The role of workfare in striking a balance between incentives and insurance in the labour market*

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August 2011

Abstract

Unemployment insurance tends to distort incentives in the labour market affecting job search and wage formation adversely. We show in a search-matching model that this moral hazard problem can be reduced by attaching workfare (activation) requirement to the eligibility conditions for claiming unemployment benefits. Even in the case where workfare has no effects on human capital or productivity it is possible to improve labour market performance so as to create more jobs and lower unemployment by the introduction of workfare and also to improve welfare. Hence the incentive structure in the labour market can be improved by workfare policies rather than benefit reductions.

*Comments from Florian Langowsky, Michael Rosholm and Martin Werding as well as participants at the CESifo area conference on Employment and Social Protection and at seminars in Copenhagen, Kiel, and Stockholm are gratefully acknowledged. We thank Mark Strøm Kristoffersen for very constructive research assistance. This paper is part of the CESifo project on "How to Construct Europe".

1 Introduction

Active labour market or workfare[1] policies have become an important and widely used policy tool (see e.g. European Commission (2007), OECD (2010))[2]. It is generally perceived that associating activation or workfare requirements to the eligibility conditions for unemployment benefits or social assistance strengthens the incentive structure and thus improves labour market performance. From a policy perspective this may be a more attractive route than the alternative of reducing benefit levels.

Despite the popularity of active labour market policies the theoretical support for this type of policy is less clear. In a seminal paper Besley and Coate (1992) showed in a model focussing on labour supply along the intensive margin that workfare in a poverty alleviation programme can work to screen deserving from non-deserving claimants of benefits, but it will imply lower welfare for the target group. The screening effect arises because workfare introduces an opportunity cost of claiming benefits (in terms of time foregone) which is larger to individuals with a higher market wage. Hence, workfare can contribute to target redistribution towards groups with low earnings ability. However, the utility of the target group decreases since the disutility from participation in the programme dominates the gains from the higher benefit level, if not the self-selection constraint is violated. Hence, either such policies are motivated by policy objectives formulated only in terms of income (consumption possibilities) or the welfare rationale is hard to justify. A number of authors have challenged this conclusion by allowing for productive effects of workfare (see Chambers (1989), Besley and Coate (1995), Brett (1998)), heterogenous preferences (see Cuff (2000)), or unemployment risk (see Kreiner and Tranæs (2005)). However, these papers do not consider the implications for unemployment. This is studied in the important contribution by Frederiksson and Holmlund (2006) who in a search-matching framework compares various labour market policies and conclude that workfare policies are inferior to other types of labour market policies like time dependence in benefits and sanctions and monitoring. However, this analysis focus on the screening effect of workfare and rests

[1]We use the term workfare programme as a general term for activties for unemployed to raise their employment chances. These programmes varies from short job search courses over subsidized employment to class room training.

[2]Denmark is often highligthed in this context since the combination of flexible hiring and firing rules, a relatively generous social safety net, and an extensive use of active labour market policies is seen as the main ingredients of the so-called flexicurity model. In particular policy reforms strenthening workfare elements are important in accounting for the succes of the model, see e.g. Andersen and Svarer (2007))

on two specific features. First, unemployed individuals can freely choose between either an unemployment benefit, possibly associated with workfare requirements, or an unconditional unemployment assistance. Second, there is a group of non-workers (high disutility from work) and the analysis is considering a separating equilibrium where unemployed workers self-select for the benefit scheme with workfare while non-workers self-select for the unemployment assistance. Satisfying the self-selection constraints restrain the design of workfare since a too demanding policy may violate these constraints. The results are thus critically dependent on the requirement of ensuring a particular unconditional benefit level only for the so-called non-workers. We do not find that this structure for social transfers capture existing systems very well, and that the screening problem associated with the non-workers is the key policy problem.

In this paper the focus is on the moral hazard problem arising in an unemployment insurance scheme[3]. It is analysed whether workfare, as an element of an unemployment benefit scheme, can be used to strengthen the incentive structure as an alternative to resorting to benefit cuts including sanctions or time-dependent unemployment insurance profiles.[4] The perspective of this paper is primarily positive in the sense that we analyse how introduction of workfare policies affects labour market performance. We do, however, also consider workfare policies from a normative perspective and find that it may lead to a welfare improvement (utilitarian).

We consider a search-matching model[5] in which unemployment benefits create a moral hazard problem affecting search incentives adversely. We consider to what extent workfare elements attached to the benefit scheme can counteract the negative incentive effects. Specifically, can workfare policies improve labour market performance for given benefits? The unemployment scheme is assumed to be tax financed, and the costs of active labour market policies are explicitly taken into account. To focus on the incentive effects, we disregard human capital considerations; that is, human capital is assumed not to depreciate over the unemployment spell[6], and workfare programmes do not affect human capital.

[3]Besley and Coate (1992) briefly discuss the moral hazard case but with a focus on minimizing the costs of poverty alleviation.

[4]See e.g. Boone et al. (2007) for an analysis of optimal unemployment insurance with monitoring and sanctions and e.g. Fredriksson & Holmlund (2001) for an analysis of optimal unemployment insurance with a time-dependent unemployment insurance profile.

[5]The model is similar to models used to discuss various labour market and policy aspects as in e.g. Van der Linden (2005), Frederikson and Holmlund (2006), Cahuc and Le Barbanchon (2008), and Kolm and Tonin (2011).

[6]For an analysis of the implications of this for labour market policies, see e.g. Pavoni and Violanti

We also disregard heterogeneity among workers. Both assumptions are made to keep the analysis as simple as possible and to show that there is a case for workfare policies even without appealing to such effects which are known to strengthen the case for workfare.

We consider the implications of workfare policies both analytically and in numerical simulations. The key issue addressed is the effects of workfare requirements on search effort undertaken by individuals in different positions in the labour market and thus total job search as well as wage determination for employment and unemployment (open and total). We consider the distributional consequences in terms of the fraction of workers in different states (employment, unemployed, and workfare) and their income and utility levels. We focus on two dimensions of workfare policies, namely the transition rate from unemployment to activation and the work requirement. The two are not in general equivalent, and therefore, the composition of workfare policies is of importance for search effort and wage determination, and thus unemployment.

A key finding of the present analysis is that workfare affects individuals in all three possible labour market states, that is, as employed, unemployed and in workfare. The direct effect on programme participants is a *locking-in* effect. That is, programme participants have less time for job search and this may hamper their chances of finding a job.[7] However, in a labour market with flows in and out of unemployment, the introduction of workfare programmes will affect not only those participating but also those anticipating that they, with some probability, may become unemployed and end up in a workfare programme. Thus, both unemployed (not in activation) and employed are affected. Unemployed search more to reduce the risk of ending up in workfare (being less attractive than passively receiving benefits). It follows that overall job search is affected by two counteracting effects, namely the possible negative locking-in effect for those in workfare and the positive *threat or motivation effect* for the unemployed. Moreover, and perhaps less obvious, even the direct effect of workfare on total search effort is not pivotal since there is also a *wage effect*[8]. This wage effect arises because workfare affects the outside option of employed workers; that is, the outside option deteriorates, and this tends to lead

(2007) and Sørensen (2009).

[7]Subsequent job search may also be affected. It is possible that participants will search more actively since they will perceive the chances of finding a job to have improved. Oppositely, the types of jobs searched may be narrowed to the extent that the individual tries to match jobs to specific training.

[8]A wage effect is also identified by Holzner, Meier, and Werding (2010) in an efficiency wage model. They show that workfare lessens the non-shirking condition which in turn leads to a downward shift in the wage curve.

to wage moderation. Wage moderation boosts job creation, which, in turn contributes to increase employment (matching) in the labour market. It is an implication that the direct effect of workfare policies on programme participants may be a poor indicator of how such policies affect labour market performance. We show that if the unemployment insurance scheme does not include workfare conditions it is always possible to introduce this instrument so as to lower (open and total) unemployment for given benefit level. Moreover, this may be welfare improving even under a utilitarian criterion. Hence, there is theoretical support for the conjecture that workfare makes it possible to combine an incentive structure conducive for job search and creation with a generous unemployment insurance scheme.

There is a vast empirical literature assessing the effects of various forms of active labour market policies. Two recent surveys by Kluve (2010) and Card et al. (2010) summarize more than 100 different evaluations of workfare programmes. The main emphasis in the surveys is on the direct treatment effects of workfare. The treatment effect is typically divided into a locking-in effect during participation in a workfare programme and the programme effect after completing the programme. Whereas the majority of the studies find that most programmes have significant locking-in effects and hence tends to prolong unemployment for participants, the sign of the programme effect is more generally dependent on which type of workfare programme is investigated. There are typically positive effects of private job creation programmes and measures aiming at enhancing job search efficiency, whereas training programmes and public sector employment are less efficient instruments which may even have a negative effect on employment. A recent literature identifies ex ante or treat effects of workfare programmes (see e.g. Black et al. (2003), Geerdsen (2006), Rosholm & Svarer (2008), and van den Berg et al. (2009)). The analysis in this paper points to an important potential shortcoming of these studies, namely that they are partial in nature and therefore overlook important general equilibrium effects of active labour market policies.

The paper is organized as follows: Section 2 sets up the basic model and considers the three effects of workfare, i.e., the locking-in effect, the threat effect, and the wage effect. The overall effects of the two dimensions of workfare (intensity and work requirement) are worked out in section 3, and section 4 provides a numerical illustration of how workfare policies affect labour market performance. Finally, section 5 offers a few concluding remarks and discusses the empirical evidence on the mechanisms analysed in the paper as well as possible extensions of the analysis.

2 A search model with workfare

This section develops a very stylized search model[9] to bring forth some basic effects of workfare as an instrument in labour market policies. Agents are homogeneous but differ in their labour market status, and frictions are associated with transition between labour market states. The model is in continuous time, and all probabilities below are parameters of the underlying Poission process for the random variables.

2.1 Workers

Consider a labour market regime in which the unemployed are entitled to a benefit b when unemployed. Unemployed persons may be required to participate in activation programmes to remain eligible for the benefit. The activation requirement may either be imposed after having claimed unemployment benefits for a certain period of time or at the discretion of the labour market authorities. As argued by Fredriksson and Holmlund (2001, 2006), a fixed time duration can be approximated by a system in which there is a stochastic transition from passive benefits to workfare. A scheme where the activation (duration and type of activity) is decided at the discretion of the authorities would thus, seen from an individual perspective, be a stochastic workfare scheme. The probability that an unemployed is required to participate in workfare[10] - the workfare intensity - is denoted p_{au} $(0 \leq p_{au} \leq 1)$ and the work requirement in number of hours is l_a. These two dimensions of workfare (l_a, p_{au}) are exogenous policy instruments.

The introduction of workfare into the search-matching framework is designed to capture the main components of existing workfare programmes although in a very simplified way. Most OECD countries rely on workfare or active labour market programmes to assist unemployed to become employed (OECD (2010)). In many countries unemployed are not activated until they unsuccessfully have had some time to search for a job. The early phase of an unemployment spell is labelled unemployment in the following. After this initial period a variety of active labour market measures are employed ranging from meetings with case workers over short job search courses to more long-term activities such as class room training or subsidized employment in either the private or public sector. For many unemployed a sequence of these activities are enacted and we therefore treat the period after unemployment as being in activation. This does not imply that the un-

[9] The model structure is closely related to Frederiksson and Holmlund (2001, 2006).

[10] As modelled, here transition from activation is only to employment.

employed are activated all the time, but that they are subjected to different measures until they regain employment. To simplify we describe the period with a constant level of workfare requirement.

Agents search for jobs with intensity s_u when unemployed, and s_a when in activation. The wage rate is denoted w, the tax rate τ, and the work requirement in a job l_e (exogenous). The replacement rate is denoted by $\lambda = b/w$. The instantaneous utility depends on consumption (= disposable income) and leisure ($F_e \equiv 1 - l_e$,$F_i \equiv 1 - l_i - s_i$, $i = a, u$) where the time endowment has been normalized to unity, l denotes time worked, and s time spent searching for jobs), i.e.,

$$\begin{array}{rl} h(w\left[1-\tau\right], 1-l_e) & \text{if employed} \\ h(b[1-\tau], 1-s_u) & \text{if receiving unemployment benefits} \\ h(b[1-\tau], 1-s_a-l_a) & \text{if in activation programme} \end{array}$$

where h is an increasing and concave function. The benefit level is the same in unemployment and activation reflecting the situation in e.g. the Danish labour market, which we will use as illustration in the numerical part of the paper.

Assuming a constant interest rate ρ, it follows that the value functions (in Steady State) associated with the three labour market states are

$$\rho V^E = h(w\left[1-\tau\right], 1-l_e) + p_{ue}\left[V^U - V^E\right] \tag{1}$$

$$\rho V^U = h(b[1-\tau], 1-s_u) + \alpha s_u\left[V^E - V^U\right] + p_{au}\left[V^A - V^U\right] \tag{2}$$

$$\rho V^A = h(b[1-\tau], 1-s_a-l_a) + \alpha s_a\left[V^E - V^A\right] \tag{3}$$

where job offers arrive with probability αs_u for unemployed, and αs_a for workers in activation programmes. α is the job arrival rate conditional on search, and it is endogenous, see below. Note that the employment probabilities are the same for the two groups, provided they exert the same search effort. Hence, there are no human capital differences between the two groups nor any change in human capital from participating in activation.[11] There

[11] These human capital effects may be either positive via forms of training or maintenance of human and social capital, or negative in terms of duration dependent depreciation of these. There is a vast empirical literature studying the effects of different workfare programmes. In recent surveys by Card et al. (2010) and Kluve (2010) most of these studies are summarized. The effects of programmes vary across type of activity. For many programmes the effects are either negative or zero. The most promising results are found for subsidized employment in the private sector, but these constitutes a very limited fraction of all workfare programmes. We therefore follow the empirical literature and impose that participating in a workfare programme does not increase the changes of finding employment.

is an exogenous job separation rate p_{ue}.

From (1), (2) and (3), we have that the value functions for the three labour market states can be written as

$$\begin{aligned}
[\rho + p_{ue}]\, V^E &= h(w\,[1-\tau]\,, 1-l_e) + p_{ue} V^U \\
[\rho + \alpha s_u + p_{au}]\, V^U &= h(b[1-\tau], 1-s_u) + \alpha s_u V^E + p_{au} V^A \\
[\rho + \alpha s_a]\, V^A &= h(b[1-\tau], 1-s_a-l_a) + \alpha s_a V^E.
\end{aligned}$$

To see the role of activation, it is useful to note that the pay-off as unemployed can be written

$$V^U = \frac{\rho + \alpha s_u}{\rho + \alpha s_u + p_{au}} \widehat{V}^U + \frac{p_{au}}{\rho + \alpha s_u + p_{au}} V^A < \widehat{V}^U \tag{4}$$

where $\widehat{V}^U$ is the pay-off to unemployed in the absence of activation ($p_{au} = 0$) given as

$$\widehat{V}^U = \frac{h(b[1-\tau], 1-s_u) + \alpha s_u V^E}{(\rho + \alpha s_u)}.$$

The pay-off for an unemployed (4) is thus a convex combination of the payoffs in the absence of activation and under activation. Hence, workfare can be interpreted as a randomized tightening of the eligibility condition in the unemployment insurance scheme in the sense that with probability p_{au} the individual is required to participate in activation to remain eligible for benefits.

The participation constraint is that employed are always better off than the unemployed

$$V^E - V^U > 0.$$

Note that it is implied that the unemployed are always better off than participants in activation programmes

$$V^U - V^A > 0 \qquad \text{for} \quad p_{au} > 0, \quad l_a > 0.$$

It is obvious that the work requirement ($l_a > 0$) worsens the situation for those in activation. However, it also affects the position for unemployed since there is a possible transition into activation (the threat effect), cf. (5) below. A change in the transition rate from unemployment into activation (p_{au}) does not directly affect those in activation, but it has an effect on the unemployed because it increases the likelihood of changing status from being unemployed to being in activation, cf. (5).

$$\frac{\partial V^A}{\partial l_a}\mid_{V^U,V^E} = -h'_F(b[1-\tau], 1-s_a-l_a) < 0 \qquad \frac{\partial V^A}{\partial p_{au}}\mid_{V^U,V^E} = 0$$

$$\frac{\partial V^U}{\partial l_a}\mid_{V^A,V^E} = \frac{p_{au}}{\rho+\alpha s_u+p_{au}}\frac{\partial V^A}{\partial l_a} < 0 \qquad \frac{\partial V^U}{\partial p_{au}}\mid_{V^A,V^E} = \frac{\rho+\alpha s_u}{[\rho+\alpha s_u+p_{au}]^2}\left[V^A - \widehat{V}^U\right] < 0. \tag{5}$$

A key question for policy design is whether workfare elements can release any incentive effects different from a benefit reduction, cf. the introduction. Although from a utility perspective, there is equivalence between benefit reductions and workfare elements[12], the effects differ across the three groups in the labour market: employed, unemployed and activated. This is so for two reasons. First, although a reduction in benefits would affect unemployed and activated in similar ways, this is not the case for workfare policies. The reason is that workfare has no direct effect on the instantaneous utility for the unemployed ($h(b[1-\tau], 1-s_u)$) but a prospective effect via the risk of ending up in activation (the threat or motivation effect). Hence, changes in work requirements (l_a) and benefits (b) would not have similar effects for the two groups, see below. Secondly, benefit changes and workfare requirements affect search incentives differently. The reason is that work requirements affect the marginal cost of search directly, whereas benefits have an effect via an income effect (see below).

The above suggests that the incentive effects of changing the incidence of workfare (p_{au}) and the work requirement (l_a) differ between the unemployed and those in activation programmes. To see the difference, consider the marginal rate of substitution of the two instruments for given pay-off gains ($V^E - V^U$ and $V^E - V^A$) and search effort (s_u and s_a). As shown in Appendix B, we have that

$$\left.\frac{dp_{au}}{dl_a}\right|_{V^E-V^U=\text{constant}} = -\frac{p_{au}}{\rho+\alpha s_a}\frac{h'_F(b[1-\tau], 1-s_a-l_a)}{V^U-V^A} < 0$$

$$\left.\frac{dp_{au}}{dl_a}\right|_{V^E-V^A=\text{constant}} = \frac{\rho+p_{ue}+p_{au}+\alpha s_u}{p_{ue}}\frac{h'_F(b[1-\tau], 1-s_a-l_a)}{V^U-V^A} > 0$$

which gives the marginal rate of substitution between the transition rate and the workfare requirement for the unemployed and those on workfare, respectively. The intuition for the negative rate of substitution for unemployed is straightforward; increasing the work requirement makes the state of unemployment less attractive due to the possibility of

[12] Since instantaneously utility is increasing in the benefit level and decreasing in the work requirement, it follows that a given utility level can be attained by increasing the work requirement and the benefit level or vice versa.

being transferred to activation, and this can be compensated for a lower incidence of activation. Therefore, for the unemployed, the two instruments are substitutes. For those in activation, the situation is different. A higher work requirement would affect utility negatively, and for the utility differences to be unchanged, the state of employment has to be less attractive, which is the case (due to the risk of job loss) if unemployment is more likely to lead to activation, i.e., p_{au} is higher. Hence, for those on workfare, the two instruments are complements. For both types it is also seen that the marginal rate of substitution depends both on the incidence of workfare (p_{au}) and the work requirement (l_a), suggesting that there may be non-linearities in the effects of the two dimensions.

2.2 Search effort

Individuals choose search effort, taking all macro variables (w, τ, α) as given. Hence the search effort is determined by[13]

$$h'_F(b[1-\tau], 1-s_u) = \alpha\left[V^E - V^U\right] \tag{6}$$

$$h'_F(b[1-\tau], 1-s_a-l_a) = \alpha\left[V^E - V^A\right]. \tag{7}$$

The LHS gives the marginal costs of search and the RHS the marginal gain as the product of the job finding probability α and the utility gain from shifting into employment.

Since $V^U - V^A > 0$, it follows that

$$h'_F(b[1-\tau], 1-s_a-l_a) > h'_F(b[1-\tau], 1-s_u)$$

and therefore

$$s_u < s_a + l_a,$$

i.e., those in activation spend more time in total (activation plus search) than the unemployed (search only), but it is generally ambiguous whether search activity is highest for the unemployed or those in activation ($s_a \lesseqqgtr s_u$).

An important question is whether workers in activation searches less than other unemployed workers. This is the so-called locking-in effect. It follows from (7) that no unambiguous statements can be made due to two counteracting effects. First, activation is time consuming, and this tends to increase the marginal costs of search, and therefore, to lower search effort. Second, activation requirements make activation less attractive

[13] The second order conditions are fulfilled given the concavity of the h-function.

than unemployment ($V^U - V^A > 0$), and therefore, workers in activation have more to gain by becoming employed, which tends to make them search more. Hence, in general it is ambiguous whether there is a locking-in effect. Unemployed and those in activation also react differently to changes in work requirement (see Appendix C for proof of signs)

$$\frac{\partial s_u}{\partial l_a} = \frac{-1}{h''_F(b[1-\tau], 1-s_u)} \frac{\partial \alpha \left[V^E - V^U\right]}{\partial l_a} > 0 \tag{8}$$

$$\frac{\partial s_a}{\partial l_a} = \frac{-1}{h''_F(b[1-\tau], 1-s_a+l_a)} \frac{\partial \alpha \left[V^E - V^A\right]}{\partial l_a} - 1 \lesseqgtr 0. \tag{9}$$

Strengthening the work requirement induces the unemployed to exert more search effort since it increases the marginal gain from becoming employed ($\frac{\partial \alpha \left[V^E - V^U\right]}{\partial l_a} > 0$). A similar effect is present for those in activation, but it is counteracted by the extra time spent in activation. Hence, it is possible that strengthened activation requirements may increase the search effort of unemployed - a threat effect - while decreasing the search effort of those in activation - a locking-in effect. The overall effect on search is therefore ambiguous.

For the incidence or risk of being on workfare (p_{au}), we also find a difference in how it affects the unemployed and those in activation since we have (for proof of signs see Appendix C)

$$\frac{\partial s_u}{\partial p_{au}} = \frac{-1}{h''_F(b[1-\tau], 1-s_u)} \frac{\partial \alpha \left[V^E - V^U\right]}{\partial p_{au}} > 0 \tag{10}$$

$$\frac{\partial s_a}{\partial p_{au}} = \frac{-1}{h''_F(b[1-\tau], 1-s_a+l_a)} \frac{\partial \alpha \left[V^E - V^A\right]}{\partial p_{au}} < 0 \tag{11}$$

i.e. a higher rate of transition into activation induces the unemployed to search more for jobs because the alternative is now less attractive. Oppositely, the search effort of those already in activation decreases since getting a job becomes less attractive (due to the risk of losing it again and ending up in activation).

2.3 Matching

Hiring and transitions into employment are determined via a matching mechanism given as

$$m(s, v)$$

where s denotes effective search and v the vacant jobs (see below).[14] The matching function is assumed to be increasing in both arguments and to display constant returns to scale. Effective or total search is determined by

$$s = s_u u + s_a a$$

where u is the fraction of the population being on unemployment benefits, and a is the fraction in activation. The job finding rate is

$$\alpha = \frac{m(s,v)}{s} = m(1,\theta)$$

where $\theta = \frac{v}{s}$, and hence $\alpha(\theta)$, $\alpha'_\theta(\theta) > 0$. Firms fill vacancies at the rate $q = \frac{m(s,v)}{v} = m(\theta^{-1},1)$, $q'_\theta(\theta) < 0$.

Inflow and outflow into jobs balance in equilibrium, i.e.,

$$[1-u-a]\, p_{ue} = \alpha \left[s_u u + s_a a\right] \tag{12}$$

as they also do for activation, i.e.

$$\alpha s_a a = p_{au} u. \tag{13}$$

2.4 Firms and vacancies

An employed worker produces an output y[15], while the cost of having an unfilled vacancy is ky $(k > 0)$. The value functions for a filled (E) and vacant (V) job, respectively, are

$$\rho J^V = -ky + q(J^E - J^V)$$
$$\rho J^E = y - w + p_{ue}(J^V - J^E).$$

Vacancies are created up to the point where (free entry) $J^V = 0$, implying the following relationship between the wage rate (w) and labour market tightness (θ)

$$w = \left[1 - (\rho + p_{ue})\frac{k}{q(\theta)}\right] y.$$

This relation implies that the higher the wage rate (w), the higher the rate at which firms are filling jobs $(q(\theta))$; i.e. a high wage is associated with a low θ and thus less job

[14] Expressed in per capita terms, i.e., the population is $N = E + U + A$, and $e = E/N, u = U/N$, and $a = A/N$.

[15] We consider a small open economy where output prices are exogenous and set to 1.

creation (fewer vacancies relative to total search effort). Note for later reference that this implies that the job finding rate α is decreasing in the wage rate. The value of a filled job is

$$J^E = \frac{ky}{q}. \tag{14}$$

2.5 Wage determination

The wage rate is assumed to be set in a Nash-bargain between workers and the firm, i.e.,

$$w = \arg\max \ \left[V^E - V^U\right]^\beta \left[J^E - J^V\right]^{1-\beta}$$

where $\beta(\in[0,1])$ is the (exogenous) bargaining power and V^U is taken as given. The first order condition reads

$$\beta\frac{\frac{\partial V^E}{\partial w}}{V^E - V^U} + (1-\beta)\frac{\frac{\partial J^E}{\partial w}}{J^E} = 0$$

where it has been used that $J^V = 0$. The first order condition can be written

$$\Psi(w, \tau, V^E - V^U, q) \equiv \beta\frac{h_w(w\left[1-\tau\right], 1-l_e)}{V^E - V^U} + (1-\beta)\frac{-1}{J^E} = 0 \tag{15}$$

and the second-order condition is

$$\frac{\partial\Psi(w, \tau, V^E - V^U, q)}{\partial w} < 0.$$

Workfare releases a wage effect. To see this, note that

$$\frac{\partial\Psi(w, \tau, V^E - V^U, q)}{\partial p_{au}} = -\beta\frac{h_w(w\left[1-\tau\right], 1-l_e)}{[V^E - V^U]^2}\frac{\partial\left(V^E - V^U\right)}{\partial p_{au}} < 0$$
$$\frac{\partial\Psi(w, \tau, V^E - V^U, q)}{\partial l_a} \equiv -\beta\frac{h_w(w\left[1-\tau\right], 1-l_e)}{[V^E - V^U]^2}\frac{\partial\left(V^E - V^U\right)}{\partial l_a} < 0.$$

Using this and the second-order condition, it follows that

$$\frac{\partial w}{\partial p_{au}} < 0, \frac{\partial w}{\partial l_a} < 0, \tag{16}$$

i.e., an increase in both the intensity and work requirement of workfare tends to lower the wage rate. In other words, both of these changes worsen the outside option and therefore exert a downward pressure on wages. A lower wage tends to make firms create more vacancies, which, in turn, improves matches etc. The wage effect of workfare may thus be important on a par with the direct search effect.

2.6 Public sector

The policy instruments of the government are the benefit level (b), the intensity of workfare (p_{au}), the work requirement (l_a) and the tax rate (τ). The budget constraint for the public sector is

$$\tau w(1-u-a)+\tau\lambda w(u+a)=bu+(b+c)a$$

where c is the cost of activation programmes.[16]

It is shown in Appendix D that the model has a well-defined equilibrium, and conditions ensuring a unique equilibrium are given.[17]

Finally, note that this model has the usual search-matching externalities as well as the moral hazard problem (common pool problem) from a tax financed unemployment benefit scheme.

3 Workfare policies and labour market policies

Our main interest is to explore how workfare policies can affect labour market performance, in particular open (u) and total ($u+a$) unemployment. To clarify the way in which the two dimensions of workfare ($z=l_a, p_{au}$) affect unemployment (u), activation (a) and total unemployment ($t=u+a$) it is useful to note that (see Appendix E):

$$\begin{aligned}
\frac{\partial u}{\partial z} &= -\Omega_{up}\frac{\partial p_{au}}{\partial z}-\Omega_{uu}\frac{\partial \alpha s_u}{\partial z}+\Omega_{ua}\frac{\partial \alpha s_a}{\partial z}\\
\frac{\partial a}{\partial z} &= \Omega_{ap}\frac{\partial p_{au}}{\partial z}-\Omega_{au}\frac{\partial \alpha s_u}{\partial z}-\Omega_{aa}\frac{\partial \alpha s_a}{\partial z}\\
\frac{\partial (u+a)}{\partial z} &= \Omega_{tp}\frac{\partial p_{au}}{\partial z}-\Omega_{tu}\frac{\partial \alpha s_u}{\partial z}-\Omega_{ta}\frac{\partial \alpha s_a}{\partial z}
\end{aligned}$$

where all parameters Ω are positive, except Ω_{tp} which is ambiguously signed, see Appendix E.

The effects of workfare policies (l_a, p_{au}) thus run through effects on the transition rate into activation (note,$\frac{\partial p_{au}}{\partial z}=1$ for $z=p_{au}$, and $=0$ otherwise), and the job finding rate

[16]To the extent that activation programmes result in some output, c could be interpreted as the net costs.

[17]Multiple equilibria cannot be ruled out due to the non-linearities in the model. Potentially, there could be multiple equilibria in the sense of having e.g. an equilibrium with a low tax, low unemployment and high search effort, or an equilibrium with a high tax, high unemployment and low search effort. Since multiplicity of equilibria is not essential for our analysis, we impose fixed taxes in the analytical part. In the numerical part of the paper we allow for endogeneous taxes.

(effective search) for the unemployed ($\frac{\partial \alpha s_u}{\partial z}$) and those in activation ($\frac{\partial \alpha s_a}{\partial z}$). The table below show how changes in these three factors influence the measure of labour market performance $(u, a, u+a)$

Table 1: Sign table for labour market performance and incidence of workfare and search

	Unemployed	Activated	Total unemployment
	u	a	$u+a$
Incidence of workfare ($\frac{\partial p_{au}}{\partial z} > 0$)	$\frac{\partial u}{\partial z} < 0$	$\frac{\partial a}{\partial z} > 0$	$\frac{\partial (u+a)}{\partial z} \lesseqgtr 0$
Job search of unemployed ($\frac{\partial \alpha s_u}{\partial z} > 0$)	$\frac{\partial u}{\partial z} < 0$	$\frac{\partial a}{\partial z} < 0$	$\frac{\partial (u+a)}{\partial z} < 0$
Job search for activated ($\frac{\partial \alpha s_a}{\partial z} > 0$)	$\frac{\partial u}{\partial z} > 0$	$\frac{\partial a}{\partial z} < 0$	$\frac{\partial (u+a)}{\partial z} < 0$

A higher incidence of workfare reduces open unemployment but increases the number in activation, and hence the effect on total unemployment is ambiguous. More search by unemployed tends to reduce both unemployment and the number of activated (fewer transit from unemployment into activation) and hence total unemployment falls. More search by the activated increases open unemployed and lowers the number of activated, and the latter dominates such that total unemployment falls.

Turning to the key transmission mechanism running via job finding rates we have that it can be decomposed into a wage and a search effect since

$$\frac{\partial \alpha(w) s_i}{\partial z} = \frac{\partial \alpha(w)}{\partial w}\frac{\partial w}{\partial z} s_i + \alpha(w)\frac{\partial s_i}{\partial z} \qquad for \quad i = u, a, \;\; z = p_{au}, l_a. \tag{17}$$

Hence, we have that the wage effect ($\frac{\partial \alpha}{\partial w}\frac{\partial w}{\partial z} > 0$ since $\frac{\partial \alpha}{\partial w} < 0$ and $\frac{\partial w}{\partial z} < 0$, cf. Appendix D) unambiguously increases the effective job finding rate for both unemployed and activated, whereas the direct search effect, as shown above, is more complicated and depends on the dimensions of workfare considered. The work requirement increases search for the unemployed ($\frac{\partial s_u}{\partial l_a} > 0$) but has an ambiguous effect for the activated ($\frac{\partial s_u}{\partial l_a} \lesseqgtr 0$)(see (8) and (9)), while increasing the propensity of activation leads to more search for the unemployed ($\frac{\partial s_u}{\partial p_{au}} > 0$) and less for the activated ($\frac{\partial s_a}{\partial p_{au}} < 0$)(see (10) and (11)). This underscores why a focus on the direct search effect of workfare policies may miss an important element of why workfare policies affect labour market performance, namely the wage effect.

A more transparent result on the effect of workfare policies can be gained by considering the introduction of workfare policies into a given unemployment insurance scheme (given benefit level b) which does not feature workfare as an eligibility condition. Introduction of workfare always contribute to improve labour market performance in the sense of lowering both open and total unemployment. To see this, we neutralize the unambiguous direct wage effect (assuming a constant wage) and consider search effects only. It can be shown (see Appendix E) that increasing the incidence of workfare (p_{au}) leads to a fall in unemployment, i.e., $\frac{\partial u}{\partial p_{au}}\Big|_{p_{au}=0} < 0$, and lowers total unemployment $\frac{\partial (u+a)}{\partial p_{au}}\Big|_{p_{au}=0} < 0$ provided that the workfare requirement is not too large, i.e. $l_a < \bar{l}_a$. Similarly, increasing the activity requirement from an initial level of zero leads to a decrease in unemployment ($\frac{\partial u}{\partial l_a}\Big|_{l_a=0} < 0$) and an increase in the number of unemployed on workfare ($\frac{\partial a}{\partial l_a}\Big|_{l_a=0} > 0$), but an overall decrease in the fraction of non-employed ($\frac{\partial (u+a)}{\partial l_a}\Big|_{l_a=0} < 0$) provided that the incidence of workfare is not too large, i.e. $p_{au} < \overline{p}_{au}$. Note that these results indicate that the overall effects of changes in the elements of workfare depend critically on the total policy package, that is, the incidence (p_{au}) and the work requirement (l_a).

Finally, it may be questioned whether an introduction of workfare policies is tantamount to a two-tier benefit scheme with a transition from a high (b) to a low ($b_L < b$) benefit level since this will also induce an incentive effect to search more actively for jobs for those in the first tier (entitlement effect). It is relatively straightforward to show that there is no equivalence between a workfare policy and a two-tier benefit scheme.[18] The intuition is that the two schemes will affect utility and search incentives differently.

4 Numerical illustrations

To analyse how workfare policies affect labour market performance, we provide in this section numerical illustrations of the main effects of the two dimensions of workfare policies. We report the results by means of simulations of the model. To emphasize the wage effect of workfare, we present the results allowing for a decomposition between the total

[18] For this to be the case, the equilibrium attained for a given workfare policy (b, p_{au}, l_a) should be replicated for a scheme with a transition from high benefits (b) to low benefits (b_L) with some probability p_T , i.e., (b, p_T, b_L). Comparing the two schemes for the same transition rate ($p_{au} = p_T$), there should exist benefit levels making the utility level and search effort the same under the two schemes. This requires for utility

$$h(b[1-\tau], 1-s_a-l_a) = h(b_L[1-\tau], 1-s_a)$$

and the search

$$h'_F(b[1-\tau], 1-s_a-l_a) = h'_F(b_L[1-\tau], 1-s_a).$$

Clearly, in general there is no level of b_L satisfying both conditions.

equilibrium effect (termed the full model) and the effect arising when the wage is kept fixed. This can be interpreted as a decomposition of the total equilibrium effect into a search and a wage component, cf. the effects of workfare outlined above. Turning an eye to the empirical side, it might also be more realistic to consider a world where wages are more rigid than in the theoretical model presented in this paper.

In the spirit of Fredriksson & Holmlund (2006), we let the instantaneous utility for type $i = e, u, a$ be given by

$$u_i = \ln c_i + \ln f_i$$

where c denotes consumption and f denotes leisure. Specifically, the utility functions for the three types of agents amount to:

$$\begin{aligned} u_e &= \ln w[1-\tau] + \ln(1-l_e) \\ u_u &= \ln \lambda w[1-\tau] + \ln(1-s_u) \\ u_a &= \ln \lambda w[1-\tau] + \ln(1-s_a-l_a). \end{aligned}$$

Unemployment insurance benefits are proportional to the wage[19]and represented by the replacement ratio $\lambda < 1$, where $\lambda = b/w$.

Again, following among others Fredriksson & Holmlund (2006), the matching function is assumed to be Cobb-Douglas of the form $m = As^{1-\eta}v^{\eta}$, with $\eta = 0.5$ and $A = 0.25$. Also, in the tradition of the search literature, we impose the Hosios condition (Hosios, 1990) and set $\beta = \eta = 0.5$. Note that this removes inefficiencies arising from search and matching externalities, implying that the remaining inefficiency arises from the moral hazard problem in a tax financed unemployment insurance scheme.[20]

Time is quarterly, and to match the features of the Danish labour market, we set the cost of activating the unemployed in workfare programmes, $c = 0.025$, which corresponds to around 3 % of GDP. Unemployment insurance in Denmark is relatively generous, and to accommodate this, the replacement rate is set to $\lambda = 0.5$. We discount utility at $\rho = 0.003$[21] and assume that workers spend 40 % of their time at work, $l_e = 0.4$. The

[19]In Denmark, benefits are indexed to wages.

[20]Kolm and Tonin (2007) consider the efficiency properties in the presence of an earned income tax credit financed by a proportional income tax. They find that if the Hosios condition holds, the tax causes an inefficiency. If the Hosios condition is not fulfilled and employment is inefficiently low, an earned income tax credit (and the implied tax) may ensure efficiency.

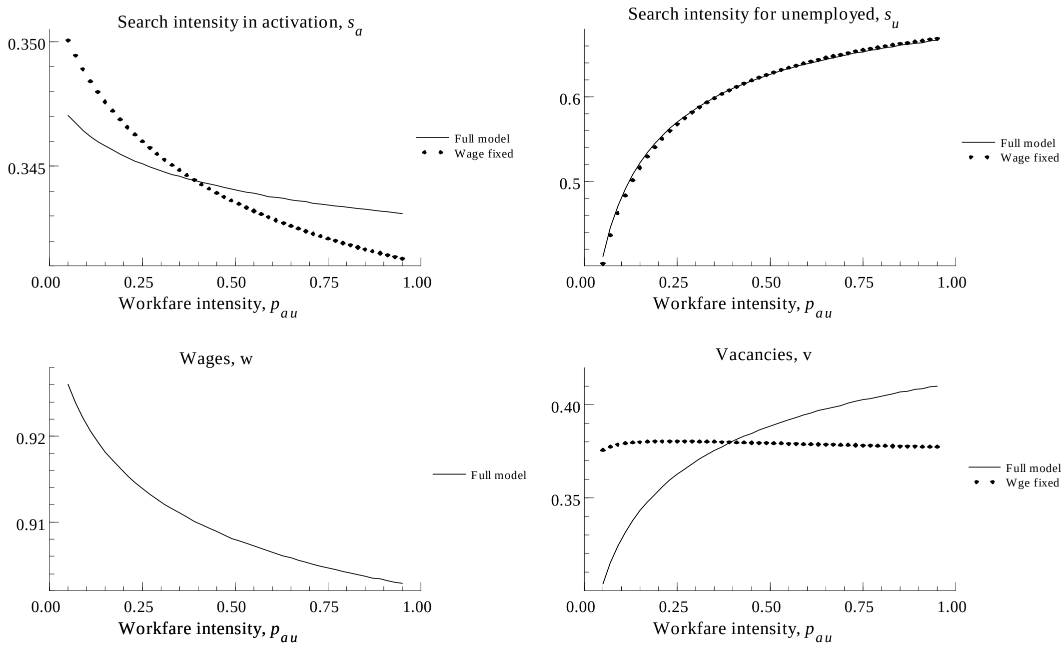
[21]We have also performed the simulations for higher interest rates. The findings, which are available upon request, give rise to the same conclusions.

exogenous exit rate from employment, $p_u e = 0.03$, is set to fit the unemployment rate at around 8 % in Denmark in the period before increased use of workfare programmes (see e.g. Andersen & Svarer, 2007). Finally, output is set to $y = 1$, vacancy costs are set to $k = 0.2$.

4.1 Intensity of workfare (p_{ua})

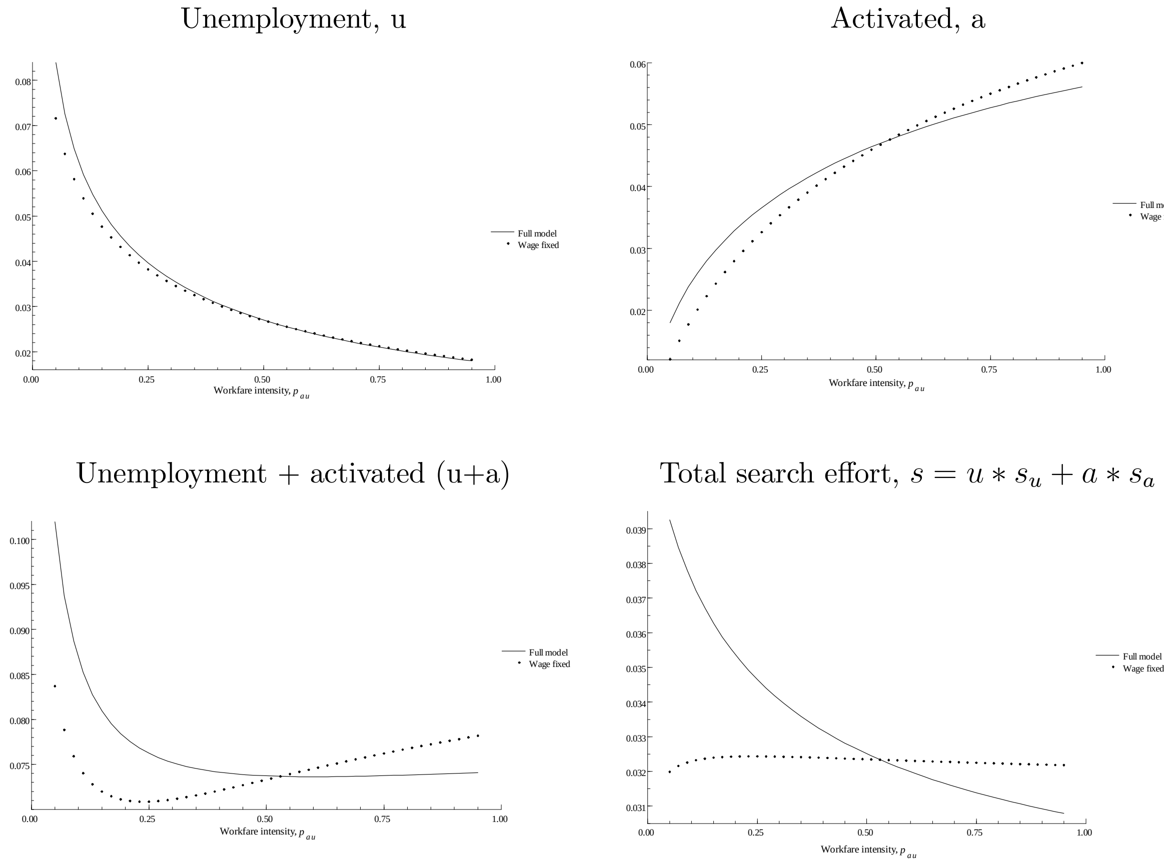
We start by considering the intensity of workfare, i.e. the probability by which unemployed are required to participate in some activation programme to remain eligible for unemployment benefits. We assume for the moment that the workfare requirement corresponds to full time employment ($l_a = l_e = 0.4$).

Figure 1: Effect of workfare intensity on search, wages and vacancies



In Figure 1, we have that unemployed search more than those in activation. Increasing the workfare intensity (p_{au}) induces the unemployed to search more (threat effect) and those in activation to search less (locking in effect), cf (11). Search intensity is basically similar across the simulations with fixed and flexible wages which reflects that the replacement rate is constant. Lower wages and a higher search intensity by the unemployed increase the incentive for employers to create jobs, and the vacancy rate increases. As the search intensities are basically unaffected by the wage drop, the vacancy increase is accordingly mainly driven by the possibility for employers to earn a higher profit per vacancy since the wage cost is reduced (productivity of labour is constant). This is emphasized by the difference in the vacancies for the model where wages are fixed and the model where wages are determined endogenously.

Figure 2: Effect of workfare intensity on labour market status

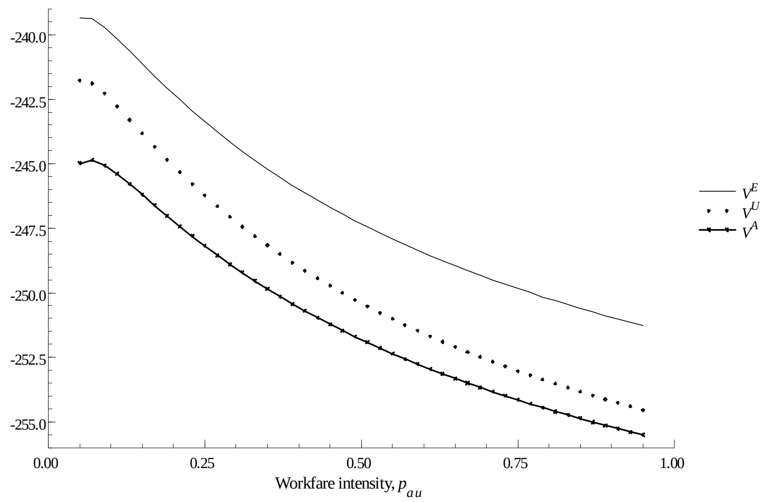


In Figure 2, we see that the increased number of vacancies lowers unemployment and that the drop in unemployment is larger when the wage effect is included. It is, however, also worth noticing that the wage effects is not necessary for the drop in unemployment. The effect on search activity is sufficient to drive unemployment down for low but realistic levels for workfare intensity. Not surprisingly, the number of activated increases, and in the model without the wage effect, it increases even more. Considering total unemployment, we confirm the analytical result derived in Section 3 that increasing the intensity of workfare from an initial low level leads to a decrease in total unemployment. At a high intensity of workfare, a further increase may increase total unemployment. Again, the figures show the importance of allowing for wage effects. In addition, for realistic levels of workfare intensity, total unemployment is lowered by the introduction of workfare. In the Danish labour market, which is the country in the world which uses most resources on workfare programmes, the annual workfare intensity reached a level of 40 % at its peak in 2002 (see Andersen & Svarer (2007)). It is therefore hard to imagine quarterly workfare intensities in excess of around 10 %. To sum up, the figures show that for the given parameter values at a low, but realistic, level of intensity of workfare, the threat and wage effects dominate, while at higher levels the locking-in effect dominates. Total search effort decreases for the particular values of the parameters applied. This finding could very well turn around if the workfare requirement was lowered, giving the activated more time to search.[22]

Note that since the wage decreases, it follows that the economic net gain from finding a job is reduced, and yet unemployment falls. The reason is that workfare makes claiming of benefits less attractive. Considering welfare, we have that the pay-offs in all three labour market states develop similarly; that is, the distributional profile is not changed much, cf. Figure 3.

[22] In the preferred case with $p_{au} = 0.1$ and $l_a = 1$ the hazard rate out of unemployment is 0.34 for the unemployed and 0.25 for the activated. This corresponds to an appr. 30 % lower hazard rate in activation and is consistent with the empirical findings for Denmark presented in Rosholm & Svarer (2008). Also, the hazard rates translates to an expected duration of unemployment of app. 3 quarters. This is close to Rosholm and Svarer (2008) who show that mean unemployment (without activation) is 35 weeks. So in this sense the numerical analysis is close to observed outcomes although the model is highly stylized. Regarding the threat effect it is even more difficult to attach a precise number to the empirical effect. In Rosholm & Svarer (2008) (and the related literature) it is found that the threat effect increases up to the time of activation. The effect is time varying. This feature is not accomodated by the theoretical model since it will imply cumbersome non-linearities in the model. In the preferred model (with $p_{au} = 0.1$ and $l_a = 1$) and 10 % increase in the "hazard into activation" p_{au} implies an increase in the hazard out of unemployment of 3 %. Although we do not have a precise empirical counterpart to this number it does not seem completely off.

Figure 3: Pay-offs in labour market states: employment, unemployment and activation

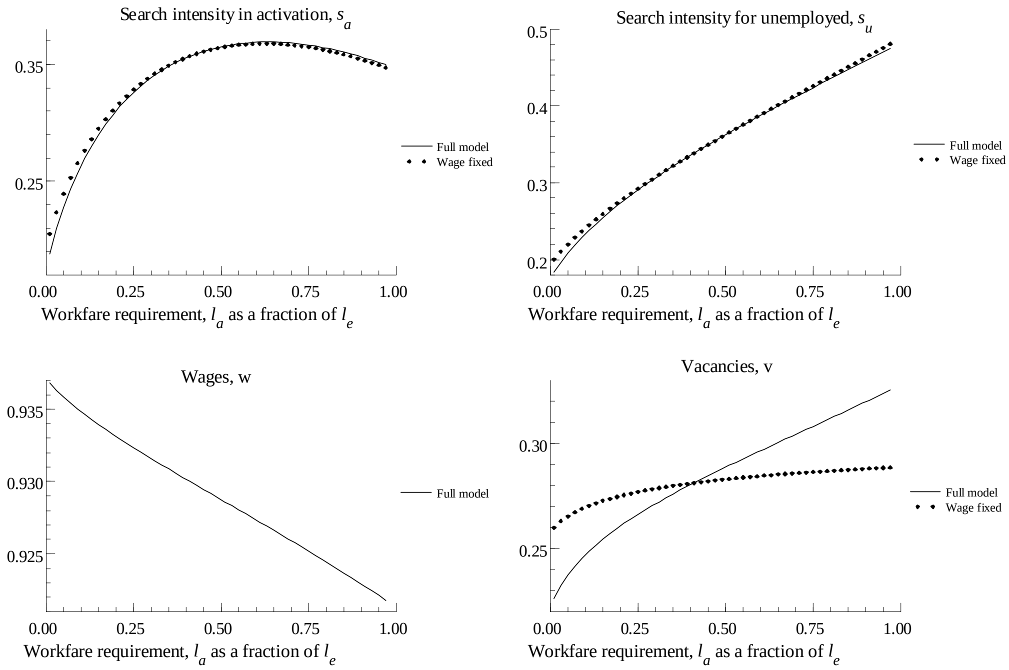


Finally, note that the locking-in effect of workfare on the search effort of the activated and the fact that more people are in activation are both poor indicators of the direction in which unemployment (open and total) moves due to workfare policies. This in turn points to the problems in assessing labour market policies from a partial perspective. The same applies when relating total search to unemployment.

4.2 Work requirements in workfare (l_a)

In this section, the workfare intensity (p_{au}) is fixed at 0.1 which basically corresponds to the current workfare intensity in the Danish labour market, and we show how increasing the work requirement (l_a) affects various labour market outcomes. In Figure 4, it is shown that increasing the workfare requirement increases search activity for the activated at moderate levels of workfare requirement, but it lowers search activity at higher levels of workfare requirement. This pattern reveals that at moderate levels the threat effect dominates, whereas the locking-in effect dominates at higher levels of l_a. The search intensity for the unemployed is strictly increasing in l_a due to the threat effect. The deterioration of the outside option of workers causes a reduction in the wage which induces more vacancies. Notice, the model with fixed wages also gives rise to increased job creation. These effects follow from the increased search intensity of the unemployed and activated which implies that firms fill their vacancies faster, and due to lower expected vacancy costs they create more job openings.

Figure 4: Effects of workfare requirement on search intensity, wages and vacancies



As shown in Figure 5, the increase in vacancies alongside the more intensive search effort by the unemployed lowers open unemployment. For the chosen parameter values, the number of activated is decreasing in the workfare requirement. This result is not general and follows from a combination of increased search intensity and enhanced job creation. In the present version, the locking-in effect is not strong enough to counteract the forces that lower the number of activated.

Figure 5: Effects of workfare requirement on labour market status

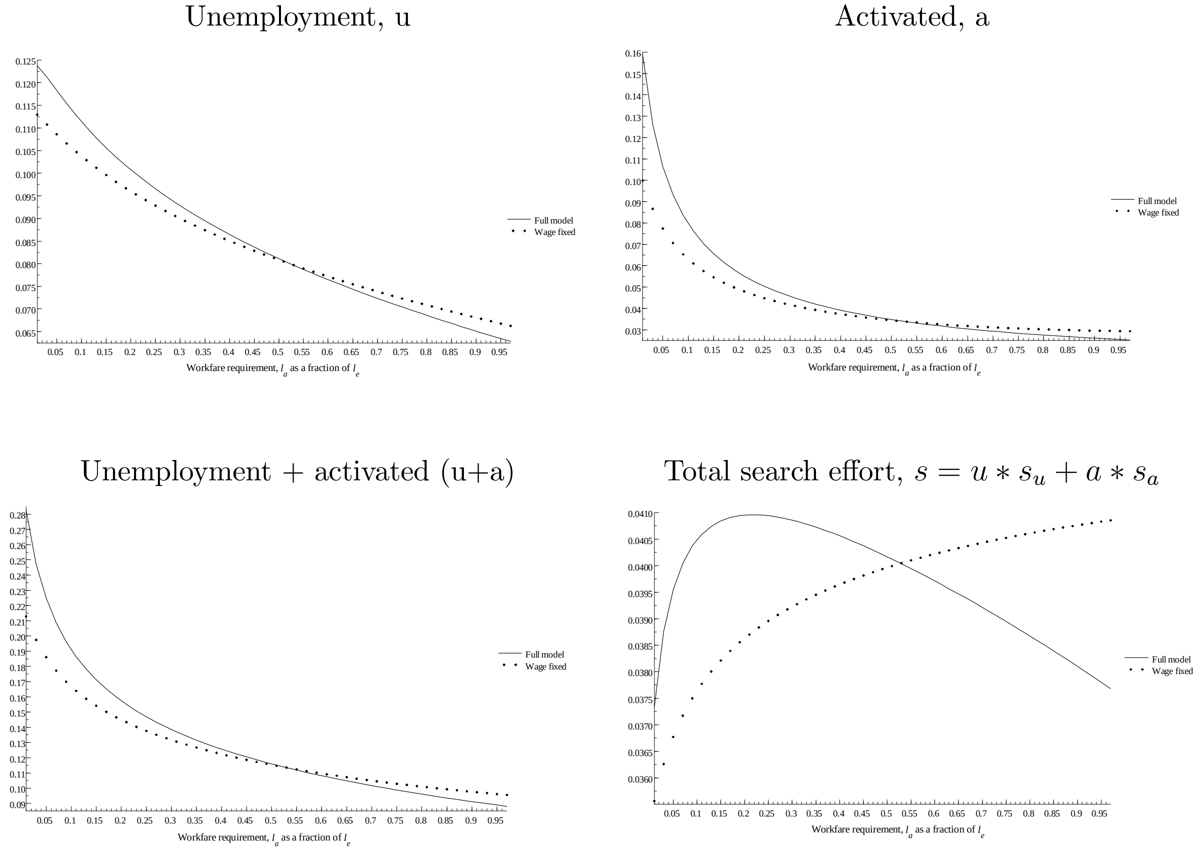
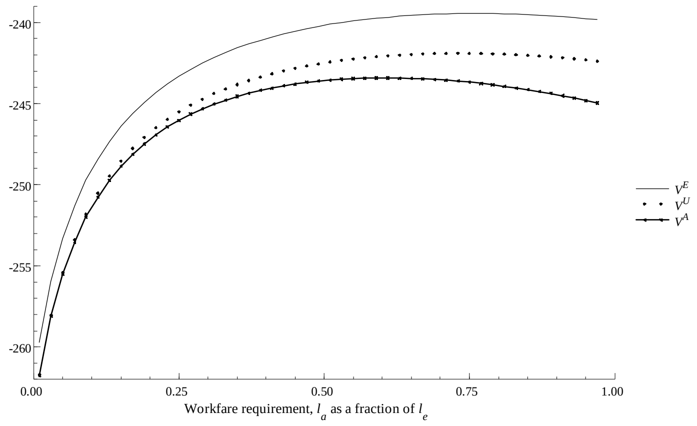


Figure 5 also shows that a higher work requirement tends to lower unemployment via the threat and wage effects. In general, a non-monotone relationship may arise, but for the particular parameter values chosen here, we get that total unemployment is unambiguously decreasing in the work requirement.

Figure 6 shows that, for the given parameter values, increasing the work requirement increases welfare for agents in all three labour market states at moderate levels of workfare requirement. The fact that welfare is improving in the use of workfare shows that the inefficiency caused by the tax financed benefit scheme is counteracted by the introduction of the workfare condition in the benefit scheme. For high, but not unrealistic levels of workfare, i.e. when workfare is similar to full-time employment, there is a non-proportional drop in the value of being activated compared to the two other states in the labour market.

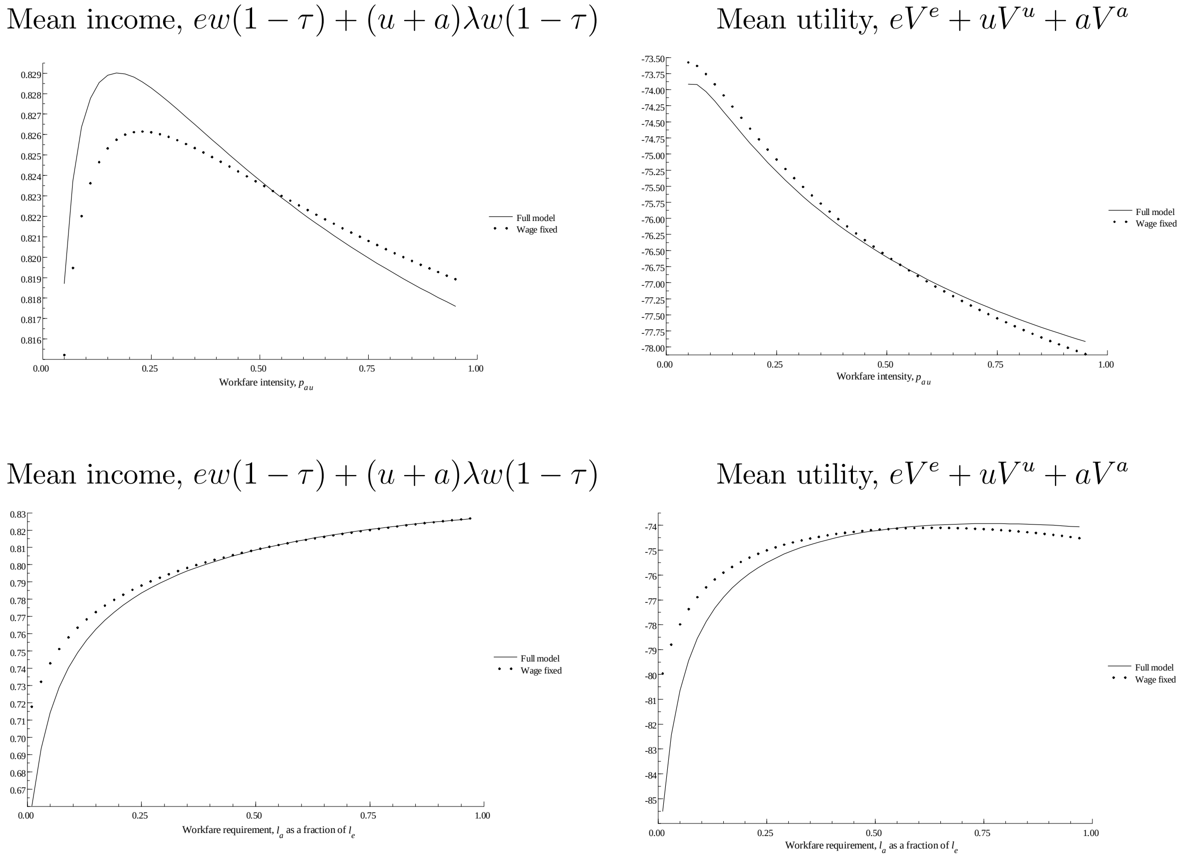
Figure 6: Pay-offs in labour market states: employment, unemployment and activation



4.3 Income and welfare effects

As indicated above, workfare influences both the expected income and expected utility of agents in the model. To summarize the main effects, Figure 7 shows how mean income and mean utility for workers are affected as workfare is intensified. The mean income profiles are increasing at low levels of both workfare intensity and workfare requirement, suggesting that the effects of the wage decrease are countered by the increase in employment. The expected utility profile is declining for increasing workfare intensity, but increasing in workfare requirement.

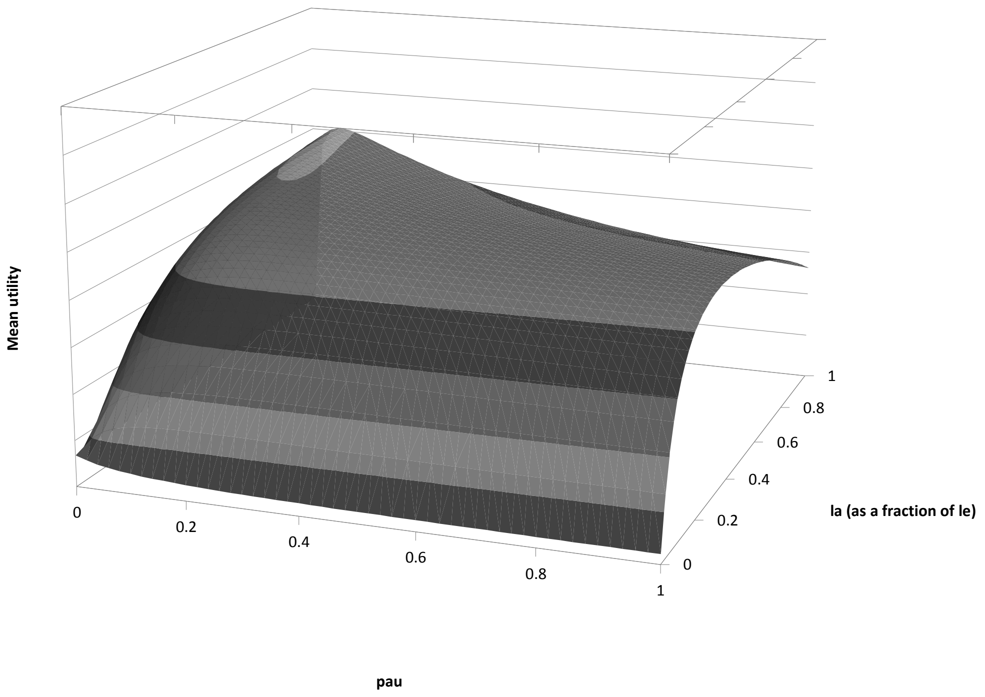
Figure 7: Effects of workfare requirement on mean labour income and mean utility



Based on these findings it may be questioned whether the introduction of workfare in an unemployment insurance scheme can be welfare improving. Assuming that the social welfare function is utilitarian we show in Figure 8 total utility ($eV^E + uV^U + aV^A$) depending on the two dimensions of workfare. It is seen that welfare can be improved by introducing workfare into the unemployment insurance scheme. The optimal workfare policy has a relatively low workfare intensity (p_{au}) but a high workfare requirement (l_a). Intuitively, this implies that a significant threat effect is maintained by making participation in workfare programmes relatively demanding, while minimizing locking-in effects by only imposing few to the requirement. The finding that welfare is maximized at levels where l_a is relatively large compared to p_{au} is consistent with the search effects of the two measures. Increasing both measures will increase search for the unemployed, but for those in activation the effect of increasing workfare intensity is unambiguously to lower search, whereas the effect of increasing work requirement is ambiguous in terms of search. On one hand it releases a threat effect which tends to increase search and on the other hand there is a locking-in effect arising from more effort in activation that tend to decrease search. In the simulations in the paper the treat effect dominates the locking in effect at all levels of l_{au} (see figure 4). It is therefore no surprise that welfare is maximized at high levels of l_{au}. Clearly, the magnitude of these effects depend on the parametric assumptions imposed in the numerical part and the finding does not necessarily generalize to other parametric settings. The illustration do, however, show that workfare may be welfare improving even with ex ante identical agents.

It is far from obvious or unproblematic that a utilitarian welfare function is the proper outset for analysing and understanding workfare policies. The debate on redistribution policy in general and labour market policies in particular is much influenced by process arguments and work norms. At the core of policy discourse on workfare policies is the issue of rights and duties, and it is usually interpreted from a dual perspective involving individuals and society. The individual has the right to social protection but also the duty to provide an effort to become self-supporting, while society has the duty to provide for the "unlucky" but also the right to ask for an effort in return. In this context work norms are an important element, and it is often argued that workfare is both justified by and supporting work norms. None of this has a place under a utilitarian criterion focussing entirely on end results outcomes (consequentialist), cf .e.g Konow (2003). It is well beyond the scope of this paper to go into a detailed analysis of these issues, the primary purpose has been to present a positive analysis of how workfare policies affect labour market performance, in particular open and total unemployment. It is an important issue for future research to analyse the welfare and political justification for workfare policies, and also to compare this type of policy to other labour market policies aiming at affecting employment such as time-dependent benefits and sanctions.

Figure 8: Utilitarian welfare depending on the parameters of workfare policies



5 Concluding remarks

The effect of workfare policies on the incentive structure (the moral hazard problem) has been studied in an equilibrium search model. Although agents are ex ante identical they differ ex post since some are employed, some are unemployed, and some are in activation. As a consequence workfare releases a locking-in-, a threat-, and a wage effect, i.e., it affects the position of all three groups in the labour market. It was found that a change in workfare - both the intensity and the work requirement - may shift the trade-off between insurance and incentives in the labour market. In the analysis, benefits where kept constant[23], and it was shown that workfare could be used to improve the incentive structure, creating more jobs and lowering (open and total) unemployment. From a political-economy perspective this is interesting since it shows that the incentive structure and thus labour market performance can be improved without taking resort to benefit reductions. From a welfare perspective this is also interesting since we have shown

[23] In the numerical analysis, the replacement rate is constant.

that a workfare policy may lead to a welfare improvement (utilitarian criterion)[24].

Empirical assessments of workfare policies tend to focus on the search effects for those in activation, but the present analysis shows that the wage effect may also be important. It is also an implication of the analysis that partial results - theoretical and empirical - of the effects of workfare policies may be a poor metric for the overall effects due to the interplay between the three effects of workfare policies. Taking the simulations presented here at face value implies that policy makers wishing to increase GDP and hence to reduce total unemployment should adopt moderate levels of workfare intensity but more strict work requirements. Moreover, this may increase utilitarian welfare. Intuitively, such a "workfare package" maintains a significant threat effect by making participation in workfare programmes relatively demanding while minimizing locking-in effects by only imposing few unemployed to workfare requirements. This matches well with empirical evidence pointing to workfare having a strong locking-in effect (see e.g. Heckman et al. (1999), Card et al. (2010), and Kluve (2010)).

While workfare policies are popular among policy makers, recent theoretical work has questioned the support for such policies. The larger part of this literature has considered the targeting or selection problem arising in relation to labour supply on the incentive margin or unemployment insurance, while this paper has focussed on the moral hazard problem. It is implied by our findings that workfare policies may have different effects than other labour market policies. Hence, it is an important issue for further research to compare workfare policies to other labour market policies like duration dependent benefit levels, monitoring and sanctions etc.. In this context it should be noted that the present analysis relies on some simplifying assumptions which it will be important to loosen. Workfare has thus been assumed to be "unproductive", but many workfare programmes have the explicit aim of improving the qualifications of the participants. This implies that their job-finding rates may improve which strengthen the case for workfare policies. However, the presence of such policies may also weaken job-search incentives since unemployed wait for programme activities to improve their job finding possibilities. Hence, the net effects are not obvious (see e.g. Sørensen (2009)). Moreover, it is important to consider

[24] In relation to poverty alleviation schemes, Besley and Coate (1992) conclude, that welfare unambiguously declines. Note that Cuff (2000) shows that heterogeneity in preferences may create situations where there is a welfare case for workfare policies even from a utilitarian perspective. It has been shown that this may be the case if workfare activities are productive (Chambers (1989) and Betts (1998)). Kreiner and Tranæs (2005) show in an unemployment insurance context with an adverse selection problem, that it is Pareto-improving to introduce workfare.

a more general specification of the social safety net since unemployment benefits in most countries have an upper time duration after which there is a transition to some form of social assistance. In some countries there are no workfare requirements associated with social assistance while in others there are. It is an open question whether it is optimal to have workfare in both unemployment insurance and social assistance schemes.

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Appendix

A: Utility gains

We have from (1), (2) and (3)

$$(\rho + p_{ue} + \alpha s_u)\left(V^E - V^U\right) = h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_u) + p_{au}\left(V^U - V^A\right)$$
$$(\rho + \alpha s_a)\left(V^E - V^A\right) = h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_a-l_a) + p_{ue}(V^U - V^E)$$

which can be written

$$(\rho + p_{ue} + \alpha s_u)\left(V^E - V^U\right) = h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_u) + p_{au}\left(V^U - V^E + V^E - V^A\right)$$
$$(\rho + \alpha s_a)\left(V^E - V^A\right) = h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_a-l_a) + p_{ue}(V^U - V^E)$$

yielding

$$(\rho + p_{ue} + p_{au} + \alpha s_u)\left(V^E - V^U\right) = h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_u) + p_{au}\left(V^E - V^A\right)$$
$$(\rho + \alpha s_a)\left(V^E - V^A\right) = h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_a-l_a) + p_{ue}(V^U - V^E).$$

It follows that

$$\left(V^E - V^U\right) = \frac{h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_u) + \frac{p_{au}[h(w[1-\tau], 1-l_e) - h(b[1-\tau], 1-s_a-l_a)]}{\rho+\alpha s_a}}{\rho + p_{ue} + p_{au} + \alpha s_u + \frac{p_{au}p_{ue}}{\rho+\alpha s_a}} \tag{18}$$

$$\left(V^E - V^A\right) = \frac{h(w\left[1-\tau\right], 1-l_e) - h(b[1-\tau], 1-s_a-l_a) - \frac{p_{ue}[h(w[1-\tau], 1-l_e) - h(b[1-\tau], 1-s_u)]}{\rho+p_{ue}+p_{au}+\alpha s_u}}{\rho + \alpha s_a + \frac{p_{ue}p_{au}}{\rho+p_{ue}+p_{au}+\alpha s_u}}. \tag{19}$$

B: Marginal rates of substitution

Consider combinations of the transition probability (p_{au}) and work requirement (l_a) leaving the utility gain of employed relative to unemployed unchanged and the utility gain of those on unemployment benefits relative to those on workfare unchanged, i.e.,

$$V^E - V^U = \text{constant}$$
$$V^E - V^A = \text{constant}$$

or

$$d\left(V^E - V^U\right) = 0 = d(V^E - V^A)$$

we have from (18)

$$\left(1+\frac{p_{ue}}{\rho+\alpha s_a}\right)\left(V^E-V^U\right)dp_{au}$$
$$=\frac{dp_{au}}{\rho+\alpha s_a}\left[h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_a-l_a)\right]+\frac{p_{au}}{\rho+\alpha s_a}h_F'(b[1-\tau],1-s_a-l_a)dl_a$$

$$\frac{p_{au}}{\rho+\alpha s_a}h_F'(b[1-\tau],1-s_a-l_a)dl_a$$
$$=\left[\left(1+\frac{p_{ue}}{\rho+\alpha s_a}\right)\left(V^E-V^U\right)-\frac{1}{\rho+\alpha s_a}\left[h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_a-l_a)\right]\right]dp_{au}.$$

Using that

$$\left(\rho+\alpha s_a\right)\left(V^E-V^A\right)-p_{ue}(V^U-V^E)=h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_a-l_a)$$

we get

$$\frac{p_{au}}{\rho+\alpha s_a}h_F'(b[1-\tau],1-s_a-l_a)dl_a$$
$$=\left[\left(1+\frac{p_{ue}}{\rho+\alpha s_a}\right)\left(V^E-V^U\right)-\frac{1}{\rho+\alpha s_a}\left[\left(\rho+\alpha s_a\right)\left(V^E-V^A\right)-p_{ue}(V^U-V^E)\right]\right]dp_{au}$$
$$=\left[\left(V^A-V^U\right)\right]dp_{au}.$$

Hence,

$$\left.\frac{dp_{au}}{dl_a}\right|_U=-\frac{p_{au}}{\rho+\alpha s_a}\frac{h_F'(b[1-\tau],1-s_a-l_a)}{V^U-V^A}<0.$$

Similarly, we have from (19)

$$\left[\frac{p_{ue}\left(\rho+p_{ue}+p_{au}+\alpha s_u\right)-p_{ue}p_{au}}{\left(\rho+p_{ue}+p_{au}+\alpha s_u\right)^2}\right](V^E-V^A)dp_{au}$$
$$=h_F'(b,1-s_a-l_a)dl_a+\frac{p_{ue}}{\left(\rho+p_{ue}+p_{au}+\alpha s_u\right)^2}\left[h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_u)\right]dp_{au}$$

$$\left[\left(\rho+p_{ue}+\alpha s_u\right)\left(V^E-V^A\right)-\left[h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_u)\right]\right]\frac{p_{ue}}{\left(\rho+p_{ue}+p_{au}+\alpha s_u\right)^2}dp_{au}$$
$$=h_F'(b,1-s_a-l_a)dl_a$$

using that

$$\left(\rho+p_{ue}+p_{au}+\alpha s_u\right)\left(V^E-V^U\right)=h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_u)+p_{au}\left(V^E-V^A\right)$$

we get

$$\left[(\rho+p_{ue}+\alpha s_u+p_{au})\left(V^E-V^A\right)-(\rho+p_{ue}+p_{au}+\alpha s_u)\left(V^E-V^U\right)\right]\frac{p_{ue}}{(\rho+p_{ue}+p_{au}+\alpha s_u)^2}dp_{au}$$

$$=h_F'(b[1-\tau],1-s_a-l_a)dl_a$$

$$-\left(V^A-V^U\right)\frac{p_{ue}}{\rho+p_{ue}+p_{au}+\alpha s_u}dp_{au}=h_F'(b[1-\tau],1-s_a-l_a)dl_a.$$

Hence,

$$\left.\frac{dp_{au}}{dl_a}\right|_A=\frac{\rho+p_{ue}+p_{au}+\alpha s_u}{p_{ue}}\frac{h_F'(b[1-\tau],1-s_a-l_a)}{V^U-V^A}>0.$$

C: Impact effects of changes in workfare policies

To see the effects of workfare policies, it is useful to consider the impact effects of changes in the two elements of workfare, namely the transition probability (p_{au}) and work requirement (l_a) on the utility gains for given macro variables (w,α,τ). We have

$$(\rho+p_{ue}+p_{au}+\alpha s_u)\left(V^E-V^U\right)=h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_u)+p_{au}\left(V^E-V^A\right)$$

$$(\rho+\alpha s_a)\left(V^E-V^A\right)=h(w\left[1-\tau\right],1-l_e)-h(b[1-\tau],1-s_a-l_a)+p_{ue}(V^U-V^E).$$

Hence,

$$\begin{aligned}\frac{\partial\left(V^E-V^U\right)}{\partial l_a}&=\frac{p_{au}}{\rho+p_{ue}+p_{au}+\alpha s_u}\frac{\partial\left(V^E-V^A\right)}{\partial l_a}\\ \frac{\partial\left(V^E-V^A\right)}{\partial l_a}&=\frac{1}{\rho+\alpha s_a}h_F'(b,1-s_a-l_a)-\frac{p_{ue}}{\rho+\alpha s_a}\frac{\partial\left(V^E-V^U\right)}{\partial l_a}\\ \frac{\partial\left(V^E-V^U\right)}{\partial p_{au}}&=\frac{1}{\rho+p_{ue}+p_{au}+\alpha s_u}\left[\left(V^U-V^A\right)+p_{au}\frac{\partial\left(V^E-V^A\right)}{\partial p_{au}}\right]\\ \frac{\partial\left(V^E-V^A\right)}{\partial p_{au}}&=-\frac{p_{ue}}{\rho+\alpha s_a}\frac{\partial\left(V^E-V^U\right)}{\partial p_{au}}.\end{aligned}$$

Hence, we have

$$\begin{array}{ll}\frac{\partial\left(V^E-V^U\right)}{\partial l_a}>0 & \frac{\partial\left(V^E-V^U\right)}{\partial p_{au}}>0\\ \frac{\partial\left(V^E-V^A\right)}{\partial l_a}>0 & \frac{\partial\left(V^E-V^A\right)}{\partial p_{au}}<0.\end{array}$$

D: Equilibrium

The model can be summarized by the following 10 equations in the following endogenous variables: $V^E-V^U, V^E-V^A, s_a, s_u, a, u, w, q, \alpha, \theta$.

Pay-off gains:

$$V^E - V^U = \frac{h(w\,[1-\tau]\,, 1-l_e) - h(b[1-\tau], 1-s_u) + p_{au}\left(V^U - V^A\right)}{\rho + p_{ue} + \alpha s_u} \tag{20}$$

$$V^E - V^A = \frac{h(w\,[1-\tau]\,, 1-l_e) - h(b[1-\tau], 1-s_a-l_a) + p_{ue}(V^U - V^E)}{\rho + \alpha s_a}. \tag{21}$$

Search effort:

$$h'_F(b[1-\tau], 1-s_u) = \alpha\left[V^E - V^U\right] \tag{22}$$

$$h'_F(b[1-\tau], 1-s_a-l_a) = \alpha\left[V^E - V^A\right]. \tag{23}$$

Inflow and outflows:

$$[1-u-a]\,p_{ue} = \alpha\,(\theta)\,[s_u u + s_a a] \tag{24}$$

$$\alpha\,(\theta)\,s_a a = p_{au} u. \tag{25}$$

Job creation and wage setting:

$$w = \left[1 - (\rho + p_{ue})\,\frac{k}{q}\right] y \tag{26}$$

$$0 = \beta\frac{u_w(w(1-\tau), 1-l_e)}{V^E - V^U} - (1-\beta)\frac{q}{ky}. \tag{27}$$

Note that the job creation equation (26) requires $q > (\rho + p_{ue})\,k$ and that it is a condition on wage formation (27) that $w \leq y$. Job-finding and job-filling rates:

$$\alpha = \frac{m(s,v)}{s} = m(1,\theta), m_\theta > 0 \tag{28}$$

$$q = \frac{m(s,v)}{v} = m(\theta^{-1}, 1), q'_\theta(\theta) < 0; \theta = \frac{v}{s}. \tag{29}$$

Note that it is required that $0 \leq \alpha \leq 1$ and $0 \leq q \leq 1$. Using (26), (28) and (29), we have

$$q = \widetilde{q}(w) \qquad \frac{\partial \widetilde{q}(w)}{\partial w} > 0$$

$$\alpha = \widetilde{\alpha}(w) \qquad \frac{\partial \widetilde{\alpha}(w)}{\partial w} < 0$$

$$\theta = \widetilde{\alpha}(w) \qquad \frac{\partial \widetilde{\theta}(w)}{\partial w} < 0.$$

Search activities can from (22) and (22) be written

$$s_u = \Phi\left(\widetilde{\alpha}(w)\left[V^E - V^U\right]\right)$$
$$s_a = \Phi\left(\widetilde{\alpha}(w)\left[V^E - V^A\right]\right) - l_a$$

and using this in (20) and (21), it follows that utility gains can be written

$$V^E - V^U = F(w, \tau, \widetilde{\alpha}(w), p_{au}, l_a)$$
$$V^E - V^A = G(w, \tau, \widetilde{\alpha}(w), p_{au}, l_a).$$

Finally, the wage equation (26) can now be written

$$\beta \frac{u_w(w(1-\tau), 1-l_e)}{V^E - V^U} - (1-\beta)\frac{q(\theta)}{ky} = 0$$

or [25]

$$\begin{aligned} 0 &= \Psi(w, \tau, \widetilde{q}(w), V^E - V^U) \qquad \Psi_w < 0, \Psi_{V^E - V^U} < 0 \\ &= \Psi(w, \tau, \widetilde{q}(w), F(w, \tau, \widetilde{\alpha}(w), p_{au}, l_a)). \end{aligned} \tag{30}$$

For a given tax rate τ, the equilibrium wage is found as the solution to (30). If the function Ψ is monotonously decreasing in the wage rate, it follows that the equilibrium is unique. This implies that $\frac{\partial w}{\partial p_{au}} < 0$, $\frac{\partial w}{\partial l_a} < 0$. Note that an equilibrium where $0 < u + a < 1$ is ensured since if $u + a = 0$, we have $u = a = 0$, implying that $\widetilde{\alpha}(w) = p_{ue}$ which is inconsistent with (29), and for $u + a = 1$, we have $\widetilde{\alpha}(w) = 0$ which is also inconsistent with (29).

Note that endogenizing the tax rate would introduce a non-linearity in the model which potentially could imply multiple equilibria since

$$\tau = \frac{bu + (b+c)a}{w(1-u-a)}.$$

For this reason, the tax rate is assumed constant in the theoretical analysis (assuming that the budget is balanced via a lump sum tax on firms), but the tax rate is endogenized in the numerical examples.

E: Effects of changes in workfare policies

In this appendix, we consider how changes in workfare policies (p_{au}, l_a) affect unemployment (u), activation (a) and total unemployment $(u + a)$.

[25] Where $\Psi_w < 0$ follows from the second order condition.

We have from (12) and (13) that

$$[1-u-a]\,p_{ue} = \alpha\,[s_u u + s_a a]$$
$$\alpha s_a a = p_{au} u.$$

Hence,

$$\begin{aligned} 0 &= (p_{ue}+\alpha s_u)\frac{\partial u}{\partial z} + (p_{ue}+\alpha s_a)\frac{\partial a}{\partial z} + u\frac{\partial \alpha s_u}{\partial z} + a\frac{\partial \alpha s_a}{\partial z} \\ \alpha s_a\frac{\partial a}{\partial z} + a\frac{\partial \alpha s_a}{\partial z} &= p_{au}\frac{\partial u}{\partial z} + u\frac{\partial p_{au}}{\partial z} \end{aligned}$$

and it follows that

$$\begin{aligned} 0 &= (p_{ue}+\alpha s_u)\frac{\partial u}{\partial z} + \frac{p_{ue}+\alpha s_a}{\alpha s_a}\left[p_{au}\frac{\partial u}{\partial z} + u\frac{\partial p_{au}}{\partial z} - a\frac{\partial \alpha s_a}{\partial z}\right] + u\frac{\partial \alpha s_u}{\partial z} + a\frac{\partial \alpha s_a}{\partial z} \\ \frac{\partial u}{\partial z} &= \frac{-\frac{p_{ue}+\alpha s_a}{\alpha s_a}u\frac{\partial p_{au}}{\partial z} - u\frac{\partial \alpha s_u}{\partial z} + a\frac{p_{ue}}{\alpha s_a}\frac{\partial \alpha s_a}{\partial z}}{(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}. \end{aligned}$$

Considering the effect on activation, we have

$$\begin{aligned} \frac{\partial a}{\partial z} &= \frac{1}{\alpha s_a}\left[p_{au}\frac{\partial u}{\partial z} + u\frac{\partial p_{au}}{\partial z} - a\frac{\partial \alpha s_a}{\partial z}\right] \\ &= \frac{1}{\alpha s_a}\left[\frac{p_{au}\left[-\frac{p_{ue}+\alpha s_a}{\alpha s_a}u\frac{\partial p_{au}}{\partial z} - u\frac{\partial \alpha s_u}{\partial z} + a\frac{p_{ue}}{\alpha s_a}\frac{\partial \alpha s_a}{\partial z}\right] + \left[(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}\right]\left[u\frac{\partial p_{au}}{\partial z} - a\frac{\partial \alpha s_a}{\partial z}\right]}{(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}\right] \\ &= \frac{1}{\alpha s_a}\left[\frac{(p_{ue}+\alpha s_u)\,u\frac{\partial p_{au}}{\partial z} - up_{au}\frac{\partial \alpha s_u}{\partial z} - [p_{ue}+\alpha s_u + p_{au}]\,a\frac{\partial \alpha s_a}{\partial z}}{(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}\right] \end{aligned}$$

and combining the two, the effect on total unemployment is found to be

$$\frac{\partial u}{\partial z} + \frac{\partial a}{\partial z} = \frac{\left[\frac{\alpha(s_u-s_a)}{\alpha s_a}\right]u\frac{\partial p_{au}}{\partial z} - \left[1+\frac{p_{au}}{\alpha s_a}\right]u\frac{\partial \alpha s_u}{\partial z} - \left[\frac{\alpha s_u + p_{au}}{\alpha s_a}\right]a\frac{\partial \alpha s_a}{\partial z}}{(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}.$$

In summary we thus have

$$\begin{aligned} \frac{\partial u}{\partial z} &= \frac{-\frac{p_{ue}+\alpha s_a}{\alpha s_a}u\frac{\partial p_{au}}{\partial z} - u\frac{\partial \alpha s_u}{\partial z} + a\frac{p_{ue}}{\alpha s_a}\frac{\partial \alpha s_a}{\partial z}}{(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}} \\ \frac{\partial a}{\partial z} &= \frac{1}{\alpha s_a}\left[\frac{(p_{ue}+\alpha s_u)\,u\frac{\partial p_{au}}{\partial z} - up_{au}\frac{\partial \alpha s_u}{\partial z} - [p_{ue}+\alpha s_u + p_{au}]\,a\frac{\partial \alpha s_a}{\partial z}}{(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}\right] \\ \frac{\partial (u+a)}{\partial z} &= \frac{\left[\frac{\alpha(s_u-s_a)}{\alpha s_a}\right]u\frac{\partial p_{au}}{\partial z} - \left[1+\frac{p_{au}}{\alpha s_a}\right]u\frac{\partial \alpha s_u}{\partial z} - \left[\frac{\alpha s_u + p_{au}}{\alpha s_a}\right]a\frac{\partial \alpha s_a}{\partial z}}{(p_{ue}+\alpha s_u) + \frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}. \end{aligned}$$

Which in compact form can be written

$$\frac{\partial u}{\partial z} = -\Omega_{up}\frac{\partial p_{au}}{\partial z} - \Omega_{\mathrm{u}\,u}\frac{\partial \alpha s_u}{\partial z} + \Omega_{ua}\frac{\partial \alpha s_a}{\partial z}$$
$$\frac{\partial a}{\partial z} = \Omega_{ap}\frac{\partial p_{au}}{\partial z} - \Omega_{au}\frac{\partial \alpha s_u}{\partial z} - \Omega_{aa}\frac{\partial \alpha s_a}{\partial z}$$
$$\frac{\partial (u+a)}{\partial z} = \Omega_{tp}\frac{\partial p_{au}}{\partial z} - \Omega_{tu}\frac{\partial \alpha s_u}{\partial z} - \Omega_{ta}\frac{\partial \alpha s_a}{\partial z}$$

where

$$\Omega_{up} = \frac{\frac{p_{ue}+\alpha s_a}{\alpha s_a}}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}} > 0 \qquad \Omega_{\mathrm{u}\,u} = \frac{u}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}} > 0 \qquad \Omega_{ua} = \frac{a\frac{p_{ue}}{\alpha s_a}}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}}$$

$$\Omega_{ap} = \frac{1}{\alpha s_a}\left[\frac{(p_{ue}+\alpha s_u)u}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}\right] > 0 \quad \Omega_{au} = \frac{1}{\alpha s_a}\left[\frac{up_{au}}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}}\right] > 0 \quad \Omega_{aa} = \frac{1}{\alpha s_a}\left[\frac{[p_{ue}+\alpha s_u}{(p_{ue}+\alpha s_u)+}\right.$$

$$\Omega_{tp} = \frac{\left[\frac{\alpha(s_u-s_a)}{\alpha s_a}\right]u}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}} \lesseqgtr 0 \qquad \Omega_{tu} = \frac{\left[1+\frac{p_{au}}{\alpha s_a}\right]u}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}p_{au}} > 0 \qquad \Omega_{ta} = \frac{\left[\frac{\alpha s_u+p_{au}}{\alpha s_a}\right]a}{(p_{ue}+\alpha s_u)+\frac{p_{ue}+\alpha s_a}{\alpha s_a}}$$

In the following, we consider the incremental introduction of workfare policies into a benefit scheme without workfare. We neutralize the unambiguous wage effect and focus on the search effects which in general are ambiguous to show that they are unambiguous for an incremental introduction of workfare. The question addressed is thus whether a marginal introduction of workfare will lower unemployment (both open and total, i.e. u and $u+a$) if starting with a benefit scheme without workfare elements. This can happen in one of two ways, either having an initial situation where $(p_{au}, l_a) = (0, l_a)$ and then rising p_{au} marginally, or having an initiation situation $(p_{au}, l_a) = (p_{au}, 0)$ and then rising l_a marginally. The wage rate w and thus the job finding rate α are constant to focus on the search effect.

(I) Incidence of workfare

Note that for $p_{au} = 0$, we have $a = 0$, and hence

$$\alpha s_a \left.\frac{\partial a}{\partial p_{au}}\right|_{p_{au}=0} = u$$
$$(p_{ue}+\alpha s_u)\left.\frac{\partial u}{\partial p_{au}}\right|_{p_{au}=0} = -(p_{ue}+\alpha s_a)\left.\frac{\partial a}{\partial p_{au}}\right|_{p_{au}=0} + u\alpha\left.\frac{\partial s_u}{\partial p_{au}}\right|_{p_{au}=0}.$$

Implying

$$\left.\frac{\partial u}{\partial p_{au}}\right|_{p_{au}=0} = -\frac{(p_{ue}+\alpha s_a)\frac{u}{\alpha s_a} + u\alpha \left.\frac{\partial s_u}{\partial p_{au}}\right|_{p_{au}=0}}{(p_{ue}+\alpha s_u)} < 0$$

where $\frac{\partial s_u}{\partial p_{au}}\,|_{p_{au}=0} > 0$ follows from (10).

$$\left.\frac{\partial (u+a)}{\partial p_{au}}\right|_{p_{au}=0} = -\frac{u\alpha\left.\frac{\partial s_u}{\partial p_{au}}\right|_{p_{au}=0}}{(p_{ue}+\alpha s_u)} + \left[1-\frac{(p_{ue}+\alpha s_a)}{(p_{ue}+\alpha s_u)}\right]\frac{u}{\alpha s_a}.$$

Note that this is negative for $l_a = 0$, hence, there exists a $\bar{l}_a$ such that $\left.\frac{\partial(u+a)}{\partial p_{au}}\right|_{p_{au}=0} < 0$.

(II) work requirement

Note that if $l_a = 0$, agents are similarly situated as unemployed and in activation ($s_a = s_u$), and we have

$$0 = (p_{ue} + \alpha s_u)\left[\frac{\partial u}{\partial l_a} + \frac{\partial a}{\partial l_a}\right] + u\alpha\frac{\partial s_u}{\partial l_a} + a\alpha\frac{\partial s_a}{\partial l_a}$$

$$\alpha s_a\frac{\partial a}{\partial l_a} + a\alpha\frac{\partial s_a}{\partial l_a} = p_{au}\frac{\partial u}{\partial l_a}$$

hence

$$0 = (p_{ue} + \alpha s_u)\frac{\partial u}{\partial l_a} + \frac{(p_{ue} + \alpha s_u)}{\alpha s_a}\left[p_{au}\frac{\partial u}{\partial l_a} - a\alpha\frac{\partial s_a}{\partial l_a}\right] + u\alpha\frac{\partial s_u}{\partial l_a} + a\alpha\frac{\partial s_a}{\partial l_a}$$

or

$$\left[(p_{ue} + \alpha s_u) + \frac{(p_{ue} + \alpha s_u)}{\alpha s_a}p_{au}\right]\frac{\partial u}{\partial l_a} = \left[\frac{(p_{ue} + \alpha s_u)}{\alpha s_a} - 1\right]a\alpha\frac{\partial s_a}{\partial l_a} - u\alpha\frac{\partial s_u}{\partial l_a}.$$

We have $\frac{(p_{ue}+\alpha s_u)}{\alpha s_a} - 1 > 0$, and hence $\frac{\partial u}{\partial l_a} < 0$ has as a sufficient condition $\alpha\frac{\partial s_a}{\partial l_a} < 0$. Note that (8) implies that $\alpha\frac{\partial s_a}{\partial l_a} < 0$. For $p_{au} = 0$, we have $a = 0$, and $\frac{\partial u}{\partial l_a} < 0$ and $\frac{\partial u}{\partial l_a} + \frac{\partial a}{\partial l_a} < 0$. Moreover, there is a $\overline{p}_{au}$ such that for $p_{au} < \overline{p}_{au}$, we have $\frac{\partial u}{\partial l_a} < 0$ and $\frac{\partial u}{\partial l_a} + \frac{\partial a}{\partial l_a} < 0$.