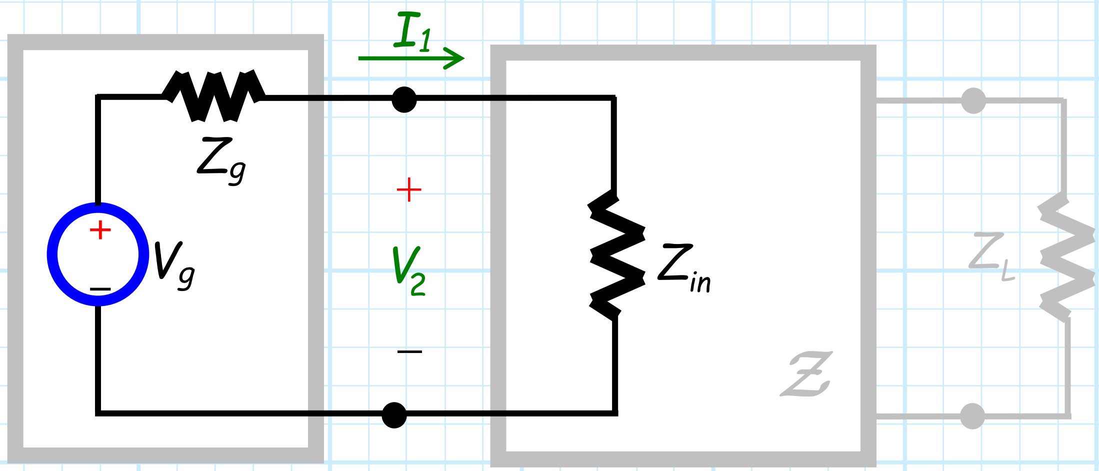
Source Transformers

Q: *So we **combined** the two-port network with the **load**, in order to make an equivalent **input impedance**.*

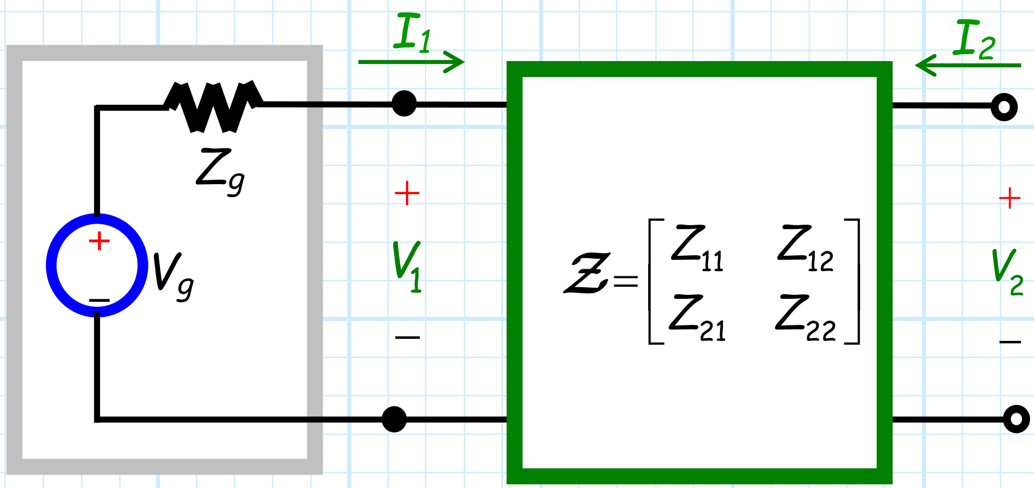


*Couldn't we have **alternatively** combined the **source** and the two-port network?*

A: What a **great** idea! Let's try it and **see what happens**.

A new one-port network

For this case, you simply need to determine an **equivalent source** for the following one-port network:



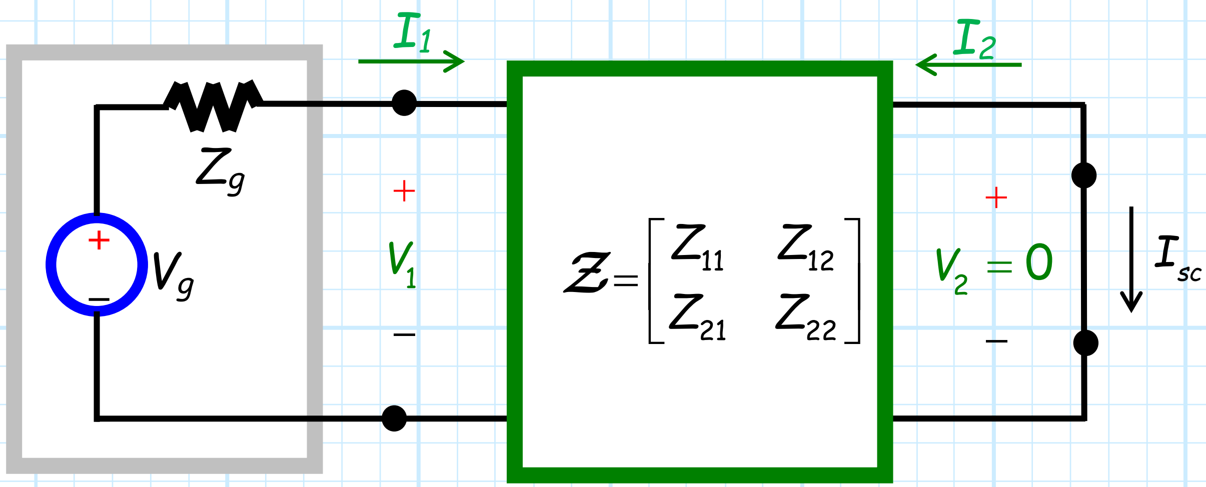
Q: *Yikes! How do **I** accomplish that?*

A: **You** already know how! Just determine:

a) the **short-circuit** output **current** I_{sc}

b) the **open-circuit** output **voltage** V_{oc}

Four equations and four unknowns



From the **trans-impedance** parameters of the two-port device, we know →

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

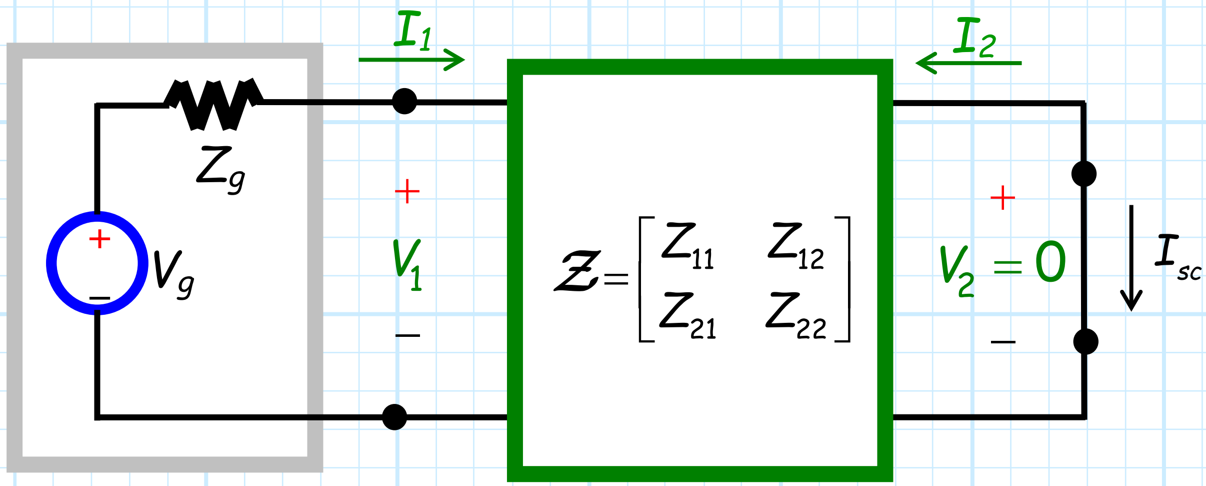
And for the **source** →

$$I_1 = \frac{V_g - V_1}{Z_g}$$

And of course for the **short-circuit** →

$$V_2 = 0$$

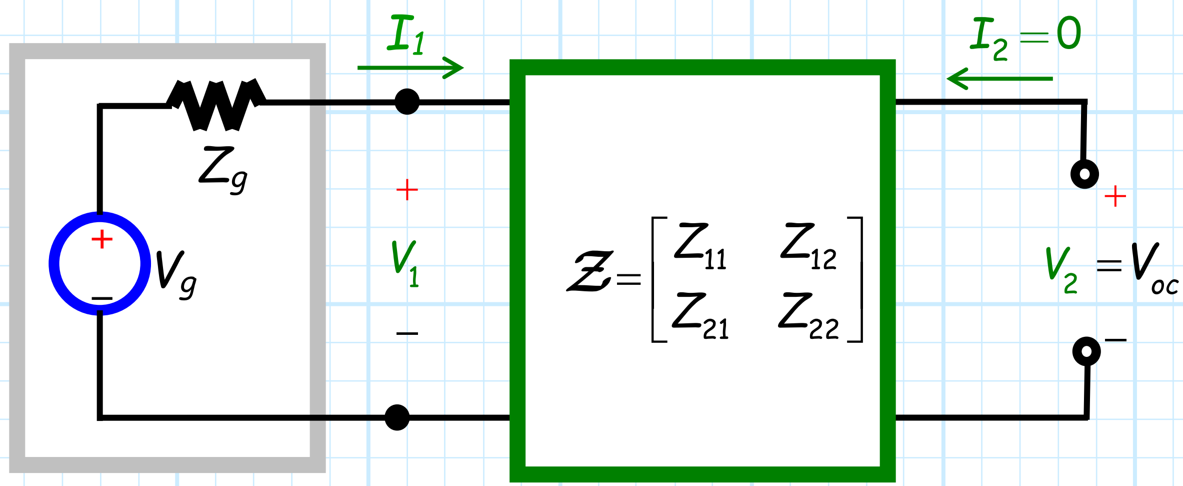
The short-circuit output current



With some algebraic **elbow grease** we can determine the **short-circuit** current:

$$I_{sc} = -I_2 = V_g \left[\frac{Z_{22}}{Z_{21}} \left(Z_{11} + Z_g \right) - Z_{12} \right]^{-1}$$

Now for an open-circuit at the output



From the **trans-impedance** parameters of the two-port device, we know →

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

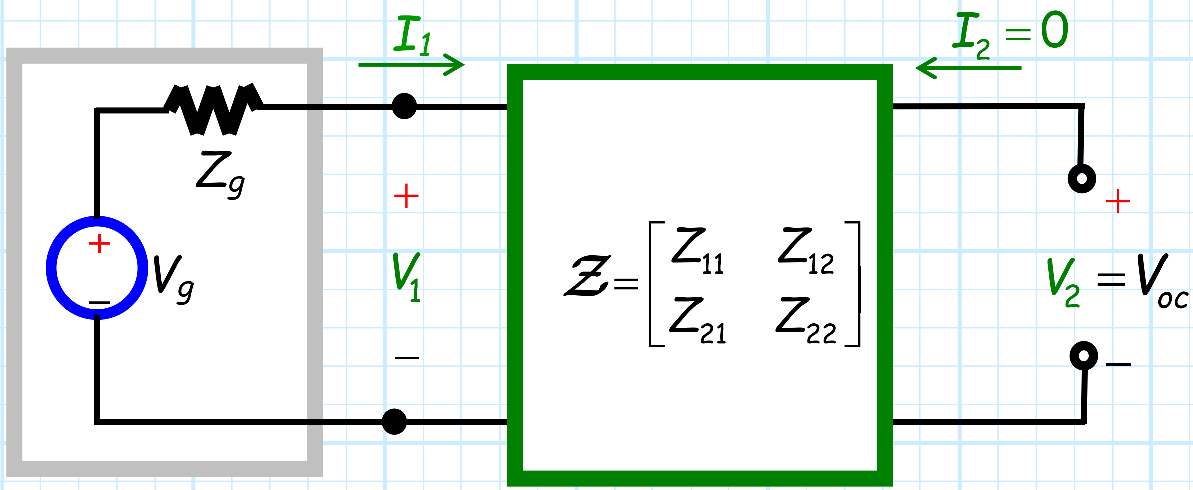
And for the **source** →

$$I_1 = \frac{V_s - V_1}{Z_1}$$

And of course for the **open-circuit** →

$$I_2 = 0$$

The open-circuit output voltage

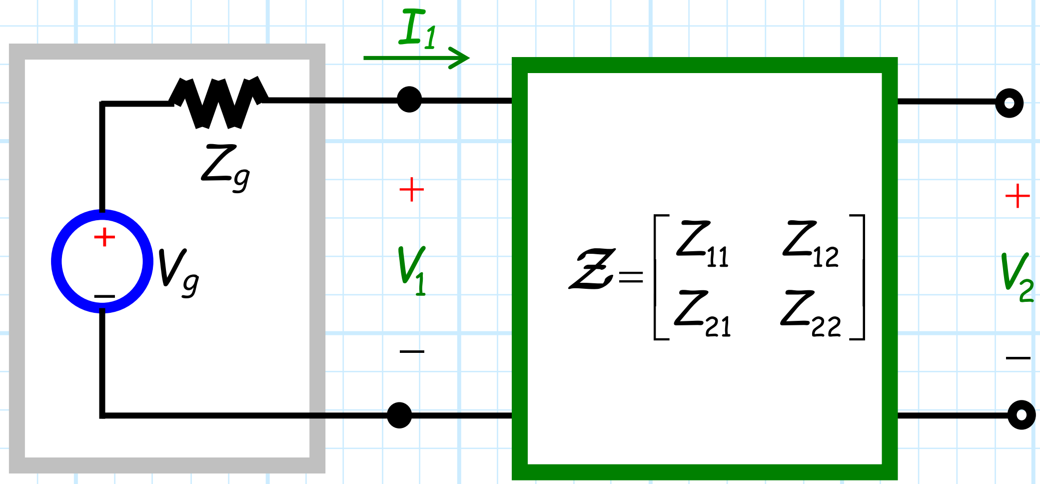


With even **more** algebraic elbow grease, we can determine the **open-circuit** voltage:

$$V_{oc} = V_2 = V_g \left(\frac{Z_{21}}{Z_{11} + Z_g} \right)$$

Thevenin's equivalent

Thus, the Thevenin's equivalent of **this** circuit→



has a **voltage source** of value V_{out}:

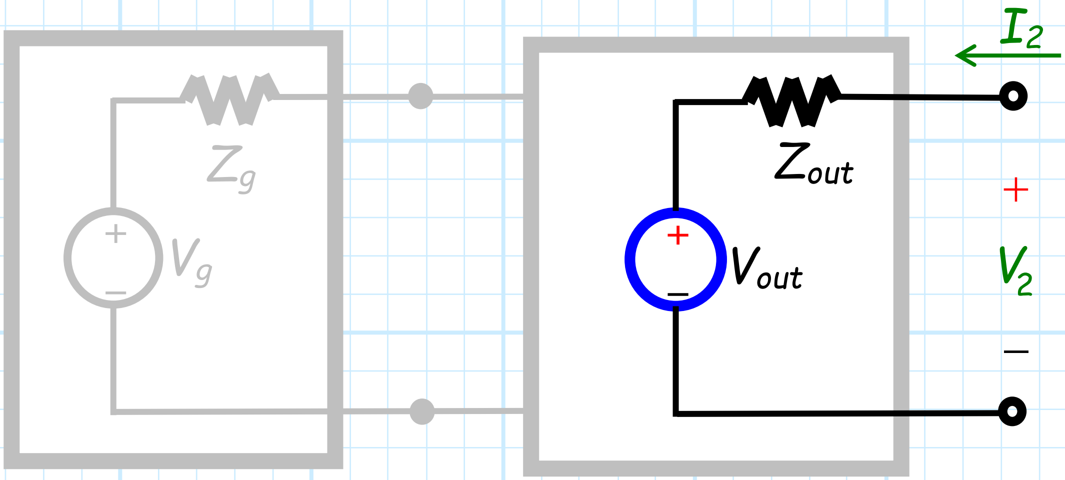
$$V_{out} = V_{oc} = V_g \left(\frac{Z_{21}}{Z_{11} + Z_g} \right)$$

and an output **impedance** Z_{out}:

$$Z_{out} = \frac{V_{oc}}{I_{sc}} = \left(\frac{Z_{21}}{Z_{11} + Z_s} \right) \left(\frac{Z_{22}}{Z_{21}} (Z_{11} + Z_s) - Z_{12} \right) = Z_{22} - \frac{Z_{21} Z_{12}}{Z_{11} + Z_g}$$

A source transformer!!!!!

In this case, the two-port device can be viewed as a **source transformer**.

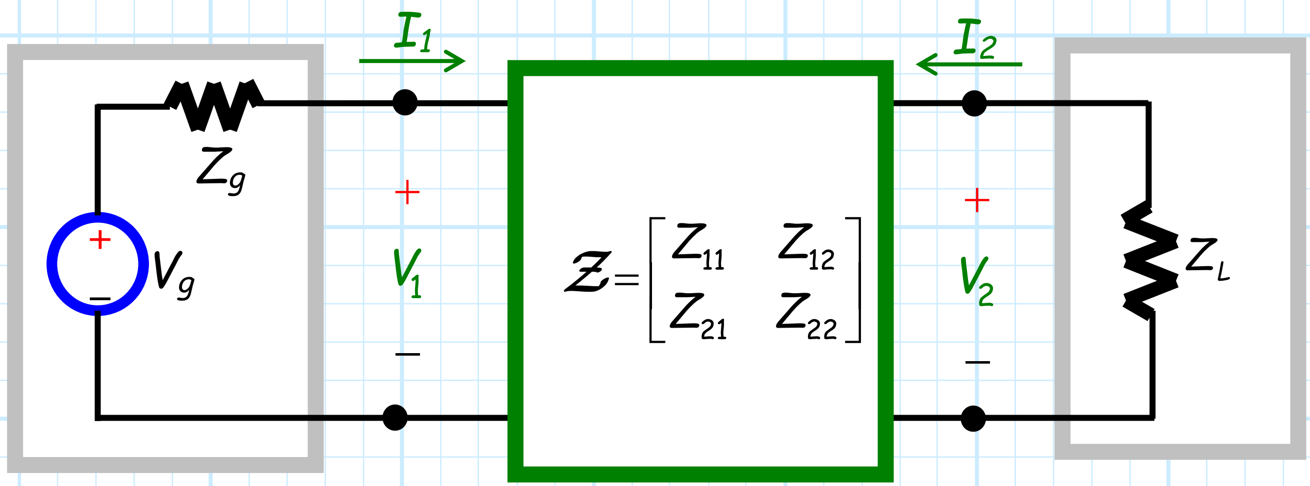


$$V_{out} = V_g \left(\frac{Z_{21}}{Z_{11} + Z_g} \right) \qquad\qquad Z_{out} = Z_{22} - \frac{Z_{21}\, Z_{12}}{Z_{11} + Z_g}$$

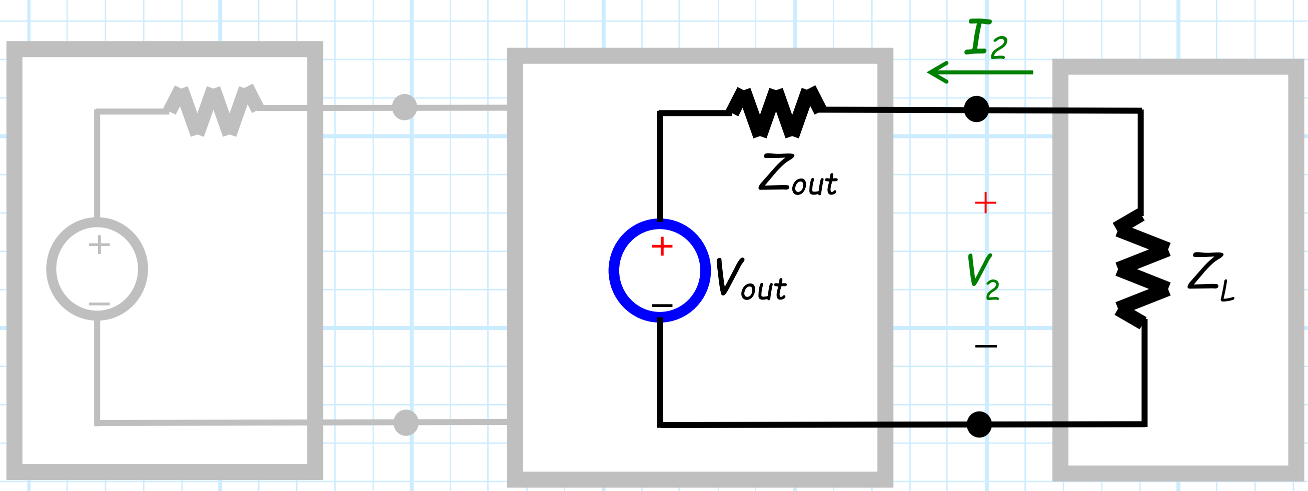
→Note that **BOTH** the voltage source **AND** the impedance are transformed!

An alternate equivalent circuit

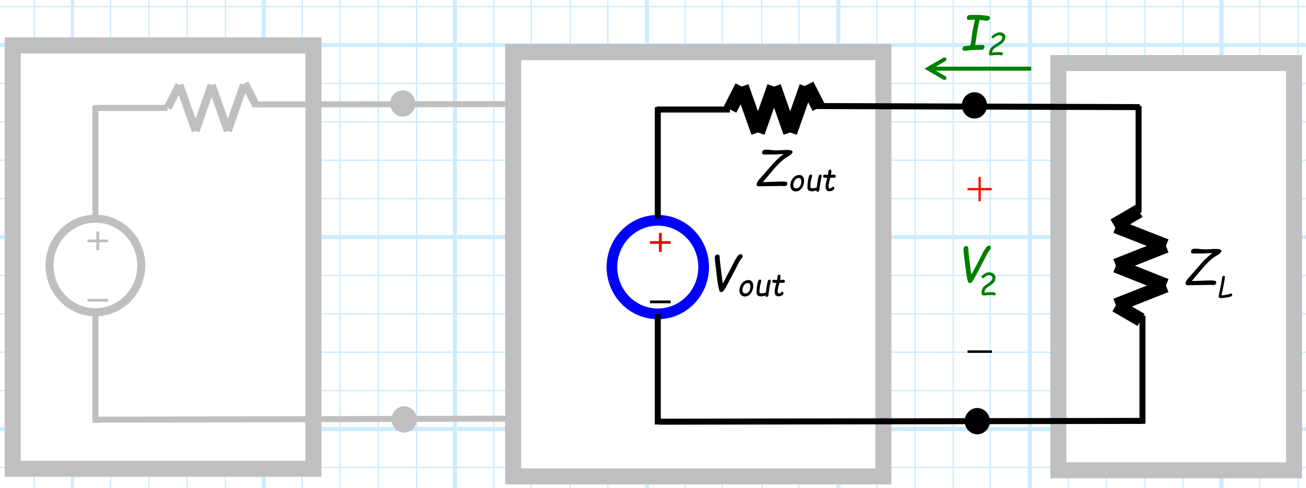
An equivalent circuit to **this**:



is therefore **this**:



Evaluate the output port values



Evaluating this **equivalent** circuit, we can **quickly** determine:

$$I_2 = \frac{V_{out}}{Z_{out} + Z_L} \qquad\qquad V_2 = V_{out}\left(\frac{Z_L}{Z_{out} + Z_L}\right)$$

Whew! We passed the sanity check!

As a **sanity check**, we can **insert** these results:

$$V_{out} = V_g\left(\frac{Z_{21}}{Z_{11}+Z_g}\right) \qquad Z_{out} = Z_{22} - \frac{Z_{21}\,Z_{12}}{Z_{11}+Z_g}$$

into these:

$$I_2 = \frac{V_{out}}{Z_{out}+Z_L} \qquad V_2 = V_{out}\left(\frac{Z_L}{Z_{out}+Z_L}\right)$$

and find that results to be the **same** as before!

$$I_2 = -V_g \frac{Z_{21}}{\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}Z_{21}} \qquad V_2 = V_g \frac{Z_L\,Z_{21}}{\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}Z_{21}}$$