$$S = [s_1, \dots, s_n]^T \; ; \quad s = z_2 + c_2 z_1 + \int_0^t c_1 z_1 \, d\tau \; ،$$

$$1_n^T S = 0 \; ; \quad U = -\mathrm{diag}\{h_1, \dots, h_n\} \, \mathrm{sgn}(LS) \; ;$$

$$\dot{s} = \dot{z}_2 + c_1 z + c_2 z_2 \; ;$$

Part 1) فاز رسیدن به سطح لغزش : $V_c = \frac{1}{2} S^T S$

$$\Rightarrow \dot{V}_c = S^T \dot{S} = -S^T L \, \mathrm{diag}\{h_1, \dots, h_n\} \, \mathrm{sgn}(LS) + S^T L \delta + c_1 S^T z_1$$
$$+ c_2 S^T z_2 = -\sum_{i=1}^{n} h_i \, |[LS]_i| + S^T L \delta + c_1 S^T z_1 + c_2 S^T z_2$$

$$[Ls]_i = \sum_{j=1}^{n} a_{ij}(s_i - s_j) \;;$$

$$\|Ls\|_2 = (s^T L^2 s)^{1/2} \;;\; 1_n^T (L^{1/2} s) = 0 \;;\; 1_n^T s = 0$$

$$\Rightarrow -\sum_{i=1}^{n} h_i \,|[Ls]_i| \leqslant -\underline{h}\,\|Ls\|_1$$

$$\leqslant -\underline{h}\,\lambda_2(L)\|s\|_2$$

$$\leqslant -h\,\frac{\lambda_2(L)}{\sqrt{n}}\,\|s\|$$

$$; \quad \underline{h} = \frac{\left[\bar{E} + \underline{\varepsilon} + \sqrt{n}\,D\,\lambda_{max}(L)\right]}{\lambda_2(L)/\sqrt{n}}$$

$$\underline{\varepsilon} = \min\{\varepsilon_i\} \;;$$

$$s^T L s \leqslant \|s^T L s\|$$

$$\leqslant \sqrt{n}\,D\,\lambda_{max}(L)\|s\|_2$$

$$\leqslant \sqrt{n}\,D\,\lambda_{max}(L)\|s\|_1$$

$$\Rightarrow \dot{V}_c \leqslant -\underline{h}\,\frac{\lambda_2(L)}{\sqrt{n}}\|s\|_1 + \sqrt{n}\,D\,\lambda_{max}(L)\|s\|_1 + c_1 s^T z_1 + c_2 s^T z_2$$

$$\leqslant \sum_{i=1}^{n}\left(-\underline{h}\,\frac{\lambda_2(L)}{\sqrt{n}} + \sqrt{n}\,D\,\lambda_{max}(L) + \bar{E}\right)|s_i|$$

$$\bar{E} = \max\{|c_1 z_{i,1} + c_2 z_{i,2}|\}$$

$$\Rightarrow \dot{V}_c \leqslant -\sum \underline{\varepsilon}\,|s_i| \leqslant -\underline{\varepsilon}\,\|s\|_1 \leqslant -\underline{\varepsilon}\,\|s\|_2 \leqslant -2^{1/2}\,\underline{\varepsilon}\,V_c^{1/2} \;;$$

$\underline{\varepsilon} = \min\{\varepsilon_i\} \Rightarrow$ طبق لم ۱ تماماً خطاهای سیستم به صفر می‌رسد.

Part 2) فاز لغزش:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -c_1 z_1 - c_2 z_2 \end{cases} \qquad c_1, c_2 > 0.$$

با توجه به اینکه سیستم فوق هرویتز است، لذا خطاها به صورت نمایی همگرا می‌شوند.