# Physics 313-Fall 2023 Homework 5 Due 5:00 pm Oct. 4, 2023

1. (a) (2 pts) According to observer $\mathcal{O}$, a certain particle has a momentum 715MeV/c and a total relativistic energy of 1012MeV. What is the rest mass of this particle?

**Solution:**

$$(1012)(1.602 \times 10^{-13}) \text{ J} = m_0 c^2 \implies m_0 = 1.80 \times 10^{-27} \text{ kg} \quad (1)$$

$$(1012) \text{ MeV} = m_0 c^2 \implies m_0 = 1.12 \times 10^{-14} \text{ MeV/c}^2 \quad (2)$$

(b) (2 pts) An observer $\mathcal{O}'$ in a different frame of reference measures the momentum of this particle to be 863MeV/ c. What does $\mathcal{O}'$ measure for the total relativistic energy of the particle?

**Solution:**

1. Given that $E^2 = (pc)^2 + (mc^2)^2$:

$$\begin{aligned} E'^2 &= (715c)^2 + (1.12 \times 10^{-14} c^2)^2 \\ &= 2.59 \times 10^{11} \end{aligned}$$

2. Two tau particles collide and become an electron and muon. The mass of a tau is 1777MeV/c$^2$, the mass of a muon is 106MeV/c$^2$, and the mass of an electron is 0.511 MeV/c$^2$. The initial state taus have velocities of $-0.75$c and 0.75c.

(a) (2 pts) Convert the electron mass to kg and compare to the known electron mass. (The point is to show Energy $/c^2$ is a unit of mass and the above masses are correct. It's often a much more natural unit for elementary particles.)

**Solution:**

1. Given that the known electron mass (according to google) is $9.1\times10^{-31}$ kg, and 1MeV = $1.602 \times 10^{-}13$ J :

$$\frac{(0.511)(1.602 \times 10^{-13})}{c^2} = m \implies m = 9.1 \times 10^{-31} \text{ kg}$$

(b) (5 pts) Calculate the energy and momentum of both the electron and the muon. (You can assume that this is one dimensional, elastic scattering.)

**Solution:**

1. Finding initial momentum using: $p = \gamma(u)mu^2$

$$p_i = (1777)\gamma(0.75c)(0.75c - 0.75c) = 0$$

2. Finding final energy & momentum using: $E^2 = (pc)^2 + (mc^2)^2$

$$\begin{aligned} p_i = p_f &\implies p_f = 0 \\ E = mc^2 &\implies \begin{aligned} E_{\mu f} &= 9.526 \times 10^{18} \\ E_{ef} &= 4.593 \times 10^{16} \\ E_f &= 9.572 \times 10^{18} \end{aligned} \\ E_i = E_f &\implies E_i = 9.572 \times 10^{18} \\ &\implies E_\tau = 4.786 \times 10^{18} \end{aligned}$$

(c) (3 pts) Is the sum of particle masses the same between the initial and final states? Is mass conserved?

**Solution:** No, $2(1777) \neq 0.511 + 106$

(d) (3 pts) Calculate the invariant mass of both the initial and final state systems. Is invariant mass of the initial and final states conserved? Is the invariant mass the same of the sum of initial or final state masses?

3. Momentum of light:

(a) (2 pts) Find the momentum of a gamma ray with an energy of 9MeV. Express momentum in both kgm/s and eV/c.

**Solution:**

1. $E = |p|c \implies p = 9 \times 10^6$ eV/c
2. $E = |p|c \implies p = \frac{9 \times 1.062 \times 10^{-13}}{c} = 4.81$ kgm/s