PHSX 536: Homework #1

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Problem 1

1.12 (1e: 1.6)
12. For the circuit in Figure 1.37 above, what is the ratio of $R_2 : R_1$ such that the voltage across A and B is $\frac{1}{2}V_0$? What is the ratio of $R_2 : R_1$ such that the voltage across A and B is $\frac{1}{10}V_0$?

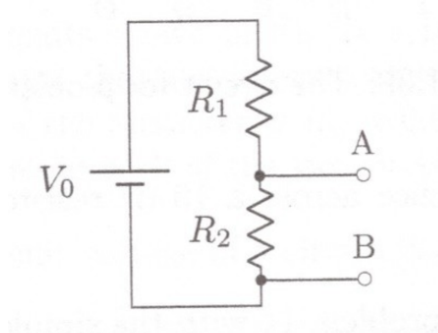


Figure 1: The circuit for problems 6 and 12.

Solution:

(a) $V_{AB} = \frac{1}{2}V_0$:

$$
\begin{aligned}
V_{AB} &= \frac{1}{2}V_0 = V_0\frac{R_2}{R_1 + R_2} \\
\frac{1}{2}(R_1 + R_2) &= R_2 \\
\cancel{\frac{1}{2}}R_1 &= \cancel{\frac{1}{2}}R_2 \\
\frac{R_2}{R_1} &= 1
\end{aligned}
$$

(b) $V_{AB} = \frac{1}{3}V_0$:

$$\begin{aligned}\frac{1}{3}V_0 &= V_0\frac{R_2}{R_1+R_2}\\ \frac{1}{3}(R_1+R_2) &= R_2\\ \frac{1}{3}R_1 &= \frac{2}{3}R_2\\ \frac{1}{2} &= \frac{R_2}{R_1}\end{aligned}$$

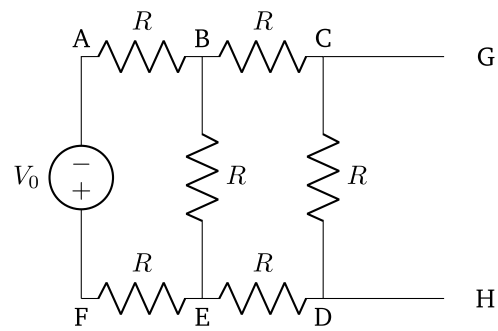
(c) $V_{AB} = \frac{1}{10}V_0$:

$$\begin{aligned}\frac{1}{10}V_0 &= V_0\frac{R_2}{R_1+R_2}\\ \frac{1}{10}R_1 &= \frac{9}{10}R_2\\ \frac{R_2}{R_1} &= \frac{10}{90} = \frac{1}{9}\end{aligned}$$

Problem 2

1.14 (1e: 1.8)
14. We now attach two output terminals to the circuit from problem 13. The resulting circuit is shown in Figure 1.39.

Figure 1.39



(a) What is the voltage between the terminals G and H?

Solution:

We get 3 equations:

$$\begin{aligned} V_0 - I_1R - I_1R - I_1R + I_2R = 0 \\ -I_2R - I_2R - I_2R - I_2R + I_1R = 0 \\ V_{GH} = I_2R \end{aligned}$$

Which can be written in standard form:

$$\begin{aligned} -3I_1R + I_2R = -V_0 \\ I_1R - 4I_2R = 0 \\ I_2R = V_{GH} \end{aligned}$$

Which are linear, so in matrix form this is:

$$\begin{bmatrix} -3R & R \\ R & -4R \\ 0 & R \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -V_0 \\ 0 \\ V_{GH} \end{bmatrix}$$

Solving the upper two rows gives $I_1R = \frac{4V_0}{11}$ and $I_2R = \frac{V_0}{11}$. By this and the third equation,

$$\boxed{V_{GH} = \frac{1}{11}V_0}$$

(b) What current flows from G to H?

Solution:

The current flowing on this branch is I_2, which we solved in part (a) to equal:

$$\boxed{I_2 = \frac{1}{11}V_0}$$

(c) If we connect a wire from G to H, what current flows through the wire and what is the voltage between G and H?

Solution:

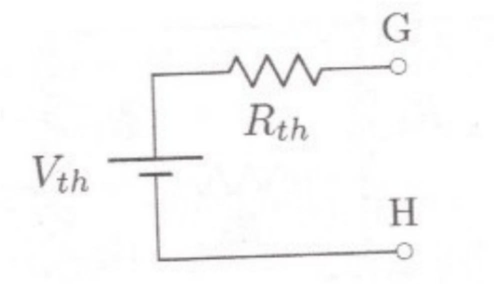
In this configuration, we have a third loop. Our equations are then:

$$\begin{aligned} -3I_1R + I_2R &= -V_0 \\ I_1R - 4I_2R + I_3R &= 0 \\ I_3R &= 0 \end{aligned}$$

From these, $I_1R = \frac{5}{14}V_0$, $I_2R = \frac{1}{14}V_0$, and $I_3R = \frac{1}{14}V_0$. The voltage between GH is zero, though, since that part of the circuit now lies on ground- there is no resistor between the two points.

Problem 3

1.16 (1e: 1.9)
16. Replace the circuit from problem 14 with the simpler one shown in Figure 1.40. V_{th} is a voltage source and R_{th} is a new resistance. What are the values of V_{th} and R_{th} such that you get the same answer to the current and voltage questions as in problem 14?



Solution:
In the configuration of Figure 1.39, we would start by replacing the battery with a short circuit. Then we have resistor AB in parallel with resistor BE, and this is in series with BC:

$$\left(\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}+R\right)=\frac{R}{2}+R=\frac{3}{2}R$$

Now, this is in parallel with CD, and in series with FE and ED:

$$\left(\frac{2}{3R}+\frac{1}{R}\right)^{-1}+2R=\frac{5}{3}R+2R=\frac{11}{3}R$$

So,

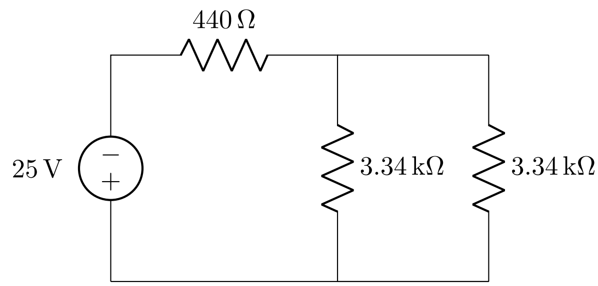
$$\boxed{R_{th}=\frac{11}{3}R}$$

I already used mesh analysis to find the voltage between G and H,

$$\boxed{V_{th}=\frac{1}{11}V_0}$$

Problem 4

1.D.51 (1e: 1.D36)
51. Your lab partner builds the voltage-divider circuit shown in Figure 1.60 with the resistors shown. These are all $\frac{1}{8}$-Watt resistors which are rated with a 20% tolerance. Explain why this is a poorly designed circuit, giving numerical values for the problems you find.



Solution:

1. This is an unusual configuration for a voltage divider, as it has two parallel resistors.
2. The equivalent resistance in the parallel resistors is

$$\left(\frac{1}{3.3}+\frac{1}{3.3}\right)^{-1} = 1.65\text{ k}\Omega$$

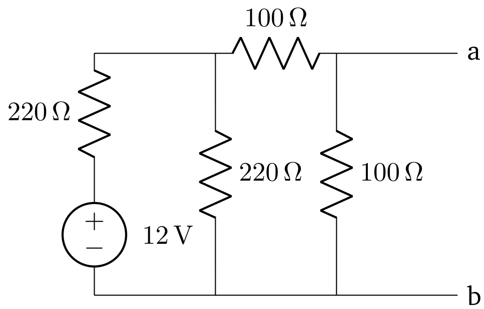
   which is much lower than any of the two resistors on their own in series.
3. In this configuration, the voltage divider will give an output voltage of

$$V_{out} = 25\frac{1.65}{0.440+1.65} = 19.7\text{ V}$$

   which is very close to the input voltage.

Problem 5

Exp. 2 Circuit Problem
5. For the following circuit, find the Thevenin equivalent circuit as seen by a load across ab:



Solution:
Replacing the voltage source with a short and calculating the equivalent resistance:

$$R_{th} = \left(\left(\left(\frac{1}{220} + \frac{1}{220}\right)^{-1} + 100\right)^{-1} + \frac{1}{100}\right)^{-1}$$
$$R_{th} = \left(\frac{1}{210} + \frac{1}{100}\right)^{-1}$$
$$\boxed{R_{th} = 67.74 \text{ k}\Omega}$$

I feel most comfortable with mesh analysis, so I will use this to determine V across ab, which if we make current in the first loop with the voltage source I_1, and the second loop I_2, then,

$$\begin{aligned} -220I_1 - 220I_1 + 220I_2 &= -12 \\ -100I_2 - 100I_2 - 220I_2 + 220I_1 &= 0 \\ V_{ab} &= 100I_2 \end{aligned}$$

In matrix form:

$$\begin{bmatrix} -440 & 220 \\ 220 & -420 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}$$

RREF gives $I_1 = \frac{63}{1705}$ A and $I_2 = \frac{3}{155}$ A. Therefore

$$\boxed{V_{ab} = 1.94 \text{ V}}$$