EECS 622: Homework #7

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Problem 1

The total voltage along a transmission line is:

$$V(z) = 14e^{-j(\pi/2)z} + 2e^{j(\pi/2)z}$$

The line impedance at location $z = 1$ on this same line is:

$$Z(z = 1) = 7.5\Omega$$

Determine the value of total current at transmission line location $z = 1$.
In other words, determine the value $I(z = 1)$.

Solution:

$$Z(z, \omega) \equiv \frac{V(z, \omega)}{I(z, \omega)} \tag{1}$$

Using (1),

$$\begin{aligned} I(z = 1, \omega) &= \frac{V(z = 1, \omega)}{Z(z = 1, \omega)} \\ &= \frac{-14j + 2j}{7.5} \\ &= -1.6j \text{ A} \end{aligned}$$

Problem 2

A certain transmission line has $\beta = \pi/2$ (radians/m).
We know that the reflection coefficient function at location $z = 1$ is:

$$\Gamma(z = 1) = -0.5$$

and the line impedance at location $z = 0$ is:

$$Z(z = 0) = 120\Omega$$

and the total voltage at location $z = -1$ is:

$$V(z=-1) = j4\,\mathrm{V}$$

Determine the function $I(z)$ that describes the time-harmonic current along this transmission line.

Solution:
I just need to fit boundary conditions to the wave equations.

$$Z(z,\omega) \equiv \frac{V(z,\omega)}{I(z,\omega)} = Z_0\left(\frac{e^{-j\beta z}+\Gamma_0 e^{+j\beta z}}{e^{-j\beta z}-\Gamma_0 e^{+j\beta z}}\right) \tag{1}$$

$$\Gamma \equiv \Gamma_0 e^{+j2\beta z} \tag{2}$$

$$\Gamma_0 \equiv \frac{V_0^-}{V_0^+} \tag{3}$$

$$I(z,\omega) = \frac{V^+}{Z_0}e^{-j\beta z} - \frac{V^-}{Z_0}e^{+j\beta z} \tag{4}$$

It's possible to find Γ_0 using (2):

$$\Gamma_0 = 0.5e^{j\pi(0-1)} = 0.5$$

Then, with (3) the components can be related by a scalar:

$$0.5 = \frac{V_0^-}{V_0^+} \implies 0.5V_0^+ = V_0^-$$

This lets me derive V^+, V^- components by substituting for one in the definition of V:

$$\begin{aligned}
V(z=-1) &= V^+e^{-j(\pi/2)(-1)} + V^-e^{j(\pi/2)(-1)} \\
j4 &= V^+e^{-j(\pi/2)(-1)} + 0.5V^+e^{j(\pi/2)(-1)} \\
j4 &= jV^+ - 0.5jV^+ \\
j4 &= 0.5jV^+ \\
V^+ &= 8\ \mathrm{V}
\end{aligned}$$

Consequently, $V^- = 4\ \mathrm{V}$. $Z(z=0) \neq Z_0$, and it is instead given by (1):

$$120 = \frac{1+\Gamma_0}{1-\Gamma_0} = 3Z_0$$

Finally, (4) gives the solution we wanted:

$$I(z,\omega) = \frac{8}{40}e^{-j\pi z/2} - \frac{4}{40}e^{j\pi z/2}\ \mathrm{A}$$