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Exercises - Chapter 06

Problem 01

$$G_0 = \begin{bmatrix} \frac{g_1}{s-1} & g_2 \\ g_1 & \frac{g_2}{s+2} \end{bmatrix}$$

$$\det(I+G_0) = \det\left(\begin{bmatrix} \frac{s-1+g_1}{s-1} & g_2 \\ g_1 & \frac{s+2+g_2}{s+2} \end{bmatrix}\right)$$

$$= \left(\frac{s-1+g_1}{s-1}\right)\left(\frac{s+2+g_2}{s+2}\right) - g_1g_2$$

$$= \frac{(1-g_1g_2)s^2 + (1+g_1+g_2-g_1g_2)s + (2g_1-g_2-g_1g_2-2)}{(s-1)(s+2)}$$

Assuming thatg_1, g_2 have no effect on $\Delta_0(s)$:

$$\Delta_0(s) = (s-1)(s+2)$$

$$\Delta_f(s) = \Delta_0(s)\det(I+G_0)(1-g_1g_2)s^2 + (1+g_1+g_2-g_1g_2)s + (2g_1-g_2-g_1g_2-2)$$

A necessary and sufficient condition for the closed-loop system to be BIBO stable and asymptotically stable is that all the roots of the numerator polynomial are negative.

Problem 02

G1:

$$G_1 = \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{-1}{s-1} & \frac{1}{s+2} \end{bmatrix} = \frac{1}{(s+1)(s-1)(s+2)}\begin{bmatrix} (s-1)(s+2) & 0 \\ -(s+1)(s+2) & (s-1)(s+1) \end{bmatrix}$$

$$\xrightarrow{Smith-McMillan} \frac{1}{(s+1)(s-1)(s+2)}\begin{bmatrix} 1 & 0 \\ 0 & (s-1)^2(s+1)(s+2) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{(s+1)(s-1)(s+2)} & 0 \\ 0 & (s-1) \end{bmatrix}$$

$$\Delta_1(s) = (s+1)(s-1)(s+2)$$

$$\det(I + G_1) = \det\left(\begin{bmatrix} \frac{s+2}{s+1} & 0 \\ \frac{s-2}{s-1} & \frac{s+3}{s+2} \end{bmatrix}\right) = \frac{s+3}{s+1}$$

$$\Delta_f(s) = (s-1)(s+2)(s+3)$$

Given that not all the poles are on the left side of the imaginary axis, the system is not asymptotically stable and not BIBO stable.

G2:

$$G_2 = \frac{1}{s^2 + 0.6s + 1}\begin{bmatrix} 0.5(s+2) & 4(s+0.1) \\ 0.3(s+8.3) & 3.2(s+1.25) \end{bmatrix}$$

$$\frac{1}{s^2 + 0.6s + 1}\begin{bmatrix} 0.5(s+2) & 4(s+0.1) \\ 0.3(s+8.3) & 3.2(s+1.25) \end{bmatrix} = \frac{1}{(s^2 + 0.6s + 1)}\begin{bmatrix} 1 & 0 \\ 0 & \frac{100s^2 - 1220s + 751}{250} \end{bmatrix}$$

$$\Delta_2(s) = (s^2 + 0.6s + 1)$$

$$\det(I + G_2) = \frac{250s^4 + 1225s^3 + 2495s^2 + 755s + 2251}{10(5s^2 + 3s + 5)^2}$$

$$\Delta_f(s) = \frac{250s^4 + 1225s^3 + 2495s^2 + 755s + 2251}{50(5s^2 + 3s + 5)}$$

Since not all the poles are on the left side of the imaginary axis, the system is neither asymptotically stable nor BIBO stable.

Problem 03

Problem 04

To analyze the stability of the closed-loop system, we first obtain the eigenvalues and then plot the Nyquist plots under the mappings of $\lambda_i(s)$ (generalized Nyquist diagram).

Also:

$$N(0, \lambda(s), \Gamma_s) = \sum_{i=1}^{n} N(0, \lambda_i(s), \Gamma_s) \qquad , \qquad \lambda(s) = \prod_{i=1}^{n} \lambda_i(s)$$

To plot the set of Nyquist diagrams under the mappings of $\lambda_i(s)$, we can obtain $\lambda(s)$ and then plot the Nyquist diagram under the mappings of $\lambda(s)$.

$\lambda(s)$ =

```
          2
  100 s   - 1220 s + 751
  ----------------------
             2          2
   10 (5 s   + 3 s + 5)
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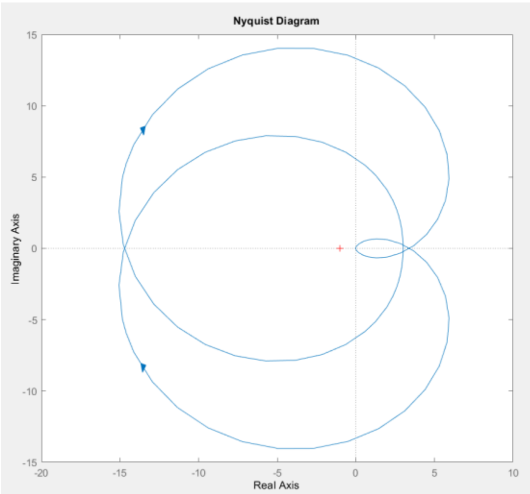
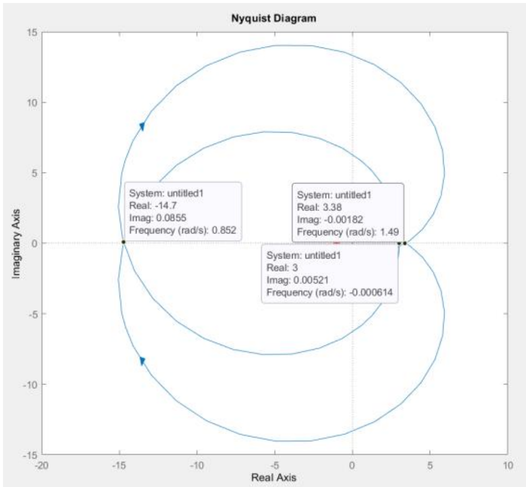
with the Nyquist diagram:





Consider the special case $k_1 = k_2$ and take the critical point as $(-\frac{1}{k}, 0)$. Given that the open-loop system does not have any roots with a positive real part, if the critical point is not encircled in the clockwise direction by the Nyquist plot, the closed-loop system is stable for that value of ***k***.

According to the Nyquist plot, the critical point $(-\frac{1}{k}, 0)$ is not encircled only in the intervals [3.38 , ∞) and (-∞ , -14.7]. Consequently, for the following values of ***k***, the closed-loop system is stable:

$$-0.296 < k < 0.068$$

Problem 05

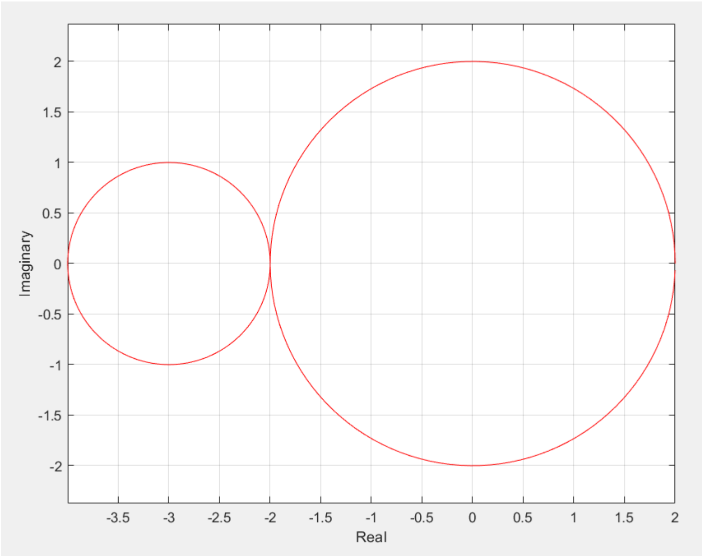
$$Z = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda & -1 \\ 2 & \lambda + 3 \end{bmatrix} \qquad det(\lambda I - A) = \lambda^2 + 3\lambda + 2 = 0$$

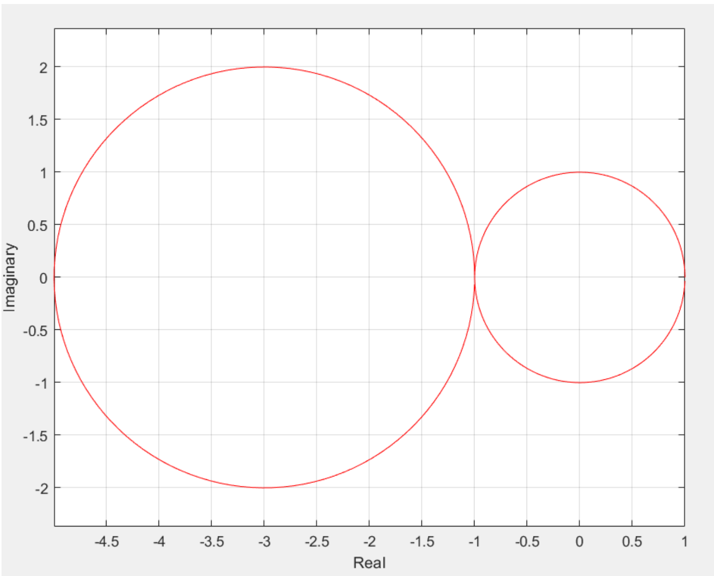
$$\lambda_1 = -1 \quad , \quad \lambda_2 = -2$$

Now, we study the Gershgorin circle theorem:

- Column Gershgorin Circles:



- Row Gershgorin Circles:



It is observed that in both figures, the eigenvalues are located inside the circles.