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Assignment 03

Section 01

Note: Since the system dimension is too large, refer to the source code `.mlx` files in order to be able to understand what has been done. This report in pdf format can not be useful.

```
sys =

  A =
                 x1          x2          x3          x4          x5          x6          x7          x8
x9
   x1       -0.5944      0.8008      -9.791     -0.8747   5.077e-05           0           0           0
0
   x2        -0.744       -7.56     -0.5294       15.72   -0.000939           0           0           0
0
   x3             0           0           0           1           0           0           0           0
0
   x4         1.041      -7.406           0      -15.81  -7.284e-18           0           0           0
0
   x5      -0.05399      0.9985         -17           0           0           0           0           0
0
   x6             0           0           0           0           0     -0.8726      0.8789      -16.82
9.791
   x7             0           0           0           0           0      -2.823      -16.09       3.367
0
   x8             0           0           0           0           0       0.702       0.514      -2.775
0
   x9             0           0           0           0           0           0           1     0.05406  -
4.088e-24
   x10            0           0           0           0           0           0           0       1.001  -
7.573e-23

                x10
   x1             0
   x2             0
   x3             0
   x4             0
   x5             0
   x6             0
   x7             0
   x8             0
   x9             0
   x10            0



B =
             u1      u2      u3      u4
   x1    0.4669       0       0       0
   x2    -2.703       0       0       0
   x3         0       0       0       0
   x4    -133.7       0       0       0
   x5         0       0       0       0
   x6         0       0       0   5.302
   x7         0       0  -156.5  -5.008
   x8         0       0    11.5  -82.04
   x9         0       0       0       0
   x10        0       0       0       0
```

```
C =
           x1       x2       x3       x4       x5       x6       x7       x8       x9      x10
   y1  0.9985  0.05399        0        0        0        0        0        0        0        0
   y2   -3176  0.05874        0        0        0        0        0        0        0        0
   y3       0        0        0        1        0        0        0        0        0        0
   y4       0        0        1        0        0        0        0        0        0        0
   y5       0        0        0        0       -1        0        0        0        0        0
   y6       0        0        0        0        0  0.05882        0        0        0        0
   y7       0        0        0        0        0        0        1        0        0        0
   y8       0        0        0        0        0        0        0        1        0        0
   y9       0        0        0        0        0        0        0        0        1        0
   y10      0        0        0        0        0        0        0        0        0        1


D =
        u1  u2  u3  u4
   y1    0   0   0   0
   y2    0   0   0   0
   y3    0   0   0   0
   y4    0   0   0   0
   y5    0   0   0   0
   y6    0   0   0   0
   y7    0   0   0   0
   y8    0   0   0   0
   y9    0   0   0   0
   y10   0   0   0   0

  Continuous-time state-space model.
```

Two Left MFDs with different orders for the given transfer matrix:

```
D_l =
```

$$\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = 800000\,s\,(425352958651173079329218259289710264320000000000000000\,s^4 + 8395446556673393770568380053399480279038057000000000000\,s^3 + 3133553370480528645586884446550772615530070822400000000\,s^2 + 1000030875400283381689846837145931457119893904111601000\,s + 2056501285833854226978819710270327768117173696943204 99)\ ($$

```
N_l =
```

$$
\left(\begin{array}{c}
-800000\,s\,\sigma_2\,\sigma_1\,\left(\frac{5397891\,\sigma_7}{64\,\sigma_1}-\frac{7339209878835036160\,\sigma_6}{\sigma_1}+\frac{14593497\,\sigma_4}{160000\,\sigma_1}+\frac{992098000396353536\,\sigma_5}{25\,\sigma_1}+\frac{25207931\,\sigma_{11}}{800000\,\sigma_1}-\frac{9323993\,\sigma_3}{320\,\sigma_1}\right)\\
-800000\,s\,\sigma_2\,\sigma_1\,\left(\frac{23344347095823810560000\,\sigma_6}{\sigma_1}-\frac{268272750\,\sigma_7}{\sigma_1}+\frac{7938711\,\sigma_4}{80000\,\sigma_1}+\frac{1079382043772583936\,\sigma_5}{25\,\sigma_1}+\frac{13712853\,\sigma_{11}}{400000\,\sigma_1}+\frac{92679650\,\sigma_3}{\sigma_1}\right)\\
800000\,s\,\left(\frac{23345\,(8445595358960555564531635435497062400000\,s^3+10855165090987593516819228857362853943347 5\,s^2+48724323701897590799718010243717783 37\,s)}{\sigma_1}+\frac{13515\,(600846102098577084639013372096937984000000\,s^3+28951059545283809073872207558963659430290 0\,s^2-2634434413572950779587017493690480769\,s)}{\sigma_1}-588018818534\right.\\
800000\,s\,\left(\frac{23345\,(8445595358960555564531635435497062400000\,s^2+10855165090987593516819228857362853943347 5\,s+48724323701897590799718010243717783 37)}{\sigma_1}+\frac{13515\,(600846102098577084639013372096937984000000\,s^2+28951059545283809073872207558963659430290 0\,s-2634434413572950779587017493690480769)}{\sigma_1}-5880188185346\right.\\
800000\,s\,\left(\frac{378794281757798596837638865017634816\,(3374375000\,s^3+125289393750\,s^2+1615046440050\,s+14237437886479)}{125\,\sigma_1}-\frac{433881306241316533709079599510454272\,(785221841875\,s^2+80604795590475\,s+47213878844947)}{5\,\sigma_1}+21929341263468186062371768090440 49\right.\\
0\\
0\\
0\\
0\\
0
\end{array}\right.
$$

where

$\sigma_1 = 40564819207303340847894502572032000000000000000\,s^5 + 97211155341150018141528301743720366080000000000\,s^4 + 101957379637021499311537126197088367935488000000\,s^3 + 62693622854787074258492782245554315806766489125\,s^2 + 52427141761094003530039785754349338763271368045\,s + 30184053009977380127591849909788514995473$

$\sigma_2 = 4253529586511730793292182592897102643200000000000000\,s^4 + 83954465566733937705683800533994802790380570000000000\,s^3 + 313355337048052864558688444655077261553007082240000000\,s^2 + 100003087540028338168984683714593145711989390411160 1000\,s + 2056501285833854226978819710270327768117173696943 20499$

$\sigma_3 = 649037107316853453566312041152512 0000000\,s^4 + 151679971979948652098447124017342054400000\,s^3 + 1531382659316887244764515978269887456870400\,s^2 - 25350825258260666871112362774582219060568\,s + 767304506223960736428423966974224412301$

$\sigma_4 = 6490371073168534535663120411525120000000000\,s^4 + 10647064323268590793683209247882267852800000\,s^3 + 6690293610570527297702964460438212208230400\,s^2 + 66152940596678575538206888660739425240712533\,s + 58314471186917596224117313223752781249 0$

$\sigma_5 = 115993126735485660561408000000 0\,s^3 + 698419172380070084395348787200\,s^2 + 5154603495631853748589364955 44\,s + 710769616388546172544396257$

$\sigma_6 = 645414681650949791940608 0000\,s^3 + 28150882533310975242036838400\,s^2 + 5493055941282904000908200725 68\,s + 2175672230047977903808 9145$

$\sigma_7 = 10394978310786724912318053651098632192000\,s^3 + 2484348788181685323918023434107383321722 88\,s^2 + 941271873374923773091204181925145002345239\,s + 8297348196164137517094906219569036347 0$

$\sigma_8 = 68056473384187692692674921486353642291200 00000\,s^3 + 115441473542662215606876790141928548728800912000\,s^2 + 112437928355456320220871361887489033154818102544\,s + 18810804479436341367116905776208754395078453 4551$

$\sigma_9 = 21267647932558653966460912964485513216000000000000\,s^3 + 7757587259880094620806291307140210630646285000000\,s^2 + 302619595535410005947439126097692822427633767 50000\,s - 790241196523143251021671458114809867028393966147$

$\sigma_{10} = 170311324643929700963418991019599989833728000000000\,s^3 + 28889228754051219455620909767499159556184186628000 00\,s^2 + 281375915709529441352725063281000090245836171974 7600\,s + 47074038209789444271203444997218622442398708818269 83$

$\sigma_{11} = 241441803921869484726668079308734464000000\,s^3 - 149341189370019337078093246881427619840000 0\,s^2 + 178583821350712348528409585709373310209800\,s - 53926815486035234675707112623131978669 67$

$\sigma_{12} = 920533\,(3738427153585160194224499280897263513108480000000\,s^2 + 152466385631397373973050617909955991248500441250 00\,s + 116725724505794430130850811804575310576158893999883)$

$\sigma_{13} = 27030000000\,s^2 + 2141999078000\,s + 25657021992961$

$\sigma_{14} = 250000000000\,s^2 + 918846710000\,s + 3571676539263$

$\sigma_{15} = 43770010433489933147592238296125466719268096000000\,s^2 + 90733715857835300440609991280223643151960233958800\,s + 585298881035639211448533535018681266325268657726089$

$\sigma_{16} = 174905136597362370220174548219928860688384 0000\,s^2 + 362572291140201000761772905404098652750257115 2\,s + 233885666747508176403047183071336485711447508 73$

$\sigma_{17} = 716081705889249879050738939514227229982720 0000000\,s^2 + 1072334038966867952523875998361063431677133375950 00\,s - 3177850281744776919706866134264281980379917599503$

$\sigma_{18} = 143088735290254623886349022425058532917248000000000\,s^2 + 22726203835273667187514155567023654445845448275360 0\,s - 352896272639522089918833395769548402005000321110 71$

Checking for minimality:

-> The first MFD is **minimal** since the following matrix is full rank.

$$
\mathrm{rank}[D(s)\quad N(s)] = \mathrm{full}
$$

Dl2 =

$$
\begin{pmatrix}
\sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1
\end{pmatrix}
$$

where

$\sigma_1 = 800000\,s^2\,(s+1)\,(4253529586511730793292182592897102643200000000000000\,s^4 + 83954465566733937705683800533994802790380570000000000\,s^3 + 313355337048052864558688444655077261553007082240000000\,s^2 + 1000030875400283381689846837145931457119893904111601000\,s + 205650128583385422697881971027032776811717369694 3$

Nl2 =

$$
\begin{pmatrix}
-800000\, s^2\, (s+1)\, \sigma_2\, \sigma_1\, \left(\frac{5397891\, \sigma_7}{64\, \sigma_1} - \frac{7339209878835036160\, \sigma_6}{\sigma_1} + \frac{14593497\, \sigma_4}{160000\, \sigma_1} + \frac{992098000396353536\, \sigma_5}{25\, \sigma_1} + \frac{25207931\, \sigma_{11}}{800000\, \sigma_1} - \frac{9323993\, \sigma_3}{320\, \sigma_1}\right) \\
-800000\, s^2\, (s+1)\, \sigma_2\, \sigma_1\, \left(\frac{23344347095823810560000\, \sigma_6}{\sigma_1} - \frac{268272750\, \sigma_7}{\sigma_1} + \frac{7938711\, \sigma_4}{80000\, \sigma_1} + \frac{1079382043772583936\, \sigma_5}{25\, \sigma_1} + \frac{13712853\, \sigma_{11}}{400000\, \sigma_1} + \frac{92679650\, \sigma_3}{\sigma_1}\right) \\
800000\, s^2\, (s+1)\, \left(\frac{23345\, (844559535896055556453163543549706240000\, s^3 + 1085516509098759351681922885736285394334 75\, s^2 + 48724323701897590799718010243717783 37\, s)}{\sigma_1} + \frac{13515\, (6008461020985770846390133720969379840000 00\, s^3 + 2895105954528380907387220755896365943029 00\, s^2 - 2634434413572950779587017493690480769\, s)}{\sigma_1} - 58801
\\
800000\, s^2\, (s+1)\, \left(\frac{23345\, (844559535896055556453163543549706240000\, s^2 + 1085516509098759351681922885736285394334 75\, s + 48724323701897590799718010243717783 37)}{\sigma_1} + \frac{13515\, (6008461020985770846390133720969379840000 00\, s^2 + 2895105954528380907387220755896365943029 00\, s - 2634434413572950779587017493690480769)}{\sigma_1} - 588018
\\
800000\, s^2\, (s+1)\, \left(\frac{378794281757798596837638865017634816\, (3374375000\, s^3 + 125289393750\, s^2 + 1615046440050\, s + 14237437886479)}{125\, \sigma_1} - \frac{433881306241316533709079599510454272\, (785221841875\, s^2 + 80604795590475\, s + 47213878844947)}{5\, \sigma_1} + 219293412634681860623717680
\\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
$$

where

$\sigma_1 = 40564819207303340847894502572032000000000000000\, s^5 + 97211155341150018141528301743720366080000000000\, s^4 + 10195737963702149931153712619708836793548800000 00\, s^3 + 62693622854787074258492782245554315806766489125\, s^2 + 52427141761094003530039785754349338763271368045\, s + 3018405300997738012759184990978851499547 38$

$\sigma_2 = 4253529586511730793292182592897102643200000000000000\, s^4 + 8395446556673393770568380053399480279038057000000000 0\, s^3 + 3133553370480528645586884446550772615530070822400000000\, s^2 + 10000308754002833816898468371459314571198939041116 01000\, s + 20565012858338542269788197102703277681171736969432 0499$

$\sigma_3 = 6490371073168534535663120411525120000000\, s^4 + 1516799719799486520984471240173420544000 00\, s^3 + 15313826593168872447645159782698874568704 00\, s^2 - 25350825258260666871112362774582219060568\, s + 7673045062239607364284239669742244 12301$

$\sigma_4 = 6490371073168534535663120411525120000000 00\, s^4 + 1064706432326859079368320924788226785280 0000\, s^3 + 66902936105705272977029644604382122082304 00\, s^2 + 661529405966785755382068866073942524071 2533\, s + 5831447118691759622411731322375278 12490$

$\sigma_5 = 1159931267354856605614080000000\, s^3 + 6984191723800700843953487872 00\, s^2 + 5154603495631853748589364955 44\, s + 7107696163885461725443962 57$

$\sigma_6 = 6454146816509497919406080000\, s^3 + 2815088253331097524203683 8400\, s^2 + 5493055941282904000908200725 68\, s + 2175672230047977903808 9145$

$\sigma_7 = 1039497831078672491231805365109863219 2000\, s^3 + 2484348788181685323918023434107383321722 88\, s^2 + 9412718733749237730912041819251450023452 39\, s + 82973481961641375170949062195690363470$

$\sigma_8 = 680564733841876926926749214863536422912 0000000\, s^3 + 1154414735426622156068767901419285487288009 12000\, s^2 + 1124379283554563202208713618874890331548181025 44\, s + 1881080447943634136711690577620875439507845 34551$

$\sigma_9 = 212676479325586539664609129644855132160000 0000000\, s^3 + 7757587259880094620806291307140210630646285 000000\, s^2 + 3026195955354100059474391260976928224276337675 0000\, s - 790241196523143251021671458114809867028393966147$

$\sigma_{10} = 170311324643929700963418991019599989833728000 0000000\, s^3 + 2888922875405121945562090976749915955618418666 2800000\, s^2 + 28137591570952944135272506328100009024583617197476 00\, s + 4707403820978944427120344499721862244239870881826983$

$\sigma_{11} = 2414418039218694847266680793087344640000 00\, s^3 - 1493411893700193370780932468814276198400 000\, s^2 + 178583821350712348528409585709373310209800\, s - 5392681548603523467570711262313197866967$

$\sigma_{12} = 920533\, (373842715358516019422449928089726351310848 0000000\, s^2 + 152466385631397373973050617909955991248500441 25000\, s + 116725724505794430130850811804575310576158893999883)$

$\sigma_{13} = 27030000000\, s^2 + 2141999078000\, s + 25657021992961$

$\sigma_{14} = 250000000000\, s^2 + 918846710000\, s + 3571676539263$

$\sigma_{15} = 437700104334899331475922382961254667192680960000 00\, s^2 + 907337158578353004406099912802236431519602339588 00\, s + 585298881035639211448533535018681266325268657726089$

$\sigma_{16} = 174905136597362370220174548219928860688384 0000\, s^2 + 362572291140201000761772905404098652750257 1152\, s + 233885666747508176403047183071336485711447508 73$

$\sigma_{17} = 716081705889249879050738939514227229982720 0000000\, s^2 + 107233403896686795252387599836106343167713337 595000\, s - 3177850281744776919706866134264281980379917599503$

$\sigma_{18} = 143088735290254623886349022425058532917248 00000000\, s^2 + 227262038352736671875141555670236544458454482753 600\, s - 352896272639522089918833395769548402005000321110 71$

```
minimal_02 = 0
```

-> The second MFD is **not minimal** since either the following matrix is not full rank or the determinant of matrix $D(s)$ does not have the lowest order and it is higher than the previous LMFD.

The greatest common left divisor (gcld) for G is obtained as:

```
gcld =
```

$$
\begin{pmatrix}
\sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1
\end{pmatrix}
$$

where

$\sigma_1 = 800000\, s\, (4253529586511730793292182592897102643200000000000000\, s^4 + 8395446556673393770568380053399480279038057000000000 0\, s^3 + 3133553370480528645586884446550772615530070822400000000\, s^2 + 10000308754002833816898468371459314571198939041116 01000\, s + 20565012858338542269788197102703277681171736969432 0499)\ ($

and the corresponding LMFD which is minimal by the way is as:

D_l =

$$\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = 800000\, s\, (4253529586511730793292182592897102643200000000000000\, s^4 + 8395446556673393770568380053399480279038057000000000\, s^3 + 3133553370480528645586884446550772615530070822400000000\, s^2 + 1000030875400283381689846837145931457119893904111601000\, s + 205650128583385422697881971027032776811717369694320499)\ ($$

N_l =

$$\begin{pmatrix} -800000\, s\, \sigma_2\, \sigma_1 \left(\frac{5397891\, \sigma_7}{64\, \sigma_1} - \frac{7339209878835036160\, \sigma_6}{\sigma_1} + \frac{14593497\, \sigma_4}{160000\, \sigma_1} + \frac{992098000396353536\, \sigma_5}{25\, \sigma_1} + \frac{25207931\, \sigma_{11}}{800000\, \sigma_1} - \frac{9323993\, \sigma_3}{320\, \sigma_1}\right) \\ -800000\, s\, \sigma_2\, \sigma_1 \left(\frac{2334434709582381056000\, \sigma_6}{\sigma_1} - \frac{268272750\, \sigma_7}{\sigma_1} + \frac{7938711\, \sigma_4}{80000\, \sigma_1} + \frac{1079382043772583936\, \sigma_5}{25\, \sigma_1} + \frac{13712853\, \sigma_{11}}{400000\, \sigma_1} + \frac{92679650\, \sigma_3}{\sigma_1}\right) \\ 800000\, s \left(\frac{23345\ (8445595358960555564531635435497062400000\, s^3 + 1085516509098759351681922885736285394333475\, s^2 + 4872432370189759079971801024371778337\, s)}{\sigma_1} + \frac{13515\ (600846102098577084639013372096937984000000\, s^3 + 2895105954528380907387220755896365943029000\, s^2 - 2634434413572950779587017493690480769\, s)}{\sigma_1} - \frac{5880188185 34}{} \right. \\ 800000\, s \left(\frac{23345\ (8445595358960555564531635435497062400000\, s^2 + 1085516509098759351681922885736285394333475\, s + 4872432370189759079971801024371778337)}{\sigma_1} + \frac{13515\ (600846102098577084639013372096937984000000\, s^2 + 2895105954528380907387220755896365943029000\, s - 2634434413572950779587017493690480769)}{\sigma_1} - \frac{588018818534 6}{} \right. \\ 800000\, s \left(\frac{378794281757798596837638865017634816\ (3374375000\, s^3 + 125289393750\, s^2 + 1615046440050\, s + 14237437886479)}{125\, \sigma_1} - \frac{433881306241316533709079599510454272\ (785221841875\, s^2 + 80604795590475\, s + 47213878844947)}{5\, \sigma_1} + \frac{2192934126346818606237176809044049}{} \right. \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

where

$$\sigma_1 = 4056481920730334084789450257203200000000000000\, s^5 + 972111553411500181415283017437203660800000000\, s^4 + 1019573796370214993115371261970883679354880000000\, s^3 + 62693622854787074258492782245554315806766489125\, s^2 + 52427141761094003530039785754349338763271368045\, s + 30184053009977380127591849909788514995473$$

$$\sigma_2 = 4253529586511730793292182592897102643200000000000000\, s^4 + 8395446556673393770568380053399480279038057000000000\, s^3 + 3133553370480528645586884446550772615530070822400000000\, s^2 + 1000030875400283381689846837145931457119893904111601000\, s + 205650128583385422697881971027032776811717369694320499$$

$$\sigma_3 = 6490371073168534535663120411525120000000\, s^4 + 151679971979948652098447124017342054400000\, s^3 + 1531382659316887244764515978269887456870400\, s^2 - 253508252582606666871112362774582219060568\, s + 767304506223960736428423966974224412301$$

$$\sigma_4 = 6490371073168534535663120411525120000000000\, s^4 + 10647064323268590793683209247882267852800000\, s^3 + 6690293610570527297702964460438212208230400\, s^2 + 6615294059667857553820688660739425240712533\, s + 583144711869175962241173132237527812490$$

$$\sigma_5 = 115993126735485660561408000000\, s^3 + 698419172380070084395348787200\, s^2 + 515460349563185374858936495544\, s + 710769616388546172544396257$$

$$\sigma_6 = 6454146816509497919406080000\, s^3 + 28150882533310975242036838400\, s^2 + 549305594128290400090820072568\, s + 21756722300479779038089145$$

$$\sigma_7 = 10394978310786724912318053651098632192000\, s^3 + 248434878818168532391802343410738332172288\, s^2 + 941271873374923773091204181925145002345239\, s + 82973481961641375170949062195690363470$$

$$\sigma_8 = 6805647338418769269267492148635364229120000000\, s^3 + 115441473542662215606876790141928548728800912000\, s^2 + 112437928355456320220871361887489033154818102544\, s + 188108044794363413671169057762087543950784534551$$

$$\sigma_9 = 21267647932558653966460912964485513216000000000000\, s^3 + 7757587259880094620806291307140210630646285000000\, s^2 + 30261959553541000594743912609769282242763376750000\, s - 790241196523143251021671458114809867028393966147$$

$$\sigma_{10} = 1703113246439297009634189910195999898337280000000000\, s^3 + 2888922875405121945562090976749915955618418662800000\, s^2 + 2813759157095294413527250632810000902458361719747600\, s + 4707403820978944427120344499721862244239870881826983$$

$$\sigma_{11} = 241441803921869484726668079308734464000000\, s^3 - 1493411893700193370780932468814276198400000\, s^2 + 178583821350712348528409585709373310209800\, s - 5392681548603523467570711262313197866967$$

$$\sigma_{12} = 920533\ (37384271535851601942244992808972635131084800000000\, s^2 + 15246638563139737397305061790995599124850044125000\, s + 116725724505794430130850811804575310576158893999883)$$

$$\sigma_{13} = 27030000000\, s^2 + 2141999078000\, s + 25657021992961$$

$$\sigma_{14} = 250000000000\, s^2 + 918846710000\, s + 3571676539263$$

$$\sigma_{15} = 43770010433489933147592238296125466719268096000000\, s^2 + 90733715857835300440609991280223643151960233958800\, s + 58529888103563921144853353501868126632526865772608 9$$

$$\sigma_{16} = 17490513659736237022017454821992886068838400000\, s^2 + 36257229114020100076177290540409865275025711 52\, s + 2338856667475081764030471830713364857 1144750873$$

$$\sigma_{17} = 71608170588924987905073893951422722998272000000000\, s^2 + 1072334038966867952523875998361063431677133375950000\, s - 3177850281744776919706866134264281980379917599503$$

$$\sigma_{18} = 14308873529025462388634902242505853291724800000000\, s^2 + 2272620383527366718751415556702365444584544827536 00\, s - 3528962726395220899188333957695484020050003211 1071$$

D_r =

$$\begin{pmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_1 & 0 & 0 \\ 0 & 0 & \sigma_1 & 0 \\ 0 & 0 & 0 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = 800000\, s\, (4253529586511730793292182592897102643200000000000000\, s^4 + 8395446556673393770568380053399480279038057000000000\, s^3 + 3133553370480528645586884446550772615530070822400000000\, s^2 + 1000030875400283381689846837145931457119893904111601000\, s + 205650128583385422697881971027032776811717369694320499)\ ($$

N_r =

$$\begin{pmatrix} -800000\, s\, \sigma_2\, \sigma_1 \left(\frac{5397891\, \sigma_7}{64\, \sigma_1} - \frac{73392098788350361 60\, \sigma_6}{\sigma_1} + \frac{14593497\, \sigma_4}{160000\, \sigma_1} + \frac{99209800039635353 6\, \sigma_5}{25\, \sigma_1} + \frac{25207931\, \sigma_{11}}{800000\, \sigma_1} - \frac{9323993\, \sigma_3}{320\, \sigma_1}\right) \\ -800000\, s\, \sigma_2\, \sigma_1 \left(\frac{2334434709582381056 0000\, \sigma_6}{\sigma_1} - \frac{268272750\, \sigma_7}{\sigma_1} + \frac{7938711\, \sigma_4}{80000\, \sigma_1} + \frac{107938204377258393 6\, \sigma_5}{25\, \sigma_1} + \frac{13712853\, \sigma_{11}}{400000\, \sigma_1} + \frac{92679650\, \sigma_3}{\sigma_1}\right) \\ 800000\, s \left(\frac{23345\, (844559535896055556453163543549706240 0000\, s^3 + 1085516509098759351681922885736285394 33475\, s^2 + 48724323701897590799718010243717783 37\, s)}{\sigma_1} + \frac{13515\, (600846102098577084639013372096937984 000000\, s^3 + 2895105954528380907387220755896365943 02900\, s^2 - 263443441357295077958701749369048076 9\, s)}{\sigma_1} - 588018818534\cdots \right. \\ 800000\, s \left(\frac{23345\, (844559535896055556453163543549706240 0000\, s^2 + 1085516509098759351681922885736285394 33475\, s + 48724323701897590799718010243717783 37)}{\sigma_1} + \frac{13515\, (600846102098577084639013372096937984 000000\, s^2 + 2895105954528380907387220755896365943 02900\, s - 263443441357295077958701749369048076 9)}{\sigma_1} - 5880188185346\cdots \right. \\ 800000\, s \left(\frac{378794281757798596837638865017634816\, (3374375000\, s^3 + 125289393750\, s^2 + 1615046440050\, s + 14237437886479)}{125\, \sigma_1} - \frac{433881306241316533709079599510454272\, (785221841875\, s^2 + 80604795590475\, s + 47213878844947)}{5\, \sigma_1} + 21929341263468186062371768090440499\cdots \right. \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

where

$\sigma_1 = 405648192073033408478945025720320000000000000000\, s^5 + 97211155341150018141528301743720366080000000000\, s^4 + 10195737963702149931153712619708836793548800000\, s^3 + 626936228547870742584927822455543158067664891 25\, s^2 + 52427141761094003530039785754349338763271368045\, s + 30184053009977380127591849909788514995473 8\cdots$

$\sigma_2 = 42535295865117307932921825928971026432000000000000000\, s^4 + 8395446556673393770568380053399480279038057000000000000\, s^3 + 3133553370480528645586884446550772615530070822400000000\, s^2 + 10000308754002833816898468371459314571198939041116010 00\, s + 205650128583385422697881971027032776811717369694320499$

$\sigma_3 = 64903710731685345356631204115251200000000\, s^4 + 15167997197994865209844712401734205440000 0\, s^3 + 15313826593168872447645159782698874568704 00\, s^2 - 25350825258260666871112362774582219060568\, s + 76730450622396073642842396697422441 2301$

$\sigma_4 = 64903710731685345356631204115251200000000 00\, s^4 + 10647064323268590793683209247882267852800000\, s^3 + 66902936105705272977029644604382122082304 00\, s^2 + 66152940596678575538206886607394252407125 33\, s + 58314471186917596224117313223752781 2490$

$\sigma_5 = 11599312673548566056140800000000\, s^3 + 6984191723800700843953487872 00\, s^2 + 51546034956318537485893649554 4\, s + 71076961638854617254439625 7$

$\sigma_6 = 6454146816509497919406080000\, s^3 + 281508825333109752420368384 00\, s^2 + 549305594128290400090820072568\, s + 217567223004797790380891 45$

$\sigma_7 = 1039497831078672491231805365109863219 2000\, s^3 + 248434878818168532391802343410738332172288\, s^2 + 941271873374923773091204181925145002345239\, s + 829734819616413751709490621956903634 70$

$\sigma_8 = 680564733841876926926749214863536422912 0000000\, s^3 + 1154414735426622156068767901419285487288009 12000\, s^2 + 1124379283554563202208713618874890331548181025 44\, s + 18810804479436341367116905776208754395078453 4551$

$\sigma_9 = 2126764793255865396646091296448551321600000000000 00\, s^3 + 7757587259880094620806291307140210630646285000000\, s^2 + 3026195955354100059474391260976928224276337675000 0\, s - 79024119652314325102167145811480986702839396614 7$

$\sigma_{10} = 1703113246439297009634189910195999898337280000000000 0\, s^3 + 2888922875405121945562090976749915955618418662800000\, s^2 + 28137591570952944135272506328100009024583617197476 00\, s + 47074038209789444271203444997218622442398708818269 83$

$\sigma_{11} = 24144180392186948472666807930873446400 00000\, s^3 - 1493411893700193370780932468814276198400000\, s^2 + 1785838213507123485284095857093733102098 00\, s - 53926815486035234675707112623131978669 67$

$\sigma_{12} = 920533\, (373842715358516019422449928089726351310848000000 00\, s^2 + 15246638563139737397305061790995599124850044125000\, s + 11672572450579443013085081180457531057615889399988 3)$

$\sigma_{13} = 27030000000\, s^2 + 2141999078000\, s + 25657021992961$

$\sigma_{14} = 250000000000\, s^2 + 918846710000\, s + 3571676539263$

$\sigma_{15} = 437700104334899331475922382961254667192680960000 00\, s^2 + 90733715857835300440609991280223643151960233958800\, s + 58529888103563921144853353501868126632526865772608 9$

$\sigma_{16} = 174905136597362370220174548219928860688384000 0\, s^2 + 362572291140201000761772905404098652750257115 2\, s + 233885666747508176403047183071336485711447508 73$

$\sigma_{17} = 716081705889249879050738939514227229982720000000 0\, s^2 + 107233403896686795252387599836106343167713337595000\, s - 317785028174477691970686613426428198037991759950 3$

$\sigma_{18} = 143088735290254623886349022425058532917248000000 00\, s^2 + 227262038352736671875141555670236544458454482753600\, s - 352896272639522089918833395769548402005000321110 71$

Smith Form of the system is derived as:

Its compact form:

```
SmithCompact =
```

$$\begin{pmatrix} 1 & 0 & \\ 0 & 640\, s\, (8192000000000000000\, s^5 + 196316364800000000000\, s^4 + 2059012884337232281 6\, s^3 + 12660876307663884288\, s^2 + 10587576961998573568\, s + 6095620961456579) & \\ 0 & 0 & 32000000\, s\, (2048000000000000000000000000000\, s^9 + 8950169600000000000000000000000\, s^8 + 16343317982670280704000000000000\, s^7 + 14573643843553928557527040000000000\, s^6\cdots \end{pmatrix}$$

Accordingly, since the Smith-McMillan form is obtained as:

$$\text{SmithMcMillan} = \frac{\text{Smith}}{d(s)}$$; Hence:

```
SmithMcMillan =
```

$$\begin{pmatrix} \frac{1}{1600000\, s\, \sigma_1\, \sigma_2} & 0 & 0 \\ 0 & \frac{1}{2500\, \sigma_1} & 0 \\ 0 & 0 & \frac{20\, (20480000000000000000000000000\, s^9 + 8950169600000000000000000000000\, s^8 + 1634331798267028070400000000000\, s^7 + 145736438435539285575270400000000\, s^6 + 5607148936791719733597411328000000\, s^5 + 151937011126979430633377611587072000\, s^4 + 1188061288419431359089189122597601 28\, s^3 + 775445}{\sigma_1\, \sigma_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where

$$\sigma_1 = 25000000000000\, s^4 + 493440000000000\, s^3 + 1841737142500000\, s^2 + 5877653223404500\, s + 1208702821978233$$

$$\sigma_2 = 8192000000000000\, s^5 + 196316364800000000\, s^4 + 2059012884337232 2816\, s^3 + 12660876307663884288\, s^2 + 10587576961998573568\, s + 6095620961456579$$

And its compact form:

```
SmithMcMillanCompact =
```

$$\begin{pmatrix} \frac{1}{1600000\, s\, \sigma_1\, (8192000000000000\, s^5 + 196316364800000000\, s^4 + 20590128843372322816\, s^3 + 12660876307663884288\, s^2 + 10587576961998573568\, s + 6095620961456579)} & 0 & 0 \\ 0 & \frac{1}{2500\, \sigma_1} & 0 \\ 0 & 0 & 20 \end{pmatrix}$$

where

$$\sigma_1 = 25000000000000\, s^4 + 493440000000000\, s^3 + 1841737142500000\, s^2 + 5877653223404500\, s + 1208702821978233$$

The *McMillan degree* of a transfer-function matrix is the total number of poles in the diagonal elements of the matrix in its McMillan form. This number determines the order of any minimal state-space realization of the transfer-function matrix or the minimal order of coprime matrix-fraction models.

```
ans =
```

$$\begin{pmatrix} -16.048276708177811460635376526 84 \\ -0.21991506529532710642684500207 14 \\ -0.00057612988463658302696282208399979 \\ 0 \\ -11.682800845216845884441135539113 + 10.015961587777745365469364693264\, i \\ -11.682800845216845884441135539113 - 10.015961587777745365469364693264\, i \\ -1.7347041132634307164688892355445 + 3.2695613084428032499317902454879\, i \\ -1.7347041132634307164688892355445 - 3.2695613084428032499317902454879\, i \\ -0.29911108984083582404538304984457 + 0.67522531332959456429106750952528\, i \\ -0.29911108984083582404538304984457 - 0.67522531332959456429106750952528\, i \end{pmatrix}$$

```
ans =
```

$$\begin{pmatrix} -16.04827670817781146063537652684 \\ -0.2199150652953271064268450020714 \\ -1.7347041132634307164688892355445 + 3.2695613084428032499317902454879\, i \\ -1.7347041132634307164688892355445 - 3.2695613084428032499317902454879\, i \end{pmatrix}$$

```
poles_sum = 14
```

The total number of poles in McMillan form on the diagonal of the matrix is 14. Hence the McMillan order is 14.

Pole polynomials are as follows:

```
dens =
```

$$\begin{pmatrix} 1600000\, s\, (25000000000000\, s^4 + 493440000000000\, s^3 + 1841737142500000\, s^2 + 5877653223404500\, s + 1208702821978233)\, (8192000000000000\, s^5 + 196316364800000000\, s^4 + 20590128843372322816\, s^3 + 12660876307663884288\, s^2 + 10587576961998573568\, s + 6095620961456579) \\ 62500000000000000\, s^4 + 1233600000000000000\, s^3 + 4604342856250000000\, s^2 + 14694133058511250000\, s + 3021757054945582500 \\ 1 \end{pmatrix}$$

and Zero polynomials are:

```
nums
```

nums =

$$\begin{pmatrix} 1 \\ 1 \\ 20 \end{pmatrix}$$

Eigenvalues of A are the system poles. Therefore, `eig(A)` returns system poles.

```
eigs = 10×1 complex
   0.0000 + 0.0000i
 -11.6828 +10.0160i
 -11.6828 -10.0160i
  -0.2991 + 0.6752i
  -0.2991 - 0.6752i
  -0.0006 + 0.0000i
 -16.0483 + 0.0000i
  -0.2199 + 0.0000i
  -1.7347 + 3.2696i
  -1.7347 - 3.2696i
```

For each element in transfer matrix, we have zeros:

```
For the transfer matrix G(1,1) =
```

$$\frac{10393183074147886495665489231992804343808000000000000\,s^4 + 3121469489518594565991159974410976863606171943043072\,s^3 + 161594549648747308286085174068049629834716785347133440\,s^2 + 3144221676948938625490097397544364164827829108056522752\,s + 117557541015456575362180753119740001096451288\ldots}{32451855365842672678315602057625600000000000000000000\,s^5 + 7776892427292001451322264139497629286400000000000000\,s^4 + 81565903709617199449246752037131804412843183308800000\,s^3 + 501548982838296616816740333289664324008761360384000000\,s^2 + 41941713408875204174864245070215176257391715942400000\,s + 241472424079819036\ldots}$$

```
zeros are:
```

$$\begin{pmatrix} -21.961496650442156811308805729 95 \\ -0.000037388511048785716383633788709294 \\ -4.0361410213476312568943268681438 - 36.895012307614264589166054731484\,i \\ -4.0361410213476312568943268681438 + 36.895012307614264589166054731484\,i \end{pmatrix}$$

```
For the transfer matrix G(2,1) =
```

$$-\frac{240635890362669980834737314079256269617155442827329536\,s^4 + 6479517839019729651030218483992407492427553214221307084 8\,s^3 + 292989776323005456016899031005803928871933452859813658624\,s^2 + 5027334507663137336532875535584046627768378323034540343296\,s + 222712410648290284725151999644397317569\ldots}{162259276829213363391578010288128000000000000000000\,s^5 + 38884462136460007256611320697488146432000000000000000\,s^4 + 40782951854808599724623376018565902206421591654400000\,s^3 + 25077449141914830840837016664483216200438068019200000\,s^2 + 20970856704437602087432122535107588128695857971200000\,s + 1207362120399095\ldots}$$

```
zeros are:
```

$$\begin{pmatrix} -264.96892280417676246707962743877 \\ -0.000044300411142993531496177445461966 \\ -2.1487548489736813887975352838215 + 8.6156342132830388725173835596309\,i \\ -2.1487548489736813887975352838215 - 8.6156342132830388725173835596309\,i \end{pmatrix}$$

```
For the transfer matrix G(3,1) =
```

$$-\frac{5\,s\,(1084703265603291334272698998776135680000000000000\,s^3 + 86787523591651347868222915805574848734822400000\,s^2 + 53926355164459182935208055833203852942619181056\,s + 6222991010580584998857469874099753240756224)}{40564819207303340847894502572032000000000000000\,s^5 + 97211155341150018141528301743720366080000000000\,s^4 + 101957379637021499311558440046414755516053979136\,s^3 + 62693622854787077102092541661208040501095170048\,s^2 + 52427141761094005218580306337768970321739644928\,s + 30184053009977379558911693683561706012278784}$$

```
zeros are:
```

$$\begin{pmatrix} -7.3220701578311377982005964024572 \\ -0.67885271073460920082957141334377 \\ -0.00011541939236818421590548262838979 \\ 0 \end{pmatrix}$$

```
For the transfer matrix G(4,1) =
```

$$-\frac{542351632801645667136349499388067840000000000000\,s^3 + 433937617958256739341114579027874243674112000000\,s^2 + 269631775822295914676040279166019264713095905280\,s + 31114955052902924994287349370498766203781120}{40564819207303340847894502572032000000000000000\,s^5 + 97211155341150018141528301743720366080000000000\,s^4 + 101957379637021499311558440046414755516053979136\,s^3 + 62693622854787077102092541661208040501095170048\,s^2 + 52427141761094005218580306337768970321739644928\,s + 30184053009977379558911693683561706012278784}$$

```
zeros are:
```

$$\begin{pmatrix} -7.3220701578311377982005964024572 \\ -0.67885271073460920082957141334377 \\ -0.00011541939236818421590548262838979 \end{pmatrix}$$

The system matootix $S(s)$ is formed as in the following.

$$S(s) = \begin{bmatrix} \mathrm{SI} - A & B \\ -C & D \end{bmatrix}$$

```
sys_matrix =
```

$$
\begin{pmatrix}
s+\frac{743}{1250} & -\frac{1001}{1250} & \frac{9791}{1000} & \frac{8747}{10000} & -\frac{7492329572977871}{1475739525896764129280} & 0 & 0 & 0 & 0 & 0 & \frac{4669}{10000} & 0 & 0 & 0 \\
\frac{93}{125} & s+\frac{189}{25} & \frac{2647}{5000} & -\frac{393}{25} & \frac{4330373171303317}{4611686018427387904} & 0 & 0 & 0 & 0 & 0 & -\frac{2703}{1000} & 0 & 0 & 0 \\
0 & 0 & s & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1041}{1000} & \frac{3703}{500} & 0 & s+\frac{1581}{100} & \frac{4727586289695961}{649037107316853453566312041152512} & 0 & 0 & 0 & 0 & 0 & -\frac{1337}{10} & 0 & 0 & 0 \\
\frac{5399}{100000} & -\frac{1997}{2000} & 17 & 0 & s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & s+\frac{4363}{5000} & -\frac{8789}{10000} & \frac{841}{50} & -\frac{9791}{1000} & 0 & 0 & 0 & 0 & \frac{2651}{500} \\
0 & 0 & 0 & 0 & 0 & \frac{2823}{1000} & s+\frac{1609}{100} & -\frac{3367}{1000} & 0 & 0 & 0 & 0 & -\frac{313}{2} & -\frac{626}{125} \\
0 & 0 & 0 & 0 & 0 & -\frac{351}{500} & -\frac{257}{500} & s+\frac{111}{40} & 0 & 0 & 0 & 0 & \frac{23}{2} & -\frac{2051}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & -\frac{2703}{50000} & s+\frac{2782148631945593}{680564733841876926926749214863536422912} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1001}{1000} & \frac{1610598977932667}{21267647932558653966460912964485513216} & s & 0 & 0 & 0 & 0 \\
-\frac{1997}{2000} & -\frac{5399}{100000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3176 & -\frac{2937}{50000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{2941}{50000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

The system is **minimum phase** sincce there is no pole or zero on the right side plane. As a result, case 9 is discarded.

Now consider H(s) with the system matrix of a its minimal realization, have full column rank as rational matrices. For this case, rank loss in the system matrix at $s = \zeta$ corresponds to having

$$
\begin{bmatrix} \zeta I - A & B \\ -C & D \end{bmatrix} \begin{bmatrix} x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

where x_0 is the state zero direction and u_0 is the input zero direction.