Energy Flow on a Terminated Line

Now consider our complete circuit with respect to **energy flow**.

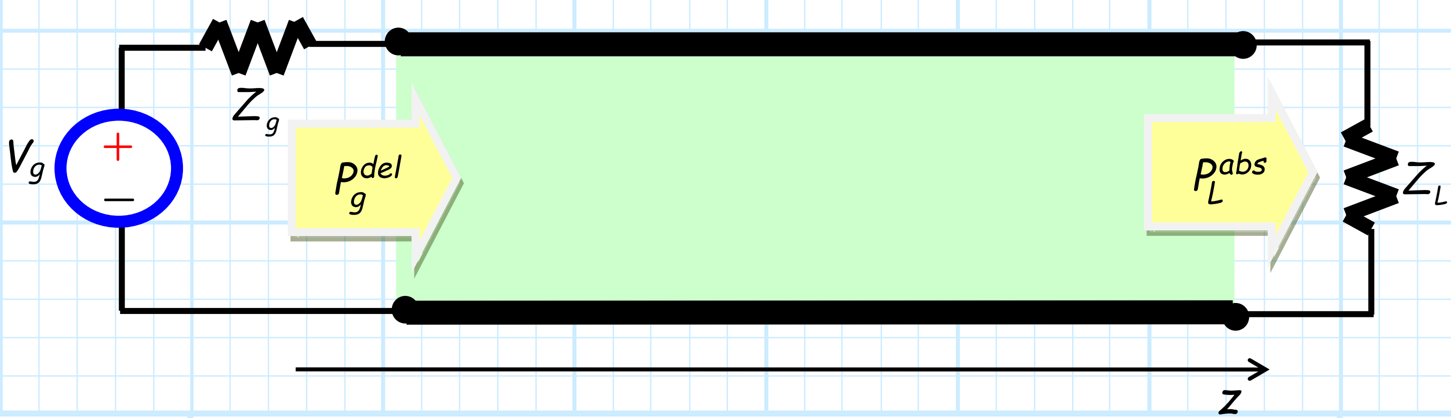
There are **four** power values that we must consider.

The **1st** is P_g^{del}, the rate at which energy is **delivered** by the source.

The **2nd** is P_L^{abs}, the rate at which energy is **absorbed** by the load.

Since the transmission line is **lossless**, we know that these two values must be **equal**:

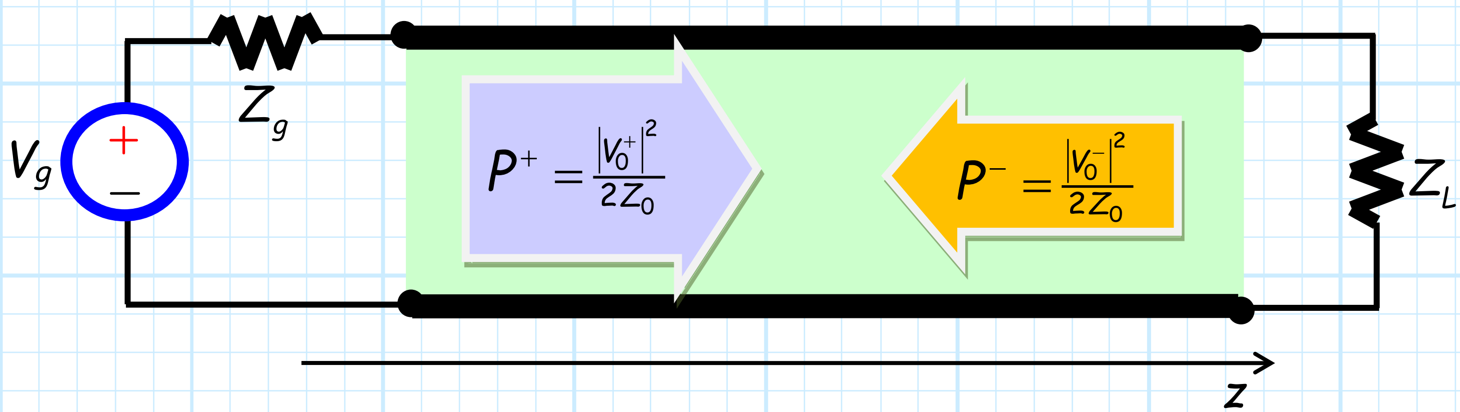
$$P_L^{abs} = P_g^{del}$$



Associated with each wave

The 3rd and 4th power values are P^+ and P^-, the rate of energy flow associated with the each transmission line **wave** function.

$$P^+ = \frac{\left|V_0^+\right|^2}{2\,Z_0} \qquad\qquad P^- = \frac{\left|V_0^-\right|^2}{2\,Z_0}$$

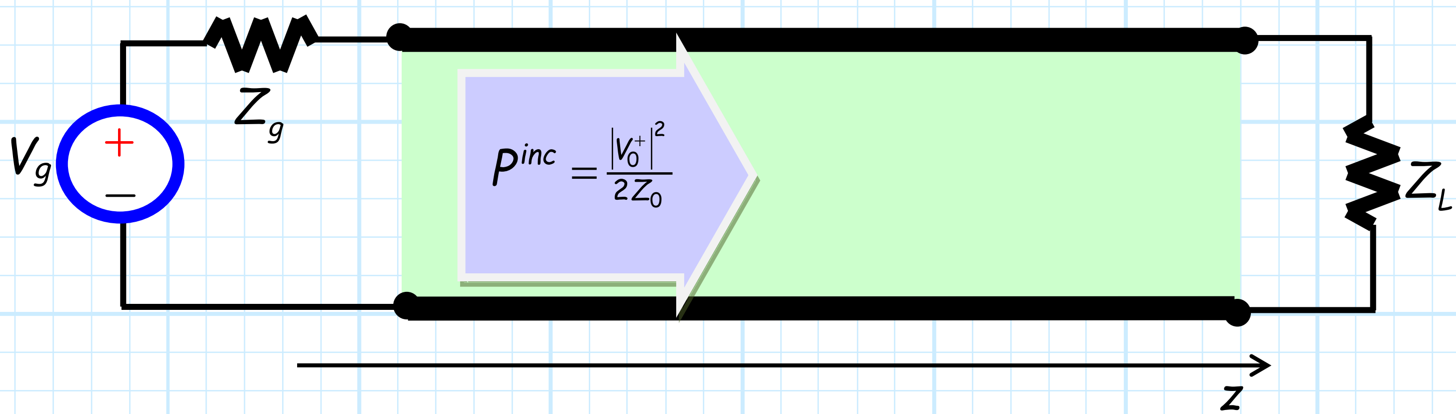


Incident on the load

For a terminated transmission line where the index z **increases** as we move **from** source **to** load, the **plus**-wave energy is flowing **toward** the load, and the minus-wave energy flows **away** from the load.

Thus, the plus-wave power is typically given the **moniker** of **incident power** P^{inc}, as this describes the rate of energy flow **incident on the load**:

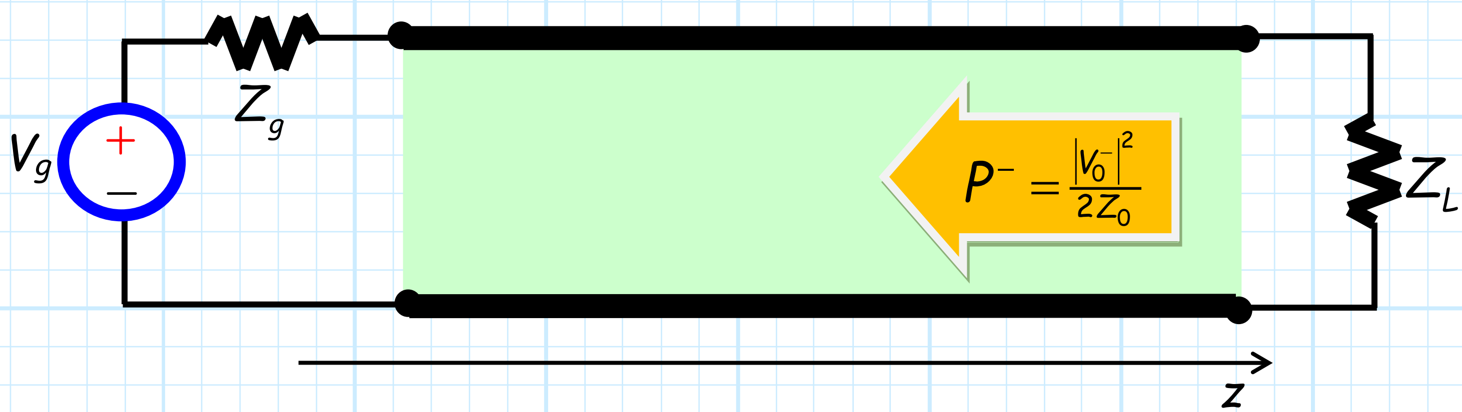
$$P^{inc} \doteq P^{+} = \frac{\left|V_0^+\right|^2}{2Z_0}$$



Reflected from the load

Likewise, the minus-wave power is typically given the **sobriquet** of **reflected power** P^{ref}, as this describes the rate of energy flow moving **away from the load**:

$$P^{ref} \doteq P^{-} = \frac{\left|V_0^{-}\right|^2}{2Z_0}$$



The load determines their ratio

Q: *So how are incident and reflected power* ***related?***

A: Recall from our **boundary condition** that:

$$\Gamma_0 = \frac{V_0^-}{V_0^+} = \Gamma_L \quad \Rightarrow \quad V_0^- = V_0^+ \, \Gamma_L$$

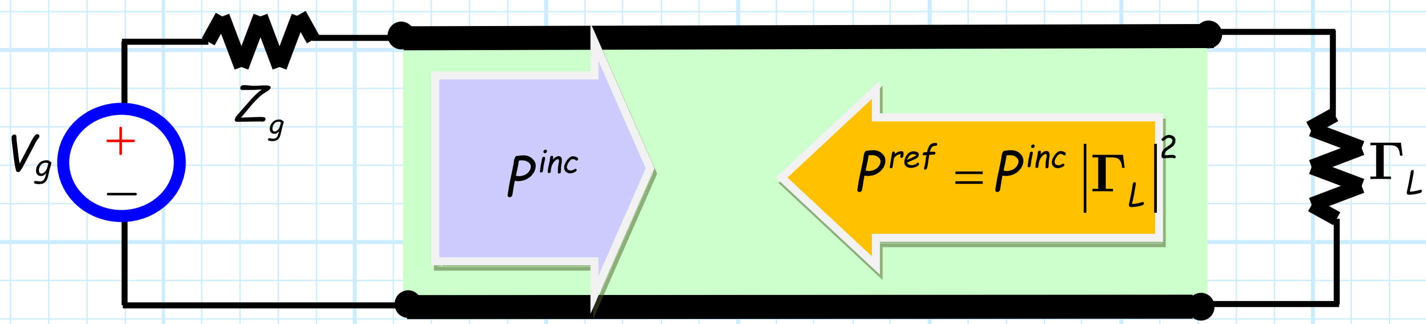
And so the **reflected power** is:

$$P^{ref} = \frac{\left|V_0^-\right|^2}{2Z_0} = \frac{\left|\Gamma_L V_0^+\right|^2}{2Z_0} = \left|\Gamma_L\right|^2 \frac{\left|V_0^+\right|^2}{2Z_0} = \left|\Gamma_L\right|^2 P^{inc}$$

An important result!

Thus we conclude that the **ratio** of reflected and incident power must be equal to the **squared-magnitude** of the **load reflection coefficient**:

$$\frac{P^{ref}}{P^{inc}} = \left|\Gamma_L\right|^2 = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|^2 \leq 1.0$$



→ Note that $P^{inc} \geq P^{ref}$!

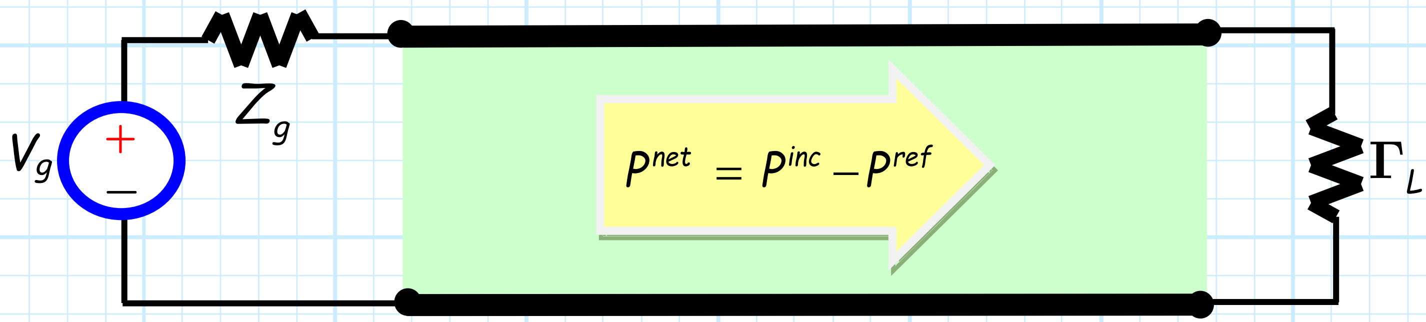
Our old friend: Conservation of Energy

Q: *OK: but how are P^{ref} and P^{inc} related to P_g^{del} and P_L^{abs}?*

A: They're related by **Conservation of Energy**!

Recall the **net** energy flow down the line is the **difference** between the power associated with the **incident** wave and that of the **reflected** wave.

$$P^{net} \doteq P^{inc} - P^{ref} \geq 0$$



→ Note that $P^{inc} = P^{net} + P^{ref}$, so that $P^{inc} \geq P^{ref}$!

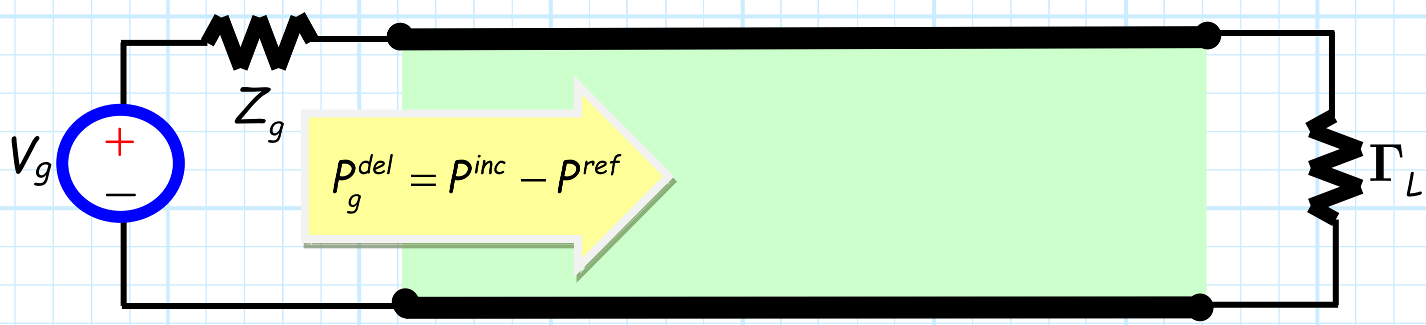
Net power is the delivered power

Recall also that for a lossless line, this **net** rate of energy flow is **constant** along the line.

→ The net power at the **beginning** of the line is the **same** as the net power at the **end**.

But, the net power at the **beginning** of the line is simply the power **delivered by the source**:

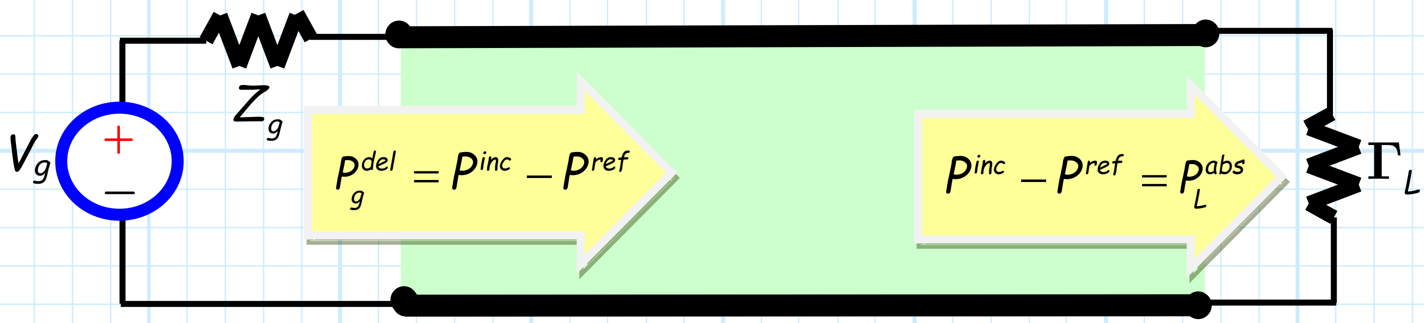
$$P_g^{del} = P^{inc} - P^{ref}$$



→ Note that $P^{inc} = P_g^{del} + P^{ref}$, so that $P^{inc} \geq P_g^{del}$ and $P^{inc} \geq P^{ref}$!

Net power is the absorbed power also

And, the net power at the **end** of the line is simply the power **absorbed by the load**.



→ Note that $P^{inc} = P_L^{abs} + P^{ref}$, so that $P^{inc} \geq P_L^{abs}$ and $P^{inc} \geq P^{ref}$!

Thus, we can conclude:

$$\boxed{P_g^{del} = P^{inc} - P^{ref} = P_L^{abs}}$$

Incident is greater than delivered!

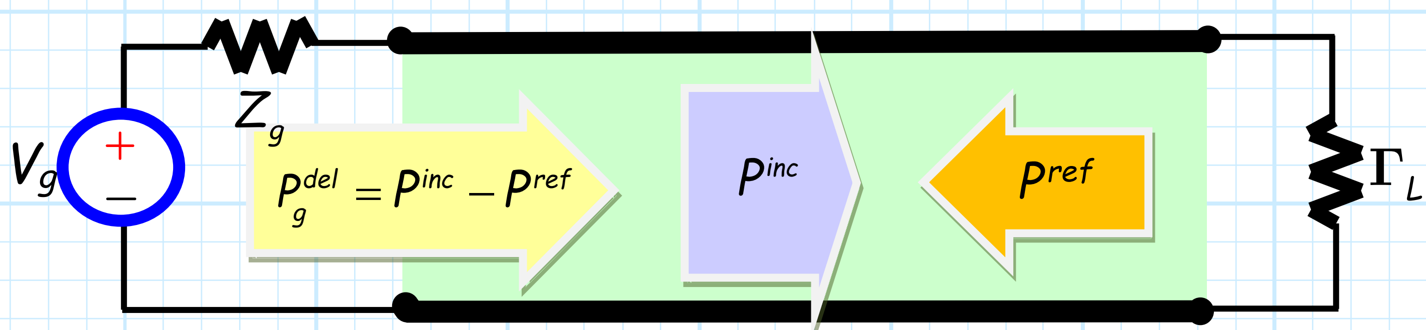
Q: *Wait! You state that the **incident** power is **greater** than the power **delivered** by the source:*

$$P^{inc} \geq P_g^{del}$$

*But, it **seems to me** that the incident power should be **equal** the power delivered by the source (i.e. $P^{inc} = P_g^{del}$)?????*

A: **Students** and engineers often **incorrectly** assume the **incident** power is just the power **delivered** by the source.

→ **But**, if the reflected power is **non-zero**, then the **incident** power **must be greater** than the **delivered** power!!!!!!!!!!!!!!!!

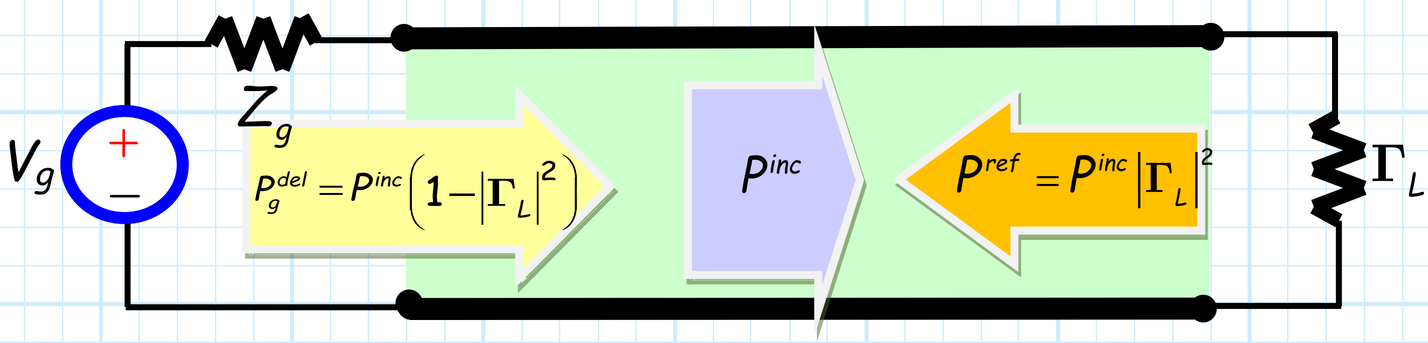


See?

To emphasize this, we express the conservation of energy equation in terms of the **load reflection coefficient**:

$$
\begin{aligned}
P_g^{del} &= P^{inc} - P^{ref} \\
&= P^{inc} - |\Gamma_L|^2 P^{inc} \\
&= P^{inc}\left(1 - |\Gamma_L|^2\right)
\end{aligned}
$$

Clearly, if $|\Gamma_L| > 0$, then $P^{inc} \geq P_g^{del}$!



Clear enough?

$$\boxed{P_g^{del} = P^{inc} - P^{ref} = P_L^{abs}}$$

$$\boxed{P^{inc} \geq P_g^{del}}$$