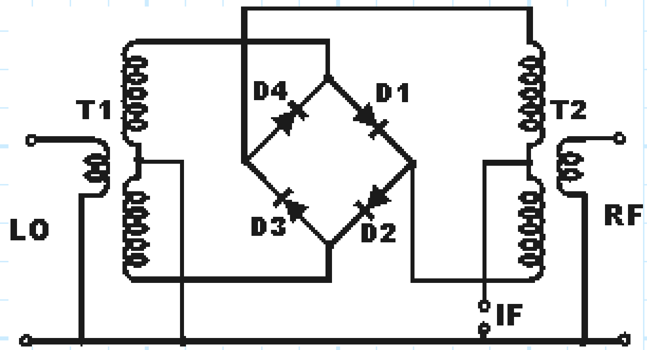
Spurious Mixer Signals

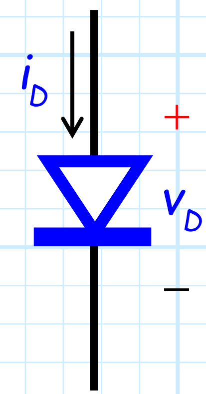
Switching mixers are generally made with semiconductor devices such as diodes and transistors.



www.radio-electronics.com/info/rf-technology-design/mixers/double-balanced-mixer-tutorial.php

These semi-conductor devices are **non-linear** devices, as their corresponding **device equations** are decidedly non-linear.

For example, as those of you who **aced EECS 312** know, the **junction diode** equation is:



$$i_D = I_s\left(e^{v_D/nV_T} - 1\right)$$

For a diode in a mixer

Now, if the voltage at the RF and LO **mixer ports** are of the form:

$$v_{RF}(t) = A_{RF} \cos\left[\omega_{RF}\, t + \varphi_{RF}\right]$$

$$v_{LO}(t) = A_{LO} \cos\left[\omega_{LO}\, t + \varphi_{LO}\right]$$

we can hypothesize that the resulting mixer diode voltages **include similar** components:

$$v_D(t) = \beta_{RF} \cos\left[\omega_{RF}\, t + \theta_{RF}\right] + \beta_{LO} \cos\left[\omega_{LO}\, t + \theta_{LO}\right] + \ldots$$

where $\beta_{RF}, \theta_{RF}, \beta_{LO}, \theta_{LO}$ are **effectively arbitrary** constants that depend on $v_{RF}(t)$, $v_{LO}(t)$, and the mixer **circuit design** and construction.

A Taylor series expansion

Now, say we expand the **non-linear** junction diode equation using a **Taylor series** as:

$$i_D = I_s\left(e^{v_D/nV_T} - 1\right) = c_1 v_D + c_2 v_D^2 + c_3 v_D^3 + \cdots$$

From the Taylor series expansion, we see that the diode will create **first-order products:**

$$c_1 v_D(t) = c_1 \beta_{RF} \cos[\omega_{RF} t + \theta_{RF}] + c_1 \beta_{LO} \cos[\omega_{LO} t + \theta_{LO}]$$

And **second-order products**:

$$\begin{aligned} c_2 v_D^2(t) &= c_2\left(\beta_{RF} \cos[\omega_{RF} t + \theta_{RF}] + \beta_{LO} \cos[\omega_{LO} t + \theta_{LO}]\right)^2 \\ &= c_2 \beta_{RF}^2 \cos^2[\omega_{RF} t + \theta_{RF}] + c_2 \beta_{LO}^2 \cos^2[\omega_{LO} t + \theta_{LO}] \\ &\quad + c_2\, 2\, \beta_{RF} \beta_{LO} \cos[\omega_{RF} t + \theta_{RF}] \cos[\omega_{LO} t + \theta_{LO}] \end{aligned}$$

Now for some trigonometry!

And **third-order** products:

$$
\begin{aligned}
c_3\, v_D^3(t) &= c_3 \left(\beta_{RF} \cos[\omega_{RF} t + \theta_{RF}] \;+\; \beta_{LO} \cos[\omega_{LO} t + \theta_{LO}]\right)^3 \\
&= c_3\, \beta_{RF}^3 \cos^3[\omega_{RF} t + \theta_{RF}] \\
&\quad + c_3\, 3\beta_{RF}^2 \beta_{LO} \cos^2[\omega_{RF} t + \theta_{RF}] \cos[\omega_{LO} t + \theta_{LO}] \\
&\quad + c_3\, 3\beta_{RF} \beta_{LO}^2 \cos[\omega_{RF} t + \theta_{RF}] \cos^2[\omega_{LO} t + \theta_{LO}] \\
&\quad + c_3\, \beta_{LO}^3 \cos^3[\omega_{LO} t + \theta_{LO}]
\end{aligned}
$$

After applying a few **trig identities**, we see that the just these **first three terms** of this Taylor Series approximation are…

Just look at this mess!

$$
\begin{aligned}
i_D(t) \cong\ & c_3\ \tfrac{1}{4}\beta_{RF}^3 \cos\left[3\,\omega_{RF} t + 3\,\theta_{RF}\right] + c_3\ \tfrac{1}{4}\beta_{LO}^3 \cos\left[3\,\omega_{LO} t + 3\,\theta_{LO}\right] \\
& + c_3\ \tfrac{3}{4}\beta_{RF}^2\,\beta_{LO} \cos\left[\left(2\omega_{RF} - \omega_{LO}\right)t + \left(2\theta_{RF} - \theta_{LO}\right)\right] \\
& + c_3\ \tfrac{3}{4}\beta_{RF}^2\,\beta_{LO} \cos\left[\left(2\omega_{RF} + \omega_{LO}\right)t + \left(2\theta_{RF} + \theta_{LO}\right)\right] \\
& + c_3\ \tfrac{3}{4}\beta_{RF}\,\beta_{LO}^2 \cos\left[\left(\omega_{RF} - 2\omega_{LO}\right)t + \left(\theta_{RF} - 2\theta_{LO}\right)\right] \\
& + c_3\ \tfrac{3}{4}\beta_{RF}\,\beta_{LO}^2 \cos\left[\left(\omega_{RF} + 2\omega_{LO}\right)t + \left(\theta_{RF} + 2\theta_{LO}\right)\right] \\
& + c_2\ \tfrac{1}{2}\beta_{RF}^2 \cos\left[2\,\omega_{RF} t + 2\,\theta_{RF}\right] + c_2\ \tfrac{1}{2}\beta_{LO}^2 \cos\left[2\,\omega_{LO} t + 2\,\theta_{LO}\right] \\
& + c_2\ \beta_{RF}\,\beta_{LO} \cos\left[\left(\omega_{RF} - \omega_{LO}\right)t + \left(\theta_{RF} - \theta_{LO}\right)\right] \\
& + c_2\ \beta_{RF}\,\beta_{LO} \cos\left[\left(\omega_{RF} + \omega_{LO}\right)t + \left(\theta_{RF} + \theta_{LO}\right)\right] \\
& + \left[c_1\,\beta_{RF} + c_3\ \tfrac{3}{4}\beta_{RF}\left(\beta_{RF}^2 + 2\beta_{LO}^2\right)\right]\cos\left[\omega_{RF} t + \theta_{RF}\right] \\
& + \left[c_1\,\beta_{LO} + c_3\ \tfrac{3}{4}\beta_{LO}\left(\beta_{LO}^2 + 2\beta_{RF}^2\right)\right]\cos\left[\omega_{LO} t + \theta_{LO}\right] \\
& + c_2\ \tfrac{1}{2}\left(\beta_{LO}^2 + \beta_{RF}^2\right) + \cdots
\end{aligned}
$$

Spurs

This non-linear device create **spurious** sinusoids at **many different frequencies**!

1st order spurious signal frequencies:

$$\omega_{RF},\ \omega_{LO}$$

2nd order spurious signal frequencies:

$$2\omega_{RF},\ 2\omega_{LO},\ \left|\omega_{RF}-\omega_{LO}\right|,\ \left(\omega_{RF}+\omega_{LO}\right)$$

3rd order spurious signal frequencies:

$$\left|2\omega_{RF}-\omega_{LO}\right|,\ \left|2\omega_{LO}-\omega_{RF}\right|,\ 3\omega_{RF},$$

$$3\omega_{LO},\ \left(2\omega_{RF}+\omega_{LO}\right),\ \left(\omega_{RF}+2\omega_{LO}\right)$$

examples of higher order terms: $\left|4\omega_{RF}-2\omega_{LO}\right|,\ 5\omega_{RF},\ 7\omega_{LO}$

We've seen these frequencies before

Q: *Wait! Aren't the sinusoids of frequencies:*

$$|\omega_{RF} - \omega_{LO}| \qquad \text{and} \qquad (\omega_{RF} + \omega_{LO})$$

*the **same** as those produced by an **ideal switching mixer**?*

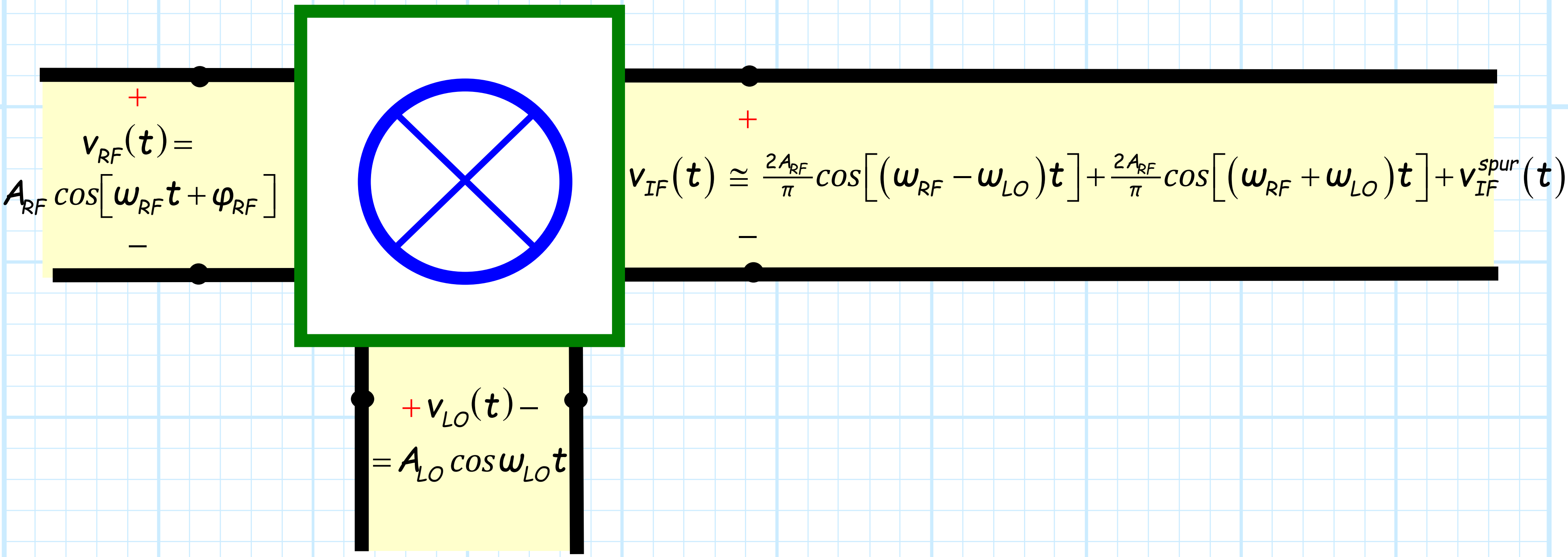
$$v_{IF}(t) \cong A_{RF}\frac{2}{\pi}\cos\left[(\omega_{RF}-\omega_{LO})t-\varphi_{RF}\right] + A_{RF}\frac{2}{\pi}\cos\left[(\omega_{RF}+\omega_{LO})t-\varphi_{RF}\right]$$

A: They are.

These "spurious" signals effectively **combine** with the two **ideal** switching mixer terms.

As such they are **not** included in the **spectrum of spurious sinusoids** created by a (non-ideal) mixer.

Spurs are at the IF port only!!!!!!



A more **accurate** expression for the IF voltage thus includes $v_{IF}^{spur}(t)$:

$$v_{IF}(t) \cong A_{RF}\frac{2}{\pi}\cos\left[(\omega_{RF}-\omega_{LO})t-\varphi_{RF}\right] + A_{RF}\frac{2}{\pi}\cos\left[(\omega_{RF}+\omega_{LO})t-\varphi_{RF}\right] + v_{IF}^{spur}(t)$$

where $v_{IF}^{spur}(t)$ represents the **spurious** mixer signals…

The IF port voltage

$$
\begin{aligned}
v_{IF}^{spur}(t) \;\cong\;& b_{3a}\cos\left[3\,\omega_{RF}t + \theta_{3a}\right] \\
&+ b_{3b}\cos\left[\left(2\omega_{RF} - \omega_{LO}\right)t + \theta_{3b}\right] \\
&+ b_{3c}\cos\left[\left(2\omega_{RF} + \omega_{LO}\right)t + \theta_{3c}\right] \\
&+ b_{3d}\cos\left[\left(\omega_{RF} - 2\omega_{LO}\right)t + \theta_{3d}\right] \\
&+ b_{3e}\cos\left[\left(\omega_{RF} + 2\omega_{LO}\right)t + \theta_{3e}\right] \\
&+ b_{3f}\cos\left[3\,\omega_{LO}t + \theta_{3f}\right] \\
&+ b_{2a}\cos\left[2\,\omega_{RF}t + \theta_{2a}\right] \\
&+ b_{2b}\cos\left[2\,\omega_{LO}t + \theta_{2b}\right] \\
&+ b_{1a}\cos\left[\omega_{RF}t + \theta_{1a}\right] \\
&+ b_{1b}\cos\left[\omega_{LO}t + \theta_{1b}\right]
\end{aligned}
$$

where **constants** b and θ depend on $v_{RF}(t)$, $v_{LO}(t)$, and the **mixer circuit**.

Spurs are generally very small, but still can be quite problematic

However, for **good** mixers, the magnitude of constants b for these **spurious terms** are relatively **small**, **i.e.**:

$$|b_m| \ll \left| A_{RF} \frac{2}{\pi} \right|$$

Thus—for **good** mixers—**most** of the signals created at the IF output will be of relatively **low** power, when compared to the **two ideal sinusoids** at frequencies $|\omega_{RF} - \omega_{LO}|$ and $\omega_{RF} + \omega_{LO}$.

But. Although these spurious IF signals are **small**, they can be **very**, **very problematic** in receiver design!!!!

The IF port spurious signal spectrum

We use **this notation** to denote the frequencies of 10 spurious sinusoids that appear at an **IF mixer port**:

$$\omega_{1a}^{spur} = \omega_{RF}$$
$$\omega_{1b}^{spur} = \omega_{LO}$$

$$\omega_{2a}^{spur} = 2\,\omega_{RF}$$
$$\omega_{2b}^{spur} = 2\,\omega_{LO}$$

$$\omega_{3a}^{spur} = 3\,\omega_{RF}$$
$$\omega_{3b}^{spur} = \left|2\omega_{RF} - \omega_{LO}\right|$$
$$\omega_{3c}^{spur} = 2\omega_{RF} + \omega_{LO}$$
$$\omega_{3d}^{spur} = \left|\omega_{RF} - 2\omega_{LO}\right|$$
$$\omega_{3e}^{spur} = \omega_{RF} + 2\omega_{LO}$$
$$\omega_{3f}^{spur} = 3\,\omega_{LO}$$

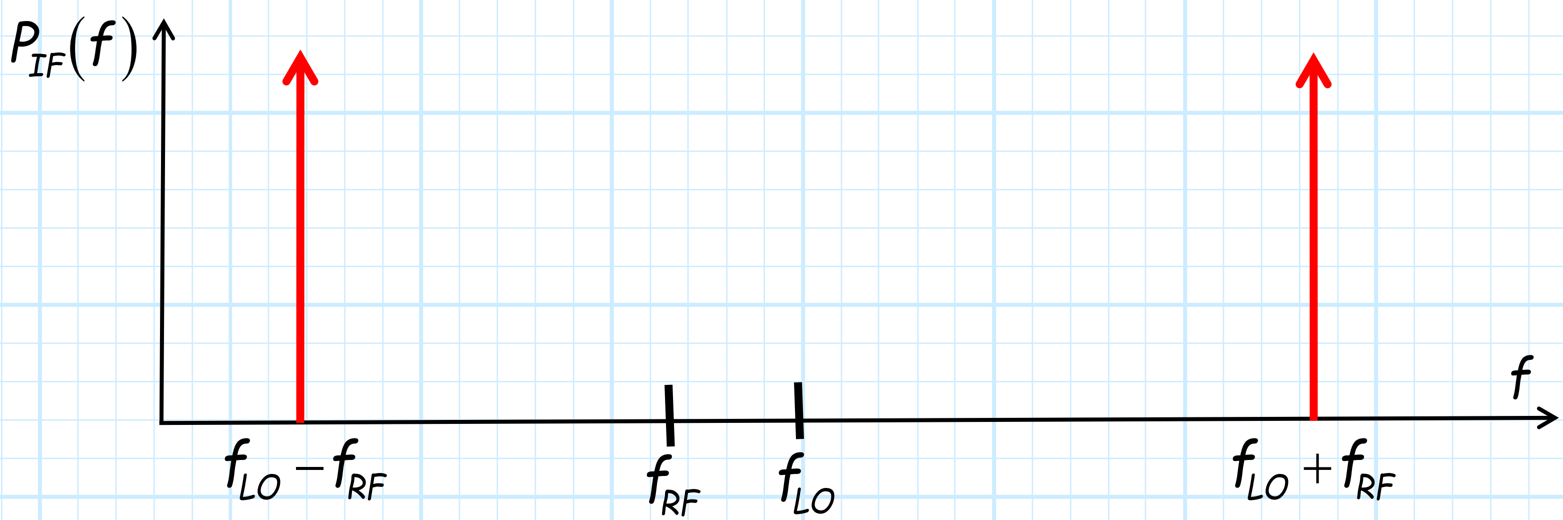
$$v_{IF}^{spur}(t) \cong \sum_{m=1}^{10} b_m \cos\left[\omega_m^{spur} t + \theta_m\right]$$

Note this includes 1st, 2nd, and 3rd order terms **only**!

Again: an IDEAL mixer

Again, the output **at the IF port** of an **IDEAL** switching mixer consists of these two significant terms:

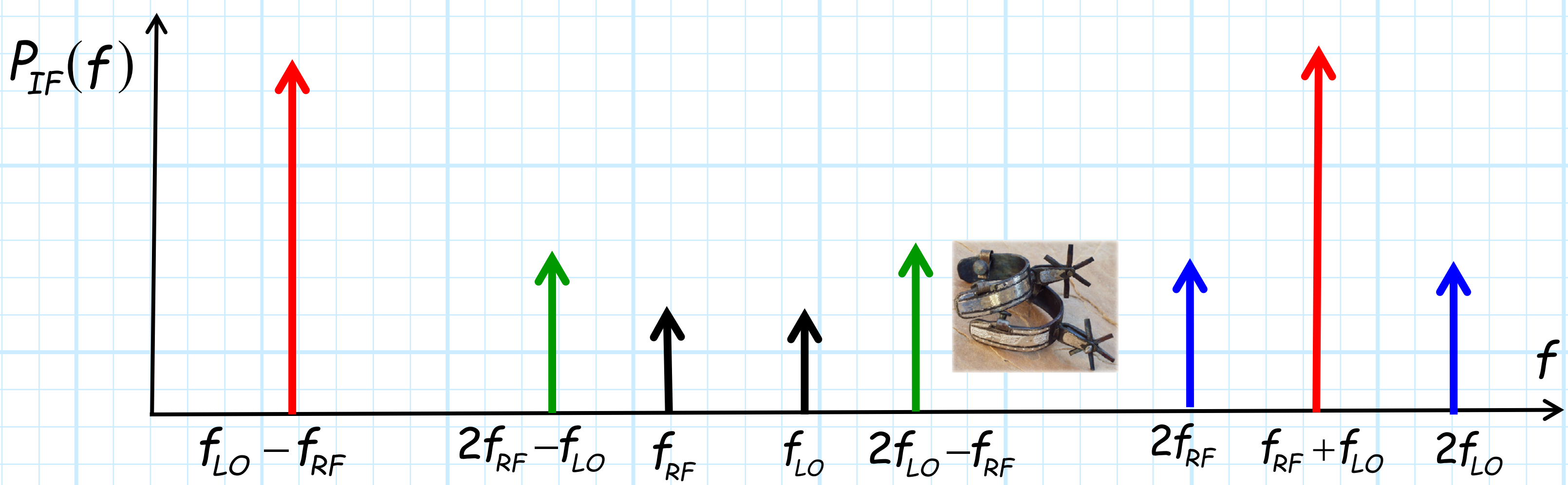
$$v_{IF}(t) \cong A_{RF}\frac{2}{\pi}\cos\left[(\omega_{RF}-\omega_{LO})t-\varphi_{RF}\right] + A_{RF}\frac{2}{\pi}\cos\left[(\omega_{RF}+\omega_{LO})t-\varphi_{RF}\right]$$



Again: mixers are not ideal!

But, the output **at the IF port** of a **real** mixer consists of the two ideal **2nd-order** terms—plus a **whole slew of spurious signals**!

$$v_{IF}(t) \;\cong\; A_{RF}\frac{2}{\pi}\cos\left[\left(\omega_{RF}-\omega_{LO}\right)t-\varphi_{RF}\right] + A_{RF}\frac{2}{\pi}\cos\left[\left(\omega_{RF}+\omega_{LO}\right)t-\varphi_{RF}\right] + v_{IF}^{spur}(t)$$

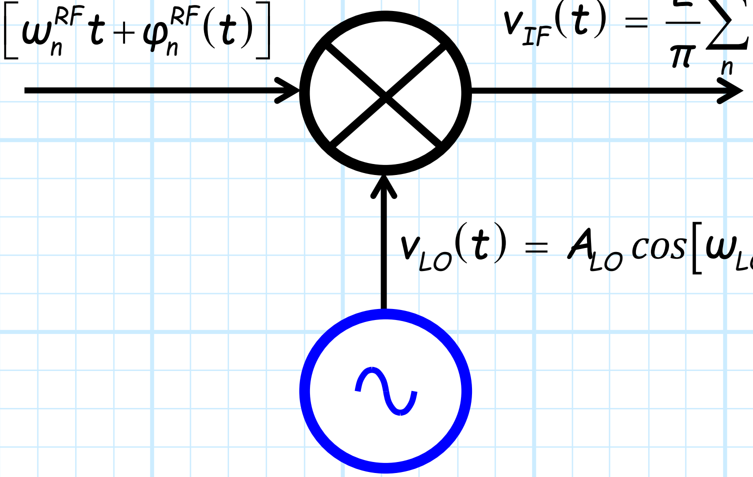


A mixer down-converts ALL these signals!

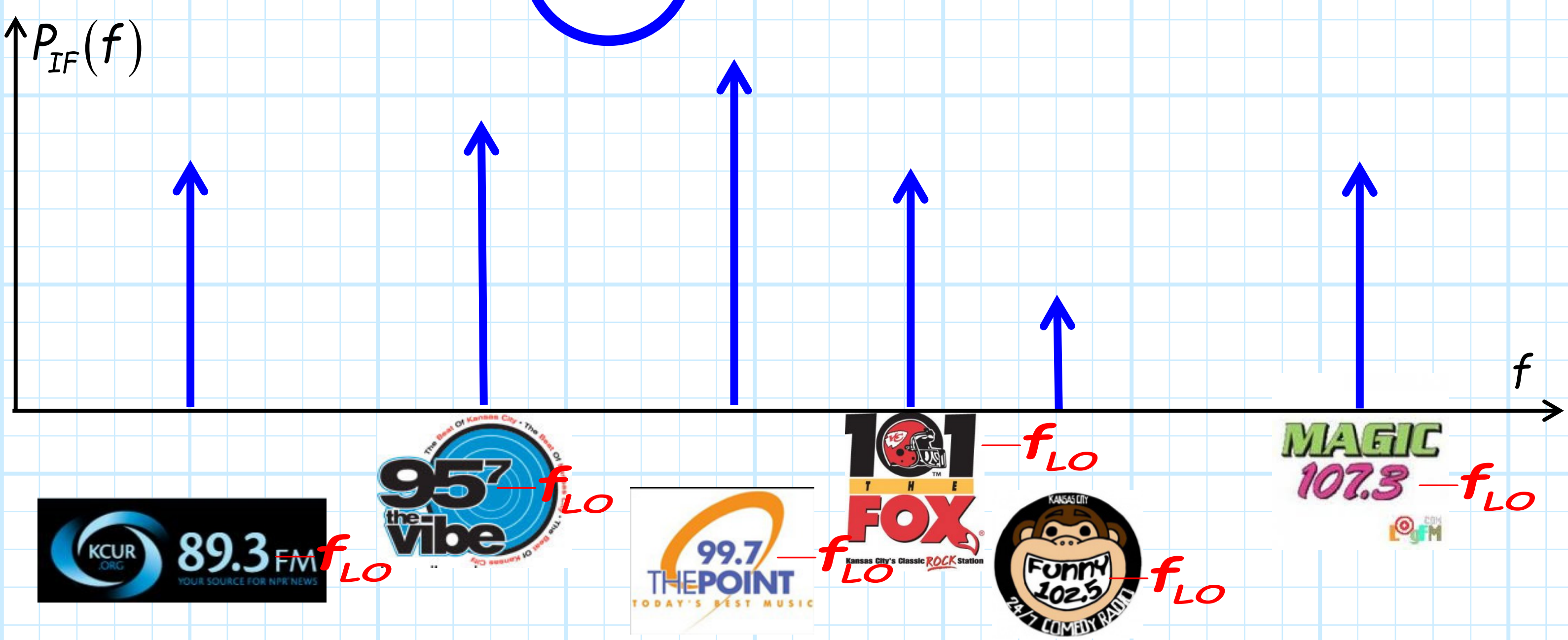
Recall that when **multiple** RF signals are incident on the RF mixer port, **all** are down-converted at the IF mixer port:

$$v_{RF}(t) = \sum_n a_n^{RF}(t)\cos\left[\omega_n^{RF} t + \varphi_n^{RF}(t)\right]$$

$$v_{IF}(t) = \frac{2}{\pi}\sum_n a_n^{RF}(t)\cos\left[\left(\omega_n^{RF} - \omega_{LO}\right)t + \varphi_n^{RF}(t)\right]$$



$$v_{LO}(t) = A_{LO}\cos\left[\omega_{LO} t\right]$$

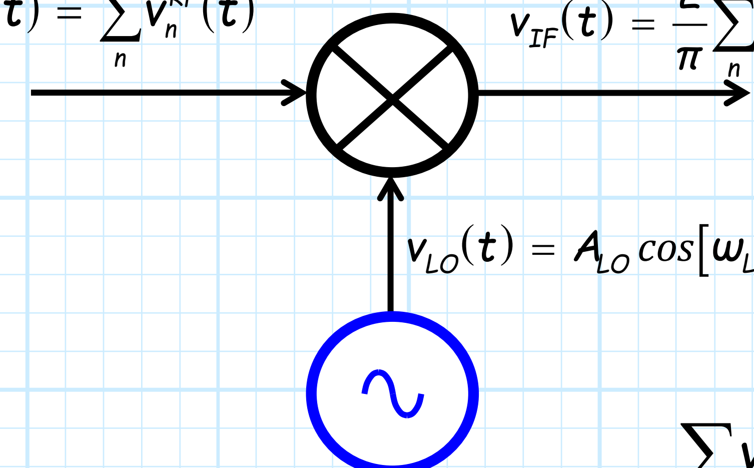


But it creates spurious signals from each as well (Doh!)

Moreover, **each** RF input signal will create **their own set of spurious signals**!!!!!!!!!!!!!

→ This can cause **super-big problems**!!!!

$$v_{RF}(t) = \sum_n v_n^{RF}(t)$$



$$v_{IF}(t) = \frac{2}{\pi}\sum_n a_n^{RF}(t)\cos\left[\left(\omega_n^{RF} - \omega_{LO}\right)t + \varphi_n^{RF}(t)\right] + \sum_n v_n^{spur}(t)$$

$$v_{LO}(t) = A_{LO}\cos\left[\omega_{LO}t\right]$$

$$\sum_n v_n^{spur}(t) \cong \sum_n \sum_{m=1}^{10} b_{mn}\cos\left[\omega_{mn}^{spur}t + \theta_{mn}\right]$$

WE'RE ALL DOOMED!!!!!!!!!!

For **example**, at the mixer IF port, we may find **multiple** 1st-order spurious signals—each at a **different frequency, e.g.**:

$$v_{1a}^{spur}(t) = \sum_n b_n^{1a} \cos\left[\omega_n^{1a} t + \theta_n^{1a}\right] = \sum_n b_n^{1a} \cos\left[\omega_n^{RF} t + \theta_n^{1a}\right]$$

And **multiple** 2nd-order spurious signals—each at a **different frequency, e.g.**:

$$v_{2a}^{spur}(t) = \sum_n b_n^{2a} \cos\left[\omega_n^{2a} t + \theta_n^{2a}\right] = \sum_n b_n^{2a} \cos\left[2\,\omega_n^{RF} t + \theta_n^{2a}\right]$$

And also **multiple** 3rd-order spurious signals—each at a **different frequency, e.g.**:

$$v_{3b}^{spur}(t) = \sum_n b_n^{3b} \cos\left[\omega_n^{3b} t + \theta_n^{3b}\right] = \sum_n b_n^{3b} \cos\left[\left(2\omega_n^{RF} - \omega_{LO}\right) t + \theta_n^{3b}\right]$$

Maybe filters will save us!!

Q: *Yikes! The signal spectrum at the mixer **IF port** seems to be a **crowed mess**. How do we deal with all these **unwanted** signals?*

A: In a word—**filters**. More on that later.

$$v_{RF}(t) = \sum_n a_n(t)\cos\left[\omega_n t + \varphi_n(t)\right]$$

$$v_{LO}(t) = A_{LO}\cos\left[\omega_{LO} t\right]$$