PHSX 711: Homework #7

November 6, 2024

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Problem 1

Exercise 11.2.2 (Shankar) – 5 points Using $T^\dagger(\epsilon)T(\epsilon) = I$ to order ϵ, deduce that $G^\dagger = G$.

Problem 2

Exercise 11.4.1 (Shankar) – 5 points Prove that if $[\Pi, H] = 0$, a system that starts out in a state of even/odd parity maintains its parity. (Note that since parity is a discrete operation, it has no associated conservation law in classical mechanics.)

Solution:

$$
\begin{aligned}
[\Pi, H] = 0 \implies H(x,p) &= H(-x,-p) \\
\Pi\,|\psi(x)\rangle &= |\psi(-x)\rangle && \text{(where } |\psi(x)\rangle = \pm\,|\psi(x)\rangle\text{)} \\
\Pi\,|\psi(x)\rangle &= \pm\,|\psi(x)\rangle
\end{aligned}
$$

$$
\begin{aligned}
\Pi\,|\psi(x,\ t)\rangle &= \Pi e^{iHt/\hbar}\,|\psi(x)\rangle \\
&= e^{iH(\pm x,\pm t)t/\hbar}\,|\psi(\pm x)\rangle && (H(x,p) = H(-x,-p)) \\
&= e^{iHt/\hbar} \pm |\psi(x)\rangle \\
&= \pm\,|\psi(x,t)\rangle
\end{aligned}
$$

Problem 3

Exercise 11.4.2 (Shankar) – 5 points A particle is in a potential

$$V(x) = V_0 \sin\left(\frac{2\pi x}{a}\right)$$

which is invariant under the translations $x \to x + ma$, where m is an integer. Is momentum conserved? Why not?
Hint: You can start this problem with:

$$\frac{d}{dt}\langle P\rangle = -\frac{i}{\hbar}\langle[P,H]\rangle$$

Whether it goes to zero or not depends on the range you used for averaging $\langle \dots \rangle$.