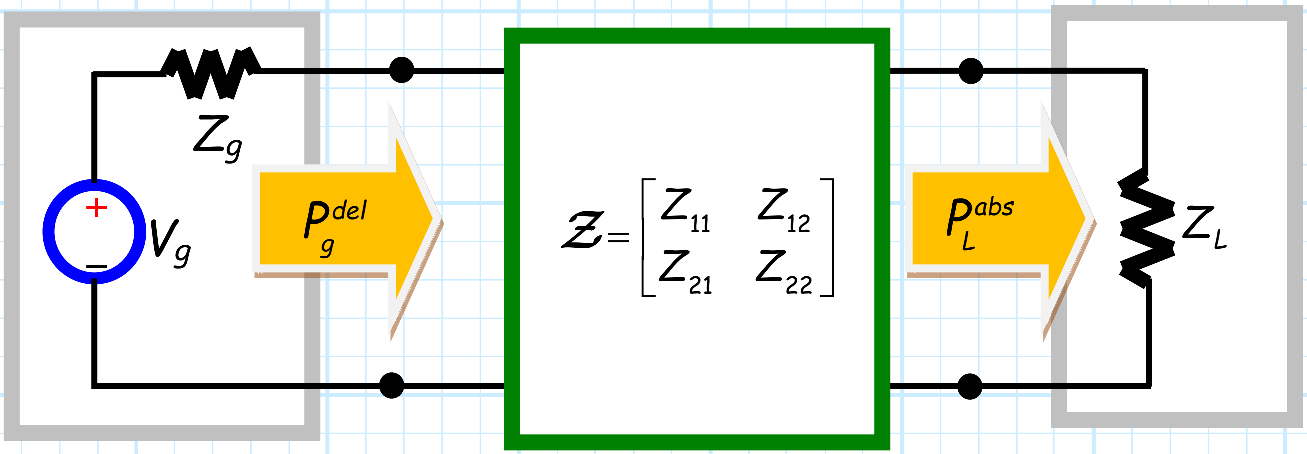
The Powers That Be

So.

We've determined the port **voltages** and port **current**—now let's determine the **rate** at which **energy** flows!

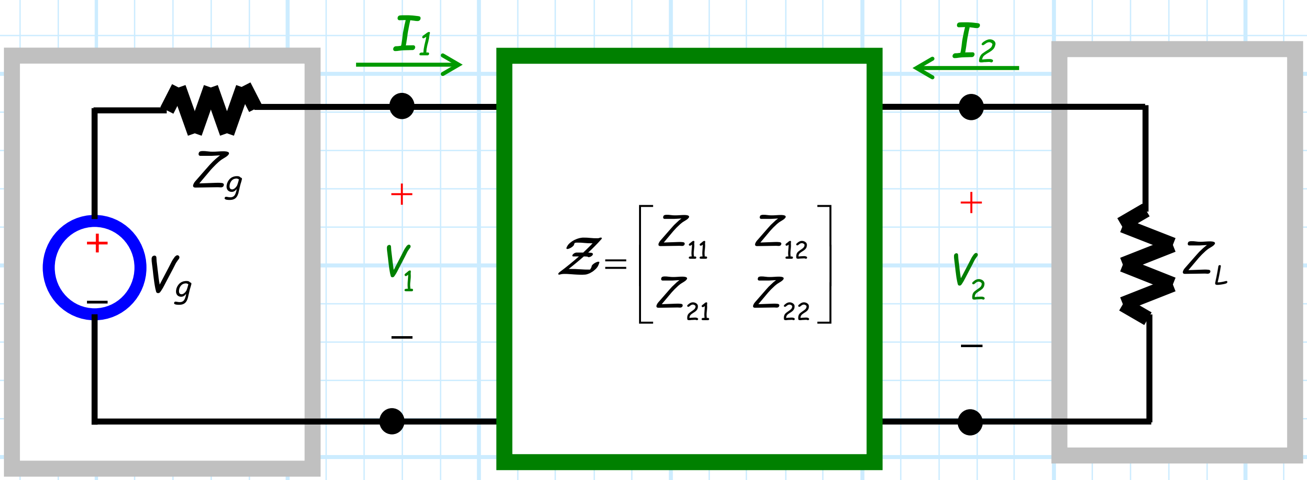


Remember this?

Recall we previously determined the voltages and currents at each of these **two** ports:

$$I_1 = V_g \frac{Z_{22}+Z_L}{\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}Z_{21}} \qquad V_1 = V_g \frac{Z_{11}\left(Z_{22}+Z_L\right) - Z_{12}Z_{21}}{\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}Z_{21}}$$

$$I_2 = -V_g \frac{Z_{21}}{\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}Z_{21}} \qquad V_2 = V_g \frac{Z_L\, Z_{21}}{\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}Z_{21}}$$



Nutin' more enjoyable than complex arithmetic

Using these results, we can determine precisely the rate (J/s) at which energy is **delivered** by the **source** to the input port:

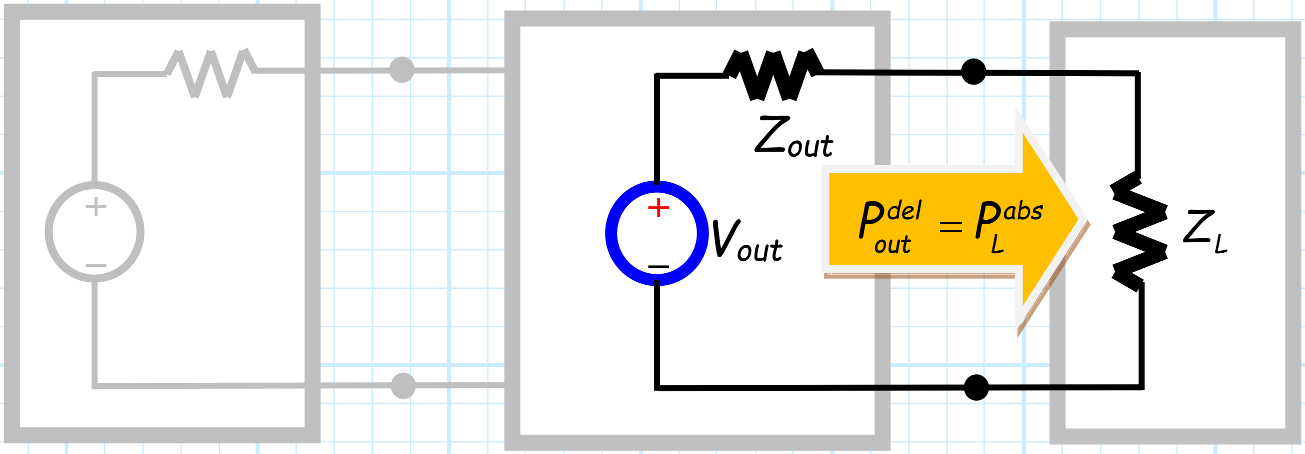
$$
\begin{aligned}
P_g^{del} &= \frac{1}{2} Re\left\{V_1 I_1^*\right\} \\
&= \frac{1}{2}\left| V_g \right|^2 \frac{Re\{Z_{11}\}\left|\left(Z_{22}+Z_L\right)\right|^2 - Re\left\{Z_{12}Z_{21}Z_{22}^*\right\} - Re\left\{Z_{12}Z_{21}Z_L^*\right\}}{\left|\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right)-Z_{12}Z_{21}\right|^2}
\end{aligned}
$$

Likewise, the rate at which energy from the output port (J/s) is **absorbed** by the **load** (note the **sign**!):

$$
\begin{aligned}
P_L^{abs} &= \frac{1}{2} Re\left\{-V_2 I_2^*\right\} \\
&= \frac{1}{2}\left| V_g \right|^2 \frac{Re\{Z_L\}\ \left|Z_{21}\right|^2}{\left|\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}\,Z_{21}\right|^2}
\end{aligned}
$$

Looking at it a different way

Alternatively, we can now replace the **left** side of the circuit with its equivalent circuit:

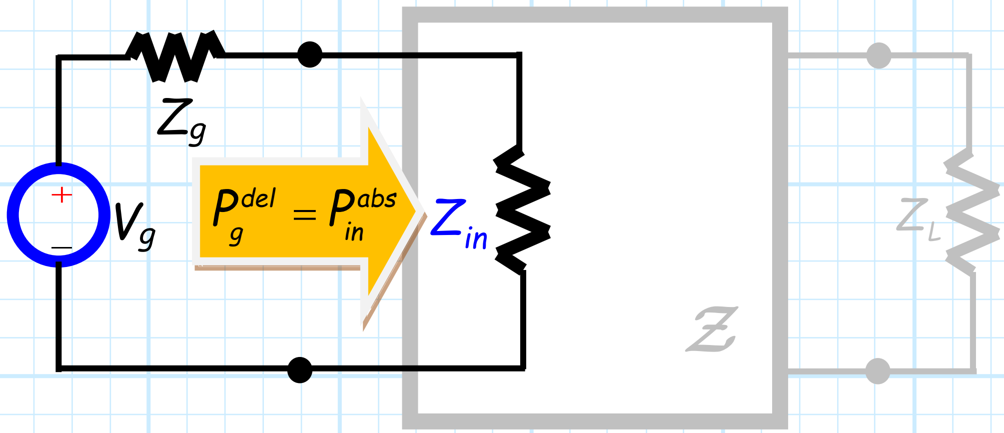


By conservation of energy, the power **absorbed by the load** is equal to the **power delivered by the equivalent source** V_{out} and Z_{out}:

$$P_{out}^{del} = \frac{1}{2} \left| V_{out} \right|^2 \frac{Re\{Z_L\}}{\left| Z_{out} + Z_L \right|^2} = P_L^{abs}$$

Or, if the right side is replaced Z_{in}

Or, replacing the **right** side of the circuit with its **equivalent**:



we see that the power delivered by the source is **equal** to the power **absorbed** by the **input impedance** Z_{in}!!

$$P_g^{del} = \frac{1}{2}\left|V_g\right|^2 \frac{Re\{Z_{in}\}}{\left|Z_g + Z_{in}\right|^2} = P_{in}^{abs}$$

Absorbed by Zin—what that means

Q: *Absorbed by the **input** impedance? What the heck does **that** mean?*

*Is this the power absorbed by the **two-port device**?*

A: Note the input impedance Z_{in} characterizes the :

two-port device, when terminated in a **load** Z_L

thus, the power absorbed by Z_{in} is in fact the rate at which energy is absorbed by the:

two-port device terminated in a **load** Z_L

In other words, the power absorbed by Z_{in} is the power absorbed by the

two-port device, **AND** the load Z_L !!!!

Either way, the results are exactly the same!

It is **essential** that **you** understand that either of these two viewpoints provide precisely the same numerical results!

$$P_g^{del} = \frac{1}{2}\left|V_g\right|^2 \frac{Re\{Z_{in}\}}{\left|Z_g + Z_{in}\right|^2}$$

$$= \frac{1}{2}\left|V_g\right|^2 \frac{Re\{Z_{11}\}\left|\left(Z_{22}+Z_L\right)\right|^2 - Re\{Z_{12}\,Z_{21}\,Z_{22}^*\} - Re\{Z_{12}\,Z_{21}\,Z_L^*\}}{\left|\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}\,Z_{21}\right|^2}$$

and:

$$P_L^{abs} = \frac{1}{2}\left|V_{out}\right|^2 \frac{Re\{Z_L\}}{\left|Z_{out} + Z_L\right|^2}$$

$$= \frac{1}{2}\left|V_g\right|^2 \frac{Re\{Z_L\}\,\left|Z_{21}\right|^2}{\left|\left(Z_{11}+Z_g\right)\left(Z_{22}+Z_L\right) - Z_{12}\,Z_{21}\right|^2}$$

Dad gummit

Q: *Now wait just **one** dog-gone second!*

*I thought you said that conservation of energy required that the **delivered** power and the **absorbed** power be **equal**.*

*But it does **not** appear to **me** that **these** results for P_g^{del} and P_L^{abs} **are** equal—shouldn't they be the **same**?*

A: Nope—for **this** case, the values of P_g^{del} and P_L^{abs} are **often very different**!

Three types of devices

* Devices or networks that **decrease** the rate of energy flow are known as **lossy**.

* Devices or networks that **increase** the rate of energy flow are known as **active**.

* Devices or networks that leave the rate of energy flow **unchanged** are known as **lossless**.