Notes for Li, Wang, Bai "A NON-PARAMETRIC APPROACH TO CONSTRAIN THE TRANSFER FUNCTION IN REVERBERATION MAPPING"

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1. Article

There are generally two classes of methods depending on the strategy adopted in solving the integral equation that links the variations of continuum and emission lines in RM. One class relies on directly inverting the integral equation, including early attempts based on the Fourier transform and regularized linear inversion (RLI) methods. The method using the Fourier transform was found to require high-fidelity data and thus has not seen much application (Maoz et al. 1991), whereas RLI seeks to discretize the integral equation and employ a differencing operator to suppress noise. The downside of this class of methods is that measurement noises cannot be self-consistently incorporated; thus, this class may be very noise sensitive in some cases. The other class relies on indirectly inferring the "best" transfer function that fits the data by supposing prior limits or models on it. The maximum entropy method and BLR dynamical modeling method belong to this class. The maximum entropy method finds the "simplest" solutions for the transfer function by maximizing the entropy on the premise of a reasonable fit to the data. However, it does not allow for straightforward uncertainty estimates or model selection.

In RM, variations of broad emission lines $f_l(t)$ are blurred echoes of continuum variations $f_c(t)$ through the transfer function $\Psi(\tau)$. Linear response is assumed for simplicity but non-linearity can be incorporated.

The idea is to write the transfer function as a sum of Gaussians

$$\Psi(\tau) = \sum_{k=1}^{K} exp\left[-\frac{(\tau-\tau_k)^2}{2\omega^2}\right]$$

where ω represents the common width of Gaussian responses, τ_k and f_k are the mean lag and weight of the kth response, and K is the number of responses, which can be regarded as a "smoothing parameter." The width is kept common for simplicity. pysically this can be understood as a group of BLR clouds each reacting to the variation in the central source.

Let y be a column vector comprised of the light curves of both the continuum and the line. A set of measurements in a campaign is equal to the realization of some underlying signals with measurement noises:

$$y = s + Lq + n$$

where s is the signal of the variations where the continuum is a DRW with covariance matrix

$$S_{cc}(\Delta t) = \sigma_d^2 exp(-\frac{|\Delta t|}{\tau_d})$$

where $\Delta = t_i - t_j$ τ_d represents the typical damping timescale, and σ_d represents the standard deviation of the process on a long timescale (»τ_d). Lq is a term that models general linearly varying trends where L is a matrix of known coefficients and q is a column vector of unknown linear fitting parameters. Lastly n is the measurement noise.

for the MCMC there are then parameters relating to the DRW process for the continuum (σ, τ_d) and for the transfer function (2K+1 since we have $f_k s, \tau_k s$ and ω). For less parameter degeneracy the central time lags τ_k are spaced in a uniform grid and the Gaussian width ω has a prior limit of $(\Delta\tau/2\Delta\tau)$ so as to to not have a too small ω and get uneccessary structures for the transfer function or too big a ω so that the Gaussian become indistinguishable. The number of Gaussian is chosen with maximum likelihood and not Bayesian methods because it is very computationally expensive.

the method reproduces simulated transfer functions well but only if the time span for observation is many times the mean lag time and the S/N and sampling cadence(ragularity of data points) is high