Physics 313 Homework 12

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December 6, 2023

1. (4 pts) Calculate the average radius of a hydrogen atom in the first excited state with $l = 1$.

Solution:

A hydrogen atom in the first excited state with $l = 1$ has an average radius:

$$R_{avg}(r) = \int_0^\infty rP(r)\ dr = \int_0^\infty r^3 \left(\frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \right)^2 \ dr$$

Numerically, this comes out to be approximately 25 nm.

2. (3 pts) What is the most probable radius of a hydrogen atom in the first excited state with $l = 1$ and $l = 0$?

Solution:

A hydrogen atom in the first excited state with $l = 0$ has a radial probability:

$$P(r) = r^2 R^2(r) = r^2 \left(\frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0} \right) e^{-r/2a_0} \right)^2$$

The radial probability is at a maximum when r is approximately 27.699 nm.

A hydrogen atom in the first excited state with $l = 1$ has a radial probability:

$$P(r) = r^2 R^2(r) = r^2 \left(\frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \right)^2$$

The radial probability is at a maximum at exactly $r = 4a_0 \approx 21.16$ nm.

3. (2 pts) What is the configuration of a magnesium atom (Z = 12)?

Solution:

A magnesium atom has 12 electrons, so the configuration is 1s, 2s, 2p, 3s. 2 electrons fill subshells 1s and 2s, 6 fill 2p, and 2 more occupy 3s.

- In s1: n=1, l=0, implying two electrons with opposite spin
- In s2: n=2, l=0, implying the same
- In 2p: n=2, l=1, implying $2l + 1 = 3 \times 2 = 6$ combinations of electrons
- In 3s: n=3, l=1, implying the same

4. (3 pts) Consider an electron in an infinite square well with a length of L = 2.0 nm. The electron is in a state with a wavefunction of the form

$$\psi(x) = e^{iKx}\frac{1}{\sqrt{L}}, \tag{1}$$

where $K = 10^{10}$ m^{-1}. What is the probability that when we measure the energy of the electron, we find it to be in the ground state?

Solution: