PHSX 631: Homework #1

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Problem 1

Permanent cylindrical magnet with axial magnetization M.

(a) Do Griffith Problem 6.9. Sketch field lines.

(b) Go beyond Griffiths Problem 6.9 by finding the magnetic field as a function of distance, z, along the symmetry axis, and outside the magnet. Assume $L = 2a$. Plot/sketch $B(z)$, $H(z)$, $M(z)$ along the axis. Hint: See Example 5.6 in Griffiths.

(Problem 6.9) A short circular cylinder of radius a and length L carries a "frozen-in" uniform magnetization $\mathbf{M}$ parallel to its axis. Find the bound current, and sketch the magnetic field of the cylinder. (Make three sketches: one for $L \gg a$, one for $L \ll a$, and one for $L \approx a$.) Compare this bar magnet with the bar electret of Prob. 4.11.

Solution, part (a):
We have equations for bound current (volume and surface respectively):

$$\mathbf{J_b} = \nabla \times \mathbf{M} = 0, \qquad \mathbf{K_b} = \mathbf{M} \times \hat{\mathbf{n}} = M\hat{\phi}$$

Or equivalently, $\mathbf{K_b} = (-M\sin\phi, M\cos\phi, 0)$.

Solution, part (b):
As we have bound current, we can use the Biot-Savart law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_S \frac{\mathbf{K_b}(\mathbf{r}) \times \hat{\boldsymbol{\scriptr}}}{|\boldsymbol{\scriptr}|^2}\, dA$$

We have a point of interest at $(0, 0, z)$. The points on the surface of the cylinder are given by $(a\cos\phi', a\sin\phi', z')$. This gives us $\boldsymbol{\scriptr} = (-a\cos\phi', -a\sin\phi', z - z')$ and magnitude $|\boldsymbol{\scriptr}| = (a^2 + (z-z')^2)^{1/2}$. Finally, the cross product is

$$\mathbf{K_b} \times \frac{\boldsymbol{\scriptr}}{|\boldsymbol{\scriptr}|} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -M\sin\phi & M\cos\phi & 0 \\ \frac{-a\cos\phi}{(a^2+(z-z')^2)^{1/2}} & \frac{-a\sin\phi}{(a^2+(z-z')^2)^{1/2}} & \frac{z-z'}{(a^2+(z-z')^2)^{1/2}} \end{vmatrix}$$

$$= M\cos\phi\left(\frac{z-z'}{(a^2+(z-z')^2)^{1/2}}\right)\hat{\mathbf{i}} + M\sin\phi\left(\frac{z-z'}{(a^2+(z-z')^2)^{1/2}}\right)\hat{\mathbf{j}} + \frac{Ma}{(a^2+(z-z')^2)^{1/2}}\hat{\mathbf{z}}$$

The x, y components of this term leave a sin and cos term in the surface integral. This will integrate to zero, as we'd expect. This means that we only need to worry about the z-component for the final result:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{-a}^{a} \frac{Ma\hat{\mathbf{z}}}{(a^2 + (z - z')^2)^{3/2}} \, dz'$$

Problem 2

Consider a square loop of wire with resistance R and size a by a. The surface normal is initially oriented parallel to a uniform magnetic field with magnitude B_0 . The loop is then rotated by 90 deg such that the normal vector is perpendicular to the magnetic field. How much charge passes through the circuit during this procedure?

Solution:
By lenz's law, the sudden magnetic flux flowing through the loop will induce a clockwise current in the loop of wire. The current will reduce as it is rotated until at 90 degrees there is no more current, since there is no flux.
We obviously need some measure of magnetic flux to work with, so I will compute that:

$$\Phi = \int \mathbf{B} \cdot d\mathbf{a} = B_0 a^2 \cos\theta$$

From here I can jump to the time derivative of this (EMF) to current using the resistance:

$$\varepsilon = \frac{d\Phi}{dt} = B_0 a^2 \frac{d}{dt} \cos\theta$$

$$I = \frac{dQ}{dt} = B_0 a^2 \frac{d}{dt} \cos\theta$$

Problem 3

Do Griffiths problem 7.17
(Problem 7.17) A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R, as shown in Fig. 7.28.

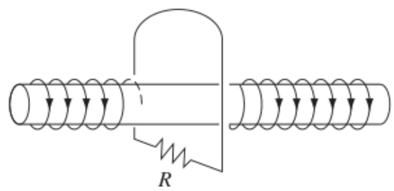

FIGURE 7.28

Problem 4

A very long cylindrical sheet of metal with radius r and length L carries a current K per unit length (azimuthal current) (units of A/m). What is the energy stored in the magnetic field in this cylinder in terms of L, R, and K?