4-2

Taking system Σ: controllable and unobservable

Applying $\underline{\bar{x}} = P\underline{x}$ would get:

$$\bar{\Sigma}: \begin{cases} \begin{bmatrix} \dot{\underline{\bar{x}}}_o \\ \dot{\underline{\bar{x}}}_{\bar{o}} \end{bmatrix} = \begin{bmatrix} \bar{A}_o & 0 \\ \bar{A}_{21} & \bar{A}_{\bar{o}} \end{bmatrix} \begin{bmatrix} \underline{\bar{x}}_o \\ \underline{\bar{x}}_{\bar{o}} \end{bmatrix} + \begin{bmatrix} \bar{B}_o \\ \bar{B}_{\bar{o}} \end{bmatrix} \underline{u} \\ \underline{y} = [\bar{C}_0 \quad 0] \begin{bmatrix} \underline{\bar{x}}_o \\ \underline{\bar{x}}_{\bar{o}} \end{bmatrix} + D\underline{u} \end{cases}$$

which includes the observable subsystem:

$$\bar{\Sigma}_O: \begin{cases} \dot{\underline{\bar{x}}}_o = \bar{A}_o\, \underline{\bar{x}}_o + \bar{B}_o\, \underline{u} \\ \underline{\bar{y}} = \bar{C}_o\, \underline{\bar{x}}_o + D\, \underline{u} \end{cases}$$

We aim to prove the controllability of $\bar{\Sigma}_O$. For that,

$$rank\{\, [sI - \bar{A}_o \quad \bar{B}_o]\, \} = full \ \text{ for any value of } s.$$

Hence:

$$\bar{A} = \begin{bmatrix} sI - \bar{A}_o & 0 \\ \bar{A}_{21} & sI - \bar{A}_{\bar{o}} \end{bmatrix} \begin{bmatrix} \underline{\bar{x}}_o \\ \underline{\bar{x}}_{\bar{o}} \end{bmatrix} \quad . \quad \bar{B} = \begin{bmatrix} \bar{B}_o \\ \bar{B}_{\bar{o}} \end{bmatrix}$$

$$rank\, [sI - \bar{A} \quad \bar{B}] = rank \begin{bmatrix} sI - \bar{A}_o & 0 & \bar{B}_o \\ \bar{A}_{21} & sI - \bar{A}_{\bar{o}} & \bar{B}_{\bar{o}} \end{bmatrix} = n$$

which is obviously held by basic linear algebra fundamentals. Rows in $[sI - \bar{A}_o \quad 0 \quad \bar{B}_o]$ are independent and therefore columns of $[sI - \bar{A}_o \quad \bar{B}_o]$ are independent (for any value of s).