Mohammad Azimi – 402123100

Inline Problems- Chapter 05

5-1

$G(s) = C(sI - A)^{-1}B = D^{-1}(s)N(s)$ $\quad\quad$ $\boldsymbol{B} = \boldsymbol{N_{Lr}}$ $\quad\quad$ $\boldsymbol{N}(\boldsymbol{s}) = \bar{\boldsymbol{L}}(\boldsymbol{s})\ \boldsymbol{N_{Lr}}$

$$D^{-1}(s)\bar{\boldsymbol{L}}(\boldsymbol{s})\ \boldsymbol{N_{Lr}} = C(sI - A)^{-1}\boldsymbol{N_{Lr}}$$

The above relationship holds for every $\boldsymbol{N_{Lr}}$**.** Therefore:

$$D^{-1}(s)\bar{\boldsymbol{L}}(\boldsymbol{s}) = C(sI - A)^{-1}$$

$$rank\ [\bar{\boldsymbol{L}}(\boldsymbol{s}) \quad D(s)] = q \quad \forall s \in ¢ \qquad \text{are Right-Coprime}$$

Also:

$$\deg\det(sI - A) = n = \deg\det D(s)$$

Therefore, an $R(s)$ is found such that:

$$(sI - A) = R(s)D(s) \quad , \quad C = R(s)L(s)$$

Due to unimodularity of $R(s)$, as a result, $(sI - A)$ and C are right coprime, and consequently, the pair $\{A, C\}$ is observable.

5-2

Some of the existing methods are as follows:

Leverrier-Faddeev Algorithm: A recursive method for computing the characteristic polynomial.

Jordan Form: Analyzing linear systems by transforming a matrix into nearly diagonal form.

Controllable and Observable Canonical Forms: Emphasizing the controllability or observability features of the system.

Hankel Realization Method:

A method in control theory used to obtain a minimal state-space representation of a system. This method utilizes the input-output behavior of the system, particularly when the system is described by its impulse response or Markov parameters. The following are the steps to perform this method:

1. **Impulse Response:** Obtain the impulse response or the Markov parameters of the system. These parameters describe the system's output in response to an impulse input at various times.

2. **Construct the Hankel Matrix:** Construct the Hankel matrix H using the impulse response data. The Hankel matrix is typically organized as follows:

$$H = \begin{bmatrix} h_1 & h_2 & h_3 & \cdots & h_n \\ h_2 & h_3 & h_4 & \cdots & h_{n+1} \\ h_3 & h_4 & h_5 & \cdots & h_{n+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_m & h_{m+1} & h_{m+2} & \cdots & h_{m+n-1} \end{bmatrix}$$

where h_i are the Markov parameters.

3. **Singular Value Decomposition (SVD):** Perform the Singular Value Decomposition (SVD) of the Hankel matrix:

$$H = U\Sigma V^T$$

where U and V are orthogonal matrices and Σ is a diagonal matrix containing the singular values.

4. **Truncate Unnecessary Values:** Determine the order of the system by examining the singular values in Σ. Retain the significant singular values and discard the others. Appropriately reduce the matrices U, Σ, and V.

5. **State-Space Matrices:** Use the reduced components of the SVD to construct the state-space matrices A, B, C and D. These matrices provide the minimal state-space representation of the system:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

where

$$A = \Sigma^{-\frac{1}{2}} U^T H_1 V \Sigma^{-\frac{1}{2}} \qquad C = e_1^T U \Sigma^{\frac{1}{2}}$$

$$B = \Sigma^{\frac{1}{2}} V^T e_1 \qquad D = h_0$$

Here, H_1 is the shifted Hankel matrix, and e_1 is a unit vector with 1 in the first position and 0 elsewhere.

The Hankel realization method is used for system identification and model reduction, providing a compact and efficient representation of the system's dynamics.