EECS 622: Homework #3

September 11, 2025

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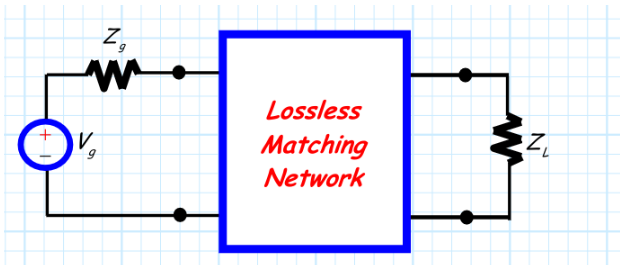
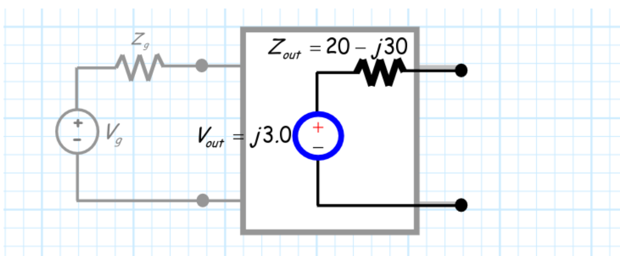
Problem 1

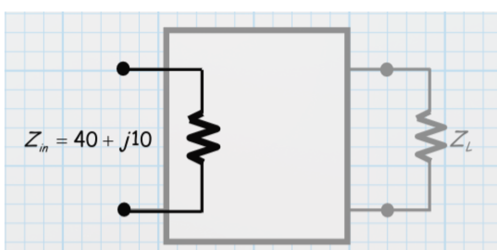
A lossless matching network was properly constructed to match a source with parameters V_g and Z_g to a load Z_L :



If we disconnect the load from the matching network, we find that we have a new equivalent source (the original source followed by the matching network), with parameters $V_{\text{out}} = j3.0$ V and $Z_{\text{out}} = 20 - j30\Omega$:



Likewise, if we disconnect the source from the matching network, we find an input impedance (the matching network followed by the load) of $Z_{\text{in}} = 40 + j10\Omega$ (turn the page!!):



(a) Determine the impedance values Z_L and Z_g.

Solution:

In general, we would need to look at the system to express a relation between $Z_g, Z_L, Z_{in}, Z_{out}$. Indeed, for the case of the matching network, we design it for the express purpose of setting the following relationship

$$\begin{cases} Z_g = Z_{in}^* = 40 - j10\ \Omega \\ Z_L = Z_{out}^* = 20 + j30\ \Omega \end{cases}$$

(b) Determine the available power of the original (i.e., V_g and Z_g) source.

Solution:

By definition, a lossless component will have invariant (available) power:

$$P_g^{avl} = P_{out}^{avl} \tag{1}$$

The right hand side of this equation is:

$$\begin{aligned} P_g^{avl} &= P_{out}^{avl} && \text{(1)} \\ &= \frac{|V_{out}|^2}{8\text{Re}\{Z_{out}\}} && \text{(Substitute known expression)} \\ &= \frac{|j3|^2}{8(20)} && \text{(Substitute numeric values)} \\ &= \frac{9}{160} \approx 0.05625 \end{aligned}$$

Now the problem is effectively solved, but I will continue to extract V_g as an exercise:

$$\begin{aligned} P_g^{avl} &= \frac{|V_g|^2}{8\text{Re}\{Z_g\}} \\ &= \frac{|V_g|^2}{8(40)} \\ &= \frac{|V_g|^2}{320}\ \frac{\text{J}}{\text{s}} \end{aligned}$$

Using (1),

$$\frac{|V_g|^2}{320} = \frac{9}{160} \implies V_g = 3\sqrt{2}\ \text{V}$$

Problem 2

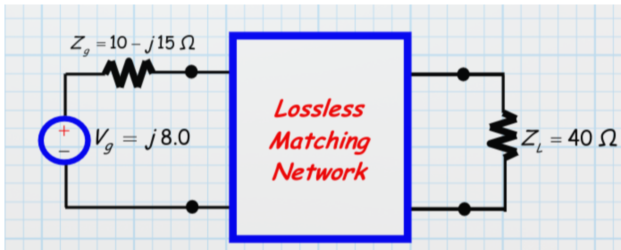
A lossless matching network was properly constructed to match a source with parameters:

$$V_g = j8.0 \text{ V} \quad \text{and} \quad Z_g = 10 - j15\Omega$$

to a load with impedance:

$$Z_L = 40\Omega$$

Determine the rate at which energy is absorbed by load Z_L.



Solution:
If and only if we have a *lossless* matching network (as we do), the following boundary condition holds:

$$P_g^{del} = P_g^{avl} = P_L^{abs} \tag{2}$$

In the same fashion as problem #1, we may compute the absorbed power without even knowing anything about the load, since all available power is absorbed

$$\begin{aligned}
P_L^{abs} &= P_g^{avl} && (2) \\
&= \frac{|V_g|^2}{8R_g} && \text{(Substitute known expression)} \\
&= \frac{64}{8(10)} && \text{(Substitute numeric values)} \\
&= \frac{4}{5}\,\frac{\text{J}}{\text{s}}
\end{aligned}$$