EECS 622: Homework #1

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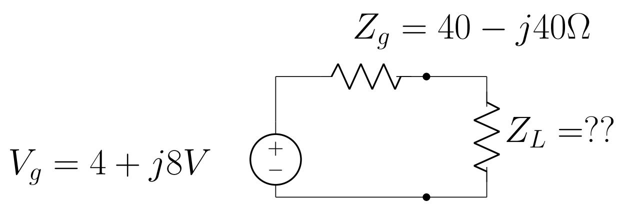
Problem 1

Consider the circuit:



Given the values of V_g and Z_g, determine:

(a) The value of load impedance Z_L that will maximize the power source delivered by this source.

Solution:

A load impedance with value:

$$Z_L = Z_g^*$$

will maximize the power delivered by the source. Therefore, we should choose Z_L equal:

$$\boxed{Z_L = 40 + j40\ \Omega}$$

(b) The value of this maximum power.

Solution:

Numerically, we will have

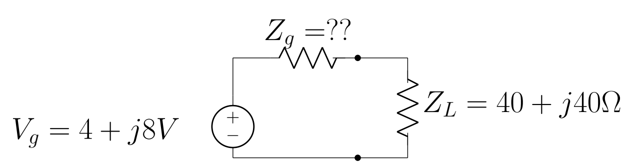
$$\begin{aligned}
P_g^{del}\Big|_{Z_L=Z_g^*} &= \frac{1}{2} I_{RMS} I_{RMS}^* R && \text{(Definition of Power for a Sinusoidal Source)} \\
&= \frac{R_g}{2}\left(\frac{V_g}{Z_L+Z_g}\right)\left(\frac{V_g^*}{Z_L^*+Z_g^*}\right)\Big|_{Z_L=Z_g^*} && \text{(Substitute } I \text{ and } R\text{)} \\
&= \frac{R_g}{2}\left(\frac{VV^*}{Z_L Z_L^* + Z_L Z_g^* + Z_L^* Z_g + Z_g Z_g^*}\right)\Big|_{Z_L=Z_g^*} && \text{(Expand)} \\
&= \frac{R_g}{2}\left(\frac{VV^*}{Z_g Z_g^* + Z_g Z_g^* + Z_g^* Z_g + Z_g Z_g^*}\right) && \text{(Apply Boundary)} \\
&= \frac{VV^*}{8}\left(\frac{R_g}{Z_g Z_g^*}\right) && \text{(Algebra)} \\
&= \frac{VV^*}{8R_g} && (Z_g Z_g^* = |R_g|^2)
\end{aligned}$$

This just shows that we have the same circuit as in the lecture. R_g is simply $\mathrm{Re}\{Z_g\} = 40\ \Omega$. Maximum power is then:

$$\boxed{P_g^{del} = \frac{(4+j8)(4-j8)}{8\cdot 40} = 0.25\ \frac{\mathrm{J}}{\mathrm{s}}}$$

Problem 2

Now consider the circuit:



Given the values of V_g and Z_g, determine:

(a) The value of source impedance Z_g that will maximize the power absorbed by load Z_L.

Solution:

It can be shown that the value of source impedance Z_g that maximizes the power absorbed by the load Z_L is-in fact-purely reactive, with value:

$$\boxed{Z_g = -jX_L}$$

Therefore, $Z_g = -j40\ \Omega$ to maximize the power absorbed by the load.

Problem 2 continued on next page...

(b) The value of this maximum power.

Solution:

With a purely reactive Z_g, we should expect total resistance to just be R_L:

$$\begin{aligned} Z_{total} = Z_g + Z_L &= (-jX_L) + (R_L + jX_L) \\ &= R_L + (jX_L - jX_L) \\ &= R_L \end{aligned}$$

Great, I can derive the power for this in the same way:

$$\begin{aligned} P_L^{abs} &= \frac{1}{2} I_{RMS} I_{RMS}^* R && \text{(Definition of Power for a Sinusoidal Source)} \\ &= \frac{V_g V_g^*}{2} \left(\frac{R_L}{R_{total}^2} \right) && \text{(Substitute I and R)} \\ &= \frac{V_g V_g^*}{2} \left(\frac{R_L}{|R_L|^2} \right) && (Z_{total} = R_L) \\ &= \frac{V_g V_g^*}{2R_L} \end{aligned}$$

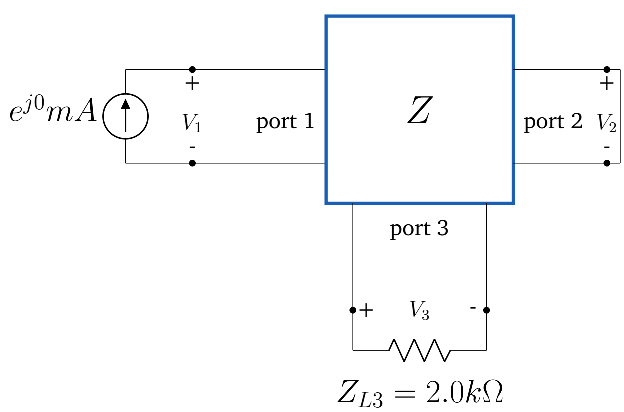
Again, what we got in class. Power delivered is then just:

$$\boxed{P_L^{abs} = \frac{(4+8i)(4-8i)}{2 \cdot 40} = 1\ \frac{\text{J}}{\text{s}}}$$

Problem 3

$$\mathcal{Z} = \begin{bmatrix} 1 & j & 1 \\ j & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \text{k}\Omega$$

I will define I pointed into Z. Determine both the complex current vector I, and the complex voltage vector V.



Solution:
We will have a system of 6 equations:

$$V_1 = I_1 + jI_2 + I_3 \tag{1}$$
$$V_2 = jI_1 + 2I_2 + 2I_3 \tag{2}$$
$$V_3 = I_1 + 2I_2 + 3I_3 \tag{3}$$
$$I_1 = -e^{j0} \tag{4}$$
$$V_2 = 0 \tag{5}$$
$$V_3 = -2I_3 \tag{6}$$

Equations 1-3 are given by the impedance matrix. Equations 4-6 are boundary conditions describing the circuit attached to the terminals. The algebra is as follows:

$$\begin{cases} V_1 = -e^{j0} + jI_2 + I_3 \\ 0 = -je^{j0} + 2I_2 + 2I_3 \\ -2I_3 = -e^{j0} + 2I_2 + 3I_3 \end{cases} \qquad \text{(Substite 4-6 into 1-3)}$$

$$\begin{cases} V_1 = -1 + jI_2 + I_3 \\ j = 2I_2 + 2I_3 \\ 1 - 5I_3 = 2I_2 \end{cases} \qquad \text{(Simplify)}$$

$$\begin{cases} V_1 = -1 + jI_2 + I_3 \\ \frac{1}{3} - \frac{j}{3} = I_3 \\ 1 - 5I_3 = 2I_2 \end{cases} \qquad \text{(Simplify eqn. 2)}$$

$$\begin{cases} V_1 = -1 + jI_2 + I_3 \\ \frac{1}{3} - \frac{j}{3} = I_3 \\ -\frac{1}{3} + \frac{5}{6}j = I_2 \end{cases} \qquad \text{(Substite eqn. 2 into 3)}$$

$$\begin{cases} -\frac{3}{2} - \frac{2}{3}j = V_1 \\ \frac{1}{3} - \frac{j}{3} = I_3 \\ -\frac{1}{3} + \frac{5}{6}j = I_2 \end{cases} \qquad \text{(Substite all into eqn. 1)}$$

To summarize:

$$V_1 = -\frac{3}{2} - \frac{2}{3}j \qquad V_2 = 0 \qquad V_3 = -\frac{2}{3} + \frac{2}{3}j$$
$$I_1 = -1 \qquad I_2 = -\frac{1}{3} + \frac{5}{6}j \qquad I_3 = \frac{1}{3} - \frac{j}{3}$$