STATISTICS III - Session 13

1. A company manufactures LED bulbs with a faulty rate of 30%. If 9 randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Sol: Probability of faulty rate = 30%.

$P = 0.30$

Given

$n = 6$ LEDs

$$P(X=2) = {}^6C_2 (0.30)^2 (0.70)^4$$

$$= \frac{6!}{2!\,4!} \times 0.09 \times 0.2401$$

$$= \frac{6 \times 5 \times 4!}{2 \times 4!} \times 0.021609$$

$$= 15 \times 0.021609$$

$$= 0.324135$$

Average Value $\mu = nP$

$= 6 \times 0.3 = 1.8$

Standard deviation $= \sqrt{nP(1-P)}$

$= \sqrt{6 \times 0.3 \times 0.7} = 1.1224$

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 question per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. what is the probability that each of them will solve 5 questions correctly? what happens in cases of 4 and 6 correct solutions? what do you infer from it? what are two main governing factors affecting their ability to solve question correctly? Give pictorial representation.

Gaurav

$n = 8$

$P = 0.75$

$P(x=5) = {}^{8}C_5 (0.75)^5 (0.25)^3$

$= \frac{8!}{5!\,3!} \times 0.2373 \times 0.01562$

$= \frac{8 \times 7 \times 6 \times 5!}{5! \times 6} \times 0.003707$

$= 0.2076$

$P(x=4) = {}^{8}C_4 (0.75)^4 (0.25)^4$

$= {}^{8}C_4 \times 0.3614 \times 0.0039$

$= 0.0988$

Barakha

$n = 12$

$P = 0.45$

$P(x=5) = {}^{12}C_5 (0.45)^5 (0.55)^7$

$= \frac{12!}{5! \times 7!} \times 0.01845 \times 0.01522$

$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{5 \times 4 \times 3 \times 2 \times 1 \times 7!} \times 0.01845 \times 0.01522$

$= 0.2224$

$P(x=4) = {}^{12}C_4 (0.45)^4 (0.55)^8$

$= \frac{12!}{4!\,8!} \times (0.45)^4 \times (0.55)^8$

$= \frac{12 \times 11 \times 10 \times 9 \times 8!}{4 \times 3 \times 2 \times 8!} \times (0.45)^4 (0.55)^8$

$P(x=6) = {}^{8}C_6 (0.75)^6 (0.25)^2$

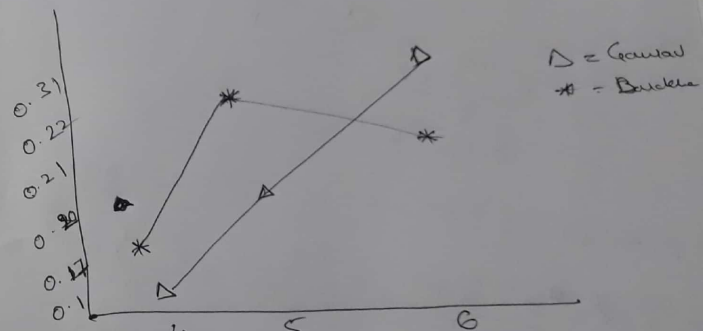
$= \frac{8!}{6!\,2!} \times (0.75)^6 (0.25)^2$

$= 0.311$

$P(x=6) = {}^{12}C_6 (0.45)^6 (0.55)^6$

$= \frac{12!}{6!\,6!} (0.45)^6 (0.55)^6$

$= 0.2122$



Problem 5 :-

Customers arrive at a rate of 72 per hour in my shop. what is probability of k customers arriving in 4 minutes?

a) 5 customers, (b) not more than 3 customers (c) more than 3 customers. Give a pictorial representation of the same.

Sol

$$P(x) = \frac{e^{-\mu}\mu^x}{x!}$$

$E(x) = V(x) = \mu$

$\mu = \frac{72}{60} = 1.2$ customer in 1 minute

k — in 4 minutes

$k = \frac{4 \times 1.2}{1} = 4.8$

$P(x=5) = \frac{e^{-4.8} \times 4.8^5}{5!}$

$= 0.17474$

$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$\frac{e^{-4.8} \times 4.8^0}{0!} + \frac{e^{-4.8} \times 4.8^1}{1!} + \frac{e^{-4.8} \times 4.8^2}{2!} + \frac{e^{-4.8} \times 4.8^3}{3!}$

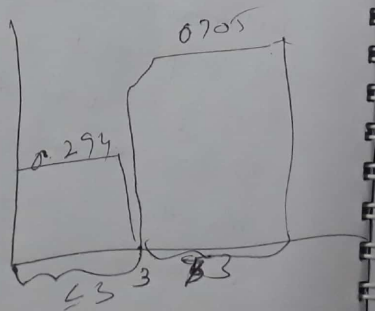
$0.00822 + 0.03949 + 0.09479 + 0.15167$

$= 0.2941$

$P(x > 3) = 1 - P(x \leq 3)$

$= 1 - 0.2941$

$= 0.7059$



Problem 4

Sol:- No of errors = 6/hr

6 / 77x60 words / hr

$= \frac{1}{770}$ — 1 — 6 errors

— 1 — 77x60

① $\mu = \frac{455}{770} = 0.591$

$P(x=2) = \frac{e^{-0.591} \times 0.591^2}{2!}$

$= 0.09671$

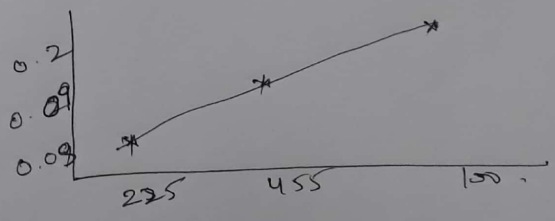
② $\mu = \frac{1000}{770} = 1.298$

$P(x=2) = \frac{e^{-1.298} \times 1.298^2}{2!}$

$= 0.2303$

③ $\mu = \frac{225}{770} = 0.3311$

$P(x=2) = \frac{e^{-0.3311} \times 0.3311^2}{2!} = 0.0393$



Problem 5

$f(x) = 0.05 \rightarrow 0 \leq x \leq 20$

$\times [0, 20]$.

$$\therefore \int_0^{10} f(x)\,dx = \int_0^{10} 0.05$$

for $[0, 10] \in [0, 20]$

$$\therefore \int_0^{10} f(x)\,dx = 0.05$$