CLL113 Major Exam, Total Marks=100,
Time 2 hours
(9/1/21)

Q1 Marks:15 done

With the help of undetermined coefficients find the finite difference formula for P^{th} order derivatives involving 5 Q points.
Provide an estimate of the optimal stepsize required for obtaining minimum error for this formula derived.

For Roll Number ending in 0: P=1, Q=backward

For Roll Number ending in 1: P=2, Q=backward

For Roll Number ending in 2: P=3, Q=backward

For Roll Number ending in 3: P=1, Q=forward

For Roll Number ending in 4: P=2, Q=forward

For Roll Number ending in 5: P=3, Q=forward

For Roll Number ending in 6-7: P=4, Q=backward

For Roll Number ending in 8-9: P=4, Q=forward

Q2 Marks:20

A reaction diffusion equation for a reactant A inside a catalytic slab can be cast in the following form

$$\frac{d^2C}{dx^2} - C = 0$$

$$C(@x=0)=M, \frac{dC}{dx}(@x = 1)=P$$

The analytical solution for the concentration is given by

$$C(x) = \frac{(M\exp(-1) + P)exp(x) + (M\exp(1) - P)\ \exp(-x)}{exp(1) + \exp\,(-1)}$$

A. Solve the concentration profile numerically by finite difference method for this ODE-BVP using central difference scheme of discretization for two equispaced interior points.

B. A. Solve the same problem as in A but with one sided difference formula for boundary conditions with same order of accuracy as done for interior points.

C. Compare the errors at each point in the domain, and comment on what type of boundary discretization lead to better results

For Roll Number ending in 0&9: M=1, P=0

For Roll Number ending in 1&8: M=2, P=0

For Roll Number ending in 2&7: M=1, P=1

For Roll Number ending in 3&6: M=2, P=2

For Roll Number ending in 4&5: M=1, P=2

Q3 Marks:25

For the same problem you solved in Q2, solve for the concentration profile by shooting method, with the help of RK2 mid point method, to find the trajectory of C.

Use h=1, tolerance=0.1

Q4 Marks:20

For the decay equation

$$\frac{dy}{dt} + \lambda y = 0$$

$$y(@t=0)=P$$

Solve the trajectory of y upto t=5 s by

A. Euler Explicit Method by taking a step-size of $h=(M/\lambda)$ s

i. M=0.5

ii. M=1.25

iii. M=2.5

B. Crank Nicholson method taking the same step sizes.

C. Explain your findings for both A and B.

MARKS (7+7+6)

For Roll Number ending in 0&5: λ=1, P=1

For Roll Number ending in 1&7: λ=2, P=2

For Roll Number ending in 2&3: λ=3, P=3

For Roll Number ending in 9&6: λ=4, P=4

For Roll Number ending in 4&8: λ=5, P=5

Q5 Marks:10 done

Judicially generate required number of equally spaced points from the function f (x) = ln(P*x) and fit a cubic polynomial using
(a) Lagrangian Interpolation
and (b) A Newton's Divided Difference Formula
to obtain f(2).

For Roll Number ending in 0&6 chose : Δx=0.6, P=1

For Roll Number ending in 1&8 chose: Δx=0.7, P=2

For Roll Number ending in 2&5 chose: Δx=0.3, P=3

For Roll Number ending in 9&3 chose: Δx=0.4, P=4

For Roll Number ending in 4&7 chose: Δx=0.5, P=5

Q6 Marks:10

For Roll Numbers ending in 0-4

Show that if $f_3(x)$ represents a 3^{rd} order **Lagrange polynomial** passing through equidistant points (x_0, x_1, x_2, x_3) or $\left(a, \frac{b-a}{3}, \frac{2(b-a)}{3}, b\right)$, the following resultant integration

$$I = \int_a^b f(x)\,dx \cong \int_a^b f_3(x)\,dx$$

yields the same expression as given by SIMPSON'S 3/8 th RULE

For Roll Numbers ending in 5-9

Derive Gauss Legendre Quadrature Formula for open numerical integration for 3 symmetric points around 0 with symmetric weightage.