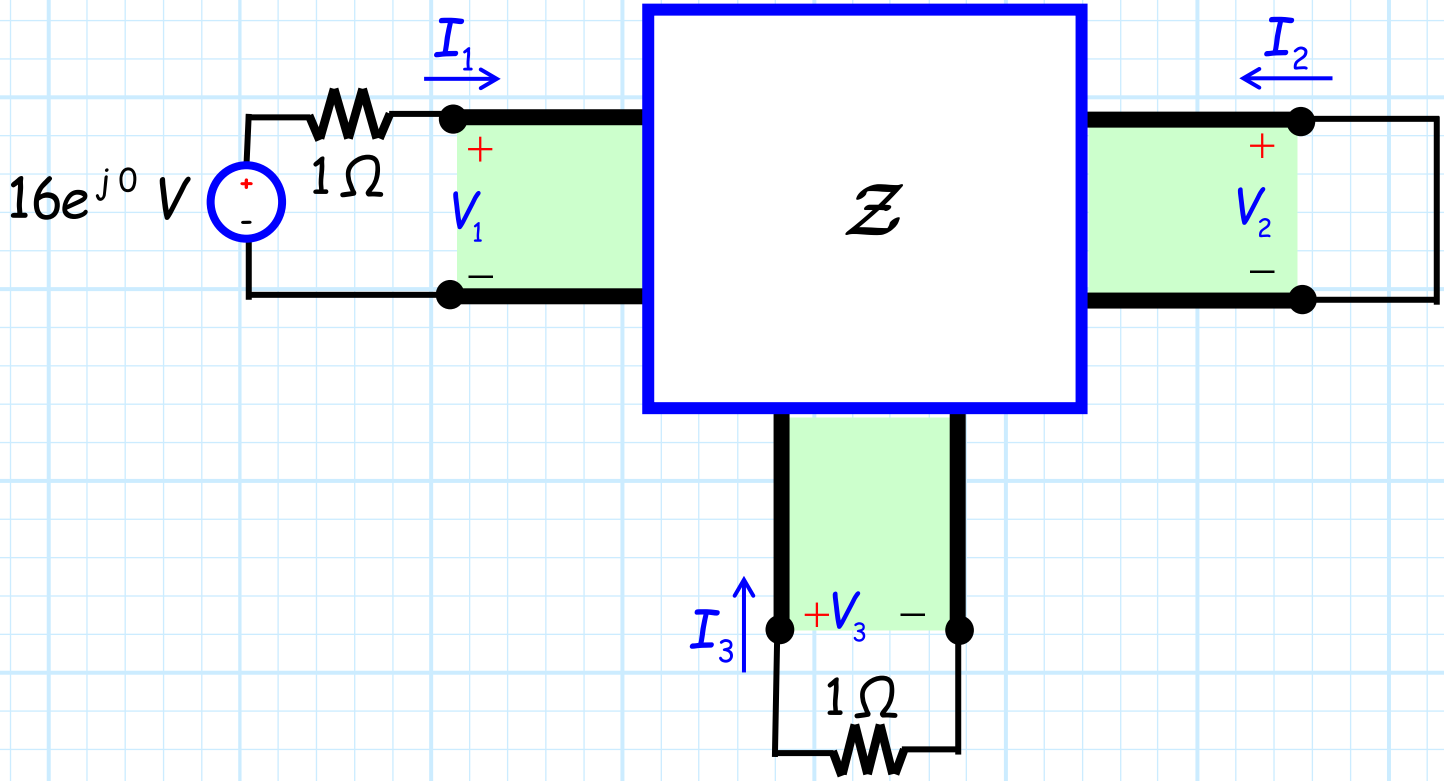
Example: Using the Impedance Matrix

Consider this circuit:

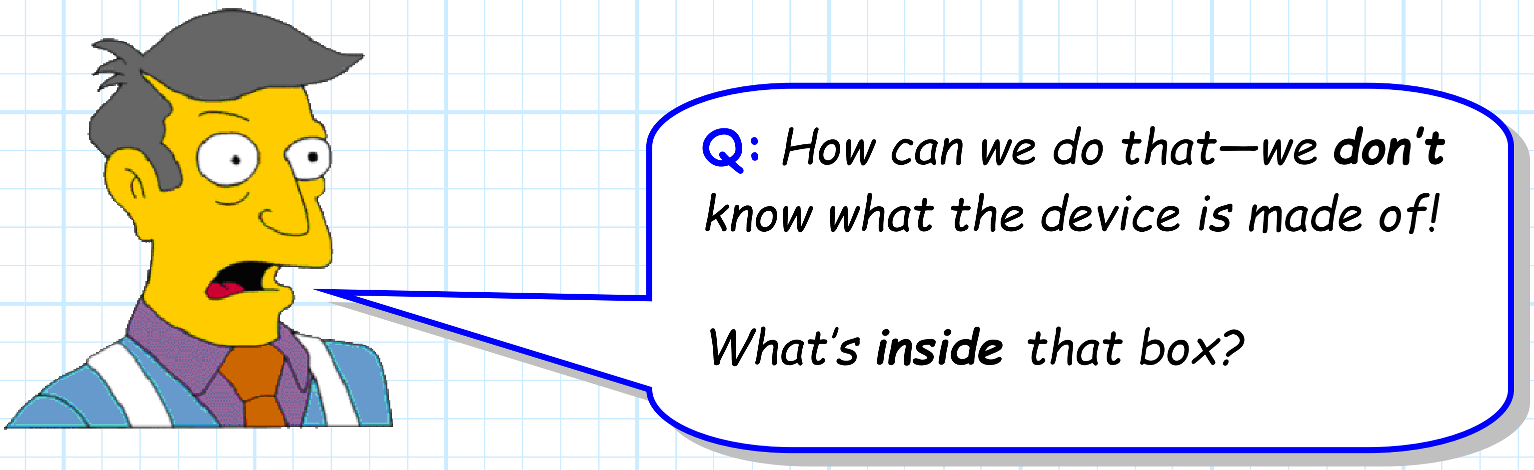


Where the 3-port **device** is characterized by this **impedance matrix**:

$$\mathcal{Z}(\omega) = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 4 \\ 2 & 4 & 1 \end{bmatrix} \quad \Omega$$

The impedance matrix is a complete description!

Let's now determine all port **voltages** V_1, V_2, V_3 and all **currents** I_1, I_2, I_3.



A: We **don't** need to know what's inside that box!

We know its impedance matrix—and that **completely** characterizes the device (or, at least, characterizes it at **one** frequency).

Thus, we have **enough information** to solve this problem!

The impedance matrix describes what's inside the box—but not what is outside!

From the **impedance matrix** we know:

$$
\begin{aligned}
V_1 &= 2I_1 + I_2 + 2I_3 \\
V_2 &= I_1 + I_2 + 4I_3 \\
V_3 &= 2I_1 + 4I_2 + I_3
\end{aligned}
$$

Q: *Wait! There are **6 unknowns** here.*

*Yet there are only **6 equations!?***

A: True!

The impedance matrix describes the device **in** the box, but it does **not** describe the devices **attached** to it.

One equation for each attached device

We require **three more equations** that describe the **three attached devices**!

1. Using KVL, the **source** at **port 1** is described by the equation:

$$V_1 = 16.0 - (1)I_1$$

2. The **short** circuit on **port 2** means that:

$$V_2 = 0$$

3. While the **load** on **port 3** leads to:

$$V_3 = -(1)I_3$$ (note the **minus sign**!)

Now there are **6** equations and **6** unknowns!

The answers are complex numbers

Combining these 6 equations, we find:

$$V_1 = 16 - I_1 = 2I_1 + I_2 + 2I_3$$

$$\therefore \; 16 = 3I_1 + I_2 + 2I_3$$

$$V_2 = 0 = I_1 + I_2 + 4I_3$$

$$\therefore \; 0 = I_1 + I_2 + 4I_3$$

$$V_3 = -I_3 = 2I_1 + 4I_2 + I_3$$

$$\therefore \; 0 = 2I_1 + 4I_2 + 2I_3$$

Solving, we find (I'll let **you** do the algebraic details!):

$$I_1 = 7.0\, e^{j0} \qquad I_2 = 3.0\, e^{j\pi} \qquad I_3 = 1.0\, e^{j\pi}$$

$$V_1 = 9.0\, e^{j0} \qquad V_2 = 0.0 \qquad V_3 = 1.0\, e^{j0}$$