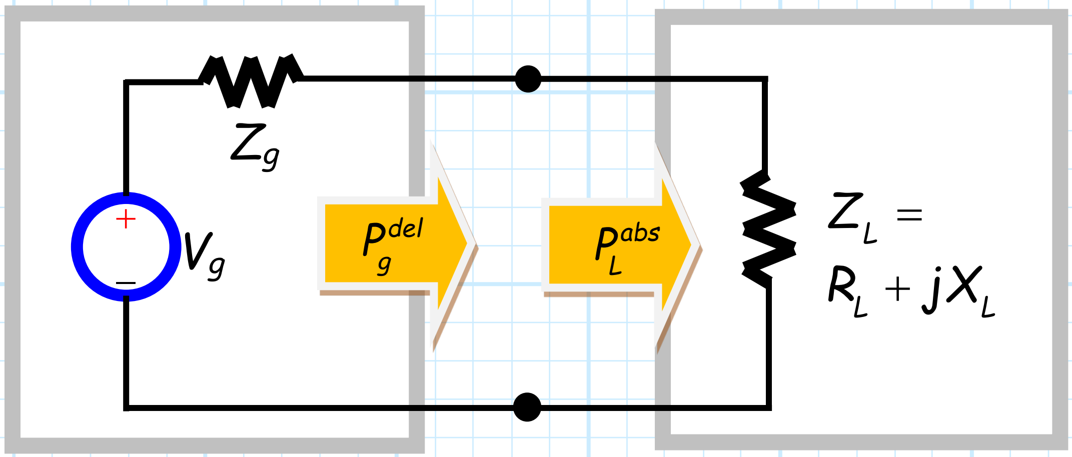
Matching Networks

Consider again the problem where a **load** is directly attached to a **source**:



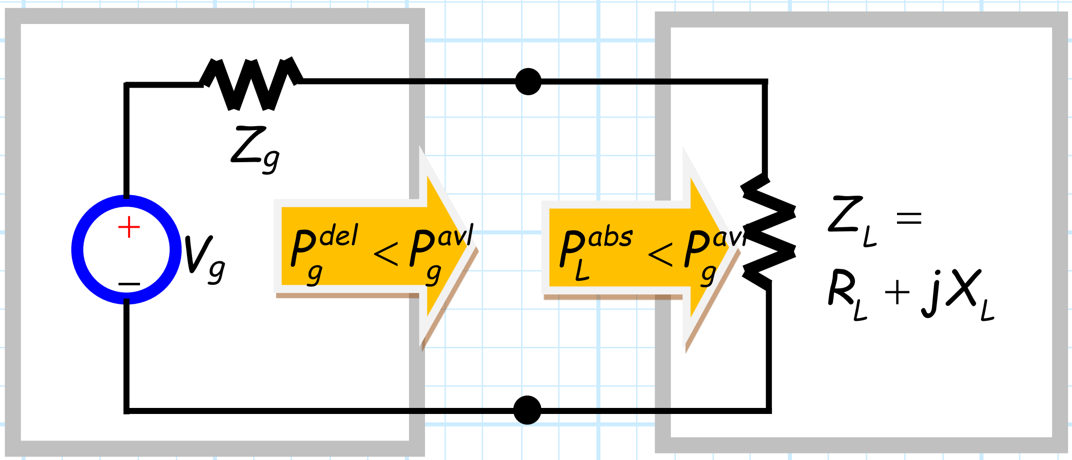
The load will **absorb energy**—at a rate equal to that **delivered** to it by the **source**.

$$P_L^{abs} = \frac{1}{2} Re\{V_L I_L^*\} = \frac{1}{2} |V_g|^2 \frac{R_L}{|Z_g + Z_L|^2} = P_g^{del}$$

Less than what's available

Generally speaking though, this absorbed/delivered power will be **less** than the power **available** from the source.

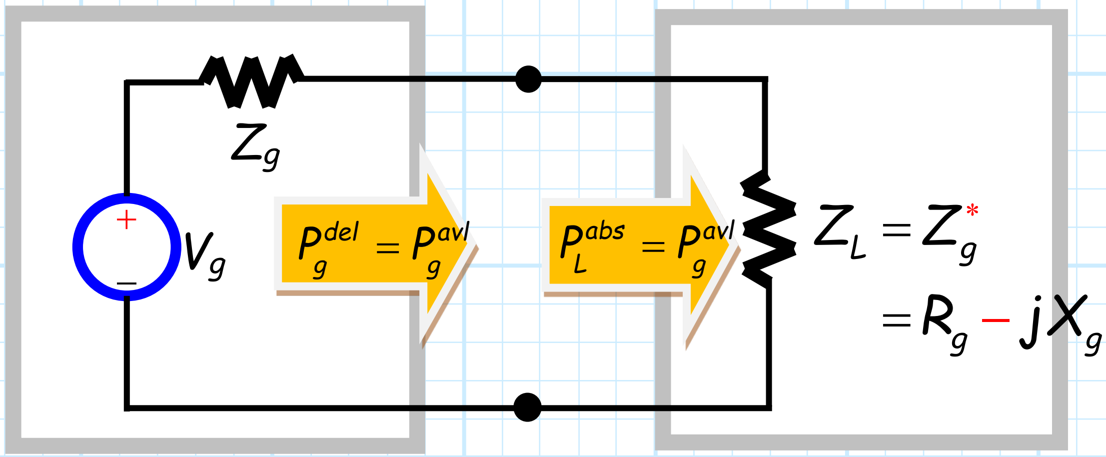
$$P_g^{avl} = \frac{|V_g|^2}{8\,R_g}$$



If we could only change Z_L…

Only if we **alter** the load impedance Z_L—such that it is a "**conjugate match**" to the source—will **all** the **available** source power be released.

$$P_g^{avl} = \frac{|V_g|^2}{8R_g}$$



…but we can't change Z_L !

Q: *But, **you** said that load impedance Z_L is usually just the **equivalent** circuit of some more **useful** device or network.*

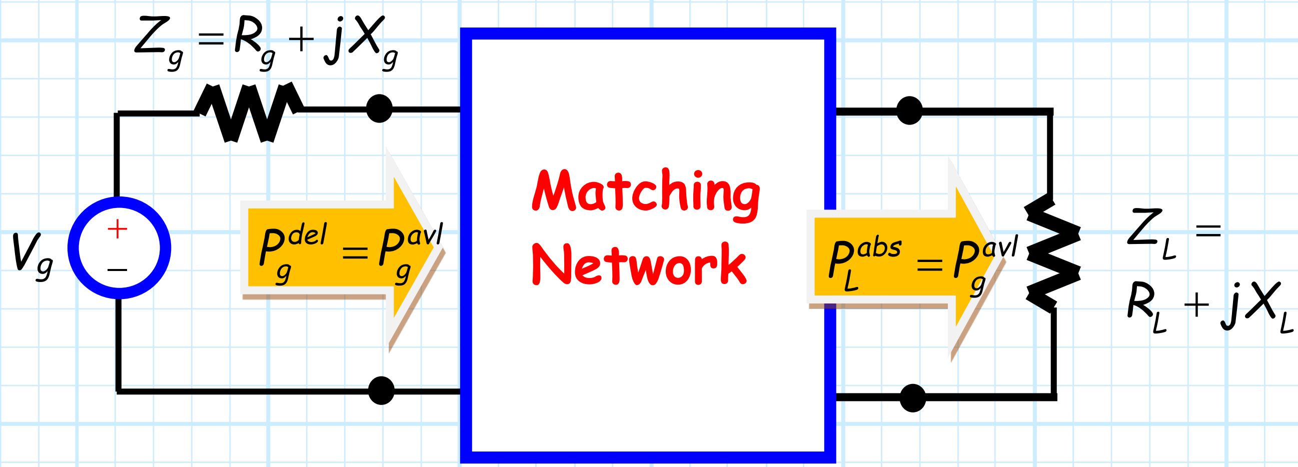
*We **don't** then typically get to "**alter**" or select this impedance—it **is** what it **is**.*

*Must we then simply **accept** the fact that the absorbed power will be **less** than the power available from the source?*

A: NO! We **can** in fact modify our circuit such that **all** available source power **is** delivered to the load—**without** in any way **altering** the impedance value of that **load**!

Eat cake; have it too!

To accomplish this, **we must insert a two-port network—a matching network—between the source and the load**:



If properly designed, we find that **all available power** from the source is indeed **absorbed** by the (unaltered) load Z_L:

$$P_g^{del} = P_g^{avl} = P_L^{abs} \quad !!!!!!$$

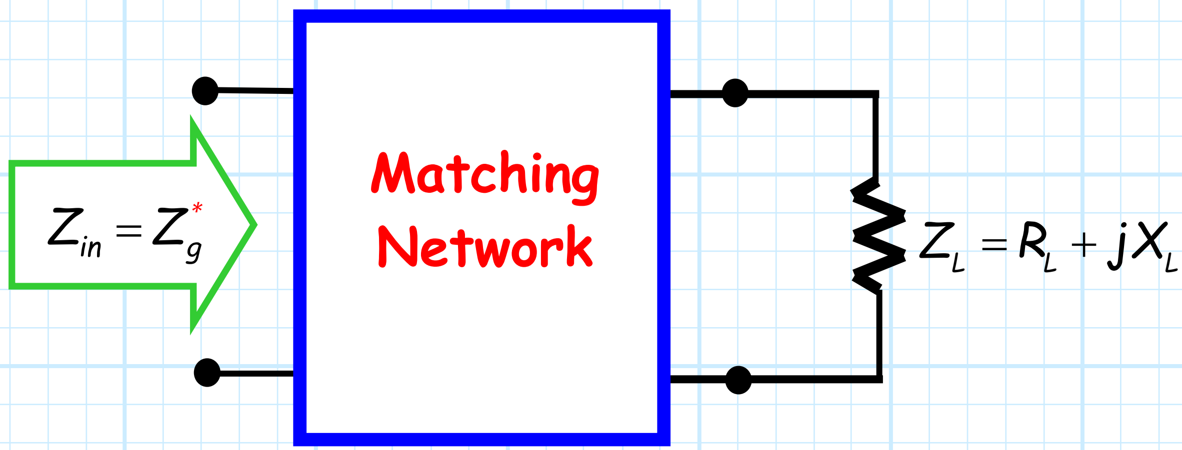
An impedance transformer

Q: *But how can this be?*

*After all, **load** impedance Z_L is **not conjugate matched** to source impedance Z_g:*

$$Z_L \neq Z_g^* \;\;!!$$

A: We can view the matching network as an **impedance transformer**—one whose sole purpose is to "**transform**" the **load** impedance Z_L into an **input** impedance Z_{in} that **is conjugate matched** to the source! I.E.:

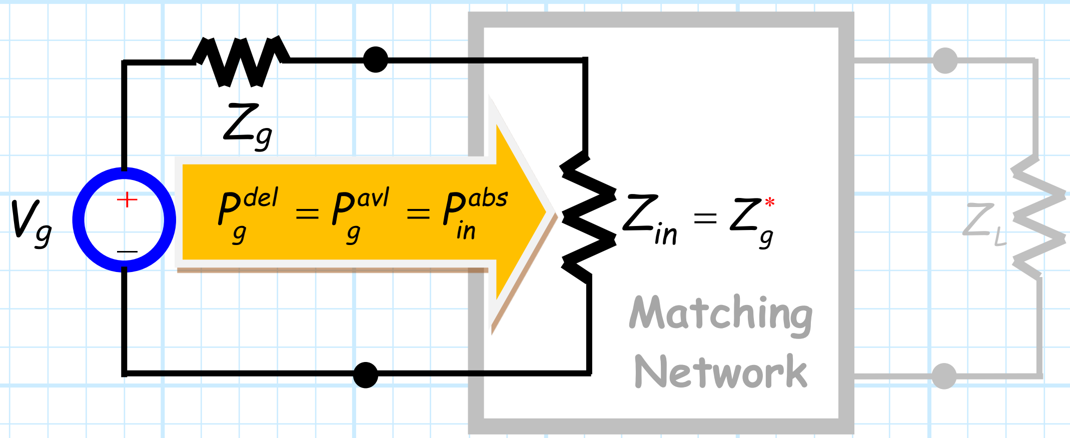


All available power is delivered to Z_{in}

Because the **input** impedance Z_{in} is **conjugate matched** to the source impedance:

$$Z_{in} = Z_g^*$$

all available source power is absorbed by the **input** impedance Z_{in} of the matching network:



Don't forget conservation of energy!

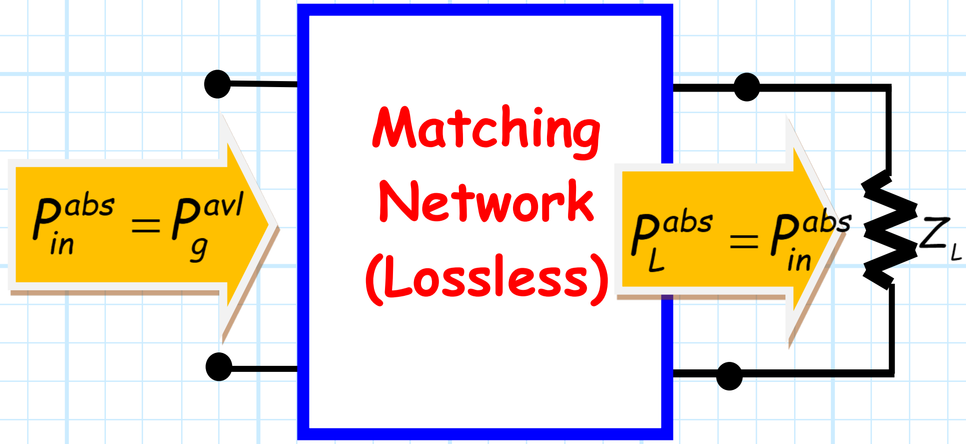
Q: Wait ***one*** *second!*

*The matching network ensures that **all** available source power is absorbed by the **input impedance (i.e.,** $P_{in}^{abs} = P_g^{avl}$**)**, but that does **not** mean (necessarily) that all this power will be absorbed by the **load** Z_L*

*Couldn't the power absorbed by the load be **much less** than the available power **(i.e.,** $P_L^{abs} < P_{in}^{abs} = P_g^{avl}$**)**???*

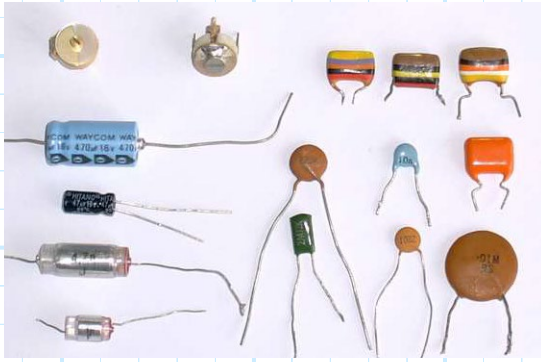
A: True enough!

So to ensure that the **available source power** is **entirely** absorbed by the **load**, the matching network must **also** be **lossless**!



Inductors and capacitors

$$\mathcal{Z}_{lossless} = \begin{bmatrix} jX_{11} & jX_{12} \\ jX_{12} & jX_{22} \end{bmatrix}$$



Therefore, we must construct our matching network entirely with **reactive elements**!

But, constructing a **proper lossless matching network** will lead to the happy condition where:

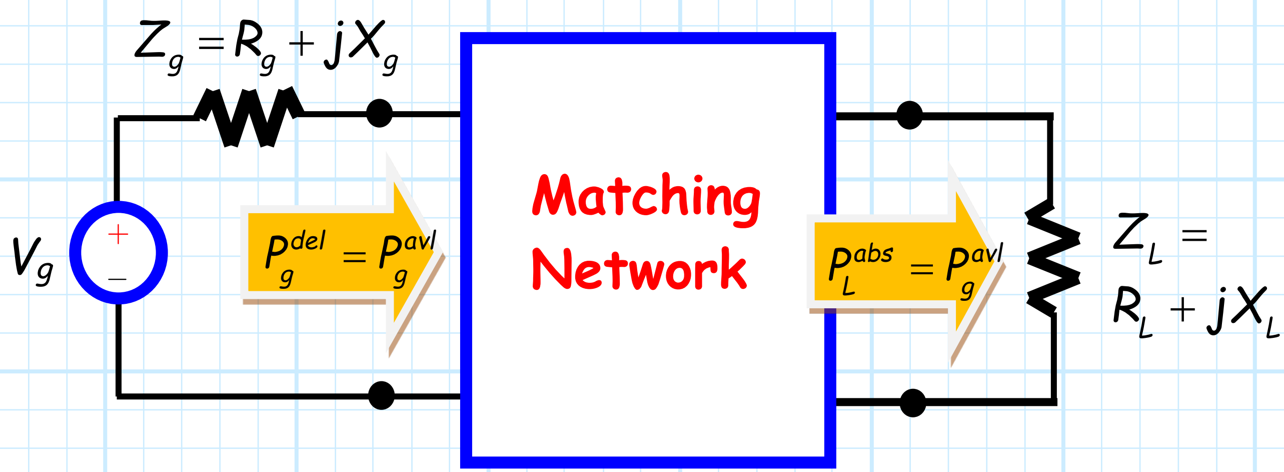
$$P_g^{del} = P_g^{avl} = P_L^{abs}$$

It depends on source and load—but does not alter them

1. The design and construction of this lossless impedance transformer will **depend** on **both source** impedance Z_g and **load** impedance Z_L.

$$Z_{in} = Z_g^* = jX_{11} + \frac{(X_{12})^2}{jX_{22} + Z_L}$$

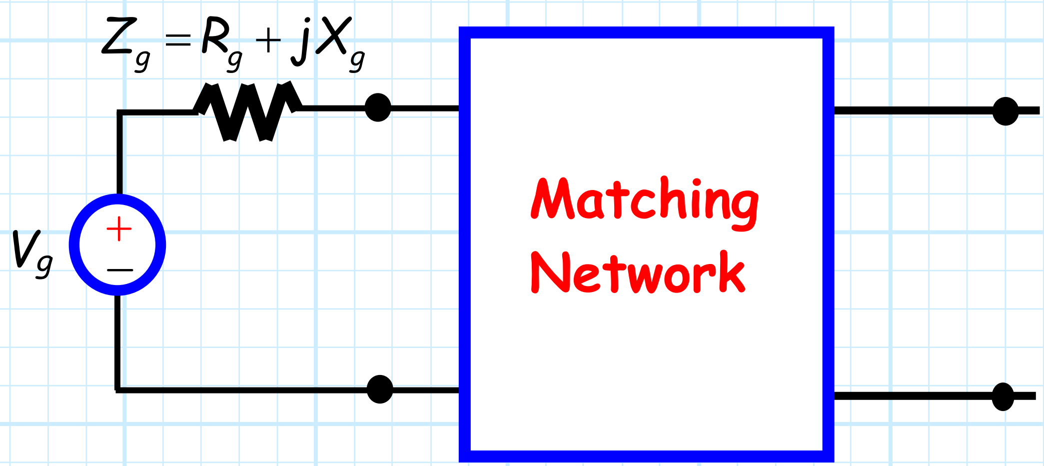
2. However, the impedance transformer does **not physically alter** either of these two quantities—the source and load are left **physically unchanged**!



A different perspective

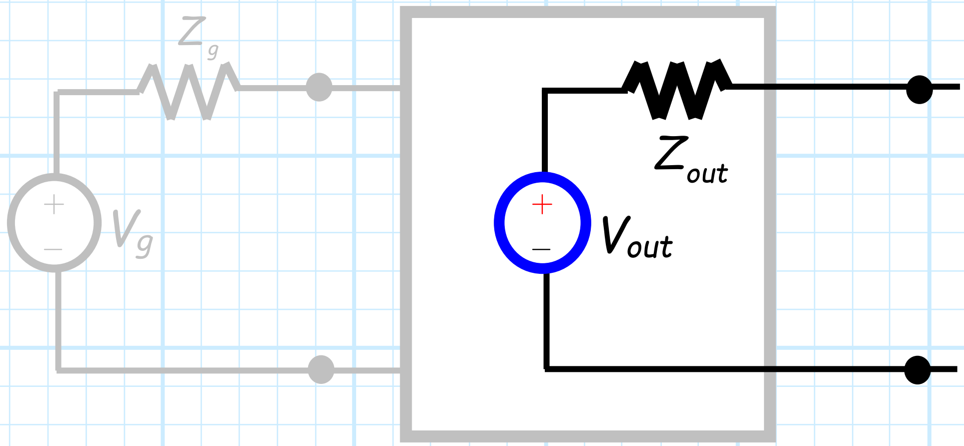
Now, let's consider the matching network from a **different perspective**.

Instead of thinking of it as an impedance transformer, think of it as a **source transformer**!



The matching network is instead a source transformer!

This **transformed** source (i.e., the original source with the matching network attached) can be expressed in terms of its **Thevenin's equivalent circuit**:



Recall that in general $V_{out} \neq V_g$ and $Z_{out} \neq Z_g$—the matching network "transforms" **both** the values of both the **impedance and** the **voltage** source.

A lossless network does NOT alter available power!

Q: *Doesn't that **also** mean that the **available power** of this "transformed" source will be **different** from the original?*

A: Remember, **because** the matching network is **lossless**, the available power of this transformed source is **identical** to the available power of the **original** source.

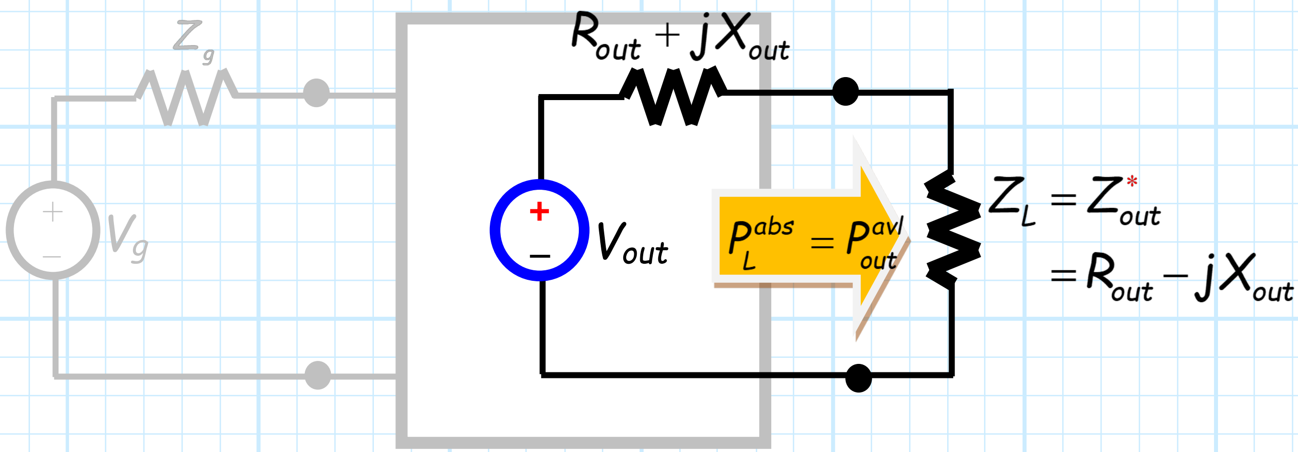
$$P_{out}^{avl} = P_g^{avl}$$

A **lossless** network (matching or otherwise) **cannot** alter the **available power** of the transformed source!

The load is matched to the transformed source

Q: *So what is the* ***purpose*** *of the* ***source transformer****; just what are we attempting to accomplish with it?*

A: We want **all** the available power of the **transformed source** to be absorbed by the **load** (i.e., want $P_L^{abs} = P_{out}^{avl}$).



This will occur **only** if the **transformed source impedance** is **conjugate matched** to the load (i.e., $Z_{out} = Z_L^*$).

All available power is delivered to the load!

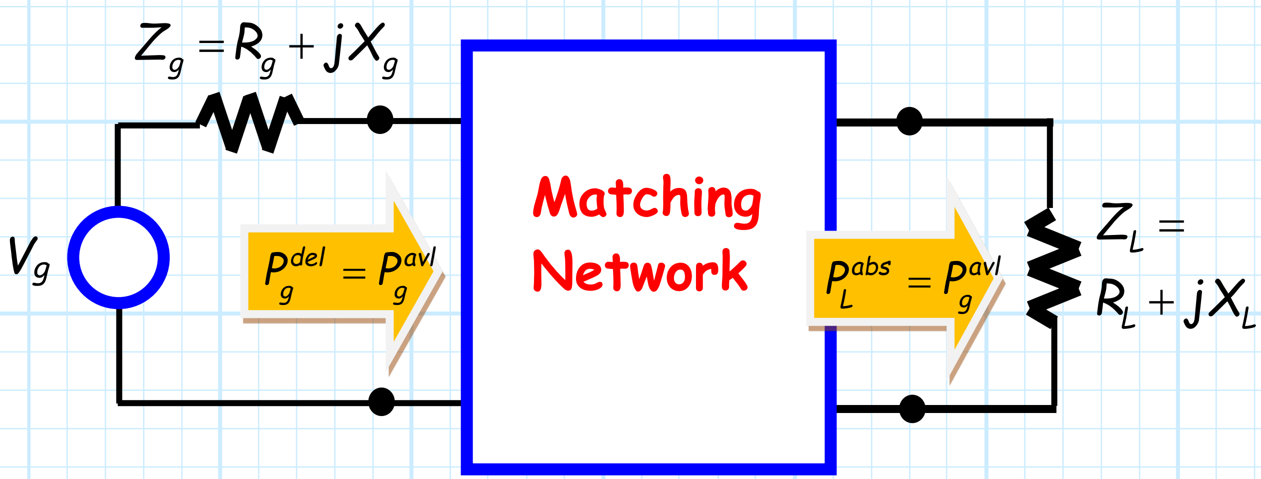
Because the matching network is **lossless**, the available power of the **transformed** source is **equal** to that of the **original**:

$$P_{out}^{avl} = P_g^{avl}$$

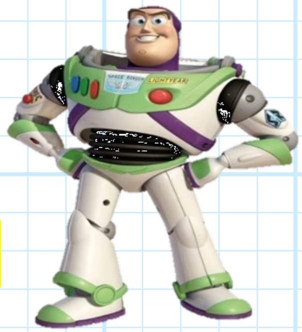
Therefore, the power **absorbed** by the load is also equal to the power available from the **original** source:

$$P_L^{abs} = P_{out}^{avl} = P_g^{avl}$$

And that's **just** what we were after!



Making R_{out} really small is not the answer



Q: Wait, *shouldn't we transform the source to one that increases the available power (preferably to* ***infinity****)?*

A: Remember, a **lossless** source transformer **cannot** alter the available power (either up or down).

If we transform the source to have a **really small** value of R_{out}, we will find that the transformed value $\left|V_{out}\right|^2$ is **proportionately just as small**.

→Thus, the available power is an **invariant**.

All we can do is to insert a lossless source transformer so that **all this** (unalterable) available power is **absorbed** by the load! I.E., by making:

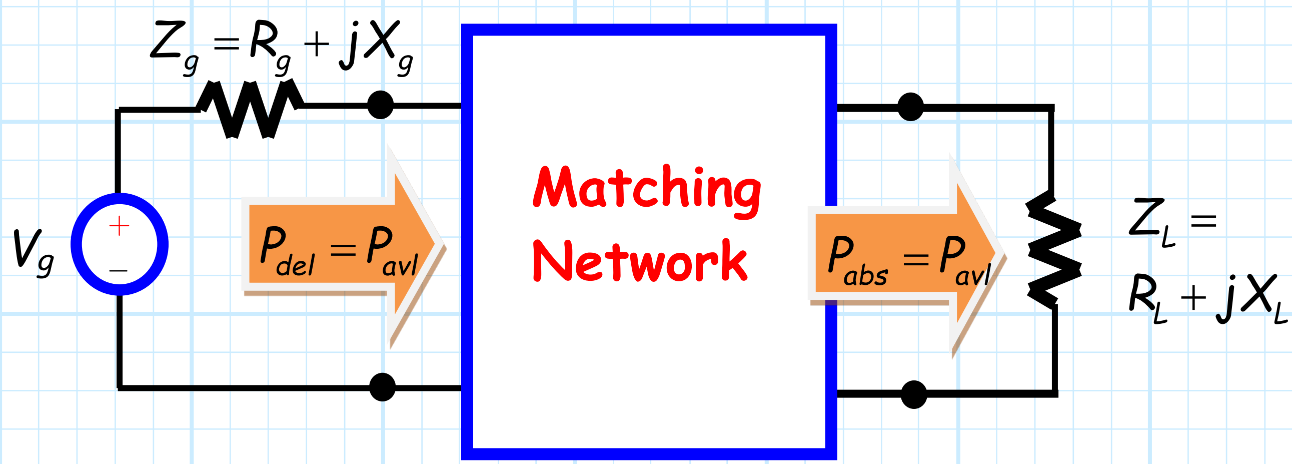
$$Z_{out} = jX_{22} + \frac{\left(X_{12}\right)^2}{jX_{11} + Z_g} = Z_L^*$$

It depends on source and load—but does not alter them

1. The design and construction of this lossless source transformer will **depend** on **both source** impedance Z_g and **load** impedance Z_L.

$$Z_{out} = jX_{22} + \frac{(X_{12})^2}{jX_{11} + Z_g} = Z_L^*$$

2. However, the source transformer does **not physically alter** either of these two quantities, the source and load are left **physically unchanged**!



Transform the source, or transform the load?

Q: *So which one is* ***better****; matching with a lossless* ***impedance*** *transformer, or matching with a lossless* ***source*** *transformer?*

A: They are the **same**—and I mean that very **literally**!

Look at the **design equation** for a matching network that implements an **impedance transformer**:

$$Z_{in} = jX_{11} + \frac{(X_{12})^2}{jX_{22} + Z_L} = Z_g^*$$

meaning that we need to **find** a lossless two-port network with trans-impedance values X_{11}, X_{12}, X_{22} that **satisfy** this equation:

$$Z_g^* = jX_{11} + \frac{(X_{12})^2}{jX_{22} + Z_L}$$

A design for the source transformer

Now consider the design equation for a matching network that implements a **source transformer**:

$$Z_{out} = jX_{22} + \frac{(X_{12})^2}{jX_{11} + Z_g} = Z_L^*$$

meaning that we need to **find** a lossless two-port network with trans-impedance values X_{11}, X_{12}, X_{22} that **satisfy** this equation:

$$Z_L^* = jX_{22} + \frac{(X_{12})^2}{jX_{11} + Z_g}$$

The same solutions satisfy both!

But now, let's take the **complex conjugate** of the **impedance** transformer design equation:

$$Z_g = jX_{11} + \frac{(X_{12})^2}{jX_{22} + Z_L^*}$$

and now solve for Z_L^*:

$$Z_L^* = jX_{22} + \frac{(X_{12})^2}{jX_{11} + Z_g}$$

This is **precisely** the design equation for the **source** transformer!!!!

Q: ???

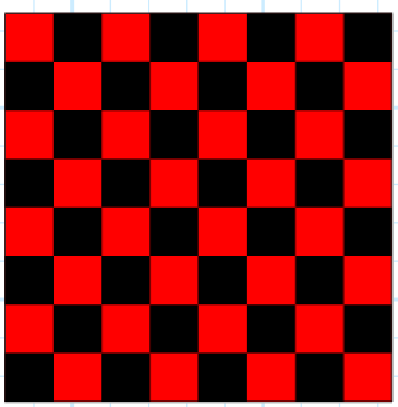
A: This means that if a lossless network has trans-impedance values X_{11}, X_{12}, X_{22} that satisfy **one** design equation, then they likewise satisfy the **other** design equation!!

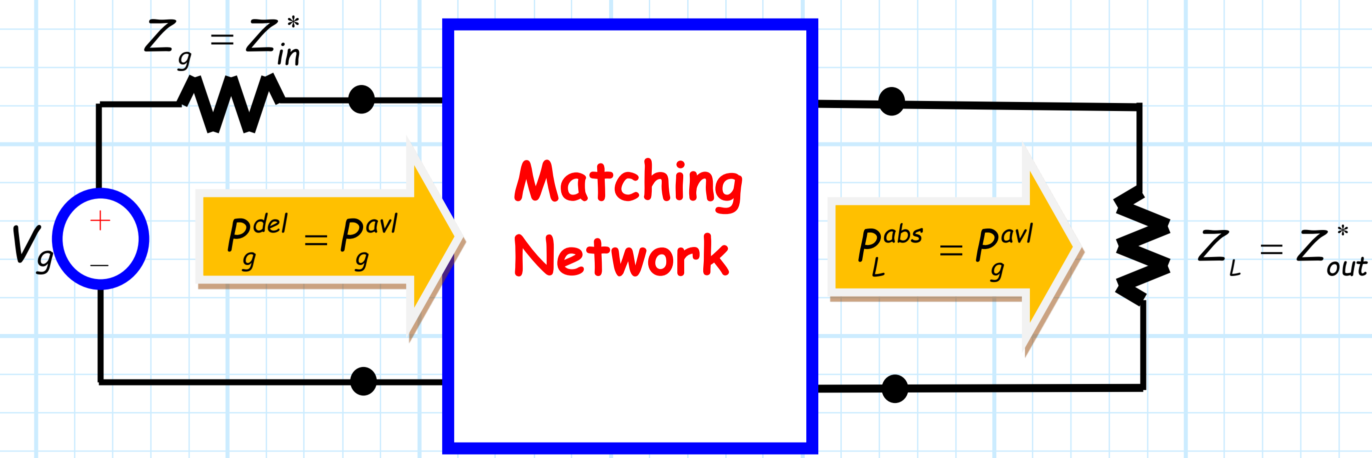
A red board with black squares, or a black board with red squares?

Q: *???*

A: An **impedance** transformer that matches **input** impedance Z_{in} to source impedance Z_g, is **likewise** a **source** transformer that matches Z_{out} to Z_L!!!

Similarly, a **source** transformer that matches **output** impedance Z_{out} to load impedance Z_L, is **likewise** an **impedance** transformer that matches Z_{in} to Z_g!

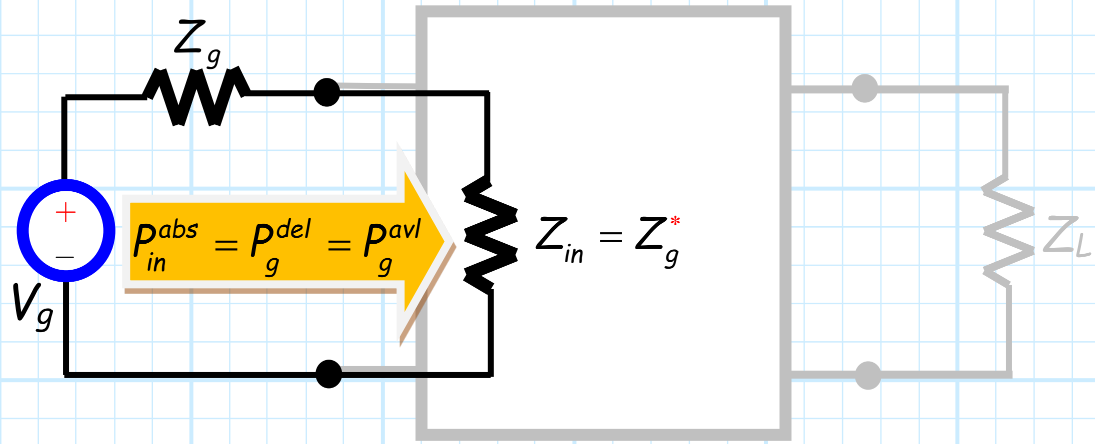




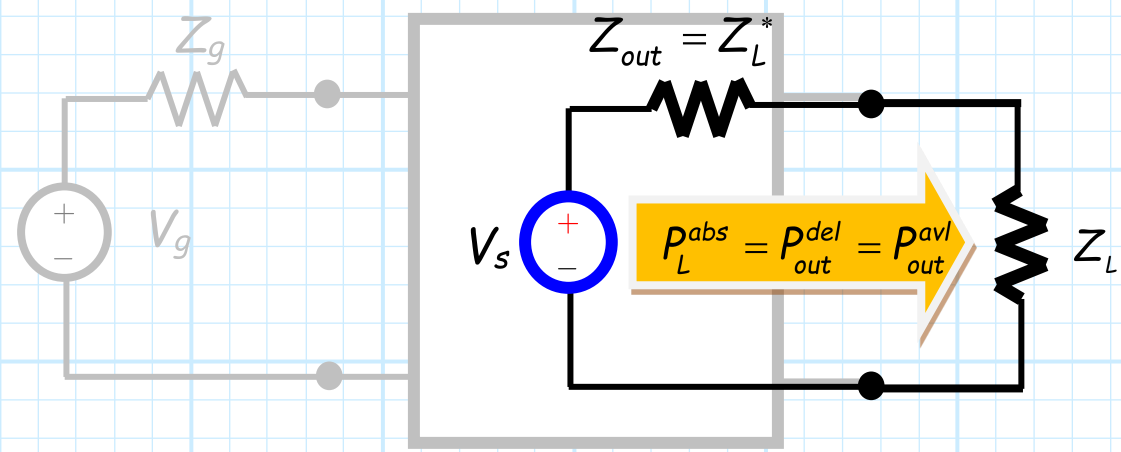
Two ways to consider the same thing

Thus, we can look at the matching network in **two equivalent ways**:

1. As a **network attached to a load**, one that "transforms" its impedance to Z_{in}—a value matched to the source impedance Z_g:



2. Or, as **network attached to a source**, one that "transforms" its impedance to Z_{out}—a value matched to the load impedance Z_L:



Make this make sense

Consider these **three** conditions:

1. $Z_{in} = Z_g^*$ (therefore $P_{in}^{abs} = P_g^{avl}$).

2. $Z_{out} = Z_L^*$ (therefore $P_L^{abs} = P_{out}^{avl}$).

3. $P_L^{abs} = P_g^{avl}$ (since $P_g^{out} = P_{out}^{avl}$).

If **any one** of these 3 conditions is true, then the **other two** conditions must be **true also**!

Thus:

* A conjugate match **anywhere** means there is a conjugate match **everywhere**!

* If $P_L^{abs} = P_g^{avl}$, then a **conjugate match** exists everywhere (and **vice versa**!).

Matching is a subset of all lossless

$$P_L^{abs} = P_g^{del} \leq P_g^{avl}$$

$$Z_{in} \neq Z_g^*$$

$$Z_{out} \neq Z_L^*$$

Lossless Networks

$$P_L^{abs} = P_g^{del} = P_g^{avl}$$

$$Z_{in} = Z_g^*$$

$$Z_{out} = Z_L^*$$

Lossless Matching Networks

Although **matching networks are lossless**, most lossless networks are **not** matching networks!