EECS 622: Homework #22

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Problem 1

At the LO port of a mixer is a single time-harmonic signal with a frequency of 120 MHz .

At the RF port of the mixer is likewise a single time-harmonic signal with a frequency of 100 MHz .

Determine the frequencies of all time-harmonic signals created at the IF port of this mixer, including all first, second, and third-order spurious signals.

Solution:

For each incoming RF frequency, we will have a series of spurs,

(a) Carrier frequencies:

$$|\omega_{RF} - \omega_{LO}| = 20 \text{ MHz}, \tag{1}$$
$$|\omega_{RF} + \omega_{LO}| = 220 \text{ MHz} \tag{2}$$

(b) First order:

$$\omega_{RF} = 120 \text{ MHz}, \tag{3}$$
$$\omega_{LO} = 100 \text{ MHz} \tag{4}$$

(c) Second order:

$$2\omega_{RF} = 240 \text{ MHz}, \tag{5}$$
$$2\omega_{LO} = 200 \text{ MHz}, \tag{6}$$
$$|\omega_{RF} - \omega_{LO}| = 20 \text{ MHz}, \quad \text{(same as carrier)} \tag{7}$$
$$(\omega_{RF} + \omega_{LO} = 220 \text{ MHz}), \quad \text{(same as carrier)} \tag{8}$$

(d) Third order:

$$|2\omega_{RF} - \omega_{LO}| = 140 \text{ MHz}, \tag{9}$$
$$|2\omega_{LO} - \omega_{RF}| = 80 \text{ MHz}, \tag{10}$$
$$3\omega_{RF} = 360 \text{ MHz}, \tag{11}$$
$$3\omega_{LO} = 300 \text{ MHz}, \tag{12}$$
$$(2\omega_{RF} + \omega_{LO}) = 340 \text{ MHz}, \tag{13}$$
$$(\omega_{RF} + 2\omega_{LO}) = 320 \text{ MHz} \tag{14}$$

Re-derivation for this taylor expansion

For my own sake, I would find it helpful to write down the entire taylor expansion from which we get the first 10 spurious signals in a mixer:

We can begin by considering the Shockley diode equation, it is simply a model for the I-V curve of a pn junction diode in terms of 1) saturation current I_s, 2) diode voltage v_D, 3) ideality n, 4) and thermal voltage V_T.

$$i_D = I_s \left(e^{\frac{v_D}{nV_T}} - 1 \right)$$

We may consider the exponential term, call it f to be that which we want to expand. The expansion of any exponential is of form:

$$e^{ax} = \sum_{k=0}^{\infty} \frac{(ax)^k}{k!}$$

So, the function will look like:

$$i_D = I_s \left(\cancel{1+} \underbrace{\left[\frac{a}{1!(nV_T)} \right]}_{c_1} v_D + \underbrace{\left[\frac{a^2}{2!(nV_T)^2} \right]}_{c_2} v_D^2 + \underbrace{\left[\frac{a^3}{3!(nV_T)^3} \right]}_{c_3} v_D^3 + \ldots \cancel{-1} \right)$$

We may then let v_D be a sinusoidal function, such that:

$$\begin{aligned} i_D \approx I_s \Bigg(& c_1 \Big[\beta_{\mathrm{RF}} \cos\{\omega_{\mathrm{RF}} t + \theta_{\mathrm{RF}}\} + \beta_{\mathrm{LO}} \cos\{\omega_{\mathrm{LO}} t + \theta_{\mathrm{LO}}\} \Big] \\ & + c_2 \Big[\beta_{\mathrm{RF}} \cos\{\omega_{\mathrm{RF}} t + \theta_{\mathrm{RF}}\} + \beta_{\mathrm{LO}} \cos\{\omega_{\mathrm{LO}} t + \theta_{\mathrm{LO}}\} \Big]^2 \\ & + c_3 \Big[\beta_{\mathrm{RF}} \cos\{\omega_{\mathrm{RF}} t + \theta_{\mathrm{RF}}\} + \beta_{\mathrm{LO}} \cos\{\omega_{\mathrm{LO}} t + \theta_{\mathrm{LO}}\} \Big]^3 \Bigg) \end{aligned}$$

And we may readily expand higher order terms, which gives the result we attained in class.