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Tutorial - 4

Q1- $T(n) = 3T(n/2) + n^2$

Ans1 -

$a = 3$

$b = 2$

as $a > 1$, $b > 1$

$\Rightarrow c = \log_b a = \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.53}$

as $f(n) > n^c$

$T(n) = \Theta\, f(n)$

$= \Theta\, (n^2)$ Ans

Q2- $T(n) = 4T(n/2) + n^2$

Ans2 -

$a = 4$

$b = 2$

as $a > 1$, $b > 1$

$\rightarrow c = \log_b a = \log_2 4 = 2 \rightarrow n^c = n^2$

so, as $f(n) = n^c$

$\rightarrow T(n) = \Theta\, (n^2 \log n)$ Ans

Q3 - $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2$

$\because$ the following (f) is exponential & not polynomial it can't be solved

Q4- $T(n) = 2^n\, T(n/2) + n^n$

$\rightarrow$ Master theorem does not apply here because 'a' is not constant

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Q5 - $T(n) = 16T(n/4) + n$

$a = 16$

$b = 4$

$\rightarrow c = \log_b a = \log_4 16 = \frac{1.204}{0.60} = 2$

So, $n^c = n^2$

as $n^c > f(n)$

$T(n) = \Theta(n^2)$

Q6 - $T(n) = 2T(n/2) + n \log n$

$a = 2$

$b = 2$

$a > 1$, $b > 1$ $\Rightarrow c = \log_b a = \log_2 2 = 1 \rightarrow n^c = 1$

$\rightarrow n^c = 1$

Assuming $f(n)$ i.e, $n \log(n) > -1$

$\rightarrow T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$= \Theta(n^1 \log^{1+1} n)$

$= \Theta(n \log^2 n)$ Ans

Q7 - $T(n) = 2T(n/2) + n/\log n$

$\because$ Master theorem ~~does not apply here because~~ applies to form that are polynomial, $n/\log n$ is not polynomial as master theorem does not apply.

Q8 - $T(n) = 2T(n/4) + n^{0.51}$

$a = 2$

$b = 4$

$\rightarrow c = \log_b a = \log_4 2 = \frac{\log(2)}{\log(4)} = \frac{0.30}{0.60} = 0.5$

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as $f(n) > n^2$

$\rightarrow T(n) = \Theta f(n) = \Theta(n^{0.51})$ Ans

Q9 - $T(n) = 0.5T(n/2) + 1/n$

$\rightarrow$ Does not apply $\because a < 1$

Q10 - $T(n) = 16T(n/4) + n!$

$a = 16$

$b = 4$

$\rightarrow c = \log_b a = \log_4 16 = \frac{\log (2)^4}{\log 4} = \frac{4 \log 2}{\log 4} = \frac{4 \times 0.30}{0.60}$

$\rightarrow c = 2$

as $n^c = n^2$

but $f(n) = n^2$

$\rightarrow T(n) = \Theta f(n)$

$= \Theta\ f(n!)$ Ans

Q11 - $T(n) = \sqrt{2}\ T(n/2) + \log n$

$a = \sqrt{2}$

$b = 2$

$c = \log_b a = \log_2 (\sqrt{2}) = \frac{\frac{1}{2} \log 2}{\log 2} = \frac{1}{2} = 0.5$

So, $n^2 = n^{0.5}$

As $f(n) = \log n$

as $f(n) < n^c$

$T(n) = \Theta(n^c) = \Theta(n^{0.5})$ Ans

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Q12 - $T(n) = \sqrt{(n)}\ T(n/2) + \log n$

$\because$ a is not a constant, following form cannot be solved.

Q13 - $T(n) = 3T(n/2) + n$

$a = 3$

$b = 2$

$\rightarrow c = \log_b a = \log_e 3 = \frac{0.69}{0.30} = 2.3$

as $n^c > f(n)$

$\Rightarrow T(n) = \Theta(n^c) = \Theta(n^{2.3})$ Ans

Q14 - $T(n) = 3T(n/3) + (\sqrt{n})$

$a = 3$

$b = 3$

$\rightarrow c = \log_b a = \log_2 3 = 1 \Rightarrow n^c = n$

as $n^c < f(n)$ i.e. $n^1 > n^{1/2}$

$\Rightarrow T(n) = \Theta(f(n))$

$= \Theta(n)$ As

Q15 - $T(n) = 4T(n/2) + (n)$

$a = 4, b = 2$

$c = \log_2 4 = 2$

$n^c = n^2 > f(n)$

$T(n) = \Theta(n^2)$

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Q16 - $T(n) = 3T(n/4) + n\log n$

$a = 3, \ b = 4$

$c = \log_4 3 = 0.79$

$n^c = n^{0.79} < f(n)$

$T(n) = \Theta(n \log(n))$

Q17 - $T(n) = 3T(n/3) + n/2$

$a = 3, \ b = 3$

$c = \log_3 3 = 1$

$n^c = n > f(n)$

$T(n) = \Theta(n)$

Q18 - $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \ b = 3$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63} < f(n)$

$T(n) = \Theta(n^2 \log n)$

Q19 - $T(n) = 4T(n/2) + n/\log n$

$a = 4, \ b = 2$

$c = \log_2 4 = 2$

$n^c = n^2 > f(n)$

$\Rightarrow T(n) = \Theta(n^2)$

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Q20- $T(n) = 64\,T(n/8) - n^2 \log n$

$a = 64$, $b = 8$

$c = \log_8 64 = 2$

$n^c = n^2 < f(n)$

$T(n) = \Theta(n^2 \log 1/n)$

Q21- $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$

$c = \log_3 7 = 1.77$

$n^c = n^{1.77} < f(n)$

$T(n) = \Theta(n^2)$

Q22- $T(n) = T(n/2) + n(2\cos n)$

$a = 1$, $b = 2$

$c = \log_2 1 = 0$

$n^c = n^0 = 1 < f(n)$

$T(n) = \Theta(n(2 - \cos n))$