Mohammad Azimi – 402123100

Assignment 03

Section 02

Note: Since the system dimension is too large, refer to the source code `.mlx` files in order to be able to understand what has been done. This report in pdf format can not be useful.

```
A = 10×10
   -1.9000         0   -0.6800         0         0         0         0 ⋯
         0   -8.9000         0    0.1812   -3.9440         0         0
   -0.0440         0   -2.8200         0         0         0         0
         0  -11.4800         0   -0.0095   -0.0664         0         0
         0   24.8700         0   -0.0205   -1.0340         0         0
         0         0         0    0.0459   -0.9989         0         0
    1.0000         0    0.0459         0         0         0         0
         0         0    1.0010         0         0         0         0
         0    1.0000         0         0         0         0         0
   11.4800         0  -24.8400         0         0         0    9.7900
```

```
B = 10×3
10³ ×
         0    0.1490    0.1050
    1.3620         0         0
         0    0.0023    0.4340
    0.0166         0         0
    0.0575         0         0
         0         0         0
         0         0         0
         0         0         0
         0         0         0
           0   -0.0000   -0.0582
```

```
C = 3×10
     0     0     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     1     0     0     0
     0     0     0     0     0     0     0     1     0     0
```

```
D = 3×3
     0     0     0
     0     0     0
     0     0     0
```

```
sys =

  A = 
             x1         x2         x3         x4         x5         x6         x7         x8         x9
   x1      -1.9          0      -0.68          0          0          0          0          0          0
   x2         0       -8.9          0     0.1812     -3.944          0          0          0          0
   x3    -0.044          0      -2.82          0          0          0          0          0          0
   x4         0     -11.48          0  -0.009451   -0.06638          0          0          0      -9.79
   x5         0      24.87          0   -0.02051     -1.034          0          0          0    -0.4497
   x6         0          0          0    0.04589    -0.9989          0          0          0      25.03
   x7         1          0    0.04593          0          0          0          0          0          0
   x8         0          0      1.001          0          0          0          0          0          0
```

x9	0	1	0	0	0	0	0	0	0
x10	11.48	0	-24.84	0	0	0	9.79	0	0

	x10
x1	0.3065
x2	0
x3	1.223
x4	0
x5	0
x6	0
x7	0
x8	0
x9	0
x10	-1.054

B =

	u1	u2	u3
x1	0	149	105
x2	1362	0	0
x3	0	2.27	434
x4	16.6	0	0
x5	57.5	0	0
x6	0	0	0
x7	0	0	0
x8	0	0	0
x9	0	0	0
x10	0	-0.0032	-58.2

C =

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10
y1	0	0	0	0	0	0	0	0	1	0
y2	0	0	0	0	0	0	1	0	0	0
y3	0	0	0	0	0	0	0	1	0	0

D =

	u1	u2	u3
y1	0	0	0
y2	0	0	0
y3	0	0	0

Continuous-time state-space model.

G =

$$\begin{pmatrix} \frac{4463001600000000000\,s^2 + 3923667130777600000\,s + 4283988382973951 5}{65536\,(50000000000\,s^4 + 497172550000\,s^3 + 5473129039010\,s^2 + 219044219249\,s + 130200898022)} & 0 & 0 \\ 0 & \frac{18638032637500000\,s^2 + 71961013707219000\,s + 63076087555920563 2}{25\,\sigma_1} & \frac{624668100000000\,s^2 + 746070918910000\,s - 420956970058820}{\sigma_1} \\ 0 & \frac{11361350000000\,s^3 + 729060332000\,s^2 + 10418443925465576\,s + 8894840397192750}{s\,\sigma_1} & \frac{2172170000000000\,s^3 + 6037218187000000\,s^2 + 3388022563926000\,s - 225689163700000}{s\,\sigma_1} \end{pmatrix}$$

where

$$\sigma_1 = 5000000000000\,s^4 + 28870000000000\,s^3 + 185818300000000\,s^2 + 295378340909500\,s - 6794168415529$$

Two Left MFDs with different orders for the given transfer matrix:

```
D_l =
```

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = 1638400\, s\ (50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022)\ (5000000000000\, s^4 + 28870000000000\, s^3 + 185818300000000\, s^2 + 295378340909500\, s - 6794168415529)$$

```
N_l =
```

$$\begin{pmatrix} 25\, s\ (4463001600000000000\, s^2 + 3923667130777600000\, s + 4283988382973 9515)\ (5000000000000\, s^4 + 28870000000000\, s^3 + 185818300000000\, s^2 + 295378340909500\, s - 6794168415529) & 0 & 0 \\ 0 & 65536\, s\ (18638032637500000\, s^2 + 71961013707219000\, s + 630760875559205632)\, \sigma_1 & 1638400\, s\ (624668100000000\, s^2 + 746070918910000\, s - \cdots \\ 0 & 1638400\ (11361350000000\, s^3 + 729060332000\, s^2 + 10418443925465576\, s + 8894840397192750)\, \sigma_1 & 1638400\ (2172170000000000\, s^3 + 6037218187000000\, s^2 + 338802256\cdots \end{pmatrix}$$

where

$$\sigma_1 = 50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022$$

Checking for minimality:

```
minimal_01 = 1
```

-> The first MFD is **minimal** since the following matrix is full rank.

$$\mathrm{rank}[D(s) \quad N(s)] = \mathrm{full}$$

The greatest common left divisor (gcld) for G is obtained as:

```
gcld =
```

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = 1638400\, s\ (50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022)\ (5000000000000\, s^4 + 28870000000000\, s^3 + 185818300000000\, s^2 + 295378340909500\, s - 6794168415529)$$

Smith Form of the system is derived as:

```
Smith =
```

$$\begin{pmatrix} 1 & 0 & \\ 0 & 16384000\, s\ (50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022) & \\ 0 & 0 & 4100096000000\, s\ (179712940646400000000000000000000000000000000000000000000000000000\, s^{15} + 41802345763158593536000000000000000000000000000000000000000000000\, s^{14} + 66632173513531884115557480873750000000000000000\cdots \end{pmatrix}$$

Its compact form:

```
SmithCompact =
```

$$\begin{pmatrix} 1 & 0 & \\ 0 & 16384000\, s\ (50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022) & \\ 0 & 0 & 4100096000000\, s\ (179712940646400000000000000000000000000000000000000000000000000000\, s^{15} + 41802345763158593536000000000000000000000000000000000000000000000\, s^{14} + 66632173513531884115557480873750000000000000000\cdots \end{pmatrix}$$

Accordingly, since the Smith-McMillan form is obtained as:

$$\mathrm{SmithMcMillan} = \frac{\mathrm{Smith}}{d(s)}$$; Hence:

```
SmithMcMillan =
```

$$\begin{pmatrix} \frac{1}{1638400\, s\ (50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022)\, \sigma_1} & 0 & \\ 0 & \frac{10}{\sigma_1} & \\ 0 & 0 & 1798926535870464000000000000000000000000000000000000\, s^7 + 13569608428777991862272000000000000000000000000000\, s^6 + 866559222921673900296571260742375000000000000000000\, s^5 + 23279259329284960474835325828528118455000\cdots \end{pmatrix}$$

where

$$\sigma_1 = 5000000000000\, s^4 + 28870000000000\, s^3 + 185818300000000\, s^2 + 295378340909500\, s - 6794168415529$$

And its compact form:

SmithMcMillanCompact =

$$\begin{pmatrix} \frac{1}{1638400\, s\, (50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022)\, \sigma_1} & 0 & \\ 0 & \frac{10}{\sigma_1} & \\ 0 & 0 & 1798926535870464000000000000000000000000000000\, s^7 + 1356960842877799186227200000000000000000000000000\, s^6 + 8665592229216739002965712607423750000000000000000\, s^5 + 232792593292849604748353258285281184550000 \end{pmatrix}$$

where

$$\sigma_1 = 5000000000000\, s^4 + 28870000000000\, s^3 + 185818300000000\, s^2 + 295378340909500\, s - 6794168415529$$

The *McMillan degree* of a transfer-function matrix is the total number of poles in the diagonal elements of the matrix in its McMillan form. This number determines the order of any minimal state-space realization of the transfer-function matrix or the minimal order of coprime matrix-fraction models.

```
Roots of the Denominators (poles):
ans =
```

$$\begin{pmatrix} -2.0198328918412763178064937434886 \\ 0 \\ 0.022676932231485033226692959482812 \\ -4.9527254525477250830147298426798 + 9.1941724407569773070399972494595\, i \\ -4.9527254525477250830147298426798 - 9.1941724407569773070399972494595\, i \\ -1.8884220201951043577100996079971 - 5.1088530320872826348108730718518\, i \\ -1.8884220201951043577100996079971 + 5.1088530320872826348108730718518\, i \\ -0.019000047452274916985270157320171 - 0.15334740050500866836476790402071\, i \\ -0.019000047452274916985270157320171 + 0.15334740050500866836476790402071\, i \end{pmatrix}$$

```
ans =
```

$$\begin{pmatrix} -2.0198328918412763178064937434886 \\ 0.022676932231485033226692959482812 \\ -1.8884220201951043577100996079971 - 5.1088530320872826348108730718518\, i \\ -1.8884220201951043577100996079971 + 5.1088530320872826348108730718518\, i \end{pmatrix}$$

```
poles_sum = 13
```

The total number of poles in McMillan form on the diagonal of the matrix is 14. Hence the McMillan order is 14.

Pole polynomials are as follows:

```
dens
```

```
dens =
```

$$\begin{pmatrix} 1638400\, s\, (50000000000\, s^4 + 497172550000\, s^3 + 5473129039010\, s^2 + 219044219249\, s + 130200898022)\, (5000000000000\, s^4 + 28870000000000\, s^3 + 185818300000000\, s^2 + 295378340909500\, s - 6794168415529) \\ 5000000000000\, s^4 + 28870000000000\, s^3 + 185818300000000\, s^2 + 295378340909500\, s - 6794168415529 \\ 1 \end{pmatrix}$$

and Zero polynomials are:

```
nums =
```

$$\begin{pmatrix} 1 \\ 10 \\ 1798926535870464000000000000000000000000000000\, s^7 + 1356960842877799186227200000000000000000000000000\, s^6 + 8665592229216739002965712607423750000000000000000\, s^5 + 232792593292849604748353258285281184550000000000000000\, s^4 + 238611830575259520751479754565606564501625000000000\, s^3 + 8042093116401104701002 \end{pmatrix}$$

Eigenvalues of A are the system poles. Therefore, `eig(A)` returns system poles.

```
eigs = 10×1 complex
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -1.8884 + 5.1089i
  -1.8884 - 5.1089i
  -2.0198 + 0.0000i
   0.0227 + 0.0000i
  -4.9527 + 9.1942i
  -4.9527 - 9.1942i
```

```
-0.0190 + 0.1533i
-0.0190 - 0.1533i
```

For each element in transfer matrix, we have zeros:

```
========================================
For the transfer matrix G(1,1) =
```

$$\frac{4463001600000000000\,s^2 + 3923667130777600000\,s + 42839883829739515}{65536\ (50000000000\,s^4 + 497172550000\,s^3 + 5473129039010\,s^2 + 219044219249\,s + 130200898022)}$$

```
zeros are:
```

$$\begin{pmatrix} -0.86809691847272541050698226293761 \\ -0.011057400176320110785235064522006 \end{pmatrix}$$

```
-----------------------------

poles are:
```

$$\begin{pmatrix} -4.9527254525477250830147298426798 + 9.1941724407569773070399972494595\,\mathrm{i} \\ -4.9527254525477250830147298426798 - 9.1941724407569773070399972494595\,\mathrm{i} \\ -0.019000047452274916985270157320171 - 0.15334740050500866836476790402071\,\mathrm{i} \\ -0.019000047452274916985270157320171 + 0.15334740050500866836476790402071\,\mathrm{i} \end{pmatrix}$$

```
========================================

For the transfer matrix G(2,2) =
```

$$\frac{18638032637500000\,s^2 + 71961013707219000\,s + 63076087555920563 2}{25\ (5000000000000\,s^4 + 28870000000000\,s^3 + 185818300000000\,s^2 + 295378340909500\,s - 6794168415529)}$$

```
zeros are:
```

$$\begin{pmatrix} -1.9304884562341726396174736819778 - 5.4877945495633389505419511 67206\,\mathrm{i} \\ -1.9304884562341726396174736819778 + 5.487794549563338950541951167206\,\mathrm{i} \end{pmatrix}$$

```
-----------------------------

poles are:
```

$$\begin{pmatrix} -2.0198328918412763178064937434886 \\ 0.022676932231485033226692959482812 \\ -1.8884220201951043577100996079971 - 5.1088530320872826348108730718518\,\mathrm{i} \\ -1.8884220201951043577100996079971 + 5.1088530320872826348108730718518\,\mathrm{i} \end{pmatrix}$$

```
========================================

For the transfer matrix G(2,3) =
```

$$\frac{624668100000000\,s^2 + 746070918910000\,s - 420956970058820}{5000000000000\,s^4 + 28870000000000\,s^3 + 185818300000000\,s^2 + 295378340909500\,s - 6794168415529}$$

```
zeros are:
```

$$\begin{pmatrix} -1.6123120908879770701543497929 66 \\ 0.41796437422051798268053769979207 \end{pmatrix}$$

```
-----------------------------

poles are:
```

$$\begin{pmatrix} -2.019832891841276317806493743488 6 \\ 0.022676932231485033226692959482812 \\ -1.8884220201951043577100996079971 - 5.1088530320872826348108730718518\,i \\ -1.8884220201951043577100996079971 + 5.1088530320872826348108730718518\,i \end{pmatrix}$$

==

For the transfer matrix G(3,2) =

$$\frac{11361350000000\,s^3 + 729060332000\,s^2 + 10418443925465576\,s + 8894840397192750}{s\,(5000000000000\,s^4 + 28870000000000\,s^3 + 185818300000000\,s^2 + 295378340909500\,s - 6794168415529)}$$

zeros are:

$$\begin{pmatrix} -0.85313280710894045446903631977535 \\ 0.39448129342231163692614811583481 + 30.290675821762095075808850899066\,i \\ 0.39448129342231163692614811583481 - 30.290675821762095075808850899066\,i \end{pmatrix}$$

----

poles are:

$$\begin{pmatrix} -2.0198328918412763178064937434886 \\ 0 \\ 0.022676932231485033226692959482812 \\ -1.8884220201951043577100996079971 - 5.1088530320872826348108730718518\,i \\ -1.8884220201951043577100996079971 + 5.1088530320872826348108730718518\,i \end{pmatrix}$$

==

For the transfer matrix G(3,3) =

$$\frac{2172170000000000\,s^3 + 6037218187000000\,s^2 + 3388022563926000\,s - 225689163700000}{s\,(5000000000000\,s^4 + 28870000000000\,s^3 + 185818300000000\,s^2 + 295378340909500\,s - 6794168415529)}$$

zeros are:

$$\begin{pmatrix} -1.9538343055254782528420058847449 \\ -0.88556401592610411594474204428371 \\ 0.060049473525315087680757145618 44 \end{pmatrix}$$

----

poles are:

$$\begin{pmatrix} -2.0198328918412763178064937434886 \\ 0 \\ 0.022676932231485033226692959482812 \\ -1.8884220201951043577100996079971 - 5.1088530320872826348108730718518\,i \\ -1.8884220201951043577100996079971 + 5.1088530320872826348108730718518\,i \end{pmatrix}$$

The system matootix $S(s)$ is formed as in the following.

$$S(s) = \begin{bmatrix} \mathrm{SI} - A & B \\ -C & D \end{bmatrix}$$

sys_matrix =

$$
\begin{pmatrix}
s+\frac{19}{10} & 0 & \frac{17}{25} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{613}{2000} & 0 & 149 & 105 \\
0 & s+\frac{89}{10} & 0 & -\frac{453}{2500} & \frac{493}{125} & 0 & 0 & 0 & 0 & 0 & 1362 & 0 & 0 \\
\frac{11}{250} & 0 & s+\frac{141}{50} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1223}{1000} & 0 & \frac{227}{100} & 434 \\
0 & \frac{287}{25} & 0 & s+\frac{5448130570019655}{576460752303423488} & \frac{3319}{50000} & 0 & 0 & 0 & \frac{979}{100} & 0 & \frac{83}{5} & 0 & 0 \\
0 & -\frac{2487}{100} & 0 & \frac{2051}{100000} & s+\frac{517}{500} & 0 & 0 & 0 & \frac{4497}{10000} & 0 & \frac{115}{2} & 0 & 0 \\
0 & 0 & 0 & -\frac{4589}{100000} & \frac{9989}{10000} & s & 0 & 0 & -\frac{2503}{100} & 0 & 0 & 0 & 0 \\
-1 & 0 & -\frac{4593}{100000} & 0 & 0 & 0 & s & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1001}{1000} & 0 & 0 & 0 & 0 & s & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & s & 0 & 0 & 0 & 0 \\
-\frac{287}{25} & 0 & \frac{621}{25} & 0 & 0 & 0 & -\frac{979}{100} & 0 & 0 & s+\frac{527}{500} & 0 & -\frac{2}{625} & -\frac{291}{5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

The system is **non-minimum phase** since there are poles or zeros on the right side plane.