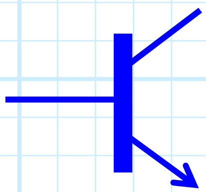
Intermodulation Distortion

The 1 dB **compression curve** shows that amplifiers are only **approximately** linear.



Actually, this should be **obvious**, as amplifiers are constructed with transistors—**non-linear** devices!

So, instead of the **ideal** case:

$$v_{out}(t) = A_{vo}\, v_{in}(t)$$

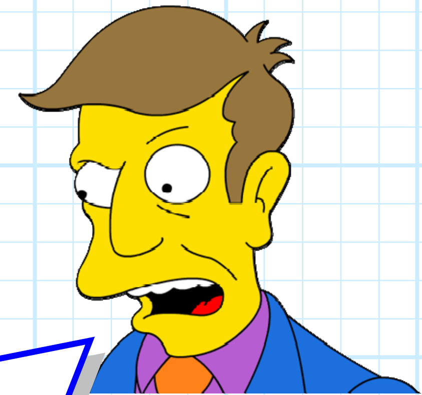
Actual amplifier behavior requires **more terms** to describe!

$$\boxed{v_{out} = A_{vo}\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots}$$

The ol' small-signal approximation

This representation is simply a **Taylor Series** representation of a **non-linear** function:

$$v_{out} = f(v_{in})$$



Q: ***Non***-*linear! But I thought an amplifier was a* ***linear*** *device?*

After all, we characterized it with an impedance ***matrix****!*

A: Generally speaking, the constants B, C, D, etc. are **very** small compared to the voltage gain A_{vo} .

Therefore, **if** v_{in} is likewise small, we can **truncate** the Taylor Series and **approximate** amplifier behavior as the linear function:

$$v_{out} \approx A_{vo}\, v_{in}$$

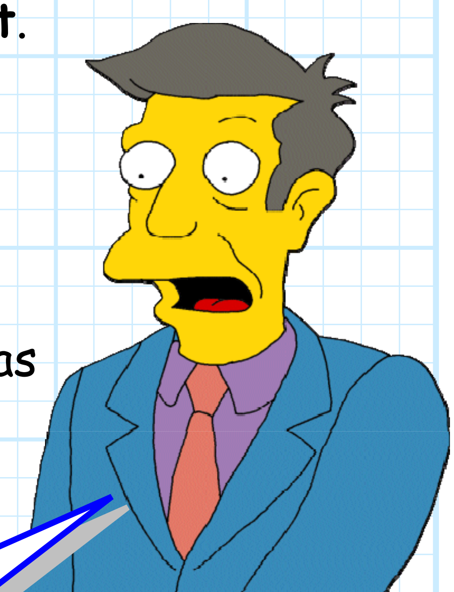
The small-signal approximation is only valid when the signal is small

BUT, as v_{in} gets large, the values v_{in}^2 and v_{in}^3 will get **really** large!

In that case, the terms $B\,v_{in}^2$ and $C\,v_{in}^3$ will become **significant**.

As a result, the output will not simply be a larger version of the input.

The output will instead be **distorted**—a phenomenon known as **Intermodulation Distortion**.



Q: *Good heavens! This sounds terrible.*

*What exactly is **Intermodulation Distortion**, and what will it do to our signal output?!?*

Where did this signal come from?

A: Say the input to the amplifier is sinusoidal, with magnitude *a:*

$$v_{in} = a\cos\omega t$$

Using our knowledge of trigonometry, we can determine the result of the second term of the output Taylor series:

$$\begin{aligned} B v_{in}^2 &= B a^2 \cos^2 \omega t \\ &= \frac{B a^2}{2} + \frac{B a^2}{2}\cos 2\omega t \end{aligned}$$

We have created a **harmonic** of the input signal!

In other words, the input signal is at a frequency ω, while the output includes a signal at **twice** that frequency (2ω).

→We call this signal a **second** order product, as it is a result of **squaring** the input signal.

It just keeps getting worse…

Note we also have a **cubed** term in the output signal equation:

$$v_{out} = A_{vo}\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

Using a **trig identity**, we find that:

$$\begin{aligned} C\, v_{in}^3 &= C\, a^3 \cos^3 \omega t \\ &= \frac{C\, a^3}{2} \cos \omega t + \frac{C\, a^3}{4} \cos 3\omega t \end{aligned}$$

Now we have produced a **second harmonic** (i.e., 3ω)!

→ As you might expect, we call this harmonic signal a **third-order** product (since it's produced from v_{in}^3).

Aren't these signals really small?

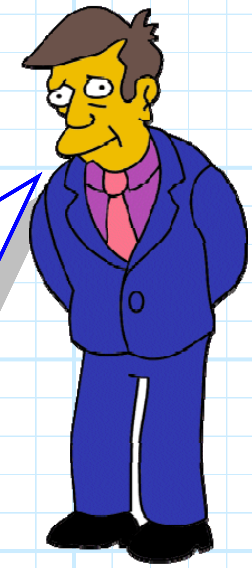
Q: *I confess that I am still a bit* ***befuddled.***

You said that values B and C are typically ***much*** *smaller that that of voltage gain* A_{vo} *.*

$$v_{out} = A_{vo}\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

Therefore it would seem that these harmonic signals would be ***tiny*** *compared to the fundamental output signal* $A_{vo}\, a \cos \omega t$ *.*

→ *Thus, I* ***don't*** *see why there's a problem!*



These products are NOT proportional to input power!

A: To understand why intermodulation distortion can be a problem in amplifiers, we need to consider the **power** of the output signals.

We know that the power of a sinusoidal signal is proportional to its **magnitude squared**.

Thus, we find that the power of **each output signal** is related to the **input** signal power as:

$$\textbf{1rst - order}\text{ output power} \doteq P_1^{out} = A_{vo}^2\, P_{in} = G\, P_{in}$$

$$\textbf{2nd - order}\text{ output power} \doteq P_2^{out} = \frac{B^2}{4} P_{in}^2 = G_2\, P_{in}^2$$

$$\textbf{3rd - order}\text{ output power} \doteq P_3^{out} = \frac{C^2}{16} P_{in}^3 = G_3\, P_{in}^3$$

where we have obviously **defined** $G_2 \doteq B^2/4$ and $G_3 \doteq C^2/16$.

These values are not unitless

Note that unlike G, the values G_2 and G_3 are **not coefficients** (i.e., **not** unitless!).

The value G_2 obviously has units of **inverse power** (e.g., mW^{-1} or W^{-1})!!

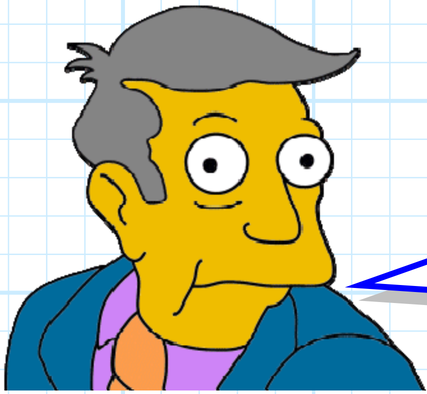
While G_3 has units of **inverse power squared** (e.g., mW^{-2} or W^{-2})!!!!

We know that typically, G_2 and G_3 are much **smaller** than G.

Thus, we are **tempted** to say that P_1^{out} is much **larger** than first harmonic power P_2^{out}, or second-harmonic power P_3^{out}.

→ But, we might be **wrong** !

Products are NOT proportional to input



Q: ***Might*** *be wrong! Now I'm more confused than ever.*

Why can't we say ***definitively*** *that the second and third order products are* ***insignificant****??*

A: Look **closely** at the expressions for the output power of the first, second, and third order products:

$$P_1^{out} = G\, P_{in} \qquad P_2^{out} = G_2\, P_{in}^{2} \qquad P_3^{out} = G_3\, P_{in}^{3}$$

This **first** order output power is of course **directly** proportional to the input power.

However, the **second** order output power is proportional to the input power **squared**, while the **third** order output is proportional to the input power **cubed**!

Like weeds, 2nd and 3rd order products grow faster!

Thus we find that **if** the input power is **small**, the second and third order products **are** insignificant.

But, as the input power **increases**, the second and third order products get **big** in a hurry!

$$P_1^{out} = G\, P_{in} \qquad P_2^{out} = G_2\, P_{in}^2 \qquad P_3^{out} = G_3\, P_{in}^3$$

For example, if we **double** the input power, the **first** order signal will of course likewise **double**.

$$P_1^{out} = G\,(2P_{in}) = 2(G\, P_{in})$$

However, the **second** order power will **quadruple**, while the **third** order power will increase **8 times**.

$$P_2^{out} = G_2(2P_{in})^2 = 4(G_2\, P_{in}^2) \qquad P_3^{out} = G_3(2P_{in})^3 = 8(G_3\, P_{in}^3)$$

Be careful with this!

For **large** input powers, the second and third order output products can in fact be **almost as large** as the first order signal!

Perhaps this can be most easily seen by expressing the above equations in **decibels**, e.g.,:

$$P_1^{out}(dBm) = G(dB) + P_{in}(dBm)$$

$$P_2^{out}(dBm) = G_2(dBm^{-1}) + 2\left[P_{in}(dBm)\right]$$

$$P_3^{out}(dBm) = G_3(dBm^{-2}) + 3\left[P_{in}(dBm)\right]$$

where we have used the fact that $\log x^n = n \log x$.

Two new operators

Note we have defined **two new operators**:

$$G_2(dBm^{-1}) \doteq 10\log_{10}\left[\frac{G_2}{\left(1/1.0mW\right)}\right] = 10\log_{10}\left[G_2\left(1.0mW\right)\right]$$

and:

$$G_3(dBm^{-2}) \doteq 10\log_{10}\left[\frac{G_3}{\left(1/1.0mW^2\right)}\right] = 10\log_{10}\left[G_3\left(1.0mW^2\right)\right]$$

Hint: Just express everything in **milliwatts!**

Two or three out for every one in!

Note the value:

$$2\left[P_{in}(dBm)\right]$$

does **not** mean the value $2P_{in}$ expressed in decibels.

The value $2\left[P_{in}(dBm)\right]$ is fact the value of P_{in} expressed in decibels—**times two**!

For **example**, if $P_{in}(dBm) = -30$, then $2\left[P_{in}(dBm)\right] = -60$.

Likewise, if $P_{in}(dBm) = 20$, then $2\left[P_{in}(dBm)\right] = 40$.

What this means is that for every **1dB** increase in **input** power P_{in} the fundamental (**first-order**) signal will increase **1dB**; the **second-order** power will increase **2dB**; and the **third-order** power will increase **3dB**.

Middle school mathematics

The statement above is evident when we look at the three power equations (in decibels), as each is an equation of a **line**.

For **example**, the equation:

$$\begin{aligned} P_3^{out}(dBm) &= 3\left[P_{in}(dBm)\right] + G_3(dBm^{-2}) \\ y \quad &= m \quad x \quad + \quad b \end{aligned}$$

describes a line with **slope** $m=3$ and "y **intercept**" $b=G_3(dBm^{-2})$ (and where $x=P_{in}(dBm)$ and $y=P_3^{out}(dBm)$).

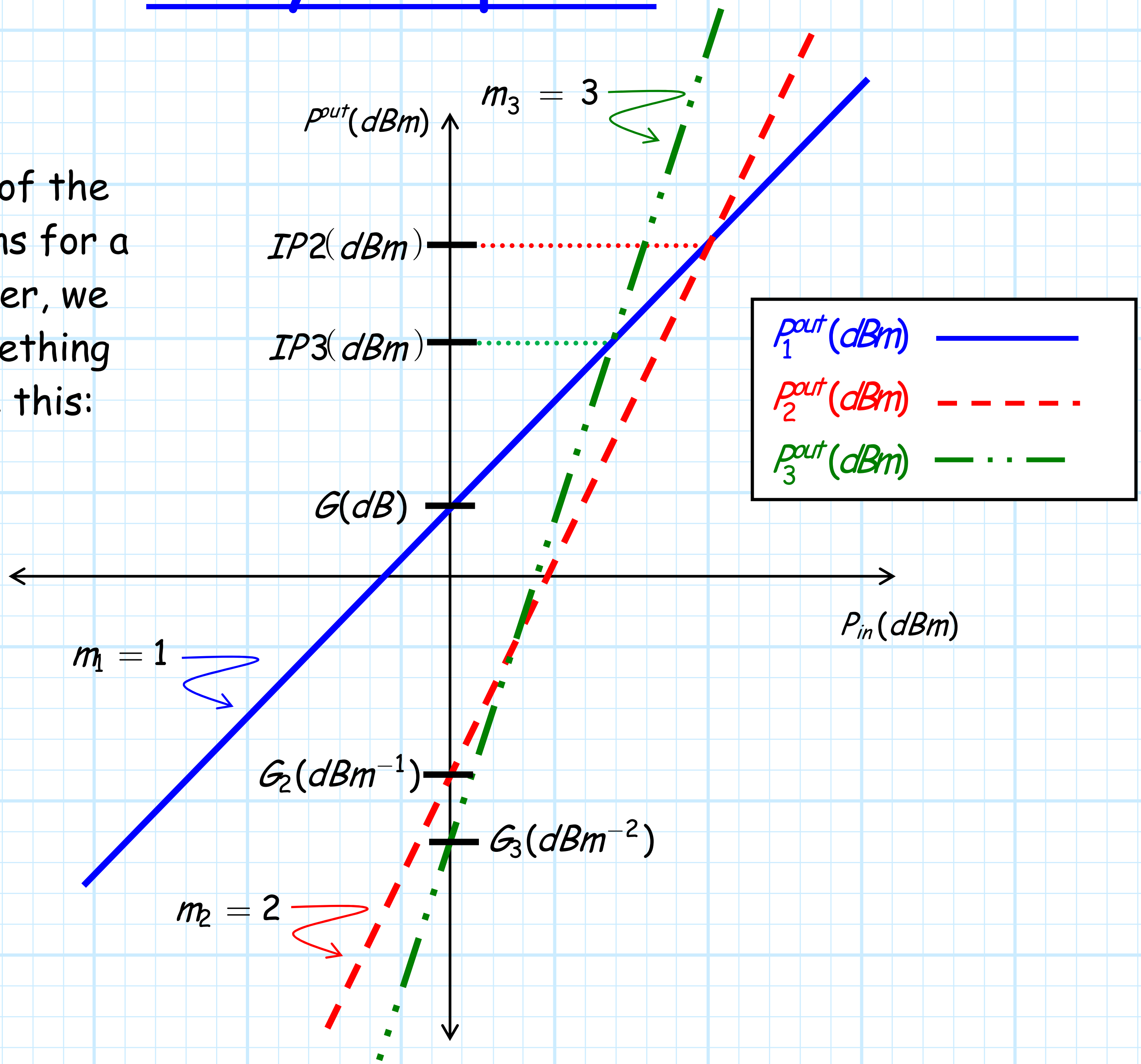
Likewise, the **second**-order expression:

$$P_2^{out}(dBm) = 2\left[P_{in}(dBm)\right] + G_2(dBm^{-1})$$

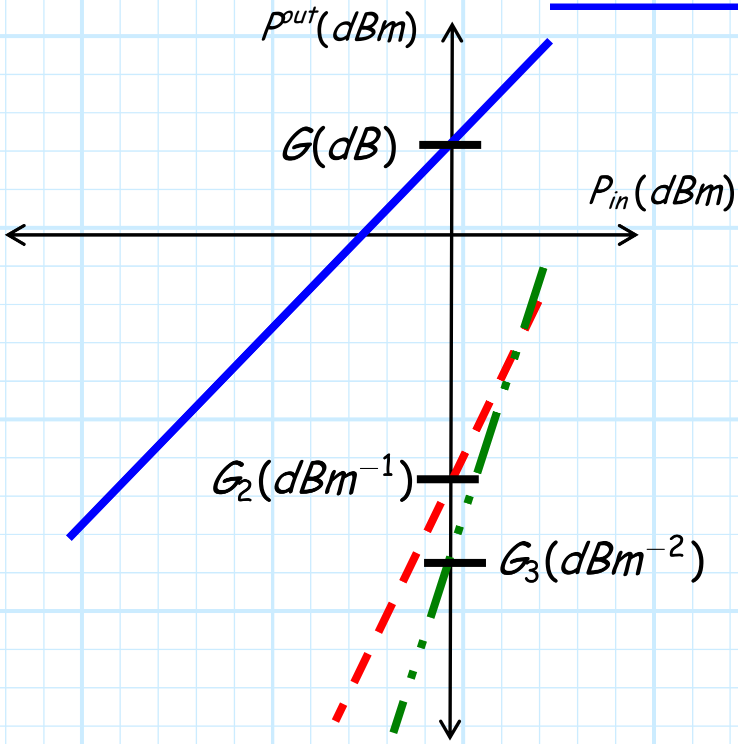
is a line with **slope** $m=2$ and "y **intercept**" $b=G_2(dBm^{-1})$.

Study this plot!!!!!

Plotting each of the three equations for a typical amplifier, we would get something that looks like this:



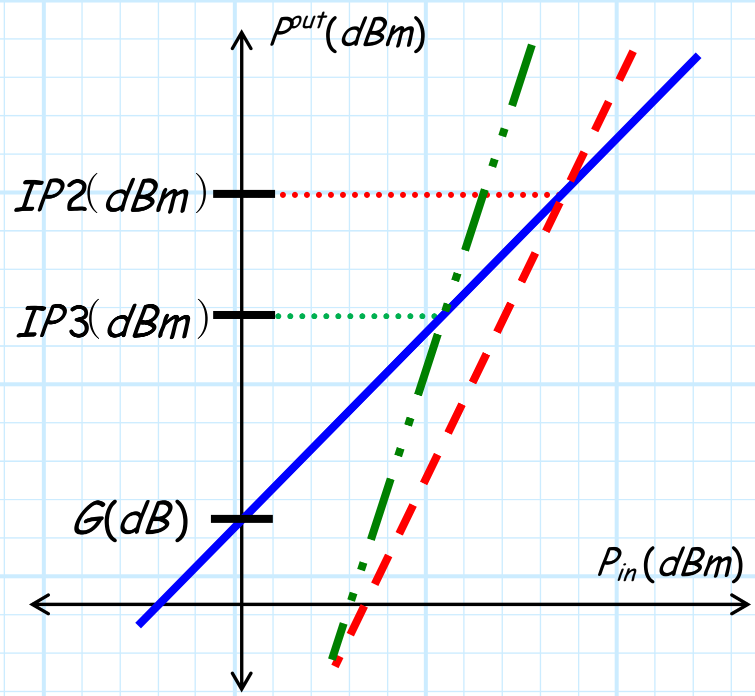
Intercept points



Note that for $P_{in}(dBm) < 0\,dBm$ (the **left** side of the plot), the second and third-order products are **small** compared to the fundamental (first-order) signal.

However, when the input power increases **beyond** 0 *dBm* (the **right** side of the plot), the second and third order products rapidly **catch up**!

In fact, they will (theoretically) become **equal** to the first order product at some large input power.



IP2 and IP3

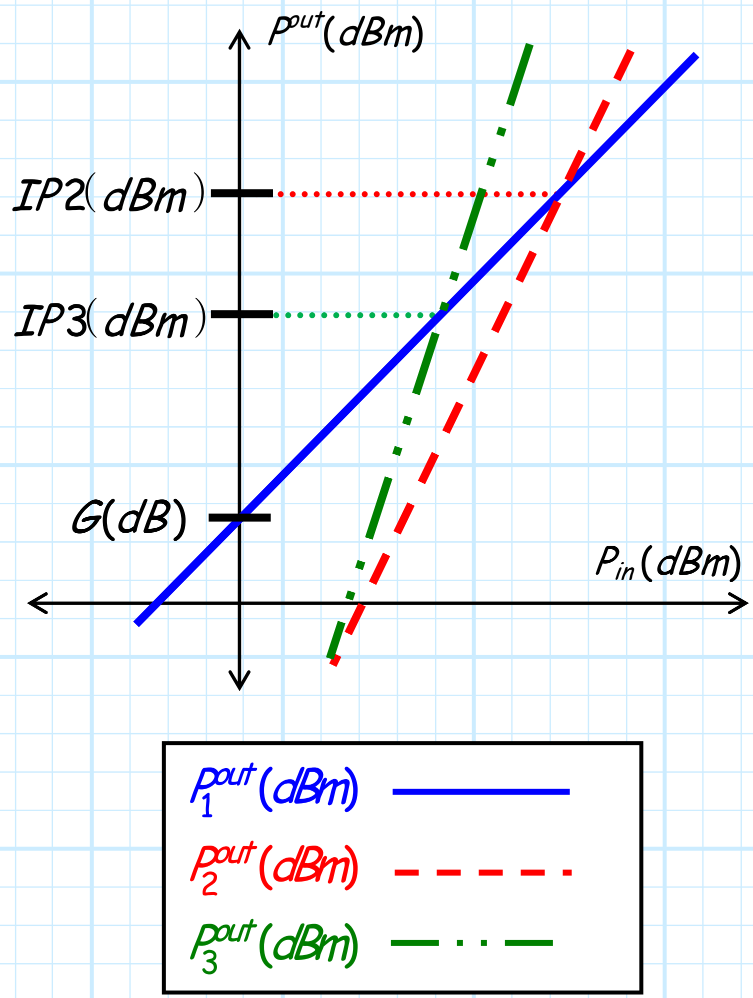
The point at which each higher order product **equals** the first-order signal is defined as the **intercept point**.

Thus, we define the **second order intercept** point as the output power **when** $P_1^{out}(dBm) = P_2^{out}(dBm)$:

$$IP2 \doteq Second\text{ - }order\ intercept\ point$$

Likewise, the **third order intercept** point is defined as the third-order output power **when** $P_1^{out}(dBm) = P_3^{out}(dBm)$:

$$IP3 \doteq Third\text{ - }order\ intercept\ point$$



Prove these results to yourself!

The intercept points of an amplifier depend on the amplifier gain G, as well as 2nd-order parameter G_2, and 3rd-order parameter G_3.

Using a little algebra, we can **you** can show that:

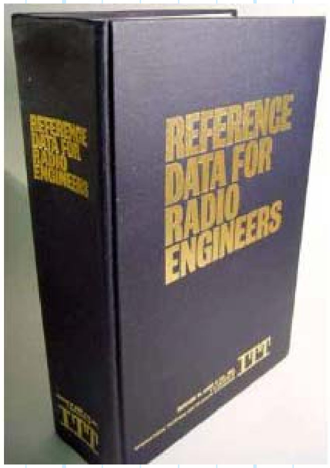
$$IP2 = \frac{(G)^2}{G_2} \quad \text{and} \quad IP3 = \sqrt{\frac{(G)^3}{G_3}}$$

Or, expressed in **decibels:**

$$IP2(dBm) = 2\,G(dB) - G_2(dBm^{-1})$$

$$IP3(dBm) = \frac{3\,G(dB) - G_3(dBm^{-2})}{2}$$

Some intercept point info



* **Radio engineers** specify the intermodulation distortion performance of a specific amplifier in terms of the **intercept points** IP2 and IP3, rather than values G_2 and G_3.

* Generally, only the **third-order** intercept point is provided by amplifier manufactures (we'll see why later).

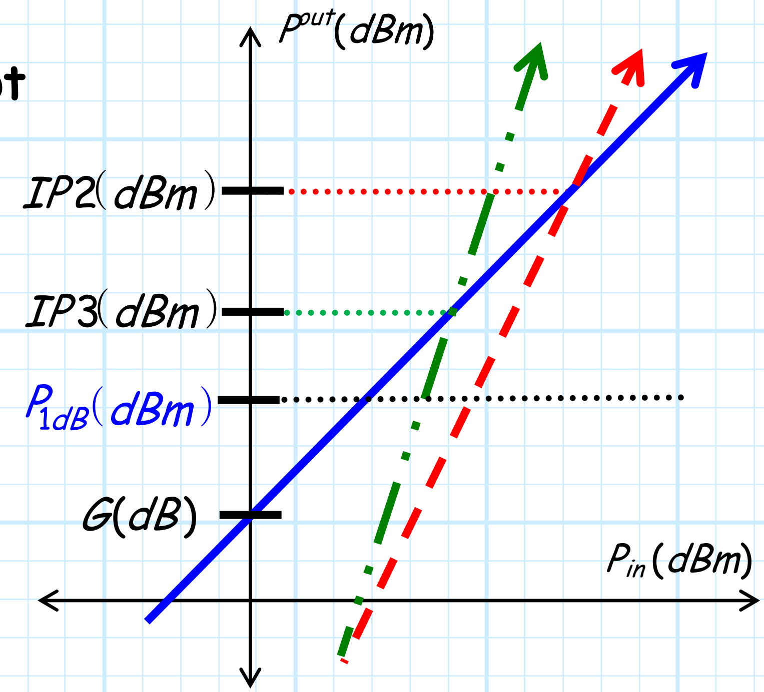
* **Typical** values of $IP3(dBm)$ for a **small-signal** amplifier range from +20 to +50.

* Note that as G_2 and G_3 **decrease**, the intercept points **increase**.

→ Therefore, the **higher** the intercept point of an amplifier, the **better** the amplifier !

Intercept points are a bit theoretical!

One other **important point**: the **intercept points** for most amplifiers are much **larger** than the **compression point**!



I.E.,:

$$IP3 \gg P_{1dB} \qquad \text{(typically)}$$

In other words the intercept points are "theoretical", in that we can **never**, in fact, increase the input power to the point that the higher order signals are **equal** to the fundamental signal power.

All signals, including the higher order signals, have a **maximum** limit that is determined by the amplifier **power supply**.