On the Atomic Nature of Charge*

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We present an apparatus designed for the observation of charged oil droplets in an electric field for the purposes of determining the atomic nature of charge by measuring the force experienced by the particle in an electric field of known strength. While it is easy to produce an electric field of known strength, there is great difficulty in the observation of particles with little charge. By careful observation of charged oil droplets, we find that charge is quantized in nature, directly observing integer multiples of charge on our oil droplets

I. INTRODUCTION

1. Experimental Measurement of Charge

At the time of the original experiment by Robert Millikan, the existence of subatomic particles had not been universally accepted, however the work of many scientists of the time led to many postulating the existence of the electron[1, 2]. Millikan's experiment was the first to confirm it, operating on the very simple principle of the motion of charged particles in an electromagnetic field; motion which is easily modelled as a combination free fall, buoyant, and coulomb interactions [3]. These forces take the form,

$$qE = mg + kv \tag{1}$$

By directly observing the motion of particles during free fall and in an electric field independently, we hope to directly determine the charge on the particle. Due to the interaction of buoyant forces on our particle, it may be extremely difficult to track their motion without sophisticated computer hardware, due to the nonlinear nature free fall velocity. For this reason, we opt to observe microscopic particles using with very low mass such that they can be assumed to always be at terminal velocity. A severe consequence of this decision is that the forces on such particles will be incredibly small–on the order of x Newtons. To accomplish this, we use non-volatile oil, introduced into the viewing chamber by an atomizer. With careful consideration of external conditions, it is possible to ascertain the charge on the oil droplets by their free fall velocity and coulomb interaction velocity alone. From (1) we can express

$$q = \frac{mg(v_f - v_i)}{Ev_f} \tag{2}$$

To eliminate m from equation (2), one uses the expres-

sion for the volume of a sphere:

$$m = \frac{4}{3}\pi a^3 \rho \tag{3}$$

Where a is the radius of the droplet and ρ is the density of oil. We assume that the oil droplets assume perfectly spherical shape due to extremely small terminal velocity. Then Stoke's law is used to relate the radius of the oil droplet to its velocity as it falls through a viscous medium.

$$a = \sqrt{\frac{9\eta v_f}{2g\rho}} \tag{4}$$

Stokes' Law, however, becomes incorrect when the velocity of fall of the droplets is less than 0.1 cm/s. Droplets having this and smaller velocities have radii, on the order of 2 microns, comparable to the mean free path of air molecules, a condition which violates one of the assumptions made in deriving Stokes' Law [4]. Since the velocities of the droplets used in this experiment will be in the range of 0.01 to 0.001 cm/s, the viscosity must be multiplied by a correction factor. The resulting effective viscosity is

$$\eta_{\text{eff}} = \eta \left(\frac{1}{1 + \frac{b}{pa}} \right) \tag{5}$$

Where b is a constant, p is the atmospheric pressure, and a is the radius of the drop as calculated by the uncorrected form of Stokes' Law, equation (5). Then we arrive at a suitable radius for our oil droplets.

$$a = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \left(\frac{b}{2p}\right) \tag{6}$$

Which in turn yields the following equation for charge

$$q = \frac{6\pi}{E} \sqrt{\frac{9\eta^3}{2g\rho \left(1 + \frac{b}{pa}\right)^3}} (v_f + v_r)\sqrt{v_f} \tag{7}$$

Due to the use of a simple air-plate capacitor to generate an electric field, electric field is straightforwardly

$$E = \frac{V}{d} \tag{8}$$

Where V is the potential difference across the parallel plates separated by the distance d

2. Considerations in Instrument Configuration

There are a variety of ways to manipulate charged particles using external electromagnetic fields [5].

We employ a uniform electric field via a simple plate capacitor due to two key advantages over magnetic fields. The electromagnetic fields created must be uniform such that it is possible to reliably measure the forces acting on particles–for this it is possible to use several devices, however magnetic fields are conventionally produced using windings of wire, dramatically increasing the complexity of optical observation of the particles inside the field. Second, the motion of charged particles in a uniform magnetic field is known to be circular, increasing the complexity in the determination of force. For these reasons, we employ a simple plate capacitor to produce a uniform electric field, as it permits the construction of a straightforward viewing chamber, shown in figure 1 . A capacitor also has the advantage of producing uniform fields which drive particles in a perfectly vertical manner, simplifying the determination of their motion.
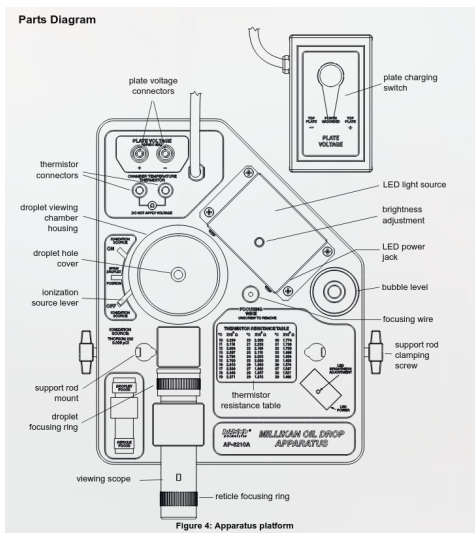

FIG. 1. A schematic of the experimental setup. The switch shown in the top right is used to switch the plates between charged and grounded mode. Droplets are monitored using the scope, and introduced between the capacitors using the port above. The sample of Thorium-232 located in the droplet chamber can be rotated to ionize the oil droplets by use of the source lever, pictured on the left.

II. DETERMINATION OF CHARGE ON OIL DROPLETS AND RESULTS

A. Experimental Setup

We select Squibb #5597 Mineral Oil which has density 886 kg/m³, provided by the manufacturer. Additionally, we use weather data in the determination of air pressure, 101 kPa, which has caveats discussed in A. Furthermore, we used an antique power supply of unknown specification provided by the physics department. A Fluke 179 Voltmeter was used to accurately measure voltage supplied to the capacitors.

With the capacitor connected to the DC power supply with 500V, we manipulated the electric field in the viewing chamber by selectively switching between ground and charged modes. With the plates grounded, we introduce oil droplets between the plates through a port above. Then, we isolate droplets which we perceive to be of low charge and record their rise and fall velocity with plates charged and grounded, respectively. By carefully controlling the conditions within the viewing chamber, we then record the time using a stopwatch for the oil droplets to move between the 0.5 mm markings in the viewing chamber.

To eliminate uncertainty about the effects of the oil droplet on the motion in the electric field, we manipulate the charge on our oil droplets by means of ionization. An alpha source, Thorium-232, is placed near the drop and can be toggled using a lever on the side of the apparatus (figure 1). Using this, we were able to modify the charge on an oil droplet to assess the effects of changes in charge on the oil droplets. This procedure was done for droplets B and C in table I. This table also serves to demonstrate the quantized nature of charge. I let charge A be the unit charge, since it is the smallest which appears in the dataset. Then, I normalize the charge on each droplet by this charge, where it can be observed that the number of charges on each droplet are round multiples of this unit charge.

Given the time constraint of this research, it is necessary to gather both high-precision and low-precision data. The high-precision population serves the purpose to test our theory that charge is quantized in nature, and to provide an accurate measure of electron charge, while the low-precision population serves to provide sufficient statistical grounds for our work.

Charge (C)	# Charges	Statistical Uncertainty	Systematic Uncertainty
A 1.695×10^{-19}	1.0 ± 0.033	3.162×10^{-20}	5.639×10^{-21}
B 1.146×10^{-18}	6.76 ± 0.21	3.536×10^{-20}	3.628×10^{-20}
C 1.167×10^{-18}	6.89 ± 0.22	3.536×10^{-20}	3.767×10^{-20}
D 4.903×10^{-19}	2.893 ± 0.093	4.472×10^{-20}	1.576×10^{-20}
E 8.033×10^{-19}	4.74 ± 0.15	3.333×10^{-20}	2.59×10^{-20}
F 5.955×10^{-19}	3.51 ± 0.11	3.536×10^{-20}	1.895×10^{-20}

TABLE I. High-precision data, collected by taking many samples of the same droplet. The corresponding statistical uncertainty associated with these are listed in the rightmost column. All droplets are round multiples (within statistical deviation) of the smallest charge in the dataset, A.

Charge (C)	# Charges	Statistical Uncertainty	Systematic Uncertainty
G 2.82×10^{-19}	1.664 ± 0.054	1.0×10^{-19}	9.216×10^{-21}
H 2.422×10^{-19}	1.429 ± 0.047	1.0×10^{-19}	7.942×10^{-21}
I 1.24×10^{-18}	7.32 ± 0.23	1.0×10^{-19}	3.905×10^{-20}
J 3.389×10^{-19}	2.0 ± 0.065	1.0×10^{-19}	1.108×10^{-20}
K 6.326×10^{-19}	3.73 ± 0.12	1.0×10^{-19}	2.025×10^{-20}
L 8.313×10^{-19}	4.9 ± 0.15	1.0×10^{-19}	2.609×10^{-20}
M 7.709×10^{-19}	4.55 ± 0.14	1.0×10^{-19}	2.421×10^{-20}
N 5.7×10^{-19}	3.36 ± 0.11	1.0×10^{-19}	1.821×10^{-20}
O 1.277×10^{-18}	7.54 ± 0.24	1.0×10^{-19}	4.06×10^{-20}
P 2.577×10^{-18}	15.2 ± 0.48	1.0×10^{-19}	8.154×10^{-20}
Q 3.151×10^{-19}	1.859 ± 0.06	1.0×10^{-19}	1.011×10^{-20}
R 8.048×10^{-19}	4.75 ± 0.15	1.0×10^{-19}	2.621×10^{-20}
S 1.184×10^{-18}	6.99 ± 0.22	1.0×10^{-19}	3.797×10^{-20}

TABLE II. Low-precision data, collected by sampling many droplets single times. The purpose of this data is to primarily provide statistical motivation to my results, however there is very large statistical uncertainty associated with these.
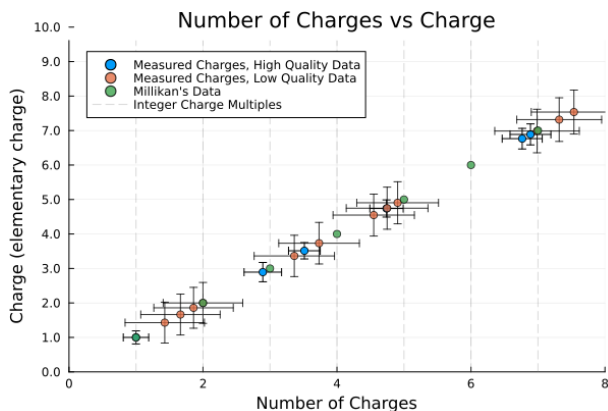

FIG. 2. A plot of the high-precision, low-precision data, and the commonly accepted values for electron charge. Units are plotted in relative charges, (elementary charge). The high-precision data has large uncertainty, primarily due to statistical sources, however the mean values in the high-precision data set lie very close to the commonly accepted values.

B. Results

Table I effectively demonstrates the quantized nature of electron charge, which is the same conclusion also found in other publications on the matter[1]. Crucially, we also observe a unit electron charge of $e = 1.695 \times 10^{-19} \pm 1.035 \times 10^{-19}$ coulombs. This is within 5.8% of the commonly accepted value A1. In the comprehensive data, we find a chi-squared parameter $\chi^2 = 0.39$, indicating that uncertainties are indeed very large. This is also clear in 2, where a summary of the data can be seen with the associated uncertainty.

III. DISCUSSION

In this paper, we demonstrate the capability to measure the charge on a microscopic drop of oil. From this, we demonstrate that charge is atomically quantized, given the results in high quality data, seen in table **??** We express charge as a function of atmospheric pressure, air viscosity, and capacitor field. We then observe the motion of the droplets to measure velocity, allowing us to calculate the charge on each droplet. The experimental observations point to the quantized nature of charge. Our results have very large uncertainty due primarily to statistical sources; despite this, our values appear to be similar to the commonly accepted value for electron charge, deviating by approximately 13%. This deviation is constant in all data, leading us to suspect that with the capacity to accurately measure parameters related to viscosity of air and air pressure, it may be possible to achieve higher quality results.

Appendix A: Discussion of Precision and Uncertainty

Throughout this work, we use linear propagation theory for the propagation of all errors via the excellent measurements library in Julia.

We preform several standard statistical assessments of our data, necessary for chi-square testing. First, uncertainty is propagated, although there is a degree of unreliability in this, as there were several values we could not associate a concrete systematic uncertainty with, such as travel distance of the droplets, as we did not have tools available to take such measurements.

Despite this, statistical uncertainty is a much more pervasive issue in our experimental procedure. We compute statistical uncertainty as

$$\sigma = \frac{1}{\sqrt{n}}$$

where n is the number of measurements taken. Furthermore, there is significant statistical error in human elements of the measurements, namely with reaction time.

This could be reduced to quantified systematic uncertainty with the aid of video recording or projector systems for viewing the droplets, however we choose to compensate by filtering data to within 2 standard deviations of the mean.

Our z-scores are given as

$$z = \left(\frac{q}{n} - e\right) \cdot \frac{1}{\sqrt{\sigma_{\text{sys}}^2 + \sigma_{\text{stat}}^2}}$$

Where q is droplet charge, n is charges on droplet, e is the commonly held electron charge, σ_{sys} is systematic uncertainty, and σ_{stat} is statistical uncertainty. Our chi-squared parameter is calculated by taking the sum of z-scores squared.

Comparing our experimental result with the accepted value of the elementary charge [1], 1.602×10^{-19} C,

$$\% \text{ error} = \left(\frac{|1.695 - 1.602|}{1.602}\right) \times 100\% \approx 5.8\% \quad \text{(A1)}$$

Appendix B: Comprehensive Data

Row	Charge (C)	# Charges	Statistical Uncertainty	Systematic Uncertainty	Z-Score
A	1.695×10^{-19}	1.0 ± 0.033	3.162×10^{-20}	5.639×10^{-21}	0.29 ± 0.18
B	1.146×10^{-18}	6.76 ± 0.21	3.536×10^{-20}	3.628×10^{-20}	0.18 ± 0.11
C	1.167×10^{-18}	6.89 ± 0.22	3.536×10^{-20}	3.767×10^{-20}	0.18 ± 0.11
D	4.903×10^{-19}	2.893 ± 0.093	4.472×10^{-20}	1.576×10^{-20}	0.2 ± 0.11
E	8.033×10^{-19}	4.74 ± 0.15	3.333×10^{-20}	2.59×10^{-20}	0.22 ± 0.13
F	5.955×10^{-19}	3.51 ± 0.11	3.536×10^{-20}	1.895×10^{-20}	0.23 ± 0.13
G	2.82×10^{-19}	1.664 ± 0.054	1.0×10^{-19}	9.216×10^{-21}	0.092 ± 0.055
H	2.422×10^{-19}	1.429 ± 0.047	1.0×10^{-19}	7.942×10^{-21}	0.093 ± 0.055
I	1.24×10^{-18}	7.32 ± 0.23	1.0×10^{-19}	3.905×10^{-20}	0.086 ± 0.05
J	3.389×10^{-19}	2.0 ± 0.065	1.0×10^{-19}	1.108×10^{-20}	0.092 ± 0.055
K	6.326×10^{-19}	3.73 ± 0.12	1.0×10^{-19}	2.025×10^{-20}	0.091 ± 0.053
L	8.313×10^{-19}	4.9 ± 0.15	1.0×10^{-19}	2.609×10^{-20}	0.09 ± 0.051
M	7.709×10^{-19}	4.55 ± 0.14	1.0×10^{-19}	2.421×10^{-20}	0.09 ± 0.052
N	5.7×10^{-19}	3.36 ± 0.11	1.0×10^{-19}	1.821×10^{-20}	0.091 ± 0.053
O	1.277×10^{-18}	7.54 ± 0.24	1.0×10^{-19}	4.06×10^{-20}	0.086 ± 0.05
P	2.577×10^{-18}	15.2 ± 0.48	1.0×10^{-19}	8.154×10^{-20}	0.072 ± 0.042
Q	3.151×10^{-19}	1.859 ± 0.06	1.0×10^{-19}	1.011×10^{-20}	0.092 ± 0.054
R	8.048×10^{-19}	4.75 ± 0.15	1.0×10^{-19}	2.621×10^{-20}	0.09 ± 0.053
S	1.184×10^{-18}	6.99 ± 0.22	1.0×10^{-19}	3.797×10^{-20}	0.087 ± 0.051

[1] R. A. Millikan, *The Electron: Its Isolation and Measurement and the Determination of Some of Its Properties* (The University of Chicago Press, Chicago, IL, 1917).

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[4] Contributors to Physics Stack Exchange, Stokes' law for small oil droplet correction factor, Physics Stack Exchange (2017), [Online; accessed 8-February-2025].

[5] D. J. Griffiths, *Introduction to Electrodynamics*, 4th ed. (Cambridge University Press, 2017).