PHSX 711: Homework #9

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Problem 1

Exercise 14.3.2 * (1) Show that the eigenvectors of $\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}$ are given by Eq. (14.3.28). (2) Verify Eq. (14.3.29).

$$|\hat{n} \text{ up}\rangle \equiv |\hat{n}+\rangle = \begin{bmatrix} \cos(\theta/2)e^{-i\phi/2} \\ \sin(\theta/2)e^{i\phi/2} \end{bmatrix}$$

$$|\hat{n} \text{ down}\rangle \equiv |\hat{n}-\rangle = \begin{bmatrix} -\sin(\theta/2)e^{-i\phi/2} \\ \cos(\theta/2)e^{i\phi/2} \end{bmatrix}$$

$$\begin{aligned} \langle \hat{n} \pm |\boldsymbol{S}|\hat{n}\pm\rangle &= \pm(\hat{\theta}/2)(\mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta) \\ &= \pm(\hbar/2)\hat{\mathbf{n}} \end{aligned}$$

Solution:

Problem 2

Exercise 14.3.3 * Using Eqs. (14.3.32) and (14.3.33) show that the Pauli matrices are traceless.

Note: for any two matrices A and B, Tr(AB)=Tr(BA).

$$[\sigma_i, \sigma_j]_+ = 0 \quad \text{or} \quad \sigma_i\sigma_j = -\sigma_j\sigma_i \quad (i \neq j)$$

$$\sigma_x\sigma_y = i\sigma_z \text{ and cyclic permutations}$$

$$\text{Tr } \sigma_i = 0, \quad i = x, y, z$$

Problem 3

Exercise 14.4.1 * Show that if $H = -\gamma L \cdot \mathbf{B}$, and $\mathbf{B}$ is position independent,

$$\frac{d\langle \mathbf{L}\rangle}{dt} = \langle \boldsymbol{\mu} \times \mathbf{B}\rangle = \langle \boldsymbol{\mu}\rangle \times \mathbf{B}$$

Hint: start with Ehrenfest's theorem:

$$\frac{d}{dt}\langle \mathbf{L}\rangle = -\frac{i}{\hbar}\langle[\mathbf{L}, H]\rangle$$

And note: $[L_i, B_j] = 0$ for any i and j because $\mathbf{B}$ is position independent. It may be easier if you look at one of the components (e.g., $\langle L_z\rangle$) of $\mathbf{L}$ first. Then, generalizes to $\langle L_x\rangle$ and $\langle L_y\rangle$.

Problem 4

Exercise 14.4.4. At $t = 0$, an electron is in the state with $s_z = \hbar/2$. A steady field $\mathbf{B} = B\hat{\mathbf{j}}$ with $B = 100$ G is turned on. How many seconds will it take for the spin to flip?