EECS 622: Homework #8

September 30, 2025

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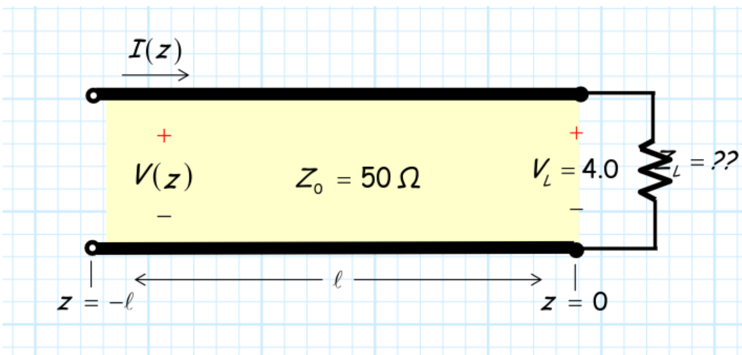
Problem 1

Consider a terminated, lossless **transmission line**:



The wave **reflected from load** Z_L has the form:

$$V^-(z) = -2e^{+j\beta z}$$

And the load voltage is:

$$V_L = 4.0$$

Determine the value of load Z_L.

Solution:
The load establishes the boundary conditions on the right, such that

$$V_L \equiv V_0, \quad I_L = I_0$$

And we could claim

$$Z_L = \frac{V_0}{I_0}$$

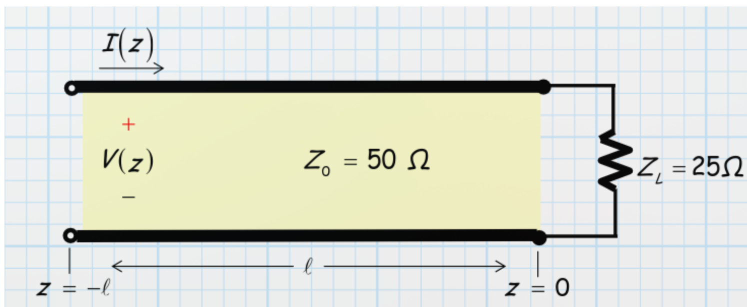
By our definition of characteristic impedance we can express I_0:

$$\begin{aligned}
I(z,\omega) &= \frac{V^+}{Z_0}e^{-j\beta z} - \frac{V^-}{Z_0}e^{+j\beta z} \\
I(z=0,\omega) \equiv I_0 &= \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \\
I_0 &= \frac{V_L - V_0^-}{Z_0} - \frac{V_0^-}{Z_0} && \text{(because } V(z) \equiv V^+(z) + V^-(z))
\end{aligned}$$

Therefore,

$$
\begin{aligned}
Z_L &= \frac{V_L}{\frac{V_L - V_0^-}{Z_0} - \frac{V_0^-}{Z_0}} \\
&= \frac{V_L}{\frac{V_L - 2V_0^-}{Z_0}} \\
&= \frac{4\text{ V } Z_0\ \Omega}{4 + 2 + 2\text{ V}} \\
&= 25\ \Omega
\end{aligned}
$$

Problem 2



The total voltage at location $z = 0$ is:

$$V(z = 0) = j\,2.0\,V$$

Determine the value of the **reflected voltage wave** at location $z = 0$ (i.e., $V^-(z = 0)$).

Solution:
We have a convenient expression for:

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{1}$$

Where,

$$\Gamma_0 = \frac{V_0^-}{V_0^+} \tag{2}$$

Substituting this into the complete wave solution I can solve for V^+

$$
\begin{aligned}
V(z,\omega) &\equiv V^+(z,\omega) + V^-(z,\omega) \\
V_0 &= V_0^+ + \Gamma_0 V_0^+ && \left(V_0^- = \Gamma V_0^+\right) \\
V_0 &= V_0^+\left(1 + \Gamma_0\right) && \text{(Algebra)} \\
V_0^+ &= \frac{V_0}{1 + \Gamma_0} && (3)
\end{aligned}
$$

Problem 2 continued on next page...

Using (3) I can eliminate V_0^+ from (2):

$$V_0^- = \frac{\Gamma_0 V_0}{1+\Gamma_0}$$

$$V_0^- = \frac{\frac{Z_L - Z_0}{Z_L + Z_0} V_0}{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$V_0^- = \frac{(-1/3)(j2)}{1+(-1/3)}$$

$$V_0^- = -\frac{j}{2} \text{ V}$$