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Exercises - Chapter 05

Problem 01

G_1 :

```
Reducible Realization for G=
```

$$\begin{pmatrix} -\frac{s}{(s+1)^2} & \frac{1}{s+1} \\ \frac{2\,s+1}{s\,(s+1)} & \frac{1}{s+1} \end{pmatrix}$$

```
---------------------------------
A = 6×6
     0     1     0     0     0     0
    -1    -2     0     0     0     0
     0     0    -1     0     0     0
     0     0     0     0     1     0
     0     0     0     0    -1     0
     0     0     0     0     0    -1

B = 6×2
     0     0
     1     0
     0     1
     0     0
     1     0
     0     1

C = 2×6
     0    -1     1     0     0     0
     0     0     0     1     2     1

D = 2×2
     0     0
     0     0
Verifying the Realization:

/          s                s        1        1   \
|  - ------------ == - --------,   ----- == ----- |
|     2                       2    s + 1    s + 1 |
|    s  + 2 s + 1      (s + 1)                    |
|                                                 |
|   2         1         2 s + 1     1        1    |
| ----- + --------- == ---------, ----- == ----- |
\ s + 1   s (s + 1)    s (s + 1)  s + 1    s + 1 /
```

G_2

```
Reducible Realization for G=
```

$$\begin{pmatrix} \frac{1}{(s-1)^2} & \frac{1}{(s-1)\,(s+3)} \\ -\frac{6}{(s-1)\,(s+3)^2} & \frac{s-2}{(s+3)^2} \end{pmatrix}$$

```
--------------------------------
A = 9×9
     0     1     0     0     0     0     0     0     0
    -1     2     0     0     0     0     0     0     0
     0     0     0     1     0     0     0     0     0
     0     0     3    -2     0     0     0     0     0
     0     0     0     0     0     1     0     0     0
     0     0     0     0     0     0     1     0     0
     0     0     0     0     9    -3    -5     0     0
     0     0     0     0     0     0     0     0     1
     0     0     0     0     0     0     0    -9    -6

B = 9×2
     0     0
     1     0
     0     0
     0     1
     0     0
     0     0
     1     0
     0     0
     0     1

C = 2×9
     1     0     1     0     0     0     0     0     0
     0     0     0     0    -6     0     0    -2     1

D = 2×2
     0     0
     0     0

Verifying the Realization:

/                 1              1                       1                 1        \
|           ------------ == --------,           ------------ == --------------- |
|            2                     2             2                (s - 1) (s + 3) |
|           s  - 2 s + 1    (s - 1)              s  + 2 s - 3                      |
|                                                                                 |
|            6                        6                s    2      s - 2          |
| - ------------------- == - ---------------,         -- - -- == --------         |
|    3      2                                2         #1   #1            2       |
\   s  + 5 s  + 3 s - 9      (s - 1) (s + 3)                      (s + 3)        /

where

             2
   #1 == s  + 6 s + 9
```

G_3

```
Reducible Realization for G=
```

$$\begin{pmatrix} \frac{s+1}{s^2} & \frac{s+1}{s^2} & \frac{1}{s} \\ 0 & 0 & \frac{1}{s} \end{pmatrix}$$

```
--------------------------------

A = 8×8
      0       1       0       0       0       0       0       0
      0       0       0       0       0       0       0       0
      0       0       0       1       0       0       0       0
      0       0       0       0       0       0       0       0
      0       0       0       0       0       0       0       0
      0       0       0       0       0       0       0       0
      0       0       0       0       0       0       0       0
      0       0       0       0       0       0       0       0

B = 8×3
      0       0       0
      1       0       0
      0       0       0
      0       1       0
      0       0       1
      0       0       0
      0       0       0
      0       0       1

C = 2×8
      1       1       1       1       1       0       0       0
      0       0       0       0       0       0       0       1

D = 2×3
      0       0       0
      0       0       0

Verifying the Realization:

/ 1    1    s + 1  1    1    s + 1  1    1 \
| - + -- == -----, - + -- == -----, - == - |
| s    2        2  s    2        2  s    s |
|     s        s       s        s          |
|                                          |
|                                   1    1 |
|     0 == 0,          0 == 0,      - == - |
\                                   s    s /
```

G_4

```
Reducible Realization for G=
```

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2\,s+6}{(s+1)\ (s+2)\ (s+3)} & 0 \end{pmatrix}$$

```
--------------------------------

A = 10×10
    -1     0     0     0     0     0     0     0     0     0
     0    -1     0     0     0     0     0     0     0     0
     0     0    -4     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0
     0     0     0     0    -2     0     0     0     0     0
     0     0     0     0     0    -4     0     0     0     0
     0     0     0     0     0     0    -1     0     0     0
     0     0     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     0    -2    -3     0
     0     0     0     0     0     0     0     0     0     0

B = 10×3
     1     0     0
     0     1     0
     0     0     1
     0     0     0
     0     1     0
     0     0     1
     1     0     0
     0     0     0
     0     1     0
     0     0     0

C = 3×10
     1     2     1     0     0     0     0     0     0     0
     0     0     0     0     1     1     0     0     0     0
     0     0     0     0     0     0     1     2     0     0

D = 3×3
     0     0     0
     0     0     0
     0     0     0
```

Verifying the Realization:

```
/   1        1                   2        2                  1        1   \
| ----- == -----,              ----- == -----,             ----- == ----- |
| s + 1    s + 1               s + 1    s + 1              s + 4    s + 4 |
|                                                                         |
|                                1        1                  1        1   |
|     0 == 0,                  ----- == -----,             ----- == ----- |
|                              s + 2    s + 2              s + 4    s + 4 |
|                                                                         |
|   1        1          2                 2 s + 6                         |
| ----- == -----, ------------ == -----------------------,     0 == 0     |
| s + 1    s + 1   2                 (s + 1) (s + 2) (s + 3)              |
\                 s  + 3 s + 2                                            /
```

Gilbert Realization for G=

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2\,s+6}{(s+1)\,(s+2)\,(s+3)} & 0 \end{pmatrix}$$

A =

$$\begin{pmatrix} -4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

B =

$$\begin{pmatrix} 0 & 0 & 1.0 \\ 0 & 1.0 & 0 \\ 0.44721359549995793928183473374626 & 0.89442719099991587856366946749251 & 0 \end{pmatrix}$$

C =

$$\begin{pmatrix} 1.0 & 0 & 2.2360679774997896964091736687313 \\ 1.0 & 1.0 & 0 \\ 0 & -2.0 & 2.2360679774997896964091736687313 \end{pmatrix}$$

D =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Verifying the Realization:

```
/  1.0       1                2.0       2                1.0       1   \
| ----- == -----,            ----- == -----,            ----- == ----- |
| s + 1    s + 1             s + 1    s + 1             s + 4    s + 4 |
|                                                                      |
|                             1.0       1                1.0       1   |
|    0 == 0,                 ----- == -----,            ----- == ----- |
|                            s + 2    s + 2             s + 4    s + 4 |
|                                                                      |
|  1.0       1     2.0     2.0             2 s + 6                     |
| ----- == -----, ----- - ----- == ----------------------,    0 == 0   |
\ s + 1    s + 1  s + 1   s + 2    (s + 1) (s + 2) (s + 3)             /
```

Based on lemma 5-1, Gilebrt Realization is Irreducible.

G_5

Reducible Realization for G=

$$\begin{pmatrix} \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

```
A = 5×5
     0     1     0     0     0
   -12    -7     0     0     0
     0     0    -1     0     0
     0     0     0    -3     0
     0     0     0     0     0

B = 5×2
     0     0
     1     0
     0     1
     1     0
     0     0

C = 2×5
     1     0     1     0     0
     0     0     0     1     0

D = 2×2
     0     0
     0     0
```

Verifying the Realization:

```
/       1                 1           1        1   \
| ------------- == ---------------, ----- == ----- |
|  2               (s + 3) (s + 4)  s + 1    s + 1 |
| s  + 7 s + 12                                    |
|                                                  |
|            1        1                            |
|          ----- == -----,              0 == 0     |
\          s + 3    s + 3                          /
```

Gilbert Realization for G=

$$\begin{pmatrix} \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

A =

$$\begin{pmatrix} -4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

B =

$$\begin{pmatrix} 1.0 & 0 \\ 1.0 & 0 \\ 0 & 1.0 \end{pmatrix}$$

C =

$$\begin{pmatrix} -1.0 & 1.0 & 1.0 \\ 0 & 1.0 & 0 \end{pmatrix}$$

D =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Verifying the Realization:

```
/  1.0     1.0            1          1.0       1   \
| ----- - ----- == --------------, ----- == ----- |
| s + 3   s + 4    (s + 3) (s + 4)  s + 1    s + 1 |
|                                                  |
|          1.0       1                             |
|         ----- == -----,             0 == 0      |
\         s + 3    s + 3                           /
```

Based on lemma 5-1, Gilebrt Realization is Irreducible.

G_6

Reducible Realization for G=

$$\begin{pmatrix} \frac{s^3 - s^2 + 1}{s^4} & \frac{1}{s^4} & -\frac{s^3 - s^2 + 2}{s^4} \\ \frac{\frac{3\,s}{2} + 1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3\,s}{2} + 2}{s^4} \\ -\frac{-s^3 + 9\,s^2 + s - 1}{s^4} & -\frac{s^2 - 1}{s^4} & -\frac{-s^3 + s + 1}{s^4} \end{pmatrix}$$

```
--------------------------------
A = 36×36
     0     1     0     0     0     0     0     0     0     0     0     0
0  ⋯
     0     0     1     0     0     0     0     0     0     0     0     0
0
     0     0     0     1     0     0     0     0     0     0     0     0
0
     0     0     0     0     0     0     0     0     0     0     0     0
0
     0     0     0     0     0     1     0     0     0     0     0     0
0
     0     0     0     0     0     0     1     0     0     0     0     0
0
     0     0     0     0     0     0     0     1     0     0     0     0
0
     0     0     0     0     0     0     0     0     0     0     0     0
0
     0     0     0     0     0     0     0     0     0     1     0     0
0
     0     0     0     0     0     0     0     0     0     0     1     0
0
      ⋮

B = 36×3
     0     0     0
     0     0     0
     0     0     0
     1     0     0
     0     0     0
     0     0     0
     0     0     0
     0     1     0
     0     0     0
     0     0     0
      ⋮

C = 3×36
    1.0000         0   -1.0000    1.0000    1.0000         0         0  ⋯
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

D = 3×3
     0     0     0
     0     0     0
     0     0     0
```

```
Verifying the Realization:
/                         3    2                                                           3    2
\
|         1    1    1    s  - s  + 1               1     1         1   1    2     s  - s
+ 2 |
|         - - -- + -- == ----------,             -- == --,       -- - - - -- == - --------
--- |
|         s    2    4          4                   4     4         2   s    4            4
|
|             s    s          s                   s     s         s        s            s
|
|
|
|                        3 s                                                      3 s
|
|                        --- + 1                                                  --- + 2
|
|                    1     2                    1    1    s + 1             2       2
|
|              #1 + -- == -------,             -- + -- == -----,      - #1 - -- == - -------
|
|                    4        4                 3    4       4              4          4
|
|                   s        s                 s    s       s              s          s
|
|
|
|                          3      2                        2                        3
|
| 1    9    1    1      - s  + 9 s  + s - 1  1    1      s  - 1  1    1    1      - s  + s
+ 1 |
| - - -- - -- + -- == - -----------------, -- - -- == - ------, - - -- - -- == - ---------
--- |
| s    2    3    4               4          4    2         4    s    3    4             4
|
\     s    s    s               s          s    s         s         s    s             s
/

where

           3
   #1 == ----
           3
          2 s
```

G_7

```
Reducible Realization for G=
```

$$\begin{pmatrix} \frac{s+3}{(s+1)^3} & \frac{s+3}{(s+1)^2} \\ \frac{s\ (s+3)}{(s+1)^3} & \frac{s\ (s+3)}{(s+1)^2} - \frac{s}{(s+2)^2} \end{pmatrix}$$

```
--------------------------------
A = 12×12
     0     1     0     0     0     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0     0     0     0     0
    -1    -3    -3     0     0     0     0     0     0     0     0     0
```

```
     0     0     0     0     1     0     0     0     0     0     0     0
     0     0     0    -1    -2     0     0     0     0     0     0     0
     0     0     0     0     0     0     1     0     0     0     0     0
     0     0     0     0     0     0     0     1     0     0     0     0
     0     0     0     0     0    -1    -3    -3     0     0     0     0
     0     0     0     0     0     0     0     0     0     1     0     0
     0     0     0     0     0     0     0     0     0     0     1     0
      ⋮

B = 12×2
     0     0
     0     0
     1     0
     0     0
     0     1
     0     0
     0     0
     1     0
     0     0
     0     0
      ⋮

C = 2×12
     3     1     0     3     1     0     0     0     0     0     0     0
     0     0     0     0     0     0     3     1    -4    -1     1     0

D = 2×2
     0     0
     0     1
```

```
Verifying the Realization:

/   s     3      s + 3                s     3      s + 3            \
|  -- + -- == --------,              -- + -- == --------           |
|  #2   #2           3               #3   #3          2            |
|             (s + 1)                            (s + 1)           |
|                                                                  |
|        2                 2                                       |
| 3 s    s      s (s + 3)  s     4    s         s (s + 3)       s    |
| --- + -- == ---------, -- - -- - -- + 1 == --------- - -------- |
|  #2   #2              3  #1   #1   #1                2          2 |
\              (s + 1)                            (s + 1)    (s + 2)  /

where

        4      3       2
   #1 == s  + 6 s  + 13 s  + 12 s + 4

        3      2
   #2 == s  + 3 s  + 3 s + 1

        2
   #3 == s  + 2 s + 1
```

G_8

Reducible Realization for G=

$$\begin{pmatrix} \frac{1}{s\ (s+2)} & \frac{2\,s-1}{s\ (s+2)} & \frac{s-1}{s\ (s+2)} \\ -\frac{1}{s+2} & -\frac{s-1}{(s+1)\ (s+2)} & \frac{1}{(s+1)\ (s+2)} \end{pmatrix}$$

```
--------------------------------
A = 11×11
     0     1     0     0     0     0     0     0     0     0     0
     0    -2     0     0     0     0     0     0     0     0     0
     0     0     0     1     0     0     0     0     0     0     0
     0     0     0    -2     0     0     0     0     0     0     0
     0     0     0     0     0     1     0     0     0     0     0
     0     0     0     0     0    -2     0     0     0     0     0
     0     0     0     0     0     0    -2     0     0     0     0
     0     0     0     0     0     0     0     0     1     0     0
     0     0     0     0     0     0     0    -2    -3     0     0
     0     0     0     0     0     0     0     0     0     0     1
      ⋮

B = 11×3
     0     0     0
     1     0     0
     0     0     0
     0     1     0
     0     0     0
     0     0     1
     1     0     0
     0     0     0
     0     1     0
     0     0     0
      ⋮

C = 2×11
     1     0    -1     2    -1     1     0     0     0     0     0
     0     0     0     0     0     0    -1     1    -1     1     0

D = 2×3
     0     0     0
     0     0     0
```

Verifying the Realization:

```
/           1         2             2 s - 1                   s - 1   \
| #1 == ---------, ----- - #1 == ---------, - #4 - #1 == --------- |
|       s (s + 2)  s + 2         s (s + 2)               s (s + 2) |
|                                                                   |
|                    1    s      s - 1            1     1          |
|     #4 == #4,      -- - -- == - -----,          -- == --         |
\                    #2   #2       #3             #2    #3         /
```

where

```
           1
   #1 == --------
          2
         s  + 2 s

          2
   #2 == s  + 3 s + 2

   #3 == (s + 1) (s + 2)

           1
   #4 == - -----
           s + 2
```

Gilbert Realization for G=

$$\begin{pmatrix} \frac{1}{s\ (s+2)} & \frac{2\,s-1}{s\ (s+2)} & \frac{s-1}{s\ (s+2)} \\ -\frac{1}{s+2} & -\frac{s-1}{(s+1)\ (s+2)} & \frac{1}{(s+1)\ (s+2)} \end{pmatrix}$$

```
--------------------------------
A =
```

$$\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
B =
```

$$\begin{pmatrix} 0.09837000848987476507346276653314 & 0.90889298922158484681785262429447 & 0.40526149036585504087219492888058 \\ 0.90755532876130218883758551373161 & 0.085128535034883484135174010780539 & -0.41121339686320935235120575147554 \\ 0 & 0.89442719099991587856366946749251 & 0.44721359549995793928183473374626 \\ 0.57735026918962576450914878050196 & -0.57735026918962576450914878050196 & -0.57735026918962576450914878050196 \end{pmatrix}$$

```
C =
```

$$\begin{pmatrix} 2.8309397043578072958161925707904 & -0.85777642208825641260766635712777 & 0 & 0.86602540378443864676372317075294 \\ -3.2303104665204843463992155682973 & -0.75172753700274328889190179459769 & 2.2360679774997896964091736687313 & 0 \end{pmatrix}$$

```
D =
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Verifying the Realization:

/ 0.5    0.5         1       2.5    0.5     2 s - 1    1.5    0.5      s - 1   \
| --- - ----- == ---------, ----- - --- == ---------, ----- - --- == --------- |
|  s    s + 2    s (s + 2)  s + 2    s     s (s + 2)  s + 2    s     s (s + 2) |
|                                                                              |
|      1.0         1        2.0     3.0       s - 1      1.0     1.0      1   |
|    - ----- == - -----,    ----- - ----- == - -----,    ----- - ----- == --   |
\      s + 2      s + 2     s + 1   s + 2        #1      s + 1   s + 2    #1   /

where

   #1 == (s + 1) (s + 2)


Based on lemma 5-1, Gilebrt Realization is Irreducible.
```

G_9

```
G =
```

$$\begin{pmatrix} \sigma_1 & \frac{s}{s^2+4\,s+4} \\ -\frac{s^2}{s^3+4\,s^2+5\,s+2}-\sigma_1 & -\frac{s}{s^2+4\,s+4} \end{pmatrix}$$

where

$$\sigma_1 = \frac{s}{(s+2)\;(s^3+4\,s^2+5\,s+2)}$$

```
Reducible Realization for G=
```

$$\begin{pmatrix} \sigma_1 & \frac{s}{s^2+4\,s+4} \\ -\frac{s^2}{s^3+4\,s^2+5\,s+2}-\sigma_1 & -\frac{s}{s^2+4\,s+4} \end{pmatrix}$$

where

$$\sigma_1 = \frac{s}{(s+2)\;(s^3+4\,s^2+5\,s+2)}$$

```
--------------------------------
A = 10×10
     0     1     0     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0     0     0
     0     0     0     1     0     0     0     0     0     0
    -4   -12   -13    -6     0     0     0     0     0     0
     0     0     0     0     0     1     0     0     0     0
     0     0     0     0    -4    -4     0     0     0     0
     0     0     0     0     0     0     0     1     0     0
     0     0     0     0     0     0    -4    -4     0     0
     0     0     0     0     0     0     0     0     0     1
     0     0     0     0     0     0     0     0    -4    -4

B = 10×2
     0     0
     0     0
     0     0
     1     0
     0     0
     0     1
     0     0
     1     0
     0     0
     0     1

C = 2×10
     0     1     0     0     0     1     0     0     0     0
     0     0     0     0     0     0     0    -1     0    -1

D = 2×2
     0     0
     0     0
```

```
Verifying the Realization:

/                s                                 \
| ---------------------------- == #2,  #1 == #1   |
|  4      3        2                               |
| s  + 6 s  + 13 s  + 12 s + 4                     |
|                                                  |
|                      2                           |
|                     s                            |
|  -#1 == - ------------------- - #2, -#1 == -#1  |
|            3      2                              |
\          s  + 4 s  + 5 s + 2                    /

where

             s
   #1 == ------------
          2
         s  + 4 s + 4

                         s
   #2 == -----------------------------
                 3      2
            (s + 2) (s  + 4 s  + 5 s + 2)
```

G_{10}

Reducible Realization for G=

$$\left(\frac{s+1}{s^2+3\,s+2} \quad \frac{s+1}{s^2+4\,s+3}\right)$$

```
---------------------------------
A = 2×2
     -2      0
      0     -3

B = 2×2
      1      0
      0      1

C = 1×2
      1      1

D = 1×2
      0      0

Verifying the Realization:

/   1          s + 1       1          s + 1    \
| ----- == ------------, ----- == ------------ |
| s + 2     2           s + 3     2            |
\          s  + 3 s + 2          s  + 4 s + 3 /
```

Gilbert Realization for G=

$$\left(\frac{s+1}{s^2+3\,s+2} \quad \frac{s+1}{s^2+4\,s+3}\right)$$

```
---------------------------------
A =
```

$$\begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix}$$

B =

$$\begin{pmatrix} 0 & 1.0 \\ 1.0 & 0 \end{pmatrix}$$

C =

$$\begin{pmatrix} 1.0 & 1.0 \end{pmatrix}$$

D =

$$\begin{pmatrix} 0 & 0 \end{pmatrix}$$

Verifying the Realization:

```
/  1.0         s + 1      1.0         s + 1    \
| ----- == ------------, ----- == ------------ |
| s + 2     2              s + 3     2         |
\          s  + 3 s + 2            s  + 4 s + 3 /
```

Based on lemma 5-1, Gilebrt Realization is Irreducible.

G_{11}

Reducible Realization for G=

$$\begin{pmatrix} \frac{1}{s+1} & \frac{1}{s+3} \\ \frac{1}{s} & \frac{2}{s+2} \end{pmatrix}$$

```
--------------------------------
A = 4×4
    -1     0     0     0
     0    -3     0     0
     0     0     0     0
     0     0     0    -2

B = 4×2
     1     0
     0     1
     1     0
     0     1

C = 2×4
     1     1     0     0
     0     0     1     2

D = 2×2
     0     0
     0     0
```

Verifying the Realization:

```
/   1        1      1        1   \
| ----- == -----, ----- == ----- |
| s + 1    s + 1  s + 3    s + 3 |
```

```
|                              |
|     1    1        2        2   |
|     - == -,     ----- == ----- |
\     s    s      s + 2    s + 2 /
```

Gilbert Realization for G=

$$\begin{pmatrix} \frac{1}{s+1} & \frac{1}{s+3} \\ \frac{1}{s} & \frac{2}{s+2} \end{pmatrix}$$

A =

$$\begin{pmatrix} -3 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

B =

$$\begin{pmatrix} 0 & 1.0 \\ 0 & 1.0 \\ 1.0 & 0 \\ 1.0 & 0 \end{pmatrix}$$

C =

$$\begin{pmatrix} 1.0 & 0 & 1.0 & 0 \\ 0 & 2.0 & 0 & 1.0 \end{pmatrix}$$

D =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Verifying the Realization:

```
/  1.0       1     1.0       1   \
| ----- == -----, ----- == ----- |
| s + 1    s + 1  s + 3    s + 3 |
|                                |
|    1.0    1      2.0       2   |
|    --- == -,    ----- == ----- |
\     s     s     s + 2    s + 2 /
```

Based on lemma 5-1, Gilebrt Realization is Irreducible.

G_{12}

Reducible Realization for G=

$$\begin{pmatrix} \frac{s}{(s+1)^2} & \frac{1}{s^2} \\ \frac{1}{s+1} & \frac{1}{s+2} \end{pmatrix}$$

```
--------------------------------
A = 6×6
     0     1     0     0     0     0
    -1    -2     0     0     0     0
     0     0     0     1     0     0
     0     0     0     0     0     0
     0     0     0     0    -1     0
     0     0     0     0     0    -2

B = 6×2
     0     0
     1     0
     0     0
     0     1
     1     0
     0     1

C = 2×6
     0     1     1     0     0     0
     0     0     0     0     1     1

D = 2×2
     0     0
     0     0

Verifying the Realization:

/       s              s         1     1    \
| ------------ == --------,    -- == --    |
|  2                     2      2     2    |
| s  + 2 s + 1    (s + 1)      s     s     |
|                                          |
|       1        1           1        1    |
|     ----- == -----,     ----- == ----- |
\     s + 1    s + 1      s + 2    s + 2 /
```

G_{13}

```
Reducible Realization for G=
```

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{s} \\ \frac{1}{s+1} & \frac{1}{(s+1)^2} \end{pmatrix}$$

```
--------------------------------
A = 6×6
     0     1     0     0     0     0
    -1    -2     0     0     0     0
     0     0     0     0     0     0
     0     0     0    -1     0     0
     0     0     0     0     0     1
     0     0     0     0    -1    -2
```

```
B = 6×2
     0      0
     1      0
     0      1
     1      0
     0      0
     0      1

C = 2×6
     1     0     1     0     0     0
     0     0     0     1     1     0

D = 2×2
     0     0
     0     0

Verifying the Realization:

/       1             1              1    1          \
| ------------ == --------,          - == -          |
|  2                     2           s    s          |
| s  + 2 s + 1    (s + 1)                            |
|                                                    |
|      1        1               1             1      |
|     ----- == -----,      ------------ == --------  |
|     s + 1    s + 1        2                     2  |
\                          s  + 2 s + 1    (s + 1)  /
```

G_{14}

Reducible Realization for G=

$$\begin{pmatrix} -\frac{(s-2)\ (-3\,s^2+5\,s+2)}{s^4-1} & -\frac{(s-2)\ (-2\,s^2+5\,s+1)}{s^4-1} \\ \frac{(s-2)\ (4\,s^2-2\,s+4)}{s^4-1} & -\frac{(s-2)\ (-3\,s^2+2\,s+5)}{s^4-1} \end{pmatrix}$$

```
---------------------------------
A = 15×15
     0     1     0     0     0     0     0     0     0     0     0     0     0
0     0
     0     0     1     0     0     0     0     0     0     0     0     0     0
0     0
     0     0     0     1     0     0     0     0     0     0     0     0     0
0     0
     1     0     0     0     0     0     0     0     0     0     0     0     0
0     0
     0     0     0     0     0     1     0     0     0     0     0     0     0
0     0
     0     0     0     0     0     0     1     0     0     0     0     0     0
0     0
     0     0     0     0     0     0     0     1     0     0     0     0     0
0     0
     0     0     0     0     1     0     0     0     0     0     0     0     0
0     0
     0     0     0     0     0     0     0     0     0     1     0     0     0
0     0
```

```
     0     0     0     0     0     0     0     0     0     0     1     0     0
0     0

B = 15×2
     0     0
     0     0
     0     0
     1     0
     0     0
     0     0
     0     0
     0     1
     0     0
     0     0

C = 2×15
     4     8   -11     3     2     9    -9     2     0     0     0     0     0
0     0
     0     0     0     0     0     0     0     0    -8     8   -10     4    10
-11     3

D = 2×2
     0     0
     0     0
```

G_{15}

```
Reducible Realization for G=
```

$$\begin{pmatrix} \dfrac{s^2+6}{(s^2-1)\ (s+2)} & \dfrac{s^2+s+4}{(s^2-1)\ (s+2)} \\ -\dfrac{-2\,s^2+7\,s+2}{(s^2-1)\ (s+2)} & -\dfrac{-s^2+5\,s+2}{(s^2-1)\ (s+2)} \end{pmatrix}$$

```
----------------------------------
A = 12×12
     0     1     0     0     0     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0     0     0     0     0
     2     1    -2     0     0     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0     0     0     0     0
     0     0     0     0     0     1     0     0     0     0     0     0
     0     0     0     2     1    -2     0     0     0     0     0     0
     0     0     0     0     0     0     0     1     0     0     0     0
     0     0     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     0     0     2     1    -2     0     0     0
     0     0     0     0     0     0     0     0     0     0     1     0

B = 12×2
     0     0
     0     0
     1     0
     0     0
     0     0
     0     1
     0     0
```

```
     0     0
     1     0
     0     0

C = 2×12
     6     0     1     4     1     1     0     0     0     0     0     0
     0     0     0     0     0     0    -2    -7     2    -2    -5     1

D = 2×2
     0     0
     0     0

Verifying the Realization:

/                  2     2                               2     2           \
|             6   s     s  + 6                 s    4   s     s  + s + 4   |
|          - -- - -- == ------,             - -- - -- - -- == ----------   |
|            #1   #1      #2                  #1   #1   #1        #2       |
|                                                                          |
|               2           2                        2         2           |
| 7 s    2   2 s       - 2 s  + 7 s + 2  5 s    2   s       - s  + 5 s + 2 |
| --- + -- - ---- == - ----------------, --- + -- - -- == - -------------- |
\  #1   #1    #1              #2          #1   #1   #1            #2       /

where

            3      2
   #1 == - s  - 2 s  + s + 2

           2
   #2 == (s  - 1) (s + 2)
```