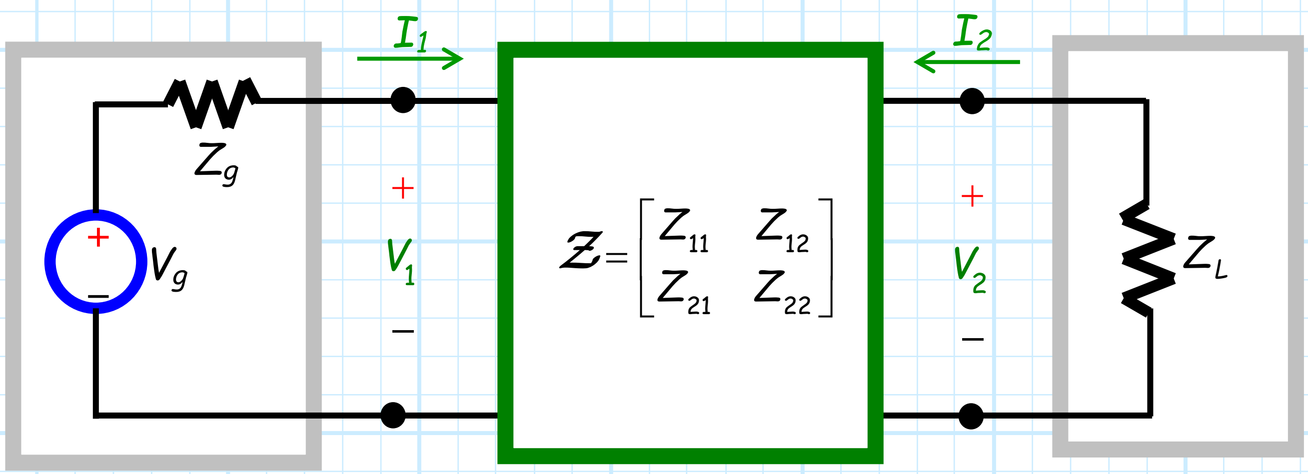
Impedance Transformers

Q: *It took an awful lot of **algebra** to determine these results:*

$$I_1 = V_g \frac{Z_{22} + Z_L}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12}Z_{21}} \qquad V_1 = V_g \frac{Z_{11}(Z_{22} + Z_L) - Z_{12}Z_{21}}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12}Z_{21}}$$

$$I_2 = -V_g \frac{Z_{21}}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12}Z_{21}} \qquad V_2 = V_g \frac{Z_L Z_{21}}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12}Z_{21}}$$

*for **this** circuit:*



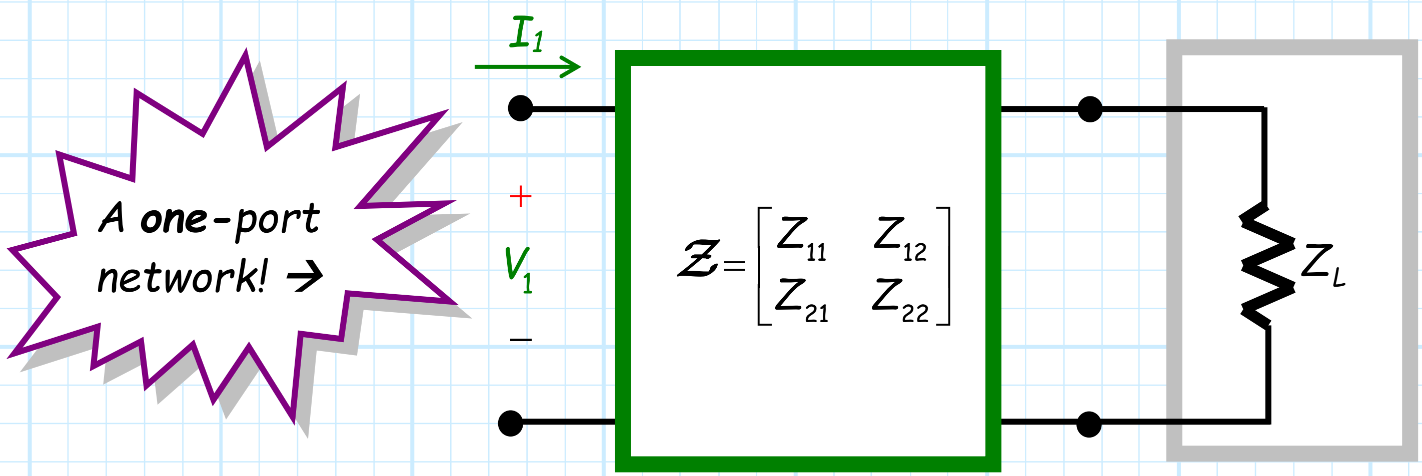
*Isn't there some **easier** way??*

A new one-port network

A: Yes, (arguably) there **is** an easier way!

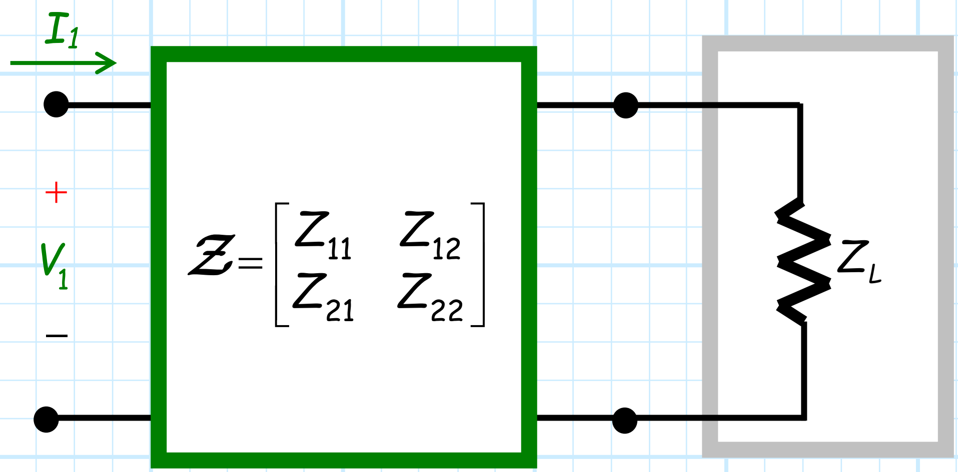
Instead of analyzing the entire circuit together, we can **divide** it into **two** more manageable pieces.

For example, we can first **combine** the **two-port network** and the **load** (leaving the source for **later**):



Just *one* port on this circuit!

Think carefully about the circuit below:



We have **terminated** the output port of the two-port network with a **load impedance** Z_L.

→ Thus, we have created a **one-port** network!!!

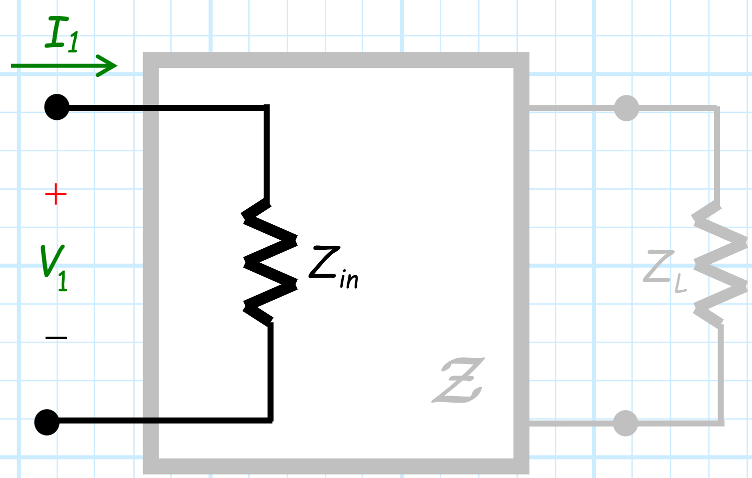
Every one-port has an impedance

Q: *A one-port network; **so what**?*

A: So what?!

Remember, a **one-port** network is completely characterized by just **one** impedance parameter—its **input impedance**.

An **equivalent circuit** to the two-port and load is thus →



Its **input impedance** is therefore:

$$Z_{in} \doteq \frac{V_1}{I_1}$$

Input Impedance is NOT Z_{11} !

Q: *Hey, does this mean the input impedance is equal to the trans-impedance parameter Z_{11}? After all, isn't $V_1/I_1 = Z_{11}$!*

A: **NOOO!!!!!** This is absolutely **false**. Do **not** make this mistake!

Remember, impedance parameter Z_{11} is defined as the ratio V_1/I_1—**if** and **when** port 2 is **open-circuited**!

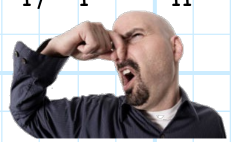
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad \text{(when port 2 is \textbf{open-circuited})}$$

Look at the circuit—port 2 has a **load** Z_L attached to it (**not an open-circuit**), and so:

$$I_2 \neq 0 \qquad \text{and} \qquad \frac{V_1}{I_1} = Z_{11} + Z_{21}\left(\frac{I_2}{I_1}\right) \neq Z_{11}$$

V_1/I_1 is NOT Z_{11}!

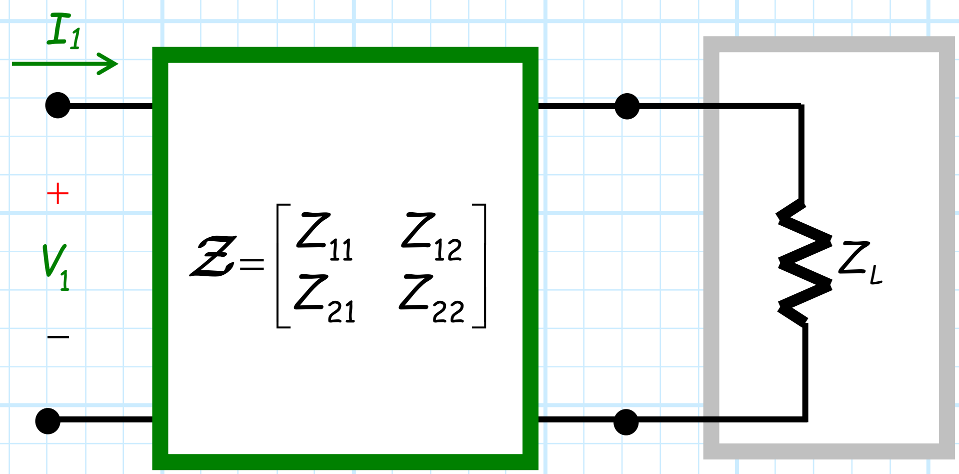
Note the incorrect assertion $V_1/I_1 = Z_{11}$ *does not pass the "smell test".*



That is, it is reasonable to believe that the **input** impedance of a two-port network terminated in some load will in fact depend on **all** circuit parameters, namely:

1. **trans-impedance** parameters Z_{11}, as well as Z_{12}, Z_{21}, Z_{22}

2. **and** the impedance Z_L of the **load**.



→ We will find that that is **indeed** the case—the input impedance depends on **all 5** of these circuit parameters.

So what *is* the input impedance?

Q: *So just what **is** Z_{in}? How do we **find** its value?*

A: Let's write down all the **circuit equations** that we know. For the **2-port network**:

$$V_1 = Z_{11}\, I_1 + Z_{12}\, I_2 \qquad V_2 = Z_{21}\, I_1 + Z_{22}\, I_2$$

And for the **load impedance**:

$$V_2 = -Z_L\, I_2$$ (note the minus sign!)

With a little algebraic elbow grease, we can solve for V_1 in terms of I_1, and then find:

$$\boxed{Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}\, Z_{21}}{Z_{22} + Z_L}}$$

←It is **quite evident** that $Z_{in} \neq Z_{11}$!

The Sanity Check

Note that this result passes the **smell test**, as Z_{in} indeed depends on **all 5** circuit parameters.

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}\,Z_{21}}{Z_{22}+Z_L}$$

However, we can **also** apply a **sanity check** to this result!

Recall that by definition of trans-impedance parameters, $V_1/I_1 = Z_{11}$—if and when **port 2** is terminated in an **open circuit**.

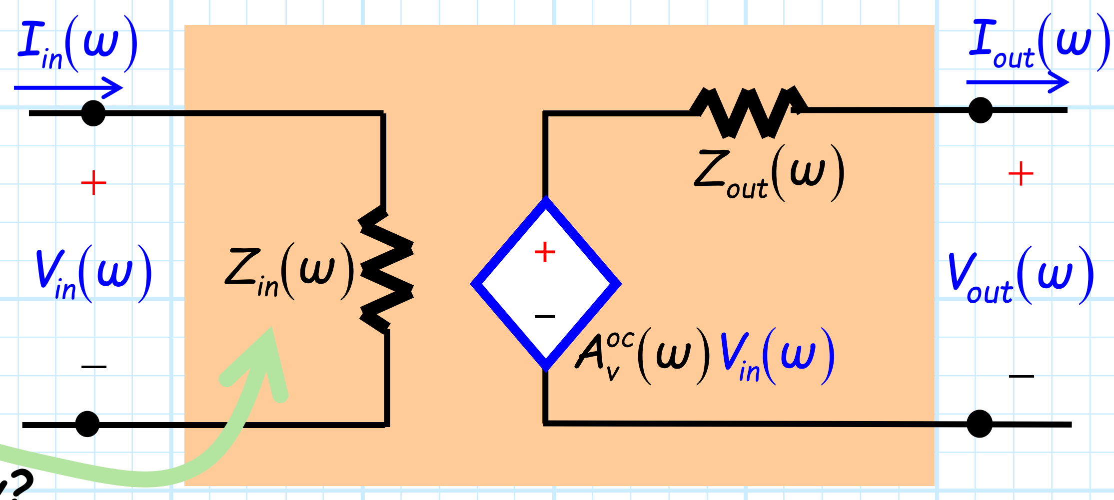
In other words we **should** find that our result reduces to $Z_{in} = Z_{11}$ when $Z_L = \infty$. Let's **see** if it does:

$$Z_{in}\big|_{Z_L=\infty} = Z_{11} - \frac{Z_{12}\,Z_{21}}{Z_{22}+\infty} = Z_{11}$$

→ Our calculation **passed** this sanity check!

Your instructors were not incompetent

Q: Wait! *In **EECS 412** we learned that the **input impedance** of an amplifier is **independent of the load** impedance attached at its output.*



Were we taught incorrectly?

A: An amplifier is a **rare example** of a **unilateral** 2-port device.

For a unilateral device $Z_{12} \cong 0$, and thus the resulting **input** impedance is **independent** of Z_L:

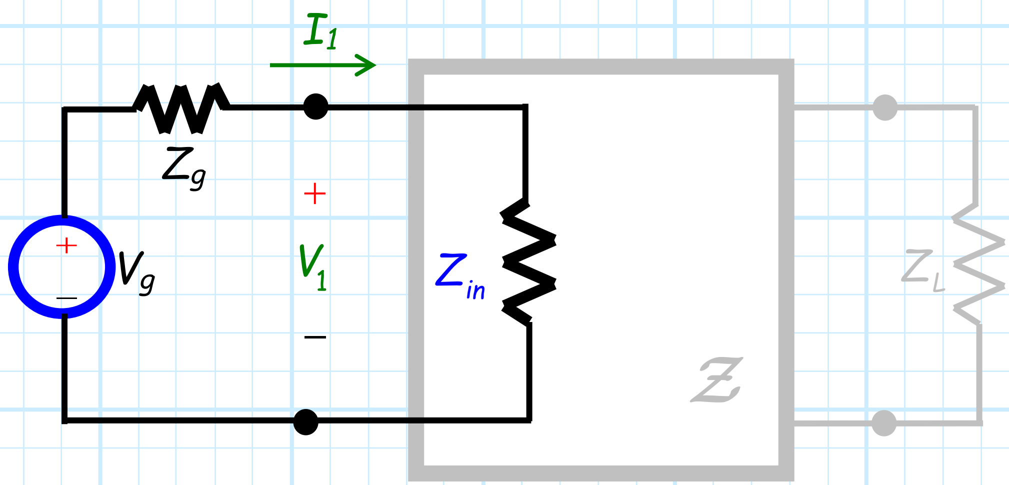
$$Z_{in}\Big|_{Z_{12}=0} = Z_{11} - \frac{Z_{12}\,Z_{21}}{Z_{22}+Z_L} = Z_{11}$$

That's why the input impedance of an amplifier is a **device parameter**!

Our equivalent circuit

Q: *OK, but I don't see why finding Z_{in} is at all **helpful**.*

A: Well, let's now attach the **source**, and then view our **new** equivalent circuit (quite **simple**—don't you think?):



We of course analyzed this circuit **earlier**—it's just a source connected to a load(Z_{in} for this case).

We find for instance:

$$I_1 = \frac{V_g}{Z_g + Z_{in}} \qquad V_1 = V_g\left(\frac{Z_{in}}{Z_g + Z_{in}}\right)$$

Sanity Check #2

Now we insert **this** finding:

$$Z_{in} = Z_{11} - \frac{Z_{12}\, Z_{21}}{Z_{22} + Z_L}$$

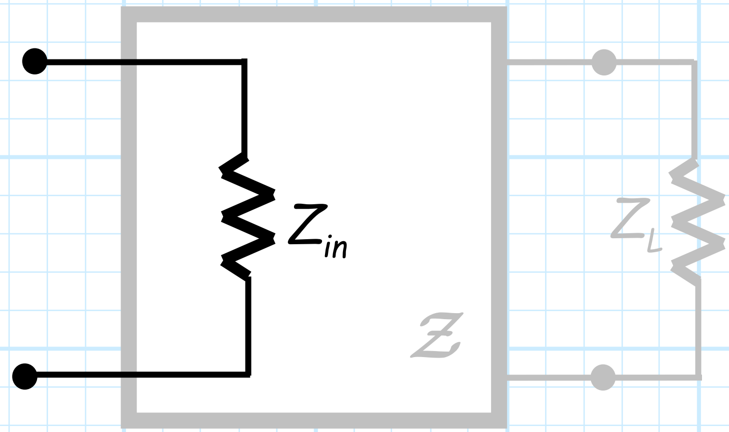
into the simple results of the previous page, we confirm the results of our earlier analysis:

$$I_1 = V_g \frac{Z_{22} + Z_L}{\left(Z_{11} + Z_g\right)\left(Z_{22} + Z_L\right) - Z_{12}Z_{21}}$$

$$V_1 = V_g \frac{Z_{11}\left(Z_{22} + Z_L\right) - Z_{12}Z_{21}}{\left(Z_{11} + Z_g\right)\left(Z_{22} + Z_L\right) - Z_{12}Z_{21}}$$

An impedance transformer

We can in this situation view our 2-port device as an impedance transformer!



I.E., the 2-port device "**transforms**" a load impedance Z_L into **new** impedance value Z_{in}.

Every 2-port device is an **impedance transformer**!

Sometimes this is the **primary** purpose of the device, but **usually** this is a **secondary** characteristic of two-port devices.