Esercizio 1

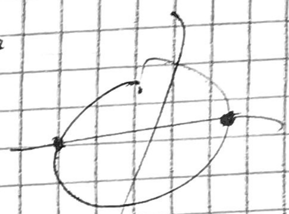
$$f_n(x) = \sin\left(\left(\frac{x}{2\pi}\right)^n\right)$$

$$\lim_{n\to\infty} \sin\left(\left(\frac{x}{2\pi}\right)^n\right) =$$ IL $\sin x$ CONVERGE SE $x < \infty$ $\Rightarrow \left(\frac{x}{2\pi}\right)^n$ CONVERGE PUNTUALM. A 1 o 0

$$\left(\frac{x}{2\pi}\right)^n = \begin{cases} 1 & \text{se } x = 2\pi \longrightarrow \sin 1 \\ 0 & \text{se } |x| < 2\pi \longrightarrow \sin 0 = 0 \\ \nexists & \text{se } x \leq -2\pi \\ \infty & \text{se } x > 2\pi \end{cases}$$

d

NON CONVERGE



II

$+x^2 + 3\sin 2x$

PARI $\to b_k = 0 \ \forall k$

$\Longrightarrow$ ALLORA CONSIDERO SOLO LO SVILUPPO DI x^2 SICCHÉ $b_k = 0 \ \forall k \in \mathbb{N}$

$b_2 = 3 \to b$

III

$$\int_\gamma \pi^2 z e^{z\pi} \, dz = \pi^2 \left[\frac{z e^{z\pi}}{\pi} - \int_\gamma \frac{e^{z\pi}}{\pi} \right] = \pi \left[z e^{z\pi} - \frac{e^{z\pi}}{\pi} \right]_{\gamma(a)}^{\gamma(b)}$$

$$= \left[\pi z e^{z\pi} - e^{z\pi} \right]_{i}^{2i} = \pi \cdot 2i \cdot e^{i2\pi} - e^{i2\pi} - \pi i e^{\pi i} + e^{i\pi}$$

$$= 2\pi i - 1 + \pi i - 1 = 3\pi i - 2$$

d

IV) $f(z) = 6iz^3 e^{\frac{1}{z^2}}$

$\downarrow$

$$6iz^3 \cdot \sum_{n\geq 0} \left(\frac{1}{z^2}\right)^n \frac{1}{n!} = 6i \sum_{n\geq 0} \frac{1}{n!} \cdot \frac{1}{z^{2n-3}}$$

$\downarrow$ per $n=2$

$$6i \cdot \frac{1}{2!} \cdot \frac{1}{z^{4-3}} = \frac{6i}{2} \frac{1}{z}$$

$\boxed{C_{-1} = 3i} \to \oint$

Esercizio II

$$f(z) = \frac{(z+i)\sin^2 z}{z^6(1+z^2)} = \frac{\sin^2 z \cdot (z+i)}{z^6(z-i)(i+z)}$$

singolarità in $0, -i, +i$

$-i$ eliminabile $\to \operatorname{res}(\ ) = 0$

i polo semplice $\to \operatorname{res}() = \lim_{z\to i} \frac{\cancel{(z-i)} \sin^2 z \cancel{(z+i)}}{\cancel{(z-i)}\cancel{(z+i)} z^6} = \frac{\sin^2 i}{i^6}$

$= -\sin^2 i$

0 polo di ordine 4 $\to \operatorname{res}(\ ) = \lim_{z\to 0} \cancel{z^4} \frac{\sin^2 z}{z^{\cancel{6}2}(z^2+1)} = \cancel{\frac{\sin^2 z}{z^2}}^{\,1} \frac{z+i}{z^2+1}$

$= i$

Esercizio III

$$\frac{1}{s(s+1)^2}$$

$$\int_0^t \mathcal{L}^{-1}\left[\frac{1}{s}\right]\cdot \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right] d\tau = \int_0^t 1\cdot \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right] d\tau =$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right] \longrightarrow \text{SING. IN } -1 \text{ E BASTA}$$

$$\operatorname{res}\left(\frac{e^{st}}{(s+1)^2}, -1\right) = \frac{1}{1!}\lim_{s\to -1} \frac{d}{ds}\left(e^{st}\right) = \lim_{s\to -1} t e^{st} = t e^{-t}$$

$$\int_0^t \tau e^{-\tau}\, d\tau = \left[-\tau e^{-\tau} + \int_0^t e^{-\tau}\right] = -\tau e^{-\tau} - e^{-\tau}\Big|_0^t$$

$$-t e^{-t} - e^{-t} - (0 - 1) = -t e^{-t} - e^{-t} + 1$$