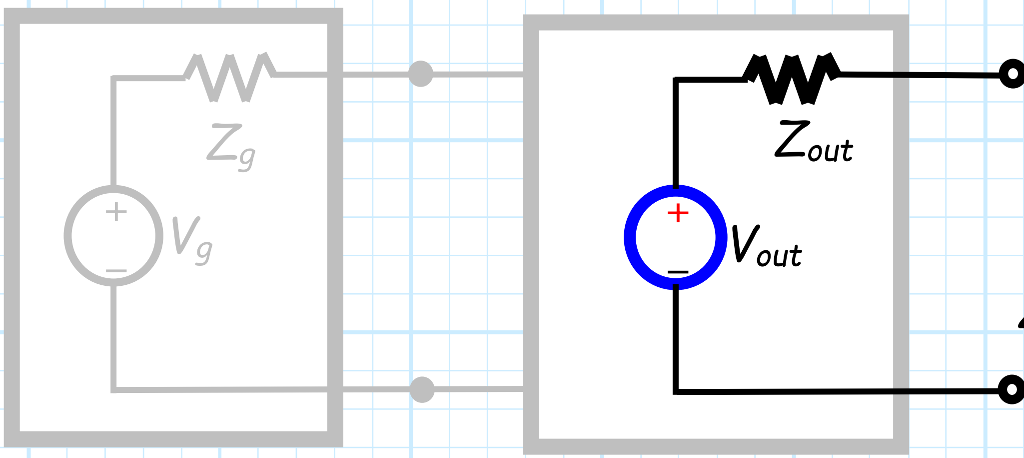
Available Power Transformation

Q: *So, you said that a two-port device can be viewed as a* ***source transformer****, which provides a* ***new equivalent source****:*



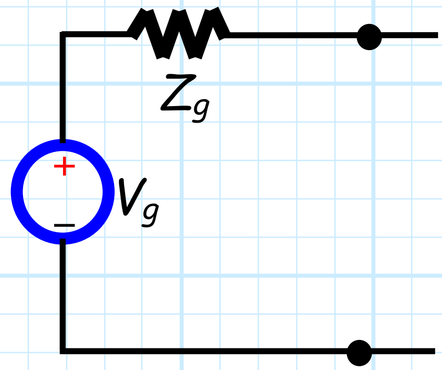
$$V_{out} = V_g \left(\frac{Z_{21}}{Z_{11} + Z_g} \right)$$

$$Z_{out} = Z_{22} - \frac{Z_{21}\, Z_{12}}{Z_{11} + Z_g}$$

but, just what is the ***available power*** P_{out}^{avl} *of this* ***transformed*** *source??*

Transformed available power

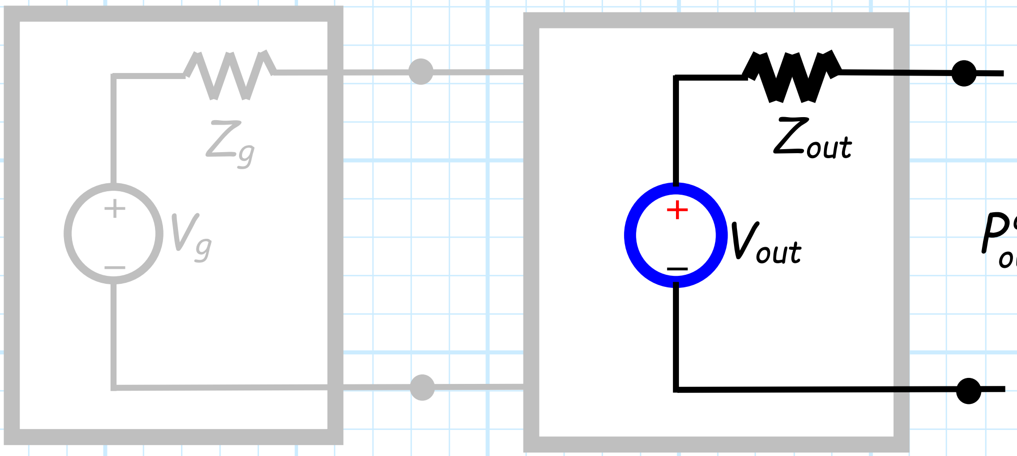
A: Recall available power P_{avl} is a parameter of the **source only**.



For the "**original**" source this value is:

$$P_g^{avl} = \frac{|V_g|^2}{8\,Re\{Z_g\}} = \frac{|V_g|^2}{8\,R_g}$$

Thus, for the **transformed** source (i.e., $V_g \rightarrow V_{out}$ and $Z_g \rightarrow Z_{out}$) the available power is likewise:



$$P_{out}^{avl} = \frac{|V_{out}|^2}{8\,Re\{Z_{out}\}} = \frac{|V_{out}|^2}{8\,R_{out}}$$

A lossy network reduces available power

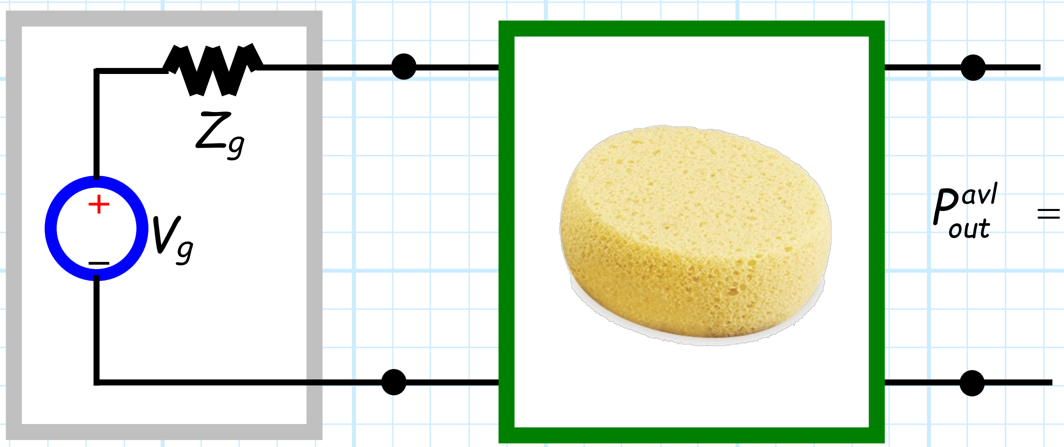
Q: *But how **are** these values related?*

*Is one value **different** than the other?*

A: It all depends on the **characteristics** of the **two-port** network!

If the two-port device is **lossy**, we find that available power of the transformed source is **less** than the original:

$$P_g^{avl} = \frac{|V_g|^2}{8\,R_g}$$

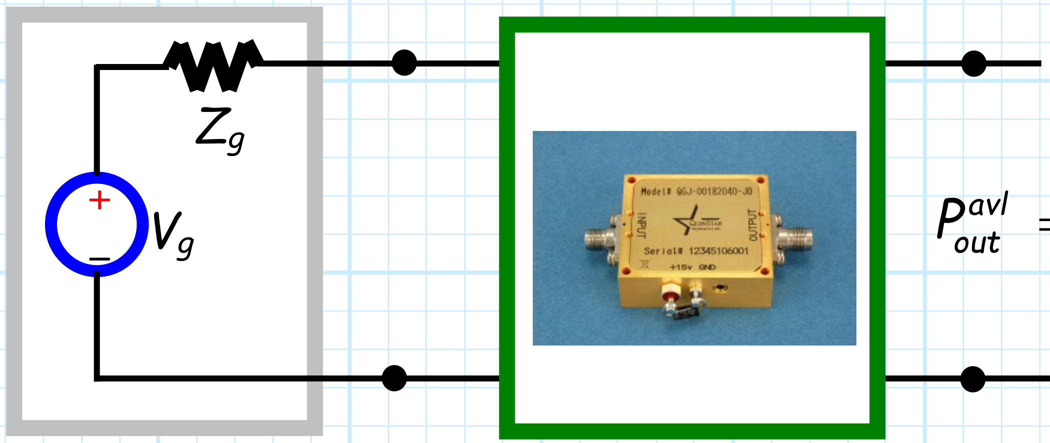


$$P_{out}^{avl} = \frac{|V_{out}|^2}{8\,R_{out}} < P_g^{avl}$$

An active device can increase available power

Conversely, **if the two-port device is an active device (e.g., an amplifier) then the available power of the transformed source can be greater than the original**:

$$P_g^{avl} = \frac{\left|V_g\right|^2}{8\,R_g}$$



$$P_{out}^{avl} = \frac{\left|V_{out}\right|^2}{8\,R_{out}} > P_g^{avl}$$

Q: *I **think** I see where this is going!*

*If the two-port device is **lossless**, is the available power of the transformed source **equal** to that of the original?*

A: Let's **find out**!

Lossless means purely reactive

Recall for a **lossless** device, the elements of its impedance matrix are **purely imaginary** (i.e., reactive).

$$\mathcal{Z}_{lossless} = \begin{bmatrix} jX_{11} & jX_{12} \\ jX_{12} & jX_{22} \end{bmatrix}$$

For this **lossless** case, the **transformed** source has the form:

$$V_{out} = V_g \left(\frac{jX_{21}}{jX_{11} + Z_g} \right)$$

$$Z_{out} = jX_{22} + \frac{(X_{21})^2}{jX_{11} + Z_g} = \frac{(X_{21})^2 Z_g^*}{\left|R_g + j(X_g + X_{11})\right|^2} + j\frac{\left(X_{22} - (X_{21})^2 X_{11}\right)}{\left|R_g + j(X_g + X_{11})\right|^2}$$

Preparing to determine P_{avl}...

Using these results, we **further** determine:

$$|V_{out}|^2 = |V_g|^2 \left| \frac{jX_{21}}{jX_{11} + Z_g} \right|^2 = |V_g|^2 \frac{(X_{21})^2}{\left|R_g + j(X_g + X_{11})\right|^2}$$

and:

$$Re\{Z_{out}\} = \frac{(X_{21})^2 \, Re\{Z_g^*\}}{\left|R_g + j(X_g + X_{11})\right|^2} = R_g \frac{(X_{21})^2}{\left|R_g + j(X_g + X_{11})\right|^2}$$

...and we've seen this result before!

Thus, the available power of the **transformed** source is:

$$P_{out}^{avl} = \frac{\left|V_{out}\right|^2}{8\,R_{out}} = \left|V_g\right|^2 \frac{\left(X_{21}\right)^2}{\left|R_g + j\left(X_g + X_{11}\right)\right|^2} \frac{\left|R_g + j\left(X_g + X_{11}\right)\right|^2}{8\,R_g\left(X_{21}\right)^2} = \frac{\left|V_g\right|^2}{8\,R_g}$$

Look at this result!

It says that although a source transformed by a **lossless** two-port device exhibits a **different** voltage source ($V_{out} \neq V_g$), and a **different** source impedance ($Z_{out} \neq Z_g$)—it exhibits precisely the **same available power**:

$$P_{out}^{avl} = \frac{\left|V_g\right|^2}{8\,R_g} = P_g^{avl} \quad !!!!!! \quad \text{(for **lossless** transformer!)}$$

But won't a smaller R_{out} result in greater available power?

Q: *Wait a second! According to **this** result:*

$$R_{out} = R_g \frac{(X_{21})^2}{\left|R_g + j(X_g + X_{11})\right|^2}$$

*the transformed output resistance could be much **greater** than the original (i.e., $R_{out} > R_g$).*

*You said earlier that a **larger** source resistance results in a **reduction** in available power (and **vice versa**).*

➔ *How then could the available power remain the **same**?*

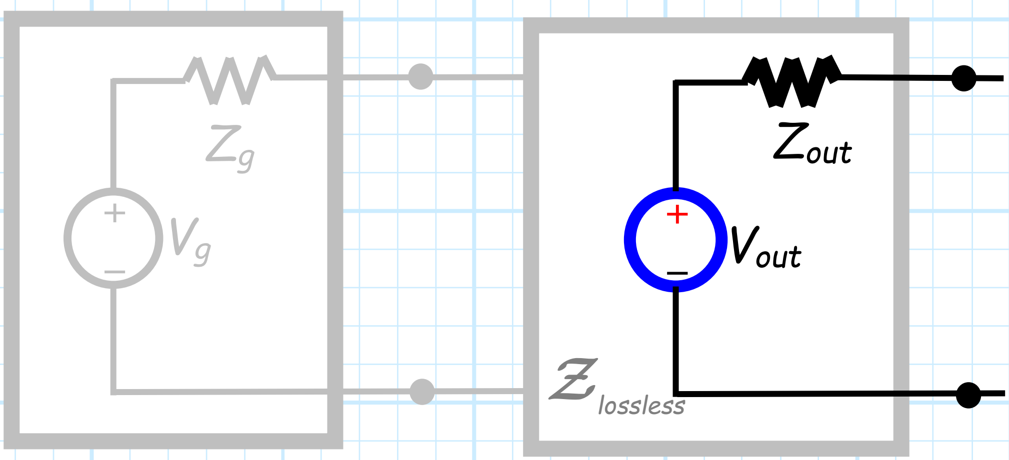
Can't transform Z_g without transforming V_g also

A: For a **given** (i.e., **fixed**) voltage source, an increase (decrease) in source resistance **does** result in a decrease (increase) in available power—just **look** at the math→

$$P_g^{avl} = \frac{|V_g|^2}{8 R_g}$$

The important thing to remember in this circumstance is that the **voltage** source is **not** fixed!

A two-port device transforms **both** the voltage source V_g **and** the source impedance Z_g!

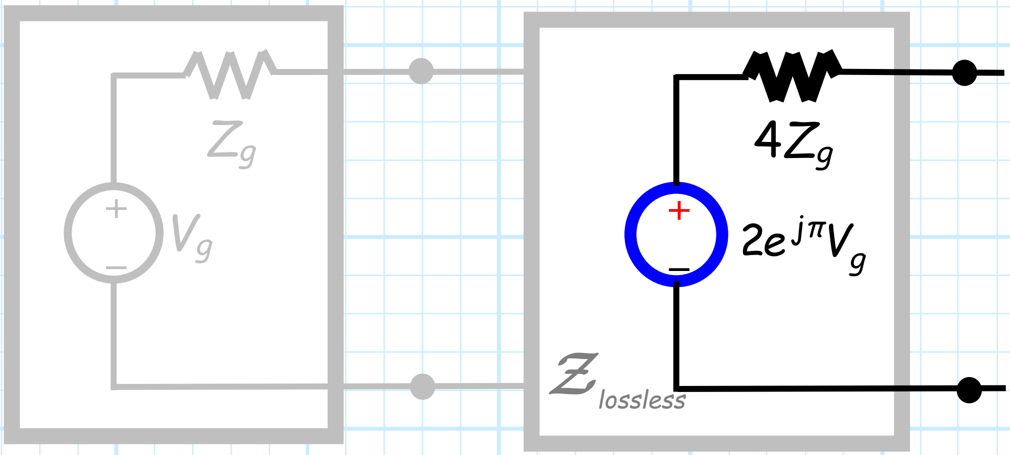


$$Z_{out} = Z_{22} - \frac{Z_{21} Z_{12}}{Z_{11} + Z_g}$$

$$V_{out} = V_g \left(\frac{Z_{21}}{Z_{11} + Z_g} \right)$$

An example

For a **lossless** two-port device, we find that **if** the transformed source resistance is **four** times (say) that of the original (i.e., $R_{out} = 4R_g$), then the **voltage** source will likewise be transformed such that its magnitude is **doubled** (i.e., $|V_{out}|^2 = 4|V_g|^2$).



$$
\begin{aligned}
P_{out}^{avl} &= \frac{|2e^{j\pi}\,V_g|^2}{8(4R_g)} \\
&= \frac{4|V_g|^2}{32R_g} \\
&= \frac{|V_g|^2}{8R_g} \\
&= P_g^{avl}
\end{aligned}
$$

The transformed **available** power is thus **unaltered** (i.e., $P_{out}^{avl} = P_g^{avl}$)!