PHSX 536: Homework #9

April 2, 2025

Grant Saggars

Problem 1

Consider the circuit shown in Figure 6.34, which is built from four resistors and an op-amp.

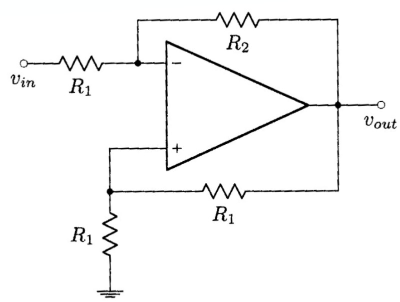


Figure 6.34: The circuit for problem 3.

(a) In terms of v_{in}, v_{out}, R_1 and R_2, what is the voltage at the non-inverting input to the op-amp (v_+)?

Solution:

Writing the loop equations, I find

$$v_{in} - v_{out} = i_1(R_1 + R_2) \tag{1}$$
$$v_{in} - i_1 R_1 = v_- \tag{2}$$
$$v_{out} = i_2(R_1 + R_1) \tag{3}$$
$$v_{out} - i_2 R_1 = v_+ \tag{4}$$
$$v_- = v_+ \tag{5}$$

To eliminate i_2, I'll substitute in $i_2 = \frac{v_{out}}{R_1+R_1}$ from equation (3) into equation (4).

$$v_+ = v_{out}\left(1 - \frac{R_1}{R_1 + R_1}\right) = v_{out}\left(1 - \frac{1}{2}\right) = \frac{1}{2}v_{out}$$

(b) In terms of v_{in}, v_{out}, R_1 and R_2, what is input current to the circuit (the current coming in through R_1)?

Solution:

We're looking for i_1, which is given by

$$v_{in} = i_1 R_1 - v_- \qquad \text{Equation (2)}$$
$$i_1 = \frac{v_{in} - v_-}{R_1}$$
$$i_1 = \frac{v_{in} - \frac{1}{2}v_{out}}{R_1} \qquad \text{Equation (5)}$$

(c) In terms of v_{in}, R_1 and R_2, what is the output voltage (v_{out}) of the circuit?

Solution:

$$v_{out} = v_{in} - i_1(R_1 + R_2) \qquad \text{Equation (1)}$$
$$v_{out} = v_{in} - \frac{v_{in} - \frac{1}{2}v_{out}}{R_1}(R_1 + R_2)$$
$$v_{out}R_1 = v_{in}R_1 - \left(v_{in} - \frac{1}{2}v_{out}\right)(R_1 + R_2)$$
$$v_{out}R_1 = -v_{in}R_2 + \frac{v_{out}R_1}{2} + \frac{v_{out}R_2}{2}$$
$$v_{out}R_1 - v_{out}R_2 = -2v_{in}R_2$$
$$v_{out}\left(R_1 - R_2\right) = -2v_{in}R_2$$
$$v_{out} = -\frac{2v_{in}R_2}{R_1 - R_2}$$

Problem 2

(16.5) The op-amp circuit shown in Figure 6.36 has an input resistor, R_i, and a feedback impedance, Z_F, built from a resistor, R_f, in parallel with a capacitor, C_f. The circuit has an input voltage $v_{in}(t)$, and an output voltage $v_{out}(t)$. You may assume that the op-amp is correctly biased with the appropriate DC voltage such that it is *on*.

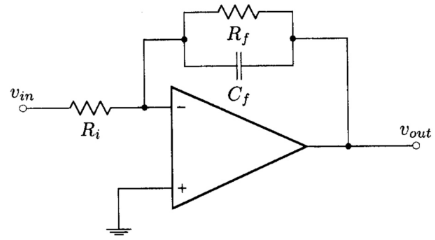


Figure 6.36: The circuit for problems 7 and 8. Note that the non-inverting input is grounded.

(a) The resistor, R_f, and capacitor, C_f, in the feedback loop are in parallel with each other. Which one will dominate in the limit of low frequencies and which will dominate in the limit of high frequencies?

Solution:

Impedance of a capacitor is inversely proportional to frequency, so at low frequencies it will have high impedance, while at high frequencies it will have low resistance. This means that the resistor dominates at low frequency and the capacitor at high frequency.

(b) Write the impedance of the parallel pair, Z_f, as R_f times a dimensionless quantity.

Solution:

$$\begin{aligned} Z_f &= \frac{R_f Z_{C_f}}{R_f + Z_{C_f}} \\ &= \frac{R_f(1/j\omega C_f)}{R_f + (1/j\omega C_f)} \\ &= \frac{R_f(1/j\omega C_f)}{R_f + (1/j\omega C_f)} \frac{j\omega C_f}{j\omega C_f} \\ &= \frac{R_f}{R_f(j\omega C_f) + 1} \end{aligned}$$

Problem 2 continued on next page…

(c) What is the gain, G, of the circuit in the low-frequency regime?

Solution:

This is a multiplier op-amp where $v_{out} = i(R_i + Z_f)$, and $i = \frac{v_{in}}{R_i}$, so

$$v_{out} = \underbrace{\left(1 + \frac{Z_f}{R_i}\right)}_{=\text{ gain}} v_{in}$$

For a low frequency, $\lim_{\omega \to 0}(Z_f) = R_f$, so the gain will be

$$G = 1 + \frac{R_f}{R_i}$$

(d) What is the gain, G, of the circuit in the high-frequency regime?

Solution:

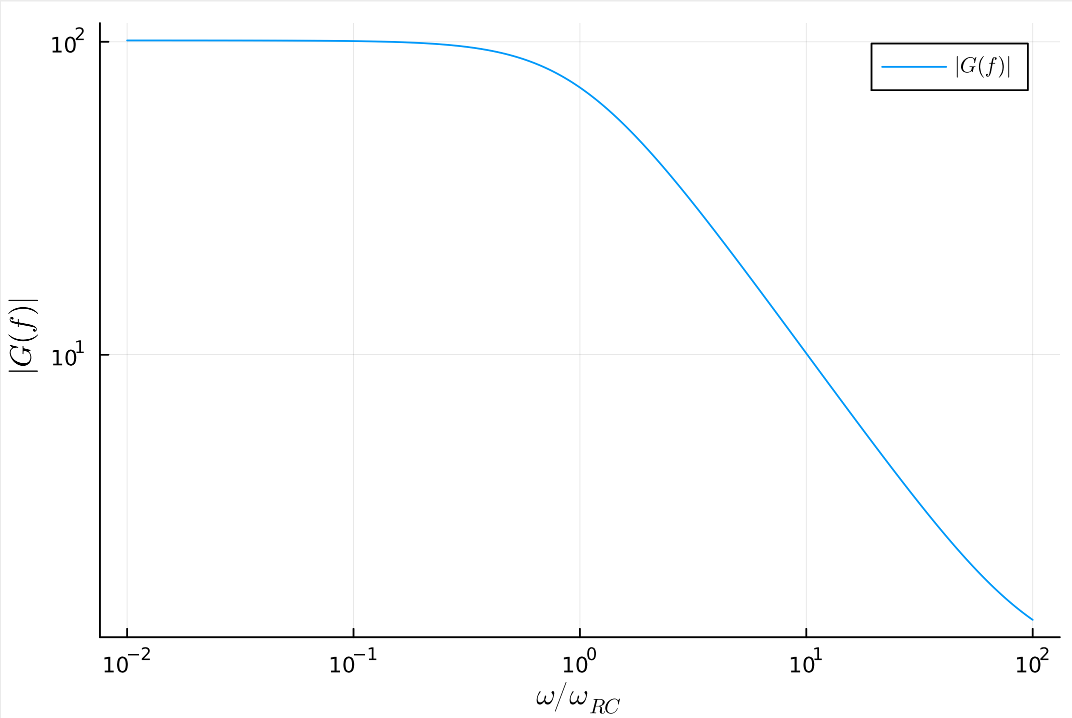
At $\lim_{\omega \to \infty}(Z_f) = 0$, so gain is

$$G = 1 + \frac{0}{R_i} = 1$$

Problem 2 continued on next page…

(e) Assume that $R_f = 100R_i$ and using your results from (c) and (d), plot the gain, $|G(f)|$, as a function of ω/ω_{RC} on a log-log plot, where $\omega_{RC} = 1/R_f C_f$.

Solution:



(f) For an arbitrary time-dependent input voltage, $v_{in}(t)$, what will the input current, $i_i(t)$, be?

Solution:

$$i_i(t) = \frac{v_{in}}{R_i}$$

(g) What will be the current in the feedback loop (through the parallel R-C pair)?

Solution:

$$i_f = \frac{v_{out}}{Z_f} = \frac{1 + v_{in}(Z_f/R_i)}{Z_f}$$