Mohammad Azimi – 402123100

Inline Problems- Chapter 06

6-1

$$\boldsymbol{G_1(s)} = \begin{bmatrix} \frac{\mathbf{1}}{\boldsymbol{s}} & \frac{\mathbf{1}}{\boldsymbol{s+1}} \\ \frac{\mathbf{1}}{\boldsymbol{s-1}} & \frac{\mathbf{1}}{\boldsymbol{s-2}} \end{bmatrix} \qquad \boldsymbol{G_2(s)} = \begin{bmatrix} \mathbf{0} & \boldsymbol{k_1} \\ \boldsymbol{k_2} & \mathbf{0} \end{bmatrix}$$

For asymptotic stability, all roots of the following expression must have negative real parts:

$$\Delta_1(s)\Delta_2(s)\det\big(I + G_1(s)G_2(s)\big)$$

In these expressions, $\Delta_1(s)$ and $\Delta_2(s)$ represent the characteristic polynomials of the matrices G1(s)G_1(s)G1(s) and $G_1(s)$ and $G_2(s)$ respectively. These polynomials correspond to the poles of the transfer matrices of the mentioned matrices. The best way to obtain them is by using the Smith-McMillan form.

For the matrix $G_2(s)$, which specifically has no poles, $\Delta_2(s) = 1$. $\Delta_2(s)$ is obtained as follows:

$$G_1(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s+1} \\ \frac{1}{s-1} & \frac{1}{s-2} \end{bmatrix}$$

$$= \frac{1}{s(s+1)(s-1)(s-2)} \begin{bmatrix} (s-1)(s+1)(s-2) & s(s-1)(s-2) \\ s(s+1)(s-2) & s(s-1)(s+1) \end{bmatrix}$$

$$\xrightarrow{smith-McMillan} \begin{cases} D_0 = 1 \\ D_1 = 1 \\ D_2 = \begin{vmatrix} (s-1)(s+1)(s-2) & s(s-1)(s-2) \\ s(s+1)(s-2) & s(s-1)(s+1) \end{vmatrix} \end{cases}$$

And we have D_2:

$$D_2 = (2s-1)(s)(s+1)(s-1)(s-2)$$

$$\Rightarrow \quad \frac{D_1}{D_0} = 1 \qquad , \qquad \frac{D_2}{D_1} = (2s-1)(s)(s+1)(s-1)(s-2)$$

Therefore, the Smith-McMillan form of the above matrix is as follows:

$$M = \frac{1}{s(s+1)(s-1)(s-2)} \begin{bmatrix} 1 & 0 \\ 0 & (2s-1)(s)(s+1)(s-1)(s-2) \end{bmatrix}$$

$$\Rightarrow \; M = \begin{bmatrix} \frac{1}{s(s+1)(s-1)(s-2)} & 0 \\ 0 & (2s-1) \end{bmatrix}$$

Thus, the characteristic polynomial, which is the product of the poles of this Smith-McMillan matrix, is as follows:

$$\Delta_1(s) = s(s+1)(s-1)(s-2)$$

$$\det\big(I + G_1(s)G_2(s)\big) =$$

```
G1 =
```

$$\begin{pmatrix} \frac{1}{s} & \frac{1}{s+1} \\ \frac{1}{s-1} & \frac{1}{s-2} \end{pmatrix}$$

```
G2 =
```

$$\begin{pmatrix} 0 & k_1 \\ k_2 & 0 \end{pmatrix}$$

Therefore:

$$\Delta_1(s)\Delta_2(s)\det\big(I + G_1(s)G_2(s)\big) =$$

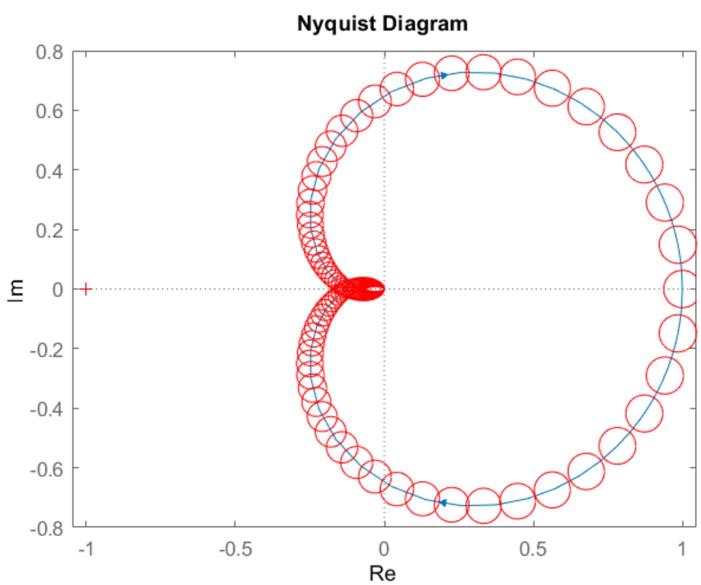
$$2\,s + k_1\,k_2 - 2\,k_1\,s + 2\,k_2\,s - k_1\,s^2 + k_1\,s^3 - 3\,k_2\,s^2 + k_2\,s^3 - s^2 - 2\,s^3 + s^4 - 2\,k_1\,k_2\,s$$

6-2

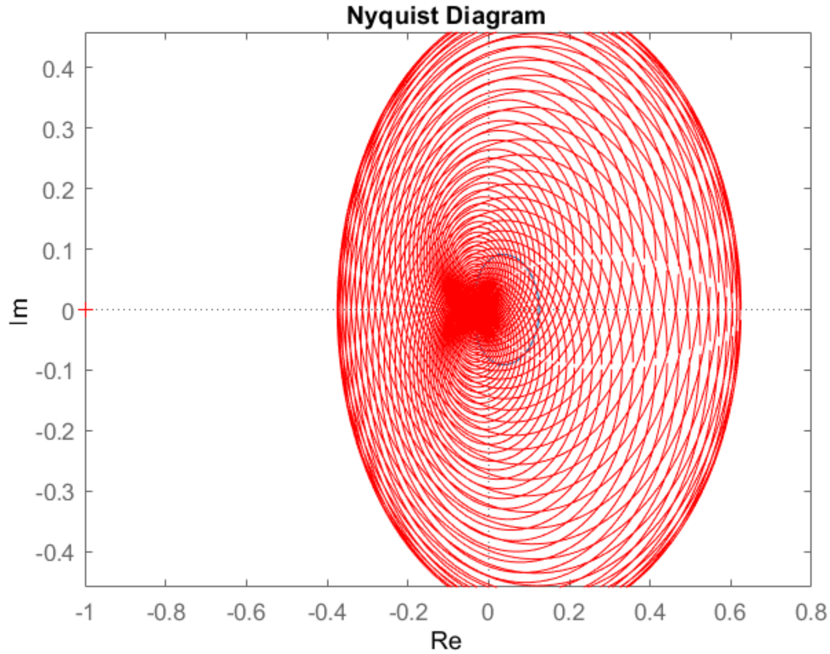
```
G =
```

$$\begin{pmatrix} \dfrac{1}{(s+1)^3} & \dfrac{1}{2\,(s+2)^3} \\ \dfrac{1}{2\,(s+1)^3} & \dfrac{1}{(s+2)^3} \end{pmatrix}$$

First, we plot the Gershgorin bands. The Gershgorin band corresponding to g_{11}, considering the elements of the first column, is as follows:



For g_{22}, it is as follows:



According to the points discussed in the book, since none of the Gershgorin circles encompass the negative point -1, we can use the method of Gershgorin bands.

```
-1.0000 + 0.0000i
-1.0000 - 0.0000i
-1.0000 + 0.0000i
-1.0000 + 0.0000i
-1.0000 - 0.0000i
-1.0000 + 0.0000i
-2.0000 + 0.0000i
-2.0000 + 0.0000i
-2.0000 - 0.0000i
-2.0000 + 0.0000i
-2.0000 + 0.0000i
-2.0000 - 0.0000i
```

Based on the above figure, the number of unstable open-loop poles is zero, and the band does not encircle the point -1. Therefore, the closed-loop system is asymptotically stable and BIBO stable.