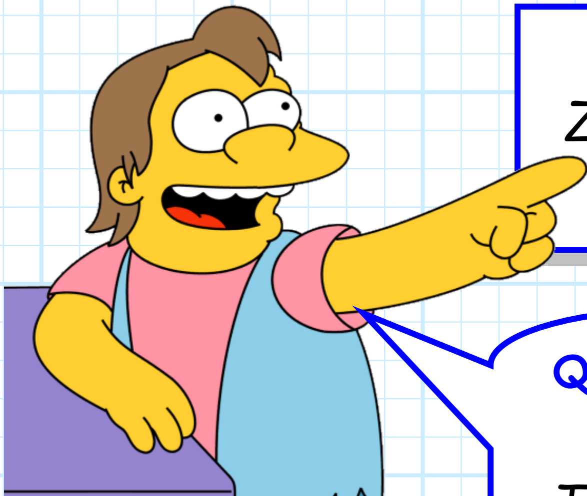
Line Impedance

Now let's define **line impedance** $Z(z,\omega)$, a **complex** function which is simply the ratio of the complex **total voltage** and complex total **current**:



$$\boxed{Z(z,\omega) = \frac{V(z,\omega)}{I(z,\omega)}}$$

Q: *Hey! I know what **this** is!*

*The ratio of the voltage to current is simply the **characteristic impedance** Z_0, right ???*

A: NO! The line impedance $Z(z,\omega)$ is (generally speaking) **NOT** the transmission line **characteristic** impedance Z_0 !!!

→ It is **unfathomably important** that you understand this!!!! ←

Why line impedance is not Z_0

To see why **line** impedance $Z(z,\omega)$ is different than **characteristic** impedance Z_0, recall that:

$$V(z,\omega) = V^+(z,\omega) + V^-(z,\omega) \quad \text{and} \quad I(z,\omega) = \frac{V^+(z,\omega) - V^-(z,\omega)}{Z_0}$$

Therefore, **line** impedance $Z(z,\omega)$ is:

$$\boxed{Z(z,\omega) = \frac{V(z,\omega)}{I(z,\omega)} = Z_0\left[\frac{V^+(z,\omega) + V^-(z,\omega)}{V^+(z,\omega) - V^-(z,\omega)}\right] \neq Z_0}$$

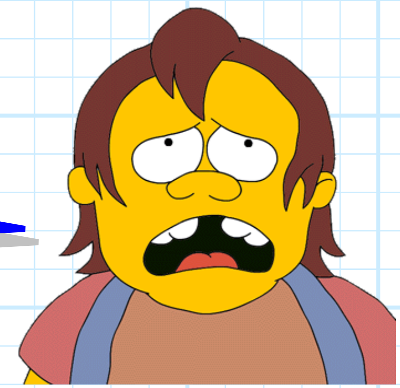
Or, more specifically, we can write:

$$Z(z,\omega) = Z_0\left[\frac{V_0^+(\omega)e^{-j\beta z} + V_0^-(\omega)e^{+j\beta z}}{V_0^+(\omega)e^{-j\beta z} - V_0^-(\omega)e^{+j\beta z}}\right]$$

What then is Z_0 ?

Q: *I'm confused! Isn't:*

$$V^+(z,\omega)/I^+(z,\omega) = Z_0 \;???$$



A: Yes! That **is** true!

The ratio of the voltage to current for **each** of the two propagating waves **is** $\pm Z_0$.

$$\frac{V^+(z,\omega)}{I^+(z,\omega)} = Z_0 \qquad\qquad \frac{V^-(z,\omega)}{I^-(z,\omega)} = -Z_0$$

However, the ratio of the **sum** of the two voltages (i.e., **total** voltage) to the **sum** of the two currents (i.e., **total** current) is **not** equal to Z_0 (generally speaking)!

There is no inconsistency here

Note the expression for **line** impedance:

$$Z(z,\omega) = \frac{V(z,\omega)}{I(z,\omega)} = Z_0\left[\frac{V^+(z,\omega)+V^-(z,\omega)}{V^+(z,\omega)-V^-(z,\omega)}\right]$$

is **consistent** with this definition of **characteristic** impedance (i.e., $Z_0 = V^+(z,\omega)/I^+(z,\omega)$)!

Say that $V^-(z)=0$, so that only **one** wave (i.e., $V^+(z,\omega)$) is propagating on the line.

In this case, the ratio of the **total** voltage to the total current is simply the ratio of the voltage and current of the **one** remaining wave—the **characteristic impedance** Z_0!

$$\frac{V^+(z,\omega)}{I^+(z,\omega)} = Z(z,\omega)\Big|_{V^-(z)=0} = Z_0\left[\frac{V^+(z,\omega)+0}{V^+(z,\omega)-0}\right] = Z_0$$

Let's Summarize!!

Q: *So, it appears to me that characteristic impedance Z_0 is a **transmission line parameter**, depending **only** on the transmission line values L and C.*

*Whereas **line impedance** is $Z(z,\omega)$ depends the magnitude and phase of the two propagating waves $V^+(z,\omega)$ and $V^-(z,\omega)$—values that depend **not only** on the transmission line, but also on the two things **attached** to either **end** of the transmission line!*

Right !?

A: **Exactly!** Moreover, note that characteristic impedance Z_0 is simply a **real-valued number**.

Whereas, line impedance $Z(z,\omega)$ is a **complex function** of position z .