MAS Course – Assignment 02 – Containment

Problem 02

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```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0         0   -0.2003   -0.2003         0         0
         0         0    0.2003         0   -0.2003         0
         0         0         0         0         0   -1.6129

B = 6×2
         0         0
         0         0
         0         0
    0.9441    0.9441
    0.9441    0.9441
  -28.7097   28.7097
```

```
cvx_begin
    variable P(6,6) symmetric
    A*P + P*A' - 2*B*B' <= 0
cvx_end
```

```
Calling SDPT3 4.0: 25 variables, 4 equality constraints
------------------------------------------------------------

 num. of constraints =  4
 dim. of linear var  = 21
 dim. of free   var  =  4 *** convert ublk to lblk
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|1.3e+01|6.2e+01|1.4e+06| 0.000000e+00  0.000000e+00| 0:0:01| chol  1  1
 1|0.702|0.928|4.0e+00|4.9e+00|1.6e+05| 0.000000e+00 -1.213948e+04| 0:0:02| chol  1  1
 2|0.953|0.926|1.8e-01|5.9e-01|1.0e+04| 0.000000e+00 -4.162457e+03| 0:0:02| chol  1  1
 3|0.979|0.981|3.9e-03|8.5e-02|4.2e+02| 0.000000e+00 -2.737813e+02| 0:0:02| chol  1  1
 4|0.985|0.960|5.7e-05|2.6e-02|2.2e+01| 0.000000e+00 -1.245507e+01| 0:0:02| chol  1  1
 5|0.978|0.885|1.6e-06|9.0e-03|3.0e+00| 0.000000e+00 -2.104276e-01| 0:0:02| chol  1  1
 6|1.000|0.972|4.2e-07|2.2e-03|2.5e-01| 0.000000e+00  3.691781e-01| 0:0:02| chol  1  1
 7|1.000|0.914|3.3e-08|7.5e-04|3.9e-02| 0.000000e+00  1.408172e-01| 0:0:02| chol  1  1
 8|1.000|1.000|7.1e-08|1.8e-04|1.3e-02| 0.000000e+00  3.077093e-02| 0:0:02| chol  1  1
 9|0.988|0.920|1.3e-09|6.5e-05|1.3e-03| 0.000000e+00  1.231694e-02| 0:0:02| chol  1  1
10|1.000|1.000|6.5e-09|6.3e-05|3.0e-04| 0.000000e+00  3.103276e-03| 0:0:02| chol  2  2
11|1.000|0.925|8.6e-11|1.6e-05|4.5e-05| 0.000000e+00  1.104349e-03| 0:0:02| chol  1  2
12|0.989|0.988|1.1e-12|2.2e-06|1.1e-06| 0.000000e+00  1.271363e-05| 0:0:02| chol  1  2
13|1.000|0.988|2.1e-13|5.3e-08|2.4e-08| 0.000000e+00  1.464649e-07| 0:0:02| chol  1  2
14|1.000|0.988|2.2e-14|1.2e-09|5.9e-10| 0.000000e+00  1.649318e-09| 0:0:02|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 14
 primal objective value =  0.00000000e+00
 dual   objective value =  1.64931818e-09
 gap := trace(XZ)       = 5.88e-10
 relative gap           = 5.88e-10
 actual relative gap    = -1.65e-09
 rel. primal infeas (scaled problem)   = 2.15e-14
```

```
 rel. dual     "         "        "       = 1.19e-09
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "         "        "       = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.0e+03, 4.5e+01, 8.7e+01
 norm(A), norm(b), norm(C) = 9.1e+00, 1.0e+03, 1.0e+00
 Total CPU time (secs)  = 1.95
 CPU time per iteration = 0.14
 termination code       =  0
 DIMACS: 2.2e-14  0.0e+00  1.2e-09  0.0e+00  -1.6e-09  5.9e-10
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +0
```

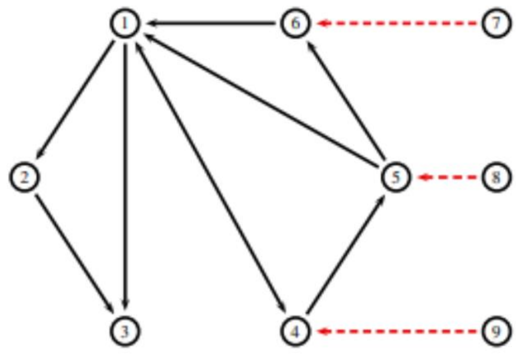
```
P = 6×6
10^3 ×
         0         0   -0.0050   -0.0022   -0.0015    0.0008
         0         0    0.0050   -0.0015   -0.0022   -0.0008
   -0.0050    0.0050    0.0069   -0.0008    0.0008   -0.0112
   -0.0022   -0.0015   -0.0008   -0.0032   -0.0010    0.0012
   -0.0015   -0.0022    0.0008   -0.0010   -0.0032   -0.0012
    0.0008   -0.0008   -0.0112    0.0012   -0.0012   -1.0177
```

Since P is not Positive-Definite, we use the pole placement method to obtain the feedback gains.

```
rng(42)
F = place(A,B,-5*rand(1,6))
```

```
F = 2×6
   -2.7894    4.3008    1.4031   -5.2719    7.5168    0.1748
   -4.5983    6.1103    2.5947   -8.9450   11.1907    0.4739
```

Define the MAS with the given topology:




```
NumFollowers = 6

NumLeaders = 3

NumAgents = 9

NumStates = 6
```

and the Laplacian Matrices:

```
L_Followers = 6×6
     3     0     0    -1    -1    -1
    -1     1     0     0     0     0
    -1    -1     2     0     0     0
    -1     0     0     1     0     0
     0     0     0    -1     1     0
    -1     0     0     0     0     1

L_total = 9×9
     3     0     0    -1    -1    -1     0     0     0
    -1     1     0     0     0     0     0     0     0
    -1    -1     2     0     0     0     0     0     0
    -1     0     0     2     0     0     0     0    -1
     0     0     0    -1     2     0     0    -1     0
    -1     0     0     0     0     2    -1     0     0
     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0

lambda_min = 1

c_th = 1

c = 1.5000
```

Define the MAS and agents' dynamics and the initial conditions.

```
agents = repmat(struct("x_dot", [], "x", []), NumFollowers, 1);
for agent=1:NumFollowers
    agents(agent).x = rand(6,1)*2
    agents(agent).x_dot = zeros(6,1)
end
```

agents = 9×1 struct

Fields	x_dot	x
1	[0;0;0;0;0;0]	[0.1162;1.7324;1.2022;1.4161;0.0412;1.9398]
2	[0;0;0;0;0;0]	[1.6649;0.4247;0.3636;0.3668;0.6085;1.0495]
3	[0;0;0;0;0;0]	[0.8639;0.5825;1.2237;0.2790;0.5843;0.7327]
4	[0;0;0;0;0;0]	[0.9121;1.5704;0.3993;1.0285;1.1848;0.0929]
5	[0;0;0;0;0;0]	[1.2151;0.3410;0.1301;1.8978;1.9313;1.6168]
6	[0;0;0;0;0;0]	[0.6092;0.1953;1.3685;0.8803;0.2441;0.9904]
7	[0;0;0;0;0;0]	[1;0;1;0;1;0]
8	[0;0;0;0;0;0]	[0;1;0;1;0;1]
9	[0;0;0;0;0;0]	[1;1;1;1;1;1]

Put together everything and form a collective structure for the MAS dynimics.

```
allStates = [];
allStates_dot = [];
for agent=1:NumAgents
    allStates = [allStates; agents(agent).x];
    allStates_dot = [allStates_dot; agents(agent).x_dot];
end
```

All the above results are based on:

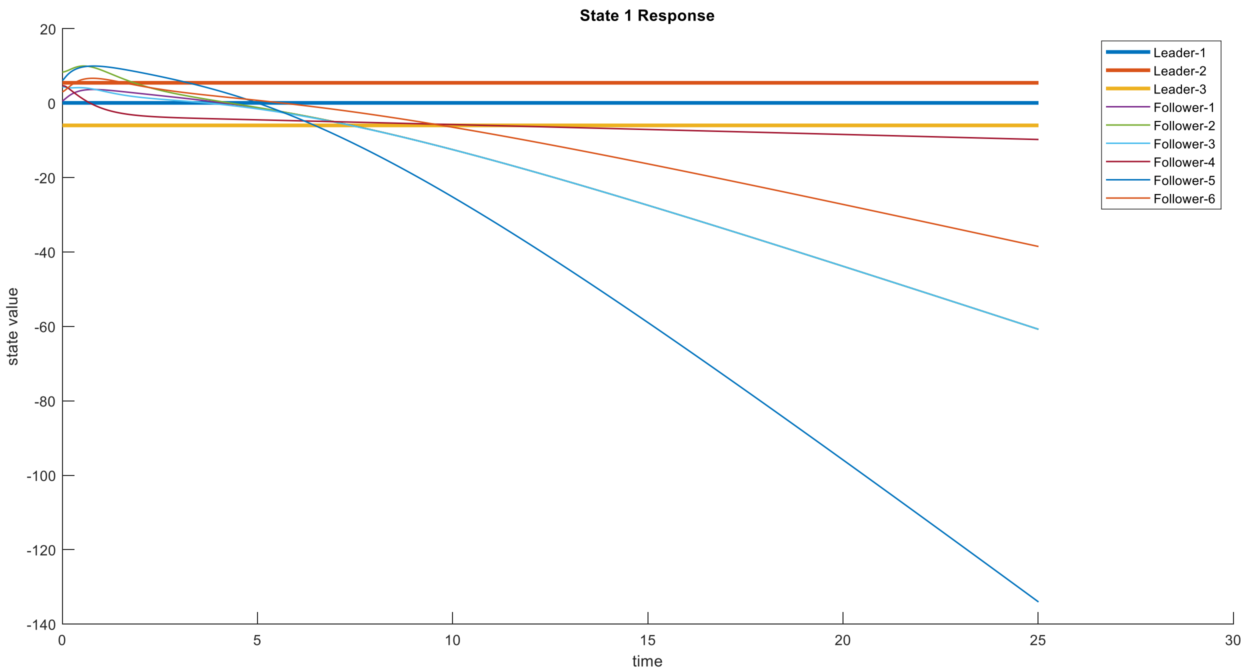
$$\dot{x}_i = Ax_i + Bu_i \qquad i = 1, \dots, 9$$

$$u_i = cF \sum_{j \in \mathcal{F} \cup \mathcal{R}} a_{ij}(x_i - x_j) \qquad i \in \mathcal{F}$$

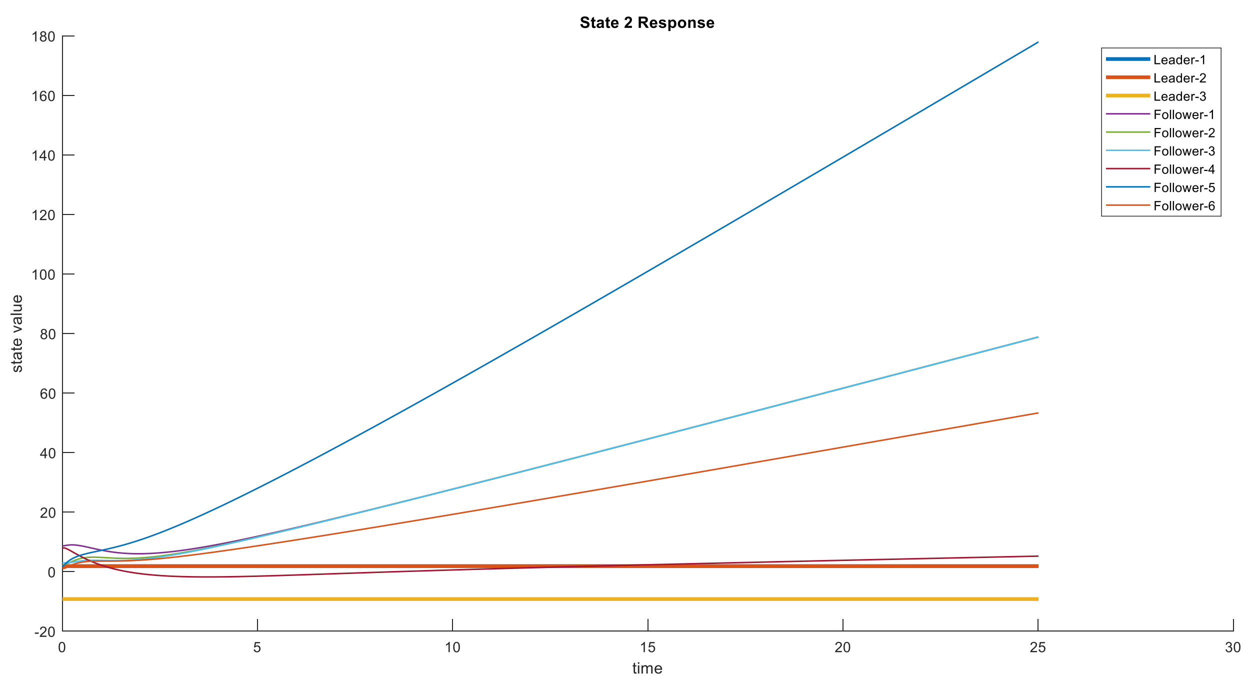
$$c_{th} = \frac{1}{\min\{Re(\lambda_i)\}}$$

Solving the system using the Euler's method obtains the following results:

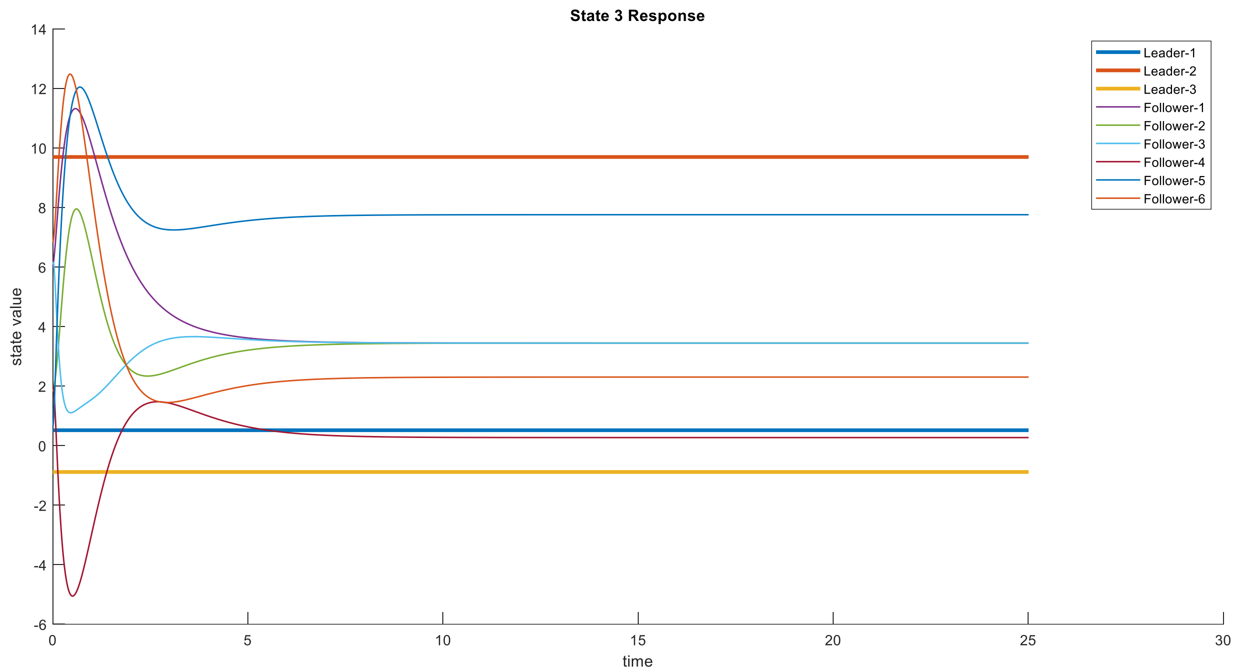
- **state 1:** unstable and diverging



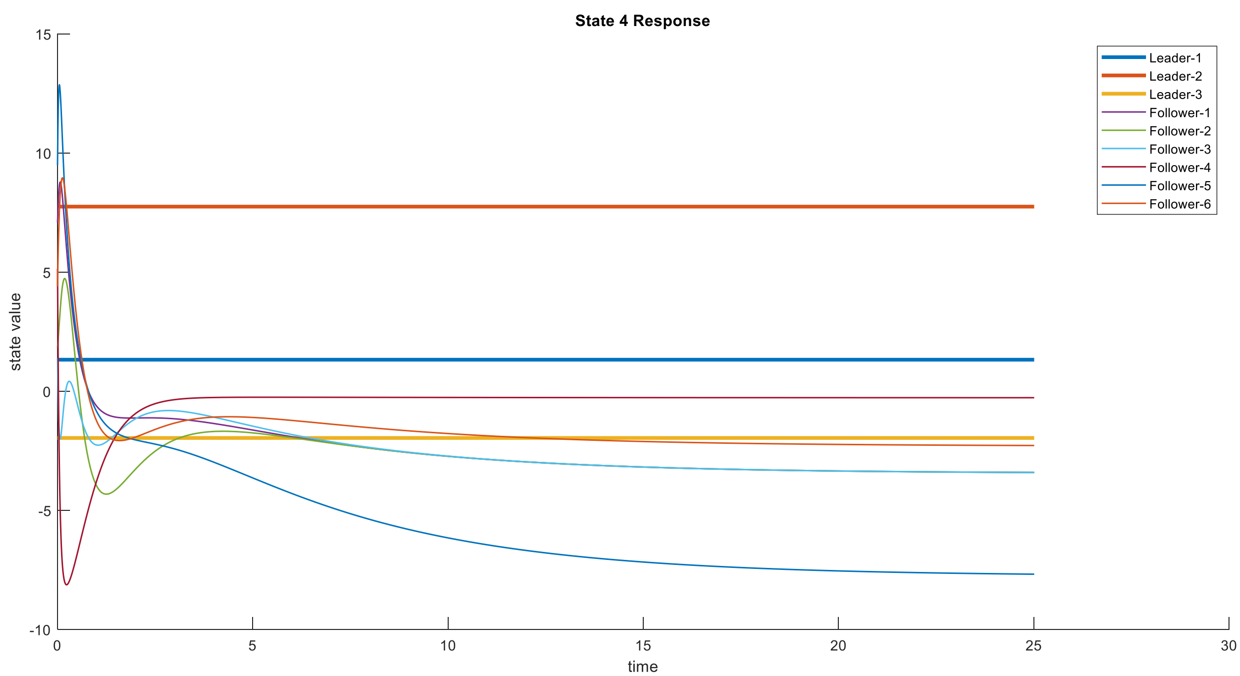
- **state 2:** unstable and diverging



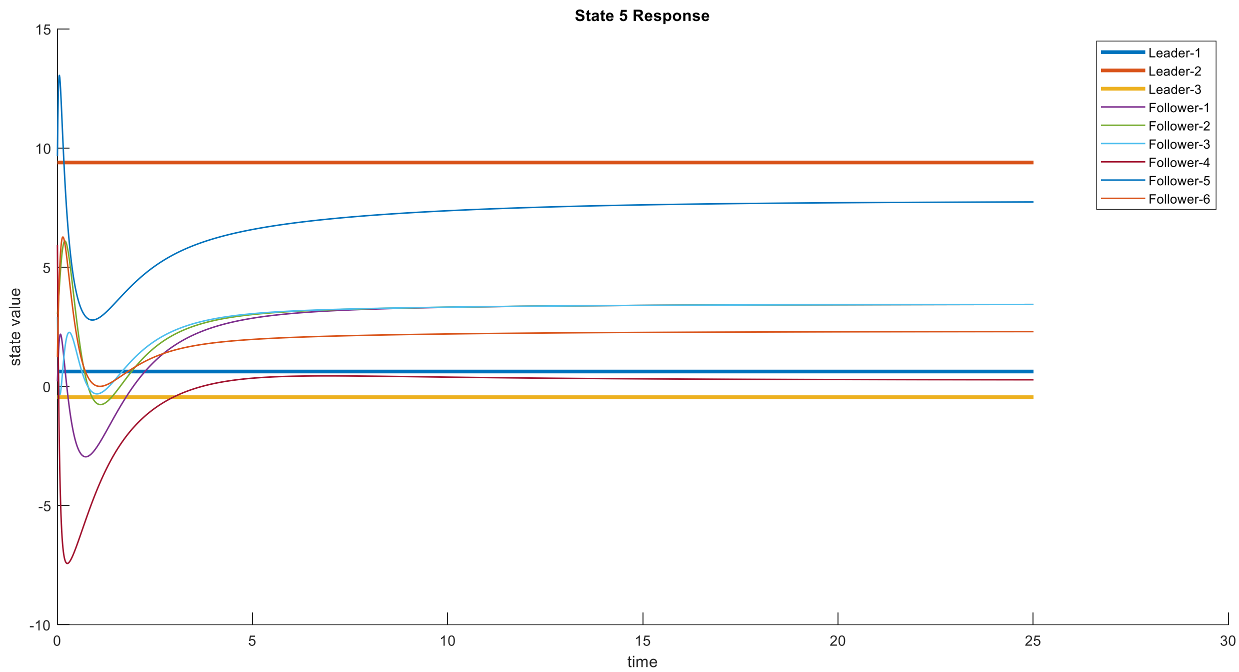
- **state 3:** stable and converging within the leaders' convex hull



- **state 4:** stable and converging but not within the leaders' convex hull



- **state 5:** stable and converging within the leaders' convex hull



- **state 6:** stable and converging within the leaders' convex hull