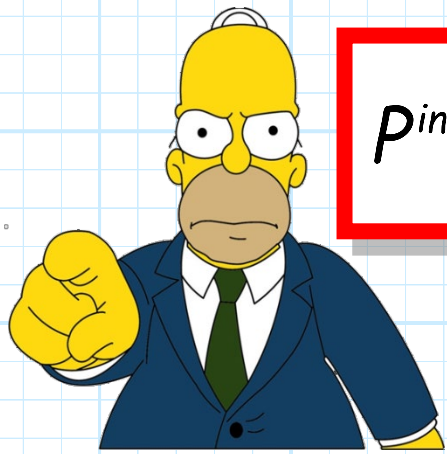
Incident and Reflected Power with "Matched" Source or Load

Q: *So. You've made a big, gigantic,* ***dramatic*** *deal about the fact that the* ***incident*** *power along a transmission line is* ***greater*** *than that* ***delivered*** *by the source:*



$$\boxed{P^{inc} \geq P_g^{del} = P_L^{abs}}$$

But. Doesn't the ***inequality*** $\geq$ *mean that the incident power* ***could be equal*** *to the delivered power of the source?*

A: For the **right circumstance**, yes; the two **could** be equal ($P^{inc} = P_g^{del}$).

When there's no reflected power

Q: *The **right circumstance**?*

***What** circumstance is the **right** circumstance in order for $P_g^{del} = P^{inc}$?*

A: Sigh.

Just **look** at the relationship:

$$P_g^{del} = P^{inc} - P^{ref}$$

Hopefully it is apparent, that **when** (and **only** when!) the **reflected** power is **zero** ($P^{ref} = 0$), the **incident** and **delivered** power will be **equal**:

$$P_g^{del} = P^{inc} \quad \textit{when} \quad P^{ref} = 0$$

When the load reflection coefficient is zero

Q: *OK; then under* ***what circumstance*** *is the reflected power zero?*

A: Sigh.

Just **look** at the relationship:

$$P^{ref} = P^{inc} \left|\Gamma_L\right|^2$$

Clearly, the reflected power will be zero **if** $\left|\Gamma_L\right|^2 = 0$ (i.e., if $\Gamma_L = 0$):

$$P_g^{del} = P^{inc} \quad \textit{when} \quad \Gamma_L = 0$$

When the load is "matched"

Q: *Well then. Under what* ***@$%*^!&%*** *circumstance is Γ_L zero?!?*

A: Sigh.

Recall the definition of Γ_L:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

From the **numerator**, it is apparent that Γ_L is zero if (and **only** if!) the **load** impedance Z_L is **numerically equal** to the **characteristic** impedance Z_0 of the transmission line:

$$\Gamma_L\Big|_{Z_L=Z_0} = \frac{Z_L - Z_0}{Z_L + Z_0}\Bigg|_{Z_L=Z_0} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$$

A special case

Q: *Hey; wasn't $Z_L = Z_0$ one of the **"special cases"** that we studied earlier?*

A: It was!

Recall that when $Z_L = Z_0$, the **input impedance** of our terminated transmission line is likewise **numerically equal** to Z_0—**regardless** of line length ℓ!

We now see **why** this is true.

Matched load means no reflected wave

When $Z_L = Z_0$, then $\Gamma_L = 0$, and so:

$$V_0^- = \Gamma_L V_0^- = (0) V_0^- = 0$$

Meaning the **reflected** wave (if $Z_L = Z_0$) is **zero**:

$$V^-(z) = V_0^- e^{j\beta z} = 0$$

So, our **total** voltage and current along the transmission line is that of the plus (i.e., **incident**) **wave only**:

$$V(z) = V^+(z) \qquad \text{and} \qquad I(z) = I^+(z)$$

The **line impedance** function is thus (when $Z_L = Z_0$):

$$Z(z) = \frac{V(z)}{I(z)} = \frac{V^+(z)}{I^+(z)} = Z_0$$

"Matched" load means…

In summary, for a "**matched**"* load:

a) the reflected wave is **zero**:

$$V^{-}(z) = 0 \qquad \text{if} \qquad Z_L = Z_0$$

b) the power of the **reflected** wave is likewise **zero**:

$$P^{ref} = \frac{\left|V^{-}(z)\right|^2}{2Z_0} = 0 \qquad \text{if} \qquad Z_L = Z_0$$

c) meaning the incident power will be **equal** to the delivered power:

$$P^{inc} = P_g^{del} = P_L^{abs} \qquad \text{if} \qquad Z_L = Z_0$$

* does **not** imply a conjugate match!

Now for the "matched" source

Q: *Things sure seem to **simplify** if the **load** is "matched".*

*What about a **"matched" source**?*

*Do **any** of the power relationships simplify then?*

A: They sure do!

Recall for a **"matched" source**, the incident wave becomes **causal** with respect to the **source**:

$$V^+(z) = V_0^+ e^{-j\beta z} = \left(\frac{1}{2} V_g e^{-j\beta\ell}\right) e^{-j\beta z} = \frac{V_g}{2} e^{-j\beta(z+\ell)}$$

In other words, for this **special case** (and **only** for this special case!), the **indent** wave is dependent on the **source only** (the value of load Z_L is irrelevant).

All I can say is: wow!

Recall the power associated with the **incident wave** is:

$$P^{inc} = \frac{\left|V^{+}(z)\right|^2}{2Z_0}$$

so that for a **"matched" source**:

$$P^{inc} = \frac{\left|V^{+}(z)\right|^2}{2Z_0} = \frac{1}{2Z_0}\left|\frac{V_g}{2}e^{-j\beta(z+\ell)}\right|^2 = \frac{\left|V_g\right|^2}{8Z_0} \quad \leftarrow \text{WOW!!!!}$$

Q: *I don't see **why** this is "wow". Am I **missing** something?*

A: Apparently you are. Look **closer** at the above result.

Do I have to explain everything?

Remember, this result is for the **special case** where the source impedance is a **real value**, numerically equal to Z_0:

$$Z_g = Z_0 + j0$$

Therefore:

$$R_g = Re\{Z_g\} = Z_0$$

The **source** resistance is **numerically equal** to transmission line characteristic impedance Z_0.

Thus, the **power** of the **incident** wave can be alternatively written as:

$$P^{inc} = \frac{|V_g|^2}{8\,Z_0} = \frac{|V_g|^2}{8\,R_g} \leftarrow \text{WOW!!!!}$$

The incident power is the available power (wow)!

Q: *This result looks vaguely **familiar**; haven't we seen this **before**?*

A: Sigh. This result is the **available power of the source**!!!!!!!

Therefore:

$$P^{inc} = \frac{|V_g|^2}{8\,Z_0} = \frac{|V_g|^2}{8\,R_g} = P_g^{avl} \qquad \leftarrow \text{Wow!}$$

For the special case $Z_g = Z_0 + j0$, the **incident power** P^{inc} is **equal** to the **available** power P_g^{avl} of the **source**.

"Matched" source means…

In **summary**, for a **"matched"*** source:

a) the power of the **incident** wave is:

$$P^{inc} = \frac{\left|V_g\right|^2}{8\,Z_0} \qquad \text{if} \qquad Z_g = Z_0$$

b) meaning the **incident** power will be equal to the **available** power of the "matched" source:

$$\boxed{P^{inc} = P_g^{avl}} \qquad \text{if} \qquad Z_g = Z_0$$

* does **not** imply a conjugate match!