# Maximum Power Transfer along a Transmission Line

**Q:** *I think I **finally** have this all figured out.*

*The power **delivered** by the source is equal to the **difference** of the **incident** and **reflected** power along the lossless transmission line:*

$$P_g^{del} = P^{inc} - P^{ref}$$

**A:** Yes, absolutely.

**Q:** *And by conservation of energy, this power is also that **absorbed** by the load:*

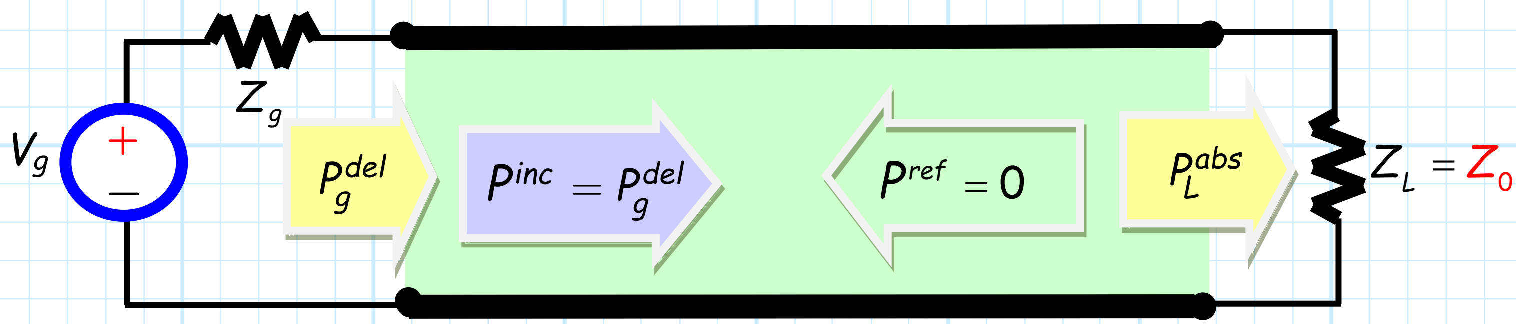
$$P_g^{del} = P^{inc} - P^{ref} = P_L^{abs}$$

**A:** I think I made that **quite clear**.

# All true!

**Q:** *Further, the "matched" load $Z_L = Z_0$ is precisely the load that reduces the **reflected** power to **zero**, so that:*

$$P_g^{del} = P^{inc} = P_L^{abs}$$




*And thus for the case where $Z_L = Z_0$—but for **just** this case—the **incident** power **is** equal to the power **delivered** by the source.*

**A:** And equal to the **power absorbed** by the load as well!!

# ACK! COMPLETELY FALSE!!!!

**Q:** ***And***, *since the* ***reflected power is zero***, *the power absorbed by matched load* $Z_L = Z_0$ *is* ***maximized***—*it equals all the* ***available power*** $P_{avl}$ *of the source, and so:*

$$P_g^{del} = P^{inc} = P_L^{abs} = P_g^{avl}$$

***Right?????***

**A: NO!!!!!!!! This last statement is absolutely false!!!!!!**

Yes, a matched load $Z_L = Z_0$ **will** minimize (to zero) the reflected power $P^{ref}$.

➔ **But**, that will **not** (generally) **maximize** the **absorbed/delivered** power!

# The load affects both waves!

**Q:** *Huh? Just **look** at the math (like you're **always** telling **us** to do!):*

$$P_g^{del} = P^{inc} - P^{ref} = P_L^{abs}$$

***Why** would minimizing $P^{ref}$ **not** maximize the values of $P_g^{del}$ and $P_L^{abs}$ ??*

**A:** Remember, the value of load $Z_L$ affects **both** the reflected ($V_0^-$) **and** the **incident ($V_0^+$) wave**!!!

$$P^{inc} = \frac{|V_0^+|^2}{2\,Z_0} = \frac{|V_g|^2}{2}\frac{Z_0}{\left|Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)\right|^2}$$

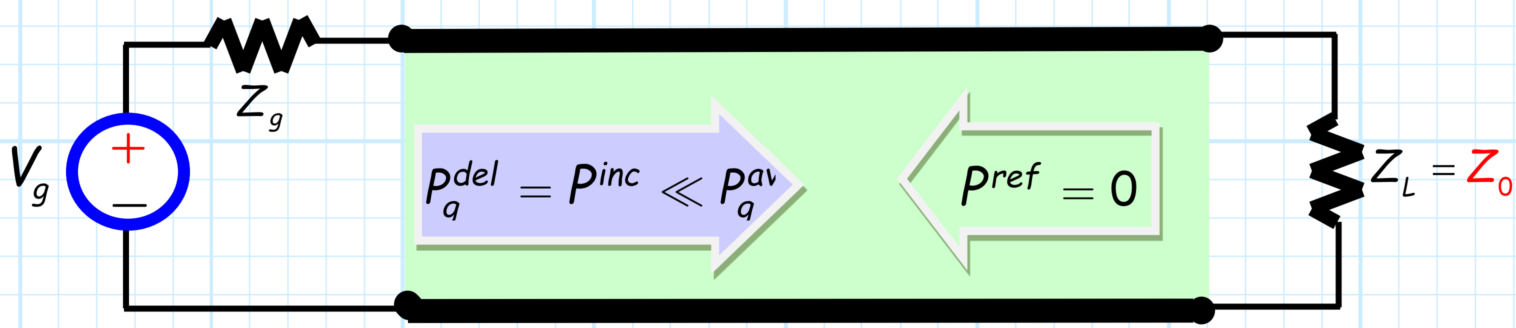
$$P^{ref} = P^{inc}\,|\Gamma_L|^2 = \frac{|V_g|^2}{2}\frac{Z_0\,|\Gamma_L|^2}{\left|Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)\right|^2}$$

# A "matched" load minimizes both reflected *and* incident power

A "matched" load of $Z_L = Z_0$ **does** *minimize* the **reflected power** ($P_{ref} = 0$).

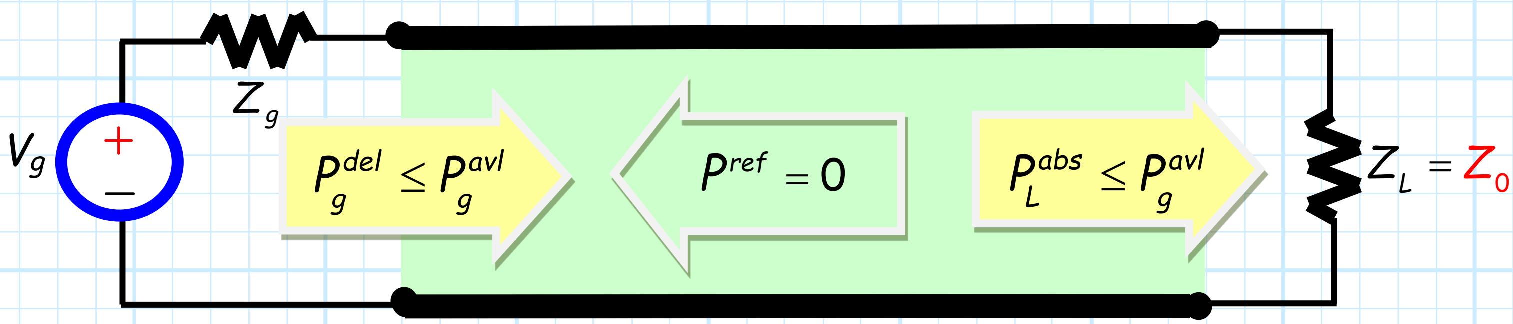
**But**, this **same** "matched" load does **not maximize** $P_{inc}$!

In fact a "matched" load actually **minimizes the incident power**!



# Minimizing reflection does NOT maximize absorption!

This means that the power absorbed by a **matched load**—and thus the power **delivered** by the source—will generally be **less** than the **available power** of the source.




If $Z_L = Z_0$, then the reflected power **is** minimized ($P^{ref} = 0$), but the absorbed power is (generally speaking) **not maximized** ($P_L^{abs} < P_g^{avl}$)!

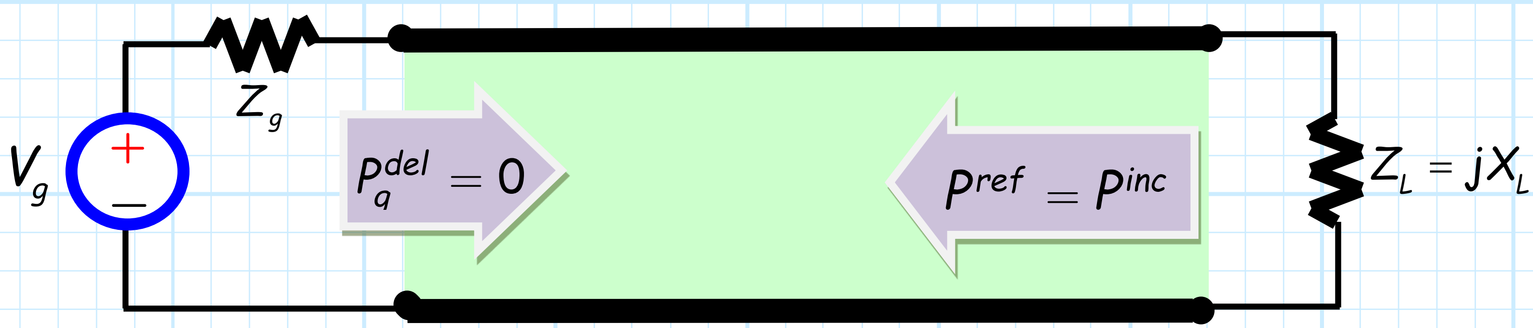
# A reactive load maximizes both incident and reflected power

**Q:** *So, the load $Z_L$ that **maximizes** its **absorbed power** is instead the load that **maximizes its incident** power $P^{inc}$ ????*

**A: Gosh no!**

The load that maximizes the incident power is in fact a **reactive load**.

- ➔ But a reactive load **cannot absorb** any energy at all!




The load that maximizes the **incident power** (i.e. $Z_L = jX_L$) is (unfortunately) the same load that maximizes the **reflected power**!

# Not minimize reflected, nor maximize incident—maximize their *difference*!

**Q:** *So, the load impedance $Z_L$ that **maximizes** its **absorbed power** is **not** the value $Z_L$ that **minimizes** $P^{ref}$.*

***Nor** is it the value $Z_L$ that **maximizes** $P^{inc}$.*

*So, just what load impedance $Z_L$ **does** maximize its absorbed power?*

**A:** The value of $Z_L$ that **maximizes the difference** between $P^{inc}$ and $P^{ref}$ !

$$P_g^{del} = P^{inc} - P^{ref} = P_L^{abs}$$

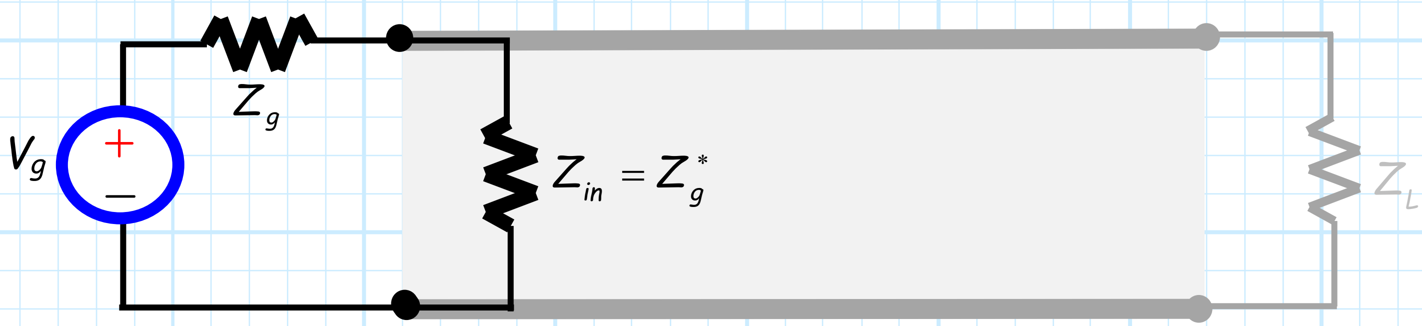
**Q:** *But what **specific value** of $Z_L$ actually does that?*

**A:** **You** know the answer!

# Conjugate match maximizes absorbed power!

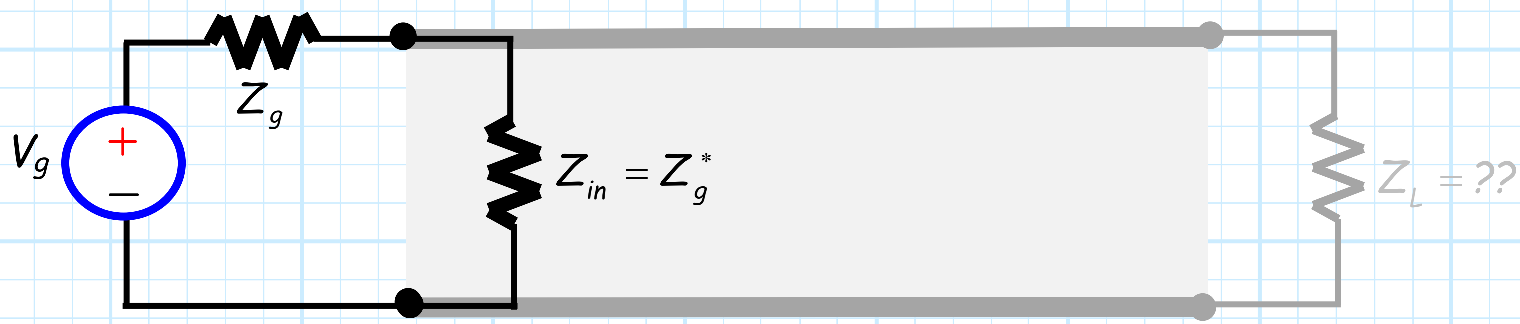
Recall that the **maximum** rate of energy absorption *i.e.*, $P_L^{abs} = P_g^{avl}$ is achieved **only** if the **input impedance** of the terminated transmission line is equal to the **complex conjugate** of the source:

$$P_L^{abs} \left( = P_g^{del} \right) = P_g^{avl} \quad \text{only if} \quad Z_{in} = Z_g^*$$



# Whatever this load is

Thus, the load that maximizes its absorbed power is **whatever** value of $Z_L$ that results in $Z_{in} = Z_g^*$!!




Note this value of load impedance $Z_L$ **depends on**:

1. the transmission line **characteristic** impedance $Z_0$ .

2. the **source** impedance $Z_g$ .

3. and—**most annoyingly**—the **line length** $\ell$.

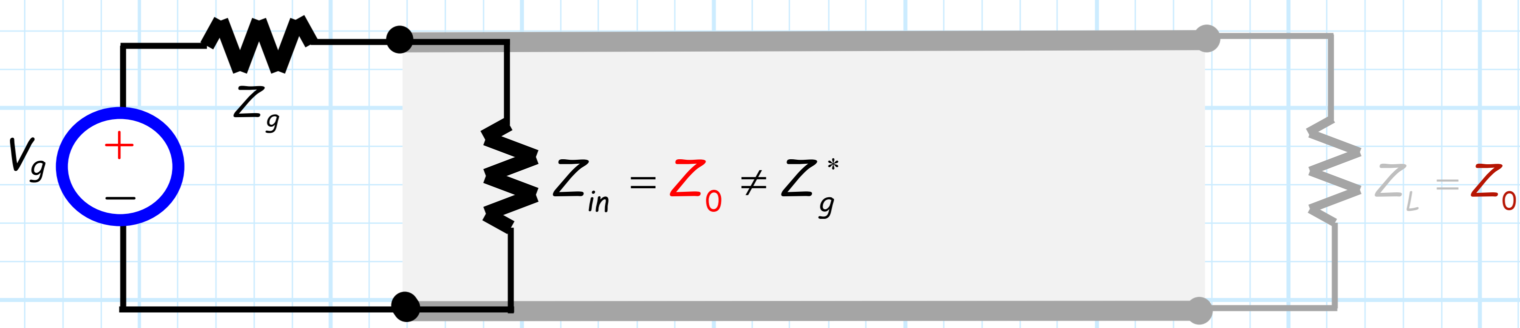
# A "matched" load is not a conjugate match!

Note for a "**matched" load** $Z_L = Z_0$, the **input impedance** is likewise numerically equal to the characteristic impedance:

$$Z_{in} = Z_0 \quad \text{if} \quad Z_L = Z_0$$

But, this input impedance is **not** typically the **complex conjugate** of the source impedance:

$$Z_g^* \neq Z_0 \text{!!!!}$$

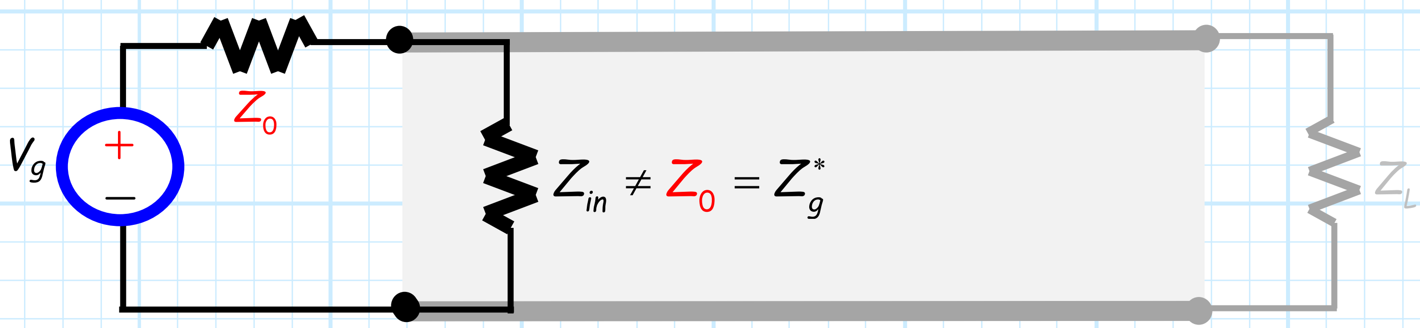


# A "matched" source is not a conjugate match either!

Likewise, for a "**matched**" **source** (where $Z_g = Z_0$)

But, this source impedance is **not** typically the **complex conjugate** of the input impedance:

$$Z_{in}^* \neq Z_0 !!!!$$



# ACK! AGAIN COMPLETELY FALSE!!!!

**Q:** *It **seems** logical that for the **conjugate matched** case, the **incident** power should be equal to its **maximum** possible value—equal to the **available** power of the source:*

$$P^{inc} = P_g^{avl} \quad \text{if} \quad Z_{in} = Z_g^* \quad \textbf{right ???}$$

**A: Nope; this statement is completely false as well !!!!!!!!!!!!!!**

# Incident is greater than delivered!

Remember, when a **conjugate match** occurs, the **delivered** (and thus absorbed) power is **equal** to all the available power of the source:

$$P_g^{avl} = P_g^{del} = P_L^{abs} \quad \text{if} \quad Z_{in} = Z_g^*$$

But, remember also that (in general), the **incident** power is **greater** than the power **delivered/ absorbed**:

$$P^{inc} \geq P_g^{del} = P_L^{abs}$$

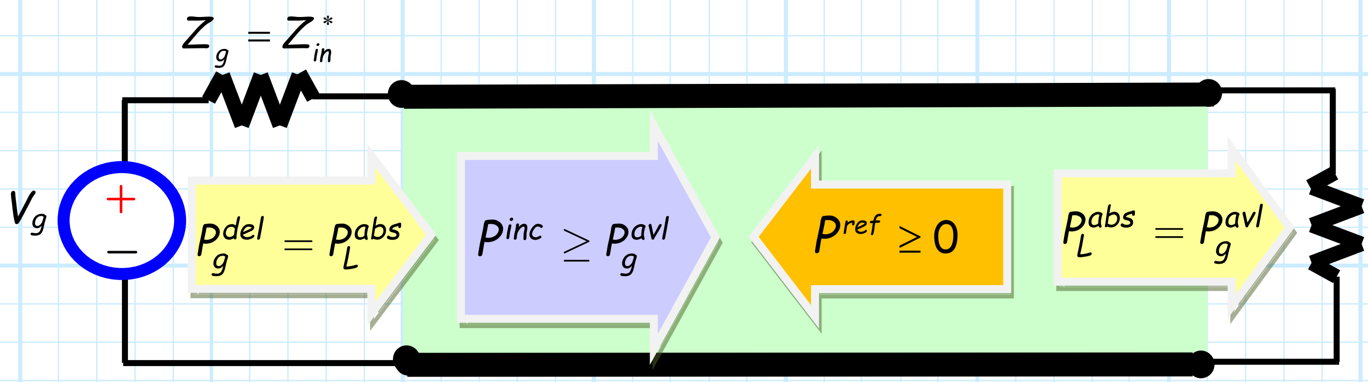
Recall that the **equality** ($P^{inc} = P_g^{del} = P_L^{abs}$) occurs **only** when $P^{ref} = 0$ (i.e., when $Z_L = Z_0$).

And recall also that the **matched load** case $Z_L = Z_0$ generally **does not create a conjugate** match ($Z_{in} = Z_0 \neq Z_g^*$)!

# So incident is greater than the available!!

We come to the conclusion that when a **conjugate match** occurs, the **incident** power is generally **greater than** the available power of the source:

$$P^{inc} \geq P_g^{avl} = P_g^{del} = P_L^{abs} \quad \text{if} \quad Z_{in} = Z_g^* \text{ !!!!!!}$$




**Q:** *Yikes! Doesn't this **violate** some sort of **conservation of energy** thing?*

**A:** Nope.

# Relax, the net power does *not* violate conservation of energy

Although the **incident** power is **greater** than that **available** ($P^{inc} \geq P_g^{avl}$), the **net power** ($P^{net} = P^{inc} - P^{ref}$) will be **equal to** $P_g^{avl}$ when a conjugate match is established.

Conservation of energy is **not violated**!

Moreover, we **frequently** find that the **incident power will exceed the power available** from the source—**even** when no conjugate match exists!

$$P^{inc} = \frac{\left|V_0^+\right|^2}{2\,Z_0} = \frac{\left|V_g\right|^2}{2} \frac{Z_0}{\left|Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)\right|^2}$$

# Weasel words: they're my specialty

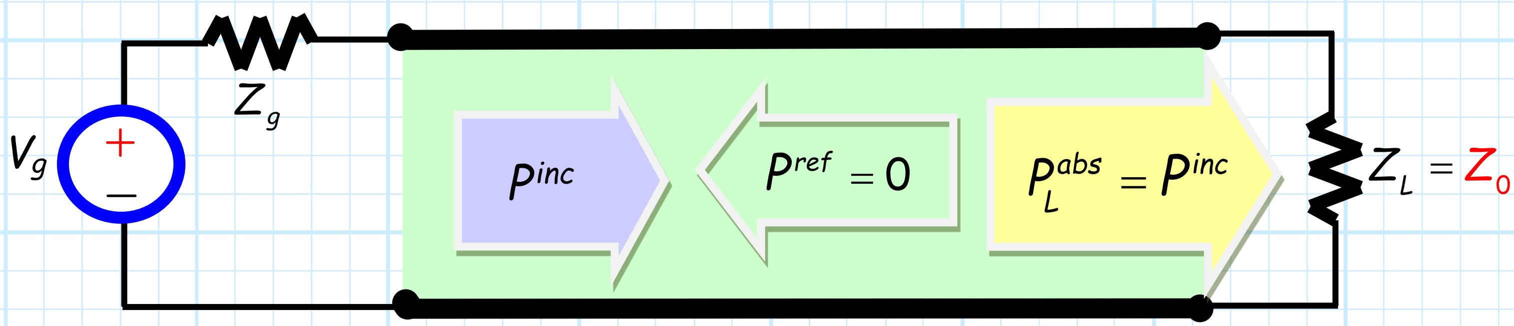
**Q:** *You peppered this discussion with **weasel words** like "**generally**".*

***Might** there be a situation where a "**matched" load** $Z_L = Z_0$ **indeed** does absorb all available power?*

**A:** Sure!

Remember, **if** the load is a "**matched" load**, then the **absorbed** power is equal to the **incident** power (the reflected power being **zero** and all):

$$P_L^{abs} = P^{inc} \quad \text{if} \quad Z_L = Z_0$$



# Deja Vu

Thus, the power **absorbed** by the "**matched**" **load** can be equal to the **available** power of the source **only if** the **incident** power is **equal** to the **available** power of the source!

$$\text{If} \quad Z_L = Z_0, \quad \text{then} \quad P_L^{abs} = P_g^{avl} \quad \text{only if} \quad P^{inc} = P_g^{avl}$$

**Q:** *Hey, didn't we discuss this case **earlier**?*

*Isn't the **incident** power equal to the **available** power when the **source impedance** is "**matched**" to the transmission line (i.e., $Z_g = Z_0$)?*

**A:** **Precisely**!

# Matched source *and* matched load

For a "**matched**" **source**, the **incident** power of the transmission line will be equal to the **available** power of the source:

$$P^{inc} = P_g^{avl} \qquad \text{if} \qquad Z_g = Z_0$$

And so if the **load** is likewise "matched", **all this incident power** will be **absorbed** by the source:

$$\boxed{P_g^{avl} = P^{inc} = P_L^{abs} \quad \text{if} \quad Z_g = Z_0 \quad \text{and} \quad Z_L = Z_0}$$

Perhaps most importantly, this is true **regardless of transmission line length** $\ell$!

# It's THE ideal situation!

Thus, the case of the "matched" load **AND** "matched" source is effectively the **ideal** case for transmission line interconnection.

Note that a **conjugate match is established**—such that **all** available source power is connected to the load.

→ And, it's all **independent of line length** $\ell$!