Notes for A. Skielboe and A. Pancoast et al. "Constraints on the broad line region from regularized linear inversion: velocity–delay maps for five nearby active galactic nuclei"

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October 2019

1. Introduction

The history and background for AGN and reverberation mapping is gone through. RLI is presented as a method for obtaining the transfer function for the BLR. The transfer equation is

$$F_l(t,\lambda) = \int_{-\infty}^{\infty} \Psi(\tau,\lambda) F_c(t-\tau) d\tau$$

2. Regularized Linear Inversion

RLI solves the transfer equation analytically to get the transfer function $\Psi(\tau,\lambda)$. Also it does not assume anything about its shape. The only assumption is that the transfer function is bounded linear operator. The solution would be restrictive if we looked at a unique solution, so instead we minimize χ^2 and to make the inversion more stable, the first order derivative for the solution will also be minimized. This is to smooth the solution at the level of noise. Ideally RLI can fir any shape, but sampling and noise gives a minimum resolution scale for the solution.

In their case with the BLR if they measure the continuum and the emission line light curves they would be able to solve for the transfer equation, but since the data we have is always discrete and not continues, we instead have a linear matrix equation

$$\boldsymbol{L}_{\Delta\lambda} = \boldsymbol{\Psi}_{\Delta\lambda} \boldsymbol{C}$$

Here $\boldsymbol{L}_{\Delta\lambda}$ is the emission-line light curve integrated over the wavelength range (spectral bin) $\Delta\lambda$, $\boldsymbol{C}$ is a matrix of continuum light curves, and $\boldsymbol{\Psi}_{\Delta\lambda}$ is the transfer function corresponding to the given wavelength range.

There are a number of assumptions for this method. First, we assume that the variations in the AGN continuum bands are correlated with the AGN ionizing continuum. Secondly, we assume that the continuum emission originates from a region negligible in size compared to the BLR. Thirdly, we assume that the continuum ionizing radiation is emitted isotropically. Lastly, we assume that the BLR structure is constant and the response linear for the duration of the campaign, such that a single linear transfer function can be calculated from the full light curves.

RLI which subtracts the mean component of the light curves to only calculate the response in the emission line to variations in the continuum, thus measuring response (rather than transfer) functions.

Following the notation of Krolik & Done (1995), and considering only variation about the mean of the light curves, we write χ^2 as

$$\chi^2 = \sum_{i=M}^{N} \frac{1}{\sigma_1^2(t_i)} \left[\delta F_1(t_i) - \sum_{j=1}^{M} [F_c(t_i - \tau_j) - \langle F_c \rangle] \Psi(t_j) \right]^2 . \quad (4)$$

This expression can be recast to matrix notation,

$$\chi^2 = (\boldsymbol{L} - \mathbf{C}\boldsymbol{\Psi})^2, \quad (5)$$

where the light curves enter as

$$C_{ij} = [F_c(t - \tau_j) - \langle F_c \rangle] / \sigma_1(t_i) \quad (6)$$

$$L_i = \delta F_1(t_i) / \sigma_1(t_i), \quad (7)$$

and the variation about the mean in the emission-line light curve is defined as

$$\delta F_1(t_i) = F_1(t_i) - \langle F_1 \rangle. \quad (8)$$

$$\mathbf{C}^{\mathrm{T}}\mathbf{C}\boldsymbol{\Psi} = \mathbf{C}^{\mathrm{T}}\boldsymbol{L}. \quad (9)$$

Although this expression looks simple, it turns out to be ill-conditioned. To remedy this, we put an extra constraint on the problem, namely that the solution should be smooth at the scale of the noise (this effectively avoids fitting the noise). To guarantee smoothness of the solution, we introduce a differencing operator $\mathbf{H}$ acting on $\boldsymbol{\Psi}$, and require that the first-order difference (the discrete version of the first-order differential) be minimized together with the ordinary χ^2. To control the weights between the χ^2 and the first-order difference, a scaling parameter κ is introduced that sets the scale of the regularization. Thus, the expression to minimize becomes

$$(\mathbf{C}^{\mathrm{T}}\mathbf{C} + \kappa\mathbf{H}^{\mathrm{T}}\mathbf{H})\boldsymbol{\Psi} = \mathbf{C}^{\mathrm{T}}\boldsymbol{L}. \quad (10)$$

This expression is more stable under inversion, while sacrificing detail by emphasizing smoothness of the solution. The question is then how to choose a suitable scale κ for the regularization.

The best choice of regularization scale depends on the signal-to-noise in the data, as well as the level of uncorrelated systematic uncertainties that result in deviations from the assumption of linear response. As a starting point for selecting a regularization scale κ, we follow the recommendation of Press (1992) in which $\kappa = \kappa_0$ is chosen to provide equal weights to the two left-hand-side terms in equation (10),

$$\kappa_0 = \frac{\mathrm{Tr}(\mathbf{C}^{\mathrm{T}}\mathbf{C})}{\mathrm{Tr}(\mathbf{H}^{\mathrm{T}}\mathbf{H})}. \tag{11}$$

They use a CAR(1) process for the continuum from Kelly et al. and try different powers for the covariance matrix though it does not affect the result strongly so CAR(1) is chosen. The covariance function is

$$C(t_1, t_2) = \sigma^2 exp\left[-\left(\frac{|t_2 - t_1|}{\tau}\right)^{\alpha}\right]$$

Where σ is the long-term standard deviation, τ is the typical timescale of variations, and the power α takes on values in the interval [1, 2]. A power of $\alpha = 1$ corresponds to the CAR(1) model, whereas $\alpha > 1$ produces smoother continuum models, with PSD slopes <-2.

For each continuum light curve we find the best-fitting Gaussian process parameters using the DNEST3 Nested Sampling code by Brewer, Partay and Csanyi (2010). The best-fitting Gaussian process is used to interpolate the continuum light curve, which we need to calculate the response for arbitrary time delays. In addition to interpolation, we use the statistical variability of the Gaussian process to generate a number of realizations of the continuum light curve that are used to estimate statistical errors on the calculated response functions. This allows us to include measurement errors in the continuum light curves that otherwise do not explicitly enter in the RLI formalism. For each continuum light curve, we generate 1000 realizations of the best-fitting Gaussian process