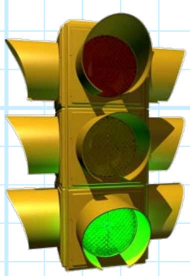
Microwave Filter Design

Recall that a **lossless** filter can be described in terms of **either** its power transmission coefficient $T(\omega)$ **or** its power reflection coefficient $|\Gamma_{in}(\omega)|^2$, as the two values are completely **dependent**:

$$|\Gamma_{in}(\omega)|^2 = 1 - T(\omega)$$

Ideally, these functions would be quite **simple**:

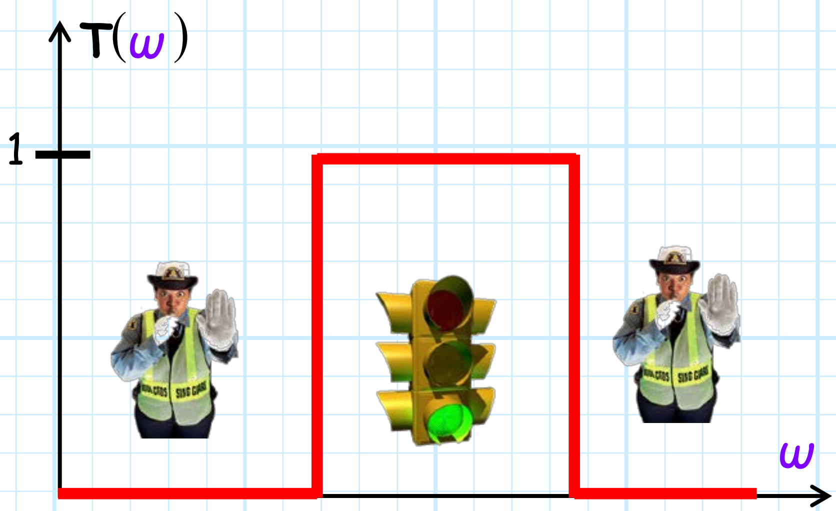


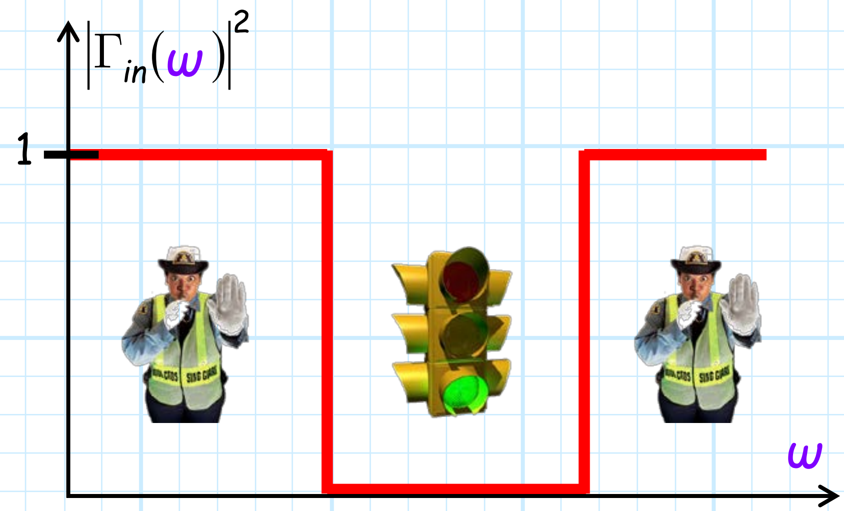
1. $T(\omega) = 1$ and $|\Gamma_{in}(\omega)|^2 = 0$ for **all** frequencies within the **pass**-band.

2. $T(\omega) = 0$ and $|\Gamma_{in}(\omega)|^2 = 1$ for **all** frequencies within the **stop**-band.

The filter of our wildest dreams

For example, the **ideal band-pass filter** would be:





Add to this a **linear phase** response, and you have the **perfect** microwave filter!

There's just one **small** problem with this **perfect** filter:

→ It's **impossible** to build!

Math: Is there anything it can't do?

Now, if we consider only possible (i.e., **realizable**) filters, we must limit ourselves to filter functions that can be expressed as **finite polynomials** of the form:

$$\mathbf{T}(\omega^2) = \frac{a_0 + a_1\,\omega^2 + a_2\,\omega^4 + \cdots}{b_0 + b_1\,\omega^2 + b_2\,\omega^4 + \cdots + b_N\,\omega^{2N}}$$

The **order** N of the (denominator) polynomial is likewise the **order** of the **filter**.

As the filter order becomes **higher**, the filter (**theoretically**) can become more and more **ideal**!

As many polynomials as there are mathematicians

There are **many**, **many** different **types** of polynomials that result in good filter responses.

→Each type has its own set of **characteristics**.

The **type** of **polynomial** likewise describes the **type** of microwave **filter**.

Now let's consider just **three** (there's **lots** more!) of the most popular **types**:

1. **Elliptical**
2. **Chebychev**
3. **Butterworth**

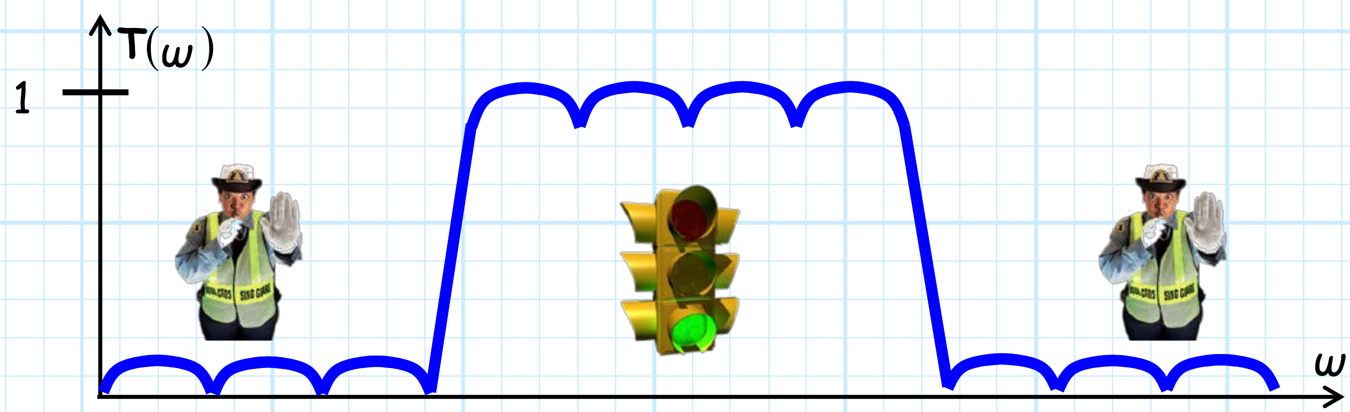
The elliptical filter

Elliptical filters have three primary characteristics:

a) They exhibit very **steep** "**roll-off**", meaning that the transition from pass-band to stop-band is very rapid.

b) They exhibit **ripple** in the **pass**-band, meaning that the value of **T** will vary slightly within the pass-band.

c) They exhibit ripple in the **stop**-band, meaning that the value of **T** will vary slightly within the stop-band.



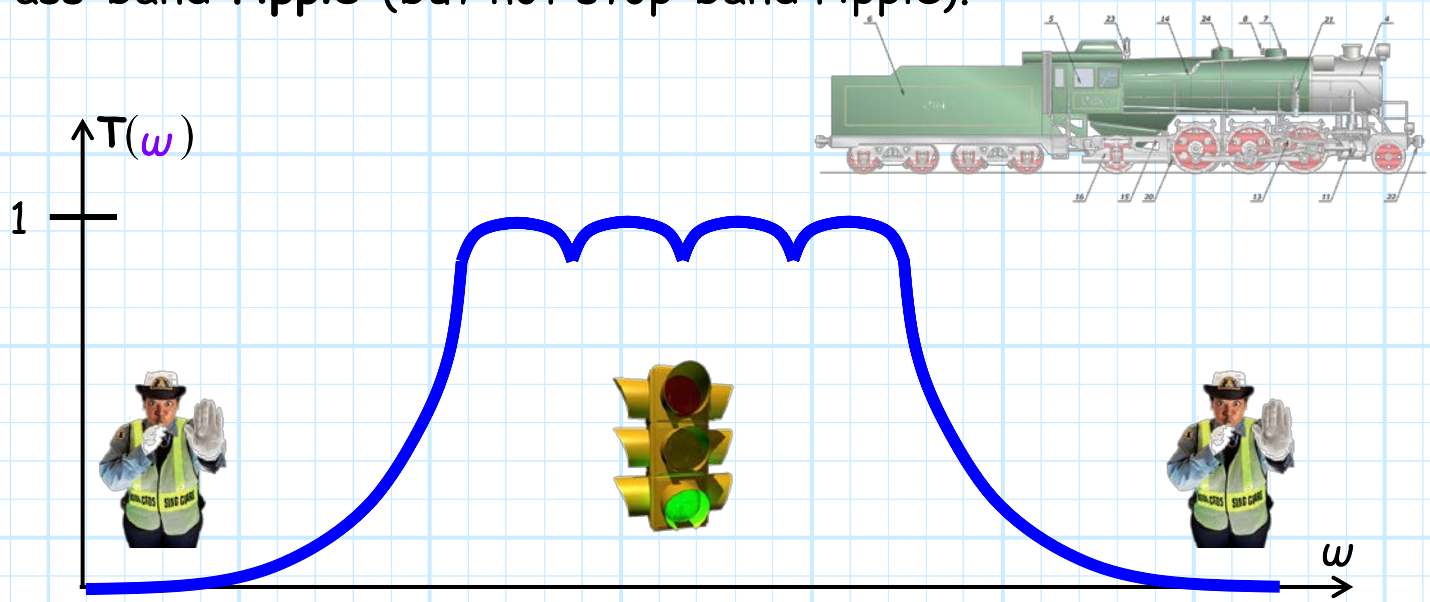
We find that we can make the roll-off **steeper** by accepting more **ripple**.

The Chebychev filter

Chebychev filters are also known as **equal-ripple** filters, and have two primary characteristics

a) **Steep** roll-off (but not as steep as Elliptical).

b) Pass-band **ripple** (but not stop-band ripple).



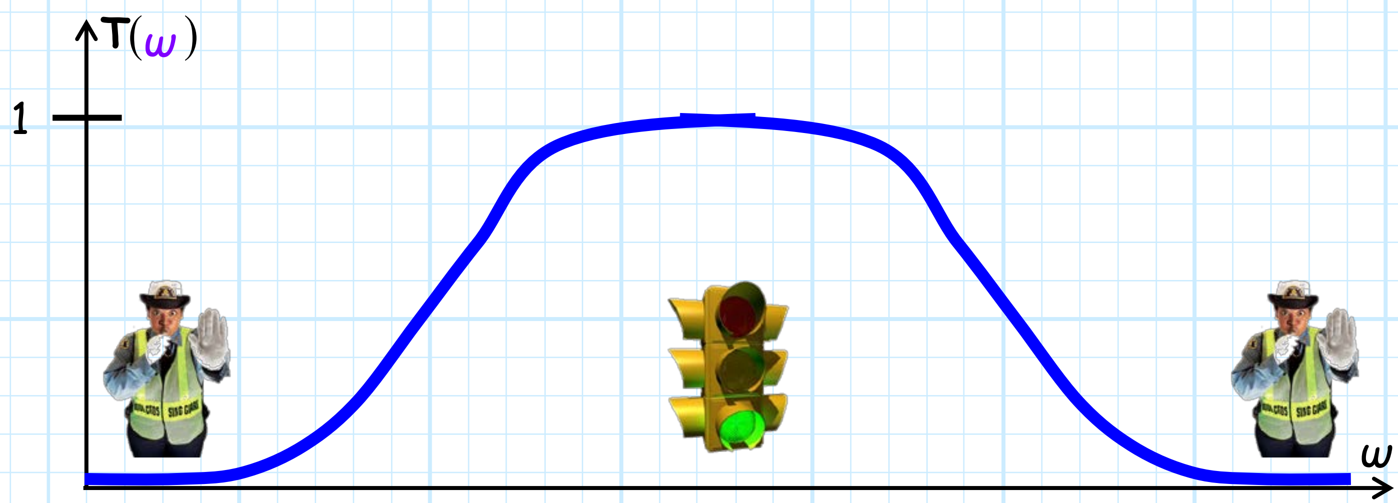
We likewise find that the roll-off can be made steeper by ***accepting*** more ripple.

The Butterworth filter

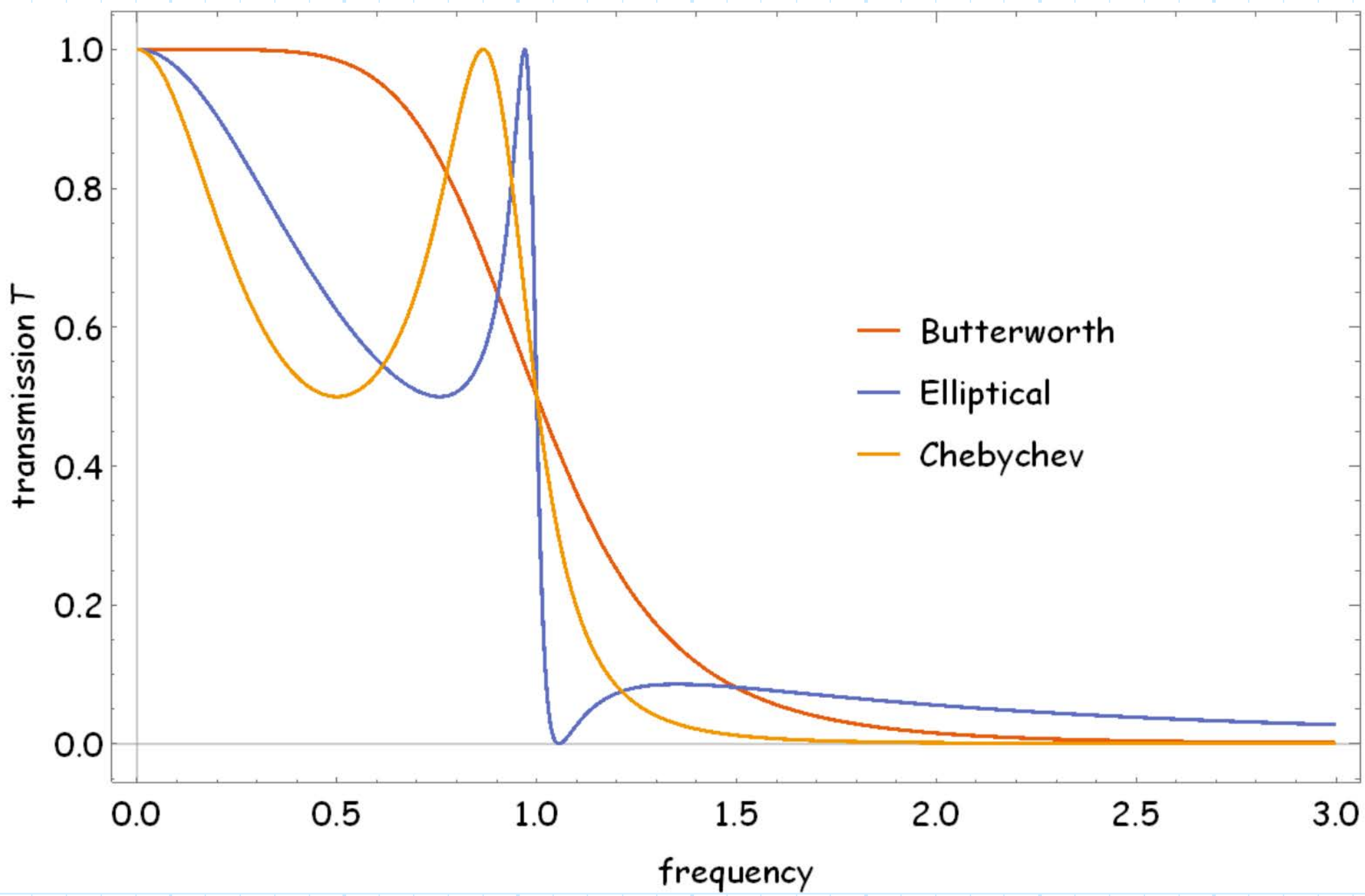
Also known as **Gaussian**, or as **maximally flat** filters, they have two primary characteristics:

a) **Gradual** roll-off .

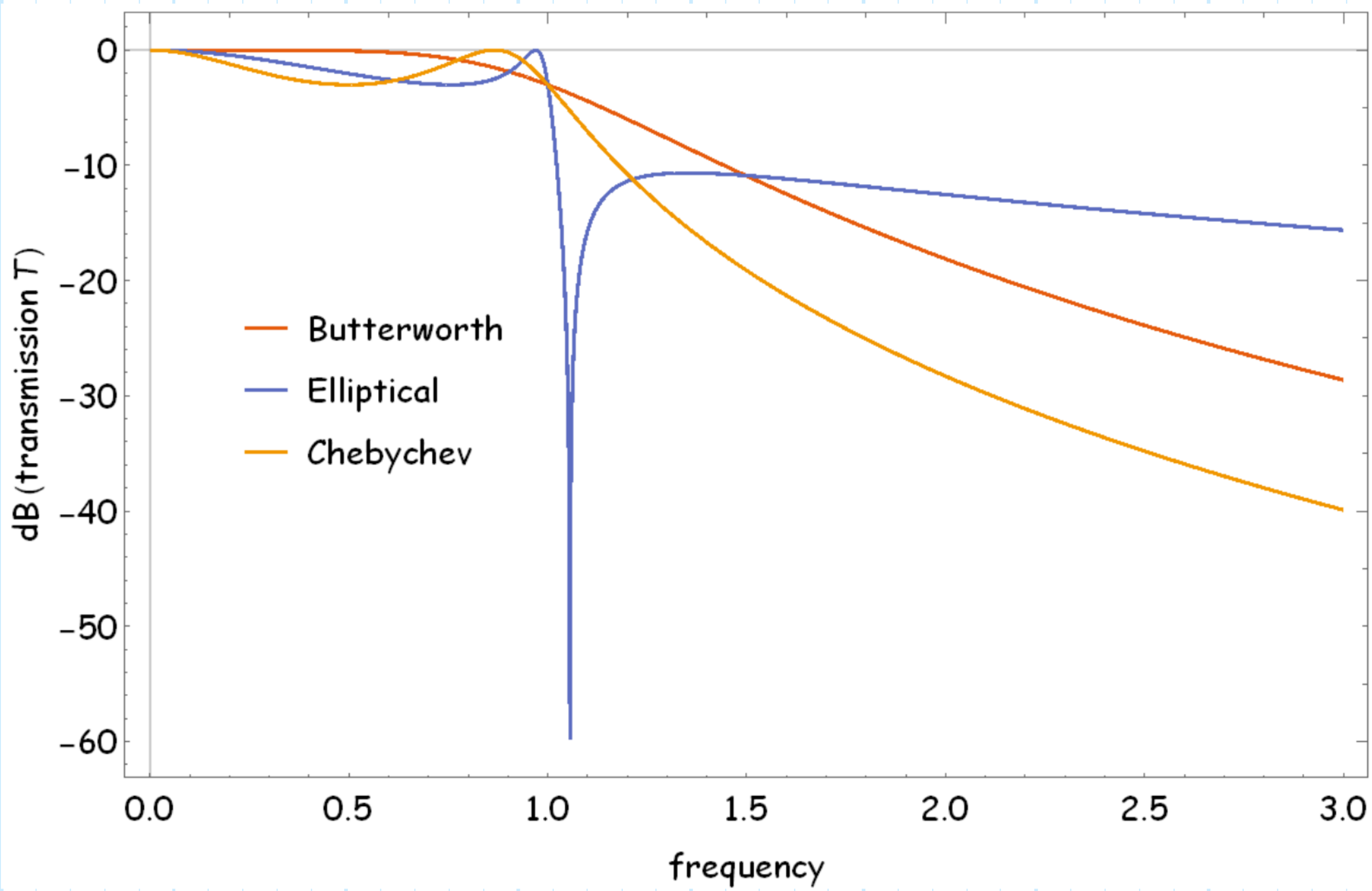
b) **No ripple**—not anywhere.



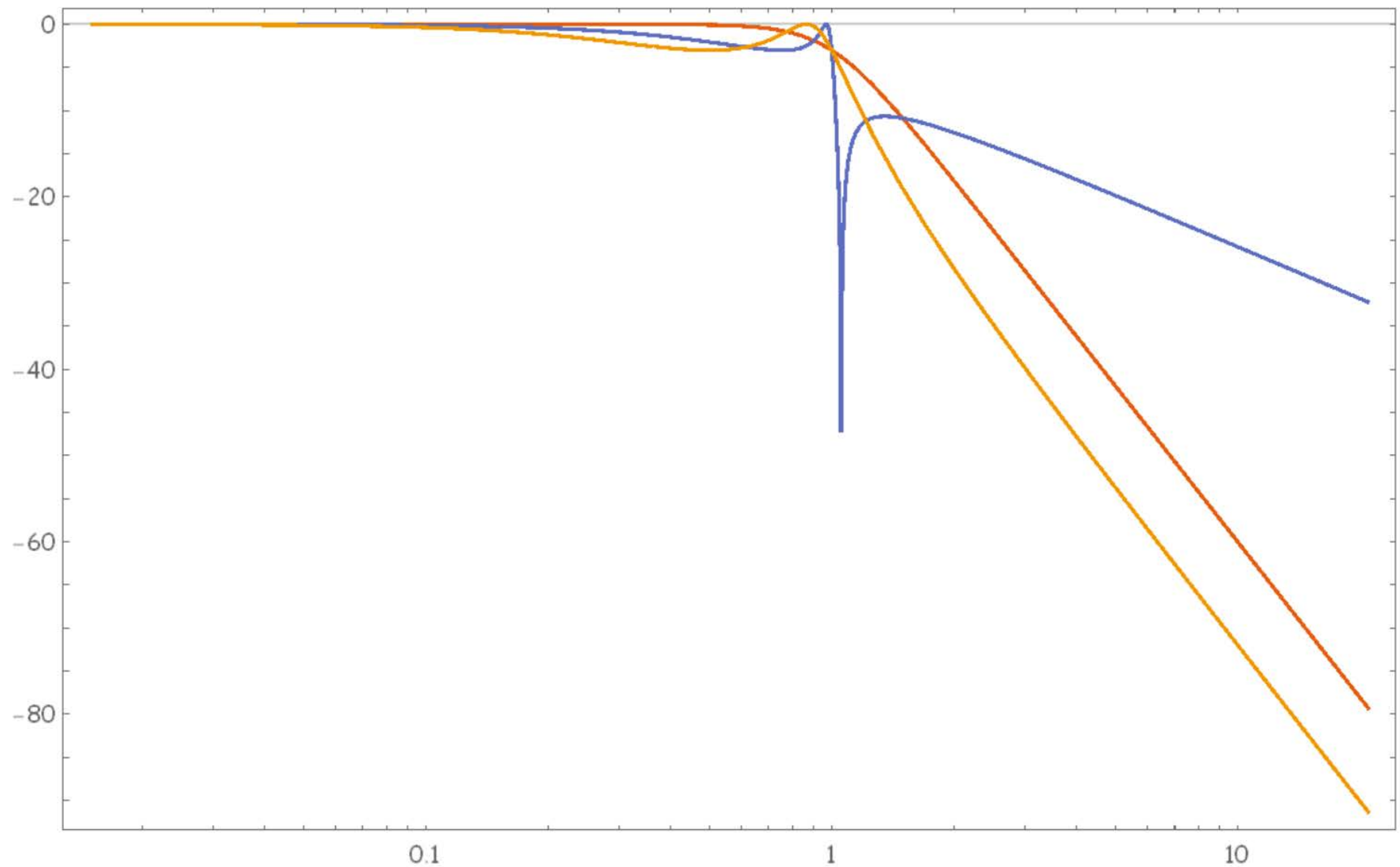
A 3rd-order low-pass example



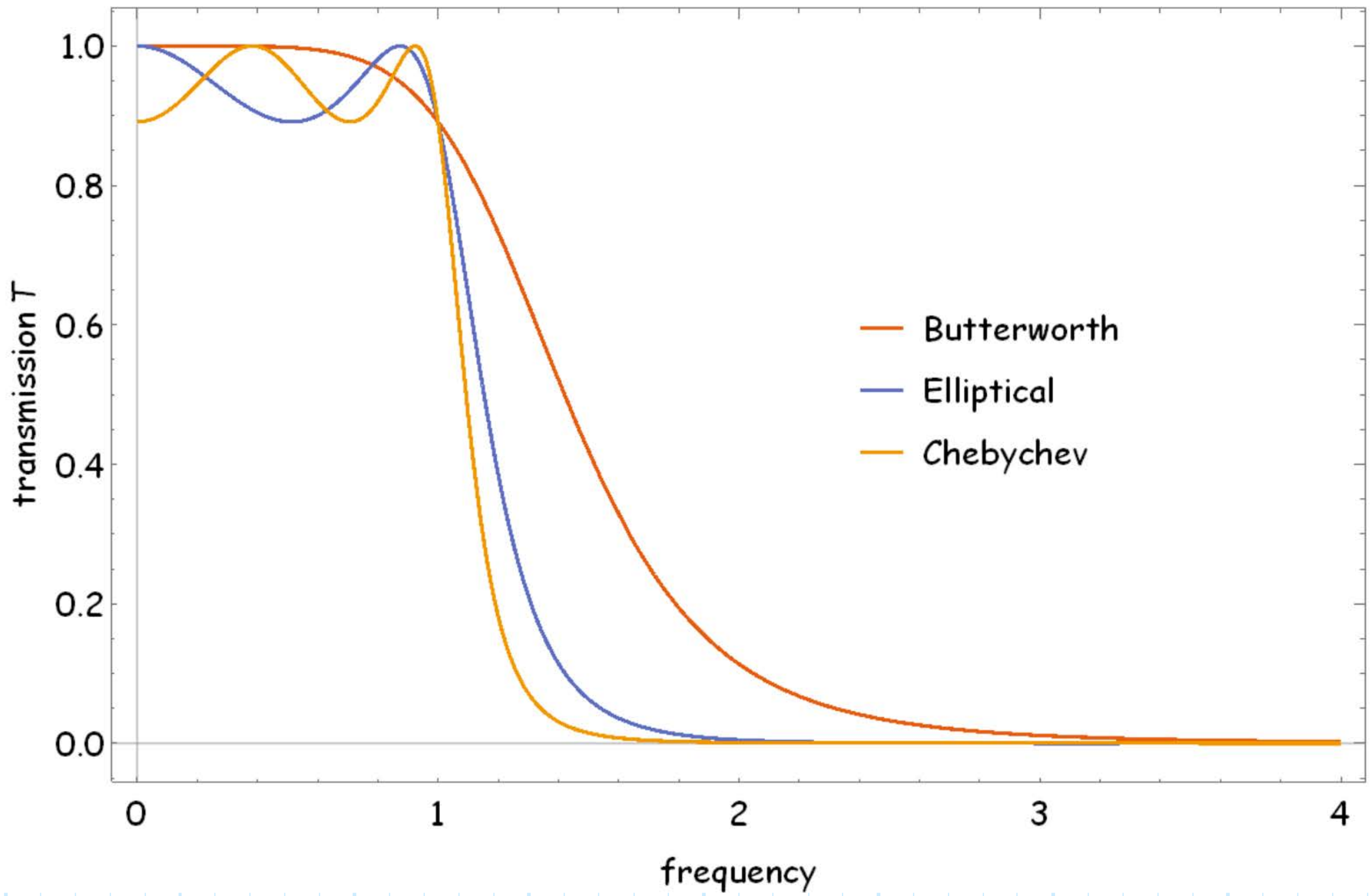
The same example "in dB"



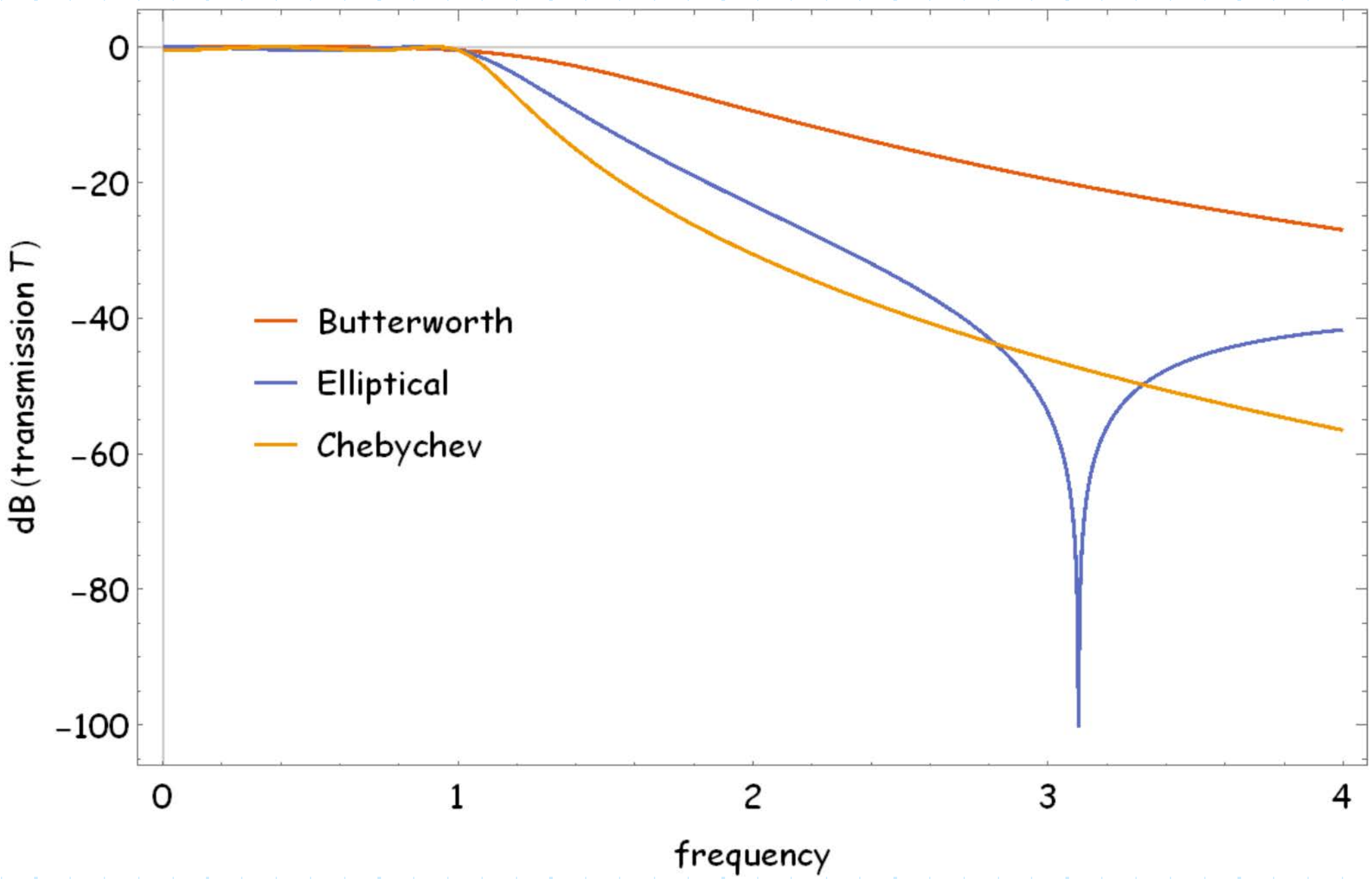
The same example in "log-log"—A Bode Plot!



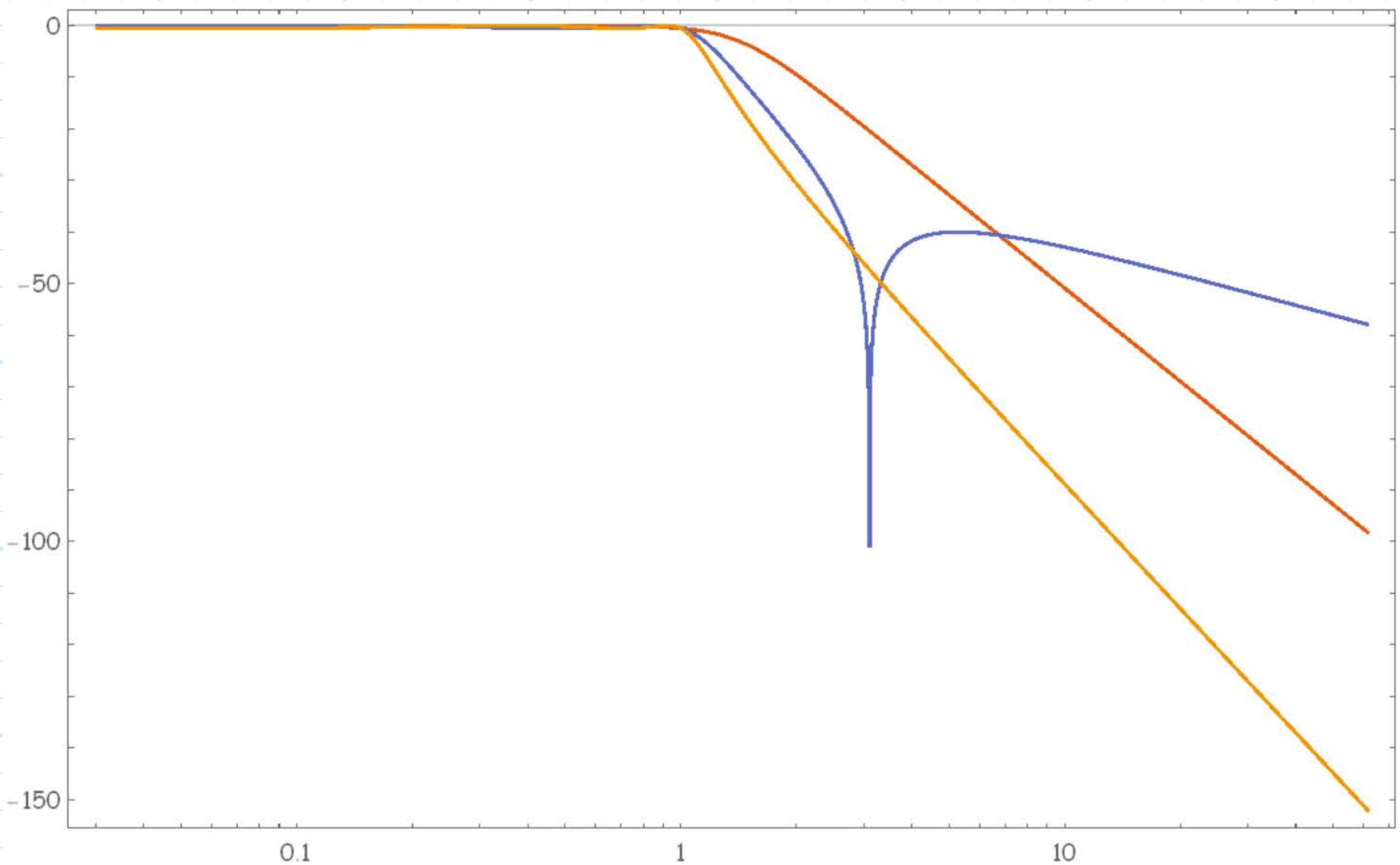
The same example, but with *less* ripple



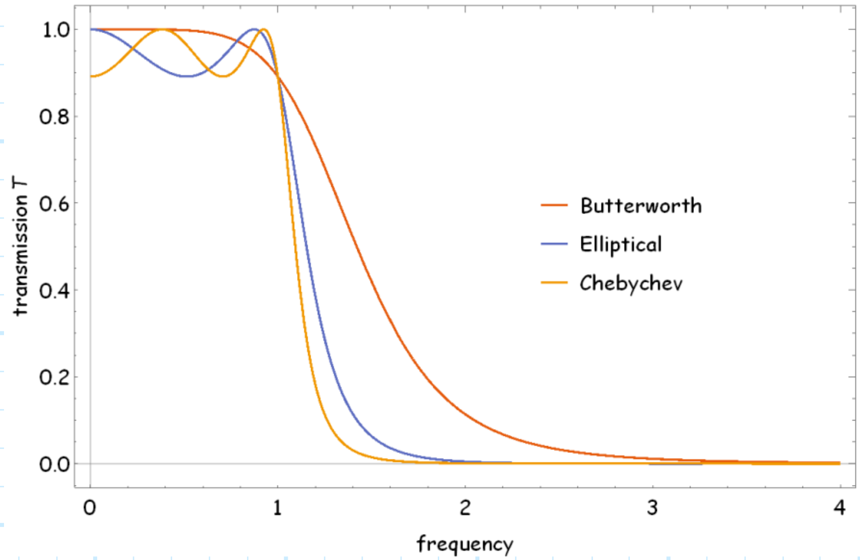
Less ripple means "slower roll-off"



The Bode Plot



Don't forget the phase!



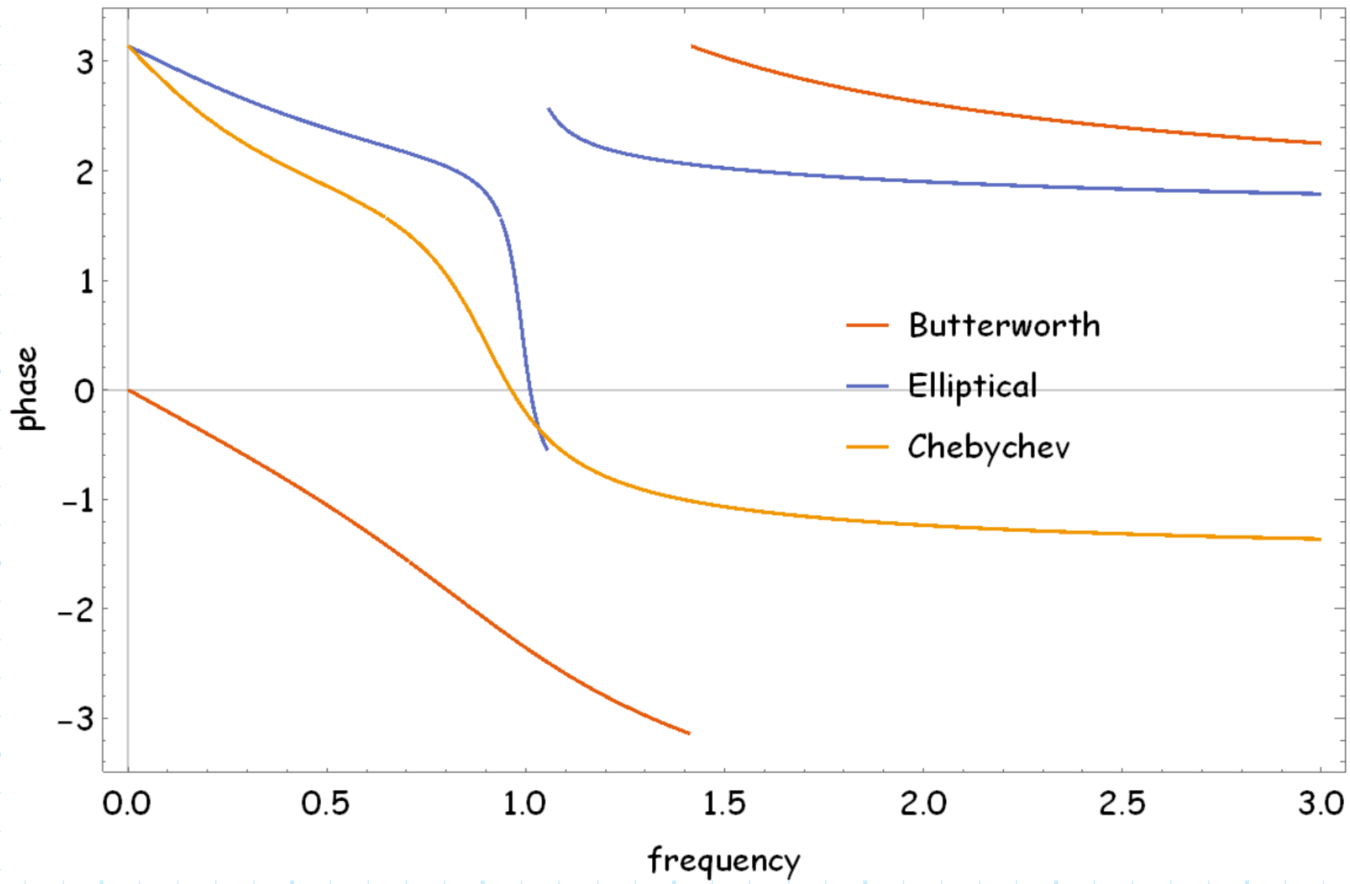
Q: *So, we always chose **elliptical or Chebychev** filters?*

*Since they have the **steepest roll-off**, they are **closest** to ideal—**right**?*

A: Oops! I forgot to talk about the **phase response** $\arg[H(\omega)]$ of these filters.

Let's examine $\arg[H(\omega)]$ for each filter type **before** we pass judgment.

Butterworth is looking better!

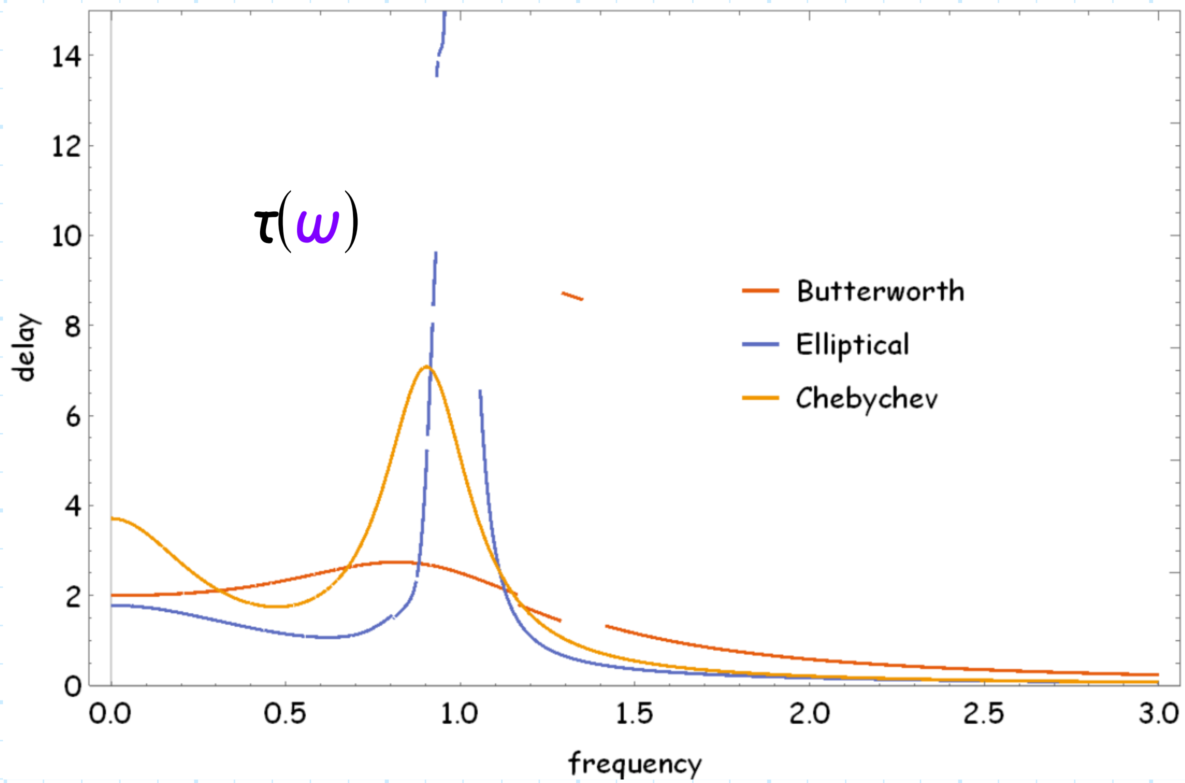


Butterworth $\arg[H(\omega)]$ → **Close** to linear phase.

Chebychev $\arg[H(\omega)]$ → **Not** very linear.

Elliptical $\arg[H(\omega)]$ → A big, non-linear **mess**!

Delay as a function of frequency



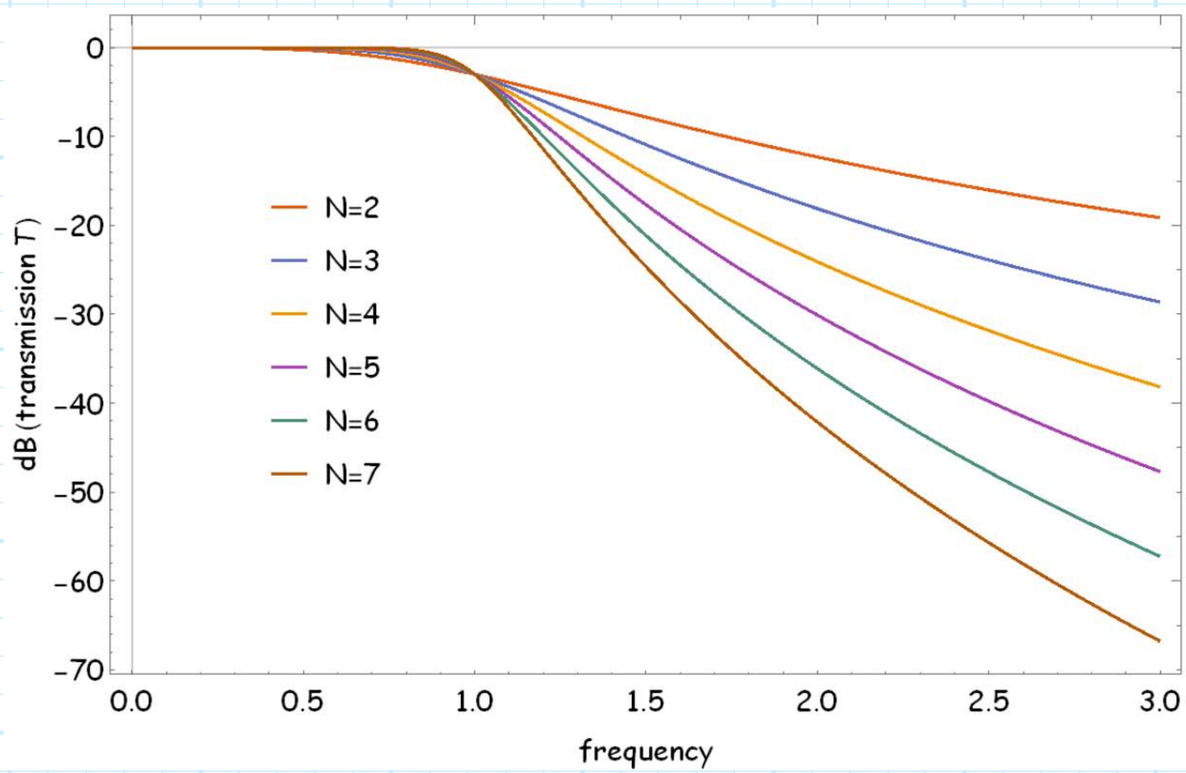
Thus, it is apparent that as a filter "roll-off" **improves**, the phase response gets **worse** (watch out for **dispersion!**).

→ There is **no** such thing as the "**best**" filter type!

Roll-off improves with increasing order…

Q: *So, Butterworth has a **fairly linear phase** response, but **slow** roll-off. Could we just use a **high-order** Butterworth filter to make the roll-off better?*

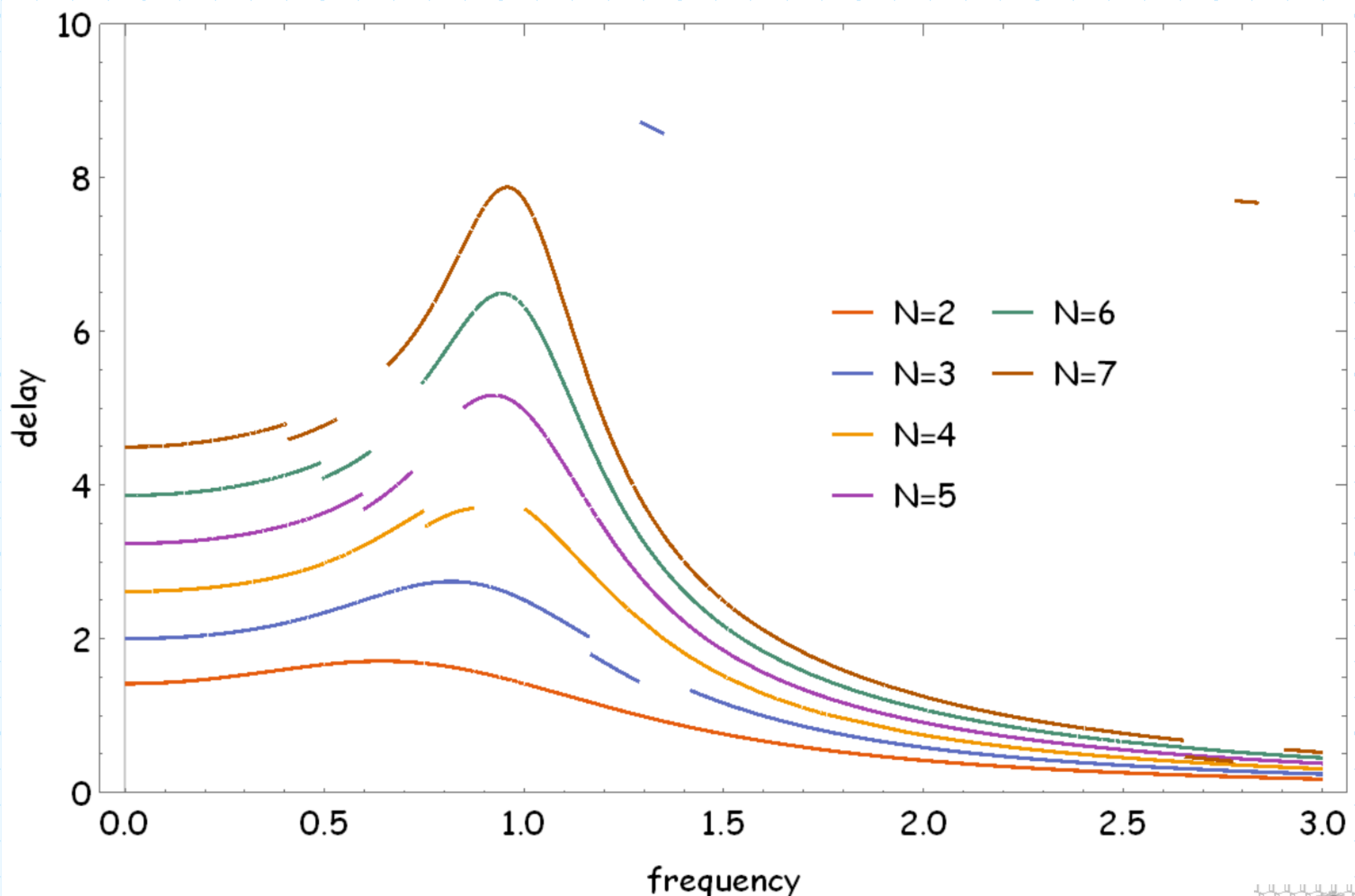
A: Certainly, a **higher filter order** does result in **steeper** roll-off:



But, **be careful…**

...but phase distortion gets worse!

...a **larger** filter order will likewise make the **phase less linear**!



→ There is **no way** to avoid the **trade-off** between linear **distortion** and **roll-off**.

A practical limit on filter order

In **addition** to making the **phase response** $\arg[H(\omega)]$ worse (i.e., more non-linear), increasing filter order will likewise:

1. increase filter **cost**, **weight**, and **size**.

2. increase filter **insertion loss** (this is bad).

3. make filter performance more **sensitive** to temperature, aging, etc.

From a **practical** viewpoint, the **order** of a filter should typically be kept to $N < 10$.

Specify type, and specify order

Q: *So exactly what **are** these filter polynomials $T(\omega)$? How do we **determine** them?*

A: Fortunately, **radio engineers** do not need to **determine** specific filter polynomials in order to **specify** (to filter manufacturers) what they want built.

Instead, radio engineers simply can specify the **type** and **order** of a filter, **saying things like**:

*"I need a **3rd-order Chebychev** filter!"*

or

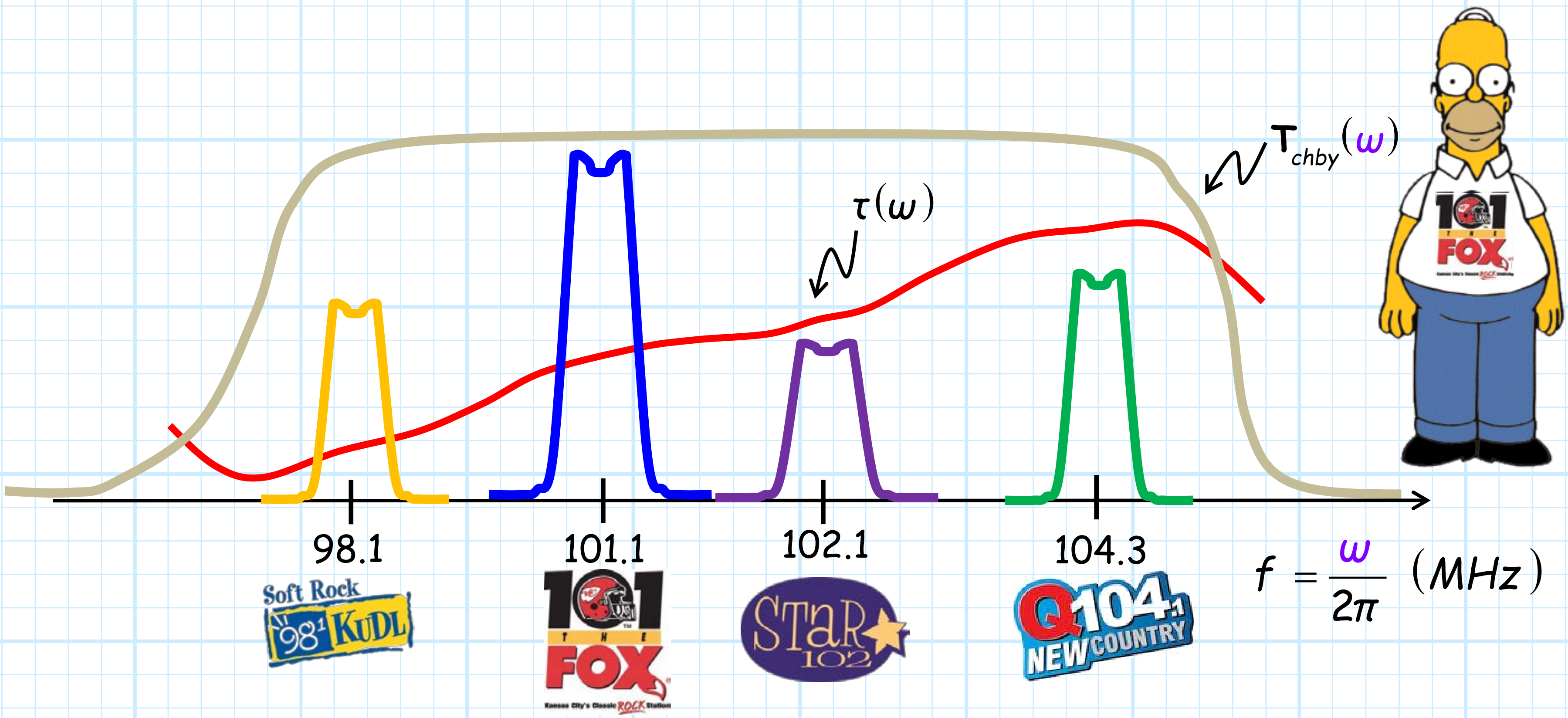
*"Get me a **5th-order Butterworth** filter!"*

or

*" I wish I'd paid **more** attention in EECS 622!"*

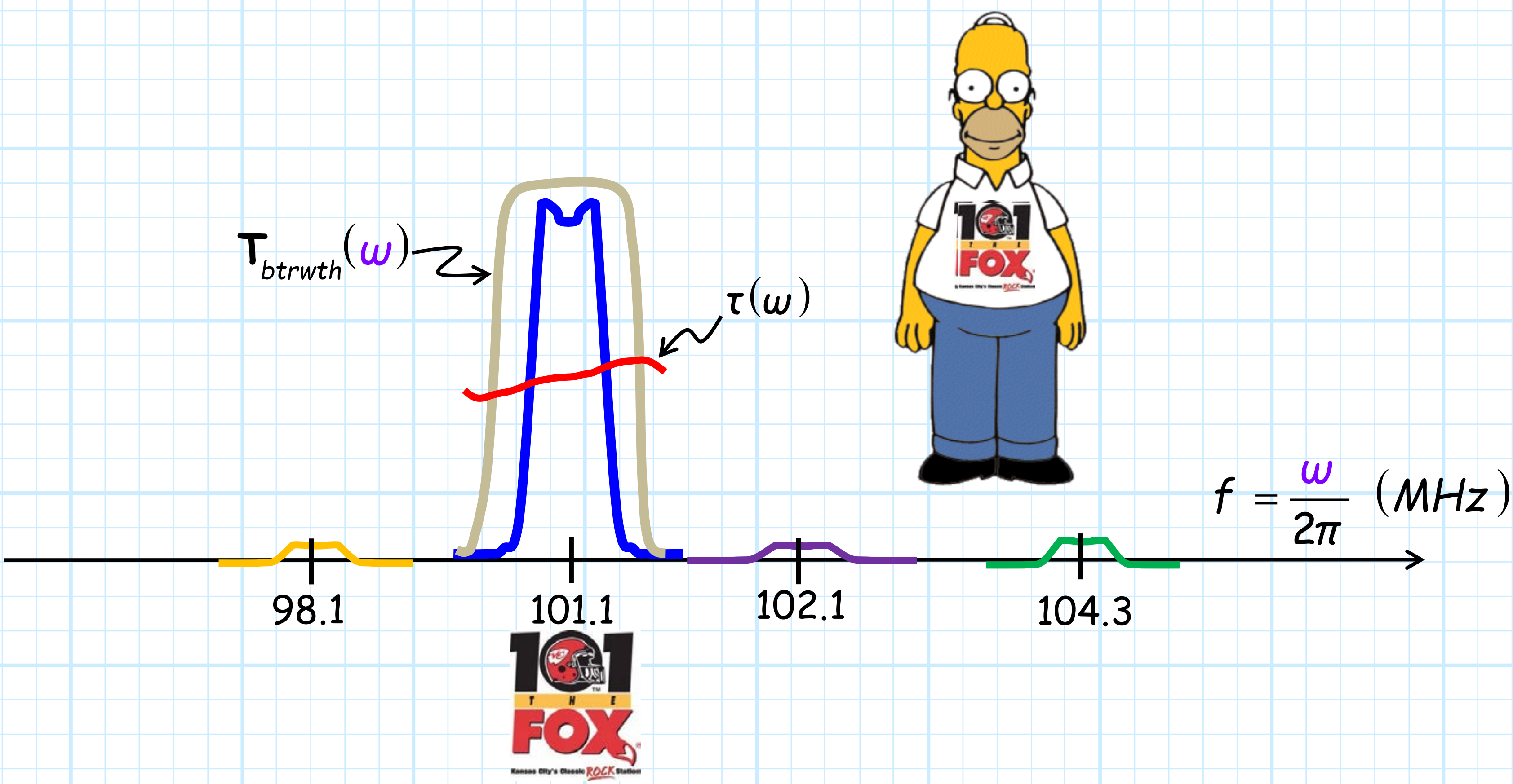
Preselectors are often Chebychev

Generally speaking, engineers are more likely to specify **Chebychev** for **wideband filters**, where the bandwidth of **filter** is much **larger** than the bandwidth of any **individual radio signal**:



IF filters are typically Butterworth

Generally speaking, engineers are more likely to specify Butterworth for **narrowband filters**, where the bandwidth of **filter** is just **slightly wider** than the bandwidth of a **single radio signal**:



As you might expect, there are many filter specifications

The most **important** filter specifications are thus:

1. Filter **bandwidth** and **center frequency**.

2. Filter **type** and **order**.

→ However, there are **many other** important filter specifications!