PHSX 671: Homework #9

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Problem 1

Electromagnetic radiation (i.e., blackbody radiation) at temperature T_i fills a thermally insulated cavity of volume V. How much work is done by the radiation if the cavity expands adiabatically $(\Delta S = 0)$ to volume rV? Express your answer in terms of T_i, V, r, c, and the Stefan-Boltzmann constant σ only. Hint: use the equation for the entropy of this system to determine how the initial and final temperatures are related.

Solution:
The adiabatic condition implies that the entropy before and after remains the sameL

$$S_i = S_f$$

And the entropy initially is $S_i = \frac{4}{3}aVT^4$. The problem is now to find a way to express work due to the change in temperature and volume.

Problem 2

A Dyson shell is a hypothetical megastructure that was originally described by Freeman Dyson as a uniform solid shell of matter around a star meant to completely encompass the star and thus capture its entire energy output. Such a structure would also provide an immense surface which many envision being used for habitation. If engineers wanted an average temperature on the inner surface of the shell to be 300 K, what must the radius of the shell be? The temperature and radius of the sun are 5800 K and 7×10^8 m, respectively.

Problem 3

The molar heat capacity at constant volume of an ideal gas of bosons of mass m at temperature T before the condensation temperature T_c is given by

$$C_V = 2R\left(\frac{T}{T_c}\right)^{\frac{3}{2}}$$

(a) What is the internal energy per mole of this gas as a function of T for $T < T_c$? (b) What is the entropy per mole of this gas as a function of T for $T < T_c$?

Problem 4

What is the chemical potential of a three-dimensional photon gas?

Problem 5

Electromagnetic radiation at temperature T has a volume V. How much work is done by the radiation if it expands isothermally to a final volume rV?

Problem 6

What is the Fermi energy of a two-dimensional electron gas in terms of the charge density σ of the gas?

Problem 7

The universe has a radius of about 14 billion light-years and contains 2.7 K background radiation left over from the Big Bang. Estimate the temperature of the universe when it had a volume of $1\ \mathrm{m}^3$. You may assume that all the energy of the present background radiation was in the universe when it had the smaller volume and ignore any coupling to matter.