PHSX 521: Review

October 4, 2024

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Polar Coordinates: We have

$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2} \\ \phi = \arctan\frac{y}{x} \end{cases}$$

Radial Time Derivative

$$\begin{aligned} \frac{d\vec{r}}{dt} &= \frac{d}{dt}\left(\cos\phi\hat{x}, \quad r\sin\phi\hat{y}\right) \\ &= \frac{d}{dt}(\cos\phi)\hat{x} + \cos\phi\frac{d\hat{x}}{dt} \\ &\quad + \frac{d}{dt}(\sin\phi)\hat{y} + \sin\phi\frac{d\hat{y}}{dt} \\ &= -\sin\phi\dot{\phi}\hat{x} + \cos\phi\dot{\phi}\hat{y} \\ &= \dot{\phi}\left(-\sin\phi\hat{x} + \cos\phi\hat{y}\right) \\ &= \dot{\phi}\hat{\phi} \end{aligned}$$

Angular Time Derivative

$$\begin{aligned} \frac{d\vec{\phi}}{dt} &= \frac{d}{dt}(-\sin\phi\hat{x} + \cos\phi\hat{y}) \\ &= -\frac{d}{dt}(\sin\phi)\hat{x]} + (-\sin\phi)\frac{d\hat{x}}{dt} \\ &\quad + \frac{d}{dt}(\cos\phi)\hat{y} + (\cos\phi)\frac{d\hat{y}}{dt} \\ &= -\cos\phi\dot{\phi}\hat{x} - \sin\phi\dot{\phi}\hat{y} \\ &= -\dot{\phi}(\cos\phi\hat{x} + \sin\phi\hat{y}) \\ &= -\dot{\phi}\hat{r} \end{aligned}$$

$$\frac{dv}{dt} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$$

Drag Forces

$$\begin{aligned} f_{\text{drag}} &= -f(v)\hat{v} \\ &= \ell v + cv^2 + \dots \\ &= \beta D v + \gamma D^2 v^2 \end{aligned}$$

Linear prop. to viscosity (β) and linear size of projectile (D). Quadratic prop. to density of medium (γ) and cross sectional area (D). The relative importance is

$$\frac{f_{\text{quad}}}{f_{\text{lin}}} = \frac{\gamma D^2 v^2}{\beta D v} = \frac{\gamma}{\beta} D v$$

Energy: Energy is conserved only under conservative forces. This also allows us to define a potential energy.

Potential Energy

Proof of Conservation

$$\begin{aligned}
\frac{\partial L}{\partial t} &= \sum_i \dot{q}_i \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i \\
&= \sum_i \frac{d}{dt}\left(\dot{q}_i \frac{\partial L}{\partial \dot{q}_i}\right) \\
&= \frac{d}{dt}\left(\sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L\right) \\
&= 0
\end{aligned}$$

Central Fields

Kepler's Central Field