Notes for RICHARD A. DAVIS "Gaussian processes""

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1. Article

Gaussian processes are stocastic processes. Gaussian processes are used for two main reasons. First, a Gaussian process is completely determined by its mean and covariance functions. This property facilitates model fitting as only the first- and second-order moments of the process require specification. Second, solving the prediction problem is relatively straightforward. The best predictor of a Gaussian process at an unobserved location is a linear function of the observed values and, in many cases, these functions can be computed rather quickly using recursive formulas.

Sometimes the assumption of stationarity is needed for simplicity where the mean vector is independent of time and the covariance vector between one value in the random vector X_{t+h} and another value X_t is independent of t for all h.

It is possible to write the likelihood of a Gaussian process if we have a stationary Gaussian time series with a mean μ and autocovariance function $\gamma(\cdot)$. With a data vector $\boldsymbol{X}_n = (X_1 ... X_n)$ and a nonsingular covariance matrix Σ_n for $\boldsymbol{X}_n$ The likelihood function as

$$L(\Sigma_n, \mu) = (2\pi)^{-n/2}(det(\Sigma_n))^{-1/2} \exp\left(-\frac{1}{2}(\boldsymbol{X}_n - \mu\mathbf{1})'\Sigma_n^{-1}(\boldsymbol{X}_n - \mu\mathbf{1})\right)$$

Where $\mathbf{1} = (1, ..., 1)'$. $'$ indicates the one step predictors. It is also possible to use the one-step prediction errors and their mean square errors to calculate the likelihood of a gaussian function.