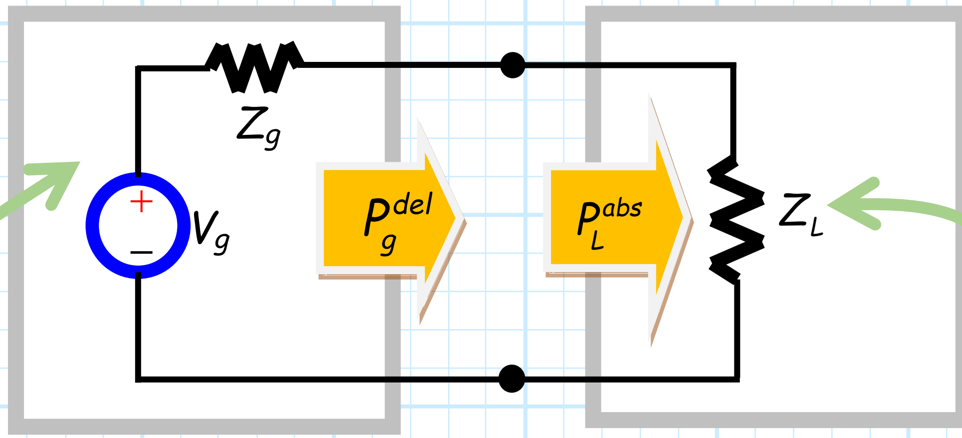
Special Cases of Source and Load Impedance

Consider again the power **absorbed** by the load (delivered by the source):

$$P_g^{del} = P_L^{abs} = \frac{|V_g|^2}{2} \frac{R_L}{|Z_g + Z_L|^2}$$



It is evident that this power transfer is dependent on **each** and **every element** of the equivalent circuit:

1. the **source parameters** V_g and Z_g,
2. as well as the **load impedance** Z_L.

You would think that maximum power transfer would not be so controversial

Q: *I assume that we want to **maximize** this power transfer.*

***How** can we maximize P_L^{abs}??*

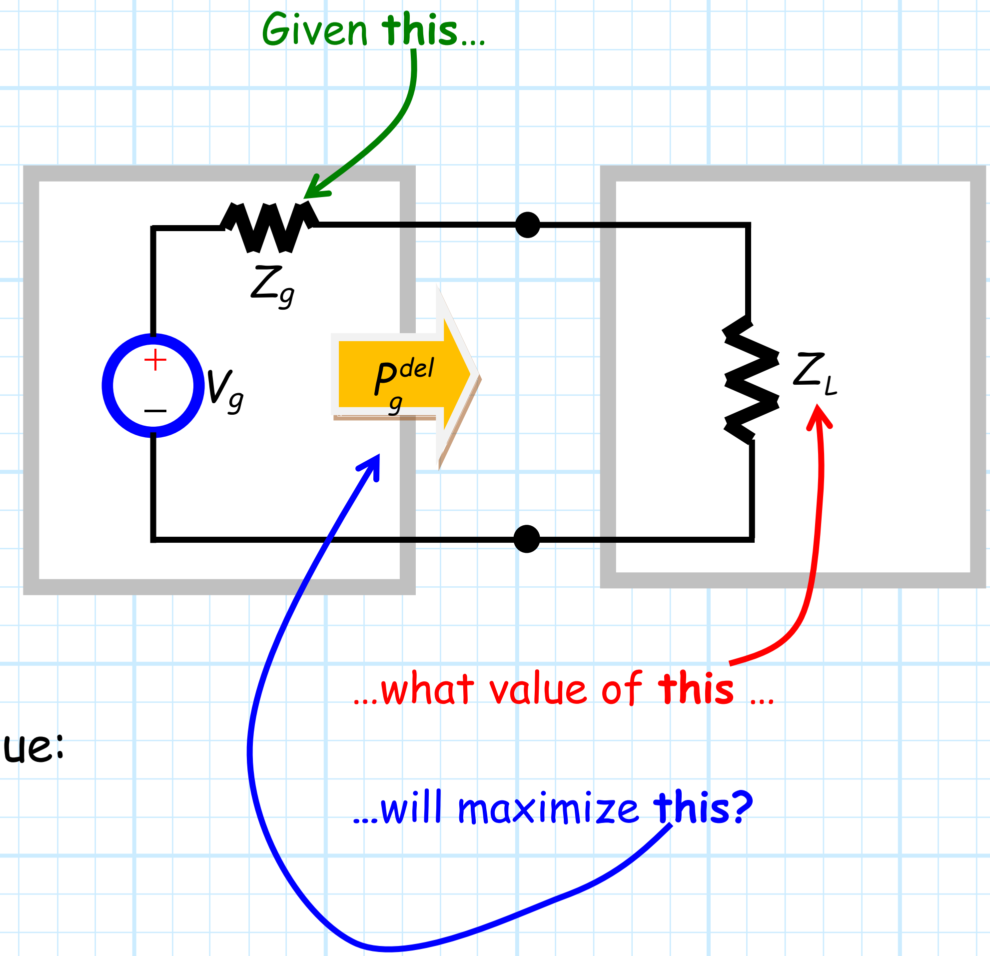
A: The answer to that question is among the **best known** in electrical engineering.

→ Unfortunately, it is also frequently **misunderstood** and misapplied—**so pay attention**!

Carefully consider this question

First, let's ask **this** question:

Q1: *What **load** impedance Z_L will maximize the power delivered by a **given source** (i.e., maximize P_g^{del})?*



A1: A **load** impedance with value:

$$Z_L = Z_g^*$$

will maximize the power delivered by the **source**.

The available power of the source

We can likewise determine the **value** of this maximum power!

For $Z_L = Z_g^*$, we find:

$$P_g^{del}\Big|_{Z_L=Z_g^*} = \frac{|V_g|^2}{2}\frac{R_g}{|Z_g+Z_g^*|^2} = \frac{|V_g|^2}{2}\frac{R_g}{|2R_g|^2} = \frac{|V_g|^2}{8R_g}$$

This maximum delivered power is **very important**; it is dubbed the **available power** P_g^{avl} **of the source**:

$$\boxed{P_g^{avl} = \frac{|V_g|^2}{8R_g}}$$

Note the available power is dependent **just on source parameters** (i.e., V_g and R_g).

And so P_g^{avl} is a parameter of the **source only**!

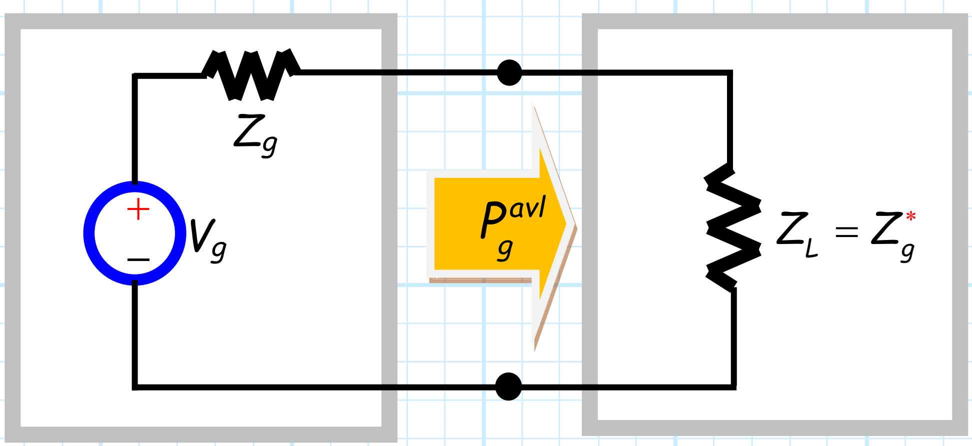
Available power is an upper limit

Available power is the **maximum**—the upper **limit** on—power that can be **delivered** by a source, i.e.:

$$P_g^{del} \leq P_g^{avl}$$

This **available** source power—the **maximum** possible—is delivered **only if** the **load impedance** is numerically equal to the complex conjugate of the source impedance (i.e., $Z_L = Z_g^*$):

$$P_g^{del} = P_g^{avl} = \frac{|V_g|^2}{8R_g} \quad \textit{iff} \quad Z_L = Z_g^*$$

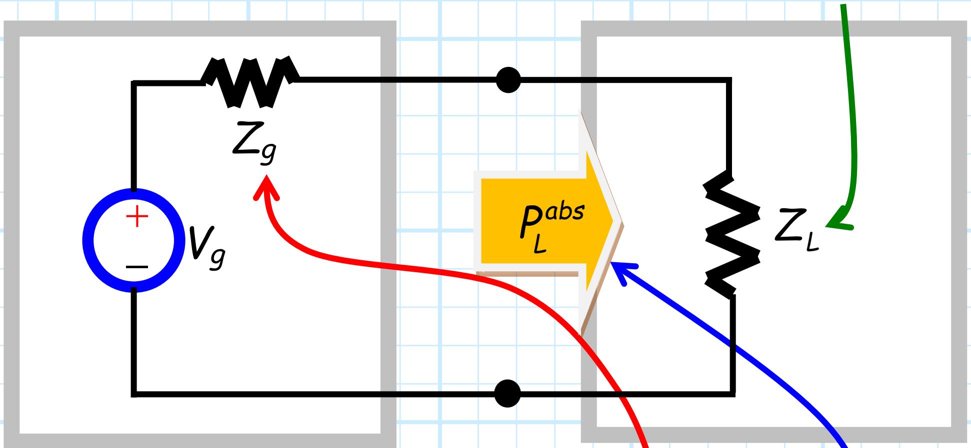


Consider this completely different question!

Now, let's ask a **completely different question**:

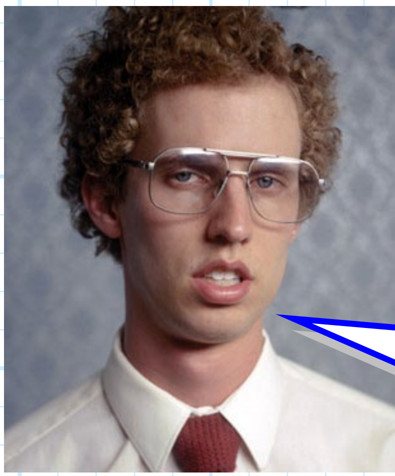
Q2: *What **source** impedance Z_g will maximize the power absorbed by a **given load** (i.e., maximize P_L^{abs})?*

Given **this**...



...what value of **this** ...

...will maximize **this?**



*Like, don't we already know the answer? Absorbed power is maximized by a **conjugate match**:*

$$Z_g = Z_L^* . \qquad \textit{Gosh!}$$

NOT!

A2: Not. So. Fast.

It can be shown that the value of **source** impedance Z_g that maximizes the power **absorbed by the load** Z_L is—in fact—**purely reactive,** with value:

$$\boxed{Z_g = -j\,X_L}$$

where X_L is the imaginary (i.e., **reactive** portion of the load ($X_L = \text{Im}\{Z_L\}$)!

The importance of asking the right question

Thus, we conclude a **correct** question—and answer—is:

Q2: *What **source** impedance Z_g will maximize the power absorbed by a **given load** (i.e., maximize P_{abs})?*

A2: The **source** impedance $Z_g = -jX_L$ will maximize the power absorbed by a **given load** (i.e., maximize P_{abs}).

Although it is **very common** for electrical engineers to assume the answer to question **Q2** is **instead** answer ***A1*** (i.e., $Z_g = Z_L^*$), this is **far** from the correct answer!

Question **Q1** and question **Q2** are **completely different**—it should be **no** surprise that **answers** ***A1*** and ***A2*** are completely different as well.

Here; I'll prove it to you

Using the **correct** solution for **Q2** ($Z_g = -jX_L$), we find the power absorbed by the load is then:

$$P_L^{abs}\Big|_{Z_g=-jX_L} = \frac{|V_g|^2}{2}\frac{R_L}{|Z_g+Z_L|^2} = \frac{|V_g|^2}{2R_L}$$

Whereas, if we enforce a **"conjugate match"**, by setting the source impedance to be $Z_g = Z_L^*$, the load **instead** absorbs energy at a rate:

$$P_L^{abs}\Big|_{Z_g=Z_L^*} = \frac{|V_g|^2}{2}\frac{R_L}{|Z_L^*+Z_L|^2} = \frac{1}{4}\frac{|V_g|^2}{2R_L}$$

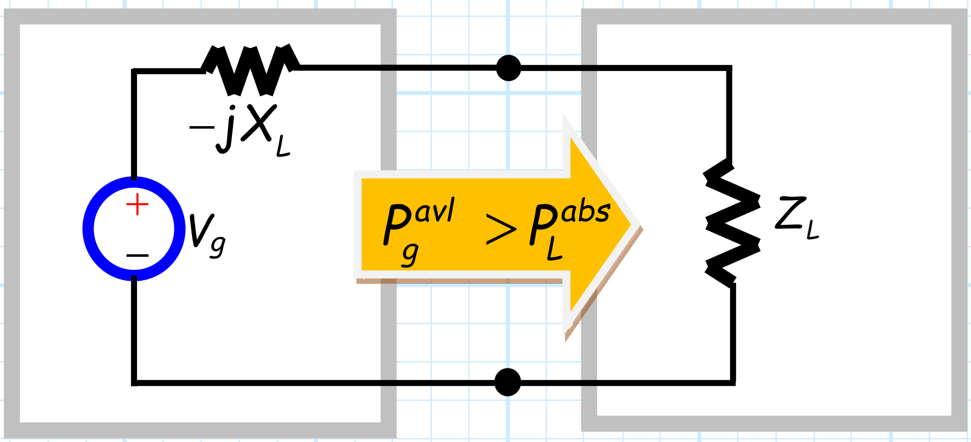
The power absorbed by the load when $Z_g = Z_L^*$ is just **25%** of the power absorbed when $Z_g = -jX_L$!

It's maximum—but far less than the available

Q: *But if $Z_L \neq Z_g^*$, isn't the absorbed power **less** than the **available** power ??*

A: Darn right it's **not**!

If $Z_g = -jX_L$, the absorbed power is **far less** than the **available power**.



Dazed and Confused

Q: *I'm so **confused!***

*I thought you said that setting $Z_g = -jX_L$ actually **maximizes** the absorbed power.*

*How can the power be maximized if it is **less** than the **available** power??*

A: Here's the deal; setting the source impedance to $Z_g = -jX_L$ (i.e., $R_g = 0$) **does** in fact maximize the power absorbed by the load.

But, it **also** increases the available power of the source—increases it all the way to **infinity** (but **not** beyond)!

$$\lim_{R_g \rightarrow 0} P_g^{avl} = \lim_{R_g \rightarrow 0} \frac{|V_g|^2}{8R_g} = \infty$$

A finite value is always less than infinity!

The maximized absorbed power (when Z_g is set to $Z_g = -jX_L$) is **finite**—therefore it is much less than the **infinite** available power!

$$P_L^{abs}\Big|_{Z_g=-jX_L} = \frac{|V_g|^2}{2R_L} \ll P_g^{del}\Big|_{Z_g=-jX_L} = P_g^{avl} = \infty$$

Contrast this with altering the value of **load** impedance Z_L (i.e., **Q1**).

Altering the **load** changes the **delivered** power P_g^{del} but does **not** alter the **available** power P_g^{avl} of the source.

Thus, the best we can do is set Z_L such that **all available power(100%)** is delivered to the load (i.e., set $Z_L = Z_g^*$).

First find the biggest plate of cookies, then eat as many of them as you can

Of course, achieving **infinite** available power is **not practical**—the available power P_g^{avl} of any **realizable** source is **finite**.

Still, engineers attempting to **maximize** the power **absorbed by a load** should:

1. Attempt to select/design/alter the **source** such that its **available power** P_g^{avl} **is maximized.**

2. Then, attach a **load** that is conjugate matched ($Z_L = Z_g^*$) to this source, such that **all available power is delivered** to the load.

1% of Bill Gate's $$$, or 100% of mine?

A problem that often arises is that a source with a **large available power** usually has likewise a very **low source impedance**,

→ This makes it is **difficult/impractical** to provide a load where $Z_L = Z_g^*$.

Engineers sometimes **erroneously** alter/design/select **another source** that it **easier** to "match", but usually this results in a dramatic **decrease in available power**!

For **example**, consider two cases:

Source	Available Power	Delivered Power
1	500 mW	200 mW
2	100 mW	100 mW

→ For **which** source is "power transfer maximized"?

Most of a lot is better than all of very little

For source 2, **100%** of the **available power** is delivered to the load—clearly the **load is a conjugate match** to the source impedance.

For source 1, **only 40%** of the available power is delivered to the load—the load is most definitely **not conjugate matched** to source impedance.

Yet, the **mismatched load** absorbs **twice** the power of the "mismatched" case!

It does so because the available power of source 1 is **five times** larger than that of source 2.

→ It's better to have **most** of **a lot,** rather than **all** of very **little**!!

Be careful!

Hence, we need to be **careful** when considering a conjugate match.

Ask yourself, what does "maximum power transfer" **really** mean?

Is **your** design problem described by **Q1** or by **Q2**?

→ Matching a **load to a source** is a **good** idea, but altering the **source** to match the **load** is typically **not**.

These questions have—and continue to—spark many **unpleasant disagreements** among electrical engineers!!