Two-Tone Intermodulation

Q: *It doesn't seem to me that this **dad-gum** intermodulation distortion is really that much of a problem.*

*I mean, the first and second **harmonics** will likely be well **outside** the amplifier bandwidth, right?*

A: True, the **harmonics** produced by intermodulation distortion typically are **not** a problem in radio system design.

There is a problem, however, that is **much worse** than **harmonic** distortion!

This problem is called **two-tone** intermodulation distortion.

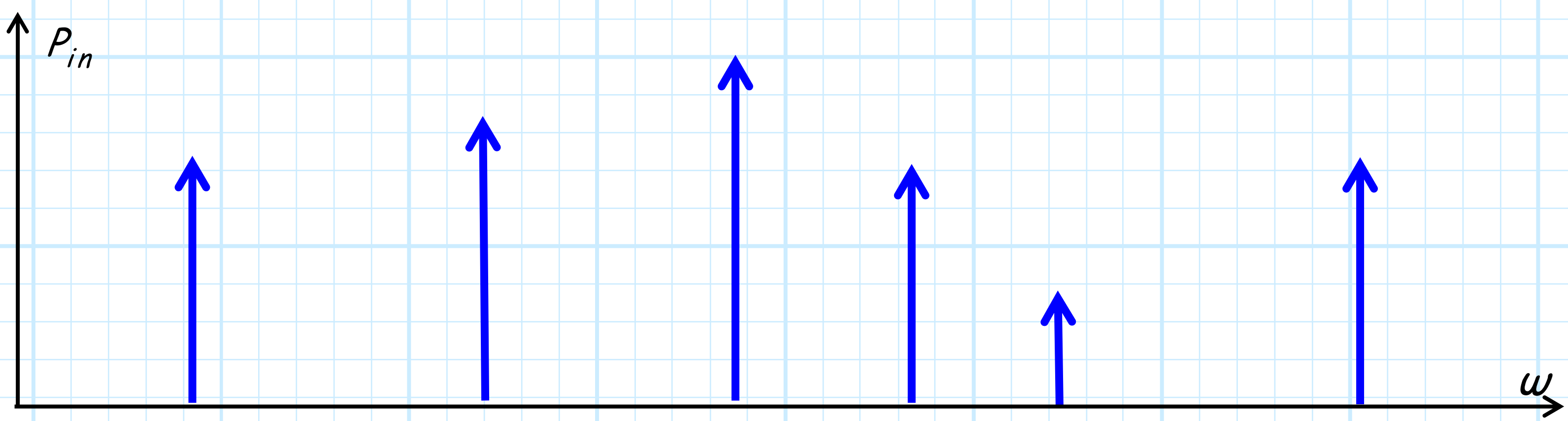
Many eigen functions at the input!

Say the input to an amplifier consists of **two** signals at **dissimilar** frequencies:

$$v_{in}(t) = a\cos\omega_1 t + a\cos\omega_2 t$$

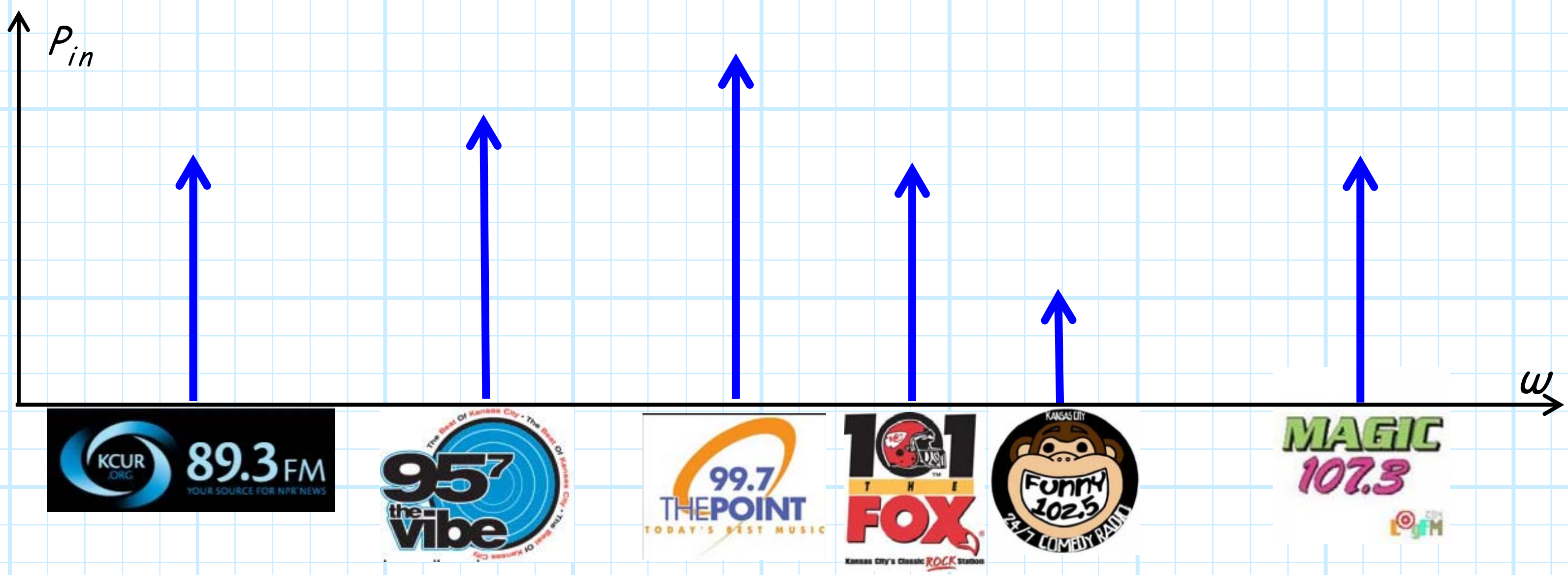
Here we will assume that both frequencies ω_1 and ω_2 are within the **bandwidth** of the amplifier, but are **not** equal to each other $(\omega_1 = \omega_2)$.

This of course is a much more **realistic** case, as typically there will be **multiple** signals at the input to an amplifier!



For example, consider the FM band

For example, the two signals considered here could represent two **FM radio stations**, operating at frequencies within the FM band (i.e., $88.1\text{ MHz} \le f_1 \le 108.1\text{ MHz}$ and $88.1\text{ MHz} \le f_2 \le 108.1\text{ MHz}$)



Harmonics: not the problem

Q: *My point exactly! Intermodulation distortion will produce those* ***dog-gone*** *second-order products:*

$$\frac{a^2}{2}\cos 2\omega_1 t \quad \text{and} \quad \frac{a^2}{2}\cos 2\omega_2 t$$

and ***gul-durn*** *third order products:*

$$\frac{a^3}{4}\cos 3\omega_1 t \quad \text{and} \quad \frac{a^3}{4}\cos 3\omega_2 t$$

but these harmonic signals will lie well ***outside*** *the FM band!*

A: True! Again, the **harmonic** signals are **not** the problem.

The problem occurs when the **two input** signals combine together to form **additional** second and third order products.

A mysterious pair of new signals

Recall an amplifier output is accurately described as:

$$v_{out} = A_v \, v_{in} + B \, v_{in}^2 + C \, v_{in}^3 + \cdots$$

Consider first the **second-order** term if **two** signals are at the input to the amplifier:

$$\begin{aligned} v_2^{out} &= B \, v_{in}^2 \\ &= B\left(a \cos \omega_1 t + a \cos \omega_2 t\right)^2 \\ &= B\left(a^2 \cos^2 \omega_1 t + 2a^2 \cos \omega_1 t \, \cos \omega_2 t + a^2 \cos^2 \omega_2 t\right) \end{aligned}$$

Note the first and third terms of the above expression are **precisely** the same as the terms we examined on the previous handout.

They result in **harmonic** signals at frequencies $2\omega_1$ and $2\omega_2$, respectively.

The **middle** term, however, is something **new**!

More than just harmonics are created!

Note **this middle term** involves the product of $\cos\omega_1 t$ and $\cos\omega_2 t$.

Again using our knowledge of **trigonometry**, we find:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos(\omega_2-\omega_1)t + a^2\cos(\omega_2+\omega_1)t$$

Note that since $\cos(-x)=\cos x$, we can **equivalently** write this as:

$$2a^2\cos\omega_1 t\ \cos\omega_2 t = a^2\cos(\omega_1-\omega_2)t + a^2\cos(\omega_1+\omega_2)t$$

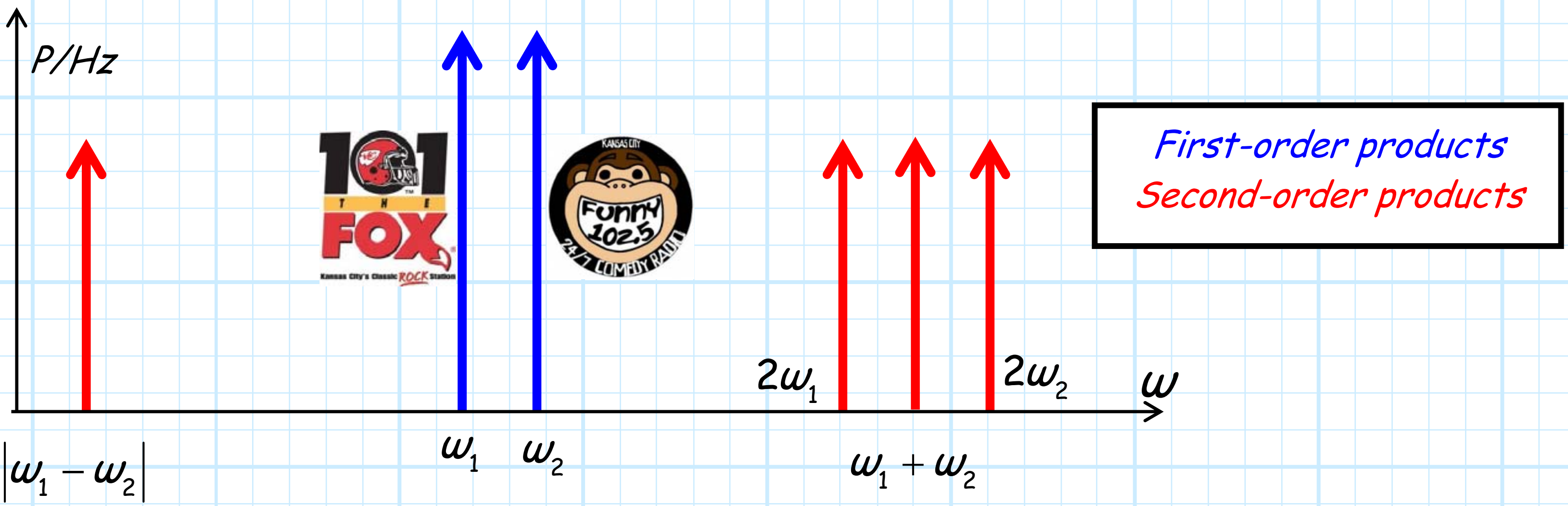
Either way, the result is obvious—we produce **two new signals**!

→These new **second-order** signals oscillate at frequencies:

$$(\omega_1+\omega_2) \quad \text{and} \quad |\omega_1-\omega_2|$$

Look at the amplifier output-Yikes!

Thus, if we looked at the **frequency spectrum** (i.e., signal power as a function of frequency) of an amplifier **output** when **two** sinusoids are at the input, we would see something like this:



Note that the new terms have a frequency that is either much **higher** than both ω_1 and ω_2 (i.e., $(\omega_1 + \omega_2)$), or much **lower** than both ω_1 and ω_2 (i.e., $|\omega_1 - \omega_2|$).

→ Either way, these **new** signals will typically be **outside** the amplifier bandwidth!

Don't forget third-order products

Q: ***Outside*** *the bandwidth! I thought you said these "two-tone" intermodulation products were some "**big problem**".*

*These sons of a gun appear to be **no more** a problem than the harmonic signals!*

A: This observation is indeed correct for **second**-order, two-tone intermodulation products.

But, we have **yet** to examine the **third**-order terms! I.E.,

$$v_3^{out} = C\, v_{in}^3 = C\left(a\cos\omega_1 t + a\cos\omega_2 t\right)^3$$

→ If we **multiply** this all out, and again apply our trig knowledge, we find that a **bunch** of new **third-order** signals are created!!

Even more mysterious new signals

Among these signals, of course, are the second **harmonics** $\cos 3\omega_1 t$ and $\cos 3\omega_2 t$.

Additionally, however, we get these **new** signals:

$$\cos(2\omega_2 - \omega_1)t \quad \text{and} \quad \cos(2\omega_1 - \omega_2)t$$

Note since $\cos(-x) = \cos x$, we can **equivalently** write these terms as:

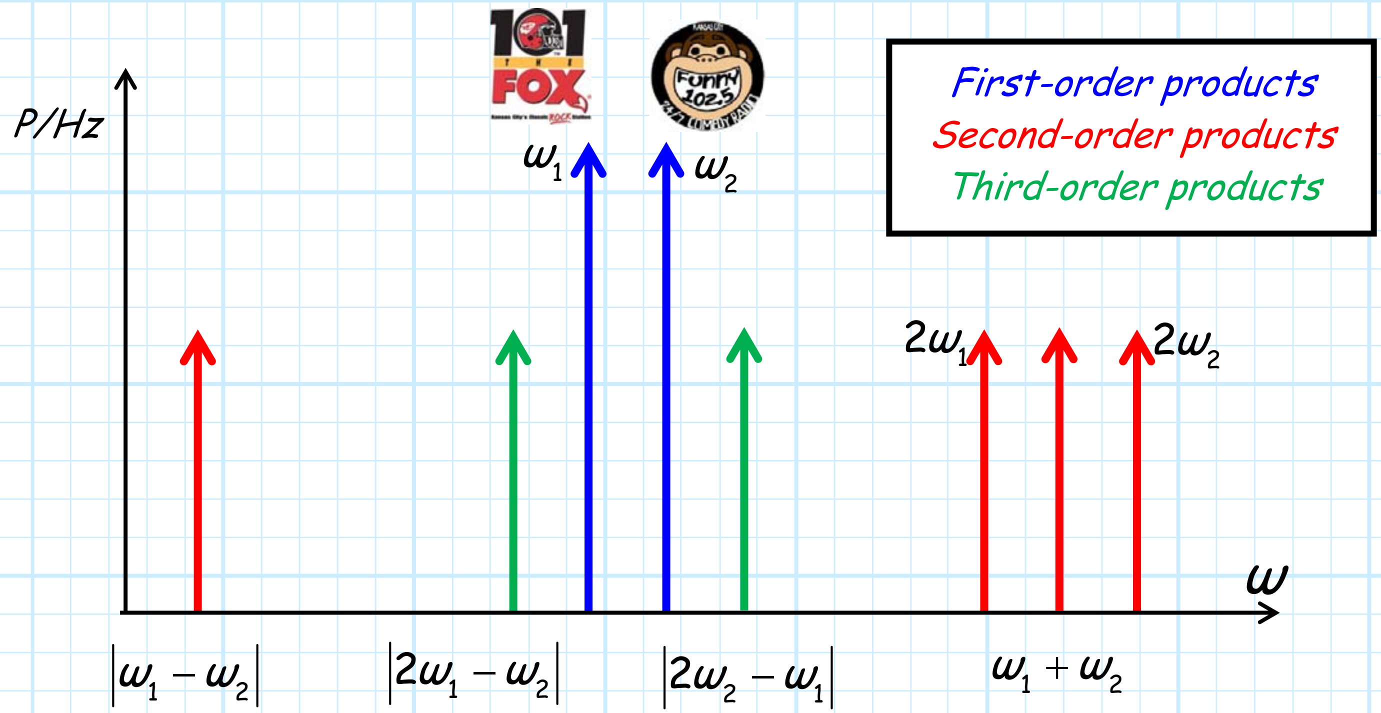
$$\cos(\omega_1 - 2\omega_2)t \quad \text{and} \quad \cos(\omega_2 - 2\omega_1)t$$

Either way, it is apparent that the **third-order** products include signals at frequencies:

$$|\omega_1 - 2\omega_2| \qquad \text{and} \qquad |\omega_2 - 2\omega_1|.$$

Look at the amplifier output—double Yikes!

Now let's look at the output spectrum with **these new** third-order products included:



Now **you** should see the problem!

These third-order products are very **close** in frequency to ω_1 and ω_2. They will likely lie **within** the bandwidth of the amplifier!

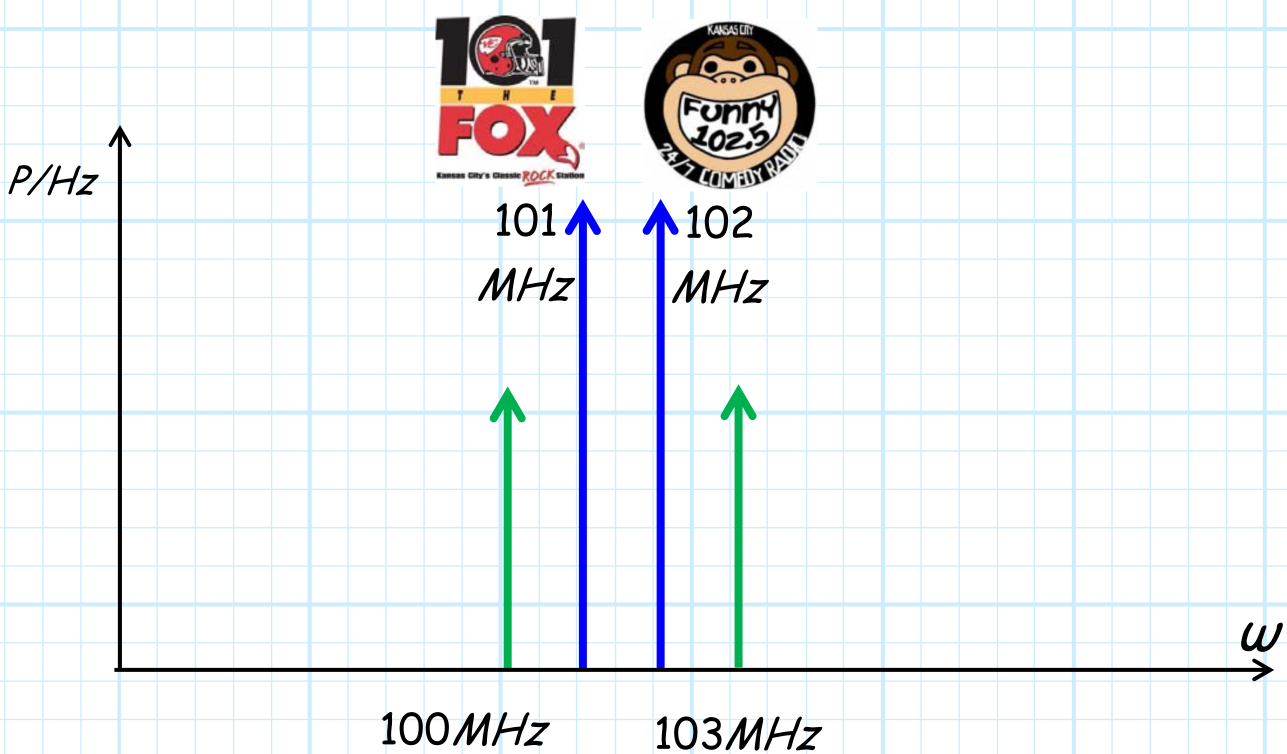
And there might be a station at 100 or 103

For **example**, if f_1 = **101 MHz** and f_2 = **102 MHz**, then:

$$2f_1 - f_2 = \mathbf{100\ MHz}\ \text{and,}$$

$$2f_2 - f_1 = \mathbf{103\ MHz}.$$

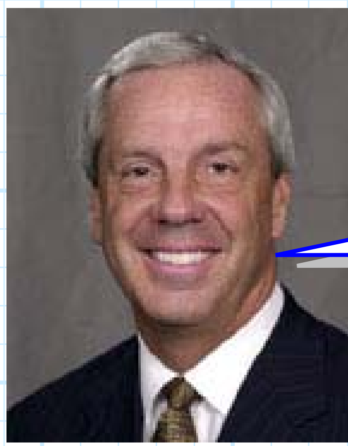
→All frequencies are **well within** the FM radio **bandwidth**!



Make sure IP3 is high!

Thus, these **third-order**, **two-tone** intermodulation products are the **most significant** distortion terms.

→This is why we are most concerned with the **third-order intercept point** of an amplifier!



*I select amplifiers with the **highest possible** 3rd order intercept point!*