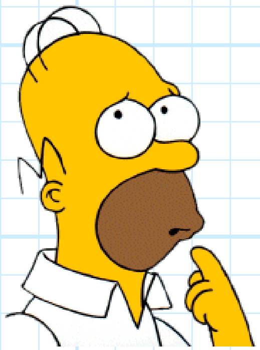
The Transmission Line Wave Equations

$$\frac{\partial V(z,\omega)}{\partial z} = -(R+j\omega L)\, I(z,\omega)$$

$$\frac{\partial I(z,\omega)}{\partial z} = -(G+j\omega C)\, V(z,\omega)$$

So **remember**, not just **any** function $I(z)$ and $V(z)$ can exist on a transmission line, but rather **only** those functions that satisfy the **telegrapher's equations**.



Our task, therefore, is to **solve** the telegrapher equations and find **all** solutions $I(z,\omega)$ and $V(z,\omega)$!

The lossless approximation

Q: *So, what functions $I(z,\omega)$ and $V(z,\omega)$* **do** *satisfy both telegrapher's equations??*

A: To make things easier, we recognize that at **high frequencies**:

$$R \ll j\omega L \quad \text{and} \quad G \ll j\omega C$$

So the **telegrapher equations** can be **approximated** as:

$$\frac{\partial V(z,\omega)}{\partial z} = -j\omega L\, I(z,\omega) \qquad \frac{\partial I(z,\omega)}{\partial z} = -j\omega C\, V(z,\omega)$$

→ This is called the **lossless approximation** (i.e., $R \cong 0$ and $G \cong 0$).

The Transmission Line Wave Equations

The complex telegrapher's equations are a pair of **coupled** differential equations:

$$\frac{\partial V(z,\omega)}{\partial z} = -j\omega L\, I(z,\omega) \qquad \frac{\partial I(z,\omega)}{\partial z} = -j\omega C\, V(z,\omega)$$

With a little mathematical elbow grease, we can **decouple** the telegrapher's equations, such that we now have **two** equations involving **one** function only:

$$\boxed{\begin{aligned} \frac{\partial^2 V(z,\omega)}{\partial z} &= -\beta^2\, V(z,\omega) \\ \frac{\partial^2 I(z,\omega)}{\partial z} &= -\beta^2\, I(z,\omega) \end{aligned}}$$

where

$$\boxed{\beta^2 \doteq \omega^2 LC}$$

These equations are known as the transmission line **wave equations**.

The (one and only) solution to the wave equations

Since these wave equations each involve only **one** unknown function, they are **easily** solved!

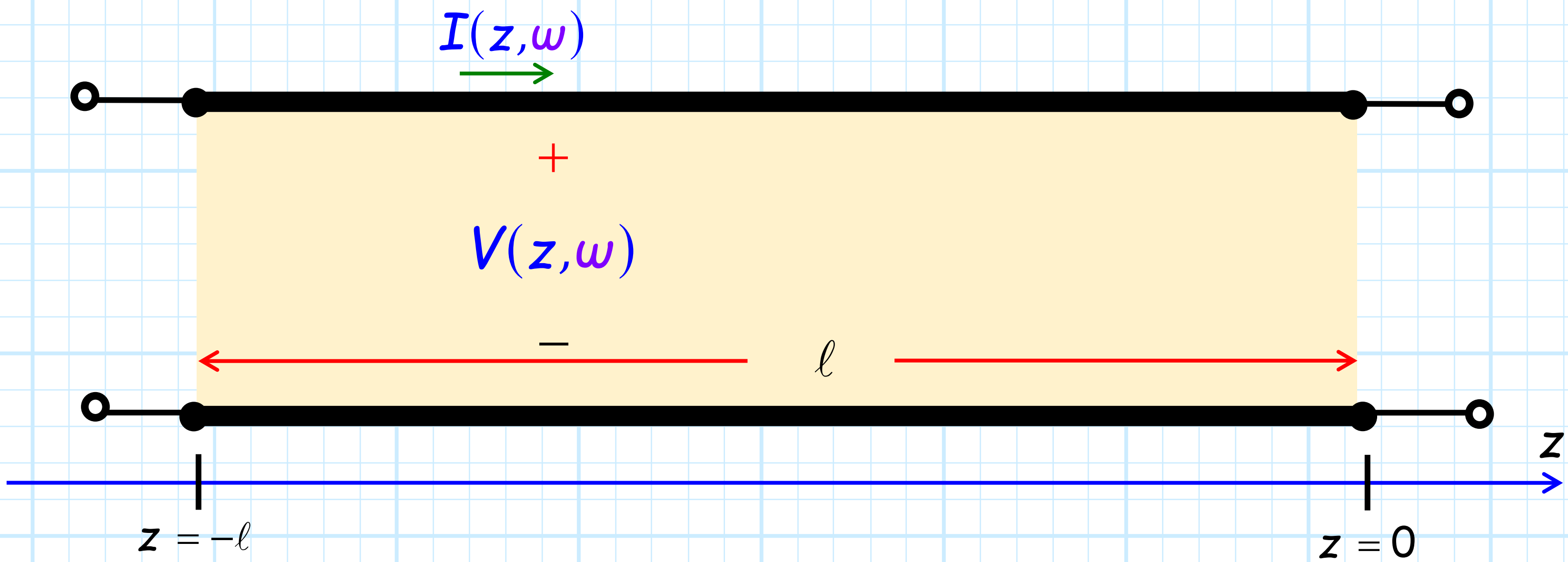
The **solutions** are:

$$\boxed{\begin{aligned} V(z,\omega) &= V_0^+(\omega)\, e^{-j\beta z} + V_0^-(\omega)\, e^{+j\beta z} \\ I(z,\omega) &= I_0^+(\omega)\, e^{-j\beta z} + I_0^-(\omega)\, e^{+j\beta z} \end{aligned}}$$

where $V_0^+(\omega)$, $V_0^-(\omega)$, $I_0^+(\omega)$, and $I_0^-(\omega)$ are **complex constants**.

→ It is **unfathomably** important that **you** understand what this result means!

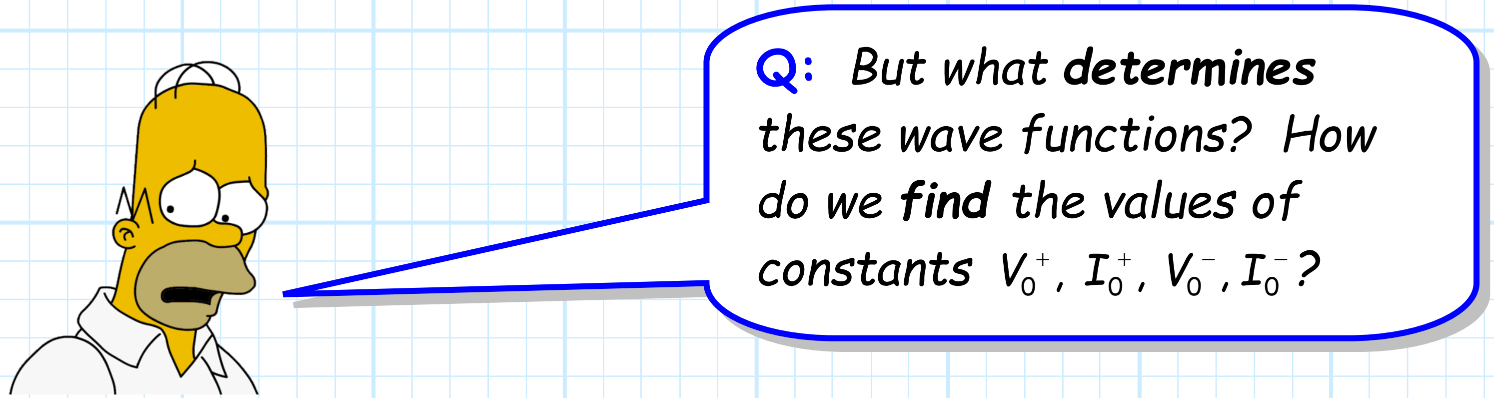
4 complex constants—and that's it!



These results mean that the functions $I(z)$ and $V(z)$—the functions describing the current and voltage at **all** points along a transmission line—can **always** be **completely** specified with just **four complex constants!!!!!**

1. $V_0^+(\omega)$
2. $V_0^-(\omega)$
3. $I_0^+(\omega)$
4. $I_0^-(\omega)$

Determining the Complex Wave Amplitudes



A: As you might expect, the voltage and current on a transmission line is determined by the devices **attached** to it on either end (e.g., active **sources** and/or passive **loads**)!

The precise values of $V_0^+(\omega), V_0^-(\omega), I_0^+(\omega), I_0^-(\omega)$ are therefore determined by satisfying the **boundary conditions** applied at **each end** of the transmission line—more on this **later**!