ES. 1

① $z = \frac{\pi}{\sqrt{2}} + i\frac{\pi}{\sqrt{2}}$ ~~$z = \sqrt{\frac{\pi}{2}}\frac{\pi}{2}$~~

$\boxed{\text{Risposta: d}}$

$$|z| = \sqrt{\frac{\pi^2}{\cancel{2}} + \frac{\pi^2}{2}} = \sqrt{\frac{\cancel{2}\pi^2}{\cancel{2}}} = \pi$$

② $z^n = |z|^n(\cos(n\varphi) + i\,\mathrm{sen}(n\varphi))$ $\boxed{\text{Risposta: c}}$

③ CR: $\frac{\partial f}{\partial x} = -i\frac{\partial f}{\partial y} \iff \frac{\partial f}{\partial x} = \frac{1}{i}\frac{\partial f}{\partial y}$

$\boxed{\text{Risposta: d}}$

④ $\mathrm{Log}(i) = \left(\underset{\nearrow 0}{\log|i|} + i\underset{\nearrow \pi/2}{\mathrm{Arg}(i)}\right) = \frac{\pi}{2}i$ $\boxed{\text{Risposta: c}}$

⑤ $\sum_{n=0}^{+\infty} \frac{(\log x)^{2n}}{(2n+1)!} \Rightarrow \frac{\sinh(\log x)}{\log x}$ poichè: $\sum \frac{1}{\log x} \cdot \frac{(\log x)^{2n+1}}{(2n+1)!}$

$\boxed{\text{Risposta: c}}$

ES. 2

(ii) $f_n(x) = \arctan(n + n^2 x)$, $x \leq 0$

per $x \leq 0$ $\arctan(n + n^2 x) \xrightarrow{n \to +\infty} -\frac{\pi}{2}$

Conv. punt. per $x \in]-\infty; 0]$

invece unif. per $g_n(x) = \sup_{x \in]-\infty; 0]} |\underset{\nearrow 0}{\arctan(0)} + \frac{\pi}{2}| = \pi/2$

(iii) $\lim_{n\to+\infty} \int_{-\pi/2}^{-\pi/4} f_n(x)\,dx$ Uso il TEO. di passaggio al limite sotto il segno di integrale.

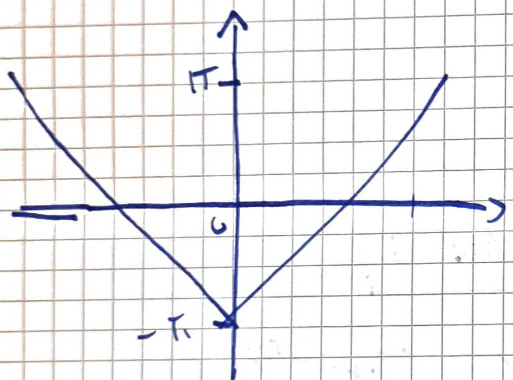
$$\int_{-\pi/2}^{-\pi/4} f(x)\,dx = \frac{\pi}{2} \ (?)$$

Es. 3

(i) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos x\,dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin x\,dx$

(ii) $f(x) = 2|x| - \pi$ è Pari $\quad \frac{a_0}{2} = -\frac{\pi}{2}$



essendo Pari $b_n = 0$, quindi

$$a_n = \frac{2}{\pi}\int_0^{\pi}(2|x|-\pi)\cos x = \frac{2}{\pi}\left[\int_0^{\pi} 2x\cos x - \int_0^{\pi}\pi\cos x\right] =$$

$$\cancel{2x\sin x}\Big|_0^{\pi} - \int_0^{\pi} 2\sin x$$

$$= \frac{2}{\pi}\left[2\cos x\Big|_0^{\pi} - \cancel{\pi\sin x}\Big|_0^{\pi}\right] = \frac{2}{\pi}\left[-2-1\right] = \boxed{-\frac{6}{\pi}}$$

$$\boxed{S(x) = -\frac{\pi}{2} + \sum -\frac{6}{\pi}\cos nx}$$

(iii)

$$S\left(\frac{7}{4}\pi\right) = -\frac{\pi}{2} + \left(-\frac{6}{\pi}\cos\left(\frac{7}{4}\pi\right)\right) = -\frac{\pi}{2} + \left(-\frac{6}{\pi}\cdot\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{2} - \frac{6\sqrt{2}}{2\pi} =$$

$$= \frac{-\pi^2 - 6\sqrt{2}}{2\pi}$$

ES. 4 – ES. 5 ??