

Mohammad Azimi – 402123100

Exercises - Chapter 02

Problem 02 – MFD

Note: MFD is discussed in chapter 03 and it is the authors' mistake to have it by the end of chapter02.

G1 =

$$\begin{pmatrix} \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)^2 (s+2)^2$$

D =

$$\begin{pmatrix} (s+1)^2 (s+2)^2 & 0 \\ 0 & (s+1)^2 (s+2)^2 \end{pmatrix}$$

N =

$$\begin{pmatrix} s & s (s+1)^2 \\ -s (s+1)^2 & -s (s+1)^2 \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} s & s (s+1)^2 \\ -s (s+1)^2 & -s (s+1)^2 \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} (s+1)^2 (s+2)^2 & 0 \\ 0 & (s+1)^2 (s+2)^2 \end{pmatrix}$$

--> D⁽⁻¹⁾ * N =

$$\begin{pmatrix} \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

==> G = D^(-1) * N as in the following:

$$\begin{pmatrix} \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} & 1 & \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} \\ \sigma_1 & \sigma_1 & 1 & \sigma_1 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{s}{(s+2)^2}$$

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)^2 (s+2)^2 & 0 \\ 0 & (s+1)^2 (s+2)^2 \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} s & s (s+1)^2 \\ -s (s+1)^2 & -s (s+1)^2 \end{pmatrix}$$

--> N * D^(-1) =

$$\begin{pmatrix} \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{pmatrix}$$

==> G = N * D^(-1) as in the following:

$$\begin{pmatrix} \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} & 1 & \frac{s}{(s+1)^2 (s+2)^2} & \frac{s}{(s+2)^2} \\ \sigma_1 & \sigma_1 & 1 & \sigma_1 & \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{s}{(s+2)^2}$$

G2 =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)(s+2)^2$$

D =

$$\begin{pmatrix} (s+1)(s+2)^2 & 0 \\ 0 & (s+1)(s+2)^2 \end{pmatrix}$$

N =

$$\begin{pmatrix} (s+2)^3 & (s+1)(s+2) \\ (s+2)^2 & (s+1)^2(s+3) \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} (s+2)^3 & (s+1)(s+2) \\ (s+2)^2 & (s+1)^2(s+3) \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} (s+1)(s+2)^2 & 0 \\ 0 & (s+1)(s+2)^2 \end{pmatrix}$$

--> D⁽⁻¹⁾ * N =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} \end{pmatrix}$$

==> G = D⁽⁻¹⁾ * N as in the following:

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} & 1 & \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} & 1 & \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} \end{pmatrix}$$

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)(s+2)^2 & 0 \\ 0 & (s+1)(s+2)^2 \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} (s+2)^3 & (s+1)(s+2) \\ (s+2)^2 & (s+1)^2(s+3) \end{pmatrix}$$

--> N * D⁽⁻¹⁾ =

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} \end{pmatrix}$$

=> G = N * D⁽⁻¹⁾ as in the following:

$$\begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+2} & 1 & \frac{s+2}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} & 1 & \frac{1}{s+1} & \frac{(s+1)(s+3)}{(s+2)^2} \end{pmatrix}$$

G4 =

$$\begin{pmatrix} (s+1)(s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)^2(s+2)$$

D =

$$\begin{pmatrix} (s+1)^2(s+2) & 0 \\ 0 & (s+1)^2(s+2) \end{pmatrix}$$

N =

$$\begin{pmatrix} (s+1)^3(s+2)^2 & (s+1)^4 \\ -(s+2)^2 & -(s+1)^2 \end{pmatrix}$$

--> G =

$$\begin{pmatrix} (s+1)(s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

Left Matrix Fraction Description:

$$N(s) =$$

$$\begin{pmatrix} (s+1)^3 (s+2)^2 & (s+1)^4 \\ -(s+2)^2 & -(s+1)^2 \end{pmatrix}$$

$$D(s) =$$

$$\begin{pmatrix} (s+1)^2 (s+2) & 0 \\ 0 & (s+1)^2 (s+2) \end{pmatrix}$$

$$\rightarrow D^{-1} * N =$$

$$\begin{pmatrix} (s+1) (s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

$$\Rightarrow G = D^{-1} * N \quad \text{as in the following:}$$

$$\begin{pmatrix} (s+1) (s+2) & \frac{(s+1)^2}{s+2} & 1 & (s+1) (s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} & 1 & -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

Right Matrix Fraction Description:

$$D(s) =$$

$$\begin{pmatrix} (s+1)^2 (s+2) & 0 \\ 0 & (s+1)^2 (s+2) \end{pmatrix}$$

$$N(s) =$$

$$\begin{pmatrix} (s+1)^3 (s+2)^2 & (s+1)^4 \\ -(s+2)^2 & -(s+1)^2 \end{pmatrix}$$

$$\rightarrow N * D^{-1} =$$

$$\begin{pmatrix} (s+1) (s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

$$\Rightarrow G = N * D^{-1} \quad \text{as in the following:}$$

$$\begin{pmatrix} (s+1) (s+2) & \frac{(s+1)^2}{s+2} & 1 & (s+1) (s+2) & \frac{(s+1)^2}{s+2} \\ -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} & 1 & -\frac{s+2}{(s+1)^2} & -\frac{1}{s+2} \end{pmatrix}$$

$$G5 =$$

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2s+4}{(s+1)(s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)(s+3)-2(s+2)^2}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)(s+2)(s+3)(s+4)$$

$$D = \begin{pmatrix} (s+1)(s+2)(s+3)(s+4) & 0 & 0 \\ 0 & (s+1)(s+2)(s+3)(s+4) & 0 \\ 0 & 0 & (s+1)(s+2)(s+3)(s+4) \end{pmatrix}$$

$$N = \begin{pmatrix} \sigma_1 & (2s+4)(s+2)(s+4) & \sigma_2 \\ 0 & (s+1)(s+3)(s+4) & \sigma_2 \\ \sigma_1 & -(s+4)((s+1)(s+3)-2(s+2)^2) & 0 \end{pmatrix}$$

where

$$\sigma_1 = (s+2)(s+3)(s+4)$$

$$\sigma_2 = (s+1)(s+2)(s+3)$$

--> G =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2s+4}{(s+1)(s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)(s+3)-2(s+2)^2}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

Left Matrix Fraction Description:

$$N(s) = \begin{pmatrix} \sigma_1 & (2s+4)(s+2)(s+4) & \sigma_2 \\ 0 & (s+1)(s+3)(s+4) & \sigma_2 \\ \sigma_1 & -(s+4)((s+1)(s+3)-2(s+2)^2) & 0 \end{pmatrix}$$

where

$$\sigma_1 = (s+2)(s+3)(s+4)$$

$$\sigma_2 = (s+1)(s+2)(s+3)$$

D(s) =

$$\begin{pmatrix} (s+1)(s+2)(s+3)(s+4) & 0 & 0 \\ 0 & (s+1)(s+2)(s+3)(s+4) & 0 \\ 0 & 0 & (s+1)(s+2)(s+3)(s+4) \end{pmatrix}$$

$$\rightarrow D^{-1} * N =$$

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2s+4}{(s+1)(s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)(s+3)-2(s+2)^2}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

$$\Rightarrow G = D^{-1} * N \quad \text{as in the following:}$$

$$\begin{pmatrix} \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{(s+1)(s+3)-2(s+2)^2}{(s+1)(s+2)(s+3)}$$

$$\sigma_2 = \frac{2s+4}{(s+1)(s+3)}$$

Right Matrix Fraction Description:

$$D(s) =$$

$$\begin{pmatrix} (s+1)(s+2)(s+3)(s+4) & 0 & 0 \\ 0 & (s+1)(s+2)(s+3)(s+4) & 0 \\ 0 & 0 & (s+1)(s+2)(s+3)(s+4) \end{pmatrix}$$

$$N(s) =$$

$$\begin{pmatrix} \sigma_1 & (2s+4)(s+2)(s+4) & \sigma_2 \\ 0 & (s+1)(s+3)(s+4) & \sigma_2 \\ \sigma_1 & -(s+4)((s+1)(s+3)-2(s+2)^2) & 0 \end{pmatrix}$$

where

$$\sigma_1 = (s+2)(s+3)(s+4)$$

$$\sigma_2 = (s+1)(s+2)(s+3)$$

$$\rightarrow N * D^{-1} =$$

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2s+4}{(s+1)(s+3)} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & -\frac{(s+1)(s+3)-2(s+2)^2}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

==> G = N * D⁽⁻¹⁾ as in the following:

$$\begin{pmatrix} \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \sigma_2 & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = -\frac{(s+1)(s+3)-2(s+2)^2}{(s+1)(s+2)(s+3)}$$

$$\sigma_2 = \frac{2s+4}{(s+1)(s+3)}$$

G8 =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2s+6}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)(s+2)(s+4)$$

D =

$$\begin{pmatrix} (s+1)(s+2)(s+4) & 0 & 0 \\ 0 & (s+1)(s+2)(s+4) & 0 \\ 0 & 0 & (s+1)(s+2)(s+4) \end{pmatrix}$$

N =

$$\begin{pmatrix} (s+2)(s+4) & 2(s+2)(s+4) & (s+1)(s+2) \\ 0 & (s+1)(s+4) & (s+1)(s+2) \\ (s+2)(s+4) & \frac{(2s+6)(s+4)}{s+3} & 0 \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2s+6}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} (s+2)(s+4) & 2(s+2)(s+4) & (s+1)(s+2) \\ 0 & (s+1)(s+4) & (s+1)(s+2) \\ (s+2)(s+4) & \frac{(2s+6)(s+4)}{s+3} & 0 \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} (s+1)(s+2)(s+4) & 0 & 0 \\ 0 & (s+1)(s+2)(s+4) & 0 \\ 0 & 0 & (s+1)(s+2)(s+4) \end{pmatrix}$$

--> D⁽⁻¹⁾ * N =

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2s+6}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

=> G = D⁽⁻¹⁾ * N as in the following:

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \frac{2s+6}{(s+1)(s+2)(s+3)}$$

Right Matrix Fraction Description:

D(s) =

$$\begin{pmatrix} (s+1)(s+2)(s+4) & 0 & 0 \\ 0 & (s+1)(s+2)(s+4) & 0 \\ 0 & 0 & (s+1)(s+2)(s+4) \end{pmatrix}$$

N(s) =

$$\begin{pmatrix} (s+2)(s+4) & 2(s+2)(s+4) & (s+1)(s+2) \\ 0 & (s+1)(s+4) & (s+1)(s+2) \\ (s+2)(s+4) & \frac{(2s+6)(s+4)}{s+3} & 0 \end{pmatrix}$$

$$\rightarrow N * D^{(-1)} =$$

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \frac{2s+6}{(s+1)(s+2)(s+3)} & 0 \end{pmatrix}$$

$$\Rightarrow G = N * D^{(-1)} \quad \text{as in the following:}$$

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} & 1 & \frac{1}{s+1} & \frac{2}{s+1} & \frac{1}{s+4} \\ 0 & \frac{1}{s+2} & \frac{1}{s+4} & 1 & 0 & \frac{1}{s+2} & \frac{1}{s+4} \\ \frac{1}{s+1} & \sigma_1 & 0 & 1 & \frac{1}{s+1} & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \frac{2s+6}{(s+1)(s+2)(s+3)}$$

$$G_9 =$$

$$\begin{pmatrix} \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)(s+3)(s+4)$$

$$D =$$

$$\begin{pmatrix} (s+1)(s+3)(s+4) & 0 \\ 0 & (s+1)(s+3)(s+4) \end{pmatrix}$$

$$N =$$

$$\begin{pmatrix} s+1 & (s+3)(s+4) \\ (s+1)(s+4) & 0 \end{pmatrix}$$

$$\rightarrow G =$$

$$\begin{pmatrix} \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

Left Matrix Fraction Description:

$$N(s) =$$

$$\begin{pmatrix} s+1 & (s+3)(s+4) \\ (s+1)(s+4) & 0 \end{pmatrix}$$

$$D(s) =$$

$$\begin{pmatrix} (s+1)(s+3)(s+4) & 0 \\ 0 & (s+1)(s+3)(s+4) \end{pmatrix}$$

$$\rightarrow D^{-1} * N =$$

$$\begin{pmatrix} \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

$$\Rightarrow G = D^{-1} * N \quad \text{as in the following:}$$

$$\begin{pmatrix} \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} & 1 & \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 & 1 & \frac{1}{s+3} & 0 \end{pmatrix}$$

Right Matrix Fraction Description:

$$D(s) =$$

$$\begin{pmatrix} (s+1)(s+3)(s+4) & 0 \\ 0 & (s+1)(s+3)(s+4) \end{pmatrix}$$

$$N(s) =$$

$$\begin{pmatrix} s+1 & (s+3)(s+4) \\ (s+1)(s+4) & 0 \end{pmatrix}$$

$$\rightarrow N * D^{-1} =$$

$$\begin{pmatrix} \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 \end{pmatrix}$$

$$\Rightarrow G = N * D^{-1} \quad \text{as in the following:}$$

$$\begin{pmatrix} \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} & 1 & \frac{1}{(s+3)(s+4)} & \frac{1}{s+1} \\ \frac{1}{s+3} & 0 & 1 & \frac{1}{s+3} & 0 \end{pmatrix}$$

$$G_{10} =$$

$$\begin{pmatrix} \frac{s^3 - s^2 + 1}{s^4} & \frac{1}{s^4} & -\frac{s^3 - s^2 + 2}{s^4} \\ \frac{\frac{3s}{2} + 1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3s}{2} + 2}{s^4} \\ -\frac{-s^3 + 9s^2 + s - 1}{s^4} & -\frac{s^2 - 1}{s^4} & -\frac{-s^3 + s^2 + s}{s^4} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$2s^4$$

D =

$$\begin{pmatrix} 2s^4 & 0 & 0 \\ 0 & 2s^4 & 0 \\ 0 & 0 & 2s^4 \end{pmatrix}$$

N =

$$\begin{pmatrix} 2s^3 - 2s^2 + 2 & 2 & -2s^3 + 2s^2 - 4 \\ 3s + 2 & 2s + 2 & -3s - 4 \\ 2s^3 - 18s^2 - 2s + 2 & 2 - 2s^2 & 2s^3 - 2s^2 - 2s \end{pmatrix}$$

--> G =

$$\begin{pmatrix} \frac{s^3 - s^2 + 1}{s^4} & \frac{1}{s^4} & -\frac{s^3 - s^2 + 2}{s^4} \\ \frac{\frac{3s}{2} + 1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3s}{2} + 2}{s^4} \\ -\frac{-s^3 + 9s^2 + s - 1}{s^4} & -\frac{s^2 - 1}{s^4} & -\frac{-s^3 + s^2 + s}{s^4} \end{pmatrix}$$

Left Matrix Fraction Description:

N(s) =

$$\begin{pmatrix} 2s^3 - 2s^2 + 2 & 2 & -2s^3 + 2s^2 - 4 \\ 3s + 2 & 2s + 2 & -3s - 4 \\ 2s^3 - 18s^2 - 2s + 2 & 2 - 2s^2 & 2s^3 - 2s^2 - 2s \end{pmatrix}$$

D(s) =

$$\begin{pmatrix} 2s^4 & 0 & 0 \\ 0 & 2s^4 & 0 \\ 0 & 0 & 2s^4 \end{pmatrix}$$

--> D⁻¹ * N =

$$\begin{pmatrix} \frac{2s^3 - 2s^2 + 2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3 - 2s^2 + 4}{2s^4} \\ \frac{3s + 2}{2s^4} & \frac{2s + 2}{2s^4} & -\frac{3s + 4}{2s^4} \\ -\frac{-2s^3 + 18s^2 + 2s - 2}{2s^4} & -\frac{2s^2 - 2}{2s^4} & -\frac{-2s^3 + 2s^2 + 2s}{2s^4} \end{pmatrix}$$

==> G = D⁻¹ * N as in the following:

$$\begin{pmatrix} \frac{s^3 - s^2 + 1}{s^4} & \frac{1}{s^4} & -\frac{s^3 - s^2 + 2}{s^4} & 1 & \frac{2s^3 - 2s^2 + 2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3 - 2s^2 + 4}{2s^4} \\ \frac{\frac{3s}{2} + 1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3s}{2} + 2}{s^4} & 1 & \frac{3s+2}{2s^4} & \frac{2s+2}{2s^4} & -\frac{3s+4}{2s^4} \\ -\frac{-s^3 + 9s^2 + s - 1}{s^4} & -\frac{s^2 - 1}{s^4} & -\frac{-s^3 + s^2 + s}{s^4} & 1 & -\frac{-2s^3 + 18s^2 + 2s - 2}{2s^4} & -\frac{2s^2 - 2}{2s^4} & -\frac{-2s^3 + 2s^2 + 2s}{2s^4} \end{pmatrix}$$

Right Matrix Fraction Description:

$$D(s) =$$

$$\begin{pmatrix} 2s^4 & 0 & 0 \\ 0 & 2s^4 & 0 \\ 0 & 0 & 2s^4 \end{pmatrix}$$

$$N(s) =$$

$$\begin{pmatrix} 2s^3 - 2s^2 + 2 & 2 & -2s^3 + 2s^2 - 4 \\ 3s + 2 & 2s + 2 & -3s - 4 \\ 2s^3 - 18s^2 - 2s + 2 & 2 - 2s^2 & 2s^3 - 2s^2 - 2s \end{pmatrix}$$

$$\rightarrow N * D^{(-1)} =$$

$$\begin{pmatrix} \frac{2s^3 - 2s^2 + 2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3 - 2s^2 + 4}{2s^4} \\ \frac{3s + 2}{2s^4} & \frac{2s + 2}{2s^4} & -\frac{3s + 4}{2s^4} \\ -\frac{-2s^3 + 18s^2 + 2s - 2}{2s^4} & -\frac{2s^2 - 2}{2s^4} & -\frac{-2s^3 + 2s^2 + 2s}{2s^4} \end{pmatrix}$$

$$\Rightarrow G = N * D^{(-1)} \text{ as in the following:}$$

$$\begin{pmatrix} \frac{s^3 - s^2 + 1}{s^4} & \frac{1}{s^4} & -\frac{s^3 - s^2 + 2}{s^4} & 1 & \frac{2s^3 - 2s^2 + 2}{2s^4} & \frac{1}{s^4} & -\frac{2s^3 - 2s^2 + 4}{2s^4} \\ \frac{\frac{3s}{2} + 1}{s^4} & \frac{s+1}{s^4} & -\frac{\frac{3s}{2} + 2}{s^4} & 1 & \frac{3s+2}{2s^4} & \frac{2s+2}{2s^4} & -\frac{3s+4}{2s^4} \\ -\frac{-s^3 + 9s^2 + s - 1}{s^4} & -\frac{s^2 - 1}{s^4} & -\frac{-s^3 + s^2 + s}{s^4} & 1 & -\frac{-2s^3 + 18s^2 + 2s - 2}{2s^4} & -\frac{2s^2 - 2}{2s^4} & -\frac{-2s^3 + 2s^2 + 2s}{2s^4} \end{pmatrix}$$

$$G_{12} =$$

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

The Lowest Common Multiplies of the given matrix denominators is:

$$(s+1)^2 (s+2)^2$$

$$D =$$

$$\begin{pmatrix} (s+1)^2 (s+2)^2 & 0 \\ 0 & (s+1)^2 (s+2)^2 \end{pmatrix}$$

$$N =$$

$$\begin{pmatrix} (s+2)^2 & (s+1)(s+2) \\ (s+1)(s+2) & (s+1)^2(s+3) \end{pmatrix}$$

$$\rightarrow G =$$

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

Left Matrix Fraction Description:

$$N(s) =$$

$$\begin{pmatrix} (s+2)^2 & (s+1)(s+2) \\ (s+1)(s+2) & (s+1)^2(s+3) \end{pmatrix}$$

$$D(s) =$$

$$\begin{pmatrix} (s+1)^2(s+2)^2 & 0 \\ 0 & (s+1)^2(s+2)^2 \end{pmatrix}$$

$$\rightarrow D^{-1} * N =$$

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

$$\Rightarrow G = D^{-1} * N \quad \text{as in the following:}$$

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} & 1 & \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} & 1 & \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

Right Matrix Fraction Description:

$$D(s) =$$

$$\begin{pmatrix} (s+1)^2(s+2)^2 & 0 \\ 0 & (s+1)^2(s+2)^2 \end{pmatrix}$$

$$N(s) =$$

$$\begin{pmatrix} (s+2)^2 & (s+1)(s+2) \\ (s+1)(s+2) & (s+1)^2(s+3) \end{pmatrix}$$

$$\rightarrow N * D^{-1} =$$

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

$$\Rightarrow G = N * D^{-1} \quad \text{as in the following:}$$

$$\begin{pmatrix} \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} & 1 & \frac{1}{(s+1)^2} & \frac{1}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} & 1 & \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+2)^2} \end{pmatrix}$$

Note: Given matrices G3, G6, G7 and G11 are non-square and the book has not discussed how to cope with non-square matrices in terms of MFD calculations. hahaha... authors ☺