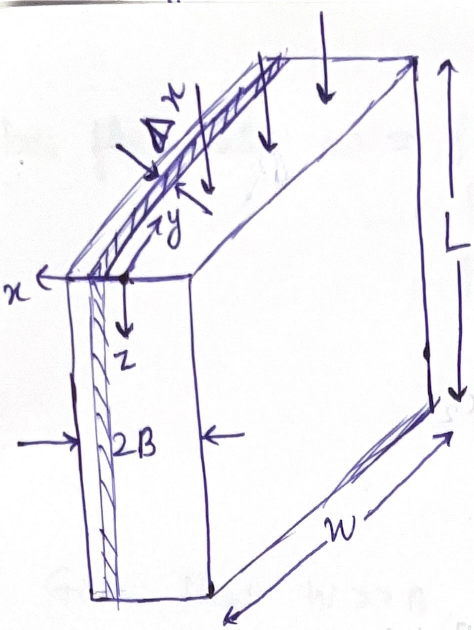
$\frac{dv_z}{dx} = 0$ at $x = 0$ $\Rightarrow$ $c_1 = 0$

$v_z = 0$ at $x = \delta$ $\Rightarrow$ $c_2 = +\frac{\rho g \cos\beta}{2\mu} \delta^2$

$$\therefore v_z = \frac{\rho g \cos\beta\, \delta^2}{2\mu} \left(1 - \left(\frac{x}{\delta}\right)^2\right)$$

(2B-3)





Q (2B-3)

let $v_z = v_z(x, y, z, t)$.

<u>Assumptions:</u>
1. Steady state flow
2. Laminar flow
3. Irrotational fluid
4. No slip condition at $x = \pm B$.
5. Constant ρ.

By eqⁿ of continuity,

$$\frac{\partial v_z}{\partial z} = 0 \Rightarrow v_z = v_z(x, y) \quad [\text{Steady state}]$$

$B << W \Rightarrow$ No shear force by y-plane.

$\therefore \quad v_z = v_z(x)$.

By shell momentum balance,

$$0 = \cancel{\rho L W}\ \cancel{\rho \Delta x W v_z \cdot v_z}\Big|_{z=0} - \cancel{\rho \Delta x W v_z \cdot v_z}\Big|_{z=L}$$

$$+ \cancel{\tau_z}\ \tau_{xz} L W\Big|_x - \tau_{xz} L W\Big|_{x+\Delta x}$$

$$+ (P_0 - P_L) \cdot W \cdot \cancel{2B} \cdot \Delta x$$

$$\therefore \frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_x}{\Delta x} = \frac{P_0 - P_L}{L}$$

lim $\Delta x \to 0$, $\quad \therefore \dfrac{d\tau_{xz}}{dx} = \dfrac{P_0 - P_L}{L}$

$$\tau_{xz} = -\mu \frac{dv_z}{dx}$$

$$\therefore \frac{d^2 v_z}{dx^2} = -\frac{(P_0 - P_L)}{L\mu}$$

$$\therefore v_z = -\frac{(P_0 - P_L)}{2L\mu} x^2 + C_1 x + C_2$$

Q

at $x = \pm B$, $v_z = 0$

$$\therefore 0 = -\frac{(P_0 - P_L)}{2L\mu} B^2 + C_1 B + C_2$$

$$0 = -\frac{(P_0 - P_L)}{2L\mu} B^2 - C_1 B + C_2$$

$$\therefore C_2 = \frac{(P_0 - P_L)}{2L\mu} B^2$$

and $C_1 = 0$

$$\therefore v_z = \frac{(P_0 - P_L)}{2L\mu} B^2 \left(1 - \left(\frac{x}{B}\right)^2\right)$$

$$\therefore \tau_{xz} = -\mu \frac{dv_z}{dx}$$

$$= -\cancel{\mu} \cdot \left(-\frac{\cancel{2}x}{\cancel{B^2}}\right) \frac{P_0 - P_L}{\cancel{2}L\cancel{\mu}} \cancel{B^2}$$

$$\boxed{\tau_{xz} = \frac{(P_0 - P_L)}{L} x}$$

b) $$v_{z,max} = \frac{(P_0 - P_L)}{2L\mu} B^2$$

$$v_{z,avg} = \frac{\int_{-B}^{B} v_z \, dx}{\int_{-B}^{B} dx} = \frac{\frac{P_0 - P_L}{2L\mu} B^2 \int_{-B}^{B} \left(1 - \left(\frac{x}{B}\right)^2\right) dx}{2B}$$

$$= \frac{P_0 - P_L}{4L\mu} B^2 \int_{-1}^{1} (1 - t^2)\, dt \qquad \boxed{t = \frac{x}{B}}$$

$$= \frac{P_0 - P_L}{4L\mu} \times B^2 \times \frac{4}{3} = \frac{P_0 - P_L}{3L\mu} B^2$$

$$\boxed{\therefore \frac{v_{z,max}}{v_{z,avg}} = \frac{2}{3}}$$

Mass flow rate $w = \rho \cdot Q$

$$= \rho \cdot \langle V_z \rangle \cdot A$$

$$= \rho \cdot \frac{(P_0 - P_L)}{3L\mu} B^2 \times 2B \times W$$

$$= \frac{2\rho W (P_0 - P_L) B^3}{3L\mu}$$

d) Given that $W >> B$, the variation of velocity in y was neglected. But when $W = B$, both are comparable and hence ~~there~~ V_z will depend on y also.

e)
$$V_z' = \frac{(P_0 - P_L) b^2}{2\mu' L}\left[\frac{2\mu'}{\mu'+\mu''} + \left(\frac{\mu'-\mu''}{\mu'+\mu''}\right)\left(\frac{x}{b}\right) - \left(\frac{x}{b}\right)^2\right]$$

$$V_z'' = \frac{(P_0 - P_L) b^2}{2\mu'' L}\left[\frac{2\mu''}{\mu'+\mu''} + \left(\frac{\mu'-\mu''}{\mu'+\mu''}\right)\left(\frac{x}{b}\right) - \left(\frac{x}{b}\right)^2\right]$$

put $\mu' = \mu'' = \mu$.

we get $V_z' = V_{z\mu} = \frac{(P_0 - P_L) b^2}{2\mu L}\left[1 - \left(\frac{x}{b}\right)^2\right]$

~~This is same as saying that both~~

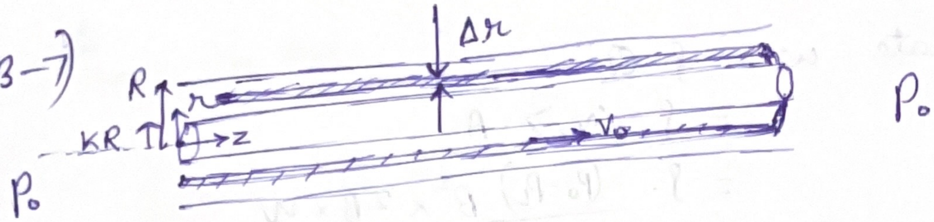
Due to laminar flow, ~~if~~ both liquids where flowing without intermixing. And now $\mu_1 = \mu_2$, ~~$\therefore$ Both~~ $\therefore$ It can be considered as a single fluid.

$\therefore$ The result $\frac{V_{z,avg}}{V_{z,max}} = \frac{2}{3}$ is valid if $\mu_1 = \mu_2$.

$\frac{x}{B}$

2

(2B-7)



Assumptions, ① constant ρ
② Laminar flow
③ Steady state.
④ No slip condition.

let $v_z = v_z(r, \theta, z, t)$.

Due to symmetry and steady state flow,

$$v_z = v_z(r, z)$$

By eq^n of continuity, $\frac{\partial v_z}{\partial z} = 0$.

$$\therefore v_z = v_z(r).$$

$\therefore$ ~~Accu.~~ By shell momentum balance,

$$\text{Rate of accu.} = 0 = \rho v_z \, \cancel{2\pi r \Delta r} \cdot v_z \Big|_{z=0} - \rho v_z \, \cancel{2\pi r \Delta r} \cdot v_z \Big|_{z=L}$$

$$+ \cancel{2\pi} r \cancel{L} \cdot \tau_{rz}\big|_r - \cancel{2\pi} r \cancel{L} \, \tau_{rz}\big|_{r+\Delta r}$$

$$+ 0$$

$$\therefore \lim_{\Delta r \to 0} \frac{r\tau_{rz}|_r - r\tau_{rz}|_{r+\Delta r}}{\Delta r} = 0.$$

$$\therefore \frac{d \, r\tau_{rz}}{dr} = 0$$

put $\tau_{rz} = -\mu \frac{dv_z}{dr}$, due to momentum flux

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = 0.$$

$$\frac{dv_z}{dr} + r\frac{d^2 v_z}{dr^2} = 0.$$

put $\frac{dv_z}{dr} = P$, $\therefore \frac{d^2 v_z}{dr^2} = \frac{dP}{dr}$

$$\therefore P + r\frac{dP}{dr} = 0$$

$$r\frac{dP}{dr} = -P$$

$$\int \frac{dP}{P} = \int -\frac{dr}{r}$$

$$\ln P = -\ln r + c_1$$

$$\therefore Pr = c_1$$

$$r\frac{dV_z}{dr} = c_1$$

$$\int dV_z = \int \frac{c_1}{r}\, dr.$$

$$V_z = c_1 \ln r + c_2$$

at $r = KR$, $V_z = V_0$

at $r = R$, $V_z = 0$.

$$\therefore 0 = c_1 \ln R + c_2$$

$$V_0 = c_1 \ln KR + c_2$$

$$V_0 = c_1 \ln K \Rightarrow c_1 = \frac{V_0}{\ln K}$$

$$\therefore c_2 = -c_1 \ln R = -\cancel{c_1} \ln R \times -\frac{V_0}{\ln K} = \frac{-V_0 \ln R}{\ln K}$$

$$\therefore V_z = \frac{V_0}{\ln K} \ln r + -V_0 \frac{\ln R}{\ln K} = \frac{V_0 \ln\left(\frac{r}{R}\right)}{\ln K}$$

a) $$\boxed{\therefore V_z = \frac{V_0 \ln\left(\frac{r}{R}\right)}{\ln K}}$$

b) $$\rho Q = \iint dQ = \rho \iint V_z \cdot r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_{KR}^{R} \frac{V_0 \ln\left(\frac{r}{R}\right)}{\ln K}\, r\,dr\,d\theta\,\rho = \rho\frac{2\pi \times V_0}{\ln K} \times R^2 \int_K^1 \ln t \cdot t\,dt$$

$$= \frac{2\pi V_0 R^2 \rho}{\ln K}\left[\ln t \cdot \frac{t^2}{2} - \frac{t^2}{4}\right]_K^1$$

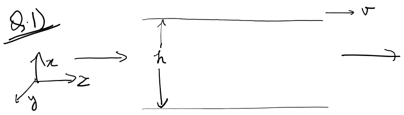
$$= \frac{2\pi V_0 R^2 \rho}{\ln K}\left(-\frac{\ln K \cdot K^2}{2} + \frac{(K^2-1)}{4}\right)$$

c) $$F = \cancel{\int_0^{2\pi}\int_0^{L}}\ 2\pi(KR) \cdot L \cdot \tau_{rz}\Big|_{r=KR}$$

$$= 2\pi KRL \times \mu \frac{dV_z}{dr}\Big|_{r=KR} = 2\pi \cancel{KR} L \mu \frac{V_0}{\ln K} \times \frac{1}{\cancel{KR}}$$

$$= \frac{2\pi L \mu V_0}{\ln K}$$

Q.1)



Step-(1) Assumptions – Steady State
Laminar Flow
Boundary Condition
Incompressible Fluid

(2) $V \longrightarrow V(x, \cancel{y}, z, \cancel{t})$ — Steady State

Laminar flow

$V(x,z)$

Now By Conservation of Mass & Incompressible flow

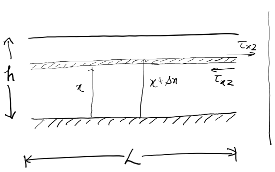
$\underline{\nabla} \cdot \underline{v} = 0 \Rightarrow \cancel{\frac{\partial v_x}{\partial x}} + \cancel{\frac{\partial v_y}{\partial y}} + \frac{\partial v_z}{\partial z} = 0$

$$\frac{\partial v_z}{\partial z} = 0$$

So v_z does not depend on z.

$V = V_z(x)$

(3) Now Taking the control volume for analysing the flow



Now for analysing the flow we see the the substantial derivative of the momentum of the C.V fluid.

$$\frac{DP}{Dt} = \frac{\partial P}{\partial t} + \underbrace{v \cdot (\nabla P)}_{\text{Rate of change of momentum due to convection}}$$

$\frac{\partial P}{\partial t} \to \sum F_i$ → Body Forces, Surface Forces

Surface Forces → Shear Force

Momentum Flux $\Longleftrightarrow$ Shear Force

Since it's a steady state so overall there is no change of velocity with respect to time so $\frac{DP}{\partial t} = 0$

Now accumulation of momentum due to convection = momentum in the C.V − momentum out of C.V

$= (\rho v_z W \cancel{\Delta x}) v_z \Big|_{x+\Delta x} - (\rho v_z \cancel{W \Delta x}) v_z \Big|_x$

$= 0$

Now there is one force acting on the C.V is shear force.

$$\sum F_i = \tau_{xz} WL\Big|_{x+\Delta x} - \tau_{xz} WL\Big|_x$$

So $0 = 0 + \tau_{xz} WL\big|_{x+\Delta x} - \tau_{xz} WL\big|_x$

$$\frac{0}{WL\Delta x} = \frac{0}{WL\Delta x} + \frac{\tau_{xz}\big|_{x+\Delta x} - \tau_{xz}\big|_x}{\Delta x}$$

$\frac{\partial(\tau_{xz})}{\partial x} = 0$, now $\tau_{xz} = \mu \frac{dv_z}{\partial x}$ → depends on x only.

So τ_{xz} also depends on x only.

$$\frac{\partial \tau_{xz}}{\partial x} = \frac{d\tau_{xz}}{\partial x} = 0$$

$$\tau_{xz} = C_1$$

$$\mu \frac{dv_z}{\partial x} = C_1$$

$$\mu v_z = C_1 x + C_2$$

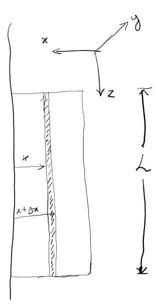
$\boxed{x = 0}$, $\boxed{C_2 = 0}$.

$\mu v = C_1 h$.

$$\boxed{C_1 = \frac{\mu v}{h}}$$

$$\boxed{v_z = \frac{\mu v x}{\mu h} = \frac{v x}{h}}$$

Q2)



① Assumptions – Laminar Flow
Incompressible Flow
Steady State
Boundary Condition

② $V \equiv v(x, \overset{0}{y}, z, \overset{0}{t})$

Let's say $v = v(x, z)$

Since the fluid is incompressible so

$$\nabla \cdot v = 0$$

$$\cancel{\frac{\partial}{\partial x} v_x} + \cancel{\frac{\partial}{\partial y} v_y} + \frac{\partial v_z}{\partial z} = 0, \quad v_x = v_y = 0$$

$$\boxed{\frac{\partial}{\partial z} v_z = 0}$$

So v does not depends on z.

③ Now take C.V as a layer of the film.

$$\text{Now } \frac{DP}{Dt} = \frac{\partial P}{\partial t} + \underline{v} \cdot (\underline{\nabla} \cdot \underline{P}), \quad \underline{P} \rightarrow \text{momentum of Control volume}$$

Since steady state is there so momentum does not change w.r.t time.

$$\frac{DP}{Dt} = 0$$

Momentum accumulation due to convection = Momentum in to the C.V – Momentum out of the C.V

$$= (\rho v_z)\cancel{v_z}\, \Delta x\, W \Big|_{z=0} - (\rho v_z)\cancel{v_z}\, \Delta x\, W \Big|_{z=L}$$

$$= 0$$

$$0 = 0 + \frac{\partial P}{\partial t} \Rightarrow 0 = 0 + \sum F_i$$

body Force   Surface Force

Here two forces are acting on the C.V, gravity and shear forces.

$$\text{gravity force} = \rho W L \Delta x g$$

$$\text{Shear force} = W L \tau_{xz}\big|_x - W L \tau_{xz}\big|_{x+\Delta x}$$

$$0 = 0 + \rho g L \Delta x W + W L \tau_{xz}\big|_x - W L \tau_{xz}\big|_{x+\Delta x}$$

$$0 = \rho g + \lim_{\Delta x \to 0} \frac{\tau_{xz}|_x - \tau_{xz}|_{x+\Delta x}}{\Delta x}$$

$$-\rho g = \frac{\partial \tau_{xz}}{\partial x}, \quad \tau_{xz} = \mu \frac{d v_z}{d x}$$

→ depends only on x.

→ depends only on x.

$$-\rho g = \frac{d \tau_{xz}}{d x} \qquad \frac{\partial \tau_{xz}}{\partial x} = \frac{d \tau_{xz}}{d x}$$

$$-\rho g x + C_1 = \tau_{xz}$$

$$x = 0, \; \tau_{xz} = 0$$

$$\boxed{C_1 = 0}. \quad \mu \frac{d v_z}{d x} = -\rho g x$$

$$\mu v_z = -\frac{\rho g x^2}{2} + C_2$$

$$\boxed{x = \delta, \; v_z = 0}$$

$$\boxed{C_2 = \frac{\rho g \delta^2}{2}}$$

$$\boxed{v_z = \frac{\rho g (\delta^2 - x^2)}{2\mu}}$$

$$v_z = \frac{\rho g \delta^2}{2\mu}\left[1 - \frac{x^2}{\delta^2}\right]$$

$$Q = \int dQ = \int v\, dA = \int v_{max}\left(1 - \frac{x^2}{\delta^2}\right) W\, dx$$

m^3/s

$$= v_{max} W \left[x - \frac{x^3}{3\delta^2}\right]_0^{\delta}$$

$$Q = \rho V_{avg} D$$

m^3/s

$$R_e = \frac{\rho V_{avg} D}{\mu}$$

$$10 = \frac{V_{avg} D}{(\mu/\rho)}$$

$$D = \frac{4A}{P} = \frac{4 \times W\delta}{W} = 4\delta$$

$$10 = \frac{4 V_{avg} \delta}{(\mu/\rho)}$$

$$= V_{max} W \left[x - \frac{x^3}{3\delta^2}\right]_0^\delta$$

$$Q = \frac{2 V_{max} W \delta}{3}$$

$$V_{avg} = \frac{Q}{W\delta} = \frac{2 V_{max}}{3}$$

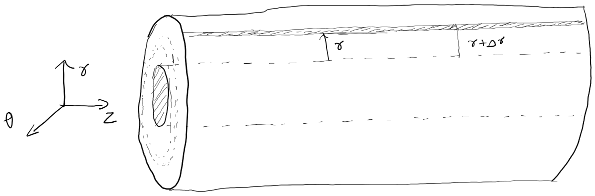
$$Q = V_{avg} W\delta$$

$$\frac{Q}{W} = V_{avg}\delta = 2.5 \frac{\mu}{\rho}$$

$$\frac{\rho g \delta^2}{3\mu} \times \delta = \_\_\_\_$$

1m — 3.28

Q.3)

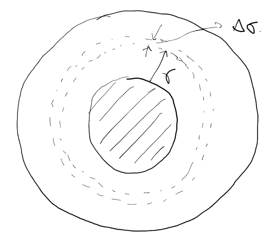


Here assumptions are – Laminar Flow, Steady Flow, Incompressible Flow, Boundary Conditi

$V \equiv (V(x))$ as $V_x = V_y = 0$

(Mass Conservation) $\nabla \cdot v = 0 = \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} + \frac{\partial v_y}{\partial y}$

$$\boxed{\frac{\partial v_z}{\partial z} = 0}$$



Here the rate of change of momentum –

$$\frac{DP}{Dt} = \frac{\partial P}{\partial t} + \underline{v} \cdot (\underline{\nabla}\, \underline{P})$$

Since the flow is steady so no change w.r.t is 0.

$$\frac{DP}{Dt} = 0$$

$$\frac{\partial P}{\partial t} = \sum F_i$$ → body force; → surface force → shear force

$\underline{v} \cdot (\underline{\nabla}\, \underline{P})$ = accumulation of momentum due to convection = Rate of change of momentum $|_{in}$ − Rate of change of momentum $|_{out}$

$$= \left(\rho\, 2\pi r dr\, \cancel{v_z}\right) v_z \Big|_{z=0} - \left(\rho\, 2\pi r dr\, \cancel{v_z}\right) v_z \Big|_{z=L}$$

$$= 0$$

$$\sum F_i = \tau_{rz} 2\pi r L \Big|_r - \tau_{rz} 2\pi r L \Big|_{r+\Delta r} + P_0 2\pi r \Delta r - P_L 2\pi r \Delta r$$

$$0 = 0 + \sum F_i = \tau_{rz} 2\pi r L \Big|_r - \tau_{rz} 2\pi r L \Big|_{r+\Delta r} + P_0 2\pi r \Delta r - P_L 2\pi r \Delta r$$

$$0 = \frac{\tau_{rz} r|_r - \tau_{rz} r|_{r+\Delta r}}{r \Delta r} + \frac{P_0 - P_L}{L}$$

$$\frac{\partial (r \tau_{rz})}{\partial r} = -\frac{(P_0 - P_L) r}{L}$$

$\tau_{rz} = \mu \frac{\partial V_z}{\partial r}$ → depends on r

$$\frac{d(r\tau_{rz})}{dr} = \frac{-(P_0 - P_L) r}{L}$$

$$\tau_{rz} = \frac{-(P_0 - P_L) r}{2L} + \frac{C_1}{r}$$

$$\mu \frac{d v_z}{\partial r} = \frac{-(P_0-P_L)\, r}{2L} + \frac{C_1}{r}$$

$$v_z = -\frac{(P_0-P_L)\, r^2}{4\mu L} + \frac{C_1}{\mu}\ln r + C_2 .$$

$$v_z\Big|_{R=R_1} = 0 = -\frac{(P_0-P_L)R_1^2}{4\mu L} + \frac{C_1 \ln R_1}{\mu} + C_2$$

$$v_z\Big|_{R=R_2} = 0 = -\frac{(P_0-P_L)R_2^2}{4\mu L} + \frac{C_1 \ln R_2}{\mu} + C_2 .$$

$$0 = \frac{(P_0-P_L)(R_2^2-R_1^2)}{4\mu L} + \frac{C_1}{\mu}\ln\left(R_1/R_2\right)$$

$$C_1 = \frac{(P_0-P_L)(R_2^2-R_1^2)}{4\, L\ln\left(R_2/R_1\right)}$$

$$C_2 = \frac{(P_0-P_L)R_1^2}{4\mu L} - C_1\frac{\ln R_1}{\mu} .$$

$$= \frac{(P_0-P_L)R_1^2}{4\mu L} - \frac{\ln R_1}{\mu}\left[\frac{(P_0-P_L)(R_2^2-R_1^2)}{4\, L\ln(R_2)-\ln(R_1)}\right]$$

$$= \left(\frac{P_0-P_L}{4\mu L}\right)\left[R_1^2 - \frac{\ln R_1\,(R_2^2-R_1^2)}{\ln(R_2)-\ln(R_1)}\right]$$

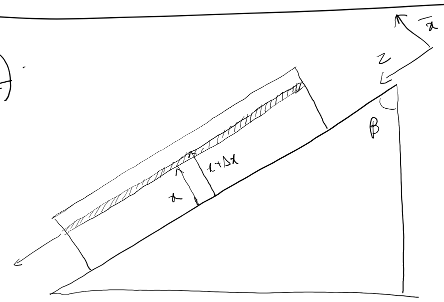
$$= \frac{(P_0-P_L)}{4\mu L}\left[\frac{R_1^2\ln R_2 - \cancel{R_1^2\ln R_1} - \ln R_1 R_2^2 + \cancel{\ln R_1 R_1^2}}{\ln R_2 - \ln R_1}\right]$$

$$C_2 = \frac{P_0-P_L}{4\mu L}\left[\frac{R_1^2\ln R_2 - R_2^2\ln R_1}{\ln R_2 - \ln R_1}\right]$$

$$v_z = -\frac{(P_0-P_L)\, r^2}{4\mu L} + \frac{\ln r}{\mu}\left[\frac{(P_0-P_L)(R_2^2-R_1^2)}{4L\ln(R_2/R_1)}\right] + \frac{P_0-P_L}{4\mu L}\left[\frac{R_1^2\ln R_2 - R_2^2\ln R_1}{\ln R_2-\ln R_1}\right]$$

$$Q = \int v_z\, dA = \int v_z\, 2\pi r\, dr$$

Q-)



$$0 = 0 + \rho W L \Delta x\, g\cos\beta + \tau_{xz} WL\Big|_{x+\Delta x} - \tau_{xz} WL\Big|_{x}$$

$$\rho g\cos\beta = \frac{\tau_{\bar{x}z}\big|_{\bar x} - \tau_{\bar x z}\big|_{\bar x+\Delta\bar x}}{\Delta x} .$$

$$\rho g\cos\beta = -\frac{\partial}{\partial x}(\tau_{xz})$$

$$-\tau_{\bar x z} = +\rho g\cos\beta\,\bar x + C_1$$

$$\bar x = \delta, \quad \tau_{\bar x z} = 0$$

$$\boxed{C_1 = -\rho g\cos\beta\,\delta}$$

$$\longrightarrow \quad -\mu\frac{\partial v_z}{\partial \bar x} = \rho g\cos\beta\,(\bar x - \delta)$$

$$-v_z = \rho g\cos\beta\left[\bar x^2 - \delta\bar x\right]$$

$\partial \bar{x}$

$$V_z = \frac{\rho g \cos\beta}{\mu}\left[\frac{\bar{x}^2}{2} - \delta \bar{x}\right]$$

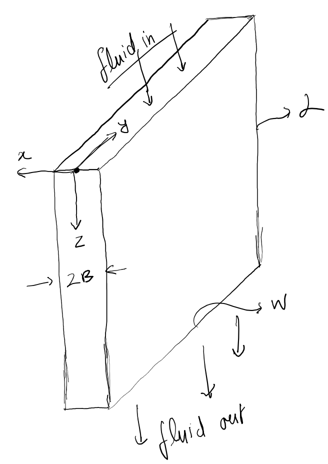
$$\boxed{V_z = \frac{\rho g \cos\beta}{\mu}\left[\frac{\bar{x}}{\delta} - \frac{(\bar{x})^2}{2\delta^2}\right]}$$

$$V_{avg} = \frac{\int v\, dA}{\int dA} = \frac{\int_0^\delta \frac{\rho g \cos\beta}{\mu}\left(\frac{\bar{x}}{\delta} - \frac{\bar{x}^2}{2\delta^2}\right) \cancel{W} d\bar{x}}{\int_0^\delta \cancel{W} d\bar{x}} = \frac{\frac{\rho g \cos\beta}{\mu}\left[\frac{\delta}{2} - \frac{\delta}{6}\right]}{\delta}$$

$$\boxed{V_{avg} = \frac{\rho g \cos\beta}{3\mu}}$$

$$V_z = \frac{\rho g \cos\beta}{2\mu}\left[1 - \frac{x^2}{\delta^2}\right]$$

Q).



a) $O = O + \tau_{xz} WL|_x - \tau_{xz} WL|_{x+\Delta x} + (P_0 - P_L)\Delta x W + \rho g \Delta x WL$

$$O = \tau_{xz} WL|_x - \tau_{xz} WL|_{x+\Delta x} + (P_0 - P_L)\Delta x W + \rho g \Delta x WL$$

$$= -\frac{\partial \tau_{xz}}{\partial x} + \frac{P_0 - P_L}{L} + \rho g$$

$$= -\frac{\partial \tau_{xz}}{\partial x} + \frac{P_0 - P_L + \rho g L}{L}$$

$$\boxed{\frac{\partial \tau_{xz}}{\partial x} = \frac{P_0 - P_L}{L}}$$

$$\tau_{xz} = \left(\frac{P_0 - P_L}{L}\right)x + C, \quad x = 0, \ \tau_{xz} = 0$$

$$V_z = -\left(\frac{P_0 - P_L}{2\mu L}\right)x^2 + C_1$$

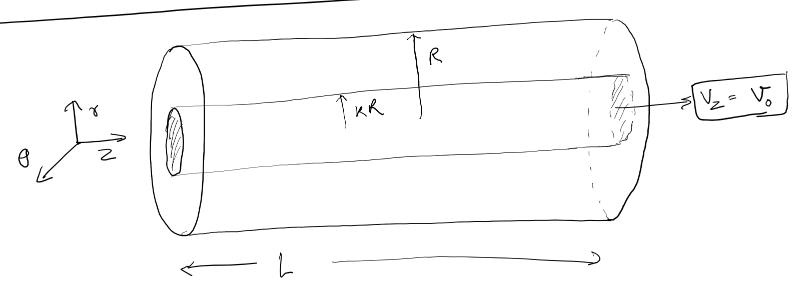
$$\boxed{V_z = \left(\frac{P_0 - P_L}{2\mu L}\right)B^2\left[1 - \frac{x^2}{B}\right]}$$

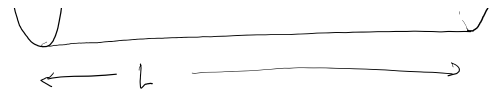
b) $V_{avg} = \dfrac{V_{max}\left[x - \frac{x^3}{3B^2}\right]_0^B}{B} = \frac{2}{3} V_{max}$.

$$\boxed{\frac{V_{avg}}{V_{max}} = \frac{2}{3}}$$

c). $Q = V_{avg} A = \dfrac{(P_0 - P_L)B^2}{3\mu L}(W\, 2B) = \dfrac{2}{3}\dfrac{(P_0 - P_L)B^3 W}{\mu L}$

Q.7)





$$0 = 0 + \tau_{rz}\, 2\pi r L\big|_r - \tau_{rz}\, 2\pi r L\big|_{r+\Delta r} + 0$$

$$0 = \frac{d}{dr}(\tau_{rz} r)$$

$$\tau_{rz} r = C_1$$

$$\mu \frac{dv_z}{dr} = \frac{C_1}{r}$$

$$\mu v_z = -C_1 \ln r + C_2$$

$$\mu V_0 = C_1 \ln KR + C_2$$

$$0 = C_1 \ln R + C_2$$

$$C_1 = \frac{\mu V_0}{\ln K} \Rightarrow C_2 = -\frac{\mu V_0 \ln R}{\ln K}$$

$$V_0 = \frac{V_0}{\ln K}\ln r - \frac{V_0 \ln R}{\ln K}$$

$$\boxed{V_0 = \frac{V_0}{\ln K} \ln r/R}$$

$$\rho Q = \rho \times \int v \cdot 2\pi r\, dr$$

$$= \rho \int \frac{V_0}{\ln K} \ln r/R \cdot 2\pi r\, dr$$

$$\tau_{rz} = \frac{\mu V_0}{\ln K\, r}$$

$$\frac{\mu_0 V_0}{\ln K\, \cancel{KR}} \times 2\pi \cancel{KR} L$$

$$\frac{2\pi L \mu_0 V_0}{\ln K}$$ → force on the length L rod