6/1/21

CLL110 : Majors

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2019CH10062
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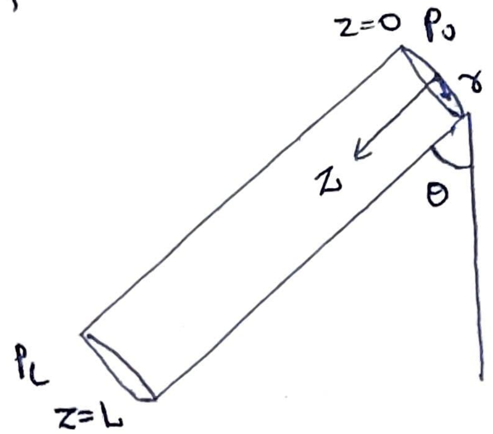
Q1



I Assumptions

Steady state
fully developed
laminar flow
Const ρ, μ
Newtonian fluid

II Velocity profile

$V_r = 0$
$V_\theta = 0$ } By intuitive postulation

different from given angle of inclination

$V_z = V_z(r, \phi, z, \cancel{t})$ (steady state)

(symmetry) $\therefore V_z = V_z(r, z)$

III Continuity theorem: $\xcancel{\frac{d\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y)} + \partial$

0

(const density)
& steady

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r V_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) = 0$$

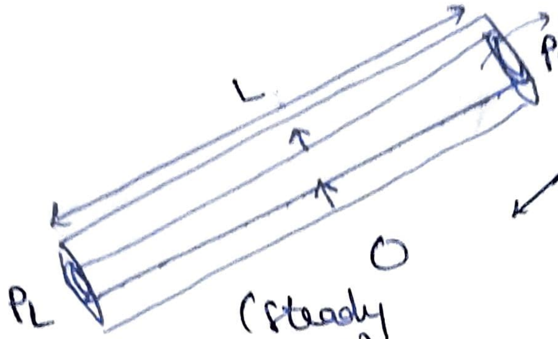
0 $\quad$ 0 $(V_r = 0)$ $\quad$ 0 $(V_\theta = 0)$ $\quad \therefore \frac{\partial}{\partial z}(\rho V_z) = 0$

hence V_z is not a func of z

$V_z = V_z(r)$

IV The only significant nonzero component of shear stress is $\overset{r}{\tau_{rz}}$: considering it as momentum flux.

V



Control volume : $2\pi r dr L$ or $2\pi r \Delta r L$

Rate of accumulation of momentum in control volume

= Rate of inflow - outflow of momentum by convection

+ Rate of inflow - outflow of viscous momentum

+ Σ forces

0 (steady state)

$$\rho V_z 2\pi r \overset{\Delta r}{dr} V_z \Big|_{z=0} - \rho V_z 2\pi r \overset{\Delta r}{dr} V_z \Big|_{z=L}$$

since V_z is independent of z $V_z|_{z=0} = V_z|_{z=L}$ making the term $= 0$

$$\tau_{rz} 2\pi r L \Big|_{r=r} - \tau_{rz} 2\pi r L \Big|_{r=r+\Delta r}$$

Pressure : $(P_0 - P_L) 2\pi r \Delta r$

Gravity : $\rho g \cos\theta \, 2\pi r \Delta r L$

$$0 = \tau_{rz} 2\pi r L \Big|_{r=r} - \tau_{rz} 2\pi r L \Big|_{r=r+\Delta r}$$
$$+ (P_0 - P_L) 2\pi r \Delta r + \rho g \cos\theta \, 2\pi r \Delta r L$$

dividing by $2\pi r \Delta r L$ & $\Delta r \to 0$

$$0 = -\frac{1}{r}\frac{d(r\tau_{rz})}{dr} + \frac{(P_0 - P_L)}{L} + \rho g \cos\theta$$

$$\frac{1}{r}\frac{d(r\tau_{rz})}{dr} = \frac{(P_0 - P_L)}{L} + \rho g\cos\theta$$

$$\frac{d}{dr}(r\tau_{rz}) = \frac{(P_0 - P_L)}{L}r + \rho g\cos\theta\, r$$

$$r\tau_{rz} = \frac{(P_0 - P_L)r^2}{2L} + \frac{\rho g\cos\theta r^2}{2} + C_1$$

$$\tau_{rz} = \frac{(P_0 - P_L)r}{2L} + \frac{\rho g\cos\theta r}{2} + \frac{C_1}{r}$$

$$\tau_{rz} = -\mu\frac{dV_z}{dr}$$

$$-\mu\frac{dV_z}{dr} = \frac{(P_0 - P_L)r}{2L} + \frac{\rho g\cos\theta r}{2} + \frac{C_1}{r}$$

$$\frac{dV_z}{dr} = \frac{-(P_0 - P_L)r}{2\mu L} \cancel{+} - \frac{\rho g\cos\theta r}{2\mu} - \frac{C_1}{r\mu}$$

$$V_z = \frac{-(P_0 - P_L)r^2}{4\mu L} - \frac{\rho g\cos\theta r^2}{4\mu} - \frac{C_1}{\mu}\ln r + C_2$$

VI Boundary Conditions : $\tau_{rz} = 0$ at $r = 0$ (symmetery)
$V_z = 0$ at $r = R$ (no slip boundary condn)

then $C_1 = 0$

$$V_z = 0 = \frac{-(P_0 - P_L)R^2}{4\mu L} - \frac{\rho g\cos\theta R^2}{4\mu} + C_2$$

$$C_2 = \frac{(P_0 - P_L)R^2}{4\mu L} + \frac{\rho g\cos\theta R^2}{4\mu}$$

$$V_z = \frac{(P_0 - P_L)R^2}{4\mu L}\left(1-\left(\frac{r}{R}\right)^2\right) + \frac{\rho g\cos\theta R^2}{4\mu}\left(1-\left(\frac{r}{R}\right)^2\right)$$

$$\tau_{rz} = -\mu\left[\frac{(P_0-P_L)\cancel{R^2}}{4\mu L}\left(\frac{-2r}{\cancel{R^2}}\right) + \frac{\rho g\cos\theta}{4\mu}(-2r)\right]$$

$$\Rightarrow \tau_{rz} = 2\mu\left[\left(\frac{P_0-P_L}{4\mu L}\right) + \left(\frac{\rho g\cos\theta}{4\mu}\right)\right] r$$

$$\Rightarrow V_z = \left(\frac{(P_0-P_L)}{L} + \rho g\cos\theta\right) \times \frac{R^2}{4\mu} \times \left(1-\left(\frac{r}{R}\right)^2\right)$$

friction factor

$$= \frac{\tau_{wall}}{\frac{1}{2}\rho(V_{zavg})^2}$$

$$\tau_{rz} = \frac{P_0 - P_L}{L}$$

$$V_{zavg} = \frac{V_{zmax}}{2}$$

$$= \frac{1}{2}\left[\left(\frac{P_0-P_L}{4\mu L}\right)R^2 + \left(\frac{\rho g\cos\theta}{4\mu}\right)R^2\right]$$

$$\tau_{rz}\big|_{r=R} = \left(\frac{P_0-P_L}{2L} + \frac{\rho g\cos\theta}{2}\right)R$$

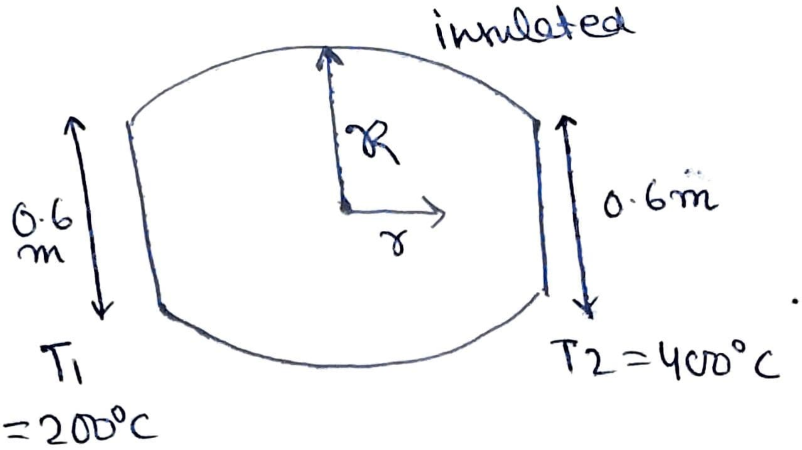
τ_{wall}

$$= \frac{\left[\frac{(P_0-P_L)}{L} + \rho g\cos\theta\right]\frac{R}{2}}{\times\frac{1}{2}\rho\times\frac{1}{4}\left[\frac{(P_0-P_L)}{L}+\rho g\cos\theta\right]^2\frac{R^4}{16\mu^2}}$$

$$= P_g \quad \rho \quad \frac{1}{\frac{\rho}{4}\left[\frac{(P_0-P_L)}{L} + \rho g\cos\theta\right]\frac{R^3}{16\mu^2}}$$

$$\Rightarrow \quad = \frac{64\mu^2}{\rho\left[(P_0-P_L) + \rho g\cos\theta\right]R^3}$$

Q2

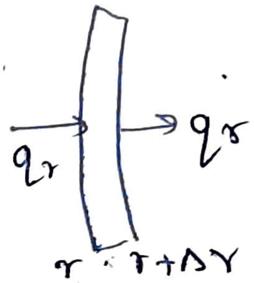


$R = 0.5\ m$

$k = 3\ cal/cms°c$

$= 300\ cal/ms°c$

Control volume



I ~~Assumptions~~

- Steady state
- Fouriers' law of conduction applicable
- k const

$$T_{avg} = \bar{T} = \bar{T}(r)$$

↙ average temp of the cross section $S =$ ~~$4\pi x^2$~~ πx^2

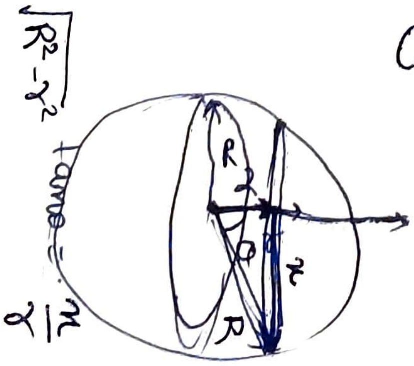
II nonzero component of heat flux = ~~A~~ q_r alone

Shell balance : $q_r S\Big|_{r=r} - q_r S\Big|_{r=r+\Delta r}$

dividing by volume $S\Delta r$ & $\Delta r \to 0$

$$-\frac{1}{S}\frac{d\, q_r S}{dr} = 0$$

$$\frac{d S q_r}{dr} = 0 \quad : \quad S q_r = C_1 \text{ : constant.}$$



$$x = \sqrt{R^2 - r^2}$$

$$S = \pi(R^2 - r^2)$$

when $x = 0.3$

$r = \sqrt{(0.5)^2 - (0.3)^2} = 0.4$

$\pi(R^2-r^2)q_r = C_1$ r goes from 0 to 0.4

fourier's law

$$q_r = -k\frac{d\bar{T}}{dr}$$

$$q_r = -k\frac{d\bar{T}}{dr} = \frac{C_1}{\pi(R^2-r^2)}$$

~~q_r~~ $$\frac{d\bar{T}}{dr} = -\frac{C_1}{k\pi(R^2-r^2)}$$

$$\bar{T} = \frac{-C_1}{k\pi}\left(\frac{\ln(r+R) - \ln(|r-R|)}{2R}\right) + C_2$$

Boundary conditions

$\bar{T} = 400$ at $r = 0.4\,m$

$\bar{T} = 200$ at $r = -0.4\,m$ {(-)ve value of r fits because log has mod & we'll get our answer appropriately}

$$400 = -\frac{C_1}{k\pi}\left(\frac{\ln(0.9) - \ln(0.1)}{1}\right) + C_2$$

$$200 = \frac{-C_1}{k\pi}\left(\ln(0.1) - \ln(0.9)\right) + C_2.$$

$$200 = -\frac{2C_1}{k\pi}\left(\ln(0.9) - \ln(0.1)\right)$$

$$C_1 = \frac{200\,k\pi}{-2(\ln 0.9 - \ln 0.1)} = -42872.26 \qquad 600 = 2C_2$$

$$C_2 = 300$$

$$T = 45.51 \left(\ln \left[\frac{(r+R)}{|r-R|} \right] \right) + 300$$

$$q_r = -k\frac{dT}{dr} = \frac{C_1}{\pi(R^2-r^2)} = \frac{-42.872.26}{\pi(0.25-r^2)}$$

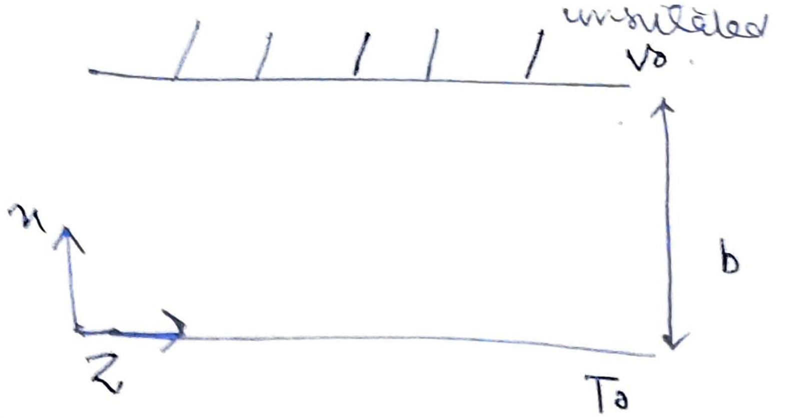
rate of heat transfer is constant at all r

$$= q_r \times \overset{S}{\cancel{4\pi r^2}} \quad \text{say at } r = 0.4$$

$$S \rightarrow \pi x^2 \quad x = 0.4$$

$$= \frac{-42{,}872.26}{\pi(0.25-0.09)} \times S = -42872.26 = C_1$$

Q3



I Assumptions

Steady state

Newtonian fully dev laminar flow const ρ, μ

T is a function of z alone

II Velocity profile $V_z = V_z(x, z)$

$V_x = 0 \quad V_y = 0$

(intuitive postulation)

Continuity : $\underbrace{\frac{\partial \rho}{\partial t}}_{\substack{0\\ \text{Steady}}} + \underbrace{\frac{\partial (\rho V_x)}{\partial x}}_{0 \; V_x = 0} + \underbrace{\frac{\partial (\rho V_y)}{\partial y}}_{0 \; V_y = 0} + \frac{\partial (\rho V_z)}{\partial z}$ $\quad \therefore \frac{\partial (\rho V_z)}{\partial z} = 0$

$V_z = V_z(x)$

III. τ_{xz} is the only significant nonzero component of momentum flux

IV Navier Stokes Eqn

$\frac{\mu}{\rho}\left(\frac{\partial^2 V_z}{\partial x^2}\right) = 0 \qquad \frac{d^2 V_z}{dx^2} = 0 \qquad V_z = C_1 x + C_2$

V Boundary conditions

$V_z = V_0$ at $x = b$ $\qquad C_1 = V_0/b$

$V_z = 0$ at $x = 0$ $\quad : \quad C_2 = 0$

$\Rightarrow V_z = V_0/_b \, x$: VELOCITY PROFILE

Q3 continued

$$\frac{dV_z}{dx} = \frac{V_o}{b}$$

Temp dependence $T = T(x)$

VI Thermal energy eqn

$$\frac{k}{\rho c_p}\left(\frac{d^2T}{dx^2}\right) + \frac{\mu}{\rho c_p}\left(\frac{dV_z}{dx}\right)^2 = 0$$

$$\frac{k d^2T}{dx^2} + \mu\frac{V_o^2}{b^2} = 0$$

$$\frac{d^2T}{dx^2} = \frac{-\mu}{K}\frac{V_o^2}{b^2} \qquad \frac{dT}{dx} = \frac{-\mu V_o^2}{K b^2}x + c_1$$

$$\Rightarrow T = \frac{-\mu V_o^2}{2Kb^2}x^2 + c_1x + c_2$$

VII Boundary condns

$T = T_o$ at $x = 0$

$\frac{dT}{dx} = 0$ at $x = b$ due to insulation no heat transfer $q_x = 0$

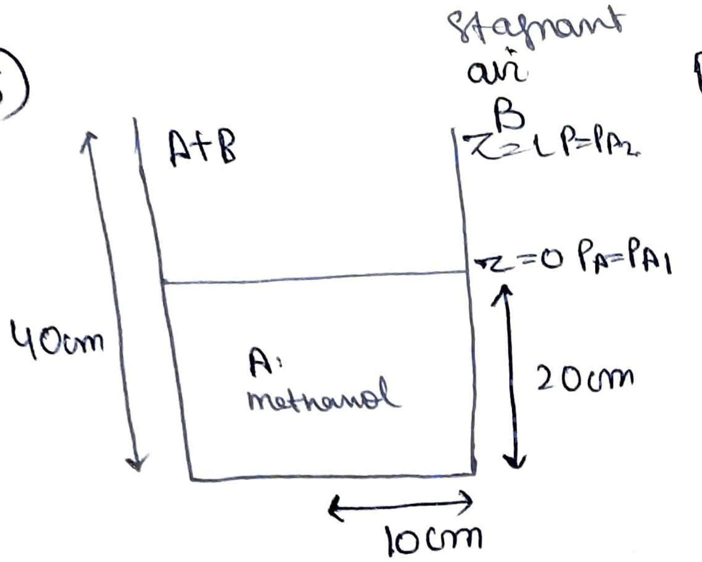
$$\hookrightarrow c_1 = \frac{\mu V_o^2}{Kb}$$

$$T_o = \frac{\mu V_o^2}{Kb}(0) + C_2 = C_2 = T_o$$

$$T = \frac{-\mu V_o^2 x^2}{2Kb^2} + \frac{\mu V_o^2}{K}\left(\frac{x}{b}\right) + T_o$$

$$\Rightarrow T = -\frac{\mu V_o^2}{2K}\left(\frac{x}{b}\right)^2 + \frac{\mu V_o^2}{K}\left(\frac{x}{b}\right) + T_o$$

Q5)



$P = 760 \text{mmHg}$

$D_{AB} = 0.15 \text{cm}^2/s$

$P_A = 160 \text{mmHg}$

$$y_{A1} = \frac{P_A^{sat}}{P} \overset{1}{\cancel{x_A}} = \frac{P_{A}^{sat}}{P} = \frac{160}{760} = \frac{4}{19}$$

I Assumptions

steady state

mass transport in x/y negligible

N_{Az} is only nonzero component.

D_{AB}, ρ const.

No chemical reaction taking place

air stagnant

II Eqn of continuity:

$$\underset{\substack{0 \\ \text{(steady state)}}}{\frac{\partial c_A}{\partial t}} + \underset{\sim}{\nabla} \cdot N_A = \overset{0 \text{ (no reaction)}}{R_A}$$

$$= \frac{dN_{Az}}{dz} = 0 \quad \therefore N_{az} = C_1$$

III

$$N_{Az} = y_A(N_{Az} + \overset{0 \text{ (stagnant)}}{N_{Bz}}) + J_{Az}^*$$

$$N_{Az} = \frac{J_{Az}^*}{(1 - y_A)}$$

IV Ficks Law:

$$J_{Az}^* = -C D_{AB} \frac{dy_A}{dz}$$

$$N_{Az} = \frac{-CD_{AB}\,Dy_A/Dz}{(1-y_A)} = C_1$$

$$\ln(1-y_A) = \frac{C_1 z}{CD_{AB}} + C_2$$

In terms of partial pr

$$\ln\left(\frac{P-P_A}{P}\right) = \frac{C_1 z}{CD_{AB}} + C_2.$$

at $z=0$ $P_A = P_{A1} = y_{A1} P = 160\text{ mm Hg}$

At $Z=L$ $P_A = P_{A2} = 0$: negligible methanol present in outside air

$$C_2 = \ln\left(\frac{P-P_{A1}}{P}\right)$$

$$C_1 = \frac{CD_{ab}}{L}\left(\frac{P-P_{A2}}{P-P_{A1}}\right)$$

$$\ln(1-y_A) = \frac{Z}{L}\ln\left(\frac{P-P_{A2}}{P-P_{A1}}\right) + \ln\left(\frac{P-P_{A1}}{P}\right)$$

rate of evaporation $= C_1 = \frac{CD_{ab}}{L}\left(\frac{P-P_{A2}}{P-P_{A1}}\right) \times$ surface area

in terms of g/s $= \frac{32 \times C \times D_{ab}}{L}\left(\frac{P-P_{A2}}{P-P_{A1}}\right) \times \pi R^2$

MW = 32 g

$$= \frac{M_A C D_{ab}}{L}\left(\frac{P-P_{A2}}{P-P_{A1}}\right) \times \pi R^2$$

Height:

molar rate of evaporation = rate of moles at which moles of A enter gas phase.

$$N_{Az}\Big|_{z=z_1} = \frac{-CD_{ab}(y_{A1}-y_{A2})}{(z_2-z_1)(y_B)_{ln}}$$

$$-\frac{S\rho^A}{M_A}\frac{dz_1}{dt} = \frac{CD_{ab}\,S\,(y_{A1}-y_{A2})}{(z_2-z_1(t))(y_B)_{ln}}$$

density of liquid

keeping $h(t) = z_1(0) - z_1(t)$

↳ distance the interface has descended in time t we get $\int_0^h (u+h)\,dh = \frac{C D_{ab} (y_{A1} - y_{A2})}{(\rho_A / M_A)(y_{Bln})} \int_0^t dt$:

keeping as $\frac{1}{2} C' \rightarrow$ const. $\quad c' = 2 \frac{C D_{ab} \left(\frac{160}{760}\right)}{\rho_A / 32 \left(\frac{700}{760}\right)}$

we get

$h(t) = u\left(\sqrt{1 + C't/u^2} - 1\right)$

keeping $h(t) = 1\,cm$

$h(t) = 1\,cm = 20\,cm \left(\sqrt{1 + C't/u^2}\right) - 1)$

Ideal gas

$C = P/RT = n/V$

$P = 1\,atm = 760\,mmhg$

$T = 300K \; (27+273)$

$C = \frac{1}{300 \times 1/12} = 1/25 \rightarrow mol/L = 4 \times 10^{-5}\,mol/cm^3$

$C = 4 \times 10^{-5}\,mol/cm^3$

rate $= \frac{32 \times 0.15 \times 4 \times 10^{-5} \times \pi (100) \times \ln\left(\frac{760}{600}\right)}{20}$

$= 2.228 \times 10^{-5}\,mol/s \times 32$

$= 71.25 \times 10^{-5}\,g/s$

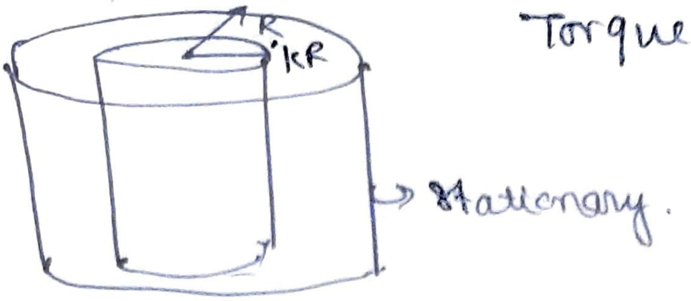
$h(t) = \frac{1 ~~cm~~}{~~400~~} = \frac{1}{20} + 1 = \left(\frac{21}{20}\right)^2 = \frac{441}{400} = 1 + \frac{C't}{u^2}$

$\frac{41}{400} = \frac{C't}{400}$

$C' = \frac{2 C D_{ab} \cdot 160}{\frac{\rho_A}{32} \times 600}$

$\Rightarrow 41/C' = t$: time

Q4)



Torque T : inner cylinder

I. Assumptions

steady state

laminar flow

fully developed

Const ρ

Power law fluid

II Velocity Profile (by intuitive postulation)

$V_r = 0$

$V_z = 0$

$V_\theta = V_\theta(r, \underbrace{\theta, z}_{0}, \underset{0 \text{ (steady state)}}{t}) = V_\theta(r, \theta)$

III Eqn of continuity

$$\underset{0}{\frac{\partial \rho}{\partial t}} + \frac{1}{r}\frac{\partial}{\partial r}\underset{0 \; V_r = 0}{(\rho r V_r)} + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}\underset{0 \; V_z = 0}{(\rho V_z)} = 0$$

hence $\frac{\partial}{\partial \theta}(\rho V_\theta) = 0$

$\boxed{V_\theta = V_\theta(r)}$

→ only significant nonzero shear stress component $= \tau_{r\theta}$

IV Eqn of motion:

$$\underset{0}{-\frac{1}{r}\frac{\partial P}{\partial \theta}} - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \tau_{r\theta}) + \overset{0}{\rho g_\theta}$$

0 Prr doesnt depend on θ (symmetry)

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \tau_{r\theta}) = 0$$

Q4 (continued)

$$\frac{1}{r^2}\frac{d}{dr}(r^2\tau_{r\theta})=0$$

$$r^2\tau_{r\theta}=C_1$$

$$\tau_{r\theta}=\frac{C_1}{r^2}$$

V Torque = force × r = shear stress × area × r

$$= -\tau_{r\theta}\Big|_{r=KR} \times 2\pi KRL \times KR$$

$$T = -2\pi K^2R^2L\,\tau_{r\theta}\Big|_{r=KR}$$

$$T = -2\pi K^2R^2L\,\frac{C_1}{K^2R^2}$$

$$C_1 = \frac{-T}{2\pi L}$$

$$\tau_{r\theta} = \frac{-T}{2\pi L r^2}$$

VI. $\tau_{r\theta} = -\mu r\frac{d}{dr}\left(\frac{V_\theta}{r}\right)$

$\left\{\frac{d}{dr}\left(\frac{V_\theta}{r}\right)\text{ is (+)ve as } \tau \text{ is (−)ve}\right\}$

$$= -m\left(r\frac{d}{dr}\left(\frac{V_\theta}{r}\right)\right)^n$$

$$= \frac{-T}{2\pi L r^2}$$

$$\left(\frac{T}{2\pi L m r^2}\right)^{1/n} = r\frac{d}{dr}\left(\frac{V_\theta}{r}\right) : \left(\frac{T}{2\pi L m}\right)^{1/n}\frac{1}{r^{2/n+1}} = \frac{d}{dr}\left(\frac{V_\theta}{r}\right)$$

$$\frac{V_\theta}{r} = \left(\frac{T}{2\pi L m}\right)^{1/n}\frac{1}{r^{2/n}} \times \frac{n}{2} + C_2$$

VII Boundary condn : $V_\theta = 0$ at $r = R$

$$C_2 = -\left(\frac{I}{2\pi Lm}\right)^{1/n} \cdot \frac{1}{(R)^{2/n}} \cdot \frac{n}{2}$$

$$\frac{V\theta}{r} = \frac{m}{2}\left(\frac{I}{2\pi Lm}\right)^{1/n}\left[\frac{1}{r^{2/n}} - \frac{1}{R^{2/n}}\right]$$

Ω

$$a = \frac{V\theta}{r}\Big|_{r=kR}$$

$$\rightarrow \Omega = \frac{n}{2}\left(\frac{I}{2\pi Lm}\right)^{1/n}\left[\frac{1}{K^{2/n}R^{2/n}} - \frac{1}{R^{2/n}}\right]$$

angular velocity