MATH 526: Homework #6

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Problem 1

Given a continuous uniform distribution with support on the interval $[A, B]$, we want to show that:

(a) $\mu = \frac{A+B}{2}$

Solution:

$$\frac{1}{b-a}\int_a^b x\,dx = \frac{a+b}{2}$$

(b) $\sigma^2 = \frac{(B-A)^2}{12}$

Solution:

$$\begin{aligned}\sigma^2 &= \langle x\rangle^2 - \langle x^2\rangle \\ &= \left(\frac{a+b}{2}\right)^2 - \frac{1}{b-a}\int_a^b x^2\,dx \\ &= \frac{(b-a)^2}{12}\end{aligned}$$

Problem 4

A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution.

(a) What is the probability that the individual waits more than 7 minutes?

Solution:

$$\int_7^{10} \frac{1}{10}\,dx = \frac{3}{10}$$

(b) What is the probability that the individual waits between 2 and 7 minutes?

Solution:

$$\int_2^7 \frac{1}{10}\, dx = \frac{1}{2}$$

Problem 5

Given a standard normal distribution, find the area under the curve that lies:

(a) to the left of $z = -1.39$;

Solution: For $\sigma = 1$, $\mu = 0$, $-1.39 \leq x$ which equals 0.0823.

(b) to the right of $z = 1.96$;

Solution: 0.0249979

(c) between $z = -2.16$ and $z = -0.65$;

Solution: 0.258-0.0154 = 0.2426

(d) to the left of $z = 1.43$;

Solution: 0.924

(e) to the right of $z = -0.89$;

Solution: 0.813

(f) between $z = -0.48$ and $z = 1.74$.

Solution: 0.959-0.316=0.643

Problem 6

Find the value of z if the area under a standard normal curve:

(a) to the right of z is 0.3622;

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Solution: z=0.353

(b) to the left of z is 0.1131;

Solution: z=-1.21

(c) between 0 and z, with $z > 0$, is 0.4838;

Solution: z=2.14

(d) between $-z$ and z, with $z > 0$, is 0.9500.

Solution: z = $\pm$ 1.96

Problem 7

Given a standard normal distribution, find the value of k such that:

(a) $P(Z > k) = 0.2946$

Solution: k=0.540

(b) $P(Z < k) = 0.0427$

Solution: k=-1.72

(c) $P(-0.93 < Z < k) = 0.7235$

Solution: k=0.900

Problem 8

Given a normal distribution with $\mu = 30$ and $\sigma = 6$, find:

(a) the normal curve area to the right of $x = 17$;

Solution: 0.00230

(b) the normal curve area to the left of $x = 22$;

Problem 8 continued on next page...

Solution: 0.0912

(c) the normal curve area between $x = 32$ and $x = 41$;

Solution: 0.967-0.631=0.336

(d) the value of x that has 80% of the normal curve area to the left;

Solution: 35.0

(e) the two values of x that contain the middle 75% of the normal curve area.

Solution: $x = \pm 36.9$

Problem 12

The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assuming that the lengths are normally distributed, what percentage of the loaves are:

(a) longer than 31.7 centimeters?

Solution: 0.198

(b) between 29.3 and 33.5 centimeters in length?

Solution: 0.960-0.363 = 0.597

(c) shorter than 25.5 centimeters?

Solution: 0.0122

Problem 18

The heights of 1000 students are normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Assuming that the heights are recorded to the nearest half-centimeter, how many of these students would you expect to have heights:

(a) less than 160.0 centimeters?

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Solution: 0.0178

(b) between 171.5 and 182.0 centimeters inclusive?

Solution: 0.861-0.332=0.529

(c) equal to 175.0 centimeters?

Solution: 0.471

(d) greater than or equal to 188.0 centimeters?

Solution: 0.025

Problem 24

A coin is tossed 400 times. Use the normal curve approximation to find the probability of obtaining:

(a) between 185 and 210 heads inclusive;

Solution: 0.841-0.0669=0.7741

(b) exactly 205 heads;

Solution: 0.779

(c) fewer than 176 or more than 227 heads.

Solution: 0.00820+0.00347=0.117

Problem 26

A process yields 10% defective items. If 100 items are randomly selected from the process, what is the probability that the number of defectives:

(a) exceeds 13?

Solution: 0.159

(b) is less than 8?

Problem 26 continued on next page...

Solution: 0.252

Problem 29

If 20% of the residents in a U.S. city prefer a white telephone over any other color available, what is the probability that among the next 1000 telephones installed in that city:

(a) between 170 and 185 inclusive will be white?

Solution: 0.118-0.00885=0.109

(b) at least 210 but not more than 225 will be white?

Solution: 0.976-0.785=0.191

Problem 32

A pharmaceutical company knows that approximately 5% of its birth-control pills have an ingredient that is below the minimum strength, thus rendering the pill ineffective. What is the probability that fewer than 10 in a sample of 200 pills will be ineffective?

Solution: 0.5

Problem 34

A pair of dice is rolled 180 times. What is the probability that a total of 7 occurs:

(a) at least 25 times?

Solution: The that a sum of 7 occurs is $\frac{1}{6}$, therefore $P(X = 7) \geq 25$ equals 0.841

(b) between 33 and 41 times inclusive?

Solution: 0.986-0.726=0.260

(c) exactly 30 times?

Solution: 0.0798

Problem 55

Computer response time is an important application of the gamma and exponential distributions. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.

(a) What is the probability that response time exceeds 5 seconds?

Solution: $\mu = \beta \implies \lambda = \frac{1}{\mu}$ which equals 1.89

(b) What is the probability that response time exceeds 10 seconds?

Solution: 0.0357