16/11/2021

24

CLL110: MAJOR EXAM

2020CH10189

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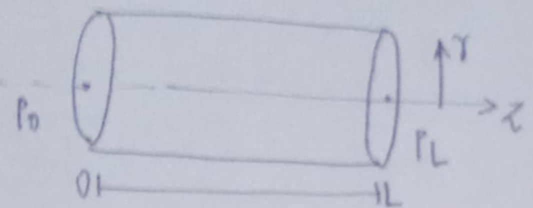
1.



Assumptions

- Steady state
- laminar
- Newtonian
- const ρ & η
- fully developed
- No slip boundary conditions

Velocity

$V_r = 0$, $V_\theta = 0$, $V_z \neq 0$

$V_z(r, \cancel{\theta}, z, \cancel{t})$

laminar steady state.

from eqn of continuity in ~~cartesian~~ cylindrical cordinate

$$\frac{\partial}{\partial z}\rho V_z = 0$$

$\therefore$ $\boxed{V_z(r)}$ only

τ

τ_{rz} only non zero shear component.

Applying Navier stokes eqn in cylindrical cordinate

Using r component

$$\underbrace{\frac{\partial P}{\partial z}}_{\text{func. of } z} - \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})}_{\text{func. of } r} = 0$$

$\rightarrow$ equating to zero $\Rightarrow$ both must be constants.

$\therefore \quad \left(-\frac{dV_z}{}\right)^n = (P_0 - P_L)\, r$

$$\frac{\partial P}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}) = C_1$$

\# $\frac{\partial P}{\partial x} = C_1 \Rightarrow \boxed{P = C_1 x + C_2}$

BCs @ $x = 0$ $P = P_0$

@ $x = L$ $P = P_L$

$\therefore$ $C_2 = P_0$

$P_L = C_1 L + P_0 \Rightarrow \boxed{C_1 = \frac{P_L - P_0}{L}}$

\# $$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}) = \left(\frac{P_L - P_0}{L}\right)$$

$$\frac{\partial}{\partial r}(r\tau_{rx}) = \left(\frac{P_L - P_0}{L}\right) r$$

$$\tau_{rx} = \left(\frac{P_0 - P_L}{2L}\right) r + \frac{C_1}{r}$$

Boundary Cond^n 1 @ $r = 0$ velocity profile symmetric so & finite

$\tau_{rx}|_{r=0} = 0$

$\therefore \boxed{C_1 = 0}$

$\tau_{rx} = m\left(-\frac{dV_z}{dr}\right)^n$ as $\boxed{dV_z \downarrow \text{ as } r \uparrow}$

$$m\left(-\frac{dV_z}{dr}\right)^n = \frac{P_0 - P_L}{2L} r$$

$$\left(-\frac{dV_z}{dr}\right)^n = \frac{(P_0 - P_L)}{2L} r$$

2020CH10139
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$$\left(-\frac{dv_z}{dr}\right)^n = \frac{(P_0 - P_L) r}{2Lm}$$

$$\boxed{\frac{dv_z}{dr} = -\sqrt[n]{\frac{P_0 - P_L}{2Lm}}\, r^{1/n}}$$

2nd B.C $r = R$, $v_z = 0$ No slip B.C.

$$v_z = -\sqrt[n]{\frac{P_0 - P_L}{2Lm}}\, \frac{r^{\frac{n+1}{n}}}{\frac{n+1}{n}} + C_2$$

$$C_2 = -\sqrt[n]{\frac{P_0 - P_L}{2Lm}}\, \frac{R^{n+1/n}}{\frac{n+1}{n}}$$

$$\boxed{v_z = \underbrace{\sqrt[n]{\frac{P_0 - P_L}{2Lm}}\, \frac{R^{\frac{n+1}{n}}}{\frac{n+1}{n}}}_{\boxed{A}} \left[1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]}$$

PART II Avg velocity

$$Q = \int_0^R (v_z)(2\pi r)\,dr = \int_0^R A\left[1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] dr\,(2\pi r)$$

$$= [2\pi A]\left[\frac{r^2}{2} - \frac{1}{R^{1+1/n}} \cdot \frac{r^{3+1/n}}{3+1/n}\right]_0^R$$

$$= [2\pi A]\left[\frac{R^2}{2} - \frac{1}{R^{1+1/n}} \cdot \frac{R^{3+1/n}}{3+1/n}\right] = 2\pi A\left[\frac{R^2}{2} - \frac{R^2}{3+1/n}\right]$$

$$\boxed{\frac{Q}{A} = v_{avg}} = \frac{2\cancel{\pi} A\left[\frac{1}{2} - \frac{1}{3+1/n}\right]\cancel{R^2}}{\cancel{\pi}\cancel{R^2}}$$

$$V_{avg} = 2A\left[\frac{1}{2} - \frac{1}{3+1/n}\right]$$

$$= \cancel{2}\sqrt[n]{\frac{P_0-P_L}{2Lm}}\frac{R^{1+1/n}}{\cancel{1+\frac{1}{n}}}\left[\frac{\cancel{1+1/n}}{\cancel{2}(3+1/n)}\right]$$

$$\boxed{V_{avg} = \sqrt[n]{\frac{P_0-P_L}{2Lm}}\frac{R^{1+1/n}}{(3+1/n)}}$$

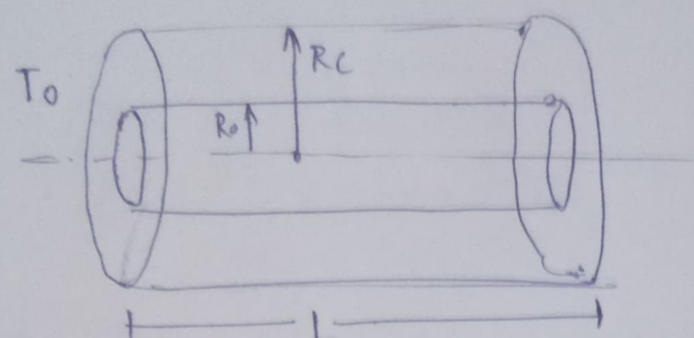
$$\boxed{\tau_W = -\tau_{@\,r=R}}$$

$$\boxed{\tau_W = -\left(\frac{P_0-P_L}{2L}\right)R}$$

$$\text{friction factor} = \frac{\tau_W}{\frac{1}{2}\rho V_{avg}^2} = \frac{-\left(\frac{P_0-P_L}{2L}\right)R}{\frac{1}{2}\rho(V_{avg})^2}$$

2.] Assumptions

- steady state
- constant k & ρ
- h_R is const
- T varies in radial direction

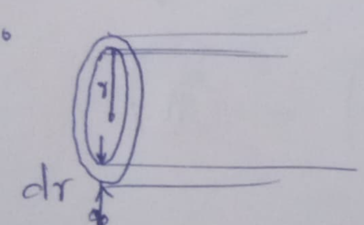


Temp

$T(r, \cancel{\theta}, \cancel{z}, \cancel{t})$ — symmetry, fully developed, steady state

also $q_r \neq 0,\ q_z = 0,\ q_\theta = 0$

C.V.



Shell Energy (heat) balance

$$2\pi r L\, q_r\big|_r - 2\pi r L\, q_r\big|_{r+dr} = 0 \quad \leftarrow \text{S.S.}$$

$$\frac{d(r q_r)}{dr} = 0$$

$$\boxed{r q_r = C_1} \Rightarrow \boxed{R_o q_o = R_c q_c = r q_r = C_1}$$

$$\boxed{q_r = \frac{R_c}{r} q_c}$$

$$q_r = -k\frac{dT}{dr} = \frac{R_c q_c}{r}$$

$$\int_{T_0}^{T_c} -k\,dT = \int_{R_0}^{R_c} \frac{-R_c q_c}{r}$$

$$T_0 - T_c = \frac{R_c q_c}{k} \ln r \Big]_{R_0}^{R_c}$$

$$T_0 = T_c + \frac{R_c q_c}{k} \ln \frac{R_c}{R_0} \quad \text{---} \; (i)$$

$\boxed{q_c = h_a (T_c - T_a)}$ From Newton's law of cooling

$$T_c - T_a = \frac{q_c}{h_a}\left(\frac{R_c}{R_c}\right) \quad \text{---} \; (ii)$$

from (i) & (ii)

$$T_0 - T_a = \underbrace{R_c q_c}_{K_1}\left(\frac{1}{R_c h_a} + \frac{1}{k}\ln\left(\frac{R_c}{R_0}\right)\right)$$

$$T_0 - T_A = K_1\left(\frac{1}{R_c h_a} + \frac{1}{k}\ln\left(\frac{R_c}{R_0}\right)\right)$$

$$\boxed{K_1 = \frac{T_0 - T_A}{\frac{1}{R_c h_a} + \frac{1}{k}\ln\left(\frac{R_c}{R_0}\right)}}$$

for optimum, optimisin $f(R_c)$

$$f(R_c) = \frac{1}{R_c h_a} + \frac{1}{k}\ln\left(\frac{R_c}{R_0}\right)$$

$$\frac{df}{dR_c} = \frac{-1}{h_a R_c^2} + \frac{1}{k R_c} = 0$$

for optimisation zero derivative = extremum.

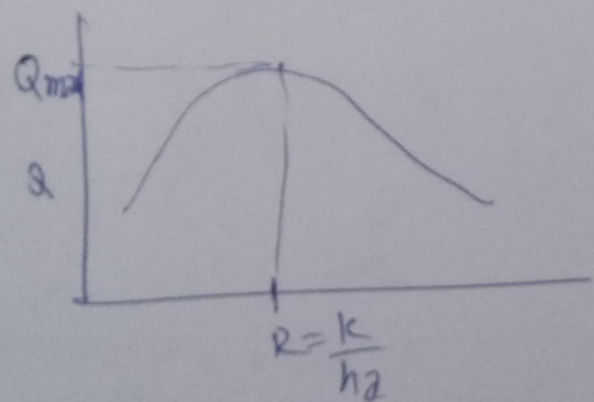
$$\boxed{R_c = \frac{k}{h_a}}$$

$$\frac{d^2 f}{dR_c^2} = -\frac{1}{k R_c^2} + \frac{2}{R_c^3 k_a}$$

$$\left.\frac{d^2 f}{dR_c^2}\right|_{@\, r = k/h_a} = \frac{-1}{k\left(\frac{k^2}{h^2}\right)} + \frac{1}{\frac{k^3}{h^3}\cdot h} \geq 0$$

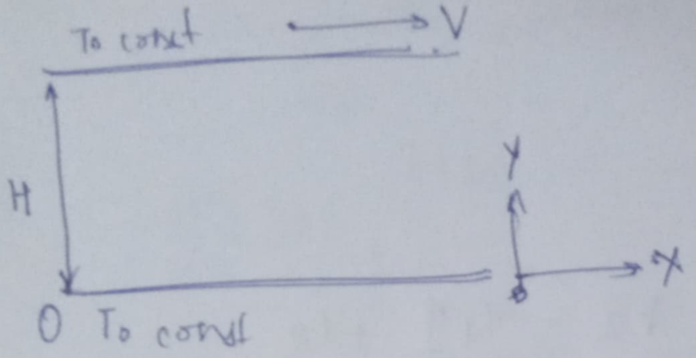
$$\Rightarrow R_c = \frac{k}{h_a} \Rightarrow$$ local minima & required answer

Heat diss profile



$\boxed{Q \propto R_c}$ as $Q = h_a(T_c - T_a)\, 2\pi R_c L$

3.]



find velocity
temp distribution
max temp
& its location

$$\boxed{k = \frac{1}{A+BT}}$$

PART I : Velocity profile:

Assumptions
- Newtonian,
- Incompressible
- steady state
- const ρ & μ
- laminar
- No slip boundary

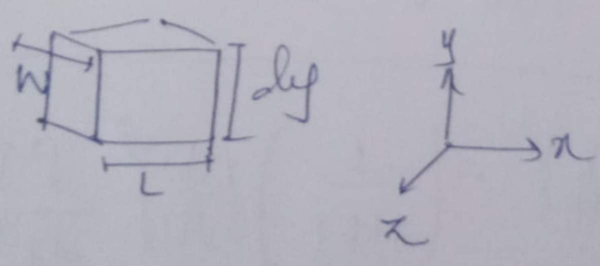
* Vel $\quad V_x \neq 0 \quad, V_z = 0, V_y = 0$

$V_x(\cancel{x}, y, \cancel{z}, \cancel{t})$

eqn of continuity — domain — s.s

* Relevant τ $\quad \tau_{yx}$ Non zero

* C.V



* shell momentum balance

$$\rho W dy V_x^2|_x - \rho W dy V_x^2|_{x+dx}$$

$$+ \tau_{yx}(WL)|_x - \tau_{yx} WL|_{x+dx} = 0.$$

$$\frac{d\tau_{yx}}{dx} = 0$$

$$\tau_{yx} = C_1$$

$$\frac{dV_x}{dy} = -\frac{C_1}{\mu} \Rightarrow V_x = -\frac{C_1 y}{\mu} + C_2$$

B.C.s @ $y = 0$ $V_x = 0$ $\Rightarrow C_2 = 0$

@ $y = H$ $V_x = V$ $\Rightarrow \boxed{C_1 = -\frac{V\mu}{H}}$

$$\boxed{V_x = \frac{Vy}{H}}$$ Velocity profile.

(B) Temperature Profile

Isothermal condition

$\therefore$ T independent of y $\Rightarrow \boxed{T_y(x)}$

Using eqn of energy for newtonian fluid

$$\rho C_p \left(V_x \cancelto{0}{\frac{dT}{dx}} \right) = k\left[\frac{d^2T}{dx^2}\right] + \mu\left(\frac{V}{H}\right)^2$$

$$\boxed{\mu\left(\frac{V}{H}\right)^2 = \left(-\frac{1}{A+BT}\right)\frac{d^2T}{dx^2}}$$

$$\boxed{-\mu\left(\frac{V}{H}\right)^2(A+BT) = \frac{d^2T}{dx^2}}$$

$$\underbrace{(A+BT) = -\left(\frac{H}{V}\right)^2 \cdot \left(\frac{1}{\mu}\right) \frac{d^2T}{dx^2}}$$

↓

solving this DE

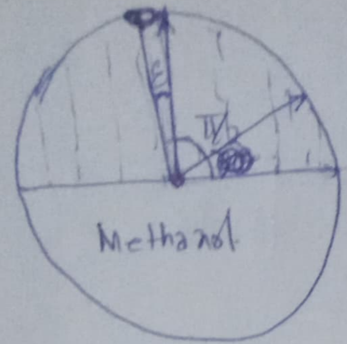
$$(A+BT) = \alpha^2 \frac{d^2T}{dx^2}$$

$$\alpha = \sqrt{-\left(\frac{H}{V}\right)^2 \left(\frac{1}{\mu}\right)}$$

↳ BC's @ $y=0 \quad T=T_0$

$y=H \quad T=T_0$

4.]



$R = 100\,cm$

$T = 27°C$

$R_{hole} = 10\,cm$

Rate of loss of methanol in gm/sec ??

$P = 760$ mm of Hg

$D_{methanol\,in\,air} = 0.1\,cm^2$

$P_{methanol}$ @ $27°C = 160$ mm Hg

using

since mass flow rate to be found dealing in x helpful

BC's

@ $\theta = \pi/2$ $\quad x_A = \frac{160}{760}$

$\frac{P_{methanol}}{P_T} = x_{methanol}$

@ $\theta = \varepsilon$ $\quad x_A = 0$

let methanol be A, air be B } for entire ques.

$\sin\varepsilon = \boxed{\varepsilon = \frac{10}{100} = \frac{1}{10}}$

Assumptions.

- const D_{AB}
- Fick's law applicable
- steady state
- No chem rxn taking place

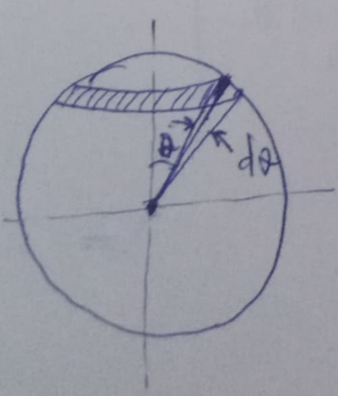
Flux Analysis

$N_r = 0$

$N_\phi = 0$

$N_\theta \neq 0$

C.V



~~surface area = $(2\pi R\sin\theta)(R\,d\theta)$~~

surface area $= \pi(R\sin\theta)^2$

Using shell mass balance

~~$N_{A\theta}(2\pi R^2 \sin\theta\, d\theta)$~~

$$\frac{N_{A\theta}(\pi)(R\sin\theta)^2\big|_{\theta=\theta} - N_{A\theta}(\pi)(R\sin\theta)^2\big|_{\theta+d\theta}}{\pi(R\sin\theta)^2(R\,d\theta)} = 0$$

$$\boxed{\frac{d\,N_{A\theta}(\sin\theta)^2}{d\theta} = 0}$$

$$N_{A\theta}(\sin\theta)^2 = c_1$$

$$N_{A\theta} = \frac{c_1}{(\sin\theta)^2}$$

still, not moving

$$N_{A\theta} = J_A^* + x_A(N_{A\theta} + \cancel{N_{B\theta}})$$

$$N_{A\theta}(1-x_A) = J_A^*$$

$$N_{A\theta} = \frac{1}{(1-x_A)} \cdot -(D_{AB}\left(\frac{1}{r} \cdot \frac{dx_A}{d\theta}\right)$$

MI

$$N_{A\theta} = \frac{1}{(1-x_A)} \cdot -(D_{AB}\left(\frac{1}{r}\frac{dx_A}{d\theta}\right) = \frac{c_1}{(\sin\theta)^2}$$

$$\boxed{\frac{dx_A}{r(1-x_A)}(-cD_{AB}) = \frac{c_1}{(\sin\theta)^2}\,d\theta}$$

~~$\boxed{r = R\sin\theta}$~~

putting BCs

$$1 = C_2\left(1-\frac{16}{76}\right)^{-\frac{D_{AB}}{C_1 R}}$$

$$13 = C_2$$

$$1 = 13\left(\frac{60}{76}\right)^{-\frac{D_{AB}}{C_1 R}}$$

$$\frac{-D_{AB}}{C_1 R} = \frac{\ln(0.0764)}{\ln(0.789)}$$

$$C_1 = \frac{-10^{-3}}{10.82}$$

$$C_1 = -9.238 \times 10^{-5}$$

$$\text{flux} = W_A = N_{Ax}\pi R^2 \sin^2\alpha$$

$$= C_1 \pi R^2$$

$$\text{mass flux} = \frac{W_A}{\text{molar mass methanol}}$$

$$= \frac{C_1 \pi R^2}{32} = \frac{-9.238 \times 10^{-5} \times \pi \times (100)^2}{32}$$

$N_\theta = 0,\ N_r = 0$ (given)

$$+\frac{CD_{AB}}{r}\ln(1-x_A) = \int c_1 (\operatorname{cosec}\theta)^2\, d\theta$$

$$\boxed{\frac{CD_{AB}}{r}\ln(1-x_A) = -c_1 \cot\theta + C_2}$$

B/Cs @ $\theta = \pi/2$ $\quad x_A = \frac{160}{760}$

$\theta = \varepsilon \quad x_A = 0$

$$\frac{CD_{AB}}{r}\ln 1 = 0 = -c_1 \cot\varepsilon + C_2$$

$$\boxed{\frac{CD_{AB}}{r}\ln\left(1-\frac{16}{76}\right) = +C_2}$$

$$\boxed{C_1 = \frac{C_2}{\cot\varepsilon} = \frac{CD_{AB}}{r}\ln\left(\frac{60}{76}\right)\cdot\frac{1}{9.9666}}$$

Ignoring MT

$$N_{A\theta} = \frac{c_1}{(\sin\theta)^2} \quad \text{by } \boxed{r = R\sin\theta}$$

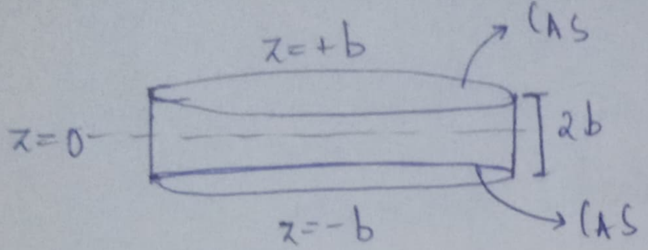
$$\frac{c_1}{(\sin\theta)^2}d\theta = \frac{-D_{AB}}{R\sin\theta}\frac{dx_A}{1-x_A}$$

$$\int \frac{c_1\, d\theta}{\sin\theta} = \frac{-D_{AB}}{R}\int\frac{dx_A}{1-x_A}$$

$$-c_1\ln(\operatorname{cosec}\theta + \cot\theta) = \frac{D_{AB}}{R}\ln(1-x_A) + \ln C_2$$

$$\operatorname{cosec}\theta + \cot\theta = C_2(1-x_A)^{\frac{-D_{AB}}{c_1 R}}$$

5. A → B



$\boxed{N_\theta = 0,\ N_r = 0}$ (given)

$\boxed{R_A = -k_1'' a\, C_A}$

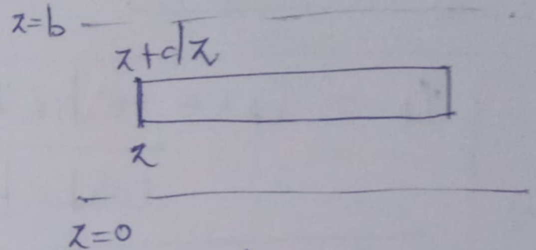
Assumptions

- Const ρ & D_{AB}
- Fick's law applicable
- steady state.

NOTE

doing calculations for 1 surface then multiplying it by 2.

$\boxed{CV}$



Shell Mass balance

$$N_{Az} \times \pi R^2 \big|_z - N_{Az} \pi R^2 \big|_{z+dz} - k'' a\, C_A \pi r^2 dz = 0$$

$$-\frac{dN_{Az}}{dz} = k'' a\, C_A$$

$$J_{Az}^* = -D_{AB} \frac{dC_A}{dz} \quad \text{(Fick's law.)}$$

$$N_{Az} = J_A^* + x_A \cancel{(N_{Az} + N_{Bz})} \qquad \boxed{N_{Az} = -N_{Bz}}$$

$$= J_A^*$$

As 1 mol A vanish to give 1 mole B

$$\underbrace{\frac{d^2 C_A}{dz^2} = \frac{k'' a}{D_{AB}} C_A}_{\text{standard 2nd order ODE}} \Big\} \rightarrow \text{here} \quad \boxed{\alpha^2 = \frac{k'' a}{D_{AB}}}$$

$$\boxed{\frac{d^2 y}{dx^2} = \alpha^2 y} \Rightarrow \text{soln.} \quad \boxed{y = c_1 \cosh \alpha x + c_2 \sinh \alpha x}$$

$C_A = C_1 \cosh z + C_2 \sinh z$

B.C's @ $z = b$ $\quad C_A = C_{AS}$

$z = -b$ $\quad C_A = C_{AS}$

$z = 0$ $\quad \frac{dC_A}{dz} = 0$ (symmetry).

$C_{AS} = C_1 \cosh b + C_2 \sinh b$

$C_{AS} = C_1 \cosh b - C_2 \sinh b$

solving this we get

$$\boxed{C_A = C_{AS} \frac{\cosh \alpha z}{\cosh \alpha b}}$$

where $\alpha = \sqrt{\frac{k''a}{D_{AB}}}$

Molar consumption of A

$$W_A = -N_{Az} \times \underbrace{2\pi R^2}_{\text{top \& bottom faces}}$$

$$= D_{AB} \times 2\pi R^2 \left. \frac{dC_A}{dz} \right|_{z=b}$$

$$\boxed{W_A = 2\pi R^2 D_{AB} \times C_{AS} \frac{\alpha \sinh \alpha b}{\cosh \alpha b}}$$

$$\boxed{W_A = 2\pi R^2 C_{AS} D_{AB} \, \alpha \tanh \alpha b}$$

for effectiveness factor

$$\eta_A = \lim_{n \to \infty} \left| \frac{W_A}{W_A^n} \right|$$

W_A calculated

W_A^n iterative catalyst

bc

$$\boxed{N_A'' = 2\pi R^2 k_1'' a C_{AS} b}$$

$$\eta_A = \frac{2\pi R^2 \cancel{C_{AS}} D_{AB} \alpha \tanh \alpha b}{2\pi R^2 k_1'' a \cancel{C_{AS}}} = \boxed{\frac{\tanh \alpha b}{\alpha b}} = \eta_A$$