PHSX 536: Homework #3

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Grant Saggars

Problem 1

(2.16) An impedance Z is built from a resistor and capacitor connected in parallel. When connected to an AC voltage source with a frequency of $f = 60Hz$, the impedance has a numerical value of $Z = 1000(1-j)\Omega$. The impedance is connected as shown in Figure 2.19, where the voltage source has an amplitude of $10V$ and at $t = 0$, the AC voltage is at a maximum.

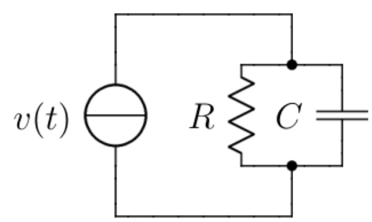


Figure 2.19: The circuit for problem 13.

(a) What are the values of R and C?

Solution:

We have an equivalent impedance in the branch:

$$Z = 1000(1-j)\ \Omega = \left(\frac{1}{R} + \frac{1}{\frac{1}{j\omega C}}\right)^{-1}$$

Where $\omega = 2\pi f = 120\pi$. The real part is R and the imaginary part is Z_C, giving the convenient relation:

$$R = 1000\,\Omega$$
$$1000 = \frac{1}{\omega C}, \quad \Rightarrow \quad C = \frac{1}{1000\omega}\text{ C}$$

(b) What is the power dissipated during one AC cycle in the impedance?

Solution:

We have a definition of power dissipation given as:

$$P = I_{\text{rms}} V_{\text{rms}} \cos\phi = I^2 R$$

$$\begin{aligned}
i_R(t) &= \frac{V}{R} = \frac{10\underline{/0^\circ}}{1000} = 0.01\underline{/0^\circ}\ \text{A} \\
i_C(t) &= \frac{V}{\frac{1}{j\omega C}} = j\omega C V = j(1000)\underline{/90^\circ}\ \text{A} \\
i(t) &= IR + IC = 0.01\underline{/0^\circ} + 0.01\underline{/90^\circ} = 0.01(1+j)\ \text{A} \\
\text{Re}(i(t)) &= 0.01(1+1)^{1/2} = 0.01\sqrt{2}\ \text{A}
\end{aligned}$$

We then have power dissipated:

$$P = I^2 R = (0.01)^2 \cdot 1000 = 0.1\ \text{W}$$

(c) What is the current in the resistor, $i_R(t)$?

Solution:

Obtained in part (b)

(d) What is the current in the capacitor, $i_C(t)$?

Solution:

Obtained in part (b)

(e) What fraction of the power from part (b) is dissipated in the resistor and the capacitor?

Solution:

All power is dissipated in the resistor. Capacitors don't dissipate power.

Problem 2

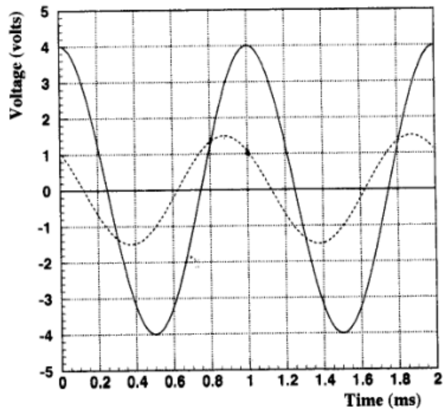


Figure 2.23: The scope trace for problem 18.

(2.18) Figure 2.23 is an image of your oscilloscope from lab. The solid curve is channel 1 and the dashed curve is channel 2. You may assume that the input to your circuit is displayed on channel 1 and the output from your circuit is on channel 2. Answer the following questions based on these measured scope traces.

(a) What is the frequency of the input signal?

Solution: There is a period of 1 second, so there is also a frequency of 1 hz.

(b) What is the "peak-to-peak" and the "RMS" voltage of the output signal?

Solution:

The it seems the waveform goes from -1.5 to 1.5 V, so the peak-to-peak is 3V. The RMS is, in general, given by the integral:

$$V_{\text{RMS}} = \sqrt{\frac{1}{T}\int_0^T V^2 \cos^2(\omega t)\, dt}$$

But for sinusoidal waveforms, it's simply

$$V_{\text{RMS}} = \frac{V_{\text{PK}}}{\sqrt{2}}$$

So the RMS is $\frac{1.5}{\sqrt{2}}$.

(c) For this particular frequency, what is the gain, $|G|$, of your circuit?

Solution:

Gain is given as output/input amplitude, so we have

$$|G| = \frac{1.5}{4} = 0.375 = 20\log_{10}(0.375)\,\text{db} = -8.52\,\text{db}$$

Problem 2 continued on next page...

(d) For this particular frequency, what is the phase difference

$$\Delta\phi = \phi_{out} - \phi_{\text{in}}$$

(in degrees) between the input and output?

Solution:

Sinusoidal waveforms are of form:

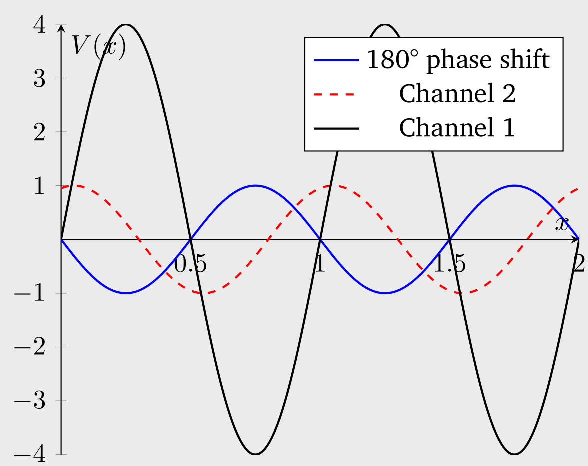
$$V(t) = \sin(\omega t + \phi) = \sin(2\pi f + \phi)$$

Channel 1 is ahead of channel 2 by 0.2 seconds, and both have the same period of 1 second. It follows that they have a frequency of 1 Hz as well. The angular frequency is then 2π rad / s. With the time difference,

$$\boxed{\Delta\phi = 2\pi \cdot 0.2 = 0.4\pi\text{rad} = 72^\circ}$$

(e) Accurately sketch what the output signal would look like if the phase difference from part (d) were 180°.

Solution:



Problem 3

(2.28) If the curve on a Bode plot is falling off at $60dB$/decade, what is the frequency dependence of the gain?

Solution:
Each pole of a bode plot indicates that it is falling off of a gain of -20 dB / decade. The frequency dependence is

$$|G| \propto \frac{1}{f^3}$$

where f is the frequency, and G is the gain.

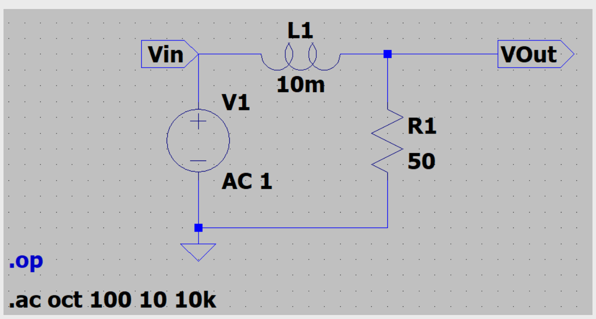
Problem 4

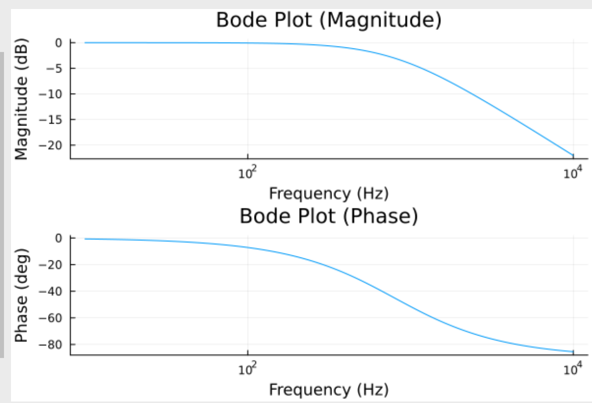
LTSpice problem: Consider the circuit shown below (this is also the circuit for Part A of Experiment #4). Use LTSpice to obtain plots of the power gain and phase shift as seen by the Ch. 2 output as a function of source frequency, with $10 \text{ Hz} \leq f_{\text{source}} \leq 10 \text{ kHz}$. Simulate a CR high-pass filter. Assume component values (C,R) = (10 μF, 50 Ω). (See video on "Bode plots with LTSpice.")

Z_1	Z_2	Type
L	R	low pass
C	R	high pass
R	C	low pass

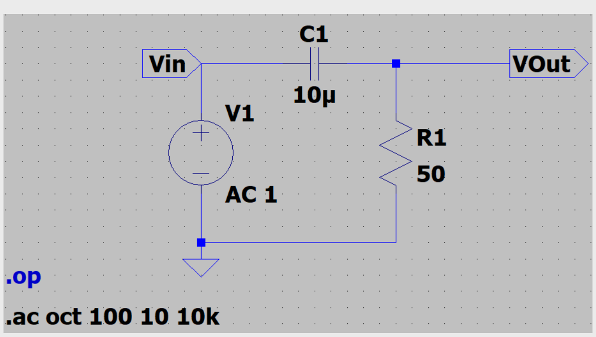
Solution:

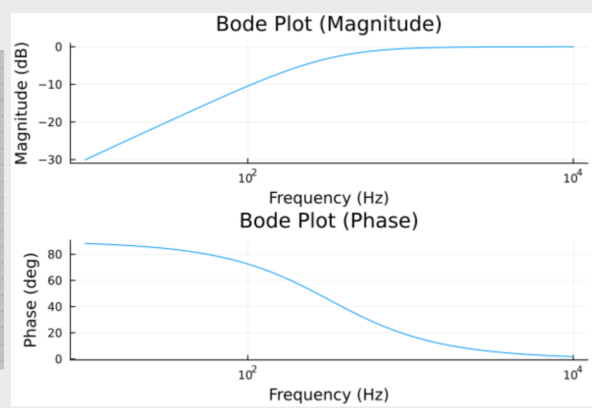
(a) LR Circuit (low-pass):



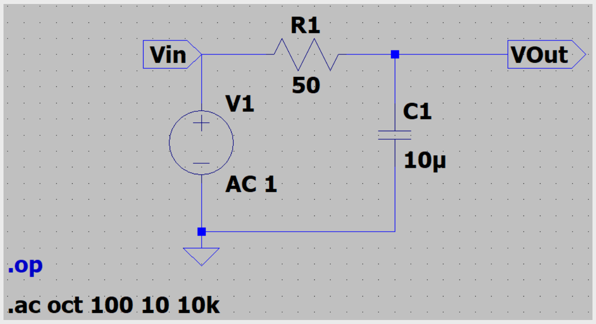


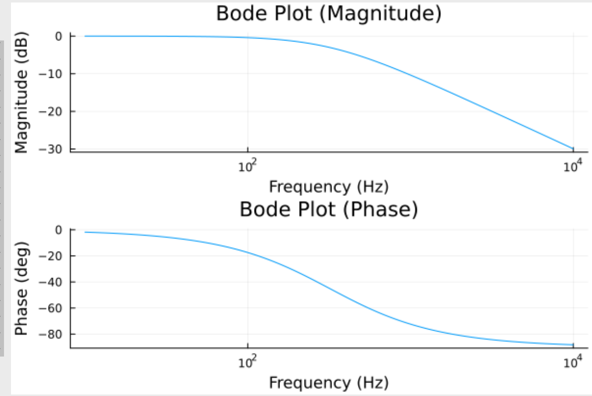
(b) CR Circuit (high-pass):





(c) RC Circuit (low-pass):

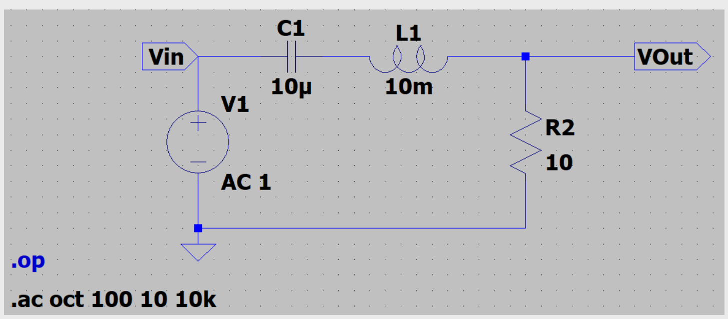




Problem 5

LTSpice problem: Set up a series LRC circuit driven by an AC signal source. For this circuit, use $C = 10\ \mu$F, $L = 10$ mH, and $R = 10\ \Omega$. Generate a Bode plot of the power gain and a plot of the phase shift for 10 Hz $\leq f_{source} \leq$ 10 kHz. (See video on "Bode plots with LTSpice.")

Solution:



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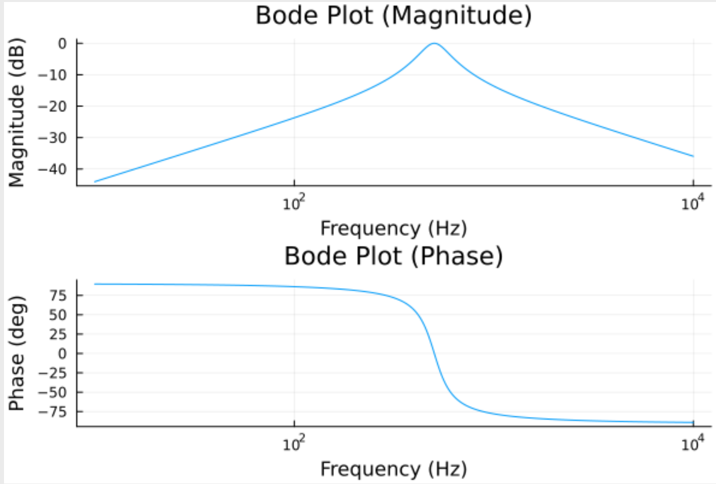
Bode Plot (Magnitude)
Magnitude (dB)
0
−10
−20
−30
−40
10^2
10^4
Frequency (Hz)
Bode Plot (Phase)
Phase (deg)
75
50
25
0
−25
−50
−75
10^2
10^4
Frequency (Hz)