# EECS 622: Homework #19

November 9, 2025

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## Problem 1

Carefully consider a phase modulated signal of the form:

$$v_o(t) = 1.0 \cos\left[2\pi t\left(4000 + Bt + At^2\right)\right]$$

Where $A$ and $B$ are some unknown constants. But, the total frequency of this signal is known to be:

$$\omega(t) = 8000\pi + 2\pi t + \pi t^2$$

Determine precisely (i.e., without any unknowns!) the relative phase $\varphi_r(t)$, and carrier frequency $\omega_0$ of this signal.

**Solution:**
The total phase is contained as the parameter of the cosine function:

$$\theta(t) = 2\pi t(4000 + Bt + At^2)$$

And we have the constraint between total phase and total frequency:

$$\omega(t) = \frac{d\theta(t)}{dt} = \underbrace{\omega_0}_{\text{carrier freq.}} + \underbrace{\omega_r(t)}_{\text{relative freq.}}$$

Which is sufficient to eliminate a term:

$$\begin{aligned}
8000\pi + 2\pi t + \pi t^2 &= \frac{d|2\pi t(4000 + Bt + At^2)|}{dt} && \left(\text{since} \quad \omega(t) = \frac{d\theta(t)}{dt}\right) \\
2\pi\left[4000 + t + \frac{1}{2}t^2\right] &= 2\pi\left[4000 + 2Bt + 3At^2\right] \\
t + \frac{1}{2}t^2 &= 2Bt + 3At^2
\end{aligned}$$

Clearly, $B = 1/2$ and $A = 1/6$. The total phase consists of the linear carrier phase and the time-varying relative phase, and a constant carrier frequency and a time-varying relative frequency, which are both related by an integral/derivative as above:

$$\begin{aligned}
\theta(t) &= 8000\pi t + \pi t^2 + \frac{\pi}{3}t^3, & \theta_0 &= 8000\pi t & \theta_r &= \pi t^2 + \frac{\pi}{3}t^3 \\
\omega(t) &= 8000\pi + \pi t + \pi t^2 & \omega_0 &= 8000\pi & \omega_r &= \pi t + \pi t^2
\end{aligned}$$

# Problem 2

An oscillator produces a sine wave with a carrier frequency of $f_0 = 6\text{GHz}$.
Due to long-term instability, this carrier frequency can ”drift” as much as $\pm 300\text{kHz}$.
Specify the accuracy of this oscillator in parts-per-million (ppm).

**Solution:**
We have defined,

$$\text{ppm}(\Delta f_r) \equiv 10^6 \left( \frac{\Delta f_r(\text{Hz})}{f_0(\text{Hz})} \right) = \frac{\Delta f_r(\text{Hz})}{f_0(\text{MHz})}$$

So,

$$\text{ppm}(\pm 300 \text{ kHz}) = 10^6 \frac{6 \times 10^5 \text{ Hz}}{6 \times 10^9 \text{ Hz}} = 100$$