MAS Course – Assignment 02 – Containment

Problem 04

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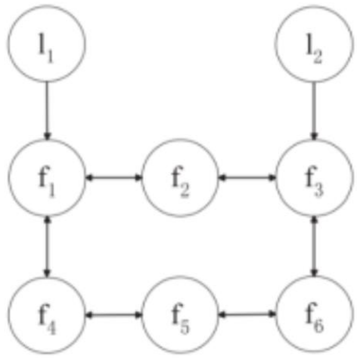
Given the following topoly, the laplacian matrix is formed easilly:




```
L = 8×8
     3    -1     0    -1     0     0    -1     0
    -1     2    -1     0     0     0     0     0
     0    -1     3     0     0    -1     0    -1
    -1     0     0     2    -1     0     0     0
     0     0     0    -1     2    -1     0     0
     0     0    -1     0    -1     2     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
```

MAS basic information:

- Leaders Count:

```
numLeaders = 2
```

- Followers Count:

```
numFollowers = 6
```

We take the following values for the system parameters:

- $\text{time step} = 0.01$
- $\rho_1 = 1$
- $\rho_2 = 1$
- $r_1 = 1 + 1 = 2$
- $r_2 = 1 + 1 = 2$
- $\alpha = 1$

Taking the initial conditions of position and velocity as random numbers along setting the seed value to 42 so that the random numbers won't change under each runtime:

```
x = 8×1
    3.7454
    9.5071
    7.3199
    5.9866
    1.5602
    1.5599
    0.5808
    8.6618

v = 8×1
     0
     0
     0
     0
     0
     0
     0
     0
```

Since the system dynamics are as follows:

$$\dot{x}_i(t) = v_i(t)$$

$$\dot{v}_i(t) = f\big(t, x_i(t), v_i(t)\big) + u_i(t) \qquad i \in \mathcal{F}$$

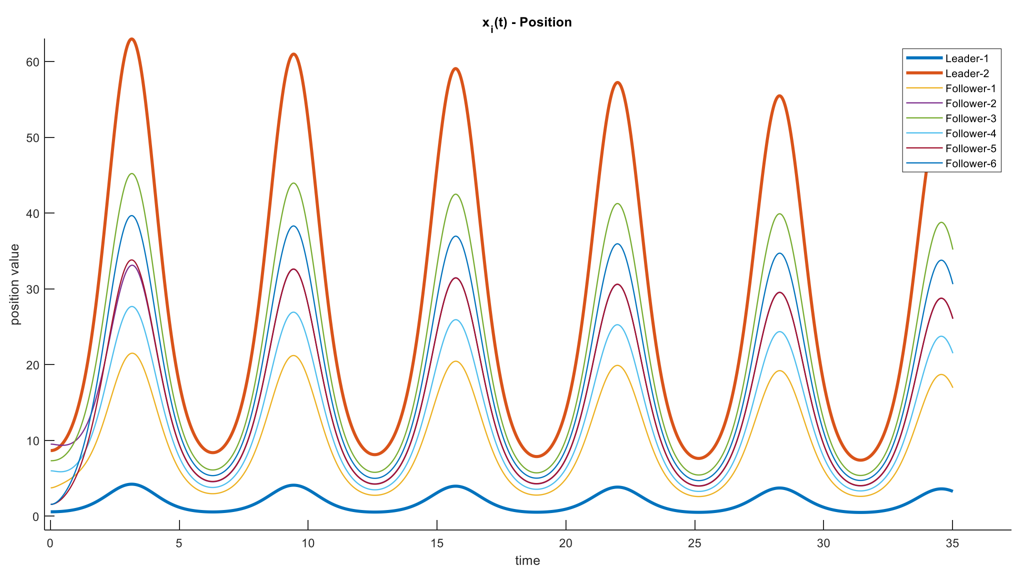
$$\dot{v}_i(t) = f\big(t, x_i(t), v_i(t)\big) \qquad i \in \mathcal{R}$$

$$u_i(t) = -\alpha \sum_{j \in \mathcal{F} \cup \mathcal{R}} a_{ij} \left[r_1 \left(x_i(t) - x_j(t) \right) + r_2 \left(v_i(t) - v_j(t) \right) \right] \qquad i \in \mathcal{F}$$

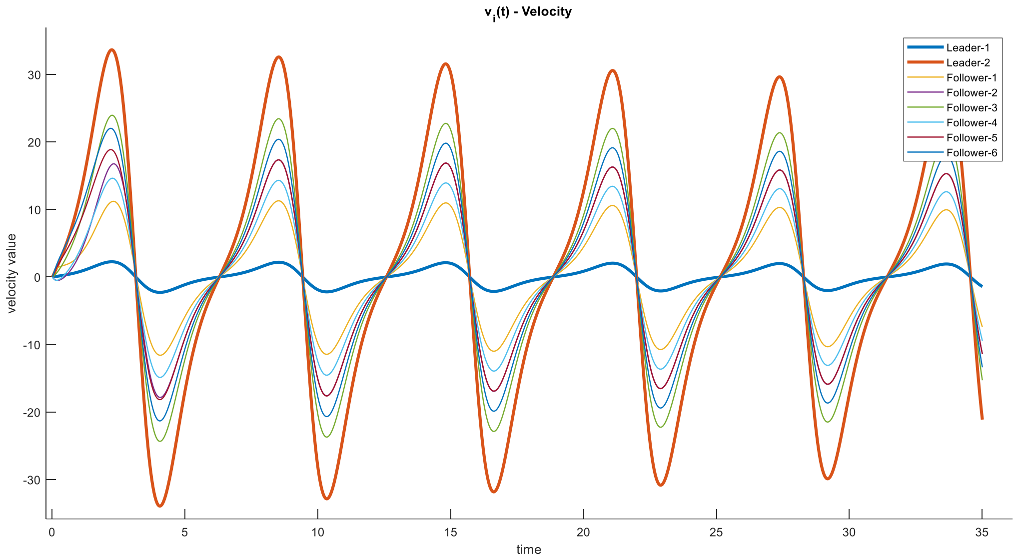
$$f(t, x, v) = [x_i \cos(t) + v_i \sin(t)]$$

solving the system with the Euler's method will lead to the following results:

- Position



- Velocity



It is seen that both position and velocity of the followers are bounded within the leaders' states boundaries.

α acts as a feedback gain in the system. the greater the α, the stronger the control signal affects on the system states. Trial and error on this system shows an upper bound value for α is 22.5 and the lower bound of α, 0.2. Also the minimum value for α in order to be sure that the containment would be achieved is:

$$\alpha > \frac{2r_2^2 + r_1}{r_2^2 \lambda_{min}(\mathcal{L}_1)}$$