EECS 622: Homework #18

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Problem 1

Consider this phase modulated signal:

$$v_o(t) = \cos\left[\omega_0\left(10^{-6}t^2 + t\right)\right]$$

Determine the relative phase and total frequency of this signal.

Solution:

(a) If there were no modulation, we would have a signal $v'(t) = \cos[\omega_0 t]$. The total phase of the modulated signal and unmodulated signal are, respectively:

$$\begin{cases} \omega_0(10^{-6}t^2 + t) \\ \omega_0 t \end{cases}$$

Which have a difference that is the relative phase (phase noise):

$$\varphi_n(t) = \omega_0(10^{-6}t^2 + t) - \omega_0 t = \omega_0 10^{-6} t^2$$

Which also checks out, since for most values of t, $\varphi_n(t) \ll 1$.

(b) Now, the total frequency is the time derivative of the total phase.

$$\omega = \frac{d|\omega_0(10^{-6}t^2 + t)|}{dt} = \omega_0(2 \times 10^{-6}t + 1)$$

Problem 2

Consider two different rates of energy flow-power P_1 and power P_2.
It is known that the sum of the two values $[P_2 + P_1]$, when expressed with the dBm operator, is :

$$dBm\,[P_2 + P_1] = 0$$

And, the difference between the two values $[P_2 - P_1]$ is, when expressed with the dBm operator, is :

$$dBm\,[P_2 - P_1] = -7$$

Determine the value $dBm\,[P_1]$

Solution:
This time I will choose to express these algebraically. We have the system of equations:

$$\begin{cases} -10\log_{10}\left[\frac{(P_2+P_1)\cdot\cancel{1\text{ mW}}}{\cancel{1\text{ mW}}}\right] &= 0 \\ -10\log_{10}\left[\frac{(P_2-P_1)\cdot\cancel{1\text{ mW}}}{\cancel{1\text{ mW}}}\right] &= -7 \end{cases}$$

$$\begin{cases} P_2 + P_1 &= 10^0 \\ P_2 - P_1 &= 10^{0.7} \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 10^0 \\ -1 & 1 & 10^{0.7} \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 10^0 \\ 0 & 1 & \frac{1}{2}(10^{0.7}+1) \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 10^0 - \frac{1}{2}(10^{0.7}+1) \\ 0 & 1 & \frac{1}{2}(10^{0.7}+1) \end{array}\right]$$

We get $P_1 \approx -2, P_2 \approx 3$. Applying the dBm operator to these gives the final result:

$$\text{dBm}[P_1] = -10\log_{10}[|-2|] = 3$$