Mohammad Azimi – 402123100

Exercises - Chapter 02

Problem 02 – MFD

Note: MFD is discussed in chapter 03 and it is the authors' mistake to have it by the end of chapter02.

G1 =

$$\begin{pmatrix} \frac{s}{(s+1)^2\ (s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)^2\ (s+2)^2$$

D =

$$\begin{pmatrix} (s+1)^2\ (s+2)^2 & 0 \\ 0 & (s+1)^2\ (s+2)^2 \end{pmatrix}$$

N =

$$\begin{pmatrix} s & s\ (s+1)^2 \\ -s\ (s+1)^2 & -s\ (s+1)^2 \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{s}{(s+1)^2\ (s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} s & s\ (s+1)^2 \\ -s\ (s+1)^2 & -s\ (s+1)^2 \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} (s+1)^2\ (s+2)^2 & 0 \\ 0 & (s+1)^2\ (s+2)^2 \end{pmatrix}$$

--> D^(-1) * N =

$$\begin{pmatrix} \frac{s}{(s+1)^2\,(s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

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==> G = D^(-1) * N    as in the following:
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$$\begin{pmatrix} \frac{s}{(s+1)^2\,(s+2)^2} & \frac{s}{(s+2)^2} & 1 & \frac{s}{(s+1)^2\,(s+2)^2} & \frac{s}{(s+2)^2} \\ \sigma_1 & \sigma_1 & 1 & \sigma_1 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{s}{(s+2)^2}$$

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Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.
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Right Matrix Fraction Description:

D(s) =
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$$\begin{pmatrix} (s+1)^2\,(s+2)^2 & 0 \\ 0 & (s+1)^2\,(s+2)^2 \end{pmatrix}$$

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N(s) =
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$$\begin{pmatrix} s & s\,(s+1)^2 \\ -s\,(s+1)^2 & -s\,(s+1)^2 \end{pmatrix}$$

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-->  N * D^(-1) =
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$$\begin{pmatrix} \frac{s}{(s+1)^2\,(s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

```
==> G = N * D^(-1)    as in the following:
```

$$\begin{pmatrix} \frac{s}{(s+1)^2\,(s+2)^2} & \frac{s}{(s+2)^2} & 1 & \frac{s}{(s+1)^2\,(s+2)^2} & \frac{s}{(s+2)^2} \\ \sigma_1 & \sigma_1 & 1 & \sigma_1 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{s}{(s+2)^2}$$

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Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.
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G2 =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)\,(s+2)^2$$

D =

$$\begin{pmatrix} (s+1)\,(s+2)^2 & 0 \\ 0 & (s+1)\,(s+2)^2 \end{pmatrix}$$

N =

$$\begin{pmatrix} (s+2)^3 & (s+1)\,(s+2) \\ (s+2)^2 & (s+1)^2\,(s+3) \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} (s+2)^3 & (s+1)\,(s+2) \\ (s+2)^2 & (s+1)^2\,(s+3) \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} (s+1)\,(s+2)^2 & 0 \\ 0 & (s+1)\,(s+2)^2 \end{pmatrix}$$

--> D^(-1) * N =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} \end{pmatrix}$$

==> G = D^(-1) * N   as in the following:

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} & 1 & \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} & 1 & \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)\,(s+2)^2 & 0 \\ 0 & (s+1)\,(s+2)^2 \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} (s+2)^3 & (s+1)\,(s+2) \\ (s+2)^2 & (s+1)^2\,(s+3) \end{pmatrix}$$

--> N * D^(-1) =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} \end{pmatrix}$$

==> G = N * D^(-1) as in the following:

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} & 1 & \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} & 1 & \frac{1}{s+1} & \frac{(s+1)\,(s+3)}{(s+2)^2} \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

G4 =

$$\begin{pmatrix} (s+1)\,(s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:
$(s+1)^2\,(s+2)$

D =

$$\begin{pmatrix} (s+1)^2\,(s+2) & 0 \\ 0 & (s+1)^2\,(s+2) \end{pmatrix}$$

N =

$$\begin{pmatrix} (s+1)^3\,(s+2)^2 & (s+1)^4 \\ -(s+2)^2 & -(s+1)^2 \end{pmatrix}$$

--> G =

$$\begin{pmatrix} (s+1)\,(s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} (s+1)^3\,(s+2)^2 & (s+1)^4 \\ -(s+2)^2 & -(s+1)^2 \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} (s+1)^2\,(s+2) & 0 \\ 0 & (s+1)^2\,(s+2) \end{pmatrix}$$

--> D^(-1) * N =

$$\begin{pmatrix} (s+1)\,(s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

==> G = D^(-1) * N   as in the following:

$$\begin{pmatrix} (s+1)\,(s+2) & \frac{(s+1)^2}{s+2} & 1 & (s+1)\,(s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} & 1 & -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)^2\,(s+2) & 0 \\ 0 & (s+1)^2\,(s+2) \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} (s+1)^3\,(s+2)^2 & (s+1)^4 \\ -(s+2)^2 & -(s+1)^2 \end{pmatrix}$$

--> N * D^(-1) =

$$\begin{pmatrix} (s+1)\,(s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

==> G = N * D^(-1)   as in the following:

$$\begin{pmatrix} (s+1)\ (s+2) & \frac{(s+1)^2}{s+2} & 1 & (s+1)\ (s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} & 1 & -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

G5 =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2\,s+4}{(s+1)\ (s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)\ (s+3)-2\ (s+2)^2}{(s+1)\ (s+2)\ (s+3)} & 0 \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)\ (s+2)\ (s+3)\ (s+4)$$

D =

$$\begin{pmatrix} (s+1)\ (s+2)\ (s+3)\ (s+4) & 0 & 0 \\ 0 & (s+1)\ (s+2)\ (s+3)\ (s+4) & 0 \\ 0 & 0 & (s+1)\ (s+2)\ (s+3)\ (s+4) \end{pmatrix}$$

N =

$$\begin{pmatrix} \sigma_1 & (2\,s+4)\ (s+2)\ (s+4) & \sigma_2 \\ 0 & (s+1)\ (s+3)\ (s+4) & \sigma_2 \\ \sigma_1 & -(s+4)\ ((s+1)\ (s+3)-2\ (s+2)^2) & 0 \end{pmatrix}$$

where

$$\sigma_1 = (s+2)\ (s+3)\ (s+4)$$

$$\sigma_2 = (s+1)\ (s+2)\ (s+3)$$

--> G =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2\,s+4}{(s+1)\ (s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)\ (s+3)-2\ (s+2)^2}{(s+1)\ (s+2)\ (s+3)} & 0 \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} \sigma_1 & (2\,s+4)\,(s+2)\,(s+4) & \sigma_2 \\ 0 & (s+1)\,(s+3)\,(s+4) & \sigma_2 \\ \sigma_1 & -(s+4)\,((s+1)\,(s+3)-2\,(s+2)^2) & 0 \end{pmatrix}$$

where

$$\sigma_1 = (s+2)\,(s+3)\,(s+4)$$

$$\sigma_2 = (s+1)\,(s+2)\,(s+3)$$

D(s) =

$$\begin{pmatrix} (s+1)\,(s+2)\,(s+3)\,(s+4) & 0 & 0 \\ 0 & (s+1)\,(s+2)\,(s+3)\,(s+4) & 0 \\ 0 & 0 & (s+1)\,(s+2)\,(s+3)\,(s+4) \end{pmatrix}$$

--> D^(-1) * N =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2\,s+4}{(s+1)\,(s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)\,(s+3)-2\,(s+2)^2}{(s+1)\,(s+2)\,(s+3)} & 0 \end{pmatrix}$$

==> G = D^(-1) * N   as in the following:

$$\begin{pmatrix} \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{(s+1)\,(s+3)-2\,(s+2)^2}{(s+1)\,(s+2)\,(s+3)}$$

$$\sigma_2 = \frac{2\,s+4}{(s+1)\,(s+3)}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)\,(s+2)\,(s+3)\,(s+4) & 0 & 0 \\ 0 & (s+1)\,(s+2)\,(s+3)\,(s+4) & 0 \\ 0 & 0 & (s+1)\,(s+2)\,(s+3)\,(s+4) \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} \sigma_1 & (2\,s+4)\,(s+2)\,(s+4) & \sigma_2 \\ 0 & (s+1)\,(s+3)\,(s+4) & \sigma_2 \\ \sigma_1 & -(s+4)\,((s+1)\,(s+3)-2\,(s+2)^2) & 0 \end{pmatrix}$$

where

$$\sigma_1 = (s+2)\,(s+3)\,(s+4)$$

$$\sigma_2 = (s+1)\,(s+2)\,(s+3)$$

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-->  N * D^(-1) =
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$$\begin{pmatrix} \frac{1}{s+1} & \frac{2\,s+4}{(s+1)\,(s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)\,(s+3)-2\,(s+2)^2}{(s+1)\,(s+2)\,(s+3)} & 0 \end{pmatrix}$$

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==> G = N * D^(-1)    as in the following:
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$$\begin{pmatrix} \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{(s+1)\,(s+3)-2\,(s+2)^2}{(s+1)\,(s+2)\,(s+3)}$$

$$\sigma_2 = \frac{2\,s+4}{(s+1)\,(s+3)}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

G8 =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2\,s+6}{(s+1)\,(s+2)\,(s+3)} & 0 \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)\,(s+2)\,(s+4)$$

D =

$$\begin{pmatrix} (s+1)\,(s+2)\,(s+4) & 0 & 0 \\ 0 & (s+1)\,(s+2)\,(s+4) & 0 \\ 0 & 0 & (s+1)\,(s+2)\,(s+4) \end{pmatrix}$$

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N =
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$$\begin{pmatrix} (s+2)\,(s+4) & 2\,(s+2)\,(s+4) & (s+1)\,(s+2) \\ 0 & (s+1)\,(s+4) & (s+1)\,(s+2) \\ (s+2)\,(s+4) & \frac{(2\,s+6)\,(s+4)}{s+3} & 0 \end{pmatrix}$$

```
--> G =
```

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2\,s+6}{(s+1)\,(s+2)\,(s+3)} & 0 \end{pmatrix}$$

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Left Matrix Fraction Description:

N(s) =
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$$\begin{pmatrix} (s+2)\,(s+4) & 2\,(s+2)\,(s+4) & (s+1)\,(s+2) \\ 0 & (s+1)\,(s+4) & (s+1)\,(s+2) \\ (s+2)\,(s+4) & \frac{(2\,s+6)\,(s+4)}{s+3} & 0 \end{pmatrix}$$

```
D(s) =
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$$\begin{pmatrix} (s+1)\,(s+2)\,(s+4) & 0 & 0 \\ 0 & (s+1)\,(s+2)\,(s+4) & 0 \\ 0 & 0 & (s+1)\,(s+2)\,(s+4) \end{pmatrix}$$

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--> D^(-1) * N =
```

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2\,s+6}{(s+1)\,(s+2)\,(s+3)} & 0 \end{pmatrix}$$

```
==> G = D^(-1) * N    as in the following:
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$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \frac{2\,s+6}{(s+1)\,(s+2)\,(s+3)}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)\ (s+2)\ (s+4) & 0 & 0 \\ 0 & (s+1)\ (s+2)\ (s+4) & 0 \\ 0 & 0 & (s+1)\ (s+2)\ (s+4) \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} (s+2)\ (s+4) & 2\ (s+2)\ (s+4) & (s+1)\ (s+2) \\ 0 & (s+1)\ (s+4) & (s+1)\ (s+2) \\ (s+2)\ (s+4) & \frac{(2\,s+6)\ (s+4)}{s+3} & 0 \end{pmatrix}$$

--> N * D^(-1) =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2\,s+6}{(s+1)\ (s+2)\ (s+3)} & 0 \end{pmatrix}$$

==> G = N * D^(-1)   as in the following:

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \frac{2\,s+6}{(s+1)\ (s+2)\ (s+3)}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

G9 =

$$\begin{pmatrix} \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)\ (s+3)\ (s+4)$$

D =

$$\begin{pmatrix} (s+1)\ (s+3)\ (s+4) & 0 \\ 0 & (s+1)\ (s+3)\ (s+4) \end{pmatrix}$$

N =

$$\begin{pmatrix} s+1 & (s+3)\ (s+4) \\ (s+1)\ (s+4) & 0 \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} s+1 & (s+3)\ (s+4) \\ (s+1)\ (s+4) & 0 \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} (s+1)\ (s+3)\ (s+4) & 0 \\ 0 & (s+1)\ (s+3)\ (s+4) \end{pmatrix}$$

--> D^(-1) * N =

$$\begin{pmatrix} \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

==> G = D^(-1) * N   as in the following:

$$\begin{pmatrix} \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} & 1 & \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 & 1 & \frac{1}{s+3} & 0 \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)\ (s+3)\ (s+4) & 0 \\ 0 & (s+1)\ (s+3)\ (s+4) \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} s+1 & (s+3)\ (s+4) \\ (s+1)\ (s+4) & 0 \end{pmatrix}$$

--> N * D^(-1) =

$$\begin{pmatrix} \frac{1}{(s+3)\ (s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

==> G = N * D^(-1)   as in the following:

$$\begin{pmatrix} \frac{1}{(s+3)\,(s+4)} & \frac{1}{s+1} & 1 & \frac{1}{(s+3)\,(s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 & 1 & \frac{1}{s+3} & 0 \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

G10 =

$$\begin{pmatrix} \frac{s^3-s^2+1}{s^4} & \frac{1}{s^4} & -\frac{s^3-s^2+2}{s^4} \\ \frac{\frac{3\,s}{2}+1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3\,s}{2}+2}{s^4} \\ -\frac{-s^3+9\,s^2+s-1}{s^4} & -\frac{s^2-1}{s^4} & -\frac{-s^3+s^2+s}{s^4} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$2\,s^4$$

D =

$$\begin{pmatrix} 2\,s^4 & 0 & 0 \\ 0 & 2\,s^4 & 0 \\ 0 & 0 & 2\,s^4 \end{pmatrix}$$

N =

$$\begin{pmatrix} 2\,s^3-2\,s^2+2 & 2 & -2\,s^3+2\,s^2-4 \\ 3\,s+2 & 2\,s+2 & -3\,s-4 \\ 2\,s^3-18\,s^2-2\,s+2 & 2-2\,s^2 & 2\,s^3-2\,s^2-2\,s \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{s^3-s^2+1}{s^4} & \frac{1}{s^4} & -\frac{s^3-s^2+2}{s^4} \\ \frac{\frac{3\,s}{2}+1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3\,s}{2}+2}{s^4} \\ -\frac{-s^3+9\,s^2+s-1}{s^4} & -\frac{s^2-1}{s^4} & -\frac{-s^3+s^2+s}{s^4} \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} 2\,s^3-2\,s^2+2 & 2 & -2\,s^3+2\,s^2-4 \\ 3\,s+2 & 2\,s+2 & -3\,s-4 \\ 2\,s^3-18\,s^2-2\,s+2 & 2-2\,s^2 & 2\,s^3-2\,s^2-2\,s \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} 2\,s^4 & 0 & 0 \\ 0 & 2\,s^4 & 0 \\ 0 & 0 & 2\,s^4 \end{pmatrix}$$

--> D^(-1) * N =

$$\begin{pmatrix} \frac{2s^3-2s^2+2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3-2s^2+4}{2s^4} \\ \frac{3s+2}{2s^4} & \frac{2s+2}{2s^4} & -\frac{3s+4}{2s^4} \\ -\frac{-2s^3+18s^2+2s-2}{2s^4} & -\frac{2s^2-2}{2s^4} & -\frac{-2s^3+2s^2+2s}{2s^4} \end{pmatrix}$$

==> G = D^(-1) * N   as in the following:

$$\begin{pmatrix} \frac{s^3-s^2+1}{s^4} & \frac{1}{s^4} & -\frac{s^3-s^2+2}{s^4} & 1 & \frac{2s^3-2s^2+2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3-2s^2+4}{2s^4} \\ \frac{\frac{3s}{2}+1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3s}{2}+2}{s^4} & 1 & \frac{3s+2}{2s^4} & \frac{2s+2}{2s^4} & -\frac{3s+4}{2s^4} \\ -\frac{-s^3+9s^2+s-1}{s^4} & -\frac{s^2-1}{s^4} & -\frac{-s^3+s^2+s}{s^4} & 1 & -\frac{-2s^3+18s^2+2s-2}{2s^4} & -\frac{2s^2-2}{2s^4} & -\frac{-2s^3+2s^2+2s}{2s^4} \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} 2s^4 & 0 & 0 \\ 0 & 2s^4 & 0 \\ 0 & 0 & 2s^4 \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} 2s^3-2s^2+2 & 2 & -2s^3+2s^2-4 \\ 3s+2 & 2s+2 & -3s-4 \\ 2s^3-18s^2-2s+2 & 2-2s^2 & 2s^3-2s^2-2s \end{pmatrix}$$

--> N * D^(-1) =

$$\begin{pmatrix} \frac{2s^3-2s^2+2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3-2s^2+4}{2s^4} \\ \frac{3s+2}{2s^4} & \frac{2s+2}{2s^4} & -\frac{3s+4}{2s^4} \\ -\frac{-2s^3+18s^2+2s-2}{2s^4} & -\frac{2s^2-2}{2s^4} & -\frac{-2s^3+2s^2+2s}{2s^4} \end{pmatrix}$$

==> G = N * D^(-1)   as in the following:

$$\begin{pmatrix} \frac{s^3-s^2+1}{s^4} & \frac{1}{s^4} & -\frac{s^3-s^2+2}{s^4} & 1 & \frac{2s^3-2s^2+2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3-2s^2+4}{2s^4} \\ \frac{\frac{3s}{2}+1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3s}{2}+2}{s^4} & 1 & \frac{3s+2}{2s^4} & \frac{2s+2}{2s^4} & -\frac{3s+4}{2s^4} \\ -\frac{-s^3+9s^2+s-1}{s^4} & -\frac{s^2-1}{s^4} & -\frac{-s^3+s^2+s}{s^4} & 1 & -\frac{-2s^3+18s^2+2s-2}{2s^4} & -\frac{2s^2-2}{2s^4} & -\frac{-2s^3+2s^2+2s}{2s^4} \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

G12 =

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)\ (s+2)} \\ \frac{1}{(s+1)\ (s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

```
The Lowest Common Multiplies of the given matrix denominators is:
```

$$(s+1)^2\ (s+2)^2$$

```
D =
```

$$\begin{pmatrix} (s+1)^2\ (s+2)^2 & 0 \\ 0 & (s+1)^2\ (s+2)^2 \end{pmatrix}$$

```
N =
```

$$\begin{pmatrix} (s+2)^2 & (s+1)\ (s+2) \\ (s+1)\ (s+2) & (s+1)^2\ (s+3) \end{pmatrix}$$

```
--> G =
```

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)\ (s+2)} \\ \frac{1}{(s+1)\ (s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

```
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Left Matrix Fraction Description:

N(s) =
```

$$\begin{pmatrix} (s+2)^2 & (s+1)\ (s+2) \\ (s+1)\ (s+2) & (s+1)^2\ (s+3) \end{pmatrix}$$

```
D(s) =
```

$$\begin{pmatrix} (s+1)^2\ (s+2)^2 & 0 \\ 0 & (s+1)^2\ (s+2)^2 \end{pmatrix}$$

```
--> D^(-1) * N =
```

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)\ (s+2)} \\ \frac{1}{(s+1)\ (s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

```
==> G = D^(-1) * N    as in the following:
```

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)\ (s+2)} & 1 & \frac{1}{(s+1)^2} & \frac{1}{(s+1)\ (s+2)} \\ \frac{1}{(s+1)\ (s+2)} & \frac{s+3}{(s+2)^2} & 1 & \frac{1}{(s+1)\ (s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

```
Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.
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Right Matrix Fraction Description:

D(s) =
```

$$\begin{pmatrix} (s+1)^2\ (s+2)^2 & 0 \\ 0 & (s+1)^2\ (s+2)^2 \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} (s+2)^2 & (s+1)\,(s+2) \\ (s+1)\,(s+2) & (s+1)^2\,(s+3) \end{pmatrix}$$

--> N * D^(-1) =

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)\,(s+2)} \\ \frac{1}{(s+1)\,(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

==> G = N * D^(-1) as in the following:

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)\,(s+2)} & 1 & \frac{1}{(s+1)^2} & \frac{1}{(s+1)\,(s+2)} \\ \frac{1}{(s+1)\,(s+2)} & \frac{s+3}{(s+2)^2} & 1 & \frac{1}{(s+1)\,(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

Since the augmented matrix of N and D is full rank, the obtained MFD is minimal.

Note: Given matrices G3, G6, G7 and G11 are non-square and the book has not discussed how to cope with non-square matrices in terms of MFD calculations.