On the Atomic Nature of Charge*

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We present an apparatus designed for the observation of charged oil droplets in an electric field for the purposes of determining the atomic nature of charge by measuring the force experienced by the particle in an electric field of known strength. While it is easy to produce an electric field of known strength, there is great difficulty in the observation of particles with little charge. By careful observation of charged oil droplets, we find that charge is quantized in nature, directly observing integer multiples of charge on our oil droplets

I. INTRODUCTION

1. *Experimental Measurement of Charge*

The motion of any charged particle in atmosphere, within an electromagnetic field are easily modelled as a combination free fall, buoyant, and coulomb interactions [1]. These forces take the form,

$$qE = mg + kv \tag{1}$$

By directly observing the motion of particles during free fall and in an electric field independently, we hope to directly determine the charge on the particle. Due to the interaction of buoyant forces on our particle, it may be extremely difficult to track their motion without sophisticated computer hardware, due to the nonlinear nature free fall velocity. For this reason, we opt to observe microscopic particles using with very low mass such that they can be assumed to always be at terminal velocity. A severe consequence of this decision is that the forces on such particles will be incredibly small–on the order of x Newtons. To accomplish this, we use non-volatile oil, introduced into the viewing chamber by an atomizer. With careful consideration of external conditions, it is possible to ascertain the charge on the oil droplets by their free fall velocity and coulomb interaction velocity alone. From (1) we can express

$$q = \frac{mg(v_f - v_i)}{Ev_f} \tag{2}$$

To eliminate m from equation (2), one uses the expression for the volume of a sphere:

$$m = \frac{4}{3}\pi a^3 \rho \tag{3}$$

Where a is the radius of the droplet and ρ is the density of oil. We assume that the oil droplets assume perfectly spherical shape due to extremely small terminal velocity. Then Stoke's law is used to relate the radius of the oil droplet to its velocity as it falls through a viscous medium.

$$a = \sqrt{\frac{9\eta v_f}{2g\rho}} \tag{4}$$

Stokes' Law, however, becomes incorrect when the velocity of fall of the droplets is less than 0.1 cm/s. (Droplets having this and smaller velocities have radii, on the order of 2 microns, comparable to the mean free path of air molecules, a condition which violates one of the assumptions made in deriving Stokes' Law.) Since the velocities of the droplets used in this experiment will be in the range of 0.01 to 0.001 cm/s, the viscosity must be multiplied by a correction factor. The resulting effective viscosity is

$$\eta_{\text{eff}} = \eta \left(\frac{1}{1 + \frac{b}{pa}} \right) \tag{5}$$

Where b is a constant, p is the atmospheric pressure, and a is the radius of the drop as calculated by the uncorrected form of Stokes' Law, equation (5). Then we arrive at a suitable radius for our oil droplets.

$$a = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \left(\frac{b}{2p}\right) \tag{6}$$

Which in turn yields the following equation for charge

$$q = \frac{6\pi}{E}\sqrt{\frac{9\eta^3}{2g\rho\left(1 + \frac{b}{pa}\right)^3}}(v_f + v_r)\sqrt{v_f} \tag{7}$$

Due to the use of a simple air-plate capacitor to generate an electric field, electric field is straightforwardly

$$E = \frac{V}{d} \tag{8}$$

Where V is the potential difference across the parallel plates separated by the distance d

2. *Considerations in Instrument Configuration*

There are a variety of ways to manipulate charged particles using external electromagnetic fields [**?**]. We

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employ a uniform electric field via a simple plate capacitor due to two key advantages over magnetic fields. The electromagnetic fields created must be uniform such that it is possible to reliably measure the forces acting on particles–for this it is possible to use several devices, however magnetic fields are conventionally produced using windings of wire, dramatically increasing the complexity of optical observation of the particles inside the field. Second, the motion of charged particles in a uniform magnetic field is known to be circular, increasing the complexity in the determination of force. For these reasons, we employ a simple plate capacitor to produce a uniform electric field, as it permits the construction of a straightforward viewing chamber, shown in figure 1 . A capacitor also has the advantage of producing uniform fields which drive particles in a perfectly vertical manner, simplifying the determination of their motion.

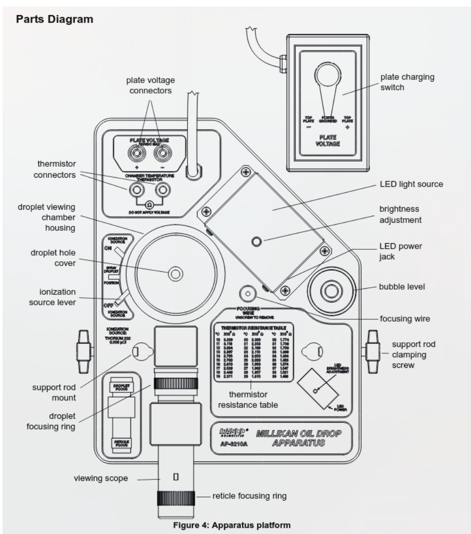

FIG. 1. A temporary schematic of the experimental setup. Will be replaced by a custom one in the next draft.

II. DETERMINATION OF CHARGE ON OIL DROPLETS

A. Observations

With the capacitor connected to the DC power supply with 500V, we manipulated the electric field in the viewing chamber by selectively switching between ground and charged modes. With the plates grounded, we introduce oil droplets between the plates through a port above. Then, we isolate droplets which we perceive to be of low charge and record their rise and fall velocity with plates charged and grounded, respectively. By carefully controlling the conditions within the viewing chamber, we then record the time using a stopwatch for the oil droplets to move between the 0.5 mm markings in the viewing chamber.

To eliminate uncertainty about the effects of the oil droplet on the motion in the electric field, we manipulate the charge on our oil droplets by means of ionization. An alpha source, Thorium-232, is placed near the drop and can be toggled using a lever on the side of the apparatus (1). Using this, we were able to modify the charge on an oil droplet to assess the effects of changes in charge on the oil droplets.

Droplet	Charge (Coulomb)	# Charges on Oil Drop
A	1.81276×10^{-19}	1.0
B	10.9941×10^{-19}	6.06482
C	12.8595×10^{-19}	7.09385

TABLE I. Measured charges on oil drops

The measured charges on the oil droplets exhibit distinct, quantized values, supporting the hypothesis that electric charge exists in discrete units rather than as a continuous variable. By analyzing the charge values obtained from multiple oil droplets, we observe that each charge closely corresponds to integer multiples of a fundamental unit of charge.

To determine this fundamental charge, we examine the measured charges in Table I. The smallest observed charge is approximately 1.81×10^{-19} C, which we identify as the unit charge. The charges of other droplets, such as Droplet B (10.99×10^{-19} C) and Droplet C (12.86×10^{-19} C), align well with integer multiples of this fundamental charge.

B. Uncertainty Analysis

Comparing our experimental result with the accepted value of the elementary charge [2], 1.602×10^{-19} C, we calculate the percentage error:

$$\text{Percentage error} = \left(\frac{|1.81 - 1.602|}{1.602} \right) \times 100 \approx 13.0\%$$

We notice a fixed error in our results such that all measured values are multiples of each other with the same 13% deviation. This implies the deviation may result from quantities we lacked sufficient means to appropriately measure: namely local atmospheric conditions. Furthermore, rudimentary means were used to time the motion of oil droplets, and we see a significant standard deviation in this degree of freedom.

todo: proper uncertainty calculations

III. DISCUSSION

The experimental observations point to the quantized nature of charge.

Appendix A: Charge Calculation Error Propagation

To start the appendixes, use the `\appendix` command. This signals that all following section commands refer to appendixes instead of regular sections. Therefore, the `\appendix` command should be used only once—to setup the section commands to act as appendixes. Thereafter normal section commands are used. The heading for a section can be left empty. For example,

```
\appendix
\section{}
```

will produce an appendix heading that says "APPENDIX A" and

```
\appendix
\section{Background}
```

will produce an appendix heading that says "APPENDIX A: BACKGROUND" (note that the colon is set automatically).

If there is only one appendix, then the letter "A" should not appear. This is suppressed by using the star version of the appendix command (`\appendix*` in the place of `\appendix`).

[1] P. Scientific, *Millikan Oil Drop Apparatus AP-8210A Instruction Manual* (n.d.), document Number: 012-13093B.

[2] R. A. Millikan, *The Electron: Its Isolation and Measurement and the Determination of Some of Its Properties* (The University of Chicago Press, Chicago, IL, 1917).