PHSX 631: Homework #8

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Problem 1

A rocket leaves Earth at a speed of $0.95c$. When a clock on the rocket says that 1.5 hour has elapsed, the rocket sends a light signal back to Earth.

(a) According to Earth clocks, when was the signal sent?

Solution:

We have a Lorentz transformation matrix given as

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad X = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}, \qquad X' = \Lambda X$$

Let the rocket be in reference frame X'. Let the stationary Earthen reference frame be X. $\beta = 0.95$ and $\gamma \approx 3.2$. For the fun of it I'll leave my units in hours, so $c = 60^2 \cdot 3 \times 10^8 \frac{\mathrm{m}}{\mathrm{hour}}$.

$$ct' = \gamma(ct - \cancelto{0}{\beta x}) \implies t = 4.8 \text{ hours}$$

(b) According to Earth clocks, how long after the rocket left, did the signal arrive back on Earth?

Solution:

In frame X', the rocket travels $d' = 0.95c \cdot 1.5 \text{ hours}$. In frame X, the rocket travels $d = 0.95c \cdot 4.8 \text{ hours}$. We want d, not d', so the return time is

$$\frac{d}{c} = d = \frac{0.95c \cdot 4.8}{c} \text{ hours} = 4.56 \text{ hours}$$

The signal arrives back at $4.8 + 4.56 = 9.36$ hours.

(c) According to the rocket observer, how long after the rocket left did the signal arrive back at Earth?

Solution:

According to the rocket observer, the signal takes time $\frac{d'}{c} = 1.425 \text{ hours}$ to return for a total of $1.425 + 1.5 = 2.925$ hours.

Problem 2

Event A happens at point $(x = 5, y = 3, z = 0)$ and at time given by $ct = 15$.
Event B occurs at $(10, 8, 0)$ and $ct = 5$. Both are specified in reference frame K.

(i) What is the invariant interval (in Minkowski space) between events A and B?

Solution:

$$s^2 = -\left(ct_B - ct_A\right)^2 + \left(x_B - x_A\right)^2 + \left(y_B - y_A\right)^2 + \left(z_B - z_A\right)^2 = -50$$

This is a timelike interval.

(ii) Is there an inertial reference frame, K', in which events A and B occur simultaneously?
If so, find the velocity difference (speed and direction) between K' and K.

Solution:

There exist only imaginary reference frames in which this is the case, evident from part (i).

(iii) Is there an inertial reference frame, K', in which events A and B occur at the same spatial point?
If so, find the velocity difference (speed and direction) between K' and K.

Solution:

We are asking the question:

$$\begin{aligned} x'_B &\stackrel{?}{=} x'_A \\ \gamma(x_B - vt_B) &\stackrel{?}{=} \gamma(x_A - vt_A) \\ v &\stackrel{?}{=} \frac{x_B - x_A}{t_B - t_A} \\ v &= -0.5 \end{aligned}$$

Correcting units, we'd have

$$v = -0.5c$$

Problem 3

A rocket is moving away from Earth at speed $0.97c$ and a gun on that rocket shoots a bullet at a speed of $0.75c$ in the same direction. What does an observer on Earth see the speed as?

Solution:
Addition in the relativistic frame is defined as

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} \approx 0.9967c$$

Problem 4

Two spacecraft are moving directly towards each other at a speed of $0.80c$, as seen by someone on a planet at rest. At what speed does one ship see the other move towards it?

Solution:

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} \approx 0.976c$$