PHSX 631: Homework #1

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Problem 1

Permanent cylindrical magnet with axial magnetization M.

(a) Do Griffith Problem 6.9. Sketch field lines.

(b) Go beyond Griffiths Problem 6.9 by finding the magnetic field as a function of distance, z, along the symmetry axis, and outside the magnet. Assume $L = 2a$. Plot/sketch $B(z)$, $H(z)$, $M(z)$ along the axis. Hint: See Example 5.6 in Griffiths.

(Problem 6.9) A short circular cylinder of radius a and length L carries a "frozen-in" uniform magnetization $\mathbf{M}$ parallel to its axis. Find the bound current, and sketch the magnetic field of the cylinder. (Make three sketches: one for $L \gg a$, one for $L \ll a$, and one for $L \approx a$.) Compare this bar magnet with the bar electret of Prob. 4.11.

Solution, part (a):
We have equations for bound current (volume and surface respectively):

$$\mathbf{J_b} = \nabla \times \mathbf{M} = 0, \qquad \mathbf{K_b} = \mathbf{M} \times \hat{\mathbf{n}} = M\hat{\phi}$$

Or equivalently, $\mathbf{K_b} = (-M\sin\phi, M\cos\phi, 0)$.

Solution, part (b):
As we have bound current, we can use the Biot-Savart law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_S \frac{\mathbf{K_b}(\mathbf{r}) \times \hat{\boldsymbol{\mathscr{r}}}}{|\boldsymbol{\mathscr{r}}|^2}\, dA$$

We have a point of interest at $(0, 0, z)$. The points on the surface of the cylinder are given by $(a\cos\phi', a\sin\phi', z')$. This gives us $\boldsymbol{\mathscr{r}} = (-a\cos\phi', -a\sin\phi', z - z')$ and magnitude $|\boldsymbol{\mathscr{r}}| = (a^2 + (z-z')^2)^{1/2}$. Finally, the cross product is

$$\mathbf{K_b} \times \frac{\boldsymbol{\mathscr{r}}}{|\boldsymbol{\mathscr{r}}|} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -M\sin\phi & M\cos\phi & 0 \\ \frac{-a\cos\phi}{(a^2+(z-z')^2)^{1/2}} & \frac{-a\sin\phi}{(a^2+(z-z')^2)^{1/2}} & \frac{z-z'}{(a^2+(z-z')^2)^{1/2}} \end{vmatrix}$$

$$= M\cos\phi\left(\frac{z-z'}{(a^2+(z-z')^2)^{1/2}}\right)\hat{\mathbf{i}} + M\sin\phi\left(\frac{z-z'}{(a^2+(z-z')^2)^{1/2}}\right)\hat{\mathbf{j}} + \frac{Ma}{(a^2+(z-z')^2)^{1/2}}\hat{\mathbf{z}}$$

The x, y components of this term leave a sin and cos term in the surface integral. This will integrate to zero, as we'd expect. This means that we only need to worry about the z-component for the final result:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{-a}^{a} \frac{Ma\hat{\mathbf{z}}}{(a^2 + (z - z')^2)^{3/2}} \, dz'$$

Problem 2

Consider a square loop of wire with resistance R and size a by a. The surface normal is initially oriented parallel to a uniform magnetic field with magnitude B_0 . The loop is then rotated by 90 deg such that the normal vector is perpendicular to the magnetic field. How much charge passes through the circuit during this procedure?

Problem 3

Do Griffiths problem 7.17

(Problem 7.17) A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R, as shown in Fig. 7.28.

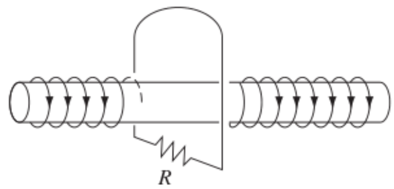

FIGURE 7.28

Problem 4

A very long cylindrical sheet of metal with radius r and length L carries a current K per unit length (azimuthal current) (units of A/m). What is the energy stored in the magnetic field in this cylinder in terms of L, R, and K?