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4-1

Taking system Σ: observable and uncontrollable

Applying $\underline{\bar{x}} = P\underline{x}$ would get:

$$\bar{\Sigma}: \begin{cases} \begin{bmatrix} \dot{\underline{\bar{x}}}_c \\ \dot{\underline{\bar{x}}}_{\bar{c}} \end{bmatrix} = \begin{bmatrix} \bar{A}_c & \bar{A}_{12} \\ 0 & \bar{A}_{\bar{c}} \end{bmatrix} \begin{bmatrix} \underline{\bar{x}}_c \\ \underline{\bar{x}}_{\bar{c}} \end{bmatrix} + \begin{bmatrix} \bar{B}_c \\ 0 \end{bmatrix} \underline{u} \\ \underline{y} = [\bar{C}_c \quad \bar{C}_{\bar{c}}] \begin{bmatrix} \underline{\bar{x}}_c \\ \underline{\bar{x}}_{\bar{c}} \end{bmatrix} + D\underline{u} \end{cases}$$

which includes the controllable subsystem:

$$\bar{\Sigma}_C: \begin{cases} \dot{\underline{\bar{x}}}_c = \bar{A}_c \, \underline{\bar{x}}_c + \bar{B}_c \, \underline{u} \\ \underline{\bar{y}} = \bar{C}_c \, \underline{\bar{x}}_c + D \, \underline{u} \end{cases}$$

We aim to prove the observability of $\bar{\Sigma}_C$. For that,

$$rank\left\{ \begin{bmatrix} sI - \bar{A}_c \\ \bar{C}_c \end{bmatrix} \right\} = full \text{ for any value of } s.$$

Hence:

$$\bar{A} = \begin{bmatrix} \bar{A}_c & \bar{A}_{12} \\ 0 & \bar{A}_{\bar{c}} \end{bmatrix} \quad . \quad \bar{C} = [\bar{C}_c \quad \bar{C}_{\bar{c}}]$$

$$rank \begin{bmatrix} sI - \bar{A} \\ \bar{C} \end{bmatrix} = rank \begin{bmatrix} sI - \bar{A}_c & \bar{A}_{12} \\ 0 & sI - \bar{A}_{\bar{c}} \\ \bar{C}_c & \bar{C}_{\bar{c}} \end{bmatrix} = n$$

which is obviously held by basic linear algebra fundamentals. Columns in $\begin{bmatrix} sI - \bar{A}_c \\ 0 \\ \bar{C}_c \end{bmatrix}$ are independent and therefore columns of $\begin{bmatrix} sI - \bar{A}_c \\ \bar{C}_c \end{bmatrix}$ are independent (for any value of s).