EECS 622: Homework #4

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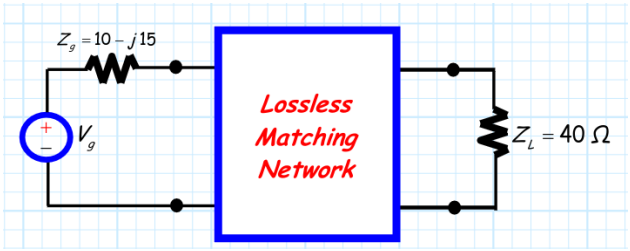
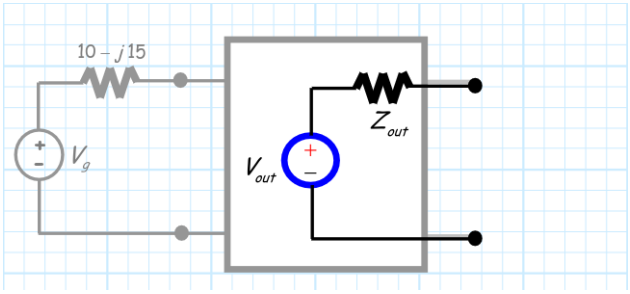
Problem 1

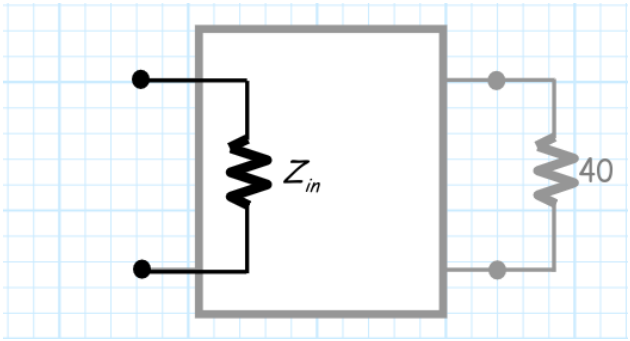
A lossless matching network was properly constructed to match a source with parameters V_g and $Z_g = 10 - j15\Omega$ to a load $Z_L = 40\Omega$:



If we disconnect the load from the matching network, we find that we have a new equivalent source (the original source followed by the matching network), with parameters V_{out} and Z_{out} :



Likewise, if we disconnect the source from the matching network, we find an input impedance (the matching network followed by the load) of Z_{in} (turn the page!!):



1. Determine the impedance values Z_{in} and Z_{out} .

Solution:

We design a matching network for the express purpose of setting the following relationship,

$$\begin{cases} Z_g = Z_{\text{in}}^* = 10 + j15\ \Omega \\ Z_L = Z_{\text{out}}^* = 40\ \Omega \end{cases}$$

2. Say the load Z_L absorbs energy at a rate of 800 mW . Determine then the value $|V_g|$.

Solution:

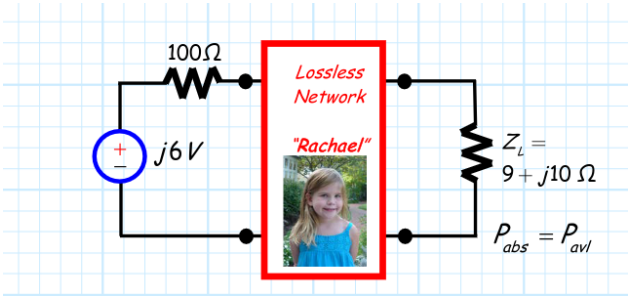
Because this is a *lossless* matching network, we may exploit the following invariance to determine the magnitude $|V_g|$.

$$\begin{aligned} P_g^{avl} &= P_L^{abs} && \text{(Lossless)} \\ \frac{|V_g|^2}{8R_g} &= P_L^{abs} && \text{(Known expression for } P^{avl}\text{)} \\ |V_g| &= \sqrt{(8R_g)P_L^{abs}} && \text{(Algebra)} \\ |V_g| &= 8\ \text{V} \end{aligned}$$

Note: $1\ \Omega = 1\frac{V^2}{W}$.

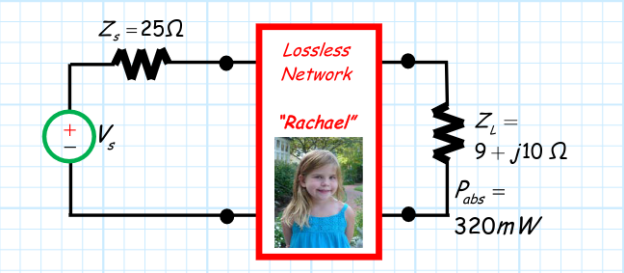
Problem 2

A promising young engineer has designed a lossless matching network.
The design matches a specific source (with $Z_g = 100\Omega$) to a specific load impedance $Z_L = 9 + j10\Omega$.



The engineer tests the design in the lab, by measuring the power absorbed by the load Z_L.
The engineer is very happy, because the design worked perfectly-the power absorbed by the load is equal to the power available from the source.
The engineer is so happy with the design that the engineer gives the matching network a name-Rachael.
Now, the engineer removes the source, and replaces it with a completely new one (i.e., $V_s \neq j6$ and $Z_s = 25\Omega$) :

Problem 2 continued on next page…



Note however, that both the load $Z_L = 9 + j10\Omega$, and the lossless network "Rachael", remain completely unchanged.
The engineer now measures the power absorbed by the load $Z_L = 9 + j10\Omega$, and finds that it is 320 mW .
Determine the magnitude $|V_s|$ of this new source.
Provide explicit justification for your answer.

Solution:
Because power on the load side was not changed, we are fortunate to have the same Z_{in} which is notably not a conjugate match.

$$I = \frac{V_s}{Z_g + Z_{in}} = \frac{V_s}{125}$$

Power is simply:

$$P_{\text{in, rms}} = \frac{1}{2}|I|^2 Z_{in} = \frac{|V_s|^2 \cdot 100}{2(125)^2}$$

Since it is lossless, this is also the power absorbed, and I can therefore express $|V_s|$:

$$P_{abs} = 0.320 = \frac{2|V_s|^2}{625} \implies |V_s| = 10$$