Lecture 11

Bionics, Mechanics

Exercises on Wheel Theory

4/11/2022

Lecture 8 24/10/25

- Do exercise 4 from lecture 10 first.

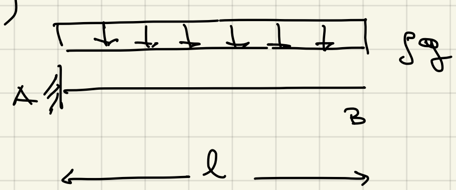
Some exercises on the linear theory

Using linear theory, compute and graph the internal forces, and compute the elastic displacements in the cases sketched below

1)

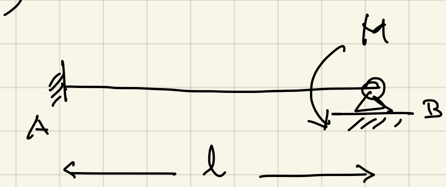
$\omega_A = ?$, $\omega_B = ?$ (rotations at the ends)

2)



$\omega_B = ?$, $w_B = ?$ (rotation and deflection at the end)

3)



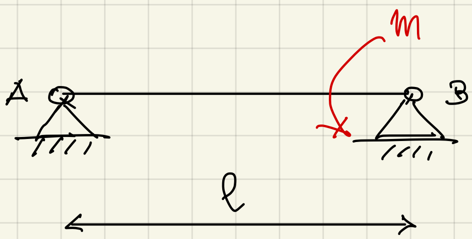
$\omega_B = ?$ (rotation at end B)

Exercise 1

Assume $M = EI\,w''$, $N = EA\,u'$.

Using linear theory, compute elastic rotations

$$\omega_A = w'(0), \quad \omega_B = w'(\ell)$$



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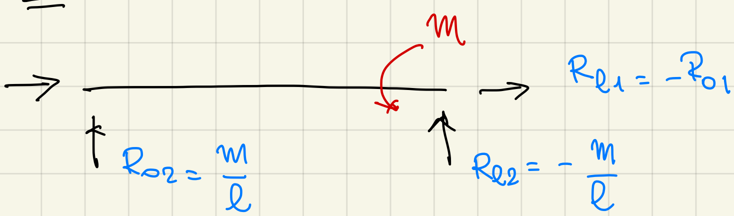

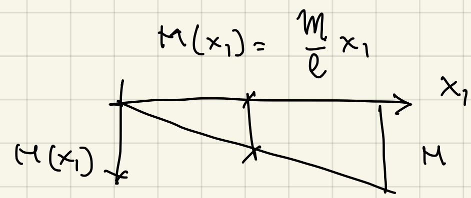

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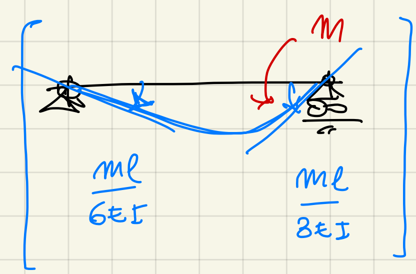
$N \equiv \text{cost} \Rightarrow u = a + b x_1, \quad u(0) = u(\ell) = 0 \Rightarrow a = b = 0 \quad \underbrace{\Rightarrow N \equiv 0 \Rightarrow R_{01} = R_{\ell 1} = 0}_{(*)}$

=

$$w'' = \frac{1}{EI}\frac{m}{\ell} x_1$$

$$w'(x_1) = w'(0) + \frac{1}{EI}\frac{m}{\ell}\frac{x_1^2}{2}$$

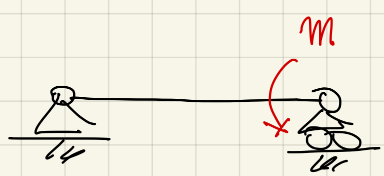
$$w(x_1) = \underbrace{w(0)}_{=0} + w'(0)\,x_1 + \frac{1}{EI}\frac{m}{\ell}\frac{x_1^3}{6}$$



$$w(\ell) = 0 = w'(0)\,\ell + \frac{m\ell^2}{6EI} \Rightarrow w'(0) = -\frac{m\ell}{6EI}$$

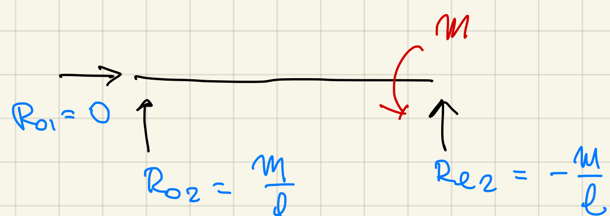
$$w'(\ell) = -\frac{m\ell}{6EI} + \frac{m\ell}{2EI} = \frac{m\ell}{3EI}$$

Remark Exercise 1 was actually



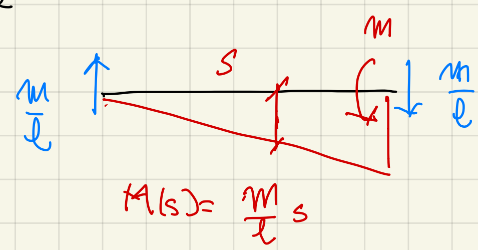
Remark This is isostatic the one above is hyperstatic.

so the free-body diagram is

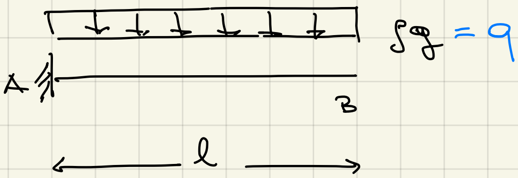


The shearing force is constant $\Rightarrow$ $M(s)$ affine

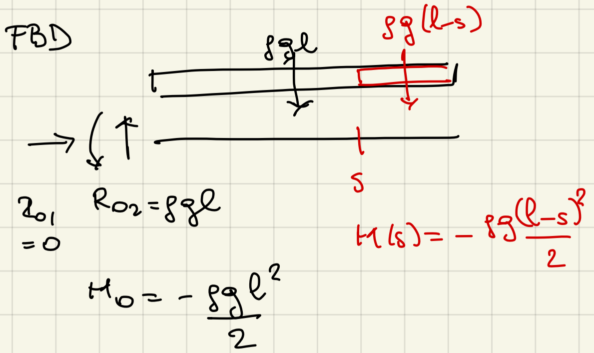
The diagram of the bending moment is a straight line:

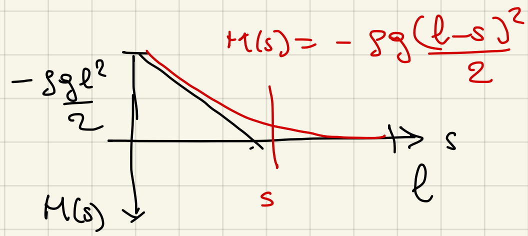


Exercise 2 $N = EA\varepsilon$, $M = EI\kappa$, linear theory: $\varepsilon = u'$, $\kappa = w''$



$\omega_B = ?$, $w_B = ?$ (rotation and deflection at the end)





$$N(s) \equiv 0 \Rightarrow \varepsilon = 0 \Rightarrow u(s) \equiv u(0) = 0$$

$$w''(s) = \frac{1}{EI} M(s) = -\frac{\rho g}{2EI}(\ell - s)^2$$

$$w'(s) = w'(0) - \frac{\rho g}{2EI}\int_0^s (\ell-\sigma)^2 \, d\sigma$$

$$= 0 - \frac{\rho g}{2EI} \left. \frac{(\sigma-\ell)^3}{3} \right|_0^s = -\frac{\rho g}{2EI}\left(\frac{(s-\ell)^3}{3} + \frac{\ell^3}{3}\right)$$

$$w(s) = w(0) - \frac{\rho g}{2EI}\left(\int_0^s \frac{(\sigma-\ell)^3}{3} d\sigma + \int_0^s \frac{\ell^3}{3} d\sigma\right)$$

$$= 0 - \frac{\rho g}{2EI}\left(\frac{1}{12}\left((s-\ell)^4 - \ell^4\right) + \frac{\ell^3}{3}s\right)$$

$$\omega_B = w'(\ell) = -\frac{\rho g}{2EI}\left(+\frac{\ell^3}{3}\right) = -\frac{\rho g \ell^3}{6EI}$$

$$w_B = w(\ell) = -\frac{\rho g}{2EI}\underbrace{\left(-\frac{1}{12}\ell^4 + \frac{1}{3}\ell^4\right)}_{\frac{-1+4}{12} = \frac{3}{12} = \frac{1}{4}} = -\frac{\rho g \ell^4}{8EI}$$