CLL:113-Major (100 Marks)

How to calculate parameter P,? My first name is JAYATI and my Roll Number is say: 20XXCH70278. Number for J=10 (10th letter of English Alphabet). So, P=10+7+8=25=2+5=7. Calculate the P specific to you.

Q1. If a microbial population is growing at the rate of $\frac{dN}{dt} = 3t^2$
Where the parameters have been non-dimensionalized. What is the population at about the end of time t=0.5?

Parameter P	1st Method step size h=0.1	2nd Method step size h=0.1	3rd Method Adaptive step size method With initial step size h=0.1
0	Heun's	Richardson's	$\varepsilon_{tol} = 0.01$
1	Heun's	Richardson's	$\varepsilon_{tol} = 0.001$
2	Heun's	Richardson's	$\varepsilon_{tol} = 0.01$
3	Midpoint	Richardson's	$\varepsilon_{tol} = 0.001$
4	Midpoint	Richardson's	$\varepsilon_{tol} = 0.01$
5	Midpoint	Richardson's	$\varepsilon_{tol} = 0.001$
6	Ralston	Richardson's	$\varepsilon_{tol} = 0.01$
7	Ralston	Richardson's	$\varepsilon_{tol} = 0.001$
8	Ralston	Richardson's	$\varepsilon_{tol} = 0.01$
9	Midpoint	Richardson's	$\varepsilon_{tol} = 0.001$

Compare your different results (through graphs and error analysis), with the obtained analytical values and explain your results. **25 Marks**

Q2. Consider the problem $\frac{dY(x)}{dx} = -\lambda Y(x) + (1+\lambda)\cos(x) - (1-\lambda)\sin(x)$ with Y (0) = 1.

The true solution is Y (x) = sin(x) + cos(x). Obtain the distribution Y(x) numerically from x=0 to π, with $h = \pi / 4$

Parameter P	Use Method	1st Parameter	2nd Parameter
0,9	Euler Explicit	$\lambda = 2/\pi$	$\lambda = 8/\pi$
1,5	Euler Implicit	$\lambda = 4/\pi$	$\lambda = 8/\pi$
2,6	Euler Explicit	$\lambda = 16/\pi$	$\lambda = 1/\pi$
3,7	Euler Explicit	$\lambda = 22/\pi$	$\lambda = 4/\pi$
4,8	Crank Nicholson	$\lambda = 8/\pi$	$\lambda = 2/\pi$

Does your numerical methods replicate the analytical results? Explain why or why not? How do they compare with each other? **20 Marks**

Q3. The temperature across a rod undergoing convective or radiative heat transfer or both are given by:
$\frac{d^2T(x)}{dx^2} = -0.01*(T_a - T) - 1\times10^{-9}\times R\times(T_a^4 - T^4)$, With the two boundary conditions: @x=0 T=T1 @x=L=10 T=T2. For R=0, the analytical solution is $T_a - T = C_1\exp(\sqrt{0.01}x) + C_2\exp(-\sqrt{0.01}x)$, where
$C_2 = [(T_a - T_1)\exp(\sqrt{0.01}L) - (T_a - T_2)] / [\exp(\sqrt{0.01}L) - \exp(-\sqrt{0.01}L)], C_1 + C_2 = (T_a - T_1)$. Find the temperature distribution numerically.

Parameter P	Use Method	Parameters
0,9,2,6	Shooting Method	T1=300,T2=400,Ta=200, R=0,L=10 Start off with discarding 2nd BC and using dT/dx=0 @x=0

1,4,5	Shooting Method	T1=300,T2=400,Ta=200, **R=1,L=10** Start off with discarding 2nd BC and using dT/dx=0 @x=0
3,7,8	Finite Difference Method, Central Difference Scheme	T1=300,T2=400,Ta=200, **R=0,L=10**

Explain your results, (through graphs and error analysis). **25 Marks**

Q4. 1.Device a numerical difference scheme through method of undetermined coefficients for the following derivative. Also find the optimal Δx below which the error increases as Δx decreases.
2.For the functions given to you, show numerically that the optimal Δx you predicted is correct for $\varepsilon_{precision} = 2\times10^{-16}$.

Parameter P	**Derivative**	**Number of unknown coefficients (n)**	**Function**
0,9	1	n=3, Forward Finite Difference	$f(x) = e^x$
1	2	n=4, Forward Finite Difference	$f(x) = e^x$
2	3	n=4, Forward Finite Difference	$f(x) = e^x$
3,8	1	n=5, Central Finite Difference	$f(x) = \sin(x)$
4,6	2	n=5, Central Finite Difference	$f(x) = \sin(x)$
5,7	3	n=5, Central Finite Difference	$f(x) = \sin(x)$

15 Marks

Q5. Numerically integrate the following functions with the schemes given below.
1. Show how the error decreases with smaller domains and repeated use(n). Show for n=1,2,3,4.
2. Compare the actual error with the estimated error.

Parameter P	**Function**	**Method**
0,9	$\int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\, dx$	Trapezoidal Method
1	$\int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\, dx$	1/3rd Simpsons Rule
2	$\int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\, dx$	3/8 Simpsons Rule
3	$\int_{-3}^{5} (4x - 3)^3\, dx$	Trapezoidal Method
4	$\int_{-3}^{5} (4x - 3)^3\, dx$	1/3rd Simpsons Rule
5	$\int_{-3}^{5} (4x - 3)^3\, dx$	3/8 Simpsons Rule
6	$\int_{1}^{2} (x + 1/x)^2\, dx$	Trapezoidal Method
7	$\int_{1}^{2} (x + 1/x)^2\, dx$	1/3rd Simpsons Rule
8	$\int_{1}^{2} (x + 1/x)^2\, dx$	3/8 Simpsons Rule

15 Marks