Trend analysis and forecasting of yearly temperature and rainfall in some African countries using Bayesian Autoregressive and Random walk models

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Abstract

In this paper, we examine the trends and forecasting of yearly rainfall and temperature data in ten African countries in the West, South, East, and North (Ghana, Nigeria, Senegal, South Africa, Namibia, Rwanda, Ethiopia, Madagascar, Egypt, and Tunisia) using Bayesian Autoregressive and Random walk models, each of order one. The performance of the two models were compared based on Deviance Information Criterion and Watanabe-Akaike Information Criterion and the results show that the random walk model provides better fit to the temperature data for all the countries, for the rainfall data, the autoregressive model provides a better fit. While the projections of the autoregressive model indicates a downward pattern for virtually all the countries, the random walk model gave patterns that are somehow consistent with the observed trend in the immediate past few years though their wider credible intervals indicate higher level of uncertainty about the predictions. The implications of the autoregressive model results obtained is that, temperature will have a decreasing pattern in the future and rainfall will have a trend consistent with the observed in the original data.

Keywords: INLA, Autoregressive model, Random walk model, Deviance Information Criterion,
Watanabe-Akaike Information Criterion and Bayesian Inference.

1. Introduction

Rainfall and temperature are some of the essential climate variables in Africa. Too much of rainfall can lead to flooding and too little rainfall which implies higher temperatures can also lead to drought. Both floods and droughts have negative impacts on our environment. Flooding tends to dislocate people and even animals and causes death in extreme conditions. Droughts on the other hand leave our lands dry and bare, dry up our underground water which leads to lack of portable drinking water and many more (1). It is therefore of great significant to study the main drivers of rainfall and temperature and their variabilities in order to be able to predict their outcomes for future benefits. Predicting rainfall compared to temperature is very difficult (1), this is because there are many factors that causes rain to fall, such as the varations in the weather patterns, however, there are certain predictions that scientists can make about the future with certainty. Global warming is one factor that affects the world's rainfall and temperature patterns, but mostly across sub-Saharan Africa, this triggers less rainfall. Such adjustments may have had big implications on our ecosystems, agriculture, and humans areas vulnerable to rainfall and temperature changes. It is, therefore, appropriate to analyze and forecast the trends of temperature and rainfall which will help give significant reference outcomes for future water resources planning and management. Global warming can also result in extreme weather, other than extremes of cold or heat. Hurricane forms, for instance, will shift. Temperatures in some African countries especially in the arid regions have been predicted to increase faster than any other part of the world (2). According to the Intergovernmental Panel on Climate Change (IPCC) 2007 fourth report, due to Africa's geographical position on the globe, it is one of the most endangered continents to changes and variability in the climate. The West African monsoon has a significant impact on the regional climate and water resources in the Sahel. Since Africa is an agriculture-based economy, it depends heavily on West African monsoon which produces a mean annual rainfall of $150-2500mm$ (3). The mean annual temperature in Africa is $25.7°C$. In the last 100 years, the global mean surface temperature has risen in a linear trend of $0.74°C$ (4). The continent of Africa is popularly recognized as hot and dry with current trends showing colder stretches than it was 100 years ago (5). These warmings are anticipated because most of Sub-Saharan Africa lies in tropical and subtropical latitudes, where year-round temperatures are high and differ further from daytime to night time than during the year (6). Although these patterns appear to be common throughout the continent, the variations in temperature are not always consistent. They differ greatly between regions and countries and within them. For instance, countries across the Nile Basin, for example, experienced an elevated temperature in the second half of the century between $0.2°C$ and $0.3°C$ per decade, while in Rwanda the temperature fell by $0.7°C$ to $0.9°C$ (7). A mean temperature increase of $-263°C$ near the surface of the earth contributes to significant rainfall increases. With the development of industrialization and rapid population growth, water resource management is becoming more important not only in Africa but worldwide. The continent has a long history of unpredictable lengths and intensities of rainfall variations (2). The truth is that rainfall continues to fluctuate, and good and poor years continue to occur. One may recognize certain general regional trends, which can be articulated in terms of variation, trends (upward or downward), and continuity, common inertia that at all times affects several climatic variables.

The specific objectives of this paper are therefore to model and forecast temperature and rainfall data from ten African countries; Senegal, Ghana, Ethiopia, Namibia, Madagascar, Tunisia, Nigeria, Rwanda, South Africa, and Egypt using Bayesian autoregressive and random walk model, and to compare the performances of the two approaches in modelling and forecasting the climate data. (2) investigated the changes in the amount of rainfall and air temperature in six Sub-Saharan African countries; Botswana, Ethiopia, Ghana, Nigeria, Uganda, and South Africa using quarterly frequency data. The study discussed seasonality, time patterns, and persistence using long memory and fractional integration techniques which are an alternative to deterministic models. The study noted that steady rainfall is expected in Africa in the future while temperatures are likely to rise.

2. Data and Methodology

2.1. Data

The data employed in this work are the yearly time series of the average amount of rainfall (mm) and temperature ($\circ C$) for ten sub-Saharan African countries: Ethiopia, Senegal, Egypt, Namibia, Tunisia, Madagascar, Ghana, Nigeria, South Africa, and Rwanda. The data span from 1901 and 2016 and were sourced from the World Bank Climate Change Knowledge Portal.

2.2. Bayesian Autoregression (AR(p)) Model

The AR(p) model is an autoregressive model with p lag terms. Autoregression occurs when variables are regressed towards their own values from the past. The lag term p shows the number of past values used at each time period. For a stochastic process $Y_t, t \in \mathbb{N}_+$, a linear autoregressive model of order p is one for which the present value Y_t of the process is described as a linear combination of its p past values $y_{t-1}, y_{t-2}, ..., y_{t-p}$ and of a white noise ϵ_t. The white noise process, $\epsilon_t, t \in \mathbb{N}_+$ is assumed to be a standard normal distribution. Linear autoregressive model can be written as;

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \cdots + \phi_p Y_{t-p} + \epsilon_t \quad (1)$$

where $\phi = (\phi_1, \phi_2, ..., \phi_p) \in \mathbb{R}^p$ and $\sigma^2_\epsilon = 1/\tau$ are the parameters of the model.

The fitting of an AR(p) model to a given time-series can be divided into three stages; estimation of the parameters for several values of p, selection of the most suitable value of p based on some statistical criteria and the use of the estimates of the parameters to predict new values for the time series using (1) recursively (8).

2.3. Parameter Estimation of Bayesian Autoregression (AR(p)) Model

The normal technique for parameter estimation of an AR(p) model starts with ascertaining the likelihood function $L(Y|\theta)$, where $Y = (Y_{p+1}, Y_{p+2}, ..., Y_N)'$ is the $N \times 1$ observed time series vector and $\theta = (\phi, \tau)$ is the vector of unknown parameters.This work will however, consider the AR(1) model. According to (8), the likelihood function has the form of probability density and it is responsible for generating the data Y given the parameters θ and it can be defined as;

$$L(Y|\theta) \propto \left(\frac{1}{\sigma_\epsilon^2}\right)^{\frac{N-p}{2}} \exp\left(-\frac{1}{2\sigma_\epsilon^2}\sum_{t=p+1}^{N}\left(Y_t - \phi_1 Y_{t-1} - \cdots - \phi_p Y_{t-p}\right)^2\right) \tag{2}$$

The maximum likelihood estimation assumes constant values for the AR(p) to be calculated, however, the Bayesian inference approach considers the parameters as random variables that follow a particular probability distribution.

Bayesian inference requires a specific prior probability density $p(\theta)$ that will express our uncertainty about θ before taking the data Y into consideration in order to determine a likelihood function.

To get posterior distribution, the maximum likelihood function has to be combined with the prior distribution using the Bayes theorem from probability theory. The posterior distribution can be written as;

$$p(\theta|Y) = \frac{p(\theta)L(Y|\theta)}{p(Y)} \tag{3}$$

where $p(Y)$ is a normalization factor to ensure that $\int_\theta p(\theta|Y)d\theta = 1$. We generally do not write the denominator $p(Y)$ because it does not depend on θ and it is difficult to estimate and therefore equation (3) is often written as;

$$p(\theta|Y) \propto p(\theta)L(Y|\theta) \tag{4}$$

The Bayesian inference makes it possible to test several hypotheses for the prior density and choose the one that yields a suitable posterior density to the problem.

2.4. Random Walk

One of the simplest and yet the most important models in time series forecasting is the random walk model.

The random walk model assumes that in each period, the variable takes a random step away from its past values and the steps are independently and identically distributed (9).

According to (9), the random walk of order one for the Gaussian vector $X = (x_1, x_2, \cdots, x_n)$ is constructed as;

$$Z_t + X_{t-1} = X_t \sim \mathcal{N}\left(0, \frac{1}{\tau}\right) \tag{5}$$

The density for X is derived from its $n-1$ first-order increment as;

$$p(X|\tau) \propto \tau^{\left(\frac{n-1}{2}\right)} \exp\left(-\frac{\tau}{2}\sum (Z_t)^2\right) \tag{6}$$

$$= \tau^{\left(\frac{n-1}{2}\right)} \exp\left(-\frac{1}{2}X^T Q X\right) \tag{7}$$

where $Q = \tau R$ and R is the structure matrix reflecting the neighbourhood structure of the model.

The random walk of order two for the Gaussian vector $X = (x_1, x_2, \cdots, x_n)$ is constructed as;

$$Z_t + 2X_{t+1} - X_{t+2} = X_t \sim \mathcal{N}\left(0, \frac{1}{\tau}\right) \tag{8}$$

The density for X is derived from its $n-2$ second-order increment as;

$$p(X|\tau) \propto \tau^{\left(\frac{n-2}{2}\right)} \exp\left(-\frac{\tau}{2}\sum (Z_t)^2\right) \tag{9}$$

$$= \tau^{\left(\frac{n-2}{2}\right)} \exp\left(-\frac{1}{2}X^T Q X\right) \tag{10}$$

This work is going to concentrate on random walk of order one which assumes independent first order increments.

2.5. Integrated Nested Laplace Approximation(INLA)

Integrated Nested Laplace Approximation (INLA) is an efficient way to compute the posterior marginals of all parameters of interest. INLA makes it possible to easily work with and also provides exact parameter and hyperparameter estimates in short computational time periods.

According to (10), the posterior marginal density for subject i in relation to parameters can be written as;

$$p(\theta_i|y) = \int p(\theta_i|\phi, y)p(\phi|y)d\theta \tag{11}$$

and the hyperparameters can also be written as;

$$p(\phi_j|y) = \int p(\phi|y)d\phi_{-j} \tag{12}$$

According to (10), normal approximation approach often yields right outputs, but some errors occur because of various factors such as position and lack of skewness.

According to (11), as the normal approximation boost, we can compute the laplace approximation as;

$$\tilde{p}_{LA}(\theta_i|\phi, y) \propto \frac{p(\theta, \phi|y)}{\tilde{p}(\theta_{-i}|\phi, y)}\Big|_{\theta_{-i}=\theta^*_{-i}}(\theta_i, \phi) \tag{13}$$

where $\tilde{p}$ is the normal approximation to $\theta_{-i}|\theta_i, \phi, y$.

This method works very well because the conditionals $p(\theta_{-i}|\theta_i, \phi, y)$ are often similar to normal but is computationally expensive.

2.6. Model Diagnostic Criteria

2.6.1. Deviance Information Criterion

In the Bayesian scheme, the Deviance Information Criterion (DIC) is used as a method for determining the best-fit model for all models. According to (12), DIC is computed as follows;

$$DIC = D(\bar{\theta}) + 2PD = \overline{D(\theta)} + PD \tag{14}$$

where $PD = \overline{D(\theta)} - D(\bar{\theta})$ is the Bayesian measure of complexity, $\overline{D(\theta)}$ is the posterior mean of the Bayesian deviance, $D(\bar{\theta})$is the posterior mean of $-2\log \cdot L(Y|\theta)$.

The rule of thumb for using DIC in model selection is that, a difference in DIC of more than 10 takes out the model with higher DIC, and a difference of less than 5 does not indicate any inference between comparative models (12).

2.7. Kullback-Leibler Divergence

The value of the Kullback-Leibler divergence (KLD) defines the difference between the standard normal and the simplifies Laplace Approximation (SLA) for each posterior. Small values suggest that the posterior distribution is well approximated by a normal distribution and there should be no need to make more computationally intense complete Laplace approximation. .

2.8. Watanabe-Akaike Information Criterion

The Wanatabe-Akaike criterion (WAIC) is a more strictly Bayesian method for estimating the probability of the out-of-sample based on the density of the log pointwise posterior prediction. It is also a model diagnostics and comparison tool in which a model with the lowest value of WAIC, just like DIC, is considered the best among competing ones.

2.9. Forecasting Procedure

Integrated Nested Laplace Approximation (INLA) requires pre-processing of the datasets it is going to use. In this work, forecasting is done by applying past observation in the year we are forecasting. For instance, for the year 2017, we used the past observation from January to December of 2016 and then add missing values for the observation we are forecasting. Overall, ten years were forecast from January 2017 to December 2026.

3. Results and Discussion

This section contains the main analysis carried out on the rainfall and temperature data and the results obtained by applying the methodology discussed in the previous chapter.

3.1. *Comparison of the two models on Temperature data*

Table 1: DIC and WAIC values for Autoregressive and Random walk fitted models for Temperature data

	Ethiopia		South Africa		Egypt		Ghana		Rwanda	
	Temperature		Temperature		Temperature		Temperature		Temperature	
	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1
DIC	240926.01	8178.37	3861027.41	117620.02	7458370.91	226604.18	425260.11	13741.73	56806.25	2619.44
WAIC	259465.98	8473.57	2066999.51	123060.79	2832867.20	237165.90	454235.93	14298.27	61465.79	2653.25

	Nigeria		Tunisia		Namibia		Senegal		Madagascar	
	Temperature		Temperature		Temperature		Temperature		Temperature	
	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1
DIC	668000.14	21079.34	9228578.75	280067.73	2800940.94	85540.34	906009.02	28264.49	869801.55	27115.78
WAIC	689022.43	21980.67	3036950.02	293140.27	1836804.64	89472.91	852693.70	29503.38	902613.56	28303.10

3.2. *Comparison of the two models on Rainfall data*

Table 2: DIC, WAIC and KLD values for Autoregressive and Random walk fitted models for Rainfall data

	Ethiopia		South Africa		Egypt		Ghana		Rwanda	
	Rainfall		Rainfall		Rainfall		Rainfall		Rainfall	
	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1
DIC	8463295.36	15408887.68	2416838.90	2681201.44	1135776.61	35787.47	14472230.00	0.00000017	18526364.74	13833276.18
WAIC	2224473.74	3046002.79	1398569.78	1442623.04	740601.65	37097.59	2833708.53	2899894.03	3957586.90	2802235.17

	Nigeria		Tunisia		Namibia		Senegal		Madagascar	
	Rainfall		Rainfall		Rainfall		Rainfall		Rainfall	
	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1	AR1	RW1
DIC	0.00000025	27616023.37	1141541.26	1272629.10	3015260.69	3353103.71	28682898.29	31684735.71	25678798.37	38433244.12
WAIC	3738591.82	3880966.48	814595.28	874945.97	1438646.63	1511214.93	3808345.08	4204185.96	4557176.00	4717268.66

Tables 1 and 2 present the WAIC and DIC values for temperature and rainfall data respectively and it can be seen that, the DIC and WAIC values for Ethiopia, South Africa, Rwanda, Egypt, Nigeria, Tunisia, Namibia, Senegal, Ghana and Madagascar are smaller for the random walk of order one model with the temperature data. This shows that the random walk of order one model is a good model of fit for the temperature data for these countries. However, the DIC and WAIC values for Ethiopia, South Africa, Nigeria, Tunisia, Namibia, Senegal and Madagascar are smaller for the autoregressive model of order one compared to the values obtained from the random walk of order one model for these countries for rainfall data. Therefore, this is also an indication that, the autoregressive model of order one fits the rainfall data of these countries well compared to the random walk model of order one.

Also, the value of the Kullback-Leibler divergence (KLD) of both models were zero which is also an indication that the posterior distribution is well approximated by a normal distribution.

3.3. Results for temperature data

Figure 1 presents the trend plot of the observed temperature data, which shows no visible trend in the Egypt, South Africa, Ghana, Ethiopia, Tunisia, Nigeria, Madagascar, Senegal and Namibia data. For the Rwanda data, it can be seen that, from 1900 to 1920, there was a decreasing trend and then an increasing trend from 1925 to 2016.
We present the graphical representation of the Random walk model of order one which has the best fit to the temperature data in order to help us understand the nature of the temperature data.

Figure 2 shows the plots of the temperatures of the ten countries from a random walk model of order one with the observed and the predicted values of the temperature. The figure presents the posterior mean and the 95% credible intervals of the random walk model of order one fit to the temperature data. Egypt experienced its highest temperature at 24.1$°C$ in the year 2010 and its lowest of 21$°C$ in the year 1911. The temperature increased from 1911 to 1935 and then hanged around 22.3$°C$ between 1935 to 1993 and then started to increase till 2010. Rwanda experienced decreasing trend in temperature from 1901 to 1917 which was also the least recorded in the country and tend to increase from 1930. South Africa's temperature has been increasing since 1910 to 2016 except the year 1954. Ghana has upward and downward trend in temperature. No visible consistency in the pattern. Its highest temperature was 28.5$°C$ in 1928 and its lowest of 26$°C$ in 1918. Ethiopia's pattern from 1901 to 1960 was a upward and downward trend and from 1960, there was consistent increasing trend in temperature. Tunisia shows increasing trend in temperature except between the years 1945 and 1975 when there was increasing and decreasing temperature. Nigeria experienced its highest temperature of 27.8$°C$ in 1938 and its lowest of 26.3$°C$ in 1974. Namibia's temperature also has an increasing trend, however, between the years 1953 and 1975, it was around 20$°C$. Madagascar has an up and down trend in temperature. Senegal's temperature hovered around 28$°C$ from 1901 to 1940, and then started increasing from 1955 to 2015.
For the next ten years, the Random walk model fitted to the temperature data shows a constant pattern of temperature for all the ten countries.
It can also been seen that, the random walk model plots looks similar to the ones fitted with AR1 model, however, the random walk model gives smoother estimates compared to the AR1 model.
Comparing the autoregressive and the random walk models on the temperature data indicates that the fitted values for both models are similar and both models fit the temperature data. However, the credible intervals for the random walk of order one model are generally wider than those of the autoregressive model of order one.
While the projections of the autoregressive model indicate a downward pattern for virtually all the countries, the random walk model gave patterns that are somehow consistent with the observed trend in the immediate past few years though with wider credible intervals indicate a higher level of uncertainty about the predictions.

3.4. Results for rainfall data

We present the graphical representation of the autoregressive model of order one which has the best fit on the rainfall data to help us understand the nature of the rainfall data.

Figure 3 presents the observed rainfall data which shows no visible trend in the Egypt, South Africa, Ghana, Ethiopia, Tunisia, Nigeria, Madagascar, Senegal and Namibia data. For the Rwanda data, it can be seen that, from 1900 to 1920, there was a decreasing trend and then an increasing trend from 1925 to 2016.

Figure 4 shows that, rainfall in Egypt has up and down trend and recorded its highest rainfall of 4.8mm in the year 1974 and the lowest of 1.2mm in the year 1910. Rwanda has a slightly increasing trend and recorded its highest rainfall of 133mm in 2011 and 75mm as the lowest in 1913. South Africa has no trend in its rainfall pattern and recorded its highest rainfall of 47mm in 1977 and its lowest of 32mm in 1990. Senegal has an increasing trend from the year 1901 to 1960, decreases and then increases again. The rainfall pattern of Ethiopia, Tunisia, Ghana, and Namibia have a cyclic pattern. Nigeria's rainfall hovered around 100mm from 1901 to 1965, and then increasing trend from 1965 to 2005. Madagascar shows slightly increasing trend up to the year 1980 and shows decreasing trend from 1980 to 2016.

It can also be seen that, for the next ten years, the pattern of rainfall in these countries using the random walk model of order one yields a constant pattern. However, Egypt's pattern increased from 2017 and becomes constant with about 2.8mm of rainfall, Rwanda's pattern decreased in 2017 and then becomes constant with 99mm of rainfall, South Africa's pattern also rises slightly in 2017 and tend to be constant with 39mm of rainfall. Ghana, Ethiopia, Namibia and Madagascar will have a constant rainfall pattern of 99mm, 68mm, 23mm, and 118mm respectively. Tunisia's pattern decrease from 2017 and then moved at a constant rate of 22mm, Nigeria's decreased from 2017 and will become constant with 97mm and the pattern for Senegal increased and becomes constant at 64mm.

4. Conclusion

We have considered the time series monthly data of rainfall and temperature in ten sub-Saharan African countries, spanning from 1901 to 2016, the sample period, which is considered long enough to explain climatic change in the western, eastern,northern and southern African regions. In particular, the countries examined are Rwanda, Ethiopia, Ghana, Nigeria, Tunisia, Madagascar, Namibia,Egypt, Senegal and South Africa. The purpose of this study is to model the rainfall and temperature data from ten African countries using Bayesian autoregressive and random walk models of order one and also to analyse the trends in the data and predict how the temperature and rainfall for the next ten years in these ten African countries will behave and compare the performances of the two models.

Model fitting was done in R using Integrated Nested Laplace Approximation. A comparison of the fitted values based on the two models' diagnostic criteria, DIC and WAIC revealed that, for the temperature data, the random walk of order one model consistently fits the data quite well for all the countries.

On the other hand, the rainfall data was well fitted by the autoregressive model of order one as it has the smallest values of DIC and WAIC for all the countries. However, both the random walk and the autoregressive models quite fit the trend of both the temperature and rainfall data quite well. Also, the random walk model on the temperature data had wider credible intervals compared to the autoregressive model and the credible intervals for both models on the rainfall data seems similar.

For the next ten years, the projection of the autoregressive model on the rainfall data for all countries shows a constant trend with a very small credible interval which is an indication that, since the credible intervals were very narrow, the rainfall pattern will hardly increase or decrease in future and this is in accord with (2). For the projection of the random walk model on the temperature data, the random walk model shows a constant trend of temperature in all the ten countries. However, there is a wider credible interval for all the countries in the projection, an indication that, with other atmospheric conditions, there can be an increasing or decreasing trend in temperature in the future.

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Appendix

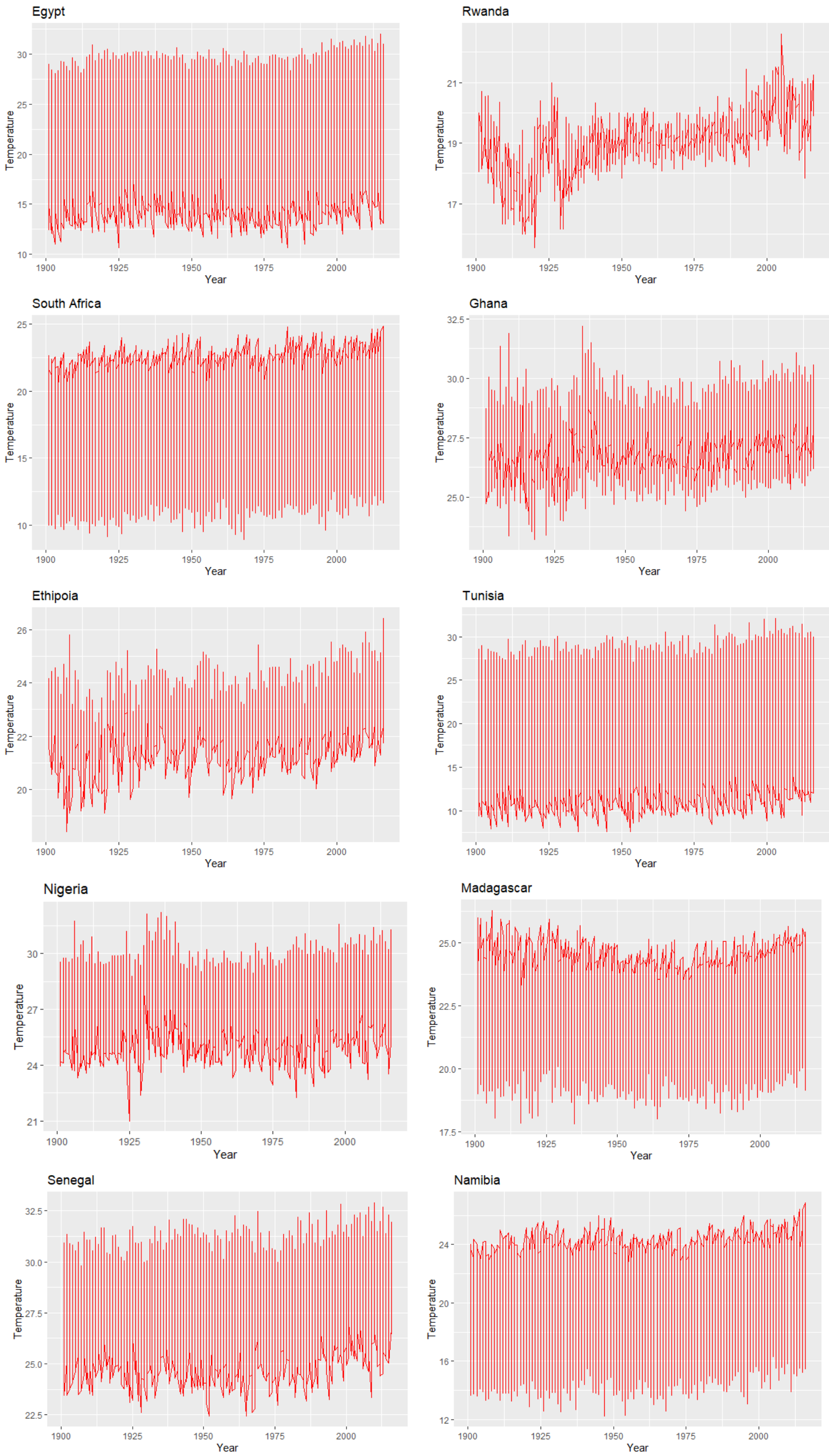


Figure 1: Plots of the observed temperature data

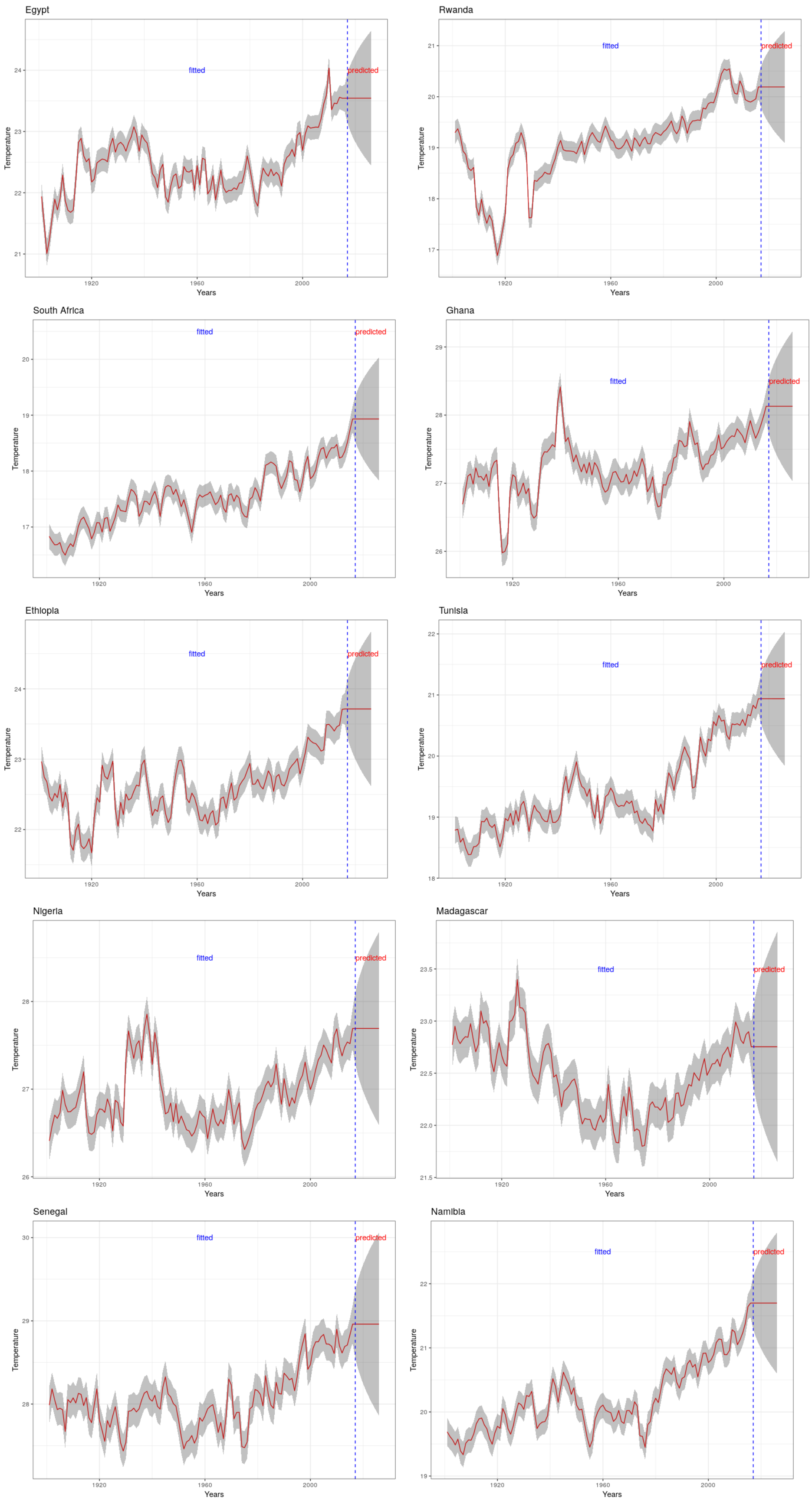


Figure 2: Results of the random walk of order 1 model fitted to temperature data

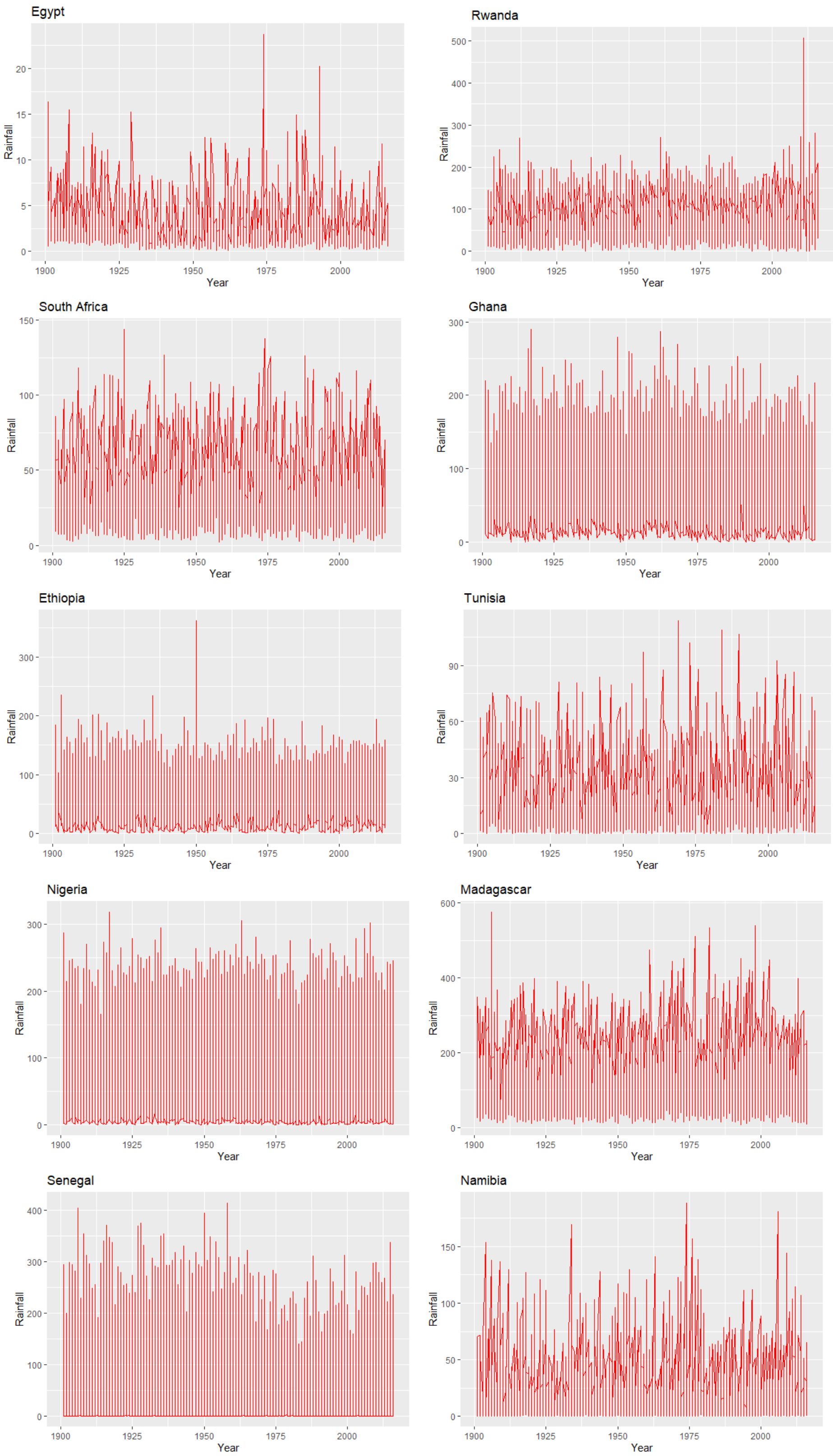


Figure 3: Plots of the original rainfall data

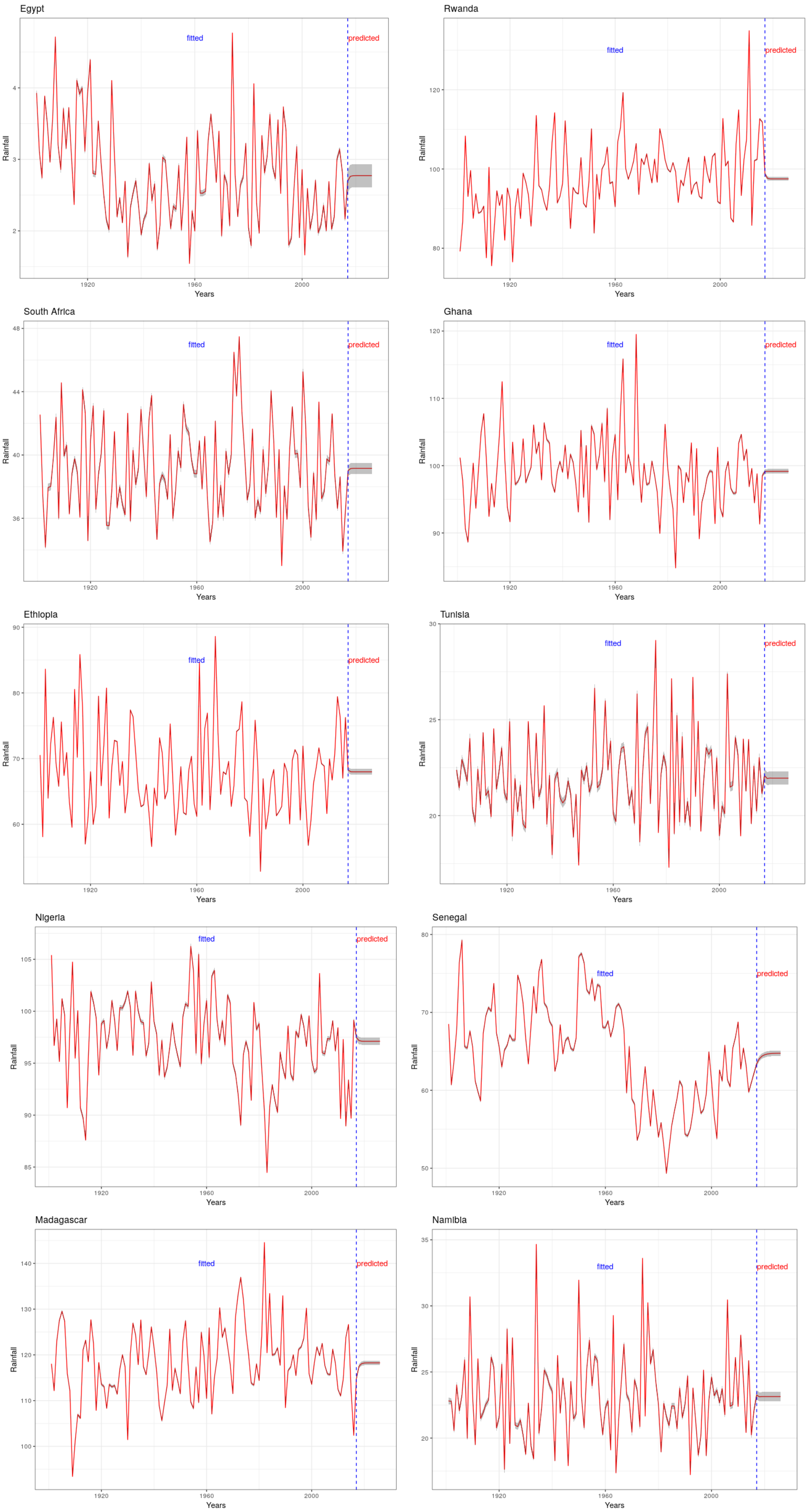
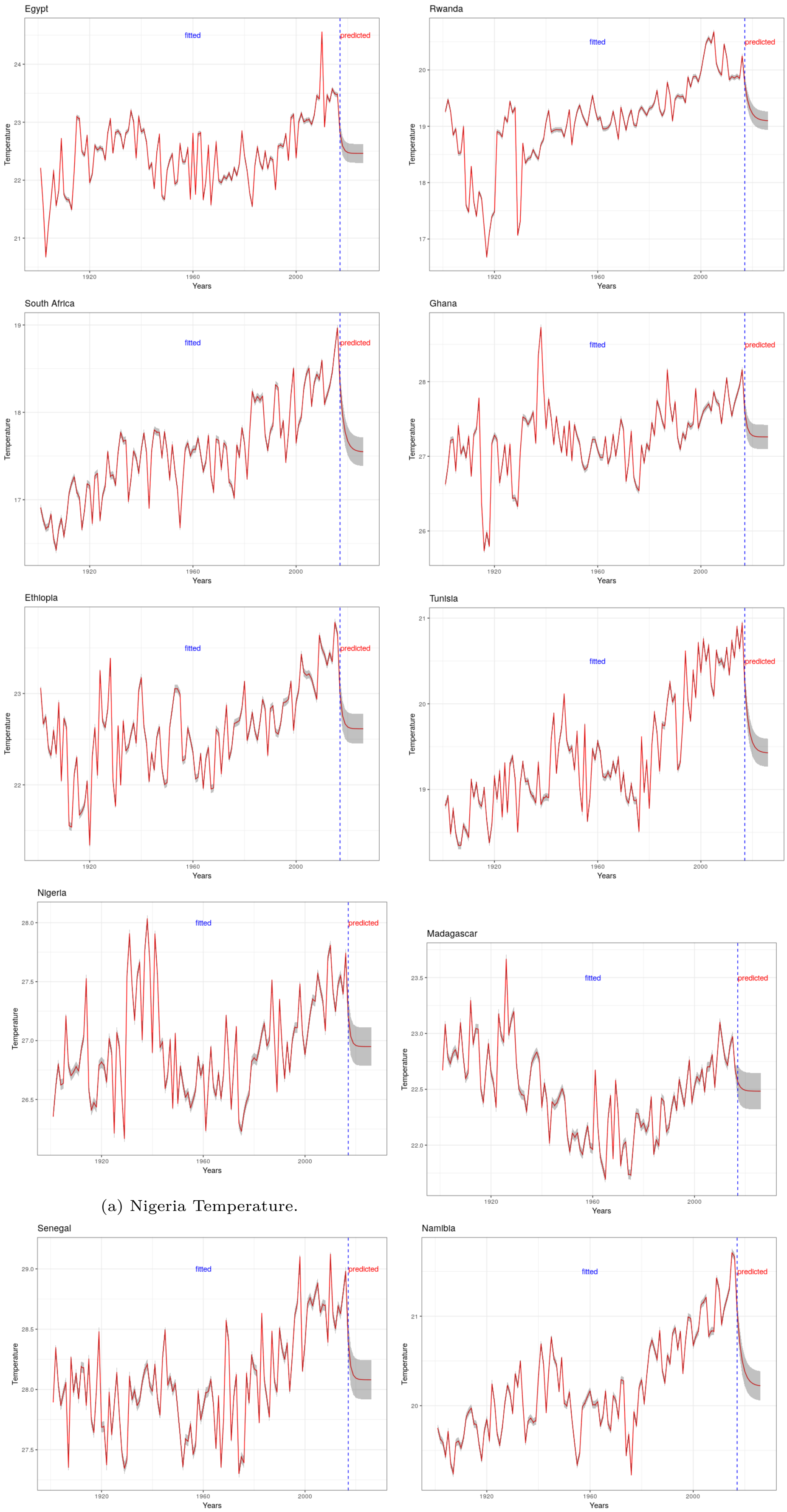


Figure 4: Results of the autoregressive of order 1 model fitted to rainfall data



(a) Nigeria Temperature.

Figure 5: Results of the autoregressive of order 1 model fitted to temperature data

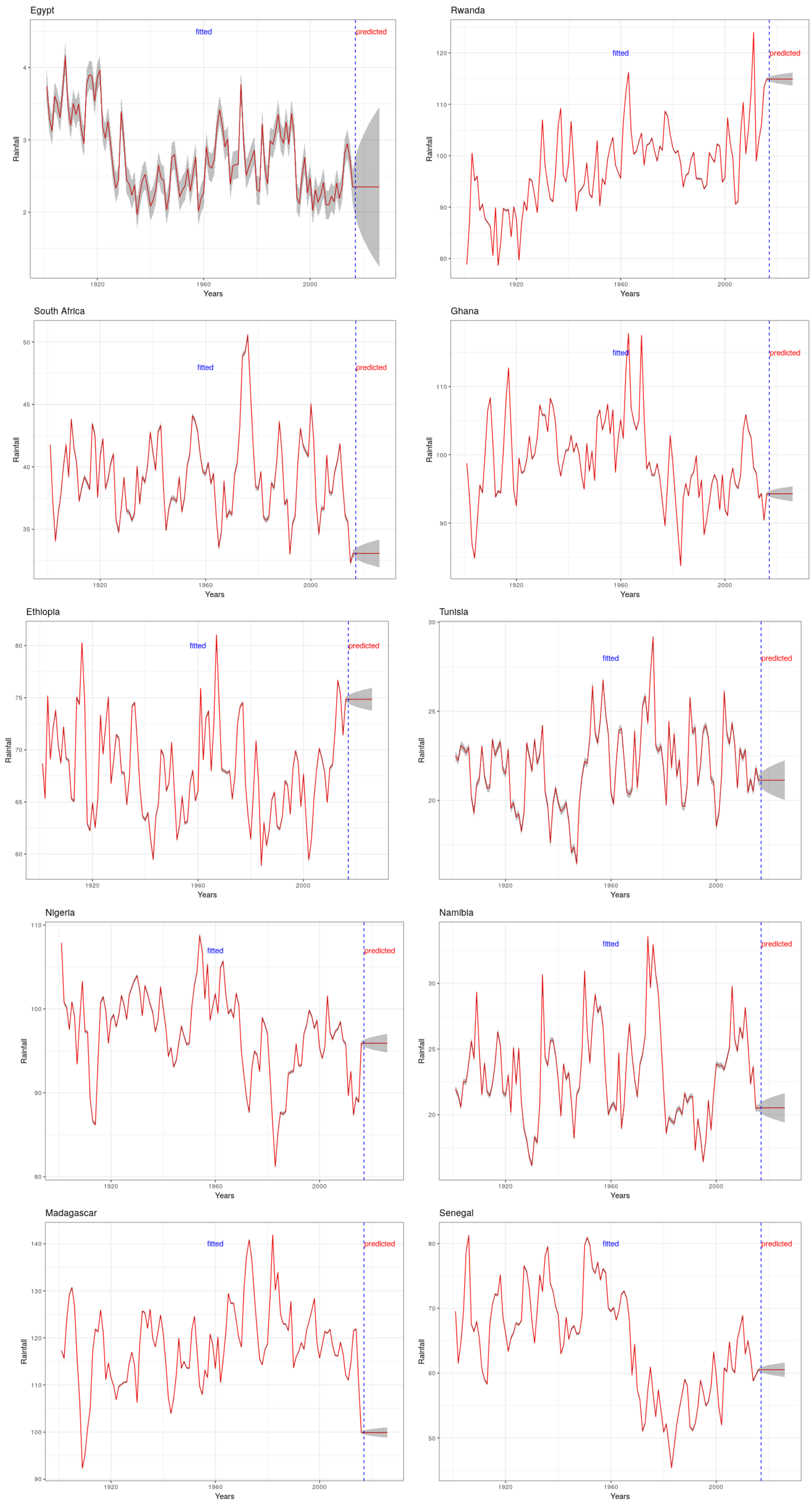


Figure 6: Results of the random walk of order 1 model fitted to rainfall data