Es. 1

1) $f(x) = |\sin x|$ $b_n = 0$ poiché $f(x)$ è pari

$\boxed{\text{risposta: b}}$

2) $z = \frac{2}{3-i}$ $\frac{2(3+i)}{(3-i)(3+i)} = \frac{6+2i}{9-i^2} = \frac{6+2i}{10} \Rightarrow \frac{3}{5} + \frac{i}{5}$

$\Downarrow$

$Re = \frac{3}{5}$; $Im = \frac{1}{5}$

$\boxed{\text{risposta: c}}$

3) $Res\left(\frac{1}{z^2}, 0\right) = \frac{1}{2z} \xrightarrow{z \to \infty} \infty$

$Res\left(\frac{1}{z}; 0\right) = 1$ $\boxed{\text{risposta: b}}$

4) $f(z) = 1 - Log(z \cdot \bar{z}) \Leftrightarrow 1 - Log(|z|)$

olom. in $\mathbb{C} \setminus \{0\}$ quindi $\boxed{\text{risposta: b}}$

5) Per def. $\cos z = \frac{e^{iz} + e^{-iz}}{2}$ quindi $\boxed{\text{risposta: b}}$

Es. 2

(i) Se γ è la frontiera di un aperto D, interamente contenuto in A, si ha:

$$\int_\gamma f(z)\, dz = 0$$

(ii) $\gamma(t)$: $\mathbb{C} \setminus \{0, 1, i, -i\}$

ES.3

(ii) $\nexists$ $\sin^2 z + \cos^2 z = 1 \Leftrightarrow$ questa A è il prol. analitico di:

$$\underbrace{\sin^2 x + \cos^2 x}_{\substack{\shortparallel \\ 1}} = 1 \qquad \text{con } x \in \mathbb{R} = B$$

ed $B \subset A$

Es.4

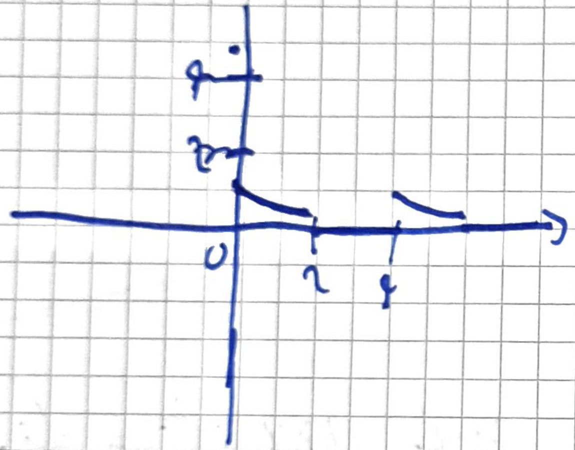
(ii) $\displaystyle\sum_{n=0}^{+\infty} \frac{3^{n+1}}{(x^2+2)^n} = \sum_{n=0}^{+\infty} 3 \cdot \left(\frac{3}{x^2+2}\right)^n$ questa conv. punt. per $\left|\frac{3}{x^2+2}\right| < 1$ ovvero

Per $|x^2+2| > 3 \rightarrow |x| > 1$

e totalmente $g_n(x) = \sup\limits_{x \in]1,+\infty[} \frac{3}{\infty} = 0$

(iii)

$$x(t) = \begin{cases} e^{-3t} & t \in]0,2] \\ 0 & t \in]2,4] \end{cases}$$



$$\mathcal{L}(s) = \frac{1}{1-e^{-2s}} \int_0^2 e^{-st} \cdot e^{-3t} =$$

$$= \frac{1}{1-e^{-2s}} \int_0^2 e^{-st-3t} = -\frac{1}{3(1-e^{-2s})} \left[e^{-(st+3t)} \right]_0^2 =$$

$$e^{-(st+3t)}$$

$$= -\frac{1}{3-3e^{-2s}} \left[e^{-(2s+6)} - e^{0} \right] = \boxed{\frac{e^{-2s} \cdot e^{6} - 1}{3-3e^{-2s}}}$$