Notes for Zeljko Ivezi and Chelsea MacLeod "Optical variability of quasars: a damped random walk"

Malte Brinch
University of Copenhagen
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1. Article

Practically all quasars spectroscopically confirmed by SDSS are photometrically variable. Damped Random Walk (DRW) seems to be a good statistical description of quasar variability for optical wavelengths.

One way to look at quasar variability is to determine the structure function, which is the standard deviation of the distribution of the magnitude difference m(t_2)−m(t_1) evaluated at many different times t1 and t2, such that time lag $\Delta t = t_2 - t_1$. The time dependence of the structure function was found to be consistent with the prediction based on a DRW model

$$SF(\Delta t) = SF_\infty[1 - exp(-\Delta t/\tau)]^{1/2}$$

for small time lags this is $SF(\Delta t) \propto \Delta t^{1/2}$ which is the same as a random walk and becomes SF_∞ for large time lags. The structure function is related to the autocorrelation function, which makes a Fourier pair with the power spectral density function (PSD).

$$PSD(f) = \frac{\tau^2 SF_\infty^2}{1 + (2\pi f \tau)^2} \tag{1}$$

So a DRW will be $1/f^2$ at high frequencies and damped with a flat PSD at low frequencies.

Light curve modelling can help us constrain τ and SF_∞. This can be done with CAR(1) processes which for a time series m(t) is described by a stochastic differential equation which includes a damping term that pushes m(t) back to its mean.

There seems to be a relationship $PSD \propto f^\alpha$ where α between -1 and 0 with α less than -1 ruled out. There also seems to be evidence that DRW works for data on timescales from months to years, whereas for a month or less there seems to be some deviation from the DRW model. For high frequencies the PSD slope is steeper than $1/f^2$.