EECS 622: Homework #15

October 30, 2025

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Problem 1

Say you require a filter with a center frequency $f_0 = 1.0\text{GHz}$, and bandwidth $\Delta f = 100\text{MHz}$.
You need this filter to attenuate a signal with frequency $f = 1.1\text{GHz}$ by at least $\mathbf{40dB}$.

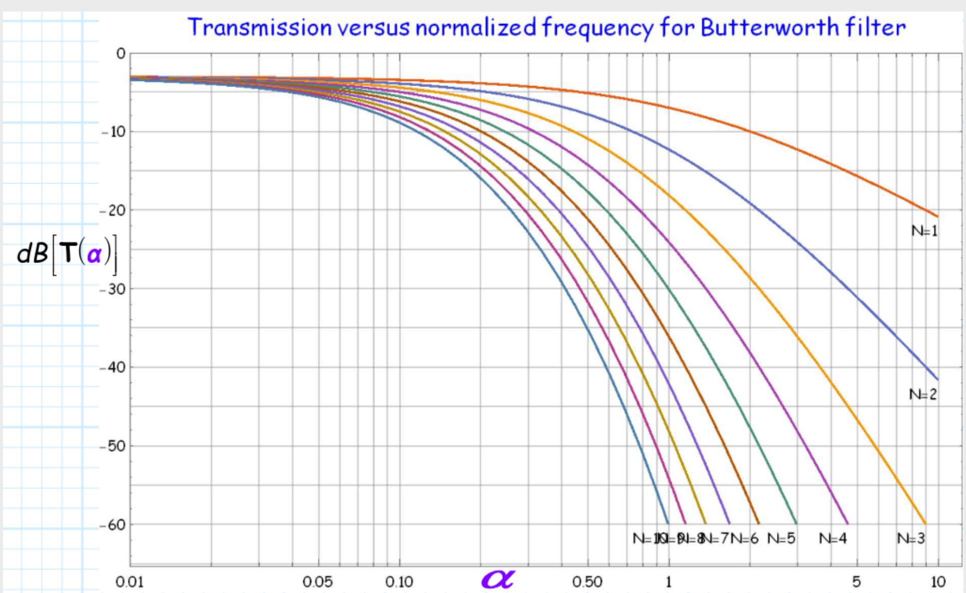
1. Determine the lowest filter order value that will achieve this requirement (i.e., 40 dB attenuation at $f = 1.1\text{GHz}$) for:

   a) a Butterworth Filter

Solution:
Before anything, we will want a normalized frequency domain.

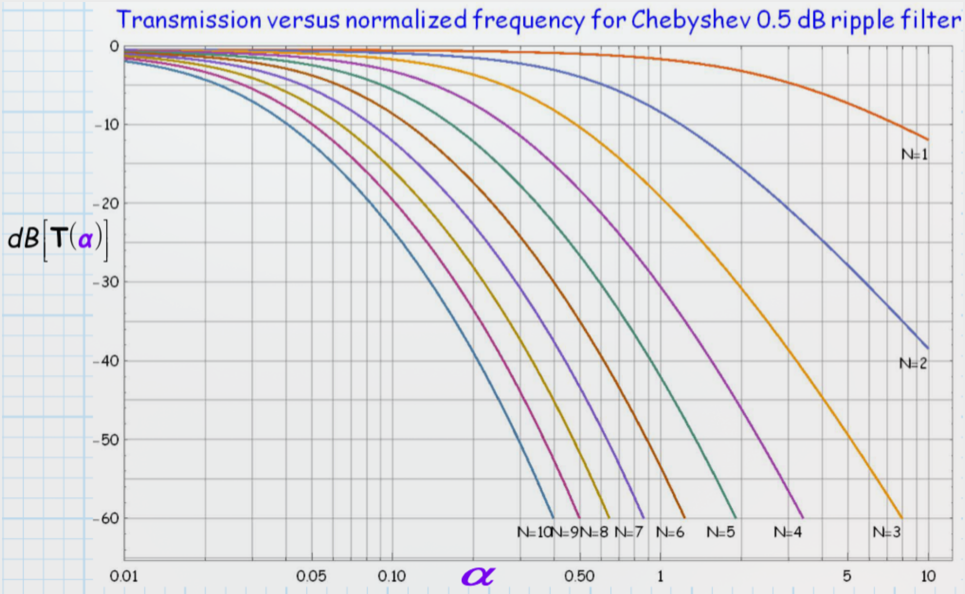
$$\begin{aligned}\alpha(f) &= \left|\frac{f_0}{f_H - f_L}\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right| - 1 \\ &= \left|\frac{1.0 \text{ GHz}}{100 \text{ MHz}}\left(\frac{f}{1.0 \text{ GHz}} - \frac{1.0 \text{ GHz}}{f}\right)\right| - 1 \\ \alpha(1.1 \text{ GHz}) &= 0.9090\ldots\end{aligned}$$

Using the normalized transmission chart from the lecture, it seems as though we could potentially get away with $N = 6$, but $N = 7$ will probably be better.



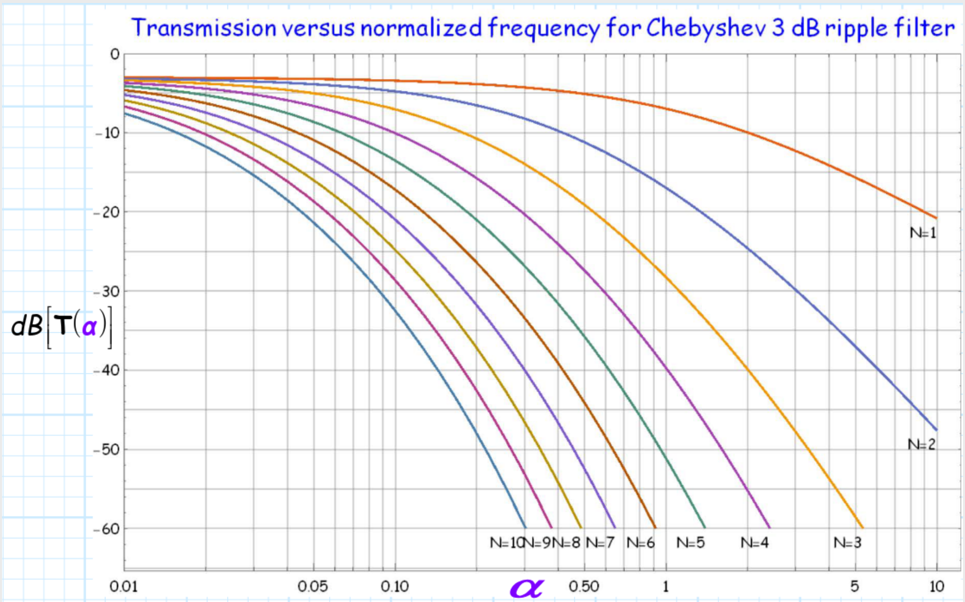
b) a Chebychev with 0.5 dB passband ripple.

Solution:
We will use the same α as in (a). We can potentially use $N = 5$, however $N = 6$ is guaranteed to achieve the desired attenuation.



c) a Chebychev with 3.0 dB passband ripple.

Solution:
We will use the same α as in (a). $N = 5$ will safely work well to achieve the desired attenuation.



Problem 1 continued on next page...

2. Using the filter orders found in part 1, determine the filter attenuation of a signal at $f = 930\text{MHz}$, for each of the three filter designs.

Solution:

We will have $\alpha(930 \text{ MHz}) = 0.45$.

(a) **Butterworth:** $N = 6$ will attenuate by -20 dB; $N = 7$ will attenuate by about -22.5 dB.

(b) **Chebychev 0.5 dB Passband Ripple:** $N = 5$ will attenuate by -25 dB; $n = 6$ will attenuate by about -26.5 dB

(c) **Chebychev 3.0 dB Passband Ripple:** $N = 5$ will attenuate by -34 dB.

Problem 2

The output of a low-pass microwave filter is terminated in a matched load.
The reflection coefficient resulting from this filter's input impedance is:

$$\Gamma_{\text{in}}(\omega) = \frac{25 \times 10^3}{25 \times 10^3 + j\left(\frac{\omega}{400}\right)}$$

Determine the 3dB cutoff frequency of this low-pass filter

Solution:
The 3 dB cutoff frequency is the point at which $P_{\text{L}}^{\text{abs}} \approx 0.5 \cdot P_{\text{g}}^{\text{avl}}$, or $V_{\text{L}}^{\text{abs}} \approx \frac{V_{\text{g}}^{\text{avl}}}{\sqrt{2}}$. We can rewrite this statement in terms of power *transmission*:

$$[T]\Big|_{3\text{dB}} = \left[\frac{P_{\text{L}}^{\text{abs}}}{P_{\text{g}}^{\text{avl}}}\right]\Bigg|_{3\text{dB}} \approx 0.5$$

Moreover, from our lesson on microwave filter design, we have derived:

$$|\Gamma_{\text{in}}(\omega)|^2 = 1 - T(\omega) \implies \Gamma_{\text{in}}(\omega) = \frac{\sqrt{2}}{2}$$

at the 3 dB point for any microwave filter. It's very straightforward to solve for ω_c to satisfy this:

$$\begin{aligned}
\left|\frac{25 \times 10^3}{25 \times 10^3 + j\left(\frac{\omega_c}{400}\right)}\right|\left|\frac{25 \times 10^3}{25 \times 10^3 - j\left(\frac{\omega_c}{400}\right)}\right| &= \frac{\sqrt{2}}{2} \\
\frac{(25 \times 10^3)^2}{(25 \times 10^3)^2 + \left(\frac{\omega_c}{400}\right)^2} &= \frac{1}{2} \\
2 \times 625 \times 10^6 = 625 \times 10^6 + \frac{\omega_c^2}{160000} & \\
625 \times 10^6 = \frac{\omega_c^2}{160000} & \\
\omega_c^2 = 625 \times 10^6 \times 160000 = 10^{14} & \\
\omega_c = 10^7 \text{ rad/s} \approx 1.6 \text{ MHz} &
\end{aligned}$$