PHSX 671: Homework #8

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Problem 1

What is the equation for the entropy of a system consisting of N spin $\frac{1}{2}$ magnetic dipole moments (μ) in an external magnetic field B ? Express your answer in terms of μ, β, and B. What are the limiting values of the entropy at high temperature and/or weak magnetic field, and at low temperature and/or strong magnetic field. Do these limits make sense?

Solution:
We derived the partition function for a system of N magnetic dipole moments in an external magnetic field:

$$Z = 2\cosh^N(B\mu\beta)$$

Mean energy for this is

$$\begin{aligned}
\bar{E} &= -\frac{\partial}{\partial\beta}\ln\left(2\cosh^N(B\mu\beta)\right) \\
&= -N\frac{\partial}{\partial\beta}\ln\left(2\cosh(B\mu\beta)\right) \\
&= -N\left(\frac{2\sinh(\beta\mu B)}{2\cosh(\beta\mu B)}(\mu B)\right) \\
&= N\mu B\tanh(\beta\mu B)
\end{aligned}$$

Which means we have entropy

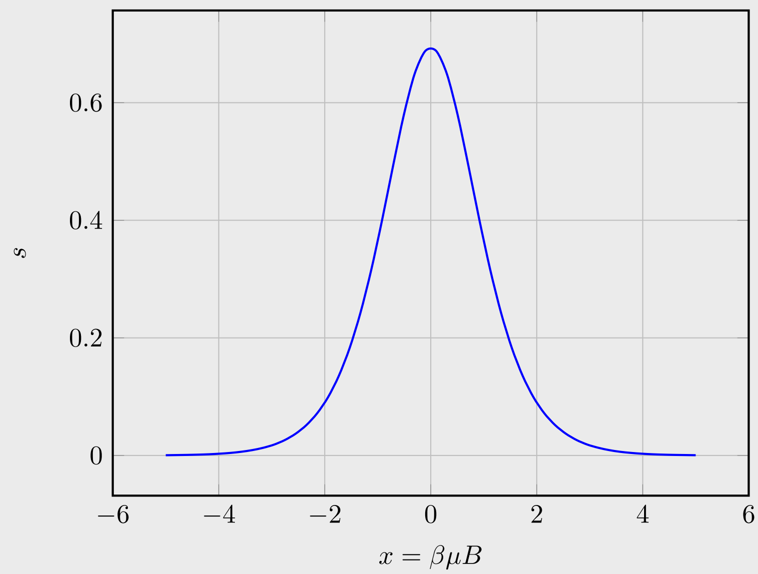
$$S = N\ln\left(2\cosh(\beta\mu B)\right) - N\beta\mu B\tanh(\beta\mu B)$$

$$s = \frac{S}{N} = \ln\left(2\cosh(\beta\mu B)\right) - \beta\mu B\tanh(\beta\mu B)$$

The plot of this slightly resembles a gaussian curve.

i. High temperature/low magnetic field limit: when $\beta \to 0$, or $B \to 0$ there are many possible configurations of the magnetic moments, which leads to a maxima in entropy.

ii. Low temperature/high magnetic field limit: when $\beta \to \infty$ or $B \to \infty$, entropy drops to zero, i.e. the system becomes totally ordered in one state.


0.6
0.4
0.2
0
s
−6
−4
−2
0
2
4
6
x = βμB