EECS 622: Homework #2

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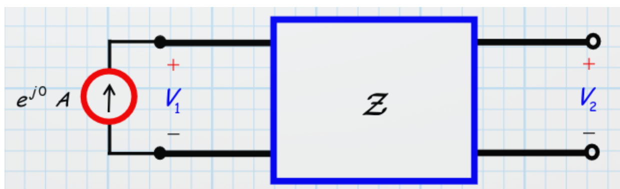
Problem 1

We start with an ideal current source, and then connect it to a two-port device with impedance matrix:

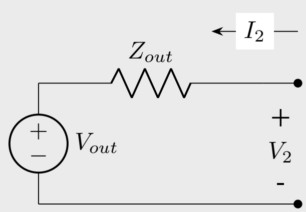
$$\mathbf{Z} = \begin{bmatrix} 20 & 40 + j30 \\ 40 + j30 & 10 \end{bmatrix} \ \Omega$$

We have thus created a **new source** (it's *not* an ideal current source anymore!)



Determine the Thevenin's equivalent (i.e., V_{out} and Z_{out}) of this new source, along with its available power.

Solution: Deriving V_{out}



The goal is to create an equivalent circuit as shown above. This is as simple as determining the Thevenin equivalent. The voltage seen by the load is $V_2 = V_{out}$. Meanwhile the resistance seen by the load is the total resistance, which may be calculated by shorting the load and expressing the current.

To find V_{out} for the thevenin equivalent, we need to find V_2 when there is an open circuit:

$$\begin{cases} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{cases}$$

subject to:

$$\begin{cases} I_1 = \frac{V_s - V_1}{Z_g} \\ I_2 = 0 \end{cases}$$

It is possible to then solve for V_2:

$$\begin{aligned}
I_1 &= \frac{V_g - Z_{11}I_1}{Z_g} \\
I_1 Z_g &= V_g - Z_{11}I_1 \\
I_1(Z_g + Z_{11}) &= V_g \\
I_1 &= \frac{V_g}{Z_g + Z_{11}} \\
V_2 &= \frac{Z_{21}V_g}{Z_g + Z_{11}}
\end{aligned}$$

Therefore,

$$\boxed{V_{out} = \frac{Z_{21}V_g}{Z_g + Z_{11}}}$$

Solution: Deriving Z_{out}
We will have a the equations for a short circuit on V_2

$$\begin{cases} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{cases}$$

Subject to:

$$\begin{cases} V_2 = 0 \\ I_1 = \frac{V_g - V_1}{Z_g} \end{cases}$$

Then I_2 will be:

$$I_2 = -\frac{Z_{21}}{Z_{22}}I_1 = -\frac{Z_{21}}{Z_{22}}\frac{V_g}{Z_g + Z_{11}\frac{Z_{12}Z_{21}}{Z_{22}}}$$

These are identical to what we have derived in class, so this is good. We will have an output impedance given as (negative sign due to definition of I_2):

$$\boxed{Z_{out} = \frac{V_2}{-I_2} = \left(\frac{Z_{21}}{Z_{11} + Z_s}\right)\left(\frac{Z_{22}}{Z_{21}}(Z_{11} + Z_s) - Z_{12}\right) = Z_{22} - \frac{Z_{21}Z_{12}}{Z_{11} + Z_g}}$$

V

Solution: Deriving Power
Beginning by substituting derived expressions into our expression for absolute power:

$$
\begin{aligned}
p_{\text{out}}^{\text{avl}} &= \frac{|Z_{21}|^2|V_g|^2}{8|Z_g+Z_{11}|^2\text{Re}\left\{Z_{22}-\frac{Z_{21}Z_{12}}{Z_{11}+Z_g}\right\}} \\
&= \frac{2500|V_g|^2}{8|Z_g+20|^2\text{Re}\left\{10-\frac{(40+j30)^2}{20+Z_g}\right\}} && \text{(Substitute given values)} \\
&= \frac{2500|V_g|^2}{8(Z_g+20)^2\left(10-\frac{2500}{20+Z_g}\right)} && \text{(Real part only, } |Z_g+20|^2=(Z_g+20)^2\text{)} \\
&= \frac{2500|V_g|^2}{8(Z_g+20)^2\cdot\frac{10(20+Z_g)-2500}{20+Z_g}} && \text{(Combine fractions in denominator)} \\
&= \frac{2500|V_g|^2}{8(Z_g+20)(10(20+Z_g)-2500)} && \text{(Cancel common factor)} \\
&= \frac{2500|V_g|^2}{8(Z_g+20)(200+10Z_g-2500)} && \text{(Expand numerator)} \\
&= \frac{2500|V_g|^2}{8(Z_g+20)(10Z_g-2300)} && \text{(Simplify)} \\
&= \frac{2500|V_g|^2}{80(Z_g+20)(Z_g-230)} && \text{(Factor out 10)} \\
&= \frac{125|V_g|^2}{4(Z_g+20)(Z_g-230)} && \text{(Simplify coefficients)}
\end{aligned}
$$

Therefore,

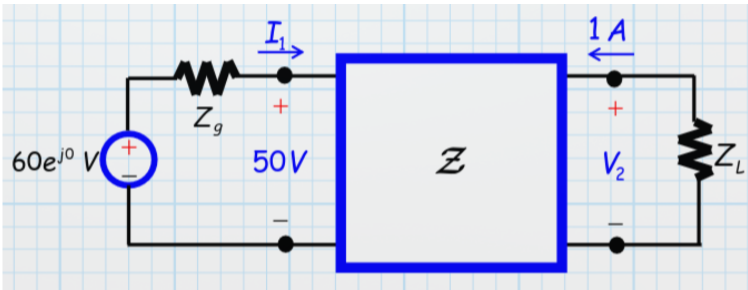
$$\boxed{P_{\text{out}}^{\text{avl}} = \frac{125|V_g|^2}{4(Z_g+20)(Z_g-230)}}$$

Problem 2

In the circuit below, the 2-port device is characterized by the impedance matrix:

$$\mathbf{Z} = \begin{bmatrix} 40 & -30 \\ -30 & 20 \end{bmatrix} \Omega$$

The voltage across **port 1** is 50 Volts, while current flowing into **port 2** is 1 Amp.



Determine the values of impedances Z_g and Z_L.

Solution:
My initial thoughts were to make the full Thevenin equivalent transformation, however, there is actually enough information given to derive numeric values from the trans-impedance parameters:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \tag{1}$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \tag{2}$$
$$I_1 = \frac{V_g - V_1}{Z_g} \tag{3}$$
$$I_2 \equiv 1 \text{ A} \tag{4}$$

I will rearrange (1) for I_1:

$$I_1 = \frac{V_1 - Z_{12}I_2}{Z_{11}} = \frac{50 - (-30)(1)}{40} = 2$$

Then, we can use (3) to extract Z_g

$$\boxed{Z_g = \left(\frac{Z_{11}}{V_1 - Z_{12}I_2}\right)(V_g - V_1) = \left(\frac{40}{50 - (-30)(1)}\right)(60e^{j0} - 50) = 5\ \Omega}$$

Now, I will want V_2 from (2) to determine Z_L by ohm's law:

$$V_2 = Z_{21}I_1 + Z_{22}I_2 = (-30)(2) + (20)(1) = -40 \text{ V}$$

The actual current passing through Z_L is in the opposite direction of I_2, so we will have:

$$\boxed{Z_L = \frac{-40}{-1} = 40\ \Omega}$$