Determining the Index of Refraction of Solutions

Grant Saggars

February 9, 2024

1 Objective

The goal of this experiment was to determine and compare the index of refraction (IOR) for two solutions. This was done using a laser and trigonometric relationships which can relate the path vector of the laser through a substance with a known IOR to the altered sum of path vectors. All that is needed to derive the IOR of such substances is the angle at which the laser is pointed at the screen, and the points at which the laser is visible on the screen.

1.1 Equipment

1. Laser (wavelength is irrelevant)
2. Screen on which the light is projected
3. Bi-plate petri-dish
4. Substance A: water
5. Substance B: a solution of water and coffee creamer

2 Setup & Theory

It is difficult to optically measure the path taken by the laser as it travels through our substances because they are transparent, so it would be more sensible to derive these path vectors trigonometrically. Measurements of position and angle of light were taken before and after inserting the pitri-dish containing the solution into the path of the laser, creating a setup which is represented in figure (1). Mathematically, this leaves us with a system with 5 degrees of freedom and 6 equations (2).

The measured values were d_{tot}, y_{tot}, d_2, and d_3. From this I expected to calculate θ_1 (although this was not possible as discussed in section 2.2). These were measured in cm, with an uncertainty of ± 0.05 cm. Three trials were conducted at different values of θ_1 for each substance.

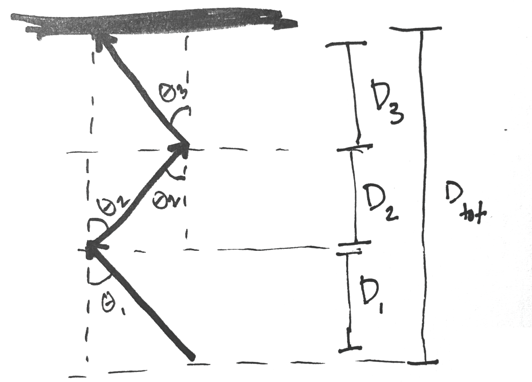

Figure 1: Setup Diagram

2.1 Relations

(1) Using Snell's Law: $(n_r \sin\theta_r = n_i \sin\theta_i)$, I relate the index of refraction between two mediums to the angles at which light travels. Air is taken to have an IOR of one. The angle θ_1 is measured from the horizontal, so $\cos(\theta_1)$ is used instead of $\sin(\theta_1)$

(2) The vertical change in medium two.

(3) Again using Snell's Law to relate the IOR of medium two to medium three. Again, air is assumed to have an IOR of one. Also, angle is again measured from the horizontal.

(4) The vertical change in medium three.

(5) The sum of vertical changes (y-components) should equal the measured y-value on the screen.

(6) The sum of the angles of incidence and refraction for any medium interface is equal to 180 degrees, since they are on the same side of the normal line.

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\begin{aligned}
\cos(\theta_1) &= n_2 \sin(\theta_2) \\
y_2 &= d_2 \left(\sin(\theta_1) - \sin(\theta_2)\right) \\
n_2 \cos(\theta_2) &= \sin(\theta_3) \\
y_3 &= d_3 \left(\sin(\theta_2) - \sin(\theta_3)\right) \\
y_{tot} &= y_1 + y_2 + y_3 \\
\pi &= \theta_1 + \theta_2 + \theta_3
\end{aligned}
$$

Figure 2: System of Equations

2.2 Calculations

I should have set up this experiment differently, since I am not capable of solving this system analytically. A better approach to setting up this experiment would have been putting the edge of the petri-dish against the screen, so that the angle made by the laser within the water could be directly calculated using a similar approach. I was, however, capable of finding numerical solutions to this system. The julia code can be found at the end of this paper. While I believe the result to be realtively reasonable, I was not capable of propagating uncertainty this way.

3 Data & Results

All samples were taken with the laser at 87 ± 0.071 cm from the screen. The petri-dish containing the water was placed at 27.125 ± 0.071 cm from the laser (measured to the far end of the dish).

My estimation of the index of refraction of mallott tap water is approximately 1.26192, and the index of refraction for the water with creamer is 1.26198. I believe these to be within the realm of plausibility, since distilled water is known to have an index of refraction of approximately 1.33. Additionally, the measurements were similar between solutions. I would have expected instrumental uncertainty to be the greatest source of error, however as mentioned previously it was not possible to calculate this.

Table 1: Measurement Data for Different Angles and Liquids

θ	**Setup**	**Fringe Distance $\pm$ 0.05 (cm)**	**Refracted Fringe Distance $\pm$ 0.05 (cm)**
1	Tap Water	3.25, 3.30, 3.40	-3.9, -4.0, 4.0
2	Tap Water	0.91, 0.90, 0.89	-2.09, -1.95, -1.95
3	Tap Water	2.05, 1.9, 1.9	-2.9, -2.94, -2.9
1	Creamer + Water	2.85, 2.75, 2.8	-3.9, -3.95, -3.95
2	Creamer + Water	1.55, 1.5, 1.57	-2.95, -2.89, -2.9
3	Creamer + Water	1.21, 1.98, 1.99	-1.99, -1.98, -1.95

n	θ_2 (rad)	θ_3 (rad)	y_2 (cm)	y_3 (cm)	$\delta f_9 5$	σ
1.26192	0.9139910	2.26194	-3.409	1.291	[1.256, 1.268]	0.00318
1.26198	0.9139369	2.26180	-3.402	1.290	[1.256, 1.268]	0.00316

Table 2: Results