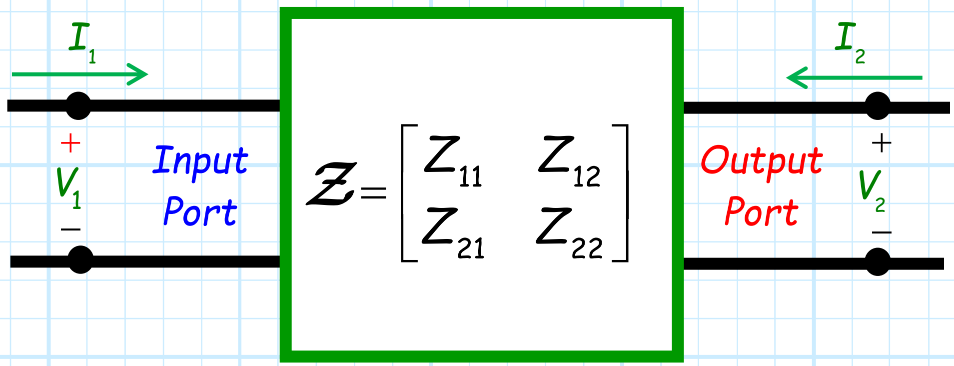
Two-port Networks

Many important microwave components are **two-port** networks (e.g., filters, amplifiers, attenuators).

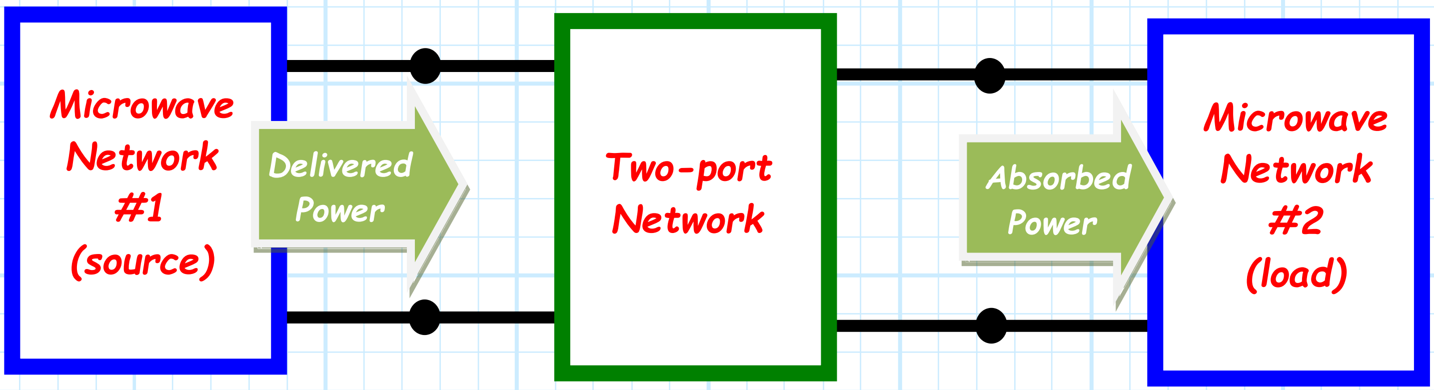


$$
\begin{aligned}
V_1 &= Z_{11} I_1 + Z_{12} I_2 \\
V_2 &= Z_{21} I_1 + Z_{22} I_2
\end{aligned}
$$

For these devices, the microwave signal power **enters** one port (i.e., the **input**) and **exits** the other (the **output**).

The two-port device does something—otherwise, what's the point?

The device typically does something to **alter** the signal as it passes from input to output (e.g., filters it, amplifies it, attenuates it).



We can thus assume that a (equivalent) **source** is connected to the **input** port, and that a (equivalent) **load** is connected to the **output** port.

Energy comes out of the source, but goes into the load

Again, the **source** network may be **quite complex**, consisting of many microwave components.

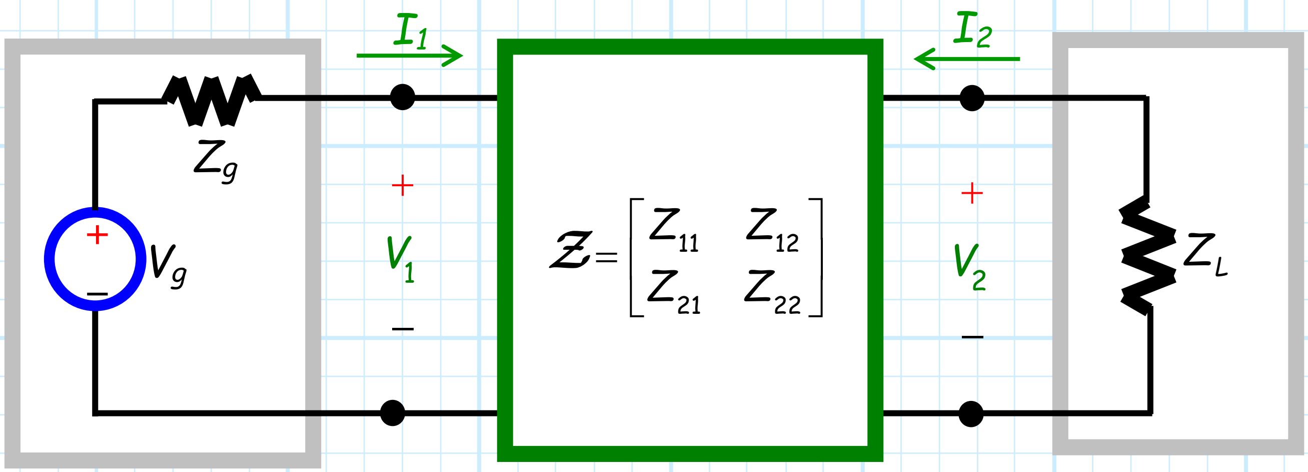
However, at least one of these components must be a **source** of microwave energy (e.g., a receive antenna or oscillator).

Likewise, the **load** network might be **quite complex**, consisting of many microwave components.

However, at least one of these components must be a **sink** of microwave energy (e.g., a transmit antenna or resistor).

The Equivalent Circuit

Now, we can use our **equivalent circuits** to model this system:

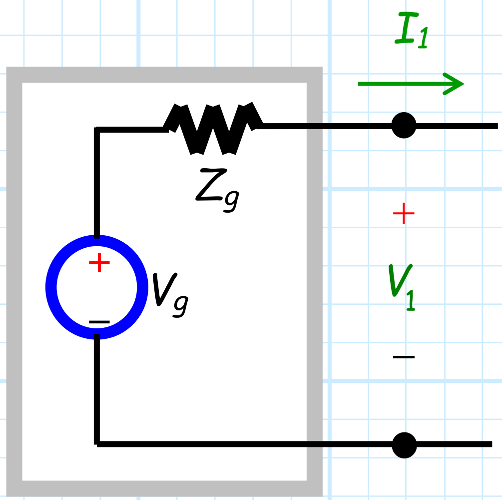


Note in this circuit there are **4 unknown values**—two voltages (V_1 and V_2), and two currents (I_1 and I_2).

→ Our job is to **determine** these 4 unknown values!

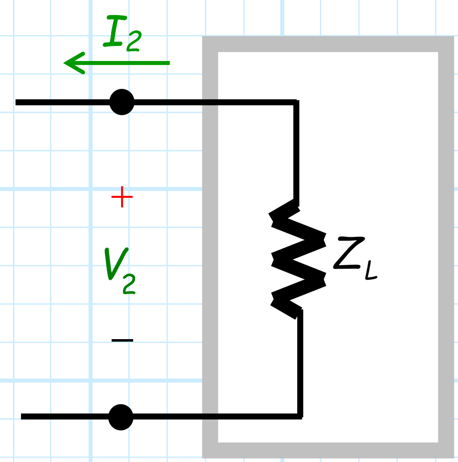
So, What Do We Know?

Let's begin by looking at the **source**, we can determine from **KVL** that:



$$I_1 = \frac{V_g - V_1}{Z_g}$$

And for the **load** we apply **Ohm's Law**:

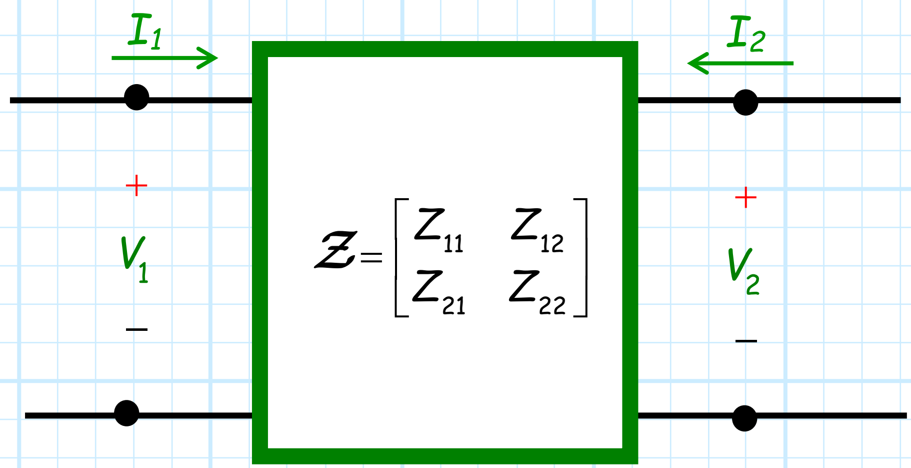


Note the **minus** sign! →

$$I_2 = -\frac{V_2}{Z_L}$$

Ohm's Law for a two-port device

Now for **two-port network**.



If we know the **impedance matrix** (i.e., **all four** trans-impedance parameters), then the voltages and currents must be related as:

$$\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}$$

We can solve for the 4 unknowns!

Now let's take stock of our results.

Notice that we have compiled **four** independent equations, involving our **four** unknown values:

$$I_1 = \frac{V_g - V_1}{Z_g}$$

$$I_2 = -\frac{V_2}{Z_L}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

We can apply a bit of **algebraic** pixie dust and solve for the unknown **complex** currents and voltages.

Apply sanity check—then forget them

Behold, the **complex** currents and voltages:

$$I_1 = V_g \frac{Z_{22}+Z_L}{(Z_{11}+Z_g)(Z_{22}+Z_L)-Z_{12}Z_{21}} \qquad V_1 = V_g \frac{Z_{11}(Z_{22}+Z_L)-Z_{12}Z_{21}}{(Z_{11}+Z_g)(Z_{22}+Z_L)-Z_{12}Z_{21}}$$

$$I_2 = -V_g \frac{Z_{21}}{(Z_{11}+Z_g)(Z_{22}+Z_L)-Z_{12}Z_{21}} \qquad V_2 = V_g \frac{Z_L\, Z_{21}}{(Z_{11}+Z_g)(Z_{22}+Z_L)-Z_{12}Z_{21}}$$