پاسخ: $z_i(t) \triangleq \beta x_i(t) + v_i(t)$ where $i = 1, \ldots, n$. Q1

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$$\begin{cases} \dot{z}_i(t) = 0 \;, & i \in R \\ \dot{z}_i(t) = -\sum_{j \in R \cup F} a_{ij}[k][z_i(t) - z_j(k)] \;, & i \in F. \end{cases}$$

$\bar{z} \triangleq \max_{i \in R} z_i$, $\bar{x} \triangleq \max_{i \in R} x_i$ & $\delta_i \triangleq x_i - \bar{x}_i$, $i \in F$.

$z_i - \bar{z}$ is bounded; $\underset{t \to \infty}{z_i - \bar{z}_i \leq 0}$; $f_i(t) = z_i - \bar{z}_i$

$\Rightarrow \dot{\delta}_i(t) + \beta \delta_i(t) = f_i(t)$; $x_i(t)$, $i \in R$ are constant.

$\Rightarrow$ ~~$\delta_i(t) = e^{-\beta(t-\tau)} f_i(t)$~~ $\Rightarrow \delta_i(t) = e^{-\beta t}\delta_i(0) + \int_0^t e^{-\beta(t-\tau)} f_i(\tau)\, d\tau$.

$$\rightarrow \lim_{t\to\infty} \delta_i(t) = \lim_{t\to\infty} e^{-\beta t}\delta_i(0) + \lim_{t\to\infty} \int_0^t e^{-\beta(t-\tau)} f_i(\tau)\, d\tau$$

$$= \lim \frac{\int_0^t e^{\beta\tau} f_i(\tau)\, d\tau}{e^{\beta t}} = \lim \frac{e^{\beta t} f_i(t)}{\beta e^{\beta t}} = \lim \frac{f_i(t)}{\beta}$$

where all limits are under "$t \to \infty$".

$$\rightarrow \left\{ \frac{\beta > 0 \quad f_i(t) > 0}{\text{as } t \to \infty} \right. \Rightarrow \delta_i(t) \leq 0 \;;\; \underset{t\to\infty}{x_i(t) \leq \bar{x}(t)} \;. \Rightarrow \underset{t\to\infty}{x_i(t) \leq \max_{j \in R} x_j(t)} \;;$$

$\rightarrow \underset{t\to\infty}{x_i(t) \geq \min_{j \in R} x_j(t)} \Rightarrow$ Therefore, all followers will converge to the convex hull formed by the leaders in any high-dimensional spaces.