PHSX 886: Homework #9

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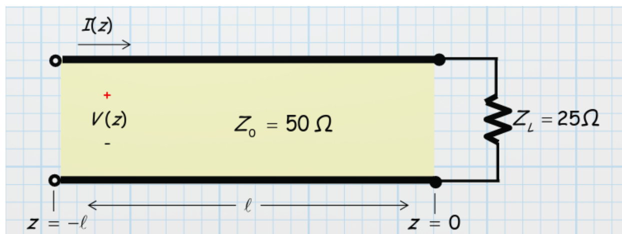
Problem 1

Consider a terminated, lossless **transmission line**:



$$Z_0 = 50\,\Omega, \quad Z_L = 25\,\Omega$$

The wave **reflected** from load Z_L has the form:

$$V^-(z) = je^{+j\beta z}$$

Determine the current flowing through load Z_L.

Solution:
By ohm's law, we have a constraint

$$V_L = I_L Z_L$$

By exploiting the reflection constraint, we may extract V_L:

$$\begin{aligned} V(z,\omega) &= V^+(z,\omega) + V^-(z,\omega) \\ &= V^-(z,\omega)\left(1 + \frac{1}{\Gamma_0}\right) \qquad \left(\Gamma_0(\omega) = \frac{V_0^-}{V_0^+}\right) \end{aligned}$$

Γ_0 we derived from ohm's law,

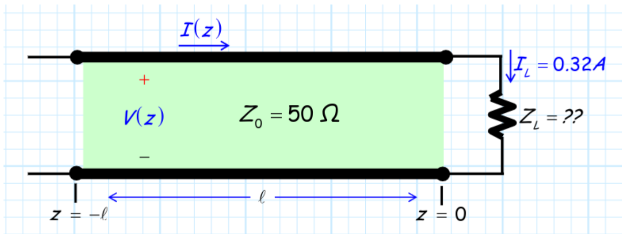
$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = -\frac{1}{3}$$

The current flowing through load Z_L is then,

$$I_L = \left.\frac{V^-(z,\omega)\left(1 + \frac{1}{\Gamma_0}\right)}{Z_L}\right|_{z=0} = -\frac{2j}{25}\ \text{A}$$

Problem 2

Consider this **lossless**, terminated transmission line:



$$Z_0 = 50\,\Omega, \quad I_L = 0.32\,A$$

The **plus-wave voltage** along this line is:

$$V^+(z) = 12e^{-j\beta z}\,V$$

and the load current is:

$$I_L = 0.32\,A$$

Determine the value of unknown load impedance Z_L.

Solution:
By ohm's law, we have a constraint

$$V_L = I_L Z_L$$

The definition of current, taken from the derivative of the voltage can be used to express Γ_0 in terms of known values

$$\begin{aligned}
I(z,\omega) &= \frac{V^+(z,\omega)}{Z_0} - \frac{V^-(z,\omega)}{Z_0} \\
I(z,\omega) &= \frac{V^+(z,\omega)}{Z_0}(1-\Gamma_0) \\
1-\Gamma_0 &= \frac{I(z,\omega)Z_0}{V^+(z,\omega)} \\
\Gamma_0 &= 1 - \frac{I(z,\omega)Z_0}{V^+(z,\omega)}
\end{aligned}$$

Problem 2 continued on next page...

Z_L we derived from ohm's law,

$$Z_L = Z_0 \frac{1+\Gamma_0}{1-\Gamma_0}$$
$$Z_L = Z_0 \frac{1+\left(1-\frac{I(z,\omega)Z_0}{V^+(z,\omega)}\right)}{1-\left(1-\frac{I(z,\omega)Z_0}{V^+(z,\omega)}\right)}$$
$$Z_L = Z_0 \frac{2-\frac{I(z,\omega)Z_0}{V^+(z,\omega)}}{\frac{I(z,\omega)Z_0}{V^+(z,\omega)}}$$
$$Z_L = \frac{2V^+(z,\omega)}{I(z,\omega)} - Z_0.$$

Substitution of numeric values nets:

$$Z_L = 25\ \Omega$$