EECS 622: Homework #17

November 4, 2025

Grant Saggars

Problem 1

Consider two different rates of energy flow-power P_1 and power P_2. It is known that:

$$dBm\left[P_1\right] = 0$$

And, it is also known that the difference between the two values (i.e., $P_1 - P_2$) is:

$$dBm\left[P_2 - P_1\right] = 0$$

Determine the value of $dBm\left[P_2\right]$.

Solution:
We simply have the set of equations:

$$-10\log_{10}\left[\frac{P_1}{1\text{ mW}}\right] = 0$$
$$-10\log_{10}\left[\frac{P_2}{1\text{ mW}} - \frac{P_1}{1\text{ mW}}\right] = 0$$

From the properties of logarithms, we know that $\log_{10}(1) = 0$. Therefore, $P_1 = 1\text{ mW}$. This must also make $P_2 = 2\text{ mW}$, since $\log_{10}\left[2 - 1\right] = 0$. Then the final solution will be:

$$\text{dBm}[2] = -10\log_{10}[2] = 3\text{ dB}$$

Problem 2

An ideal oscillator produces an output of the form:

$$v_o(t) = 1.0\cos[2\pi(1000)t]$$

This oscillator is now phase modulated, such that its output now has the form:

$$v_o(t) = 1.0\cos\left[2\pi\left(2t^2 + 1000t - 0.2\right)\right]$$

Determine an expression for the relative phase and relative frequency of this output signal.

Solution:
Relative phase is just the difference:

$$\begin{aligned}\Delta\theta(t) &= \left[2\pi\left(2t^2 + 1000t - 0.2\right)\right] - \left[2\pi(1000t)\right]\\ &= 2\pi\left(2t^2 - 0.2\right)\end{aligned}$$

I want to note for myself that absolute (instantaneous) frequency is the time derivative of the phase of the modulated signal:

$$\begin{aligned}\omega(t) &= \frac{d|\omega_0 t + \phi_n(t)|}{dt}\\ &= 2\pi\left(4t + 1000\right)\end{aligned}$$

And instead, relative frequency is the time derivative of the relative phase:

$$\Delta\omega(t) = 8\pi t$$