

Problem 1:

In each of the following situations, state whether it is a correctly stated hypothesis testing problem and why?

- ① $H_0: \mu = 25$, $H_1: \mu \neq 25$ True
- ② $H_0: \sigma > 10$, $H_1: \sigma = 0$ False $\rightarrow H_1$ should be $H_1: \sigma \leq 10$
- ③ $H_0: \bar{x} = 50$, $H_1: \bar{x} \neq 50$ True
- ④ $H_0: P = 0.1$, $H_1: P = 0.5$ False \rightarrow we can ~~test~~ test the null hypothesis
- ⑤ $H_0: S = 30$, $H_1: S > 30$ False $\rightarrow H_1: S \neq 30$

Problem 2:

The college bookstore tells prospective students that the average cost of its textbook is Rs 52 with a standard deviation of Rs 4.50. A group of smart statistics students thinks that the average cost is higher. To test the bookstore's claim against their alternative, the students will select a random sample of size 100. Assume that the mean from their random sample is Rs 52.80. Perform a hypothesis test at the 5% level of significance and state your decision.

Given $\mu = 52$ $\sigma = 4.50$ $n = 100$ $\mu_s = 52.80$
 $\alpha = 0.05$

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{52.8 - 52}{4.5 / 10} = 1.78$$

$$H_1: \mu > 52$$

$$H_0: \mu \leq 52$$

$$Z_{\alpha} = 1.645 \quad 0.9625$$

1.78 is in rejection so

Problem 3:

A certain chemical pollutant in the Gurema River has been constant for several years with mean 34 ppm (part per million) and standard deviation $\sigma = 8$ ppm. A group of factory representatives whose companies discharge liquids into the river is now claiming that they have lowered the average with improved filtration devices. A group of environmentalists will test to see if this is true at the 1% level of significance. Assume that their sample of size 50 gives a mean of 32.5 ppm. Perform a hypothesis test at the 1% level of significance and state your decision.

Sol: Given $\mu = 34$ $\sigma = 8$ $H_0: \mu = 34$

$\bar{x} = 32.5$ $n = 50$ $H_1: \mu < 34$

$$Z = \frac{32.5 - 34}{8/\sqrt{50}} = \frac{-1.5}{1.1313} = -1.325$$

$$= 0.0934$$

Reject null hypothesis.

Based on population figures and other general information on the US population, suppose it has been estimated that, on average, a family of four in the US spends about \$1135 annually on dental expenditures. Suppose further that a regional dental association wants to test its estimate if this figure is accurate for their area of country. To test this 22 families of 4 are randomly selected from the population in that area of the country and a log is kept of the family dental expenditure for one year. The resulting data are given below. Assuming that dental expenditure is normally distributed in the population. Use the data and an alpha of 0.5 to test the dental association hypothesis.

Sol

$$\text{Given } \mu = 1135$$

$$n = 22$$

$$\bar{x} = \frac{1008 + 812 + 1117 + 1323 + 1308 + 1415 + 831 + 1021 + 1287 + 851 + 930 + 730 + 699 + 872 + 913 + 944 + 954 + 987 + 1695 + 995 + 1003 + 994}{22}$$

$$= \frac{22687}{22} = 1031.227$$

Standard Deviation:-

x_i	\bar{x}	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1008	1031	-23	529
812	1031	-219	47961
1117	1031	86	7396
1323	1031	292	85264
1308	1031	277	76729
1415	1031	384	147456
831	1031	-200	40000
1021	1031	-10	100
1287	1031	256	65536
851	1031	-180	32400
930	1031	-101	10201
730	1031	-301	90601
699	1031	-332	110224
872	1031	-159	25281
913	1031	-118	13924
944	1031	-87	7569
954	1031	-77	5929
987	1031	-44	1936
1695	1031	664	440896
995	1031	-36	1296
1003	1031	-28	784
994	1031	-37	1369

$$S.D = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{121339}{22}}$$

$$= \sqrt{55153}$$

$$= 234.84$$

$$Z = \frac{1031 \cdot 227 - 1135}{234.84 / \sqrt{22}}$$

$$= \frac{-103.773}{234.84 / \sqrt{22}}$$

$$= \frac{-103.773}{4.69}$$

$$= -21.89$$

$$50.072$$

$$= -2.072$$

a report prepared by the Economic Research Department of a major bank the Department manager maintains that the average annual family income of Metropolis is \$48432. What do you conclude about the validity of the report if a random sample of 400 families shows an average income of \$48574 with a S.D of 2000?

$$\mu = 48432$$

$$n = 400$$

$$\bar{x} = 48574$$

$$\sigma = 2000$$

$$z = \frac{48574 - 48432}{2000/\sqrt{400}} = \frac{142}{2000/20} = \frac{142}{100} = 1.42$$

$$@5\% \quad 0.9719$$

failed to reject null hypothesis

Problem 6:-

Suppose that in past years the average price per sq-ft for warehouses in the US has \$32.28. A national real estate investor wants to determine whether that figure has changed now. The investor hires a researcher who randomly samples 19 warehouses that are for sale across the US and find that the mean price is \$31.67 with SD \$1. Assume that the prices of warehouse footage are normally distributed in population. If the researcher uses a 5% level of significance, what statistical conclusion can be reached.

$$H_0: \mu = 32.28$$

$$H_a: \mu \neq 32.28$$

$$\bar{x} = 31.67$$

$$s = 1.29$$

$$z = \frac{31.67 - 32.28}{\frac{1.29}{\sqrt{9}}} = \frac{-0.61}{\frac{1.29}{4.35}} = \frac{-0.61}{0.2965} = -2.051$$

Problem 13.

Acceptance region.

Sample size

α

β at $\mu = 52$

β at $\mu = 50$

- | | |
|-----------------------------|----|
| ① $48.5 < \bar{x} < 51.5$ | 10 |
| ② $48 < \bar{x} < 52$ | 10 |
| ③ $48.81 < \bar{x} < 51.9$ | 16 |
| ④ $48.42 < \bar{x} < 51.58$ | 16 |

Problem 8:

Find the t score for a sample size of 16 taken from a population with mean 10 when the sample mean is 12 and the S.D is 1.5

$$\mu = 10 \quad S = 1.5$$

$$\bar{x} = 12$$

$$t = \frac{\bar{x} - \mu}{S / \sqrt{n}} = \frac{12 - 10}{1.5 / \sqrt{16}} = \frac{2}{1.5 / 4} = 5.33$$