EECS 622: Homework #6

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Problem 1

The **reflection coefficient function** along a transmission line is:

$$\Gamma(z) = j\,0.5e^{+j(\pi)z}$$

At $z = 1$ on this same line, the minus-wave voltage is:

$$V^{-}(z = 1) = 2.0\,\text{V}$$

Determine the value of the **plus-wave voltage**, at transmission line location $z = 1$.
In other words, determine the value $V^{+}(z = 1)$.

Solution:
We have some relations between the decomposed wave components and reflection function:

$$\Gamma(z, \omega) = \frac{V^{-}(z, \omega)}{V^{+}(z, \omega)} \qquad V(z, \omega) = V(z, \omega)^{+} + V(z, \omega)^{-}$$

The reflection coefficient makes this straightforward,

$$V^{+}(z, \omega) = \frac{2.0e^{j0}}{j0.5e^{+j\pi}} = \frac{2}{0.5j} = 4j \text{ V}$$

Problem 2

For a certain transmission line, $\beta = \pi/2$ (radians/m) and $Z_0 = 50\,\Omega$.
We know that the total voltage at location $z = -1$ m on this transmission line is:

$$V(z = -1) = j\,6\,\text{V}$$

The reflection coefficient function at location $z = 1$ m is likewise:

$$\Gamma(z = 1) = -0.25$$

Determine the total current at location $z = 0$ (i.e., $I(z = 0)$) on this transmission line.

Solution:
Starting with the same relations:

$$\Gamma(z,\omega) = \frac{V^-(z,\omega)}{V^+(z,\omega)} \tag{1}$$

$$V(z,\omega) = V(z,\omega)^+ + V(z,\omega)^- \tag{2}$$

$$Z_0 = \frac{V^+(z,\omega)}{I^+(z,\omega)} \tag{3}$$

$$-Z_0 = \frac{V^-(z,\omega)}{I^-(z,\omega)} \tag{4}$$

And β is sufficient to make the additional relations:

$$V^+(z,\omega) = V_0^+(\omega)e^{-j\beta z} \qquad V^-(z,\omega) = V_0^-(\omega)e^{+j\beta z} \tag{5}$$

$$I^+(z,\omega) = I_0^+(\omega)e^{-j\beta z} \qquad I^-(z,\omega) = I_0^-(\omega)e^{+j\beta z} \tag{6}$$

It is most convenient to start with some relations since we know β

$$V_0^+ = \frac{V^+}{e^{+j\pi/2}} = jV^+ \qquad V_0^- = \frac{V^-}{e^{-j\pi/2}} = -jV^-$$

Use (1-2) extract components for a fixed $z = 1$:

$$\begin{cases} -j0.25V_0^+(z=1,\omega) = -jV_0^-(z=1,\omega) \\ j6\text{ V} = j\left(V_0^+(z=1,\omega) - V_0^-(z=1,\omega)\right) \end{cases} \qquad \text{(At z=-1)}$$

$$\implies \begin{cases} V_0^+(z=1,\omega) = 8\text{V} \\ V_0^-(z=1,\omega) = 2\text{V} \end{cases}$$

Current falls out of (3):

$$I_0^+ = \frac{V^+}{Z_0} = -\frac{8}{50} = 0.16\text{ A} \qquad I_0^- = -\frac{V^-}{Z_0} = \frac{2}{50} = 0.04\text{ A}$$

Total current I is given as the sum:

$$I(z=0,\omega) = I_0^+ + I_0^- = 0.20\text{ A}$$