The Linear Phase Filter

Q: *So, **narrowband** filters particularly need to exhibit a **constant** phase delay* $\tau(\omega)$.

*What should the **phase function*** $\arg[H(\omega)]$ *be for this **dispersionless** case?*

A: We can express this problem **mathematically** as requiring:

$$\tau(\omega) = \tau_c$$

where τ_c is some **constant**.

We get to solve a differential equation!!!!!

Recall that the definition of **phase delay** is:

$$\tau(\omega) = -\frac{\partial \arg[H(\omega)]}{\partial \omega}$$

and thus **combining** these two equations, we find ourselves with a **differential equation (yeah!)**:

$$-\frac{\partial \arg[H(\omega)]}{\partial \omega} = \tau_c$$

The **solution** to this differential equation provides us with the necessary phase function $\arg[H(\omega)]$ for a **constant** phase delay τ_c.

A rather trivial solution

Fortunately, this differential equation (like **most** differential equations!) is **easily** solved!

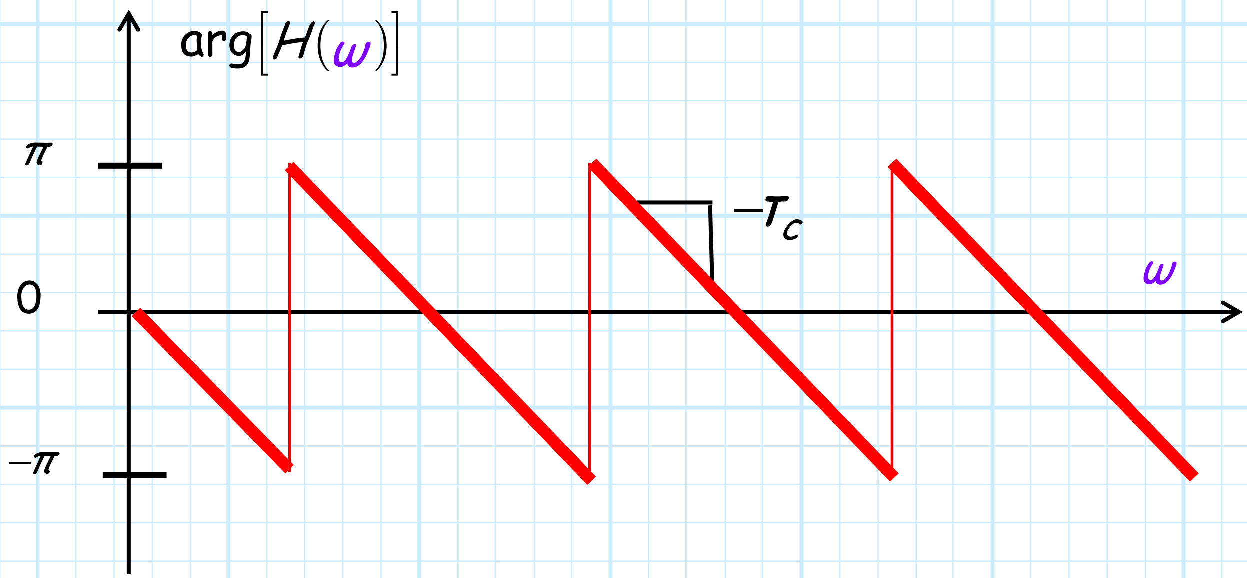
The **solution** is:

$$\boxed{\arg\left[H(\omega)\right] = -\omega T_c + \varphi_c}$$

where φ_c is an arbitrary **constant**.

Linear phase is ideal

Plotting this phase function (with $\varphi_c = 0$):



As **you likely expected**, this phase function is **linear**, such that it has a **constant slope** $(-T_c)$.

Filters with this phase response are called **linear phase filters**, and have the desirable trait that they cause **no** dispersion distortion.