EECS622: Homework #14

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Problem 1

The output of a lossless microwave filter is terminated in a matched load.

The input impedance of the filter can be expressed as a reflection coefficient :

$$\Gamma_{\text{in}}(\omega) = \frac{2 + j2\omega}{j\omega^2 - 3}$$

Say a matched source with and available power of 1 mW is connected to the filter input. This source generates a linear eigen function with frequency $\omega = 1$.

Determine the power of the wave absorbed by the matched load at the filter output.

Solution:

For this, the goal is to model the transmitted power through a filter. We should expect that for any filter power is nearly delivered in maximum or reflected from the load.

$$\Gamma_{\text{in}}(\omega)\Big|_{\omega=1} = -0.4 - 0.8j$$

We know from the lecture "microwave filter design" that a lossless filter can be described in terms of its power reflection coefficient:

$$|\Gamma_{\text{in}}|^2 = 1 - T(\omega)$$
$$(-0.4 - 0.8j)(-0.4 + 0.8j) = 1 - \frac{P_{\text{L}}^{\text{abs}}}{P_{\text{g}}^{\text{avl}}}$$
$$\frac{P_{\text{L}}^{\text{abs}}}{P_{\text{g}}^{\text{avl}}} = 0.2 \approx 7 \text{ dB}$$

Since the whole system is matched and made of lossless components, it can be taken that all power produced by the source reaches the filter, so this problem is solved:

$$\boxed{P_{\text{L}}^{\text{abs}} = 0.2 \cdot P_{\text{g}}^{\text{avl}} = 0.2 \text{ mW} \approx 7 \text{ dBm}}$$

Problem 2

The complex transfer function for a certain microwave filter has the form:

$$H(\omega) = \frac{10^7}{10^7 + \omega^2} e^{-j[\omega(0.002 + A\omega) + B]}$$

where A and B are some unknown constants.

But, it is know that the phase delay of this filter at frequency $\omega = 100$ is 0.004 seconds.

Determine precisely (i.e., without any unknowns!) the phase delay of the filter at $\omega = 200$.

Solution:

It seems we have two boundary conditions, A and B but one constraint. It seems most productive to me to start by expressing the delay τ:

$$\tau(\omega) = -\frac{\partial \arg[H(\omega)]}{\partial \omega} = \frac{\partial}{\partial \omega}\big[\omega(0.002 + A\omega) + B\big] = 0.002 + 2A\omega$$

It is fortunate that this is independent of B, we can determine A from the boundary condition given:

$$0.004 = [0.002 + 2A(100)] \implies A = 10 \times 10^{-6}$$

This makes computing $\tau(\omega = 200)$ trivial:

$$\tau(\omega = 200) = 0.002 \text{ s} + 20 \times 10^{-6} \frac{\text{s}}{\text{rad}} (200 \text{ rad}) = \boxed{6 \text{ ms}}$$