Amirkabir University of Technology
(Tehran Polytechnic)

Electrical Engineering Faculty
Control Department
MSc Program

Assignment 05
through
System Identification
Course

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Q1 - part a

```
clear; clc;
```

Load Data

Load the data and split them into estimation and validation datasets.

```
load q1_402123100.mat

u_val = u(length(u)/2+1:end);
v_val = v(length(u)/2+1:end);
z_val = z(length(u)/2+1:end);
y_val = y(length(u)/2+1:end);

u = u(1:length(u)/2);
v = v(1:length(v)/2);
z = z(1:length(z)/2);
y = y(1:length(y)/2);



%%

Ts = 0.1;
t = 0:Ts:length(u)*Ts-Ts;
N = length(y);
%%
```

Run for Estimation through Loop

```
fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 5;

for degree=1:1:100
    na = degree;
    nb = degree;
    p = na+nb+1;

```

```
    U = arx_U_builder_3(u,y,na,nb,1);
    theta_hat = inv(U'*U)*U'*y;

    y_hat = form_tf_lsim_2(theta_hat, u, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y, y_hat);

    error = y - y_hat;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    covariance = variance*inv(U'*U);
    cov = trace(covariance)/p;
    covs = [covs; cov];

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_arx, mse_arx, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_arx];
    MSEs = [MSEs; mse_arx];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
end
```

```
>>> Degree = 1 : R2=-0.013109 | MSE=4.257591 | var=4.283291 | s_hat=2128.795577 |
-------------------------------------------------------------
>>> Degree = 2 : R2=-0.002442 | MSE=4.212763 | var=4.255316 | s_hat=2106.381374 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.037217 | MSE=4.046094 | var=4.103544 | s_hat=2023.047184 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.060495 | MSE=3.948270 | var=4.020641 | s_hat=1974.134836 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.044131 | MSE=4.017040 | var=4.107403 | s_hat=2008.520016 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.100162 | MSE=3.781568 | var=3.882513 | s_hat=1890.784068 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.092951 | MSE=3.811872 | var=3.929764 | s_hat=1905.935767 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.149934 | MSE=3.572401 | var=3.698137 | s_hat=1786.200255 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.259092 | MSE=3.113667 | var=3.236660 | s_hat=1556.833659 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.344388 | MSE=2.755211 | var=2.876003 | s_hat=1377.605414 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.445626 | MSE=2.329757 | var=2.442093 | s_hat=1164.878454 |
```

```
-------------------------------------------------------------
>>> Degree = 12 : R2=0.519301 | MSE=2.020136 | var=2.126459 | s_hat=1010.068101 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.557457 | MSE=1.859788 | var=1.965949 | s_hat=929.893821 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.604787 | MSE=1.660884 | var=1.763147 | s_hat=830.442056 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.653406 | MSE=1.456559 | var=1.552835 | s_hat=728.279691 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.705157 | MSE=1.239080 | var=1.326638 | s_hat=619.539788 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.718145 | MSE=1.184494 | var=1.273649 | s_hat=592.246907 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.737369 | MSE=1.103705 | var=1.191906 | s_hat=551.852472 |
-------------------------------------------------------------
>>> Degree = 19 : R2=0.742338 | MSE=1.082824 | var=1.174429 | s_hat=541.411830 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.754741 | MSE=1.030702 | var=1.122769 | s_hat=515.350842 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.757138 | MSE=1.020626 | var=1.116659 | s_hat=510.313160 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.763412 | MSE=0.994261 | var=1.092594 | s_hat=497.130462 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.768461 | MSE=0.973042 | var=1.073998 | s_hat=486.521067 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.771023 | MSE=0.962278 | var=1.066827 | s_hat=481.138922 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.773173 | MSE=0.953242 | var=1.061517 | s_hat=476.621120 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.773585 | MSE=0.951511 | var=1.064329 | s_hat=475.755257 |
-------------------------------------------------------------
>>> Degree = 27 : R2=0.776436 | MSE=0.939529 | var=1.055650 | s_hat=469.764257 |
-------------------------------------------------------------
>>> Degree = 28 : R2=0.778222 | MSE=0.932024 | var=1.051946 | s_hat=466.012085 |
-------------------------------------------------------------
>>> Degree = 29 : R2=0.777561 | MSE=0.934798 | var=1.059862 | s_hat=467.399023 |
-------------------------------------------------------------
>>> Degree = 30 : R2=0.779526 | MSE=0.926543 | var=1.055288 | s_hat=463.271350 |
-------------------------------------------------------------
>>> Degree = 31 : R2=0.780068 | MSE=0.924266 | var=1.057512 | s_hat=462.132874 |
-------------------------------------------------------------
>>> Degree = 32 : R2=0.780750 | MSE=0.921397 | var=1.059077 | s_hat=460.698670 |
-------------------------------------------------------------
>>> Degree = 33 : R2=0.781168 | MSE=0.919643 | var=1.061944 | s_hat=459.821641 |
-------------------------------------------------------------
>>> Degree = 34 : R2=0.781139 | MSE=0.919764 | var=1.067011 | s_hat=459.881804 |
-------------------------------------------------------------
>>> Degree = 35 : R2=0.782425 | MSE=0.914357 | var=1.065684 | s_hat=457.178576 |
-------------------------------------------------------------
>>> Degree = 36 : R2=0.782382 | MSE=0.914541 | var=1.070891 | s_hat=457.270310 |
-------------------------------------------------------------
>>> Degree = 37 : R2=0.783425 | MSE=0.910155 | var=1.070770 | s_hat=455.077367 |
-------------------------------------------------------------
>>> Degree = 38 : R2=0.783855 | MSE=0.908351 | var=1.073701 | s_hat=454.175517 |
-------------------------------------------------------------
>>> Degree = 39 : R2=0.784675 | MSE=0.904903 | var=1.074707 | s_hat=452.451459 |
-------------------------------------------------------------
>>> Degree = 40 : R2=0.783923 | MSE=0.908062 | var=1.083606 | s_hat=454.031040 |
-------------------------------------------------------------
>>> Degree = 41 : R2=0.785142 | MSE=0.902940 | var=1.082662 | s_hat=451.470054 |
```

```
------------------------------------------------------------------
>>> Degree = 42 : R2=0.785832 | MSE=0.900042 | var=1.084389 | s_hat=450.021249 |
------------------------------------------------------------------
>>> Degree = 43 : R2=0.786140 | MSE=0.898746 | var=1.088071 | s_hat=449.373122 |
------------------------------------------------------------------
>>> Degree = 44 : R2=0.788278 | MSE=0.889761 | var=1.082435 | s_hat=444.880688 |
------------------------------------------------------------------
>>> Degree = 45 : R2=0.791789 | MSE=0.875008 | var=1.069691 | s_hat=437.503816 |
------------------------------------------------------------------
>>> Degree = 46 : R2=0.791525 | MSE=0.876117 | var=1.076311 | s_hat=438.058597 |
------------------------------------------------------------------
>>> Degree = 47 : R2=0.792521 | MSE=0.871932 | var=1.076459 | s_hat=435.965945 |
------------------------------------------------------------------
>>> Degree = 48 : R2=0.794132 | MSE=0.865162 | var=1.073402 | s_hat=432.581160 |
------------------------------------------------------------------
>>> Degree = 49 : R2=0.794701 | MSE=0.862769 | var=1.075772 | s_hat=431.384538 |
------------------------------------------------------------------
>>> Degree = 50 : R2=0.794747 | MSE=0.862578 | var=1.080925 | s_hat=431.288918 |
------------------------------------------------------------------
>>> Degree = 51 : R2=0.794994 | MSE=0.861536 | var=1.085058 | s_hat=430.768068 |
------------------------------------------------------------------
>>> Degree = 52 : R2=0.795239 | MSE=0.860510 | var=1.089253 | s_hat=430.254895 |
------------------------------------------------------------------
>>> Degree = 53 : R2=0.794604 | MSE=0.863178 | var=1.098191 | s_hat=431.588926 |
------------------------------------------------------------------
>>> Degree = 54 : R2=0.795664 | MSE=0.858722 | var=1.098110 | s_hat=429.360964 |
------------------------------------------------------------------
>>> Degree = 55 : R2=0.795125 | MSE=0.860989 | var=1.106669 | s_hat=430.494423 |
------------------------------------------------------------------
>>> Degree = 56 : R2=0.797638 | MSE=0.850424 | var=1.098740 | s_hat=425.212245 |
------------------------------------------------------------------
>>> Degree = 57 : R2=0.799447 | MSE=0.842822 | var=1.094574 | s_hat=421.411020 |
------------------------------------------------------------------
>>> Degree = 58 : R2=0.799319 | MSE=0.843361 | var=1.100993 | s_hat=421.680335 |
------------------------------------------------------------------
>>> Degree = 59 : R2=0.798448 | MSE=0.847023 | var=1.111579 | s_hat=423.511588 |
------------------------------------------------------------------
>>> Degree = 60 : R2=0.798671 | MSE=0.846085 | var=1.116207 | s_hat=423.042534 |
------------------------------------------------------------------
>>> Degree = 61 : R2=0.802605 | MSE=0.829552 | var=1.100202 | s_hat=414.776193 |
------------------------------------------------------------------
>>> Degree = 62 : R2=0.803049 | MSE=0.827686 | var=1.103582 | s_hat=413.843239 |
------------------------------------------------------------------
>>> Degree = 63 : R2=0.804371 | MSE=0.822131 | var=1.102052 | s_hat=411.065526 |
------------------------------------------------------------------
>>> Degree = 64 : R2=0.804588 | MSE=0.821218 | var=1.106763 | s_hat=410.608988 |
------------------------------------------------------------------
>>> Degree = 65 : R2=0.804746 | MSE=0.820554 | var=1.111862 | s_hat=410.276907 |
------------------------------------------------------------------
>>> Degree = 66 : R2=0.805957 | MSE=0.815466 | var=1.110989 | s_hat=407.732920 |
------------------------------------------------------------------
>>> Degree = 67 : R2=0.806441 | MSE=0.813431 | var=1.114289 | s_hat=406.715585 |
------------------------------------------------------------------
>>> Degree = 68 : R2=0.809084 | MSE=0.802326 | var=1.105132 | s_hat=401.163087 |
------------------------------------------------------------------
>>> Degree = 69 : R2=0.810091 | MSE=0.798091 | var=1.105389 | s_hat=399.045552 |
------------------------------------------------------------------
>>> Degree = 70 : R2=0.810535 | MSE=0.796225 | var=1.108948 | s_hat=398.112442 |
------------------------------------------------------------------
>>> Degree = 71 : R2=0.810180 | MSE=0.797719 | var=1.117254 | s_hat=398.859750 |
```

```
-------------------------------------------------------------
>>> Degree = 72 : R2=0.812909 | MSE=0.786250 | var=1.107394 | s_hat=393.124820 |
-------------------------------------------------------------
>>> Degree = 73 : R2=0.812886 | MSE=0.786348 | var=1.113807 | s_hat=393.173784 |
-------------------------------------------------------------
>>> Degree = 74 : R2=0.814573 | MSE=0.779259 | var=1.110055 | s_hat=389.629362 |
-------------------------------------------------------------
>>> Degree = 75 : R2=0.814883 | MSE=0.777955 | var=1.114548 | s_hat=388.977316 |
-------------------------------------------------------------
>>> Degree = 76 : R2=0.815250 | MSE=0.776413 | var=1.118751 | s_hat=388.206634 |
-------------------------------------------------------------
>>> Degree = 77 : R2=0.816400 | MSE=0.771578 | var=1.118229 | s_hat=385.789115 |
-------------------------------------------------------------
>>> Degree = 78 : R2=0.816650 | MSE=0.770528 | var=1.123219 | s_hat=385.264056 |
-------------------------------------------------------------
>>> Degree = 79 : R2=0.816351 | MSE=0.771784 | var=1.131648 | s_hat=385.892006 |
-------------------------------------------------------------
>>> Degree = 80 : R2=0.816732 | MSE=0.770183 | var=1.135962 | s_hat=385.091277 |
-------------------------------------------------------------
>>> Degree = 81 : R2=0.816842 | MSE=0.769720 | var=1.142018 | s_hat=384.860123 |
-------------------------------------------------------------
>>> Degree = 82 : R2=0.817458 | MSE=0.767133 | var=1.144974 | s_hat=383.566415 |
-------------------------------------------------------------
>>> Degree = 83 : R2=0.817448 | MSE=0.767174 | var=1.151913 | s_hat=383.587147 |
-------------------------------------------------------------
>>> Degree = 84 : R2=0.817803 | MSE=0.765683 | var=1.156621 | s_hat=382.841472 |
-------------------------------------------------------------
>>> Degree = 85 : R2=0.817509 | MSE=0.766917 | var=1.165527 | s_hat=383.458370 |
-------------------------------------------------------------
>>> Degree = 86 : R2=0.817134 | MSE=0.768495 | var=1.175069 | s_hat=384.247614 |
-------------------------------------------------------------
>>> Degree = 87 : R2=0.817715 | MSE=0.766054 | var=1.178545 | s_hat=383.027097 |
-------------------------------------------------------------
>>> Degree = 88 : R2=0.818974 | MSE=0.760761 | var=1.177649 | s_hat=380.380661 |
-------------------------------------------------------------
>>> Degree = 89 : R2=0.818406 | MSE=0.763150 | var=1.188707 | s_hat=381.575052 |
-------------------------------------------------------------
>>> Degree = 90 : R2=0.818136 | MSE=0.764283 | var=1.197935 | s_hat=382.141322 |
-------------------------------------------------------------
>>> Degree = 91 : R2=0.819277 | MSE=0.759488 | var=1.197930 | s_hat=379.743787 |
-------------------------------------------------------------
>>> Degree = 92 : R2=0.819191 | MSE=0.759849 | var=1.206109 | s_hat=379.924464 |
-------------------------------------------------------------
>>> Degree = 93 : R2=0.822830 | MSE=0.744557 | var=1.189389 | s_hat=372.278671 |
-------------------------------------------------------------
>>> Degree = 94 : R2=0.822746 | MSE=0.744910 | var=1.197604 | s_hat=372.454770 |
-------------------------------------------------------------
>>> Degree = 95 : R2=0.823640 | MSE=0.741152 | var=1.199275 | s_hat=370.575908 |
-------------------------------------------------------------
>>> Degree = 96 : R2=0.824142 | MSE=0.739044 | var=1.203655 | s_hat=369.521983 |
-------------------------------------------------------------
>>> Degree = 97 : R2=0.824767 | MSE=0.736418 | var=1.207242 | s_hat=368.208839 |
-------------------------------------------------------------
>>> Degree = 98 : R2=0.825567 | MSE=0.733055 | var=1.209662 | s_hat=366.527638 |
-------------------------------------------------------------
>>> Degree = 99 : R2=0.827176 | MSE=0.726292 | var=1.206465 | s_hat=363.146099 |
-------------------------------------------------------------
>>> Degree = 100 : R2=0.828320 | MSE=0.721487 | var=1.206499 | s_hat=360.743320 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 100
```

```
na = bestFitDegree;
nb = bestFitDegree;
p = na+nb+1;

BestFitU = arx_U_builder_3(u,y,na,nb,1);
BestFitModel = inv(BestFitU'*BestFitU)*BestFitU'*y;

denom = BestFitModel(1:na);
num = BestFitModel(na+1:na*2);
BestFitG = tf(num', [1 denom'], 'Ts', Ts);
BestFit_y_hat = lsim(BestFitG, u_val, t);
[r2_arx, mse_arx] = rSQR(y_val, y_hat);


%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 28 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 28
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb+1;

VarU = arx_U_builder_3(u,y,na,nb,1);
VarModel = inv(VarU'*VarU)*VarU'*y;

VarU_val = arx_U_builder_3(u_val,y_val,na,nb,1);
Var_y_hat = VarU_val*VarModel;

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | CoVariance Method=================\n")
```

```
===============Degree Extraction | CoVariance Method=================
```

```
maxCovIndex = find(covs == min(covs));
fprintf(">>> Since the minimum CovMatrix trace occurs in iteration %d ;\n", maxCovIndex)
```

```
>>> Since the minimum CovMatrix trace occurs in iteration 23 ;
```

```
fprintf("    Degree = %d \n", maxCovIndex)
```

```
    Degree = 23
```

```
na = maxCovIndex;
nb = maxCovIndex;
p = na+nb+1;

CovU = arx_U_builder_3(u,y,na,nb,1);
CovModel = inv(CovU'*CovU)*CovU'*y;

CovU_val = arx_U_builder_3(u_val,y_val,na,nb,1);
Cov_y_hat = CovU_val*CovModel;

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=5.00) occurs in iteration 18 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 18
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb+1;
AICU = arx_U_builder_3(u,y,na,nb,1);
AICModel = inv(AICU'*AICU)*AICU'*y;

AICU_val = arx_U_builder_3(u_val,y_val,na,nb,1);
AIC_y_hat = AICU_val*AICModel;

fprintf("==================================================================\n")
```

```
==================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
```

```
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.24 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 28
```

```

na = winner;
nb = winner;
p = na+nb+1;
FTestU = arx_U_builder_3(u,y,na,nb,1);
FTestModel = inv(FTestU'*FTestU)*FTestU'*y;

FTestU_val = arx_U_builder_3(u_val,y_val,na,nb,1);
FTest_y_hat = FTestU_val*FTestModel;

fprintf("=================================================================\n")
```

```
=================================================================
```

```


%%
[BestFit_r2, BestFit_mse] = rSQR(y_val, BestFit_y_hat);
[Var_r2, Var_mse] = rSQR(y_val, Var_y_hat);
[Cov_r2, Cov_mse] = rSQR(y_val, Cov_y_hat);
[AIC_r2, AIC_mse] = rSQR(y_val, AIC_y_hat);
[FTest_r2, FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.7680    | MSE : 1.2850
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.7950    | MSE : 1.1351
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Covariance Method:\n")
```

```
>>> Covariance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Cov_r2, Cov_mse)
```

```
    R2 value : 0.7953    | MSE : 1.1336
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.7845    | MSE : 1.1934
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.7950    | MSE : 1.1351
```

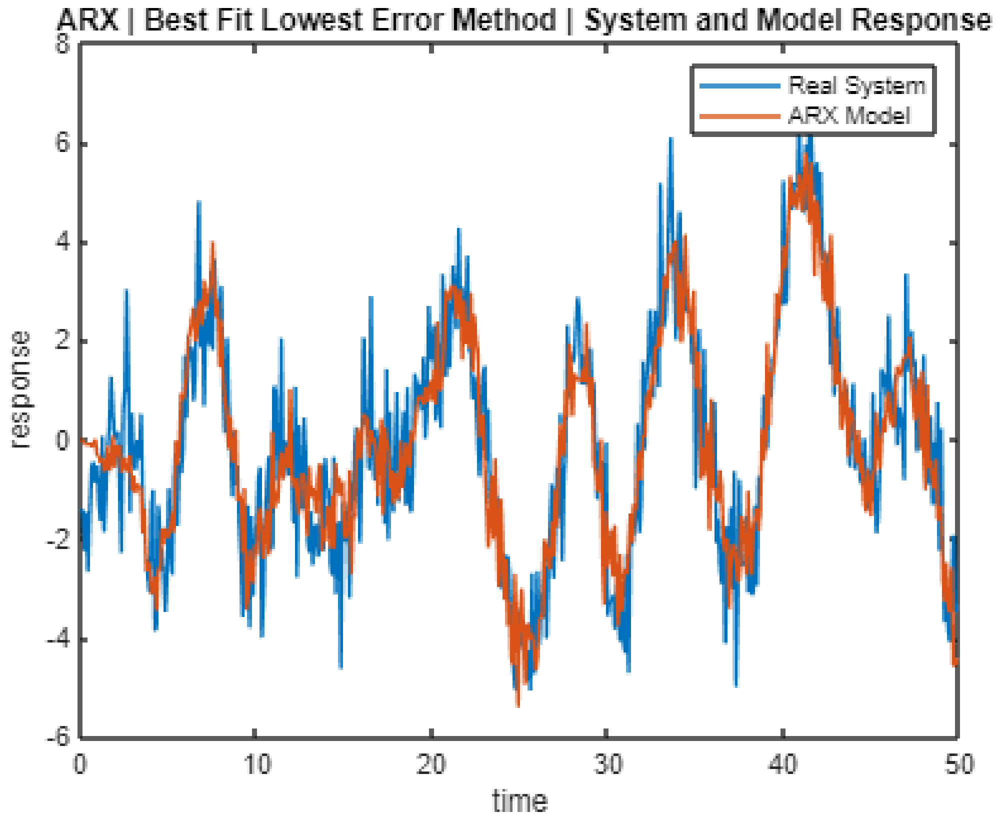
```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

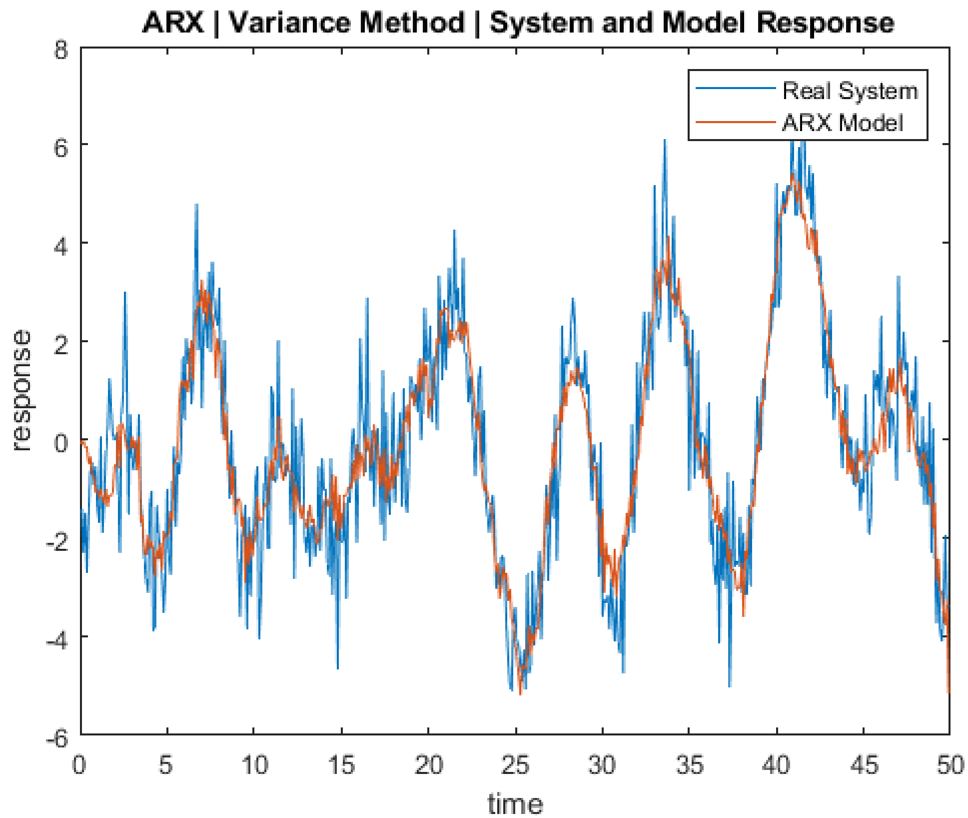
```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

```
===============================================================
```

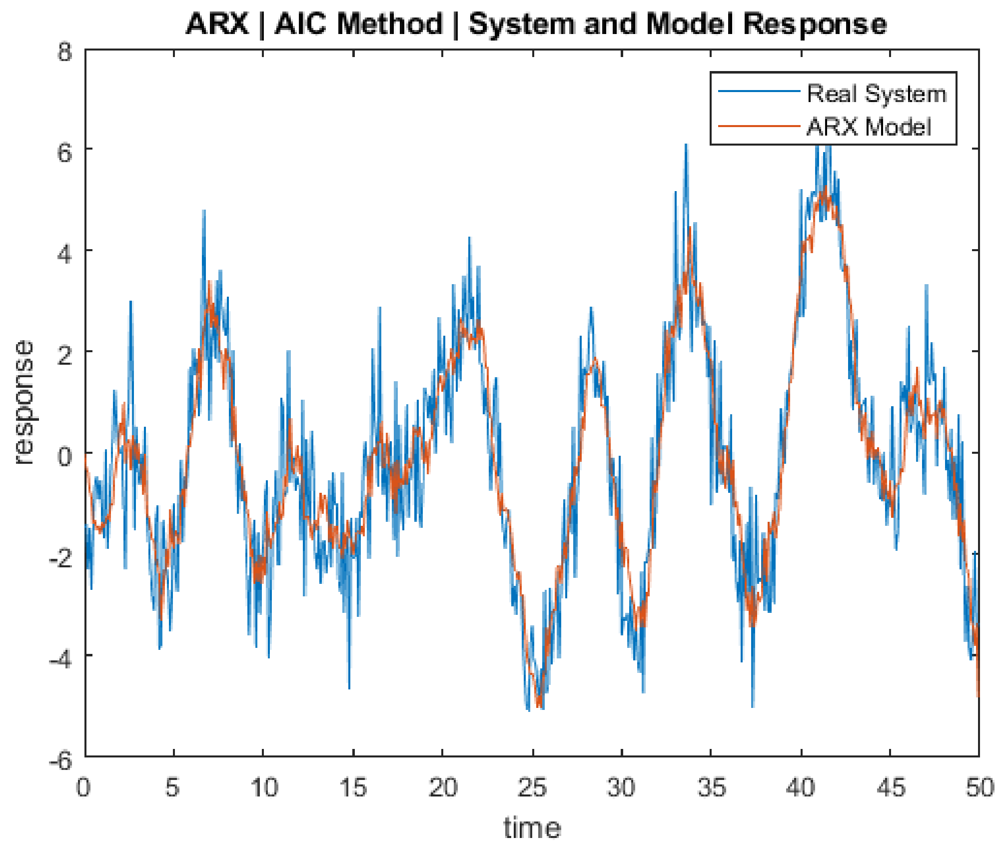
```
%%

BestFitError = y_val - BestFit_y_hat;
VarError = y_val - Var_y_hat;
CovError = y_val - Cov_y_hat;
AICError = y_val - AIC_y_hat;
FTestError = y_val - FTest_y_hat;

for k=0:N-1
    BestFit_Ree(k+1,1) = AutoCorrelate(BestFitError, k);
    Var_Ree(k+1,1) = AutoCorrelate(VarError, k);
    Cov_Ree(k+1,1) = AutoCorrelate(CovError, k);
    AIC_Ree(k+1,1) = AutoCorrelate(AICError, k);
    FTest_Ree(k+1,1) = AutoCorrelate(FTestError, k);
end

for k=0:N-1
    BestFit_Rue(k+1,1) = CrossCorrelate(u_val, BestFitError, k);
    Var_Rue(k+1,1) = CrossCorrelate(u_val, VarError, k);
    Cov_Rue(k+1,1) = CrossCorrelate(u_val, CovError, k);
    AIC_Rue(k+1,1) = CrossCorrelate(u_val, AICError, k);
    FTest_Rue(k+1,1) = CrossCorrelate(u_val, FTestError, k);
end


%%
figure(1)
plot(t,y_val,t,BestFit_y_hat)
legend('Real System','ARX Model')
title(" ARX | Best Fit Lowest Error Method | System and Model Response")
xlabel("time")
ylabel("response")
```



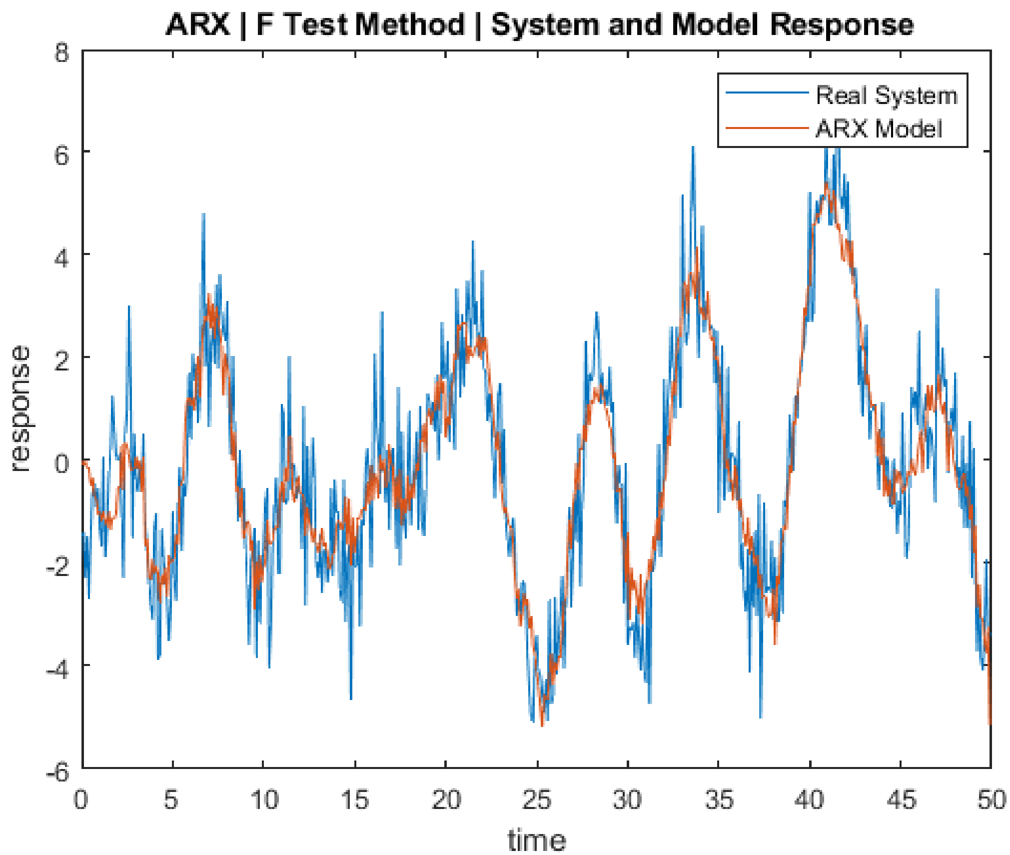

```
figure(2)
plot(t,y_val,t,Var_y_hat)
legend('Real System','ARX Model')
title(" ARX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```



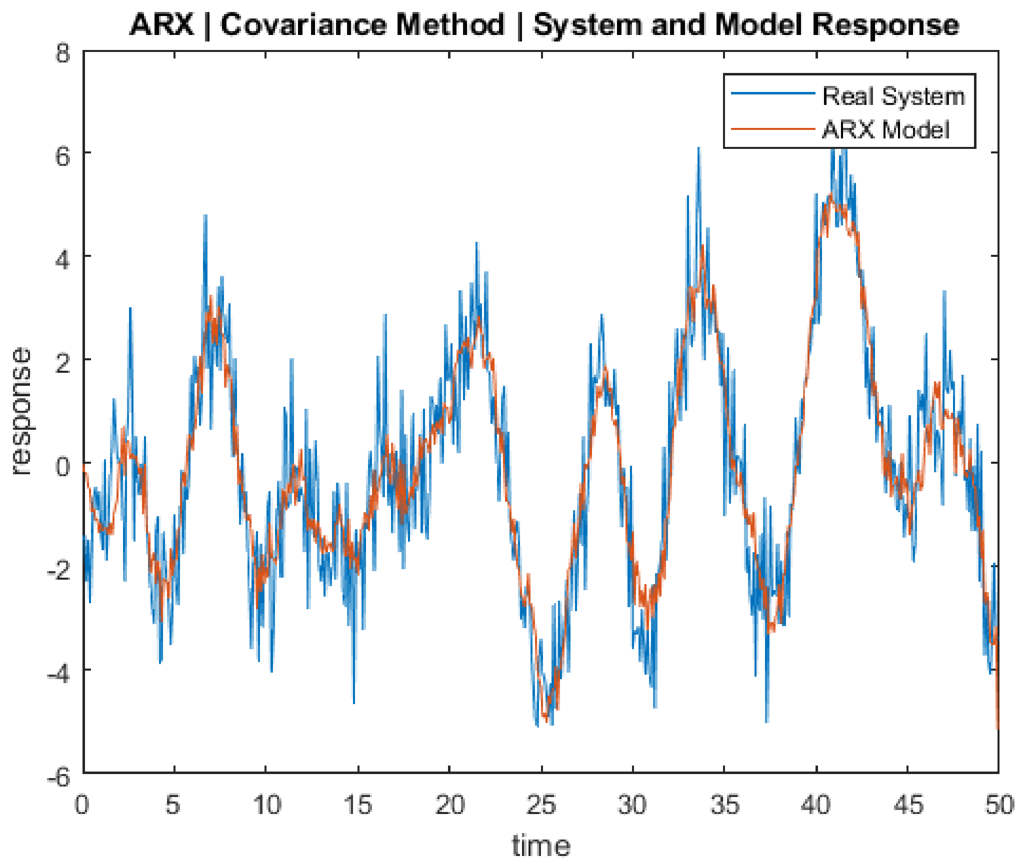

```
figure(3)
plot(t,y_val,t,AIC_y_hat)
legend('Real System','ARX Model')
title(" ARX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t,y_val,t,FTest_y_hat)
legend('Real System','ARX Model')
title(" ARX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```

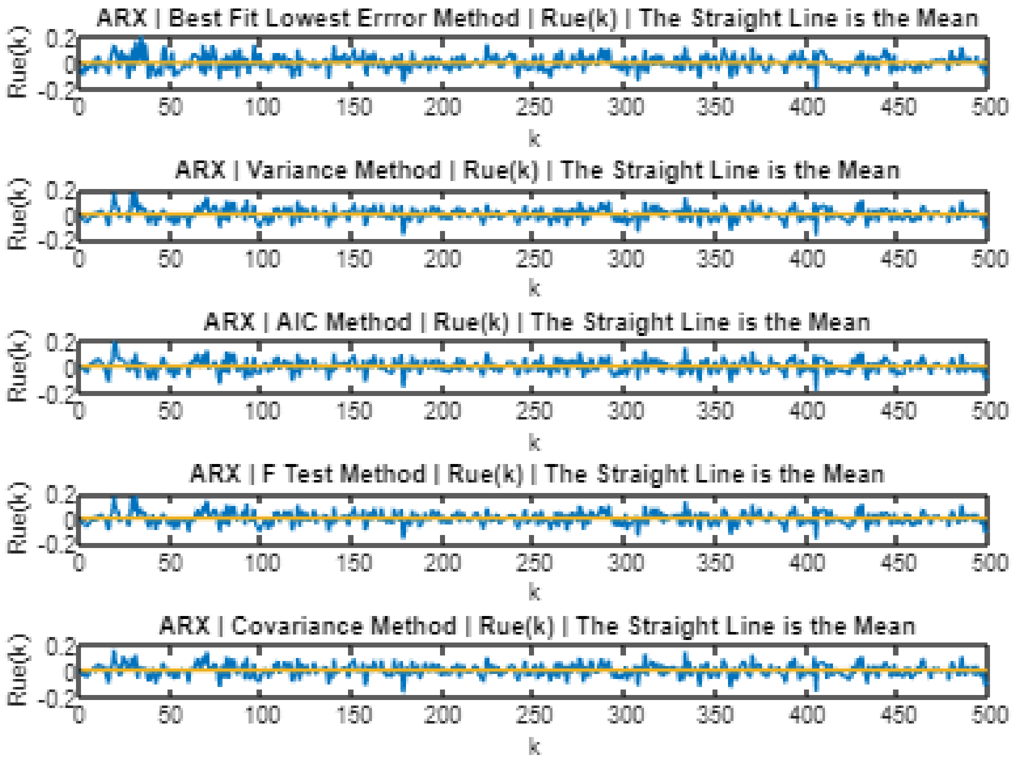



```
figure(7)
plot(t,y_val,t,Cov_y_hat)
legend('Real System','ARX Model')
title(" ARX | Covariance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
%%

figure(5)
subplot(5,1,1)
plot(1:N-1,BestFit_Ree(2:end), 1:N-1, mean(BestFit_Ree(2:end))*ones(length(1:N-1)))
title(" ARX | Best Fit Lowest Errror Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(5,1,2)
plot(1:N-1,Var_Ree(2:end), 1:N-1, mean(Var_Ree(2:end))*ones(length(1:N-1)))
title(" ARX | Variance Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(5,1,3)
plot(1:N-1,AIC_Ree(2:end), 1:N-1, mean(AIC_Ree(2:end))*ones(length(1:N-1)))
title(" ARX | AIC Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")
```

```
subplot(5,1,4)
plot(1:N-1,FTest_Ree(2:end), 1:N-1, mean(FTest_Ree(2:end))*ones(length(1:N-1)))
title(" ARX | F Test Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(5,1,5)
plot(1:N-1,Cov_Ree(2:end), 1:N-1, mean(Cov_Ree(2:end))*ones(length(1:N-1)))
title(" ARX | Covariance Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")


%%

figure(6)
subplot(5,1,1)
plot(1:N-1,BestFit_Rue(2:end), 1:N-1, mean(BestFit_Rue(2:end))*ones(length(1:N-
1)))
title(" ARX | Best Fit Lowest Errror Method | Rue(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(5,1,2)
plot(1:N-1,Var_Rue(2:end), 1:N-1, mean(Var_Rue(2:end))*ones(length(1:N-1)))
title(" ARX | Variance Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(5,1,3)
plot(1:N-1,AIC_Rue(2:end), 1:N-1, mean(AIC_Rue(2:end))*ones(length(1:N-1)))
title(" ARX | AIC Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(5,1,4)
plot(1:N-1,FTest_Rue(2:end), 1:N-1, mean(FTest_Rue(2:end))*ones(length(1:N-1)))
title(" ARX | F Test Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")


subplot(5,1,5)
plot(1:N-1,Cov_Rue(2:end), 1:N-1, mean(Cov_Rue(2:end))*ones(length(1:N-1)))
title(" ARX | Covariance Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")
```


ARX | Best Fit Lowest Errror Method | Rue(k) | The Straight Line is the Mean
Rue(k)
0.2
0
-0.2
0
50
100
150
200
250
300
350
400
450
500
k
ARX | Variance Method | Rue(k) | The Straight Line is the Mean
ARX | AIC Method | Rue(k) | The Straight Line is the Mean
ARX | F Test Method | Rue(k) | The Straight Line is the Mean
ARX | Covariance Method | Rue(k) | The Straight Line is the Mean

Q1 - part b | ARMAX

```
clear; clc;
%%

load q1_402123100.mat

u_val = u(length(u)/2+1:end);
v_val = v(length(u)/2+1:end);
z_val = z(length(u)/2+1:end);
y_val = y(length(u)/2+1:end);

u = u(1:length(u)/2);
v = v(1:length(v)/2);
z = z(1:length(z)/2);
y = y(1:length(y)/2);

%%

Ts = 0.1;
t = 0:Ts:length(u)*Ts-Ts;
N = length(y);
data = iddata(y,u,Ts);

%%

fprintf("===============Degree Extraction | Best Fit Lowest Error
Method====================\n")
```

```
===============Degree Extraction | Best Fit Lowest Error Method====================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 5;

for degree=1:1:100
    na = degree;
    nb = degree;
    nc = degree;
    p = na+nb+nc+1;
```

```
    try
        sys = armax(data, [na nb nc 1]);
        armax_y_hat = lsim(sys, u_val, t);
    catch
        break
    end
%     armax_U = armax_U_builder(na,nb,nc,u,y,error_hat);
%     armax_theta_hat = inv(armax_U'*armax_U)*armax_U'*y;
%     armax_y_hat = armax_U*armax_theta_hat;

    [r2_armax, mse_armax] = rSQR(y_val, armax_y_hat);

    error = y_val - armax_y_hat;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

%     theta = [sys.A sys.B sys.C];
%     covs = [covs; cov(theta)];

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f |\n", degree, r2_armax,
mse_armax, variance)
    fprintf("-------------------------------------------------------------\n")
    ps = [ps; p];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
    R2s = [R2s; r2_armax];
    MSEs = [MSEs; mse_armax];
    vars = [vars; variance];

end
```

```
>>> Degree = 1 : R2=0.032631 | MSE=5.357101 | var=5.400303 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.043706 | MSE=5.295767 | var=5.370961 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.821072 | MSE=0.990870 | var=1.011092 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.822038 | MSE=0.985521 | var=1.011828 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.820017 | MSE=0.996713 | var=1.029662 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.694651 | MSE=1.690961 | var=1.757756 |
-------------------------------------------------------------
```

```
>>> Degree = 7 : R2=0.510297 | MSE=2.711879 | var=2.836693 |
------------------------------------------------------------
>>> Degree = 8 : R2=0.810596 | MSE=1.048883 | var=1.104088 |
------------------------------------------------------------
>>> Degree = 9 : R2=-0.557017 | MSE=8.622456 | var=9.133957 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.751777 | MSE=1.374611 | var=1.465470 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.818417 | MSE=1.005571 | var=1.078939 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.803089 | MSE=1.090457 | var=1.177599 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.809914 | MSE=1.052660 | var=1.144196 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.804460 | MSE=1.082862 | var=1.184751 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.756354 | MSE=1.349261 | var=1.485970 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.814773 | MSE=1.025749 | var=1.137194 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.815708 | MSE=1.020572 | var=1.139031 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.815409 | MSE=1.022228 | var=1.148571 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.814727 | MSE=1.026006 | var=1.160641 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.816893 | MSE=1.014013 | var=1.154912 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.804673 | MSE=1.081682 | var=1.240461 |
------------------------------------------------------------
>>> Degree = 22 : R2=0.794331 | MSE=1.138954 | var=1.315190 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.800014 | MSE=1.107484 | var=1.287772 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.790430 | MSE=1.160556 | var=1.358965 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.804238 | MSE=1.084091 | var=1.278410 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.804315 | MSE=1.083664 | var=1.287011 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.808231 | MSE=1.061977 | var=1.270307 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.805900 | MSE=1.074887 | var=1.295045 |
------------------------------------------------------------
>>> Degree = 29 : R2=-3881.375374 | MSE=21499.835350 | var=26092.033191 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.807443 | MSE=1.066341 | var=1.303596 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.790466 | MSE=1.160361 | var=1.429016 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.796398 | MSE=1.127510 | var=1.398896 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.791481 | MSE=1.154736 | var=1.443419 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.794406 | MSE=1.138539 | var=1.433928 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.795516 | MSE=1.132391 | var=1.437045 |
```

```
-------------------------------------------------------------
>>> Degree = 36 : R2=0.792529 | MSE=1.148935 | var=1.469227 |
-------------------------------------------------------------
>>> Degree = 37 : R2=0.782962 | MSE=1.201915 | var=1.548860 |
-------------------------------------------------------------
>>> Degree = 38 : R2=0.763405 | MSE=1.310215 | var=1.701578 |
-------------------------------------------------------------
>>> Degree = 39 : R2=0.786069 | MSE=1.184706 | var=1.550663 |
-------------------------------------------------------------
>>> Degree = 40 : R2=0.788260 | MSE=1.172576 | var=1.546934 |
-------------------------------------------------------------
>>> Degree = 41 : R2=0.785208 | MSE=1.189475 | var=1.581748 |
-------------------------------------------------------------
>>> Degree = 42 : R2=0.767266 | MSE=1.288836 | var=1.727662 |
-------------------------------------------------------------
>>> Degree = 43 : R2=0.795871 | MSE=1.130428 | var=1.527606 |
-------------------------------------------------------------
>>> Degree = 44 : R2=0.792833 | MSE=1.147250 | var=1.563011 |
-------------------------------------------------------------
>>> Degree = 45 : R2=0.795873 | MSE=1.130418 | var=1.552772 |
-------------------------------------------------------------
>>> Degree = 46 : R2=0.791383 | MSE=1.155279 | var=1.600110 |
-------------------------------------------------------------
>>> Degree = 47 : R2=0.603915 | MSE=2.193441 | var=3.063465 |
-------------------------------------------------------------
>>> Degree = 48 : R2=-3928.263457 | MSE=21759.492381 | var=30647.172368 |
-------------------------------------------------------------
>>> Degree = 49 : R2=-43942891984562995193834373120.000000 |
MSE=243347139707897645138572214272.000000 | var=345663550721445517063454982144.000000 |
-------------------------------------------------------------
>>> Degree = 50 : R2=0.748257 | MSE=1.394106 | var=1.997287 |
-------------------------------------------------------------
>>> Degree = 51 : R2=0.778632 | MSE=1.225895 | var=1.771525 |
-------------------------------------------------------------
>>> Degree = 52 : R2=0.751659 | MSE=1.375262 | var=2.004755 |
-------------------------------------------------------------
>>> Degree = 53 : R2=0.776786 | MSE=1.236116 | var=1.817818 |
-------------------------------------------------------------
>>> Degree = 54 : R2=0.766203 | MSE=1.294722 | var=1.920952 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.777662 | MSE=1.231265 | var=1.843210 |
-------------------------------------------------------------
>>> Degree = 56 : R2=-117505589948385622772338917376.000000 |
MSE=650722970706478265540771053568.000000 | var=982965212547550231778469871616.000000 |
-------------------------------------------------------------
>>> Degree = 57 : R2=-58782.810794 | MSE=325533.244896 | var=496239.702586 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.762276 | MSE=1.316470 | var=2.025338 |
-------------------------------------------------------------
>>> Degree = 59 : R2=0.725349 | MSE=1.520965 | var=2.361747 |
-------------------------------------------------------------
>>> Degree = 60 : R2=-16396352922281238220099602139340047646720.000000 |
MSE=90799795031242923371457403223718892666880.000000 |
var=142319427948656661300622249849621529493504.000000 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.736022 | MSE=1.461859 | var=2.313068 |
-------------------------------------------------------------
```

```
>>> Degree = 62 : R2=0.781939 | MSE=1.207580 | var=1.929042 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.701995 | MSE=1.650291 | var=2.661760 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.769534 | MSE=1.276276 | var=2.078625 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.772066 | MSE=1.262257 | var=2.076080 |
-------------------------------------------------------------
>>> Degree = 66 : R2=-86.782837 | MSE=486.124179 | var=807.515248 |
-------------------------------------------------------------
>>> Degree = 67 : R2=-220398924433130793205\76.000000 | MSE=122052612910276172316672.000000 |
var=204786263272275417497600.000000 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.782060 | MSE=1.206909 | var=2.045609 |
-------------------------------------------------------------
>>> Degree = 69 : R2=-45374056948129993014470914236675719375274316398592.000000 |
MSE=25127265131183420684154676909205316542565893904\7936.000000 |
var=43026138923259277384362988775258003274583213762150\4.000000 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.746436 | MSE=1.404189 | var=2.429393 |
-------------------------------------------------------------
>>> Degree = 71 : R2=0.702898 | MSE=1.645294 | var=2.876387 |
-------------------------------------------------------------
>>> Degree = 72 : R2=-Inf | MSE=Inf | var=Inf |
-------------------------------------------------------------
Warning: The "C" polynomial is unstable and cannot be automatically stabilized to meet the
prescribed constraints. This can cause the estimation to fail. Make sure that the starting
polynomial value is stable and within the desired constraints.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));
fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 4
```

```
na = bestFitDegree;
nb = bestFitDegree;
```

```
nc = bestFitDegree;
p = na+nb+nc+1;

BestFitModel = armax(data, [na nb nc 1])
```

```
BestFitModel =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.921 z^-1 + 0.1017 z^-2 + 1.593 z^-3 - 0.7716 z^-4

  B(z) = -0.02185 z^-1 + 0.07827 z^-2 - 0.06685 z^-3 + 0.02848 z^-4

  C(z) = 1 - 1.964 z^-1 + 0.08861 z^-2 + 1.731 z^-3 - 0.8545 z^-4

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=4   nb=4   nc=4   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data".
Fit to estimation data: 53.61% (prediction focus)
FPE: 0.9489, MSE: 0.9044
```

```
BestFit_y_hat = lsim(BestFitModel, u_val, t);
% [armax_BestFit_r2, armax_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%
fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 3
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
p = na+nb+nc+1;

armax_VarModel = armax(data, [na nb nc 1])
```

```
armax_VarModel =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 2.842 z^-1 + 2.7 z^-2 - 0.8569 z^-3

  B(z) = 0.002931 z^-1 + 0.006859 z^-2 - 0.000951 z^-3

  C(z) = 1 - 2.916 z^-1 + 2.841 z^-2 - 0.9255 z^-3

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nc=3   nk=1
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data".
Fit to estimation data: 53.33% (prediction focus)
FPE: 0.9488, MSE: 0.9153
```

```
Var_y_hat = lsim(armax_VarModel, u_val, t);
% [armax_Var_r2, armax_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

% fprintf("===============Degree Extraction | CoVariance
Method=================\n")
%
% maxCovIndex = find(covs == min(covs),1);
% fprintf(">>> Since the maximum accuracy occurs in iteration %d ;\n",
maxCovIndex)
% fprintf("    Degree = %d \n", maxCovIndex)
% na = maxCovIndex;
% nb = maxCovIndex;
% nc = maxCovIndex;
% p = na+nb+nc+1;
%
% armax_CovModel = armax(data, [na nb nc 1]);
% Cov_y_hat = lsim(armax_CovModel, u_val, t);
%
%
fprintf("=================================================================\n")


%%
```

```
fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=5.00) occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 3
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
p = na+nb+nc+1;

armax_AICModel = armax(data, [na nb nc 1])
```

```
armax_AICModel =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 2.842 z^-1 + 2.7 z^-2 - 0.8569 z^-3

  B(z) = 0.002931 z^-1 + 0.006859 z^-2 - 0.000951 z^-3

  C(z) = 1 - 2.916 z^-1 + 2.841 z^-2 - 0.9255 z^-3

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nc=3   nk=1
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data".
Fit to estimation data: 53.33% (prediction focus)
FPE: 0.9488, MSE: 0.9153
```

```
AIC_y_hat = lsim(armax_AICModel, u_val, t);
% [armax_AIC_r2, armax_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("================================================================\n")
```

```
================================================================
```

```
%%

fprintf("==============Degree Extraction | F test Method===================\n")
```

```
==============Degree Extraction | F test Method===================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", winScore)
```

```
>>> The F test is suggesting the best model with the m=1.21 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 3
```

```
na = winner;
nb = winner;
nc = winner;
p = na+nb+nc+1;

armax_FTestModel = armax(data, [na nb nc 1]);
FTest_y_hat = lsim(armax_FTestModel, u_val, t);
% [armax_FTest_r2, armax_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
```

```
[armax_BestFit_r2, armax_BestFit_mse] = rSQR(y_val, BestFit_y_hat);
[armax_Var_r2, armax_Var_mse] = rSQR(y_val, Var_y_hat);
[armax_AIC_r2, armax_AIC_mse] = rSQR(y_val, AIC_y_hat);
% [armax_Cov_r2, armax_Cov_mse] = rSQR(y_val, Cov_y_hat);
[armax_FTest_r2, armax_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_BestFit_r2,
armax_BestFit_mse)
```

```
    R2 value : 0.8220    | MSE : 0.9855
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Var_r2, armax_Var_mse)
```

```
    R2 value : 0.8211    | MSE : 0.9909
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Cov_r2, armax_Cov_mse)
% fprintf("----------------------------------------------------------------\n")
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_AIC_r2, armax_AIC_mse)
```

```
    R2 value : 0.8211    | MSE : 0.9909
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_FTest_r2, 
armax_FTest_mse)
```

```
    R2 value : 0.8211    | MSE : 0.9909
```

```
fprintf("---------------------------------------------------------------\n")
```

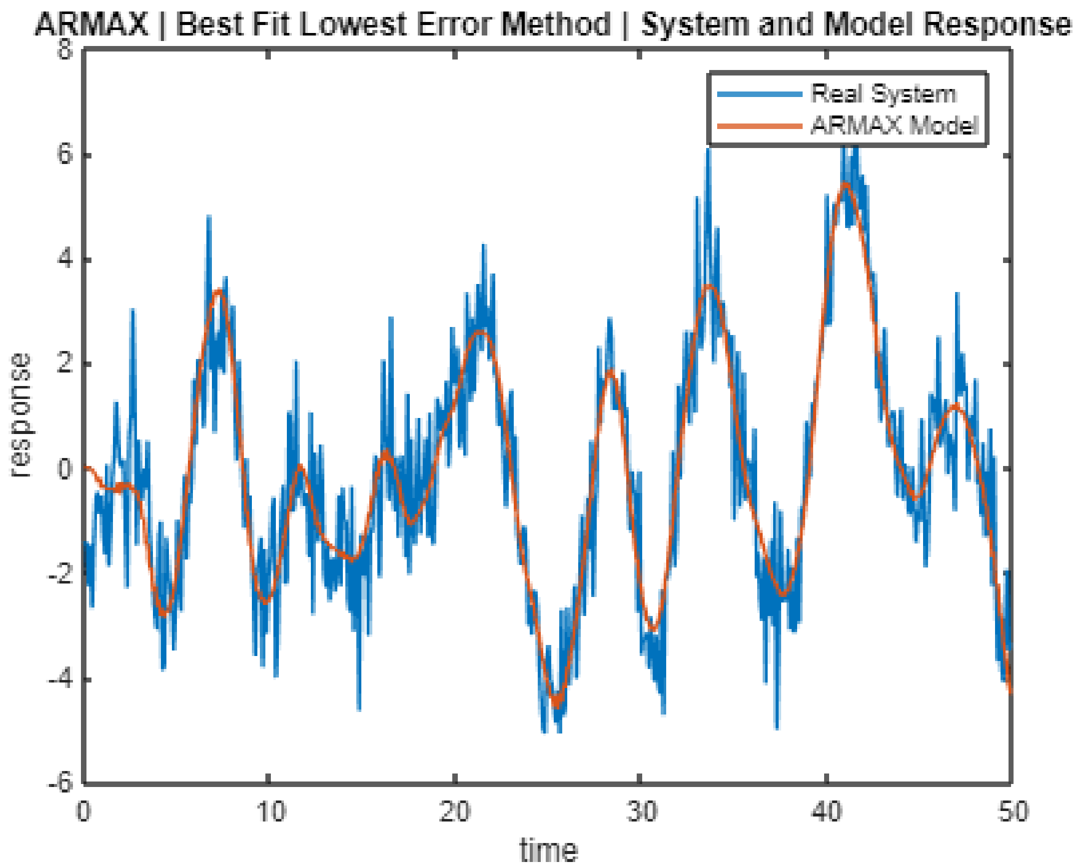
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

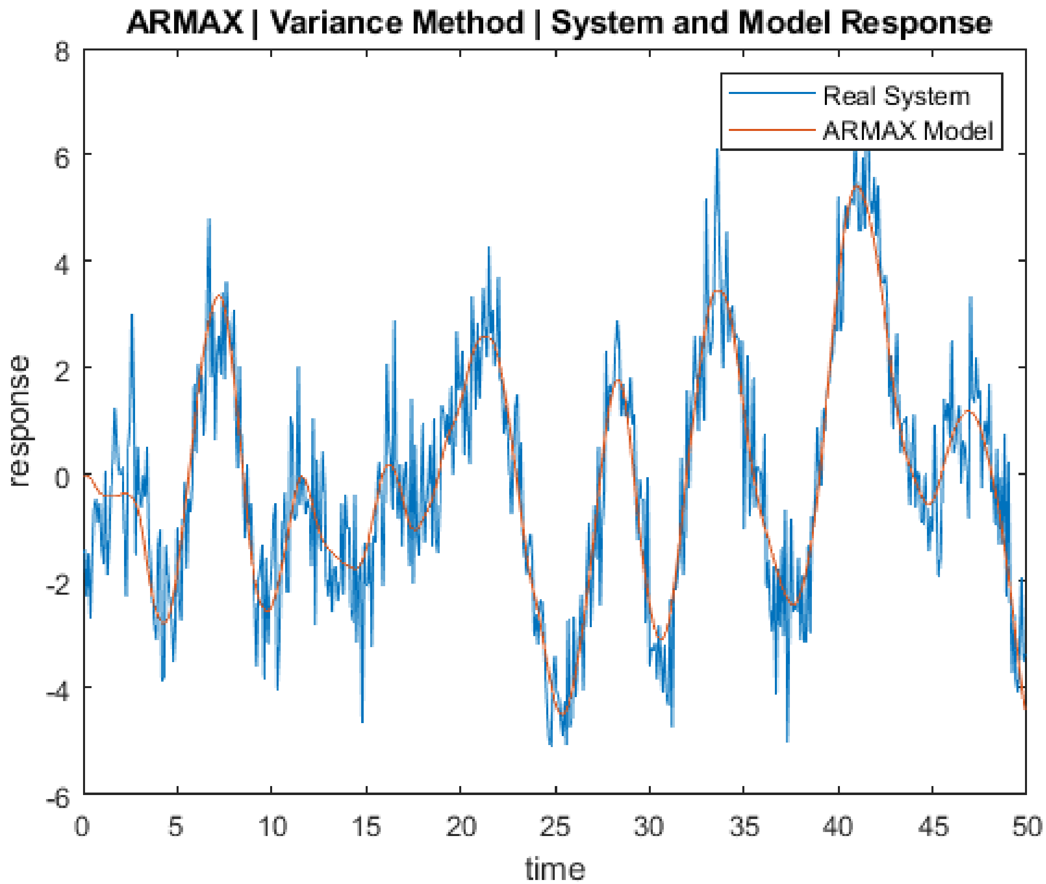
```
===============================================================
```

```
%%

armax_BestFitError = y_val - BestFit_y_hat;
armax_VarError = y_val - Var_y_hat;
armax_AICError = y_val - AIC_y_hat;
% armax_CovError = y_val - Cov_y_hat;
armax_FTestError = y_val - FTest_y_hat;

for k=0:N-1
    armax_BestFit_Ree(k+1,1) = AutoCorrelate(armax_BestFitError, k);
    armax_Var_Ree(k+1,1) = AutoCorrelate(armax_VarError, k);
    armax_AIC_Ree(k+1,1) = AutoCorrelate(armax_AICError, k);
%     armax_Cov_Ree(k+1,1) = AutoCorrelate(armax_CovError, k);
    armax_FTest_Ree(k+1,1) = AutoCorrelate(armax_FTestError, k);
end

for k=0:N-1
    armax_BestFit_Rue(k+1,1) = CrossCorrelate(u_val, armax_BestFitError, k);
    armax_Var_Rue(k+1,1) = CrossCorrelate(u_val, armax_VarError, k);
    armax_AIC_Rue(k+1,1) = CrossCorrelate(u_val, armax_AICError, k);
%     armax_Cov_Rue(k+1,1) = CrossCorrelate(u_val, armax_CovError, k);
    armax_FTest_Rue(k+1,1) = CrossCorrelate(u_val, armax_FTestError, k);
end
```

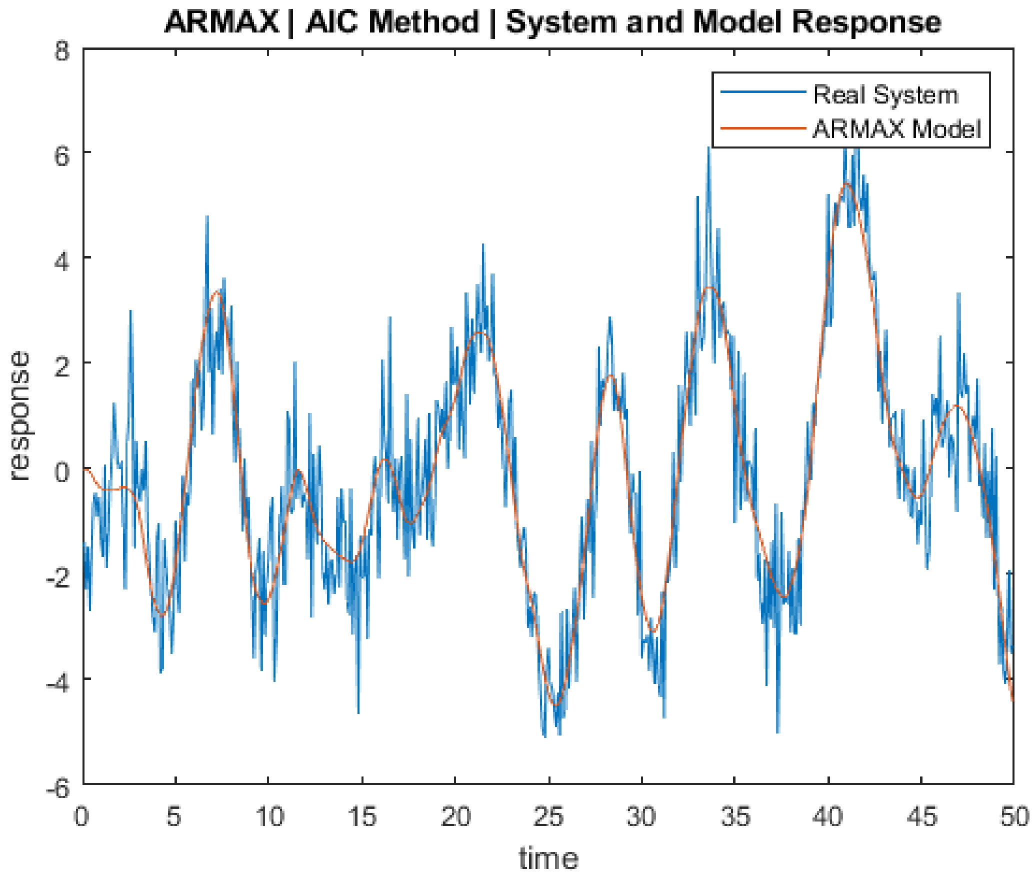
```
%%
figure(1)
plot(t,y_val,t,BestFit_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | Best Fit Lowest Error Method | System and Model Response")
xlabel("time")
ylabel("response")
```



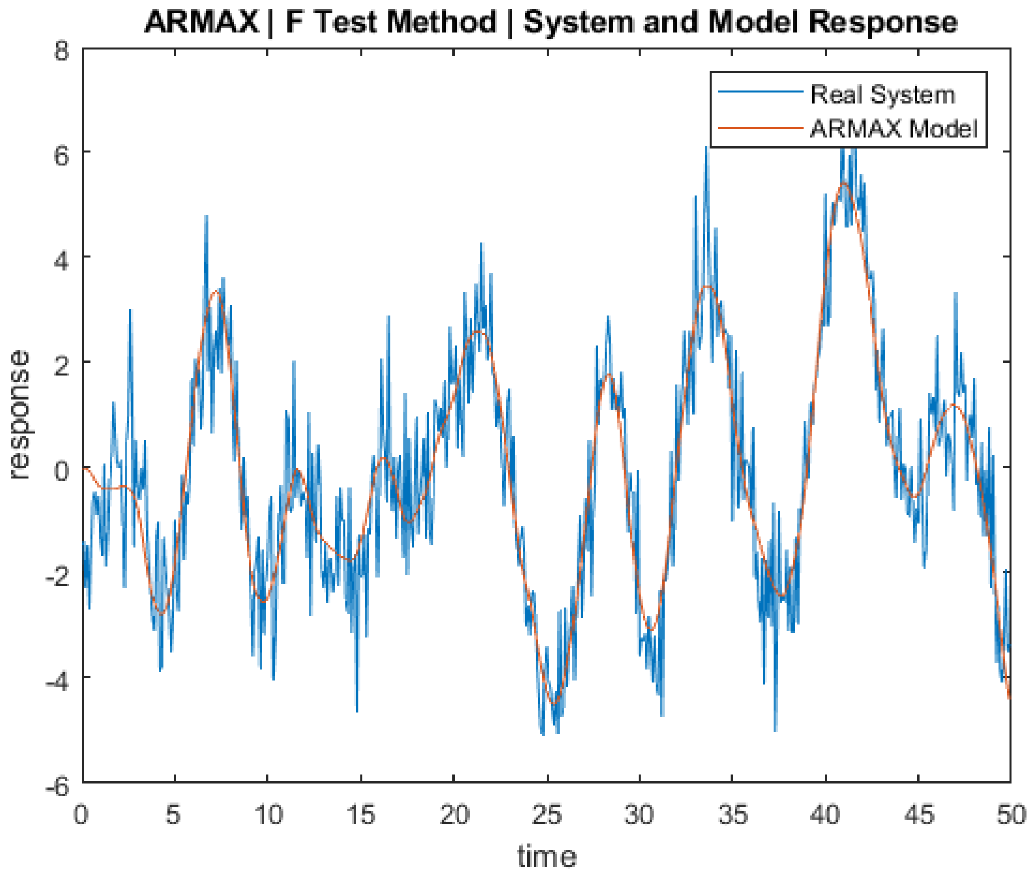

```
figure(2)
plot(t,y_val,t,Var_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(3)
plot(t,y_val,t,AIC_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

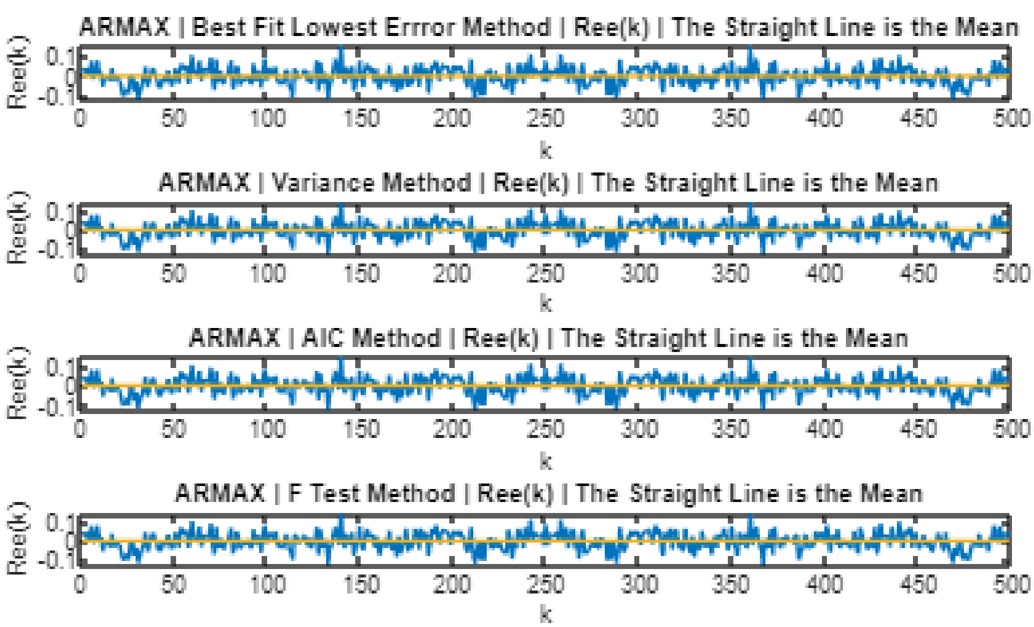



```
figure(4)
plot(t,y_val,t,FTest_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```

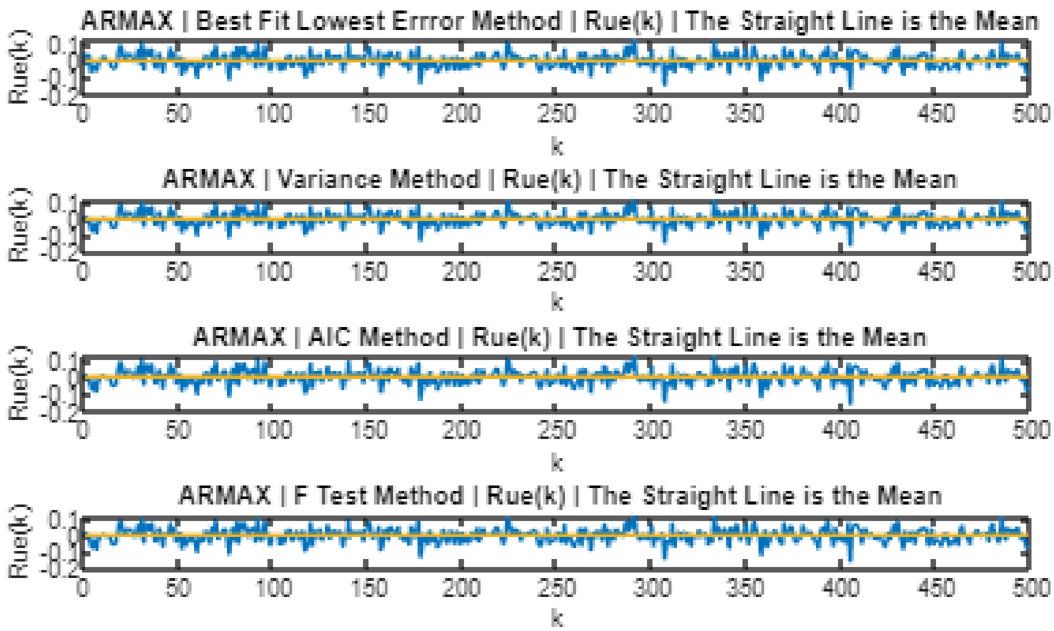



```
% figure(7)
% plot(t,y_val,t,Cov_y_hat)
% legend('Real System','ARMAX Model')
% title(" ARMAX | CoVariance Method | System and Model Response")
% xlabel("time")
% ylabel("response")

%%

figure(5)
subplot(5,1,1)
plot(1:N-1,armax_BestFit_Ree(2:end), 1:N-1,
mean(armax_BestFit_Ree(2:end))*ones(length(1:N-1)))
title(" ARMAX | Best Fit Lowest Errror Method | Ree(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(5,1,2)
```

```
 plot(1:N-1,armax_Var_Ree(2:end), 1:N-1,
mean(armax_Var_Ree(2:end))*ones(length(1:N-1)))
 title(" ARMAX | Variance Method | Ree(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree(k)")

 subplot(5,1,3)
 plot(1:N-1,armax_AIC_Ree(2:end), 1:N-1,
mean(armax_AIC_Ree(2:end))*ones(length(1:N-1)))
 title(" ARMAX | AIC Method | Ree(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree(k)")

 subplot(5,1,4)
 plot(1:N-1,armax_FTest_Ree(2:end), 1:N-1,
mean(armax_FTest_Ree(2:end))*ones(length(1:N-1)))
 title(" ARMAX | F Test Method | Ree(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree(k)")
```



```
 % subplot(5,1,5)
 % plot(1:N-1,armax_Cov_Ree(2:end), 1:N-1,
mean(armax_Cov_Ree(2:end))*ones(length(1:N-1)))
 % title(" ARMAX | CoVariance Method | Ree(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree(k)")
 %%

 figure(6)
 subplot(5,1,1)
 plot(1:N-1,armax_BestFit_Rue(2:end), 1:N-1,
mean(armax_BestFit_Rue(2:end))*ones(length(1:N-1)))
 title(" ARMAX | Best Fit Lowest Errror Method | Rue(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue(k)")

 subplot(5,1,2)
 plot(1:N-1,armax_Var_Rue(2:end), 1:N-1,
mean(armax_Var_Rue(2:end))*ones(length(1:N-1)))
 title(" ARMAX | Variance Method | Rue(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue(k)")

 subplot(5,1,3)
 plot(1:N-1,armax_AIC_Rue(2:end), 1:N-1,
mean(armax_AIC_Rue(2:end))*ones(length(1:N-1)))
 title(" ARMAX | AIC Method | Rue(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue(k)")

 subplot(5,1,4)
 plot(1:N-1,armax_FTest_Rue(2:end), 1:N-1,
mean(armax_FTest_Rue(2:end))*ones(length(1:N-1)))
 title(" ARMAX | F Test Method | Rue(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue(k)")
```




```
% subplot(5,1,5)
% plot(1:N-1,armax_Cov_Rue(2:end), 1:N-1,
mean(armax_Cov_Rue(2:end))*ones(length(1:N-1)))
% title(" ARMAX | CoVariance Method | Rue(k) | The Straight Line is the Mean")
% xlabel("k")
% ylabel("Rue(k)")
```

Q1 - part b | Box-Jenkins

```
clear; clc;
%%

load q1_402123100.mat

u_val = u(length(u)/2+1:end);
v_val = v(length(u)/2+1:end);
z_val = z(length(u)/2+1:end);
y_val = y(length(u)/2+1:end);

u = u(1:length(u)/2);
v = v(1:length(v)/2);
z = z(1:length(z)/2);
y = y(1:length(y)/2);
%%

Ts = 0.1;
t = 0:Ts:length(u)*Ts-Ts;
N = length(y);

data = iddata(y,u,Ts);
data_val = iddata(u_val,y_val,Ts);
```

Run for Estimation through Loop

```
%%

fprintf("===============Degree Extraction | Best Fit Lowest Error
Method===================\n")
```

```
===============Degree Extraction | Best Fit Lowest Error Method===================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 5;

for degree=1:1:100
    na = degree;
    nb = degree;
```

```
    nc = degree;
    nd = degree;
    nk = 1;
    p = na+nb+nc+nd+nk;

    try
        sys = bj(data,[na nb nc nd nk]);
        bj_y_hat = lsim(sys,u_val,t);
    catch
        break;
    end

    [r2_bj, mse_bj] = rSQR(y_val, bj_y_hat);

    error = y_val - bj_y_hat;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

%     covariance = ??
%     sigma2 = mean(diag(covariance));
%     temp = covariance(:,1)/sigma2;
%     inZone = find(temp <= 2/(sqrt(N)) & temp >= -2/(sqrt(N)));
%     cov = length(inZone)/length(temp);


    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f |\n", degree, r2_bj,
mse_bj, variance)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
    R2s = [R2s; r2_bj];
    MSEs = [MSEs; mse_bj];
    vars = [vars; variance];
end
```

```
>>> Degree = 1 : R2=-0.000827 | MSE=5.542386 | var=5.598370 |
--------------------------------------------------------------
>>> Degree = 2 : R2=0.772063 | MSE=1.262269 | var=1.285406 |
--------------------------------------------------------------
>>> Degree = 3 : R2=-0.031268 | MSE=5.710962 | var=5.863410 |
```

```
------------------------------------------------------------
>>> Degree = 4 : R2=0.025531 | MSE=5.396419 | var=5.586355 |
------------------------------------------------------------
>>> Degree = 5 : R2=-0.002341 | MSE=5.550767 | var=5.794120 |
------------------------------------------------------------
>>> Degree = 6 : R2=0.820928 | MSE=0.991666 | var=1.043859 |
------------------------------------------------------------
>>> Degree = 7 : R2=0.578451 | MSE=2.334457 | var=2.478193 |
------------------------------------------------------------
>>> Degree = 8 : R2=0.819133 | MSE=1.001604 | var=1.072381 |
------------------------------------------------------------
>>> Degree = 9 : R2=0.692460 | MSE=1.703098 | var=1.839199 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.783149 | MSE=1.200877 | var=1.308145 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.804669 | MSE=1.081707 | var=1.188689 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.799999 | MSE=1.107565 | var=1.227899 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.320682 | MSE=3.761930 | var=4.207975 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.763085 | MSE=1.311987 | var=1.480798 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.811265 | MSE=1.045176 | var=1.190406 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.806233 | MSE=1.073044 | var=1.233384 |
------------------------------------------------------------
>>> Degree = 17 : R2=-0.008278 | MSE=5.583646 | var=6.477548 |
------------------------------------------------------------
>>> Degree = 18 : R2=-4.200058 | MSE=28.796905 | var=33.720029 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.726308 | MSE=1.515653 | var=1.791552 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.751638 | MSE=1.375383 | var=1.641268 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.220460 | MSE=4.316939 | var=5.201132 |
------------------------------------------------------------
>>> Degree = 22 : R2=0.780143 | MSE=1.217527 | var=1.481177 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.684287 | MSE=1.748359 | var=2.147862 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.447584 | MSE=3.059172 | var=3.795499 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.786917 | MSE=1.180010 | var=1.478710 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.591935 | MSE=2.259787 | var=2.860489 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.553852 | MSE=2.470680 | var=3.159437 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.763020 | MSE=1.312347 | var=1.695539 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.758863 | MSE=1.335368 | var=1.743301 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.289497 | MSE=3.934624 | var=5.190797 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.741973 | MSE=1.428905 | var=1.905206 |
------------------------------------------------------------
```

```
>>> Degree = 32 : R2=0.700121 | MSE=1.660673 | var=2.238104 |
-------------------------------------------------------------
>>> Degree = 33 : R2=0.758500 | MSE=1.337382 | var=1.822046 |
-------------------------------------------------------------
>>> Degree = 34 : R2=0.762299 | MSE=1.316343 | var=1.813144 |
-------------------------------------------------------------
>>> Degree = 35 : R2=0.745465 | MSE=1.409563 | var=1.963180 |
-------------------------------------------------------------
>>> Degree = 36 : R2=0.749397 | MSE=1.387791 | var=1.954636 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")


%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```

bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 6
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
nd = bestFitDegree;
p = na+nb+nc+nd+1;

BestFitModel = bj(data, [na nb nc nd 1])
```

```
BestFitModel =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.02905 z^-1 + 0.05307 z^-2 + 0.0467 z^-3 - 0.007955 z^-4 - 0.07559 z^-5 + 0.06619
z^-6

  C(z) = 1 - 0.2023 z^-1 - 0.7259 z^-2 - 0.8185 z^-3 + 0.05534 z^-4 + 0.2938 z^-5 + 0.4078 z^-
6

  D(z) = 1 - 0.1828 z^-1 - 0.5818 z^-2 - 0.7698 z^-3 - 0.01036 z^-4 + 0.1084 z^-5 + 0.4625 z^-
6

  F(z) = 1 - 0.9311 z^-1 - 0.9063 z^-2 + 0.006122 z^-3 + 0.8306 z^-4 + 0.7024 z^-5 - 0.6963
z^-6
```

```
Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=6   nc=6   nd=6   nf=6   nk=1
   Number of free coefficients: 24
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data".
Fit to estimation data: 54.37% (prediction focus)
FPE: 0.9633, MSE: 0.875
```

```
BestFit_y_hat = lsim(BestFitModel, u_val, t);
% [bj_BestFit_r2, bj_BestFit_mse] = rSQR(y_val, BestFit_y_hat);



%%
fprintf("===============Degree Extraction | Variance Method===================\n")
```

```
===============Degree Extraction | Variance Method===================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n", minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 6 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 6
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
nd = minVarIndex;
nk = 1;
p = na+nb+nc+nd+nk;

bj_VarModel = bj(data, [na nb nc nd nk])
```

```
bj_VarModel =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.02905 z^-1 + 0.05307 z^-2 + 0.0467 z^-3 - 0.007955 z^-4 - 0.07559 z^-5 + 0.06619 z^-6

  C(z) = 1 - 0.2023 z^-1 - 0.7259 z^-2 - 0.8185 z^-3 + 0.05534 z^-4 + 0.2938 z^-5 + 0.4078 z^-6
```

```
  D(z) = 1 - 0.1828 z^-1 - 0.5818 z^-2 - 0.7698 z^-3 - 0.01036 z^-4 + 0.1084 z^-5 + 0.4625 z^-
6

  F(z) = 1 - 0.9311 z^-1 - 0.9063 z^-2 + 0.006122 z^-3 + 0.8306 z^-4 + 0.7024 z^-5 - 0.6963
z^-6

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=6   nc=6   nd=6   nf=6   nk=1
   Number of free coefficients: 24
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data".
Fit to estimation data: 54.37% (prediction focus)
FPE: 0.9633, MSE: 0.875
```

```
Var_y_hat = lsim(bj_VarModel, u_val, t);
% [bj_Var_r2, bj_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("==================================================================\n")
```

```
==================================================================
```

```
%%

% fprintf("===============Degree Extraction | CoVariance
Method=================\n")
%
% maxCovIndex = find(covs == max(covs));
% fprintf(">>> Since the maximum accuracy occurs in iteration %d ;\n",
maxCovIndex)
% fprintf("    Degree = %d \n", maxCovIndex)
%
% na = maxCovIndex;
% nb = maxCovIndex;
% nc = maxCovIndex;
% nd = maxCovIndex;
% nk = 1;
% p = na+nb+nc+nd+nk;
%
% bj_CovModel = bj(data,[na nb nc nd nk]);
% Cov_y_hat = lsim(bj_VarModel, u_val, t);
%
%
fprintf("==================================================================\n")
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=499.00) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
nd = minAICIndex;
nk = 1;
p = na+nb+nc+nd+nk;

bj_AICModel = bj(data,[na nb nc nd nk])
```

```
bj_AICModel =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.1297 z^-1 + 0.1835 z^-2

  C(z) = 1 - 0.1185 z^-1 - 0.615 z^-2

  D(z) = 1 - 0.2302 z^-1 - 0.5958 z^-2

  F(z) = 1 - 1.943 z^-1 + 0.9497 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data".
Fit to estimation data: 49.65% (prediction focus)
FPE: 1.1, MSE: 1.066
```

```
AIC_y_hat = lsim(bj_AICModel, u_val, t);
% [bj_AIC_r2, bj_AIC_mse] = rSQR(y_val, AIC_y_hat);
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.26 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 6
```

```
na = winner;
nb = winner;
nc = winner;
nd = winner;
nk = 1;
p = na+nb+nc+nd+nk;

bj_FTestModel = bj(data,[na nb nc nd nk])
```

```
bj_FTestModel =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.02905 z^-1 + 0.05307 z^-2 + 0.0467 z^-3 - 0.007955 z^-4 - 0.07559 z^-5 + 0.06619
z^-6
```

```
  C(z) = 1 - 0.2023 z^-1 - 0.7259 z^-2 - 0.8185 z^-3 + 0.05534 z^-4 + 0.2938 z^-5 + 0.4078 z^-
6

  D(z) = 1 - 0.1828 z^-1 - 0.5818 z^-2 - 0.7698 z^-3 - 0.01036 z^-4 + 0.1084 z^-5 + 0.4625 z^-
6

  F(z) = 1 - 0.9311 z^-1 - 0.9063 z^-2 + 0.006122 z^-3 + 0.8306 z^-4 + 0.7024 z^-5 - 0.6963
z^-6

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=6   nc=6   nd=6   nf=6   nk=1
   Number of free coefficients: 24
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data".
Fit to estimation data: 54.37% (prediction focus)
FPE: 0.9633, MSE: 0.875
```

```
FTest_y_hat = lsim(bj_FTestModel, u_val, t);
% [bj_FTest_r2, bj_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[bj_BestFit_r2, bj_BestFit_mse] = rSQR(y_val, BestFit_y_hat);
[bj_Var_r2, bj_Var_mse] = rSQR(y_val, Var_y_hat);
[bj_AIC_r2, bj_AIC_mse] = rSQR(y_val, AIC_y_hat);
[bj_FTest_r2, bj_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_BestFit_r2, bj_BestFit_mse)
```

```
    R2 value : 0.8209    | MSE : 0.9917
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Var_r2, bj_Var_mse)
```

```
    R2 value : 0.8209    | MSE : 0.9917
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_AIC_r2, bj_AIC_mse)
```

```
    R2 value : 0.7721    | MSE : 1.2623
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_FTest_r2, bj_FTest_mse)
```

```
    R2 value : 0.8209    | MSE : 0.9917
```

```
fprintf("----------------------------------------------------------------\n")
```

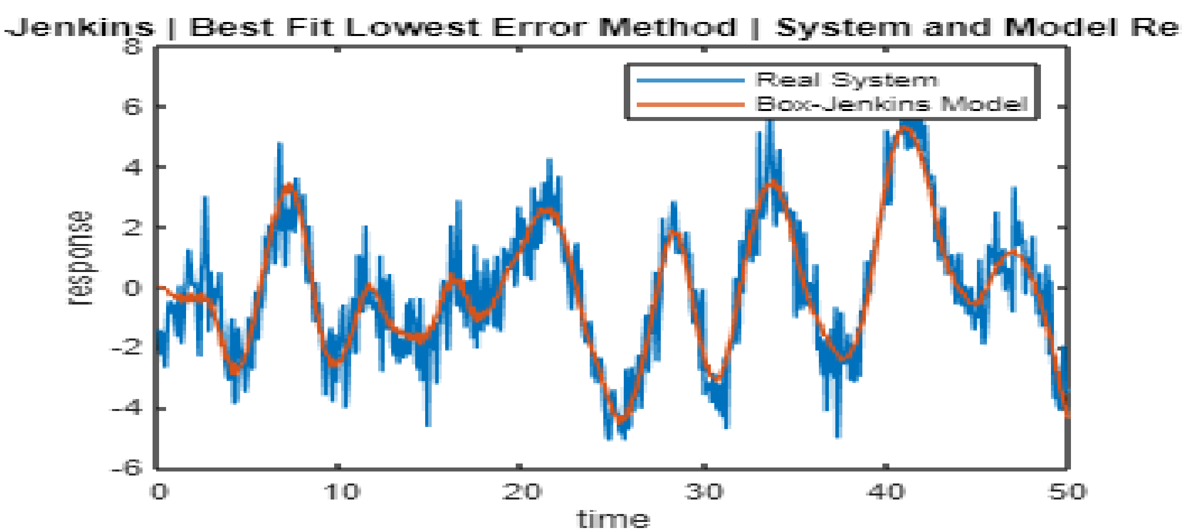
```
----------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("================================================================\n")
```

```
================================================================
```

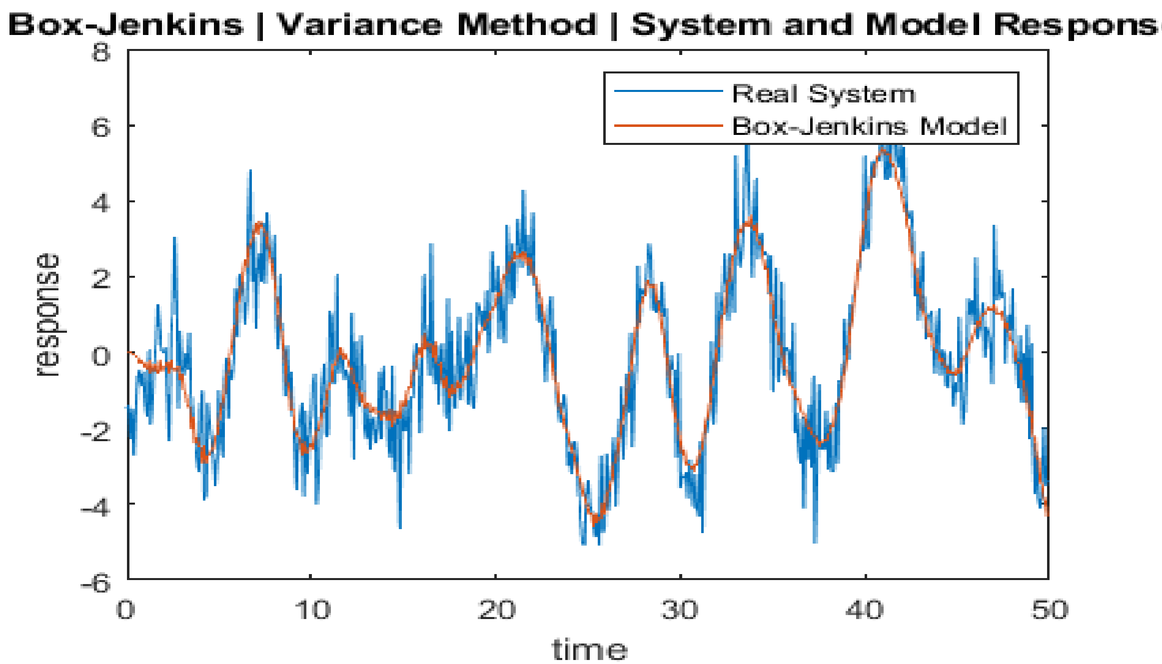
```

%%

bj_BestFitError = y_val - BestFit_y_hat;
bj_VarError = y_val - Var_y_hat;
bj_AICError = y_val - AIC_y_hat;
```

```
bj_FTestError = y_val - FTest_y_hat;

for k=0:N-1
    bj_BestFit_Ree(k+1,1) = AutoCorrelate(bj_BestFitError, k);
    bj_Var_Ree(k+1,1) = AutoCorrelate(bj_VarError, k);
    bj_AIC_Ree(k+1,1) = AutoCorrelate(bj_AICError, k);
    bj_FTest_Ree(k+1,1) = AutoCorrelate(bj_FTestError, k);
end

for k=0:N-1
    bj_BestFit_Rue(k+1,1) = CrossCorrelate(u_val, bj_BestFitError, k);
    bj_Var_Rue(k+1,1) = CrossCorrelate(u_val, bj_VarError, k);
    bj_AIC_Rue(k+1,1) = CrossCorrelate(u_val, bj_AICError, k);
    bj_FTest_Rue(k+1,1) = CrossCorrelate(u_val, bj_FTestError, k);
end




%%
figure(1)
plot(t,y_val,t,BestFit_y_hat)
legend('Real System','Box-Jenkins Model')
title(" Box-Jenkins | Best Fit Lowest Error Method | System and Model
Response")
xlabel("time")
ylabel("response")
```

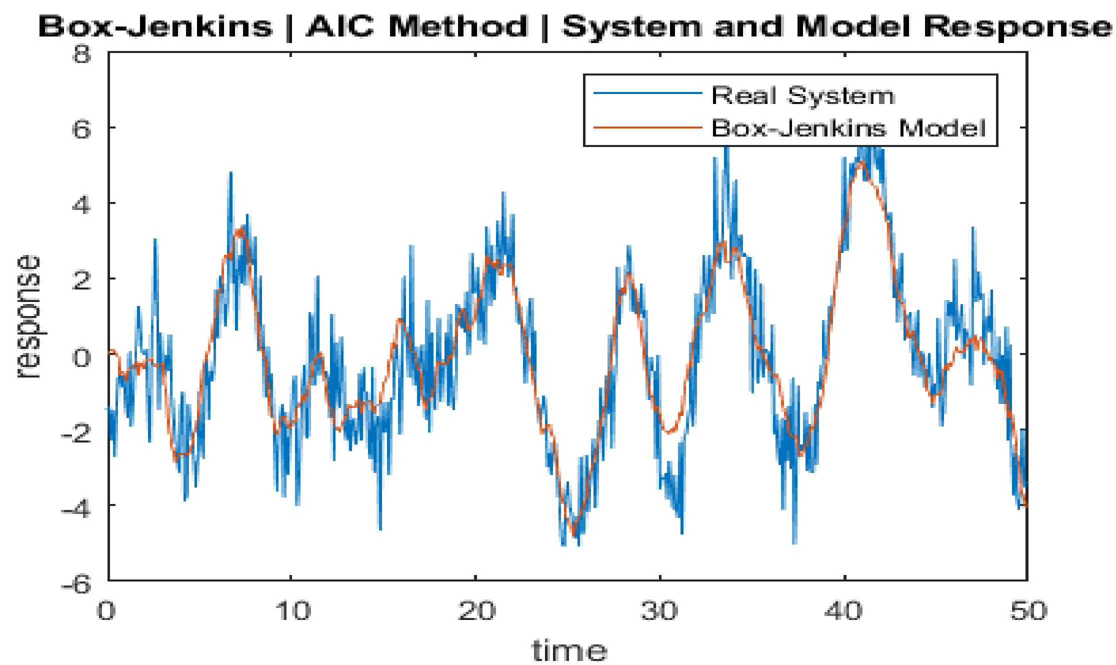



```
figure(2)
plot(t,y_val,t,Var_y_hat)
legend('Real System','Box-Jenkins Model')
```

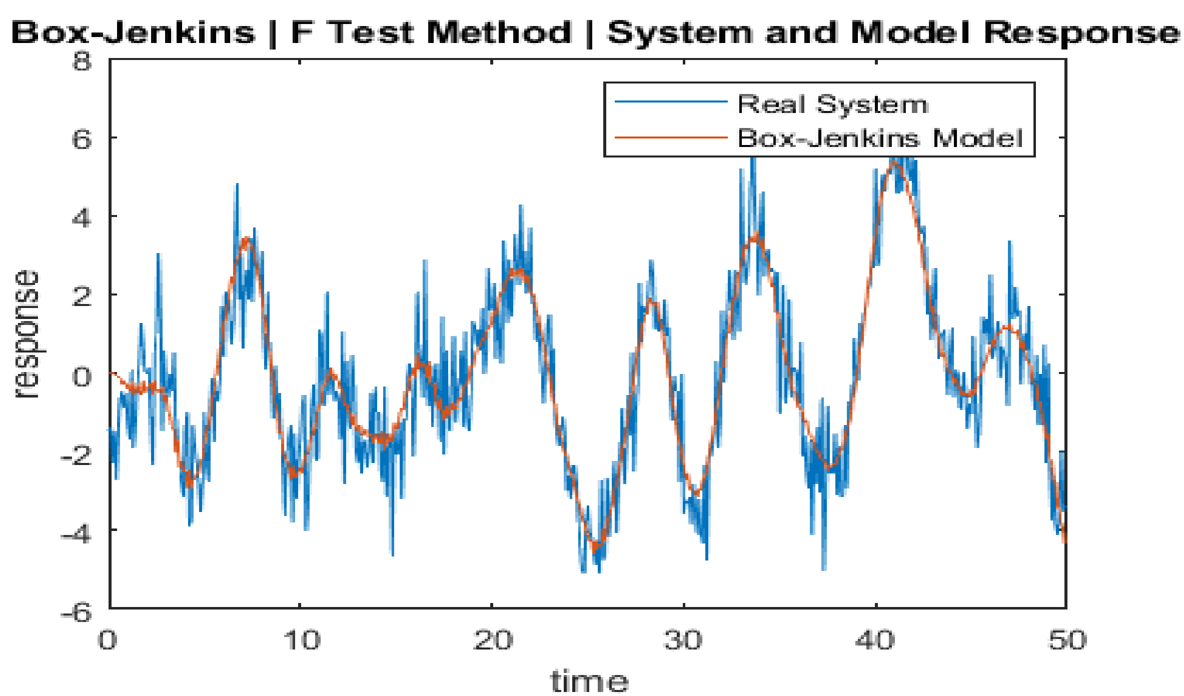
```
title(" Box-Jenkins | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```



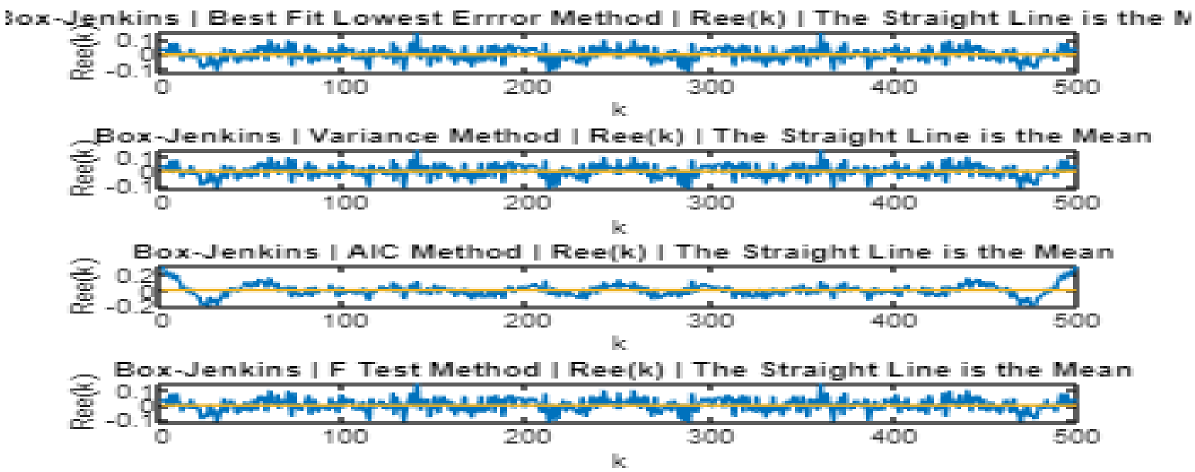

```
figure(3)
plot(t,y_val,t,AIC_y_hat)
legend('Real System','Box-Jenkins Model')
title(" Box-Jenkins | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t,y_val,t,FTest_y_hat)
legend('Real System','Box-Jenkins Model')
title(" Box-Jenkins | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



```
%%

figure(5)
subplot(4,1,1)
plot(1:N-1,bj_BestFit_Ree(2:end), 1:N-1,
mean(bj_BestFit_Ree(2:end))*ones(length(1:N-1)))
title(" Box-Jenkins | Best Fit Lowest Errror Method | Ree(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,2)
plot(1:N-1,bj_Var_Ree(2:end), 1:N-1, mean(bj_Var_Ree(2:end))*ones(length(1:N-
1)))
title(" Box-Jenkins | Variance Method | Ree(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,3)
plot(1:N-1,bj_AIC_Ree(2:end), 1:N-1, mean(bj_AIC_Ree(2:end))*ones(length(1:N-
1)))
title(" Box-Jenkins | AIC Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,4)
plot(1:N-1,bj_FTest_Ree(2:end), 1:N-1,
mean(bj_FTest_Ree(2:end))*ones(length(1:N-1)))
title(" Box-Jenkins | F Test Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N-1,bj_BestFit_Rue(2:end), 1:N-1,
mean(bj_BestFit_Rue(2:end))*ones(length(1:N-1)))
title(" Box-Jenkins | Best Fit Lowest Errror Method | Rue(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(4,1,2)
plot(1:N-1,bj_Var_Rue(2:end), 1:N-1, mean(bj_Var_Rue(2:end))*ones(length(1:N-
1)))
title(" Box-Jenkins | Variance Method | Rue(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(4,1,3)
plot(1:N-1,bj_AIC_Rue(2:end), 1:N-1, mean(bj_AIC_Rue(2:end))*ones(length(1:N-
1)))
title(" Box-Jenkins | AIC Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(4,1,4)
plot(1:N-1,bj_FTest_Rue(2:end), 1:N-1,
mean(bj_FTest_Rue(2:end))*ones(length(1:N-1)))
title(" Box-Jenkins | F Test Method | Rue(k) | The Straight Line is the Mean")
```

```
xlabel("k")
ylabel("Rue(k)")
```

Q1 - part b | Output Error

```
clear; clc;
%%

load q1_402123100.mat

u_val = u(length(u)/2+1:end);
v_val = v(length(u)/2+1:end);
z_val = z(length(u)/2+1:end);
y_val = y(length(u)/2+1:end);

u = u(1:length(u)/2);
v = v(1:length(v)/2);
z = z(1:length(z)/2);
y = y(1:length(y)/2);



%%
Ts = 0.1;
t = 0:Ts:length(u)*Ts-Ts;
N = length(y);

data = iddata(y,u,Ts);
%%



fprintf("===============Degree Extraction | Best Fit Lowest Error
Method===================\n")
```

```
==============Degree Extraction | Best Fit Lowest Error Method===================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 5;

for degree=1:1:100
    na = degree;
    nb = degree;
    nk = 1;
```

```
    p = na+nb+1;

    try
        sys = oe(data,[na nb nk]);
        oe_y_hat = lsim(sys,u_val,t);
    catch
        break;
    end

    [r2_oe, mse_oe] = rSQR(y_val, oe_y_hat);

    error = y_val - oe_y_hat;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

%     covariance = variance*inv(U'*U);
%     detUTU = det(U*U');
%     covs = [covs; covariance];

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f |\n", degree,
r2_oe, mse_oe, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_oe];
    MSEs = [MSEs; mse_oe];
%     dets = [dets; detUTU];
    vars = [vars; variance];

    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
end
```

```
>>> Degree = 1 : R2=0.258071 | MSE=4.108655 | var=4.133456 | s_hat=2054.327691 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.774381 | MSE=1.249432 | var=1.262052 | s_hat=624.715848 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.773828 | MSE=1.252498 | var=1.270282 | s_hat=626.248987 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.773307 | MSE=1.255380 | var=1.278391 | s_hat=627.690128 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.773503 | MSE=1.254297 | var=1.282512 | s_hat=627.148454 |
-------------------------------------------------------------
```

```
>>> Degree = 6 : R2=0.819775 | MSE=0.998051 | var=1.024693 | s_hat=499.025310 |
------------------------------------------------------------
>>> Degree = 7 : R2=0.819510 | MSE=0.999520 | var=1.030433 | s_hat=499.760058 |
------------------------------------------------------------
>>> Degree = 8 : R2=0.819094 | MSE=1.001824 | var=1.037085 | s_hat=500.912096 |
------------------------------------------------------------
>>> Degree = 9 : R2=0.774039 | MSE=1.251331 | var=1.300759 | s_hat=625.665267 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.728888 | MSE=1.501366 | var=1.567188 | s_hat=750.682957 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.809830 | MSE=1.053122 | var=1.103901 | s_hat=526.560848 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.813455 | MSE=1.033051 | var=1.087422 | s_hat=516.525338 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.815035 | MSE=1.024300 | var=1.082770 | s_hat=512.150073 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.815450 | MSE=1.022004 | var=1.084930 | s_hat=511.001997 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.814503 | MSE=1.027246 | var=1.095145 | s_hat=513.622887 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.810106 | MSE=1.051596 | var=1.125905 | s_hat=525.797780 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.814559 | MSE=1.026939 | var=1.104235 | s_hat=513.469343 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.814718 | MSE=1.026054 | var=1.108050 | s_hat=513.027149 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.810706 | MSE=1.048272 | var=1.136954 | s_hat=524.135960 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.813597 | MSE=1.032262 | var=1.124469 | s_hat=516.131153 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.737888 | MSE=1.451527 | var=1.588104 | s_hat=725.763435 |
------------------------------------------------------------
>>> Degree = 22 : R2=0.796763 | MSE=1.125487 | var=1.236799 | s_hat=562.743527 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.799472 | MSE=1.110483 | var=1.225699 | s_hat=555.241581 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.797271 | MSE=1.122674 | var=1.244649 | s_hat=561.336761 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.793943 | MSE=1.141104 | var=1.270717 | s_hat=570.551863 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.787355 | MSE=1.177586 | var=1.317210 | s_hat=588.793082 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.796150 | MSE=1.128881 | var=1.268405 | s_hat=564.440426 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.361116 | MSE=3.538012 | var=3.993241 | s_hat=1769.005957 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.767369 | MSE=1.288263 | var=1.460616 | s_hat=644.131488 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.775192 | MSE=1.244943 | var=1.417931 | s_hat=622.471595 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.773751 | MSE=1.252922 | var=1.433549 | s_hat=626.460978 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.749858 | MSE=1.385237 | var=1.592227 | s_hat=692.618625 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.792220 | MSE=1.150642 | var=1.328686 | s_hat=575.321124 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.774274 | MSE=1.250027 | var=1.450148 | s_hat=625.013640 |
```

```
------------------------------------------------------------
>>> Degree = 35 : R2=0.685256 | MSE=1.742992 | var=2.031459 | s_hat=871.496068 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.747975 | MSE=1.395663 | var=1.634266 | s_hat=697.831722 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.779104 | MSE=1.223279 | var=1.439152 | s_hat=611.639642 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.779498 | MSE=1.221098 | var=1.443378 | s_hat=610.548956 |
------------------------------------------------------------
>>> Degree = 39 : R2=-0.009231 | MSE=5.588923 | var=6.637676 | s_hat=2794.461730 |
------------------------------------------------------------
>>> Degree = 40 : R2=-1.523583 | MSE=13.975108 | var=16.676740 | s_hat=6987.553970 |
------------------------------------------------------------
>>> Degree = 41 : R2=-0.307603 | MSE=7.241251 | var=8.682555 | s_hat=3620.625315 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.551223 | MSE=2.485240 | var=2.994265 | s_hat=1242.619798 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.762137 | MSE=1.317237 | var=1.594718 | s_hat=658.618460 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.737532 | MSE=1.453494 | var=1.768241 | s_hat=726.747124 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.611457 | MSE=2.151673 | var=2.630407 | s_hat=1075.836445 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.651850 | MSE=1.927985 | var=2.368532 | s_hat=963.992606 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.782312 | MSE=1.205515 | var=1.488291 | s_hat=602.757734 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.752382 | MSE=1.371261 | var=1.701317 | s_hat=685.630609 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.248283 | MSE=4.162859 | var=5.190597 | s_hat=2081.429510 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.742692 | MSE=1.424923 | var=1.785618 | s_hat=712.461505 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.763749 | MSE=1.308311 | var=1.647746 | s_hat=654.155302 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.742317 | MSE=1.427000 | var=1.806329 | s_hat=713.499951 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.596762 | MSE=2.233055 | var=2.841037 | s_hat=1116.527718 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.744846 | MSE=1.412995 | var=1.806899 | s_hat=706.497495 |
------------------------------------------------------------
>>> Degree = 55 : R2=0.138754 | MSE=4.769413 | var=6.130352 | s_hat=2384.706742 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.732106 | MSE=1.483547 | var=1.916727 | s_hat=741.773441 |
------------------------------------------------------------
>>> Degree = 57 : R2=0.710902 | MSE=1.600968 | var=2.079179 | s_hat=800.484029 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.150552 | MSE=4.704076 | var=6.141091 | s_hat=2352.038018 |
------------------------------------------------------------
>>> Degree = 59 : R2=-1.220707 | MSE=12.297839 | var=16.138897 | s_hat=6148.919649 |
------------------------------------------------------------
>>> Degree = 60 : R2=0.759676 | MSE=1.330869 | var=1.755764 | s_hat=665.434660 |
------------------------------------------------------------
>>> Degree = 61 : R2=0.466294 | MSE=2.955560 | var=3.919840 | s_hat=1477.779856 |
------------------------------------------------------------
>>> Degree = 62 : R2=0.742067 | MSE=1.428380 | var=1.904507 | s_hat=714.190245 |
------------------------------------------------------------
```

```
>>> Degree = 63 : R2=0.690543 | MSE=1.713711 | var=2.297200 | s_hat=856.855426 |
-------------------------------------------------------------
>>> Degree = 64 : R2=-5.513148 | MSE=36.068542 | var=48.609894 | s_hat=18034.270754 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 6
```

```
na = bestFitDegree;
nb = bestFitDegree;

p = na+nb+1;

BestFitModel = oe(data, [na nb 1])
```

```
BestFitModel =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -0.02 z^-1 + 0.06048 z^-2 + 0.04228 z^-3 - 0.04339 z^-4 - 0.09412 z^-5 + 0.1075 z^-6

  F(z) = 1 - 1.13 z^-1 - 0.7627 z^-2 + 0.3486 z^-3 + 0.8758 z^-4 - 0.00416 z^-5 - 0.3221 z^-6

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=6   nf=6   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data".
Fit to estimation data: 52.54%
FPE: 0.993, MSE: 0.9464
```

```
BestFit_y_hat = lsim(BestFitModel, u_val, t);
```

```
[oe_BestFit_r2, oe_BestFit_mse] = rSQR(y_val, BestFit_y_hat);


%%

fprintf("===============Degree Extraction | Variance Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n", minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 6 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 6
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb;

VarModel = oe(data, [na nb 1])
```

```
VarModel =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -0.02 z^-1 + 0.06048 z^-2 + 0.04228 z^-3 - 0.04339 z^-4 - 0.09412 z^-5 + 0.1075 z^-6

  F(z) = 1 - 1.13 z^-1 - 0.7627 z^-2 + 0.3486 z^-3 + 0.8758 z^-4 - 0.00416 z^-5 - 0.3221 z^-6

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=6   nf=6   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data".
Fit to estimation data: 52.54%
FPE: 0.993, MSE: 0.9464
```

```
Var_y_hat = lsim(VarModel, u_val, t);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("==============Degree Extraction | AIC Method===================\n")
```

```
==============Degree Extraction | AIC Method===================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=5.00) occurs in iteration 6 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 6
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb+1;

AICModel =  oe(data, [na nb 1])
```

```
AICModel =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -0.02 z^-1 + 0.06048 z^-2 + 0.04228 z^-3 - 0.04339 z^-4 - 0.09412 z^-5 + 0.1075 z^-6

  F(z) = 1 - 1.13 z^-1 - 0.7627 z^-2 + 0.3486 z^-3 + 0.8758 z^-4 - 0.00416 z^-5 - 0.3221 z^-6

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=6   nf=6   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data".
Fit to estimation data: 52.54%
FPE: 0.993, MSE: 0.9464
```

```
AIC_y_hat = lsim(AICModel, u_val, t);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test 
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", 
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.26 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 6
```

```


na = winner;
nb = winner;
p = na+nb+1;

FTestModel =  oe(data, [na nb 1])
```

```
FTestModel =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -0.02 z^-1 + 0.06048 z^-2 + 0.04228 z^-3 - 0.04339 z^-4 - 0.09412 z^-5 + 0.1075 z^-6

  F(z) = 1 - 1.13 z^-1 - 0.7627 z^-2 + 0.3486 z^-3 + 0.8758 z^-4 - 0.00416 z^-5 - 0.3221 z^-6

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=6   nf=6   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data".
```

```
Fit to estimation data: 52.54%
FPE: 0.993, MSE: 0.9464
```

```matlab
FTest_y_hat = lsim(FTestModel, u_val, t);

fprintf("=================================================================\n")
```

```
=================================================================
```

```matlab
%%

[BestFit_r2, BestFit_mse] = rSQR(y_val, BestFit_y_hat);
[Var_r2, Var_mse] = rSQR(y_val, Var_y_hat);
[AIC_r2, AIC_mse] = rSQR(y_val, AIC_y_hat);
[FTest_r2, FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```matlab
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```matlab
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```matlab
fprintf("    R2 value : %.4f   | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.8198   | MSE : 0.9981
```

```matlab
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```matlab
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```matlab
fprintf("    R2 value : %.4f   | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.8198   | MSE : 0.9981
```

```matlab
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```matlab
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.8198    | MSE : 0.9981
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.8198    | MSE : 0.9981
```

```
fprintf("---------------------------------------------------------------\n")
```

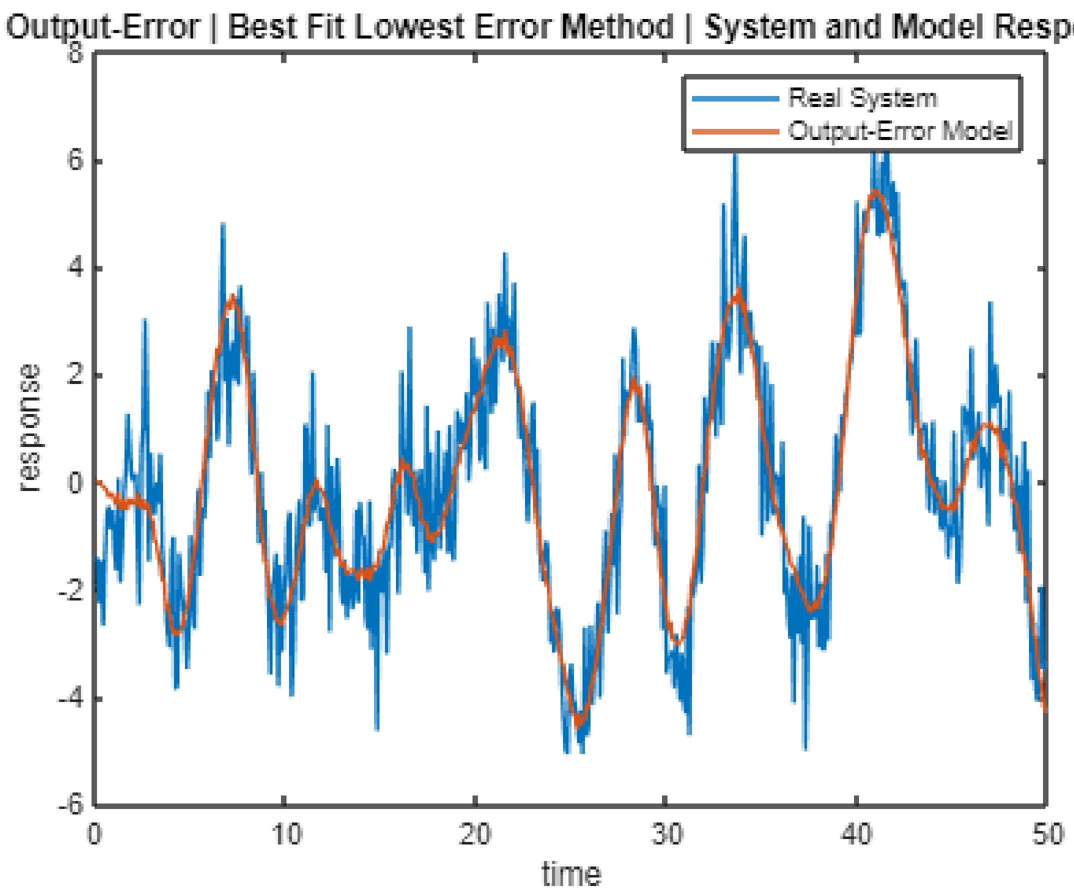
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

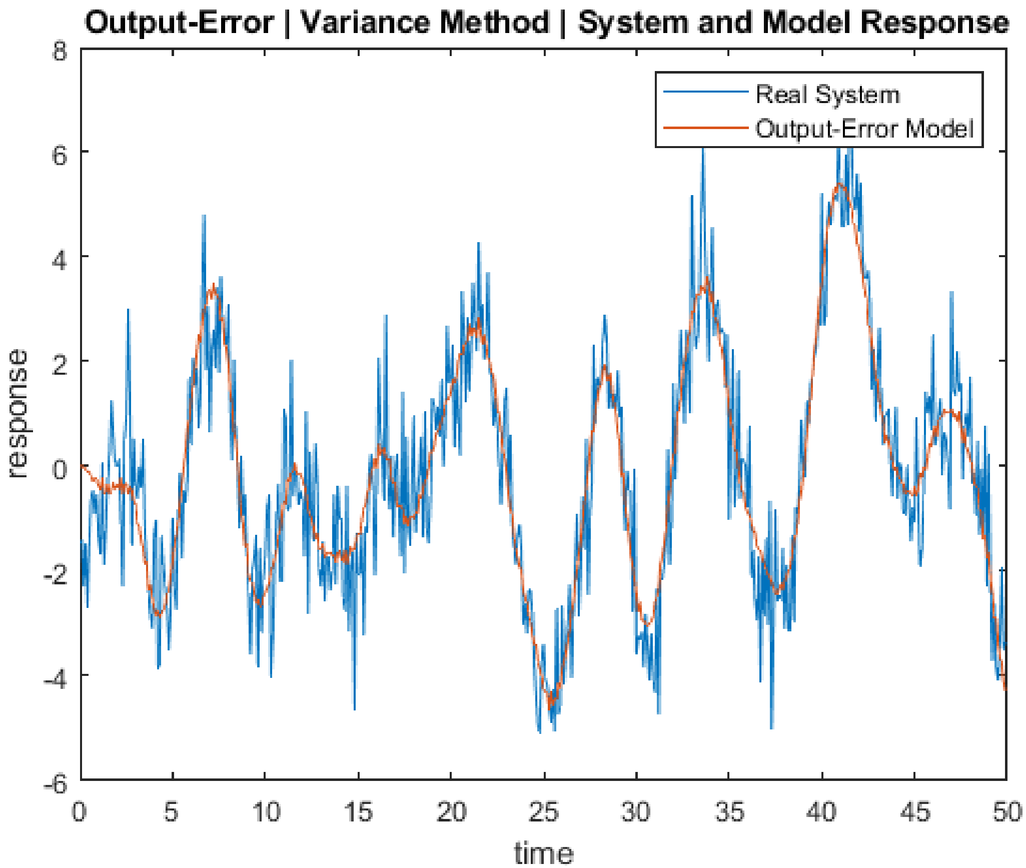
```
===============================================================
```

```



%%

BestFitError = y_val - BestFit_y_hat;
VarError = y_val - Var_y_hat;
AICError = y_val - AIC_y_hat;
FTestError = y_val - FTest_y_hat;

for k=0:N-1
    BestFit_Ree(k+1,1) = AutoCorrelate(BestFitError, k);
    Var_Ree(k+1,1) = AutoCorrelate(VarError, k);
    AIC_Ree(k+1,1) = AutoCorrelate(AICError, k);
    FTest_Ree(k+1,1) = AutoCorrelate(FTestError, k);
end

for k=0:N-1
    BestFit_Rue(k+1,1) = CrossCorrelate(u_val, BestFitError, k);
    Var_Rue(k+1,1) = CrossCorrelate(u_val, VarError, k);
    AIC_Rue(k+1,1) = CrossCorrelate(u_val, AICError, k);
    FTest_Rue(k+1,1) = CrossCorrelate(u_val, FTestError, k);
end
```

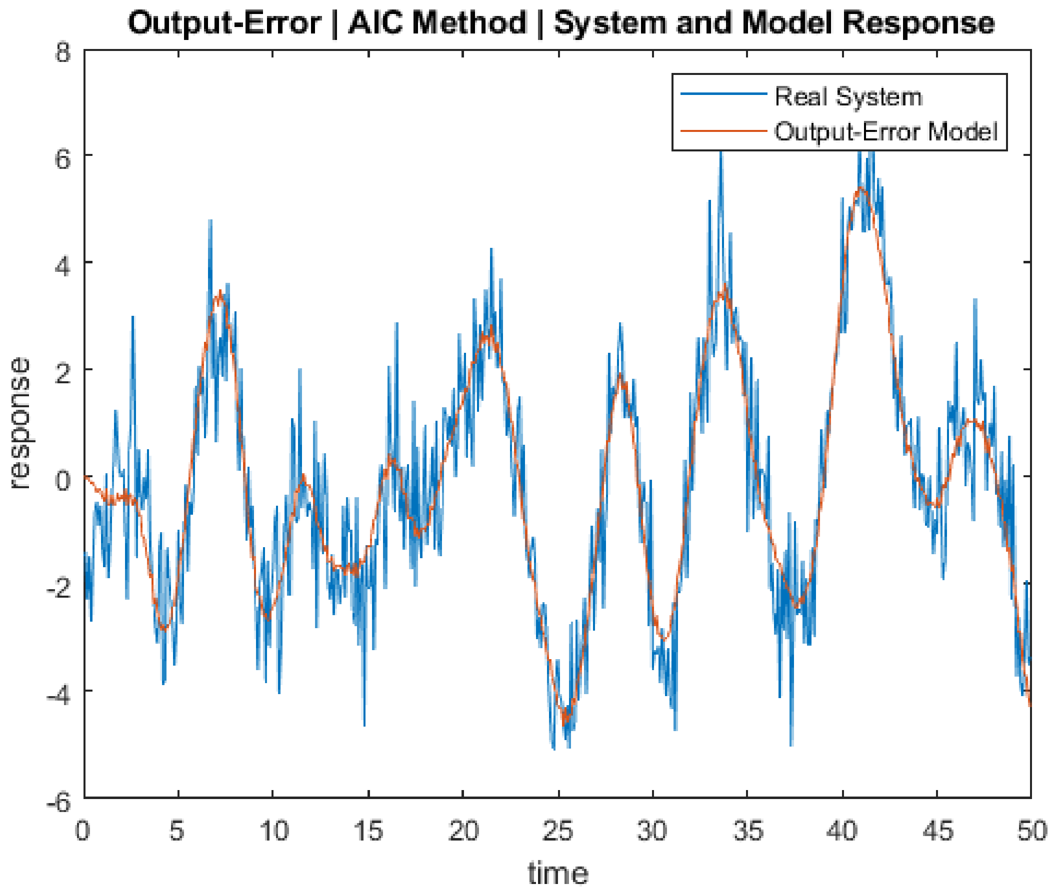
```
%%
figure(1)
plot(t,y_val,t,BestFit_y_hat)
legend('Real System','Output-Error Model')
title(" Output-Error | Best Fit Lowest Error Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



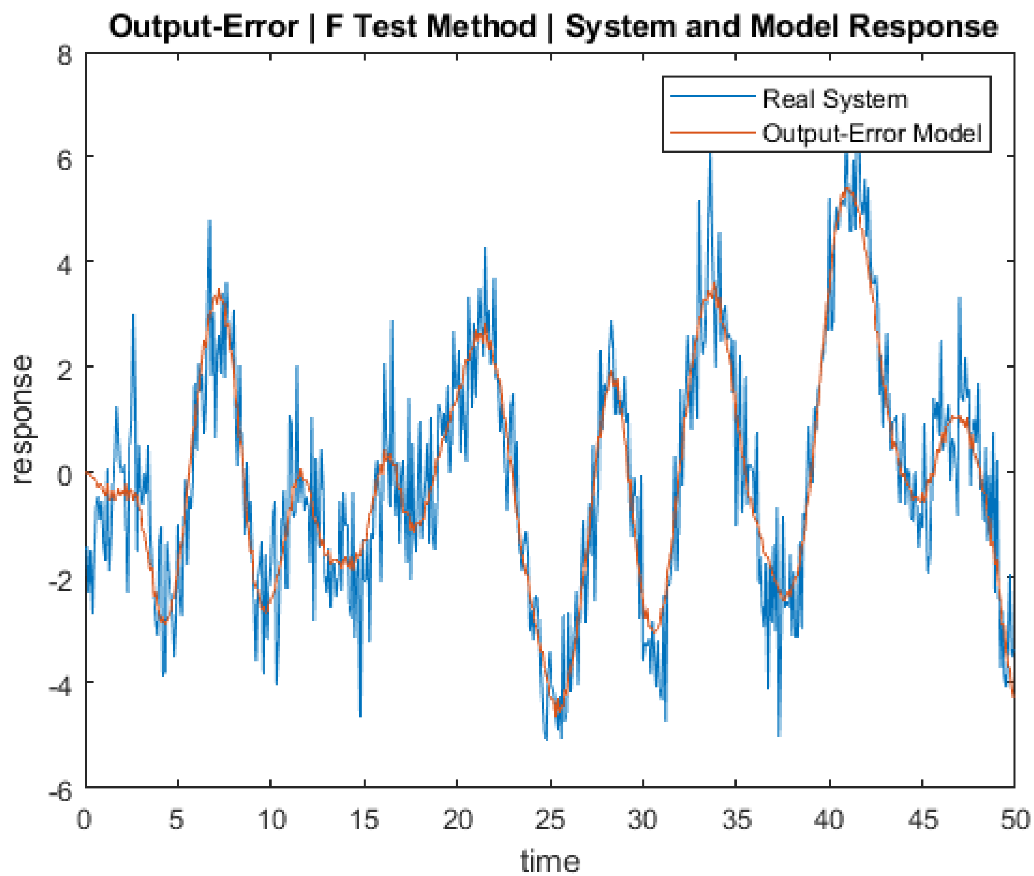

```
figure(2)
plot(t,y_val,t,Var_y_hat)
legend('Real System','Output-Error Model')
title(" Output-Error | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(3)
plot(t,y_val,t,AIC_y_hat)
legend('Real System','Output-Error Model')
title(" Output-Error | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

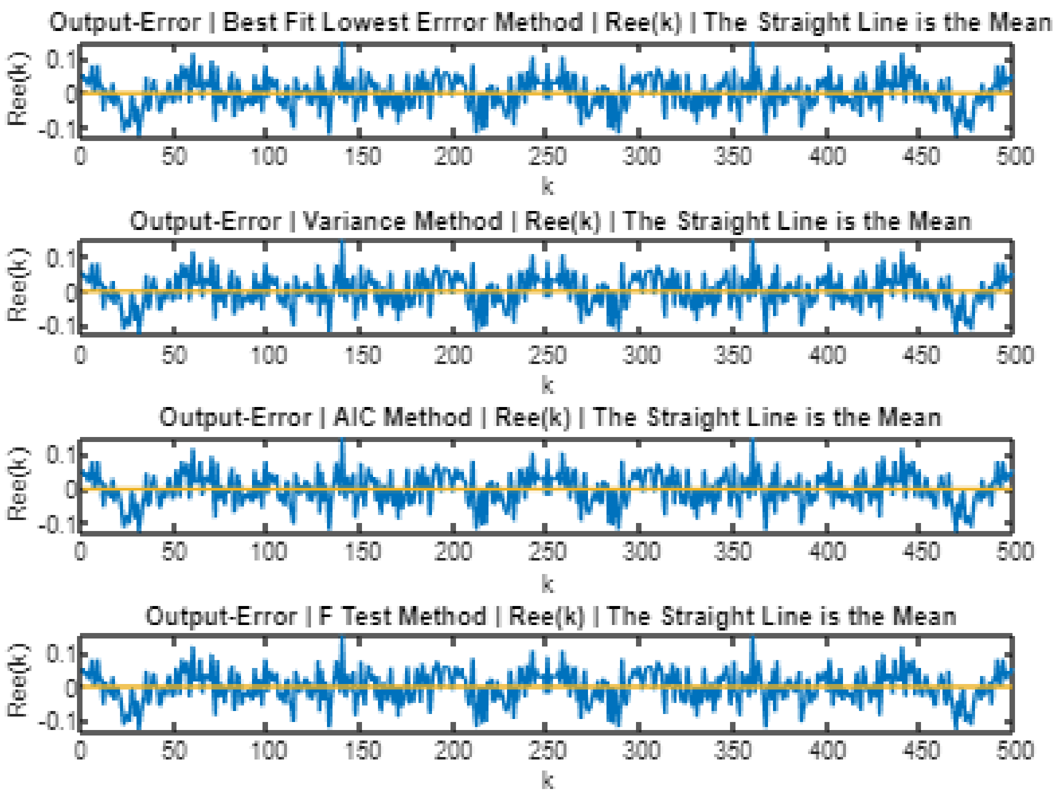



```
figure(4)
plot(t,y_val,t,FTest_y_hat)
legend('Real System','Output-Error Model')
title(" Output-Error | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



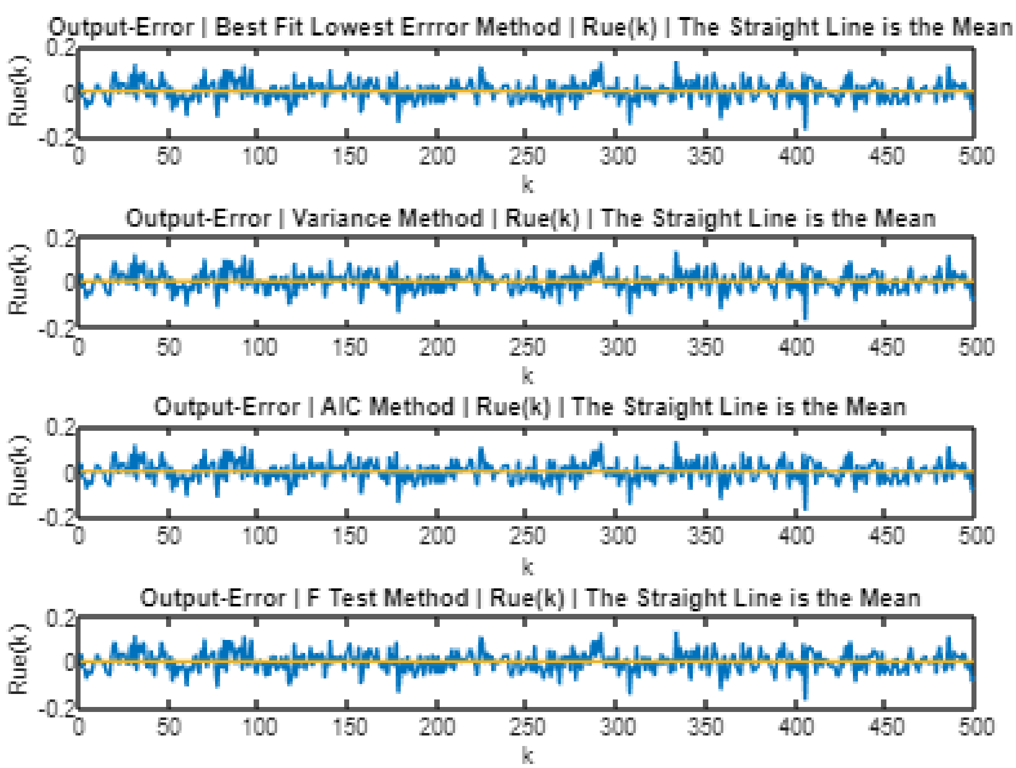

```
%%

figure(5)
subplot(4,1,1)
plot(1:N-1,BestFit_Ree(2:end), 1:N-1, mean(BestFit_Ree(2:end))*ones(length(1:N-
1)))
title(" Output-Error | Best Fit Lowest Errror Method | Ree(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,2)
plot(1:N-1,Var_Ree(2:end), 1:N-1, mean(Var_Ree(2:end))*ones(length(1:N-1)))
title(" Output-Error | Variance Method | Ree(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,3)
plot(1:N-1,AIC_Ree(2:end), 1:N-1, mean(AIC_Ree(2:end))*ones(length(1:N-1)))
```

```
title(" Output-Error | AIC Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,4)
plot(1:N-1,FTest_Ree(2:end), 1:N-1, mean(FTest_Ree(2:end))*ones(length(1:N-1)))
title(" Output-Error | F Test Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N-1,BestFit_Rue(2:end), 1:N-1, mean(BestFit_Rue(2:end))*ones(length(1:N-
1)))
title(" Output-Error | Best Fit Lowest Errror Method | Rue(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Rue(k)")
```

```
subplot(4,1,2)
plot(1:N-1,Var_Rue(2:end), 1:N-1, mean(Var_Rue(2:end))*ones(length(1:N-1)))
title(" Output-Error | Variance Method | Rue(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(4,1,3)
plot(1:N-1,AIC_Rue(2:end), 1:N-1, mean(AIC_Rue(2:end))*ones(length(1:N-1)))
title(" Output-Error | AIC Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(4,1,4)
plot(1:N-1,FTest_Rue(2:end), 1:N-1, mean(FTest_Rue(2:end))*ones(length(1:N-1)))
title(" Output-Error | F Test Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")
```



```
%%
```

Q1 - part c | RELS algorithm - Not Good at all

```
clear; clc;
%

load q1_402123100.mat

y_tilda = y-v;

u_val = u(length(u)/2+1:end);
v_val = v(length(u)/2+1:end);
z_val = z(length(u)/2+1:end);
y_val = y(length(u)/2+1:end);
y_tilda_val = y_tilda(length(u)/2+1:end);

u = u(1:length(u)/2);
v = v(1:length(v)/2);
z = z(1:length(z)/2);
y = y(1:length(y)/2);
y_tilda = y_tilda(1:length(y)/2);

Ts = 0.1;
t = 0:Ts:length(u)*Ts-Ts;
N = length(y);
data = iddata(y,u,Ts);


fprintf("===============Degree Extraction | Best Fit Lowest Error
Method====================\n")
```

```
===============Degree Extraction | Best Fit Lowest Error Method====================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 1;
iteration_number = 2000;
for degree=1:1:100
    na = degree;
    nb = degree;
    nc = degree;
```

```
    p = na+nb+nc+1;

%     UU = armax_error_U_builder(na,v);
%     dyn = inv(UU'*UU)*UU'*v;
%     vv = UU*dyn;

%     armax_U = armax_U_builder(na,nb,nc,u,y,z);
%     armax_theta_hat = inv(armax_U'*armax_U)*armax_U'*y;

    Theta0 = ones(p,1)*0.1;
    P0 = 100*eye(p);
    armax_theta_hat = run_iterative_armax(u, y, v, na, nb, nc, p, P0, Theta0,
iteration_number);

    armax_y_hat = form_tf_lsim(armax_theta_hat, u, t, na, Ts);

%     armax_y_hat = armax_U*armax_theta_hat;

    [r2_armax, mse_armax] = rSQR(y, armax_y_hat);

    error = y - armax_y_hat;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

%     covariance = variance*inv(armax_U'*armax_U);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f |\n", degree, r2_armax,
mse_armax, variance)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
    R2s = [R2s; r2_armax];
    MSEs = [MSEs; mse_armax];
%     dets = [dets; detUTU];
    vars = [vars; variance];

end
```

```
>>> Degree = 1 : R2=-0.024974 | MSE=4.307454 | var=4.342191 |
```

```
------------------------------------------------------------
>>> Degree = 2 : R2=-0.020169 | MSE=4.287258 | var=4.348132 |
------------------------------------------------------------
>>> Degree = 3 : R2=-0.020355 | MSE=4.288043 | var=4.375554 |
------------------------------------------------------------
>>> Degree = 4 : R2=-0.017210 | MSE=4.274823 | var=4.388936 |
------------------------------------------------------------
>>> Degree = 5 : R2=-0.009771 | MSE=4.243564 | var=4.383847 |
------------------------------------------------------------
>>> Degree = 6 : R2=0.007881 | MSE=4.169381 | var=4.334076 |
------------------------------------------------------------
>>> Degree = 7 : R2=0.007050 | MSE=4.172873 | var=4.364930 |
------------------------------------------------------------
>>> Degree = 8 : R2=0.019927 | MSE=4.118758 | var=4.335535 |
------------------------------------------------------------
>>> Degree = 9 : R2=0.039040 | MSE=4.038435 | var=4.278004 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.039905 | MSE=4.034801 | var=4.301494 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.067421 | MSE=3.919164 | var=4.205112 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.095761 | MSE=3.800063 | var=4.103740 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.124325 | MSE=3.680026 | var=4.000028 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.148848 | MSE=3.576964 | var=3.913528 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.164396 | MSE=3.511628 | var=3.867431 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.203824 | MSE=3.345931 | var=3.709458 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.237374 | MSE=3.204935 | var=3.576936 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.302701 | MSE=2.930401 | var=3.292585 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.307031 | MSE=2.912203 | var=3.294347 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.388797 | MSE=2.568582 | var=2.925491 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.431625 | MSE=2.388597 | var=2.739216 |
------------------------------------------------------------
>>> Degree = 22 : R2=0.475789 | MSE=2.202998 | var=2.543877 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.511954 | MSE=2.051014 | var=2.384900 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.550548 | MSE=1.888822 | var=2.211735 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.578099 | MSE=1.773040 | var=2.090850 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.604034 | MSE=1.664049 | var=1.976305 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.629947 | MSE=1.555148 | var=1.860225 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.650407 | MSE=1.469165 | var=1.770079 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.665919 | MSE=1.403974 | var=1.703852 |
------------------------------------------------------------
```

```
>>> Degree = 30 : R2=0.679151 | MSE=1.348366 | var=1.648370 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.688302 | MSE=1.309911 | var=1.613191 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.699446 | MSE=1.263080 | var=1.567097 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.706867 | MSE=1.231889 | var=1.539862 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.712695 | MSE=1.207401 | var=1.520656 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.718259 | MSE=1.184018 | var=1.502561 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.722864 | MSE=1.164666 | var=1.489342 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.726544 | MSE=1.149198 | var=1.480925 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.729811 | MSE=1.135471 | var=1.474637 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.730827 | MSE=1.131199 | var=1.480626 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.733868 | MSE=1.118418 | var=1.475486 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.735432 | MSE=1.111845 | var=1.478518 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.735613 | MSE=1.111087 | var=1.489392 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.736277 | MSE=1.108297 | var=1.497698 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.737704 | MSE=1.102298 | var=1.501768 |
------------------------------------------------------------
>>> Degree = 45 : R2=-0.240020 | MSE=5.211183 | var=7.158219 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.735458 | MSE=1.111739 | var=1.539804 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.734340 | MSE=1.116438 | var=1.559270 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.734462 | MSE=1.115924 | var=1.571724 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.731269 | MSE=1.129343 | var=1.604181 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.729904 | MSE=1.135079 | var=1.626188 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.727835 | MSE=1.143774 | var=1.652853 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.725493 | MSE=1.153617 | var=1.681657 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.722953 | MSE=1.164289 | var=1.712189 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.720380 | MSE=1.175103 | var=1.743477 |
------------------------------------------------------------
>>> Degree = 55 : R2=0.719372 | MSE=1.179338 | var=1.765477 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.718886 | MSE=1.181382 | var=1.784565 |
------------------------------------------------------------
>>> Degree = 57 : R2=0.711049 | MSE=1.214316 | var=1.851092 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.706706 | MSE=1.232567 | var=1.896257 |
```

```
------------------------------------------------------------
>>> Degree = 59 : R2=0.706793 | MSE=1.232201 | var=1.913355 |
------------------------------------------------------------
>>> Degree = 60 : R2=0.694830 | MSE=1.282477 | var=2.010151 |
------------------------------------------------------------
>>> Degree = 61 : R2=0.686849 | MSE=1.316015 | var=2.082303 |
------------------------------------------------------------
>>> Degree = 62 : R2=0.676536 | MSE=1.359356 | var=2.171495 |
------------------------------------------------------------
>>> Degree = 63 : R2=0.665380 | MSE=1.406242 | var=2.268133 |
------------------------------------------------------------
>>> Degree = 64 : R2=0.650179 | MSE=1.470121 | var=2.394334 |
------------------------------------------------------------
>>> Degree = 65 : R2=0.626037 | MSE=1.571578 | var=2.584832 |
------------------------------------------------------------
>>> Degree = 66 : R2=0.608942 | MSE=1.643419 | var=2.729932 |
------------------------------------------------------------
>>> Degree = 67 : R2=0.586715 | MSE=1.736831 | var=2.914146 |
------------------------------------------------------------
>>> Degree = 68 : R2=0.536445 | MSE=1.948089 | var=3.301846 |
------------------------------------------------------------
>>> Degree = 69 : R2=0.523120 | MSE=2.004089 | var=3.431659 |
------------------------------------------------------------
>>> Degree = 70 : R2=0.494285 | MSE=2.125268 | var=3.676934 |
------------------------------------------------------------
>>> Degree = 71 : R2=0.471396 | MSE=2.221459 | var=3.883669 |
------------------------------------------------------------
>>> Degree = 72 : R2=0.450377 | MSE=2.309793 | var=4.080906 |
------------------------------------------------------------
>>> Degree = 73 : R2=0.435760 | MSE=2.371220 | var=4.234321 |
------------------------------------------------------------
>>> Degree = 74 : R2=0.429728 | MSE=2.396569 | var=4.325937 |
------------------------------------------------------------
>>> Degree = 75 : R2=0.468459 | MSE=2.233799 | var=4.076276 |
------------------------------------------------------------
>>> Degree = 76 : R2=0.440194 | MSE=2.352585 | var=4.340562 |
------------------------------------------------------------
>>> Degree = 77 : R2=0.444595 | MSE=2.334088 | var=4.354641 |
------------------------------------------------------------
>>> Degree = 78 : R2=0.440247 | MSE=2.352362 | var=4.438419 |
------------------------------------------------------------
>>> Degree = 79 : R2=0.442903 | MSE=2.341198 | var=4.467935 |
------------------------------------------------------------
>>> Degree = 80 : R2=0.413644 | MSE=2.464160 | var=4.757065 |
------------------------------------------------------------
>>> Degree = 81 : R2=0.371898 | MSE=2.639598 | var=5.155465 |
------------------------------------------------------------
>>> Degree = 82 : R2=0.340333 | MSE=2.772252 | var=5.478759 |
------------------------------------------------------------
>>> Degree = 83 : R2=0.259104 | MSE=3.113615 | var=6.227230 |
------------------------------------------------------------
>>> Degree = 84 : R2=0.209909 | MSE=3.320359 | var=6.721374 |
------------------------------------------------------------
>>> Degree = 85 : R2=0.197033 | MSE=3.374469 | var=6.914895 |
------------------------------------------------------------
>>> Degree = 86 : R2=0.150346 | MSE=3.570673 | var=7.408035 |
------------------------------------------------------------
```

```
>>> Degree = 87 : R2=0.107942 | MSE=3.748875 | var=7.875788 |
-------------------------------------------------------------
>>> Degree = 88 : R2=-1133300.321004 | MSE=4762698.945151 | var=10133402.010959 |
-------------------------------------------------------------
>>> Degree = 89 : R2=0.079125 | MSE=3.869979 | var=8.340471 |
-------------------------------------------------------------
>>> Degree = 90 : R2=0.097412 | MSE=3.793125 | var=8.281931 |
-------------------------------------------------------------
>>> Degree = 91 : R2=0.107967 | MSE=3.748768 | var=8.293733 |
-------------------------------------------------------------
>>> Degree = 92 : R2=0.097382 | MSE=3.793251 | var=8.505046 |
-------------------------------------------------------------
>>> Degree = 93 : R2=0.199196 | MSE=3.365381 | var=7.648593 |
-------------------------------------------------------------
>>> Degree = 94 : R2=-7.828108 | MSE=37.100127 | var=85.484163 |
-------------------------------------------------------------
>>> Degree = 95 : R2=0.241451 | MSE=3.187800 | var=7.448132 |
-------------------------------------------------------------
>>> Degree = 96 : R2=0.298598 | MSE=2.947641 | var=6.984931 |
-------------------------------------------------------------
>>> Degree = 97 : R2=0.483635 | MSE=2.170024 | var=5.216404 |
-------------------------------------------------------------
>>> Degree = 98 : R2=0.538744 | MSE=1.938427 | var=4.727871 |
-------------------------------------------------------------
>>> Degree = 99 : R2=0.559782 | MSE=1.850018 | var=4.579253 |
-------------------------------------------------------------
>>> Degree = 100 : R2=0.540271 | MSE=1.932013 | var=4.854304 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));
fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 44
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
```

```
p = na+nb+nc+1;

% UU = armax_error_U_builder(na,v);
% dyn = inv(UU'*UU)*UU'*v;
% v = UU*dyn;
%
% armax_U = armax_U_builder(na,nb,nc,u,y,v);
% BestFitModel = inv(armax_U'*armax_U)*armax_U'*y;

Theta0 = zeros(p,1);
P0 = 100*eye(p);
BestFitModel = run_iterative_armax(u, y, v, na, nb, nc, p, P0, Theta0,
iteration_number);

BestFit_y_hat = form_tf_lsim(BestFitModel, u_val, t, na, Ts);
```

```
G =


  -0.003182 z^44 - 0.0002734 z^43 + 0.00382 z^42 + 0.009519 z^41 + 0.01703 z^40 + 0.0249 z^39

          + 0.03109 z^38 + 0.03747 z^37 + 0.04268 z^36 + 0.04622 z^35 + 0.04887 z^34 + 0.05066
z^33

          + 0.05009 z^32 + 0.04907 z^31 + 0.04604 z^30 + 0.0421 z^29 + 0.03908 z^28 + 0.03478
z^27

          + 0.02943 z^26 + 0.02434 z^25 + 0.02006 z^24 + 0.01522 z^23 + 0.01183 z^22 + 0.00807
z^21

          + 0.004754 z^20 + 0.0008978 z^19 - 0.001177 z^18 - 0.004088 z^17 - 0.005029 z^16

          - 0.006212 z^15 - 0.006278 z^14 - 0.007282 z^13 - 0.007331 z^12 - 0.008926 z^11 -
0.009125 z^10

          - 0.008247 z^9 - 0.006524 z^8 - 0.006547 z^7 - 0.004442 z^6 - 0.005231 z^5 -
0.005942 z^4

                                                - 0.004548 z^3 - 0.003118 z^2 - 0.001927 z -
0.001169

  ---------------------------------------------------------------------------------------------
------------

   z^44 - 0.5952 z^43 - 0.08523 z^42 - 0.07469 z^41 - 0.06434 z^40 - 0.0542 z^39 - 0.04472
z^38

           - 0.03528 z^37 - 0.02706 z^36 - 0.01939 z^35 - 0.01279 z^34 - 0.006822 z^33 -
0.001816 z^32

           + 0.002075 z^31 + 0.004675 z^30 + 0.007442 z^29 + 0.008499 z^28 + 0.01006 z^27 +
0.01038 z^26
```

```
           + 0.01014 z^25 + 0.009466 z^24 + 0.008779 z^23 + 0.007565 z^22 + 0.00692 z^21 + 
0.005597 z^20

           + 0.004413 z^19 + 0.003153 z^18 + 0.001636 z^17 + 0.0004591 z^16 - 0.0002848 z^15

           - 0.0008893 z^14 - 0.001326 z^13 - 0.001412 z^12 - 0.001629 z^11 - 0.001631 z^10

           - 0.001518 z^9 - 0.001469 z^8 - 0.001026 z^7 - 0.00182 z^6 - 0.001655 z^5 - 
0.001523 z^4

                                                  - 0.0009019 z^3 - 0.001131 z^2 - 0.001274 z - 
0.002019


Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
%%

fprintf("===============Degree Extraction | Variance Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars),1);
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n", minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 38 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 38
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
p = na+nb+nc+1;

% UU = armax_error_U_builder(na,v);
% dyn = inv(UU'*UU)*UU'*v;
% v = UU*dyn;
%
% armax_VarU = armax_U_builder(na,nb,nc,u,y,v);
% armax_VarModel = inv(armax_VarU'*armax_VarU)*armax_VarU'*y;

Theta0 = zeros(p,1);
P0 = 100*eye(p);
```

```
armax_VarModel = run_iterative_armax(u, y, v, na, nb, nc, p, P0, Theta0,
iteration_number);

Var_y_hat = form_tf_lsim(armax_VarModel, u_val, t, na, Ts);
```

```
G =


  -0.002032 z^38 + 0.0004285 z^37 + 0.004222 z^36 + 0.009968 z^35 + 0.01699 z^34 + 0.02466
z^33

          + 0.03123 z^32 + 0.03788 z^31 + 0.04301 z^30 + 0.04646 z^29 + 0.04878 z^28 + 0.0502
z^27

          + 0.04936 z^26 + 0.04812 z^25 + 0.0453 z^24 + 0.04177 z^23 + 0.03843 z^22 + 0.03433
z^21

          + 0.02929 z^20 + 0.02467 z^19 + 0.02077 z^18 + 0.01658 z^17 + 0.01324 z^16 +
0.009954 z^15

          + 0.007259 z^14 + 0.004336 z^13 + 0.003096 z^12 + 0.0007689 z^11 + 0.0001319 z^10

          - 0.000559 z^9 - 0.000746 z^8 - 0.001153 z^7 - 0.0007847 z^6 - 0.001501 z^5 -
0.0006634 z^4

                                             - 0.0002338 z^3 + 0.0002181 z^2 - 0.0005845 z +
0.0004888

  --------------------------------------------------------------------------------------------
---------

  z^38 - 0.596 z^37 - 0.08581 z^36 - 0.07514 z^35 - 0.06474 z^34 - 0.05443 z^33 - 0.04472 z^32

          - 0.03518 z^31 - 0.02657 z^30 - 0.01866 z^29 - 0.01192 z^28 - 0.005763 z^27 -
0.0006374 z^26

          + 0.003524 z^25 + 0.006403 z^24 + 0.009111 z^23 + 0.01034 z^22 + 0.01161 z^21 +
0.01179 z^20

          + 0.01147 z^19 + 0.0108 z^18 + 0.009935 z^17 + 0.008566 z^16 + 0.007537 z^15 +
0.00558 z^14

          + 0.004084 z^13 + 0.002277 z^12 + 0.0006144 z^11 - 0.0008767 z^10 - 0.002009 z^9

          - 0.003111 z^8 - 0.003984 z^7 - 0.004311 z^6 - 0.004362 z^5 - 0.003952 z^4 -
0.002929 z^3

                                                              - 0.001606 z^2 + 0.0005525 z +
0.002823


Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("==================================================================\n")
```

```
==================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=1.00) occurs in iteration 36 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 36
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
p = na+nb+nc+1;

% UU = armax_error_U_builder(na,v);
% dyn = inv(UU'*UU)*UU'*v;
% v = UU*dyn;
%
% armax_AICU = armax_U_builder(na,nb,nc,u,y,v);
% armax_AICModel = inv(armax_AICU'*armax_AICU)*armax_AICU'*y;

Theta0 = zeros(p,1);
P0 = 100*eye(p);
armax_AICModel = run_iterative_armax(u, y, v, na, nb, nc, p, P0, Theta0,
iteration_number);


AIC_y_hat = form_tf_lsim(armax_AICModel, u_val, t, na, Ts);
```

```
G =


  -0.005213 z^36 - 0.002721 z^35 + 0.000617 z^34 + 0.00662 z^33 + 0.01279 z^32 + 0.02082 z^31
```

```
           + 0.02729 z^30 + 0.0347 z^29 + 0.04007 z^28 + 0.04194 z^27 + 0.04415 z^26 + 0.04482
z^25

           + 0.04385 z^24 + 0.044 z^23 + 0.04347 z^22 + 0.03963 z^21 + 0.03666 z^20 + 0.03249
z^19

           + 0.02664 z^18 + 0.02458 z^17 + 0.0199 z^16 + 0.0153 z^15 + 0.01361 z^14 + 0.01053
z^13

           + 0.009671 z^12 + 0.008087 z^11 + 0.008273 z^10 + 0.006207 z^9 + 0.009646 z^8 +
0.008254 z^7

           + 0.009693 z^6 + 0.008453 z^5 + 0.007261 z^4 + 0.003702 z^3 + 0.002464 z^2 +
0.001336 z

                                                                                         +
0.0004916

  ---------------------------------------------------------------------------------------------
-----------

  z^36 - 0.5884 z^35 - 0.07797 z^34 - 0.0686 z^33 - 0.05872 z^32 - 0.05006 z^31 - 0.04176 z^30

          - 0.03307 z^29 - 0.02615 z^28 - 0.01908 z^27 - 0.01385 z^26 - 0.007075 z^25 -
0.002975 z^24

          + 0.003026 z^23 + 0.005214 z^22 + 0.00833 z^21 + 0.008729 z^20 + 0.01007 z^19 +
0.01018 z^18

          + 0.009549 z^17 + 0.00882 z^16 + 0.006899 z^15 + 0.006088 z^14 + 0.004746 z^13 +
0.004658 z^12

          + 0.002487 z^11 + 0.0008741 z^10 - 0.0008861 z^9 - 0.003794 z^8 - 0.004544 z^7 -
0.005758 z^6

                   - 0.006197 z^5 - 0.005812 z^4 - 0.002141 z^3 + 0.0008793 z^2 + 0.006673 z +
0.0129


Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("============================================================\n")
```

```
=================================================================
```

```
%%
fprintf("===============Degree Extraction | F test Method===================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.22 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 36
```

```
na = winner;
nb = winner;
nc = winner;
p = na+nb+nc+1;

% UU = armax_error_U_builder(na,v);
% dyn = inv(UU'*UU)*UU'*v;
% v = UU*dyn;
%
% armax_FTestU = armax_U_builder(na,nb,nc,u,y,v);
% armax_FTestModel = inv(armax_FTestU'*armax_FTestU)*armax_FTestU'*y;

Theta0 = zeros(p,1);
P0 = 100*eye(p);
armax_FTestModel = run_iterative_armax(u, y, v, na, nb, nc, p, P0, Theta0,
iteration_number);

FTest_y_hat = form_tf_lsim(armax_FTestModel, u_val, t, na, Ts);
```

```
G =

   -0.005213 z^36 - 0.002721 z^35 + 0.000617 z^34 + 0.00662 z^33 + 0.01279 z^32 + 0.02082 z^31

          + 0.02729 z^30 + 0.0347 z^29 + 0.04007 z^28 + 0.04194 z^27 + 0.04415 z^26 + 0.04482
z^25
```

```
           + 0.04385 z^24 + 0.044 z^23 + 0.04347 z^22 + 0.03963 z^21 + 0.03666 z^20 + 0.03249
z^19

           + 0.02664 z^18 + 0.02458 z^17 + 0.0199 z^16 + 0.0153 z^15 + 0.01361 z^14 + 0.01053
z^13

           + 0.009671 z^12 + 0.008087 z^11 + 0.008273 z^10 + 0.006207 z^9 + 0.009646 z^8 +
0.008254 z^7

           + 0.009693 z^6 + 0.008453 z^5 + 0.007261 z^4 + 0.003702 z^3 + 0.002464 z^2 +
0.001336 z

                                                                                         +
0.0004916

  -------------------------------------------------------------------------------------------
-----------

  z^36 - 0.5884 z^35 - 0.07797 z^34 - 0.0686 z^33 - 0.05872 z^32 - 0.05006 z^31 - 0.04176 z^30

          - 0.03307 z^29 - 0.02615 z^28 - 0.01908 z^27 - 0.01385 z^26 - 0.007075 z^25 -
0.002975 z^24

          + 0.003026 z^23 + 0.005214 z^22 + 0.00833 z^21 + 0.008729 z^20 + 0.01007 z^19 +
0.01018 z^18

          + 0.009549 z^17 + 0.00882 z^16 + 0.006899 z^15 + 0.006088 z^14 + 0.004746 z^13 +
0.004658 z^12

          + 0.002487 z^11 + 0.0008741 z^10 - 0.0008861 z^9 - 0.003794 z^8 - 0.004544 z^7 -
0.005758 z^6

                   - 0.006197 z^5 - 0.005812 z^4 - 0.002141 z^3 + 0.0008793 z^2 + 0.006673 z +
0.0129


Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("============================================================================\n")
```

```
=================================================================
```

```
%%

[BestFit_r2, BestFit_mse] = rSQR(y_val, BestFit_y_hat);
[Var_r2, Var_mse] = rSQR(y_val, Var_y_hat);
[AIC_r2, AIC_mse] = rSQR(y_val, AIC_y_hat);
[FTest_r2, FTest_mse] = rSQR(y_val, FTest_y_hat);
```

```
fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.5670   | MSE : 2.3981
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.5517   | MSE : 2.4824
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.4711   | MSE : 2.9288
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.4711   | MSE : 2.9288
```

```
fprintf("---------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> Winner:\n")
```

```
>>> Winner:
```

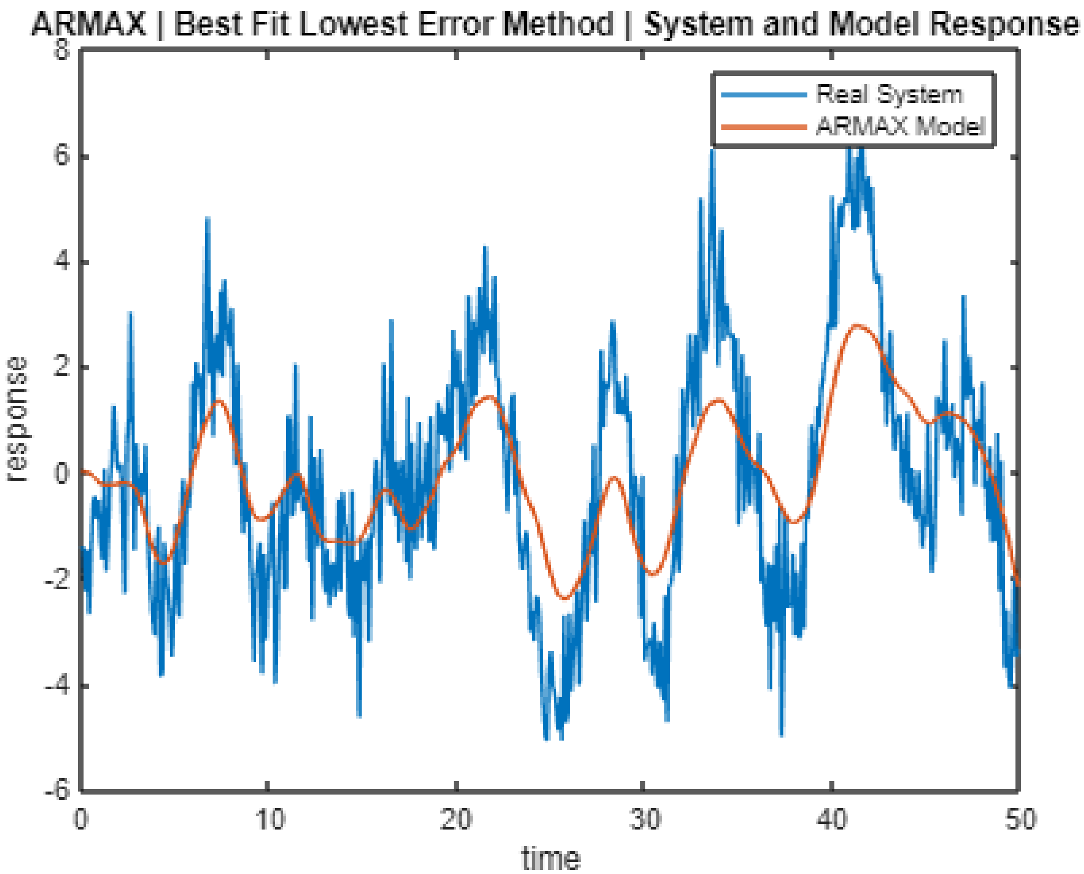
```
fprintf("    The best R2 value is \n")
```

```
    The best R2 value is
```

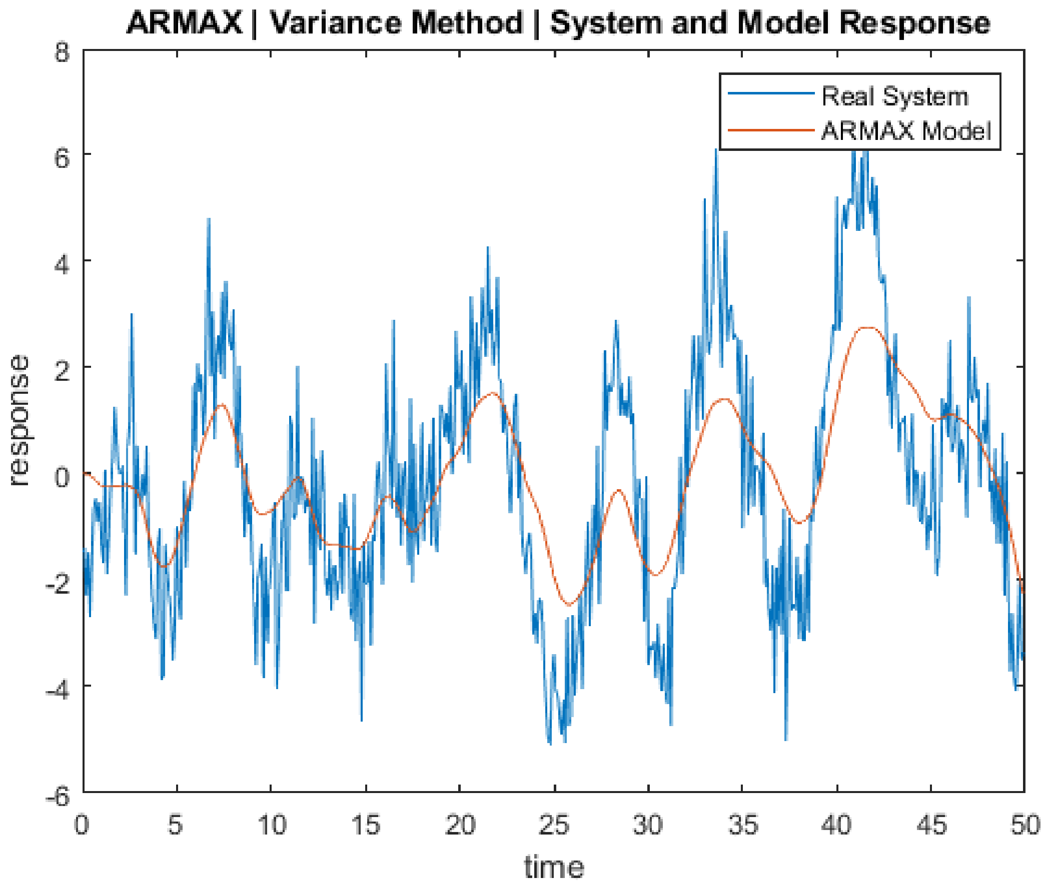
```
fprintf("================================================================\n")
```

```
============================================================
```

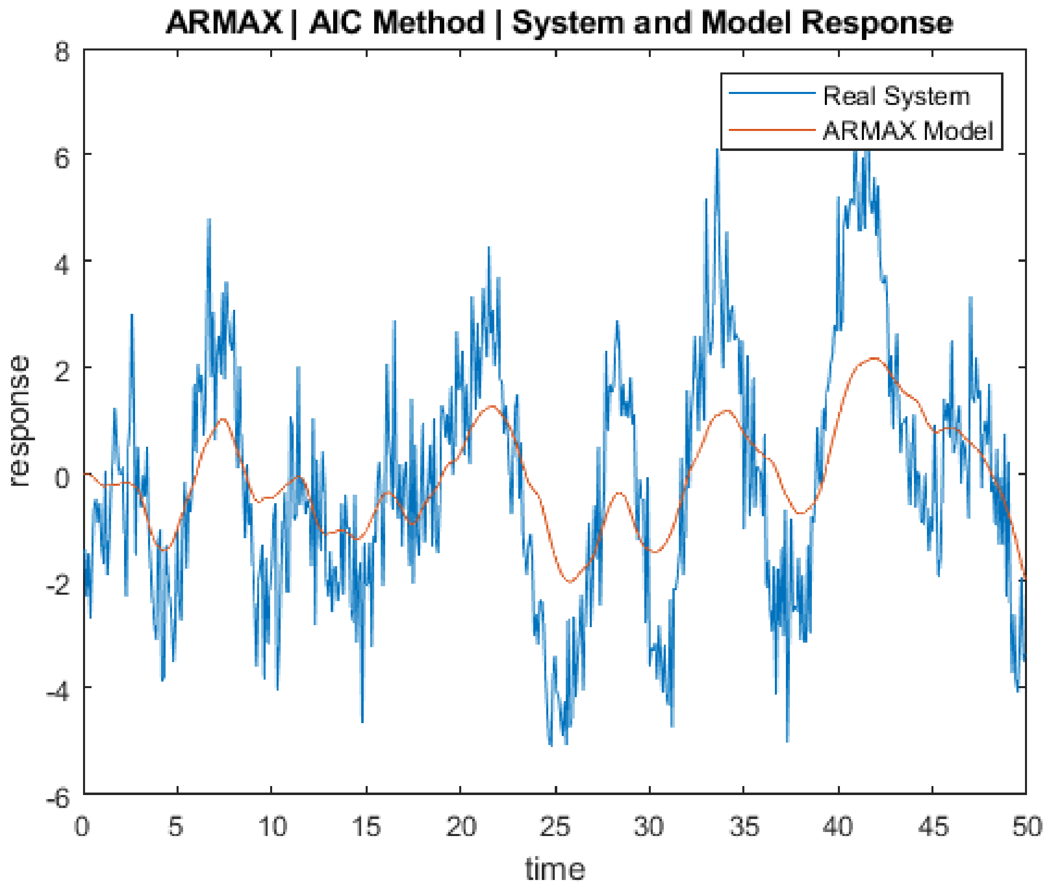
```
%%

armax_BestFitError = y_val - BestFit_y_hat;
armax_VarError = y_val - Var_y_hat;
armax_AICError = y_val - AIC_y_hat;
armax_FTestError = y_val - FTest_y_hat;

for k=0:N-1
    armax_BestFit_Ree(k+1,1) = AutoCorrelate(armax_BestFitError, k);
    armax_Var_Ree(k+1,1) = AutoCorrelate(armax_VarError, k);
    armax_AIC_Ree(k+1,1) = AutoCorrelate(armax_AICError, k);
    armax_FTest_Ree(k+1,1) = AutoCorrelate(armax_FTestError, k);
end

for k=0:N-1
    armax_BestFit_Rue(k+1,1) = CrossCorrelate(u_val, armax_BestFitError, k);
    armax_Var_Rue(k+1,1) = CrossCorrelate(u_val, armax_VarError, k);
    armax_AIC_Rue(k+1,1) = CrossCorrelate(u_val, armax_AICError, k);
    armax_FTest_Rue(k+1,1) = CrossCorrelate(u_val, armax_FTestError, k);
end

%%
figure(1)
plot(t,y_val,t,BestFit_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | Best Fit Lowest Error Method | System and Model Response")
xlabel("time")
ylabel("response")
```



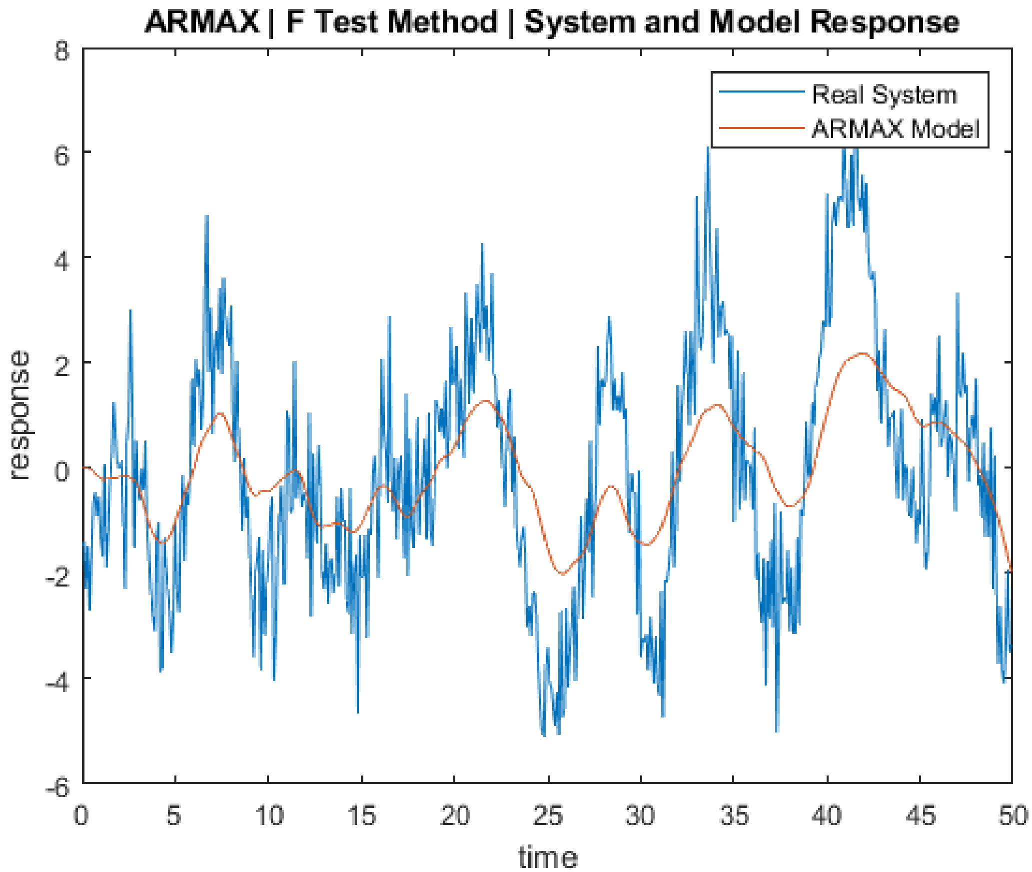

```
figure(2)
plot(t,y_val,t,Var_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(3)
plot(t,y_val,t,AIC_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

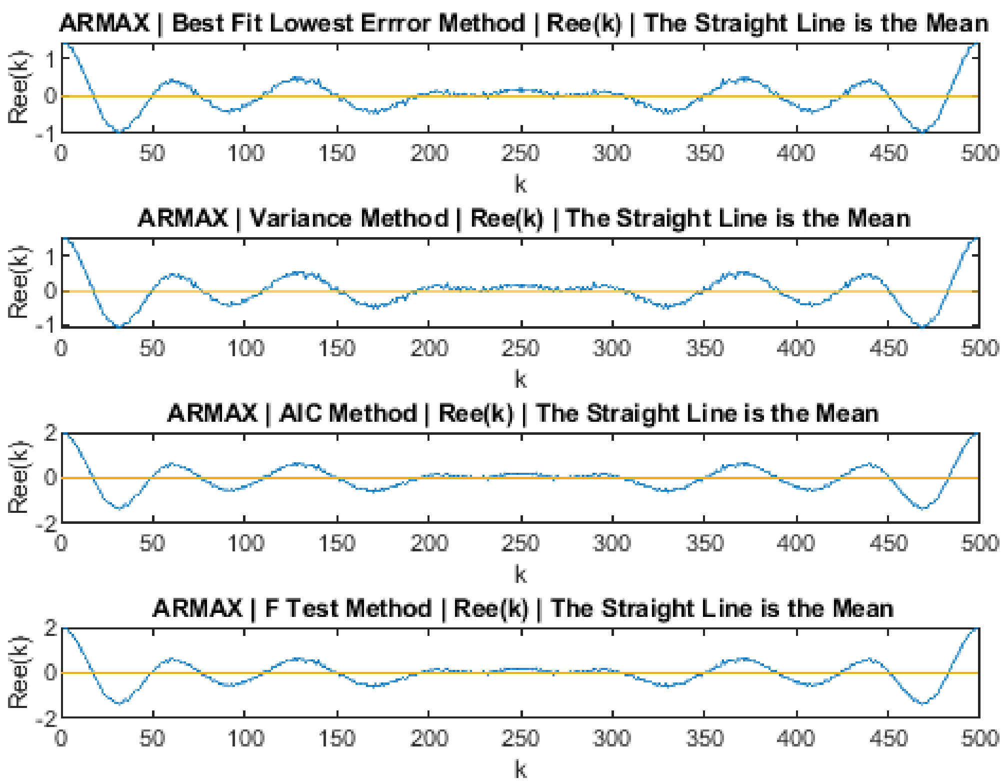



```
figure(4)
plot(t,y_val,t,FTest_y_hat)
legend('Real System','ARMAX Model')
title(" ARMAX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



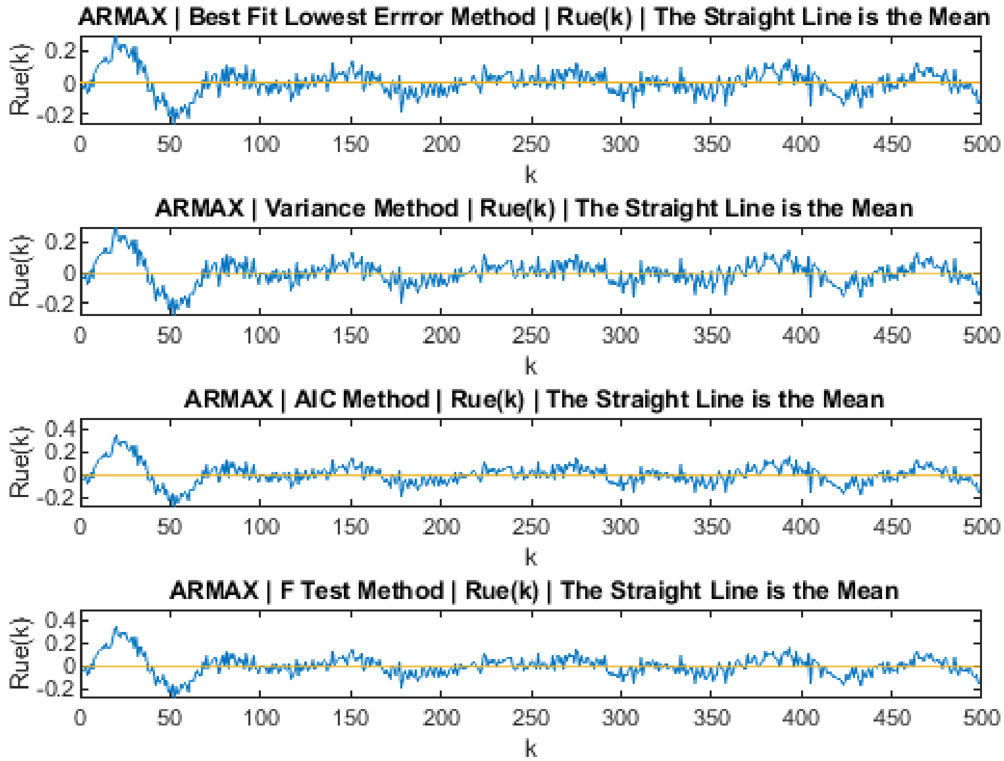

```
%%

figure(5)
subplot(4,1,1)
plot(1:N-1,armax_BestFit_Ree(2:end), 1:N-1,
mean(armax_BestFit_Ree(2:end))*ones(length(1:N-1)))
title(" ARMAX | Best Fit Lowest Errror Method | Ree(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,2)
plot(1:N-1,armax_Var_Ree(2:end), 1:N-1,
mean(armax_Var_Ree(2:end))*ones(length(1:N-1)))
title(" ARMAX | Variance Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,3)
```

```
 plot(1:N-1,armax_AIC_Ree(2:end), 1:N-1,
mean(armax_AIC_Ree(2:end))*ones(length(1:N-1)))
 title(" ARMAX | AIC Method | Ree(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree(k)")

 subplot(4,1,4)
 plot(1:N-1,armax_FTest_Ree(2:end), 1:N-1,
mean(armax_FTest_Ree(2:end))*ones(length(1:N-1)))
 title(" ARMAX | F Test Method | Ree(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree(k)")
```




```
 %%

 figure(6)
 subplot(4,1,1)
 plot(1:N-1,armax_BestFit_Rue(2:end), 1:N-1,
mean(armax_BestFit_Rue(2:end))*ones(length(1:N-1)))
```

```
 title(" ARMAX | Best Fit Lowest Errror Method | Rue(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue(k)")

 subplot(4,1,2)
 plot(1:N-1,armax_Var_Rue(2:end), 1:N-1,
mean(armax_Var_Rue(2:end))*ones(length(1:N-1)))
 title(" ARMAX | Variance Method | Rue(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue(k)")

 subplot(4,1,3)
 plot(1:N-1,armax_AIC_Rue(2:end), 1:N-1,
mean(armax_AIC_Rue(2:end))*ones(length(1:N-1)))
 title(" ARMAX | AIC Method | Rue(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue(k)")

 subplot(4,1,4)
 plot(1:N-1,armax_FTest_Rue(2:end), 1:N-1,
mean(armax_FTest_Rue(2:end))*ones(length(1:N-1)))
 title(" ARMAX | F Test Method | Rue(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue(k)")
```




```
%%

figure(7)

denom = armax_AICModel(1:minAICIndex);
num = armax_AICModel(minAICIndex+1:minAICIndex*2);
G = tf(num', [1 denom'], 'Ts', Ts);
G_AIC_poles = pole(G);

fprintf(">>> We have used  u, y and v for model estimation under ARMAX
structure. \n")
```

```
>>> We have used  u, y and v for model estimation under ARMAX structure.
```

```
fprintf("    Accordingly, the transfer function of the proper system degree is
as follows:\n")
```

```
    Accordingly, the transfer function of the proper system degree is as follows:
```

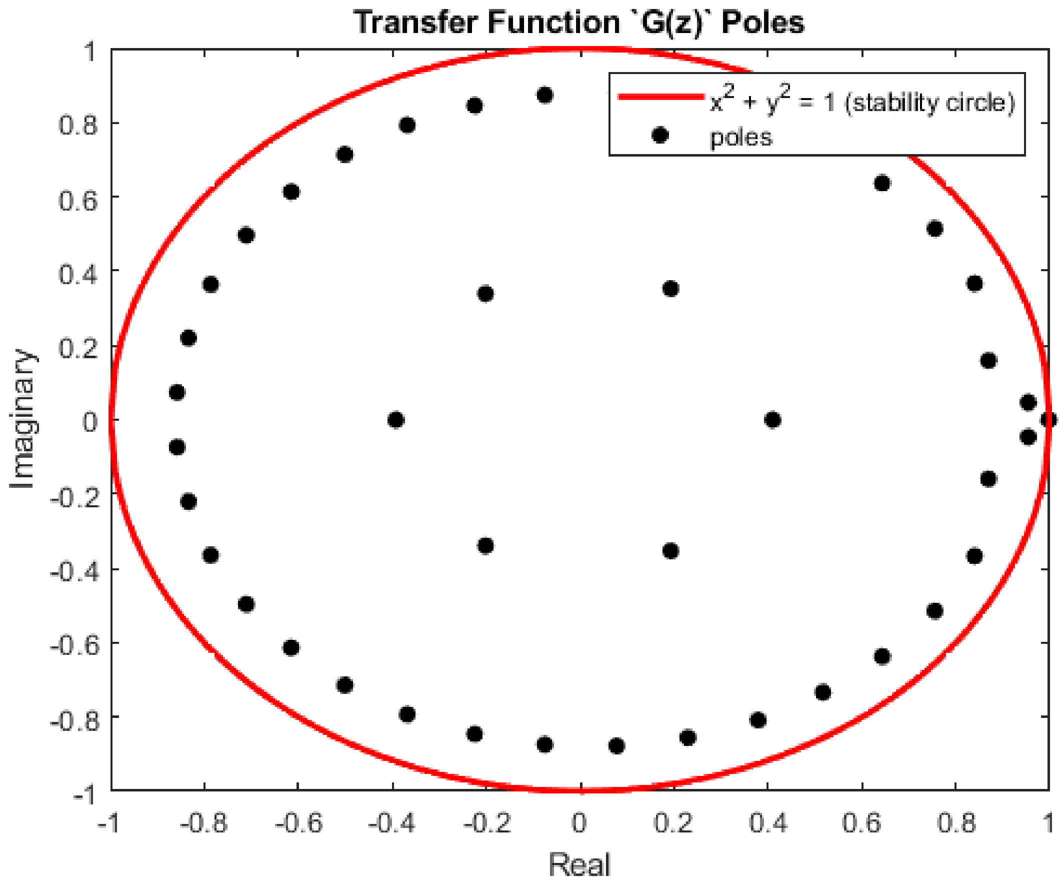
```
fprintf(" G(z) = \n")
```

```
 G(z) =
```

```
disp(G)
```

```
tf with properties:

      Numerator: {[0 -0.0052 -0.0027 6.1696e-04 0.0066 0.0128 0.0208 0.0273 0.0347 0.0401
0.0419 0.0441 … ]}
    Denominator: {[1 -0.5884 -0.0780 -0.0686 -0.0587 -0.0501 -0.0418 -0.0331 -0.0261 -0.0191
-0.0139 … ]}
       Variable: 'z'
        IODelay: 0
     InputDelay: 0
    OutputDelay: 0
             Ts: 0.1000
       TimeUnit: 'seconds'
      InputName: {''}
      InputUnit: {''}
     InputGroup: [1×1 struct]
     OutputName: {''}
     OutputUnit: {''}
    OutputGroup: [1×1 struct]
          Notes: [0×1 string]
       UserData: []
           Name: ''
   SamplingGrid: [1×1 struct]
```

```
xx = -1:0.01:+1;
yy = [sqrt(1-xx.^2), flip(-sqrt(1-xx.^2))];
xx = [xx flip(xx)];


plot(xx, yy, LineWidth=2, Color="red")
hold on
scatter(real(G_AIC_poles), imag(G_AIC_poles), 'fill', 'black')
legend("x^2 + y^2 = 1 (stability circle)", "poles")
title("Transfer Function `G(z)` Poles")
xlabel("Real")
ylabel("Imaginary")
```




```
fprintf(">>> It is shown in the figure that all the poles are in the stability circle area and \n")
```

```
>>> It is shown in the figure that all the poles are in the stability circle area and
```

```
fprintf("    hence, they are all stable. Therefre the estimated model in stable. \n")
```

```
    hence, they are all stable. Therefre the estimated model in stable.
```

Q1 - part c | Good Approach

```
clear; clc;
%%

load q1_402123100.mat

y_tilda = y-v;

u_val = u(length(u)/2+1:end);
v_val = v(length(u)/2+1:end);
z_val = z(length(u)/2+1:end);
y_val = y(length(u)/2+1:end);
y_tilda_val = y_tilda(length(u)/2+1:end);

u = u(1:length(u)/2);
v = v(1:length(v)/2);
z = z(1:length(z)/2);
y = y(1:length(y)/2);
y_tilda = y_tilda(1:length(y_tilda)/2);

Ts = 0.1;
t = 0:Ts:length(u)*Ts-Ts;
N = length(y);
data = iddata(y,u,Ts);


fprintf("===============Degree Extraction | Best Fit Lowest Error
Method===================\n")
```

```
===============Degree Extraction | Best Fit Lowest Error Method===================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 1;

for degree=1:1:100
    na = degree;
    nb = degree;
    p = na+nb;
```

```
    U = arx_U_builder_3(u,y_tilda,na,nb,1);
    theta_hat = inv(U'*U)*U'*y_tilda;
%     theta_hat = U'*y\(U'*U);

%     [theta_hat, arx_U, detUtU, rankU] = UCalc(na, u, y_tilda);

    y_hat = form_tf_lsim_2(theta_hat, u, t, na, Ts);
%     y_hat = arx_U*theta_hat;

    [r2_arx, mse_arx] = rSQR(y_tilda, y_hat);

    error = y_tilda - y_hat;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);


    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f |\n", degree, r2_arx,
mse_arx, variance)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
    R2s = [R2s; r2_arx];
    MSEs = [MSEs; mse_arx];
%     dets = [dets; detUTU];
    vars = [vars; variance];


end
```

```
>>> Degree = 1 : R2=-0.027254 | MSE=3.402397 | var=3.416062 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.341361 | MSE=2.181497 | var=2.199090 |
-------------------------------------------------------------
>>> Degree = 3 : R2=1.000000 | MSE=0.000000 | var=0.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
5.714161e-17.
>>> Degree = 4 : R2=-
13451075218161955013999236922194319069275330080424780452296474314687859099241566699520.000000
|
```

```
MSE=445516770264213160656559205465864149477620963408984449568340628703557838310697871605 76.000
000 |
var=452760945390460581306455310556563747959557498729729487606292054523611594227518390927 36.000
000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
4.142768e-17.
>>> Degree = 5 : R2=-
1842135211370272755529570028143039017524135827887064161237742992019902323628898075223204850832
8362728557551917781029152780515508339119526811264847675901969184977578034219392797939429610998
33096536064.000000 |
MSE=6101386814427567117715167015337240402901433136446865712987095980301083975398550779201682 46
5175751048227420788494405736742687134013006568317609825373357033565567189992632060912115852032
593931104419840.000000 |
var=622590491268119061125752176048671549913113184180172379105227075926727556707353863159137402
9249393270132570846282484015723560655372557101132308390593604785632505308910897601781446009725
875509883043840.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.050897e-17.
>>> Degree = 6 : R2=-
754554757341868160245711807654832903877880662776706123497239444473221426425364480.000000 |
MSE=249918161207325565149117704553233922653863982468830769473676006653308960288171622 4.000000
|
var=256063689761604068177363768425575698347609631017168959222443536026048623472227123 2.000000
|
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.266841e-18.
>>> Degree = 7 : R2=-
4034021597553796741386948696061656524649832557134692973700109561076749754099235929363310961863
4626352309272804846907624306918654704912615920116746709094355129327727456857554407384843309156
67796293793534996841308195396175738302480926824762561906193267466842560486608732 16.000000 |
MSE=13361194136299192195124125474045311562849350629468364508707308195958928326927657433737 1531
4142645945396219530296596474535369139543169794909710088005090160914540305678439686302820729822
1495210633735968078974396860335204107829212829939422548221903709477296523681033732 75136.000000
|
var=137460845023654230746207137504528363724925898484069378753274285251649670572511428735065220
7222261013964314016023487598878843768045356958526474223946963764320330757721657312106285232195
2588995680704802500768441382857828177365483243384147526974032155732487125204666186 46528.000000
|
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.798766e-18.
>>> Degree = 8 : R2=-
134312842500666378777301259459742062760606350612456985876160857814747254926135778460874155 9996
776448.000000 |
MSE=444861268154290980957847987083827304252189580054466372152307177021261574783295393200223934
8138311680.000000 |
var=459567425779226217589509624393545887550449681680003805972540258428403046778017047018118587
3632034816.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.470583e-17.
>>> Degree = 9 : R2=-559589697484673529458952092766168101 6832.000000 |
MSE=185343246285531333819020592293903576 59648.000000 |
var=192264778304493114370286171815333781 83168.000000 |
```

```
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.407054e-17.
>>> Degree = 10 : R2=-1159715909422808433885927497364536547082240.000000 |
MSE=38411270326735707004636875007309316200857 60.000000 |
var=40011739923683035093242799112877917350133 76.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.766848e-18.
>>> Degree = 11 : R2=-
16542584108743841126946896426523516357321434612022657094894702646883708241905653394538771509 03
646511273726995677534714354287315616855670294717661184.000000 |
MSE=54791148844372741862712796174476985571897115091738718043794896336740514600171973331745515 0
539918587467935276364461183394448269626816108958009052 3648.000000 |
var=57312917201226718647270335307698636210563311885325292985929537722304754727767908427811846 1
5387627648943136223348125398311041105932770553606424231936.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.134582e-20.
>>> Degree = 12 : R2=NaN | MSE=NaN | var=NaN |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.741142e-18.
>>> Degree = 13 : R2=-
1693695193920186154290350204540746334515057925895977841457499245505665901675142505992005622 48
3375672149967017956838382319340373586198949734265115153297200748809129148562237912880794816907
889084582508576672254617110708224.000000 |
MSE=56097345406893869859850875198554104874045170012763314025205928216244796212948118817764516 8
7104219686511743083451609673021939295017191180721969100012556132394803062566010452741732892081
7370486313436140211775768468838678528.000000 |
var=59174414986174970238576134052733827595918896044631258182510359548190339689142244692944668 8
1072871122851686144209865368483739585604033711285194434618471169196222505817181149748705507852
69091408586668034508086575034877870 08.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.295064e-18.
>>> Degree = 14 : R2=-
223011169080376312826374830253267570556364164050800217019064838668683066285233624315854848.000
000 |
MSE=738641440703445622901049068903897963581334889215318565861829103294788019294243252269481984
.000000 |
var=782459153287548353301579976766120108313075104581224119124600617905413859375850242388787200
.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.456647e-18.
>>> Degree = 15 : R2=-
693490923620372471995195692619183347318662965006509900884232156217344.000000 |
MSE=229693040510045615843301046321916979768556337929627151949097278950 6048.000000 |
var=244354298414942143698992846690396330414237047185609106531625231764 6848.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.347751e-18.
>>> Degree = 16 : R2=-
2464416021299908615059933518902449077620291924695093934670898790 40.000000 |
MSE=816246023897372328152723173293639806222024881328679688928067846144.000000 |
var=872057717839073112886163862698913757673547355536948123855929475072.000000 |
```

```
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.659753e-18.
>>> Degree = 17 : R2=-
8520398275436348978842133373108448583147446846862573387477371688425843319731401544022205713509
0393957315742537876917139284164608.000000 |
MSE=282206460039101775076256593982194533478782425114510822565581941444754425464926801435507755
464476189759525032160098197431937662976.000000 |
var=302796630943242273368053590666039092934970477851023071012728107654640221730227524063384934
153183420803025296259984187344071688192.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.517989e-18.
>>> Degree = 18 : R2=-
1372238781849414953869355197793143858684294624787800802239175651753984.000000 |
MSE=4545029896906554747172879403455202986556176403369492507137821726736384.000000 |
var=4897661526838959810628237348286977319528512546509520868333110243098624.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.418982e-18.
>>> Degree = 19 : R2=-
7081175204229396676719727664272774469140254950983261843859906750309010459012387234555854079149
5122054358761472.000000 |
MSE=234537555957136450661033610722324659331905044158950606030786396294516349738480057278268769
090304140329474850816.000000 |
var=253828523763134672992544799996606239136119921148958786162942924409739967593322954918777504
768897013194483040256.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
8.346454e-19.
>>> Degree = 20 : R2=-
2016790666786758936695749933288220016867468571035379623082648868446194011888423223634972823233
8295274980278639001600.000000 |
MSE=667986796291683097802770884366721993041510719425244476968478758044024747429402117222154884
93988996908611706697023488.000000 |
var=726072604664872955217802002482213551106013151906966939520549233281374062083478194301851823
10206394786069952868646912.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.111949e-19.
>>> Degree = 21 : R2=-Inf | MSE=Inf | var=Inf |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
4.027726e-19.
>>> Degree = 22 : R2=-
1821737214880425489785368259118442541726345076051913317962040534780318399137830502838645790780
360185776088704430295700757632233210415413179002109012705790614860622639136768.000000 |
MSE=603382604795629449466009846384734714987101610871264465637114462190794000050595731015004377
9134384551722424597512691917515056670973531607245741582221635794451814972833923072.000000 |
var=661603733328541070538166125565070646895835975825282547046655441155810755970459638377793378
6305860825600900223823725293078969062377565995194036691029388412913986026036264960.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.507982e-18.
>>> Degree = 23 : R2=-
164635026186188171476397651293284305554968546635823318952395417978367115264.000000 |
```

```
MSE=545292208609457389507497228853220176859785055651704420287300582202274742272.000000 |
var=600542079966362690329921136679651698092905532481780193409555440766929600512.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.930360e-18.
>>> Degree = 24 : R2=-
29643384670448580673312363150071721644319220740611456695858497608956128670406312539560345 6.000
000 |
MSE=981826715253665833557373773384962625832849233550835114270148311670672134233648470519971840
.000000 |
var=108609149917440912092644672184722203931335995408816455267464726689689371625117184327837286
4.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
6.102065e-19.
>>> Degree = 25 : R2=-
235738605454228907463688123791815110167682392809937870008855357649212205422112419467003185636 4
9170988019137324025618081856068566779436764041875440117075790362562345806541619 2.000000 |
MSE=780796333565585900621319819726223642931395651956195247818199577288305488977735003321209646
551543608684867213217347062089547486226822462487433294043639945013811194073926598656.000000 |
var=867551481739539975007837347987836056365205112964221423595416598770362304661917495776992556
228101547175039098472931197594740193123786747225777288489407061905155901038239154176.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.889858e-19.
>>> Degree = 26 : R2=-Inf | MSE=Inf | var=Inf |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.418981e-18.
>>> Degree = 27 : R2=-138325530235952667508624290302895203266655682 56.000000 |
MSE=458151801817278298419069023372531691194951925 76.000000 |
var=513623096207711132808605602177321705192386723 84.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.050038e-18.
>>> Degree = 28 : R2=-607099993266250179330128919335877278970872332288.000000 |
MSE=2010792623196442252438367022866280217998142734336.000000 |
var=2264406107203200673332488748029943827058481168384.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
5.920918e-19.
>>> Degree = 29 : R2=-
240337675257316016175508588622099860962301972814123648724235839611323796467028763045059272150 4
25714195010546061803520.000000 |
MSE=796029039439719206009076613124543903339665774140825794052394611508936002037679379484881695
45685594091130119568909926 4.000000 |
var=900485338732713838856031069495044898661323722138706310730420050272535989523211091937678297
04124790337526494770967347 2.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
7.921932e-19.
>>> Degree = 30 : R2=-17002541851363990882605884430803560248115 2.000000 |
MSE=56314587563037121303649741898231484252160 0.000000 |
var=63993849503451266823255154147710273899724 8.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.049073e-18.
```

```
>>> Degree = 31 : R2=-
2645105617204584244573444132003336634177192500676682245880330754753648690778031136704247855651
95040419966470026428416.000000 |
MSE=876092723051283548395085974203355525110301601520446647497099822328639641621905661026785884
297058772202693059957227520.000000 |
var=100010584823205877297065868456665134345040773454554313990937731215729484097855429086719808
6138712403188246432687587328.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
5.930040e-19.
>>> Degree = 32 : R2=-729216794574482335215306515339591049918224453777489920.000000 |
MSE=2415259047117566372859853495301501572523150764205408256.000000 |
var=2769792485226566882037768031328962300562167929560367104.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
6.972022e-19.
>>> Degree = 33 : R2=-4800206846586628971288630552166400.000000 |
MSE=15898897420511690606087367573897216.000000 |
var=18316702097363698525533219391012864.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
9.032953e-19.
>>> Degree = 34 : R2=-74621493976252693788753920.000000 |
MSE=247155907237082374302859264.000000 | var=286060077820697282833547264.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
6.179993e-19.
>>> Degree = 35 : R2=-645620334813531508057604564406255321484361728.000000 |
MSE=2138376908298404594972569042900251637069119488.000000 |
var=2486484777091167942067431410733057491276398592.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.298188e-18.
>>> Degree = 36 : R2=-52833618.272990 | MSE=174991686.821362 | var=204429540.679161 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
8.064624e-19.
>>> Degree = 37 : R2=-30347306032333054617493592504668518350848.000000 |
MSE=100514148872570139442924842607954817449984.000000 |
var=117974353136819437192377359694154331848704.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
7.917686e-19.
>>> Degree = 38 : R2=-1928738983497553920.000000 | MSE=6388229555435719680.000000 |
var=7533289570089291776.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
5.356527e-19.
>>> Degree = 39 : R2=-471198047735353457797159340557169063886848.000000 |
MSE=1560668043090037710994857038467134425923584.000000 |
var=1849132752476348550308426314893504488144896.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.299054e-19.
>>> Degree = 40 : R2=-
7452102891203707423115952610543813661105822339718636041310952601622283430113269375568620499554
29333117452417645402637649810106475347968.000000 |
```

```
MSE=2468231541284439375721248349977681733099037176200138804081685907981144073877361570177873575874494898421933576542597501055262034772361216.000000 |
var=29383708824814752782113155419020348833612889476601020427950801218736319859417820957645885297002728497517132692083533000959375907991060 48.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 3.324776e-19.
>>> Degree = 41 : R2=-
1456754989866832044269287654602811031105322071093605017657431238991900221441632324695114102792
3491109713888437362988864914983318591133909176987079968183410578315429114463530702650041526983
4737057792.000000 |
MSE=4824958359280013493755577214524430780001866433956394721750945004578913632948478419079201594335765017920805594278322714845833367785243385341850755492113312730992403424698999063695028628
86409424863232.000000 |
var=5771481291004800709137988731266366063373333958118846326708436167993837589777622202156130646858658968277428425699471815498637279184698794767943398729783417755004315142401198797089817189
16194094809088.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.279235e-19.
>>> Degree = 42 : R2=-114117522375265636586748620073008019656624529735627252432896.000000 |
MSE=377971791656739487971253287666098731225698578698760502640640.000000 |
var=454293018818196426201445314074307506874281120637583699738624.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 7.081892e-19.
>>> Degree = 43 : R2=-1015971516452912435403161 6.000000 |
MSE=3365027266217214673839718 4.000000 | var=4064042592049776620155699 2.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 5.229023e-19.
>>> Degree = 44 : R2=-1534403853759428533446688457039334604 8.000000 |
MSE=508214130187039931487013517269013626 88.000000 |
var=616764721100776593875153183312592240 64.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.761100e-19.
>>> Degree = 45 : R2=-
8612962668631258689611634745323564059111714985936488292839406393904242011383162000292918739423
8297129096419878464589177706402073252201325851842157851483502387849844010009225055825082954055
80915299051780068106391488509111089233920.000000 |
MSE=2852723108226867678838773480484590617270044071238678995774518565212071247201739816595455969871318972374804685183513441191846559745704414508437018887322691663865797693252411892645013332
41096412949287293103148187378902373236819886 08.000000 |
var=3478930619788862733948890610147651067097565741697075415534644401477011992252677375525195153490299330757860032512790466718173067026510533208089609052489298544852409008245449167149336104
93629794313325689731150917611418900263516241 92.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 2.931674e-19.
>>> Degree = 46 : R2=-
2830697202740007641070482878170317642845517840461464801226634202085990737428900296481343376931
5740284106305145854742845468433474417346395092145910038877215696137668721265506404870542521843
7581878362363219408130137040329951454167 04.000000 |
MSE=9375630237037653998983334387647014767257241012261660497529581135647558802913183266993385752907981555258387026574065132380077871574466594453987425487237807600645239990489218224672345020
1235121072701930778544479813310552438585950208.000000 |
```

```
var=114897429375461455360116673580517994392859920759954747471141343564344452528035848611773685
952667363026337466836369435778498783515724964237046063134472285558128959207981138206697751590
0595283724100147498296311151142523583973241651 2.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.062253e-19.
>>> Degree = 47 : R2=-
896895971486431890306084150617969451143204978232691227978230419985912866274200028841985499608 0
387735491311492537096701629959719952547293391147949544785153203967355411606968540840208831922 8
105217835803076523039707976855884683125875235580669693503154270321332868362190284744795587299 5
2693059584.000000 |
MSE=297063387126178385552507640563293263975563371820548425528605689469250215447673059864713572
592818275287833234079280736405299819889772374438477777472154221619738375190693201829139352563 4
121264787545509258902831429254057883293858887250637128584587377913188710583734200000096205234 60
148245586509824.000000 |
var=365841609761303447293112425745364805745129377222878917814431731234137274743375369876042638
816245214340675110045819155551807493319470686172413305671357524250088690357333760043497414533 5
390476420417662163925232608355942140170584984226487059269185021959641211020012871365145657920 7
624617300590592.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.335919e-19.
>>> Degree = 48 : R2=-
330499250932526196293526201854871253814077472271399068302623522367070813046686878787505839192 7
517041977944567347643678 72.000000 |
MSE=109465567965444083886060669523355001816749052468223965095868014005480117593364734873994325
912470292343835962393955716300 8.000000 |
var=135477188076044667499914816201840859593956636386146686490970580200982388292733246140462466
831739998231926064358742818816 0.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.556589e-19.
>>> Degree = 49 : R2=-
28321108441994336817689549859477497544468818956313693990204065333287307615661646151 68.000000 |
MSE=93803123978842793229084375081862195674883936817281910408666467088797134282019695493 12.0000
00 |
var=11667055221249103755637918362256358944744994718416103700475338705553638309051378958336.000
000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.529116e-19.
>>> Degree = 50 : R2=-
144519755689525817883779344819860846978349171663524308791387196690274465178420510 72.000000 |
MSE=478667866693922288943689585013286696848923625277763535676503500236338616618157342 72.000000
|
var=598334833367402844329645314297458499394270102209935248572414111207565470082940272 64.000000
|
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
4.046223e-19.
>>> Degree = 51 : R2=-16823064179341527096486483008976839516356608.000000 |
MSE=557201346179965342087678047717988479239454 72.000000 |
var=700016911808610487115654770147901745555046 4.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.734096e-19.
```

```
>>> Degree = 52 : R2=-
672471249310990638769408717369794270108104725891213516761539474983228807006571893497272006820757504.000000 |
MSE=22273105623857478415336942615924952336556764929087070338760231097136151699428254688987038429541826560.000000 |
var=28122608110931161871960004373226890886946615160005636931533488668061251579666252081791642447390965760.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.237471e-19.
>>> Degree = 53 : R2=-Inf | MSE=Inf | var=Inf |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.085338e-20.
>>> Degree = 54 : R2=NaN | MSE=NaN | var=NaN |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.425436e-20.
>>> Degree = 55 : R2=NaN | MSE=NaN | var=NaN |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.317397e-19.
>>> Degree = 56 : R2=-
197956533829228642258638450413990833407171693307487609099148429916948397954161257818416898489857333254969870646861865113626137992958261567354974017229836972504744096050803031965176739847166942031823554351457672218526770732252456013972838652313 6.000000 |
MSE=655657292624579260180475866467984504151067267394947560043412537427807428582510732733679658287250142116673340443671994896400945013126214431930224899561204487461245623449262425911268988496349962310855713646351991368743540577835061859225503268864 0.000000 |
var=844919191526519682374263172908765355138672353569044966858604055443838828039612351185637929040981326060128435972748279360057835942778833501890774360403209618473697671822696959986538580767434069759597953663532379620937422550046346890735703503667 2.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.607500e-19.
>>> Degree = 57 : R2=-
5561025186696453973398954840905425211312547172487234840991481545476588002081588364781324983673 0746648133632.000000 |
MSE=18418824817734485479986894384271167595753644659580630730336047807568585986160338934331339 0742374702437629952.000000 |
var=23858581370122389960835357749258686756292636356891143932986946975806800326175487086380840 1090942459816443904.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.082778e-19.
>>> Degree = 58 : R2=-
755345301878461929047899658272036546406306452169723411466790617453128614034943105643879783014929270238104603016380491260740344511381511092455961815112773992473991907272294 4.000000 |
MSE=250179999642529424288249819993786921259463703284454707044024809308771757270307518683977834968079366688959926100750327032816717165808559816029275944137914490496465394498273 28.000000 |
var=325755207867876845960994877953647909995884031466558878210113178608331463869300257637833826700949943556383686283533794602359711236962156380718968052617104727116403951859138 56.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
6.476159e-20.
>>> Degree = 59 : R2=-
150776036052847970307557960654961597132151806969978911507721789817584334821045582081613960001 9
```

```
62334536328300882384462848446102554820286445091130119098425132353468436236887654400.000000 |
MSE=499389465347770143853475626878365845675393772350371774323405700763612139946185141974246320
931200695747406321273796996605379564509320516066669347229472264800578626541055935250432.000000
|
var=653651132654149347596447589230785140387868848301233378204519356970675116229165915475095631
700542820351340146462333690309061524285633641241507390455269434193584533823198161010688.000000
|
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.778541e-19.
>>> Degree = 60 : R2=-197779479408407499290208441991168.000000 |
MSE=655070865796642678711244017369088.000000 | var=861935349732424463476488384020480.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
8.990989e-20.
>>> Degree = 61 : R2=-
2485050839858057266355567754803841596273267244341556334902781021265244255815535176576809725645
581402966546494075048528332665000601610621114674115054904803328.000000 |
MSE=823080539034572429707871835229571118062152675400417314884018554145752785142813949925467469
4449606577037548802236369910097832935659239246863866650975738527744.000000 |
var=108873087173885236298677164040280766792349185235184382855275165034239173821177023660120080
69183096527006581900104092886135856774547526736156305421394753093632.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
5.396270e-20.
>>> Degree = 62 : R2=-
3796343561946811033279036405047650259571341186258639732665916635824267128969686307981628185215
0632538166275622608425718273321393896670350541320257706361011739840259860546869527272279083588
566921628352512.000000 |
MSE=125739741626617510419449401803697660688628439912842801544714851817408287160009768494792019
5136027295259681445949074802031375137152221756241465183667183393255246894976877431045231314474
04903137016060837888.000000 |
var=167207103226884955043380840059710437769833720073835591644752015584718697441575962617081410
8792515608230791697751026863671423803944272539987847548270245950302298507620858820364593707928
70938105454228668416.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.134327e-19.
>>> Degree = 63 : R2=-
4058813269592434901946782176414191904535984942146240776292453588259869713473061969064111343592
0872243200.000000 |
MSE=134433073166729824271943825560312016074243490387091593270378716206324247753935353484755021
444899829448704.000000 |
var=179723359848569316465742702639231351999454212967419152256683743579624672978102601253115800
273984363167744.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.093763e-19.
>>> Degree = 64 : R2=-163119424287328891493641867170414592.000000 |
MSE=540272341780712040940159919055699968.000000 |
var=726172502393430023707517615970713600.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
6.629980e-20.
>>> Degree = 65 : R2=-
4695906555826139777753736472259315101075344982228734755589005810441782190008404290287453876505
6054086675753263909285709088470596484806780461284873192804786948964033918184577987802292440422
```

```
9864308892214448029384258404265537180856750118472990231475271739977538350235256861019340 8.0000
00 |
MSE=155534415523109842551388867268864481215487684374314998687409707106135934087255865810143970
1330490185640314449889945412857754338999673375248726103271100493181397484967905964087942914239
3716737028428294438425176948102143544628913407012843611014986672510561863679721261578344988672
.000000 |
var=210181642598797091975581410358603547371693703490182654746212765408339038405609983659413483
1304938834309404898102611241743686477762196461416663333253865243776366195120799838431891393408
7167858505083193299135695593135947166660405264298729571085583605597382010377772495864156127232
.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
4.741002e-20.
>>> Degree = 66 : R2=-
3329528060161986529993924056372893977340309286845617326082493664899236760592227218383531719434
5083439430878677618633298219553403381800178808480774304346627971499422800063391577631238731067
780167862219230412 8.000000 |
MSE=1102782167082502725291297531344102000734255998789046710217816089497290973613310647715472 01
3526174591062283030449294609063846692038711120843759330978208454179107050842559759096543430576
25929047296897319121715 2.000000 |
var=149834533570992228648476634636062597504643110827989026277540857706903504546139055978688839
1404470328346196770169304898506475482051380496094384933752161549116035724562279076490254538098
34754704399162290680627 2.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
4.238792e-19.
>>> Degree = 67 : R2=-314898936294624339480413320691611546419 2.000000 |
MSE=10429854474993247098068260266615322443776.000000 |
var=14248435075127388357895983894116662509568.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.703131e-19.
>>> Degree = 68 : R2=-
6216198708971503909668766208053678563880289137285212710417509853159728426760710342375622575086
1431364633860871179466663285806501941055848448.000000 |
MSE=2058884310157019591857077258637461865922929386706405800533574521484854795008251064314016 42
15383622093816833259101924633259471894141844062208 0.000000 |
var=282813778867722459321058611714226424923621371069220464090157022703260835691731241399054600
57151041938633181760649639346354464898943337208217 6.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.147399e-19.
>>> Degree = 69 : R2=-
6234680016455930176254190259624375131304567740714293711750780991918998189331520701433328435 2.0
00000 |
MSE=206500555495505915631605383867810045560478204165386645814033972956982686581156227389163831
296.000000 |
var=285221761734124217491142439799038868495041292131663612179021562797830868311635586474842259
456.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.199951e-19.
>>> Degree = 70 : R2=-61538703628627866437289934416281017172441219832217 60.000000 |
MSE=20382403668261754368459710441870703517664291632185344.000000 |
var=28308893983696880476537155264857811372841711586770944.000000 |
------------------------------------------------------------
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.451875e-19.
>>> Degree = 71 : R2=-
1430615576184107542333095655912042612777604981502079849623789379809535109539816653198306207334
4.000000 |
MSE=473838128665459561012982183235272504026220120266865468944967829824135285949579396314222653
60384.000000 |
var=661785095901479837957066575014758529147538179454295963671339996179516376200689311336311134
61760.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.764263e-19.
>>> Degree = 72 : R2=-
1493317325023915944373140032278506029074104429259135715084500388985160116874721668701020782680
3904649370150880024215048304695300682745308229620009007476727359570877247481793386381312.00000
0 |
MSE=494605747744201501891351326457446657628204900183782564248071037557924049015051928567865010
8101507669787960982997863228680289103011345360972072815143840742608394913840442377772413747 2.0
00000 |
var=694670994022754910932888576618194004267582543074479545500211476611383000858162666920584315
233414949062753358172336630636009842576015279784978787750775750872612323690964900232948613120.0
00000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.736962e-19.
>>> Degree = 73 : R2=-
520462504699042214863382136798014147710735600321648755079607012379653379256892425943310306253 7
850414315571899452390602679057913672379757112712160169226852787451404850110787136059965847299 1
919433398142107648.000000 |
MSE=172383820903816987003929463042175838042478568732982928212862195526821319573253829054039704
682858171500646675016112198560835494096778157073690849640577147673056961718658506172758492381 2
745998513564707580805 12.000000 |
var=243479973027990094638318450624542144127794588605908090696133044529408643465047781149773594
184827925848371009909762992317564257198839204906343007966916875244430736890760601938924424267 3
370054397690264944640 00.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.232265e-19.
>>> Degree = 74 : R2=-
57332033725686620617685906869137732589735455261954836567758789935 1040.000000 |
MSE=189891009334769967872743366532565278117440970707651297303305002575462 4.000000 |
var=269731547350525519063566052765828273784212680777211084535969328241049 6.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.729185e-19.
>>> Degree = 75 : R2=-
75696390733670478081804473324212282538144212511018324439464287234764962888731852 8.000000 |
MSE=250716102418250107425381802815144755195955738949992340688997554065474461112467456 0.000000
|
var=358165860597500063197152573829761052064484469639618499074200448194296726465544192 0.000000
|
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.402591e-19.
>>> Degree = 76 : R2=-
2582571825246348965822318291060158153623761357243647490785379546169875392796416474247809134073
1642335854181674390069283725300636318396053333225198356297095793030964882734703419443693609336
```

```
0800401457152.000000 |
MSE=855380733434282354640940799261899916423135812434956178613244301293417139931061708302592215
5347238665344030630106005008973956828566774175335981369135204833129825412397908464632212107638
05864526459437056.000000 |
var=122899530665845161051731596452728499255540194659745397516097190091898620667349462964852156
190939178090582488390190174188354875349692465671160130448940431910538927705151975382616559057 6
161522753696956416.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.062143e-19.
>>> Degree = 77 : R2=-61361865998002971183460795148604805805506 56.000000 |
MSE=20323832789146599920723188870141224599683072.000000 |
var=29369700562350579351264769217406711732633600.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.259341e-19.
>>> Degree = 78 : R2=-
698979295329303463601963773925226862649733056461625652946899366740235128934853753698771598277 1
918368716606141002067843656377524887513905985239793513325500825 6.000000 |
MSE=231510859233169593809451287362612241361697313047351019179790461995436502990743060645162432
194730513942892577578412223811925744576833223192970230766886103023616.000000 |
var=336498341908676783965477039621408035921319089133559311013401883203308684195756361363239279
806992631157243002196149583982652985656910493196273211219818561142784.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.222692e-19.
>>> Degree = 79 : R2=-Inf | MSE=Inf | var=Inf |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.610882e-19.
>>> Degree = 80 : R2=-
698972278832294288672331081318928868308691677893929095872583951784097041393018889033491333585 9
234075004960383069330409801606093392157489632630538240.000000 |
MSE=231508535279853351150843238180913694850664067611988405161411301054733637874487874339558752
054755290969726415728397943949850391835871671170673336647 68.000000 |
var=340453728352725549800813782968572094877989800164897211342211503725622090692730292256671331
762412345460583456666925540770116458870131958225925937561 60.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.003662e-19.
>>> Degree = 81 : R2=-
656750108462041742450386798429975640797622619991933975659423324516146660480347769143163008311 7
761739940064954361073025072213417221983031843228766020159520538302824514639353105867489389449 8
564660220766934007808.000000 |
MSE=217524013840629154586442923131249843069638231869466016671077692798851201983834212439629979
428193627705199109523614411218241342017278566170498292326718027444482411100519081522327005213 0
203049393364795310462730 24.000000 |
var=321781085563060905050619371705116847128257714487819614597497834570138313388853787022622797
053490294602848942393504166837861986637826994342690593719869351073793580242293139492849906957 6
992091580612610665228533 76.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.964317e-20.
>>> Degree = 82 : R2=-Inf | MSE=Inf | var=Inf |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
7.873528e-20.
```

```
>>> Degree = 83 : R2=-
145449845254862885979846267195669300055068803351561913552431134801920.000000 |
MSE=481748441982401311754163263529336336190835480756667176934387541344256.000000 |
var=721180302368864262668027780545953793957922917735533396746202909769728.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.833489e-19.
>>> Degree = 84 : R2=-
1314762204176916734242996939498708088280157533961112139752104653632082754808046237151424833651
2481555526476131994991997457652861290484698475210823972147797269863026336606743675410552970547
298304.000000 |
MSE=4354660139581009383367357922528981345387019893647625891142695854104324954830445823262195 32
8011066436243382736496020493921781393899863862781099859436092975289112849204886646386426964163
8662373376.000000 |
var=6558223101778626900152757092448150975153266927154559123630758585949699434870969715749408 49
8432730591223751608406665813845316520003944709209202687294950997554129511458517827150152496885
3850816512.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.172055e-19.
>>> Degree = 85 : R2=-284164579920594383058803001439677832768404848640.000000 |
MSE=941189338520475374628077394692170069902381547520.000000 |
var=1426044452303750513531873473129136551144271642624.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.393772e-19.
>>> Degree = 86 : R2=-
1415097517310467859192039159153133865993176349452139516258160861431923232075865625075477916732
6208.000000 |
MSE=4686983496083779941221903771682705142398043292683422793840533114812481557841100619565947 95
37680384.000000 |
var=7144791914761859213034648856124910301608900625506235082655466288039050863044703320573520 24
65251328.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.243536e-19.
>>> Degree = 87 : R2=-
4531959108043169933030744986427012969512893240357759289236111371930760193421464132476618584309
42236373678601273344.000000 |
MSE=1501042669108481405174373565779013105135874417492655968374869187871551124382326077605502 88
8115864926030783617957888.000000 |
var=2302212682681720102579037221148876669412926347334431682971238841113388678191195208080241 33
5716020484546236732407808.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
8.461970e-20.
>>> Degree = 88 : R2=-
6812172472011049788754682269960110288954030286946671058548794557666041278559486399500161762145
3048578799948764749502951999520287247325385137785055668600832.000000 |
MSE=2256278423092378992180132899844349030275723705152040541233949857174870054138294230411957 87
068344355668079874493113688035941011398966782941621527340331827200.000000 |
var=3481911146747498562337495550221262525834104665353803707830762653674988104800433549407037 17
278221858279198991397325822917225663514025340848705727953077534720.000000 |
------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.004581e-19.
```

```
>>> Degree = 89 : R2=-
948108412979940167067562597254950900101146807785782672486524938246959575170515969358241465268483397779456.000000 |
MSE=31402560104698530937757426198634541764741235971803425151119167124029320184347095424093605053968609269776384.000000 |
var=48761739293010137348716669364225957324902944273854143895031414546352447309595661535078982652818806139781 12.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.194148e-19.
>>> Degree = 90 : R2=-
16787225221247094380009371583192370049550709288161872795524309782817838451254795472829238851461261924346567694525091766206379560847447439284633 60.000000 |
MSE=55601431416944572121745759120063248256930305731835260131689444902807824621060165007150053116697662991003723173205479965460025632485674027529011 20.000000 |
var=86877236588975898272524145688872043492725765429560880575197687935350624674280852534017751441130134423403326995851653223205057177951946650786201 60.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.814907e-19.
>>> Degree = 91 : R2=-
41116990101251252508323387533373047700451347448951529083833994980844587720985554227930923204340235697225577660094561963212 80.000000 |
MSE=13618471635755387878926999640028333912859202205460242652900525973322077621632074958091449838293603663171641115729871534817 280.000000 |
var=21412691251187716968014219517566554798041306370044685482599509938555709772927738351631262647476114700360366412262215849607 168.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
1.782435e-19.
>>> Degree = 92 : R2=-2612670420586564489873017859920443425536452318324759820316442624.000000
| MSE=865349772167599796127374832293335002359658585400651679120385638 4.000000 |
var=136922432304999953496657274162929626742511533240129377897933701 12.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
4.242742e-20.
>>> Degree = 93 : R2=-
216311797100766325673801431164957921034197813685511779223781524605431898002944290004469329008465798232142453557256596911355918272214297917423723098697587295263852450741453144028821454454004982144897933144076930507253735666531835363979772689613436529671370587504 64.000000 |
MSE=716452266092895422700318167408016136278211226406280451254097483896950017522528261218331919871387492005956341303586358936304548349196111958458851386457285654786059415265885655520262123553249168631437475008287281782189863012263606583689654875489019329142422555852 80.000000 |
var=114084755747276333178104279174163757027996432005330958086351965814605870718746808339914764802863956911914624647091049801255340135377892757194764265432203928906853037671679624416795422620786687120314630981651258226744502156432948050144184492618727807367763470581 760.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
4.491210e-20.
>>> Degree = 94 : R2=-Inf | MSE=Inf | var=Inf |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.336109e-19.
>>> Degree = 95 : R2=-79392571863311151435612 16.000000 | MSE=262958325827763217031495 68.000000
| var=42412633198026332255027200.000000 |
-------------------------------------------------------------
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
3.298453e-19.
>>> Degree = 96 : R2=-
2675757774418424960832695704713964622318488182038995579497976750488050155198757879911238598656
.000000 |
MSE=8862451085387294542821220966931660246896844177798535649469619953304608515517601433306921959424.000000 |
var=1438709591783651519011400515716529076222395728706876498698859898887213109005185258486440656896 0.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.450631e-19.
>>> Degree = 97 : R2=-769555637437759242755451834396049408 0.000000 |
MSE=254886644055758695290318779032975441920.000000 |
var=416481444535553476456381571668870430720.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
6.281698e-20.
>>> Degree = 98 : R2=-
2258392041018560316131029880394034462292698965746915142061157830004398891820096126885243220930
7564623776106759699138672272693725962118134511537836856629120459726042445433882564982034031224
4567350136348904791478588348765141738183757787573914907051434558095360.000000 |
MSE=7480082534565446020890066215996184978796563427066369042760024335254287272108997574979371153707984160867272321687945539811860200053827619805802402561828585330388243334300779520516262595
187843969731498262365757863744859339055665094595989604927369031389303275520.000000 |
var=1230276732658790597322470601017510642583312693876861377559644133037618351364136099586562380317770404255498017708708436538180481614360141122677744030056476826080976858179229132514378990
804607611651614774546438037958389029799255903876576408928372816512307691520.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.506540e-19.
>>> Degree = 99 : R2=-1624855397132631147454265539054063719546880.000000 |
MSE=5381728352090977009811237069353822778818560.000000 |
var=8910146278296319983077227349807418058997760.000000 |
-------------------------------------------------------------
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =
2.751320e-19.
>>> Degree = 100 : R2=-
4040270393126447541018566303922332799885873925360142101260723222314888278961970372466912461816794430427050803 2.000000 |
MSE=1338189094437152871262937751947909351170398540522685385372759062135077293638199967474693706768431121059282944 0.000000 |
var=2230315157395254715907401594362723840654384736497701078397810599761171467683405941161607812692158964038841139 2.000000 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 3
```

```
na = bestFitDegree;
nb = bestFitDegree;
p = na+nb;

% UU = arx_error_U_builder(na,v);
% dyn = inv(UU'*UU)*UU'*v;
% v = UU*dyn;

arx_U = arx_U_builder_3(u,y_tilda,na,nb,1);
BestFitModel = inv(arx_U'*arx_U)*arx_U'*y_tilda;
BestFit_y_hat = form_tf_lsim_2(BestFitModel, u_val, t, na, Ts);
```

```
G =

  0.001601 z^2 + 0.00615 z + 0.001478
  -----------------------------------
   z^3 - 2.837 z^2 + 2.69 z - 0.8521

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars),1);
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 3
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb;

arx_U = arx_U_builder_3(u,y_tilda,na,nb,1);
arx_VarModel = inv(arx_U'*arx_U)*arx_U'*y_tilda;
Var_y_hat = form_tf_lsim_2(arx_VarModel, u_val, t, na, Ts);
```

```
G =

  0.001601 z^2 + 0.00615 z + 0.001478
  -----------------------------------
   z^3 - 2.837 z^2 + 2.69 z - 0.8521

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=1.00) occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 3
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb;

arx_U = arx_U_builder_3(u,y_tilda,na,nb,1);
arx_AICModel = inv(arx_U'*arx_U)*arx_U'*y_tilda;
```

```
AIC_y_hat = form_tf_lsim_2(arx_AICModel, u_val, t, na, Ts);
```

```
G =

  0.001601 z^2 + 0.00615 z + 0.001478
  -----------------------------------
   z^3 - 2.837 z^2 + 2.69 z - 0.8521

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.21 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 3
```

```
na = winner;
nb = winner;
p = na+nb;
```

```
arx_U = arx_U_builder_3(u,y_tilda,na,nb,1);
arx_FTestModel = inv(arx_U'*arx_U)*arx_U'*y_tilda;
FTest_y_hat = form_tf_lsim_2(arx_FTestModel, u_val, t, na, Ts);
```

```
G =

  0.001601 z^2 + 0.00615 z + 0.001478
  -----------------------------------
   z^3 - 2.837 z^2 + 2.69 z - 0.8521

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[BestFit_r2, BestFit_mse] = rSQR(y_val, BestFit_y_hat);
[Var_r2, Var_mse] = rSQR(y_val, Var_y_hat);
[AIC_r2, AIC_mse] = rSQR(y_val, AIC_y_hat);
[FTest_r2, FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.8231    | MSE : 0.9795
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.8231    | MSE : 0.9795
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.8231    | MSE : 0.9795
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.8231    | MSE : 0.9795
```

```
fprintf("---------------------------------------------------------------\n")
```

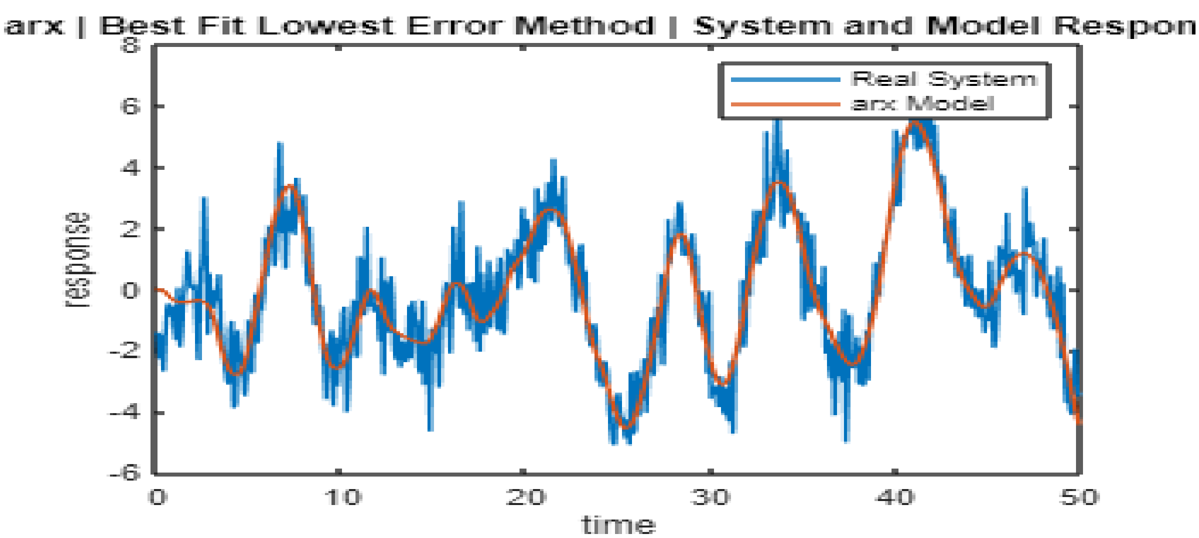
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

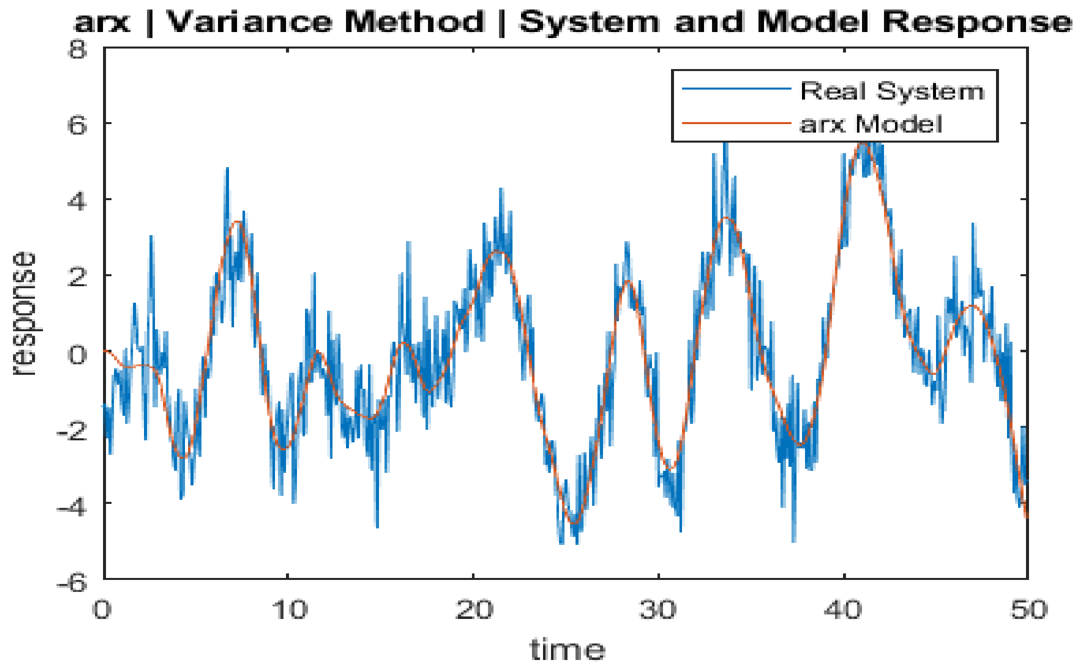
```
===============================================================
```

```
%%

arx_BestFitError = y_val - BestFit_y_hat;
arx_VarError = y_val - Var_y_hat;
arx_AICError = y_val - AIC_y_hat;
arx_FTestError = y_val - FTest_y_hat;

for k=0:N-1
    arx_BestFit_Ree(k+1,1) = AutoCorrelate(arx_BestFitError, k);
    arx_Var_Ree(k+1,1) = AutoCorrelate(arx_VarError, k);
```

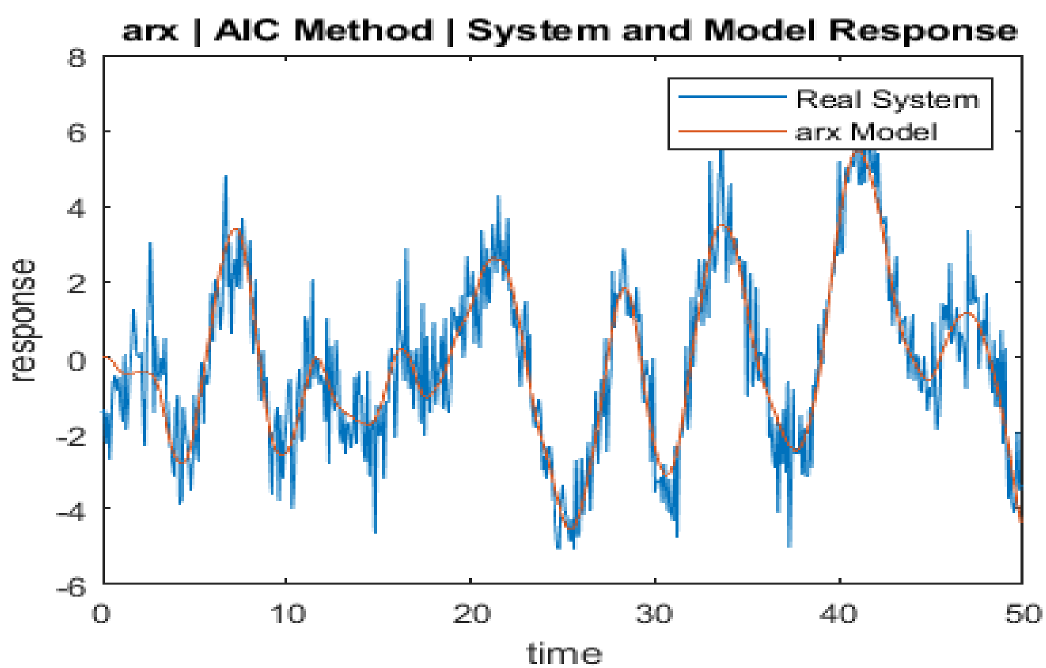
```
    arx_AIC_Ree(k+1,1) = AutoCorrelate(arx_AICError, k);
    arx_FTest_Ree(k+1,1) = AutoCorrelate(arx_FTestError, k);
end

for k=0:N-1
    arx_BestFit_Rue(k+1,1) = CrossCorrelate(u_val, arx_BestFitError, k);
    arx_Var_Rue(k+1,1) = CrossCorrelate(u_val, arx_VarError, k);
    arx_AIC_Rue(k+1,1) = CrossCorrelate(u_val, arx_AICError, k);
    arx_FTest_Rue(k+1,1) = CrossCorrelate(u_val, arx_FTestError, k);
end

%%
figure(1)
plot(t,y_val,t,BestFit_y_hat)
legend('Real System','arx Model')
title(" arx | Best Fit Lowest Error Method | System and Model Response")
xlabel("time")
ylabel("response")
```



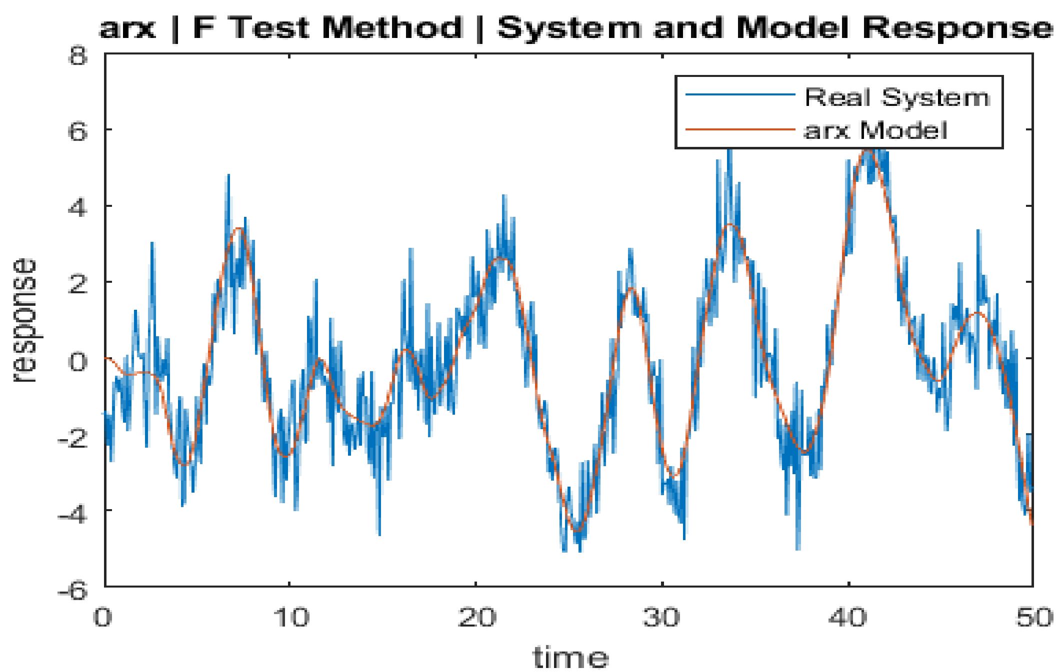

```
figure(2)
plot(t,y_val,t,Var_y_hat)
legend('Real System','arx Model')
title(" arx | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(3)
plot(t,y_val,t,AIC_y_hat)
legend('Real System','arx Model')
title(" arx | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

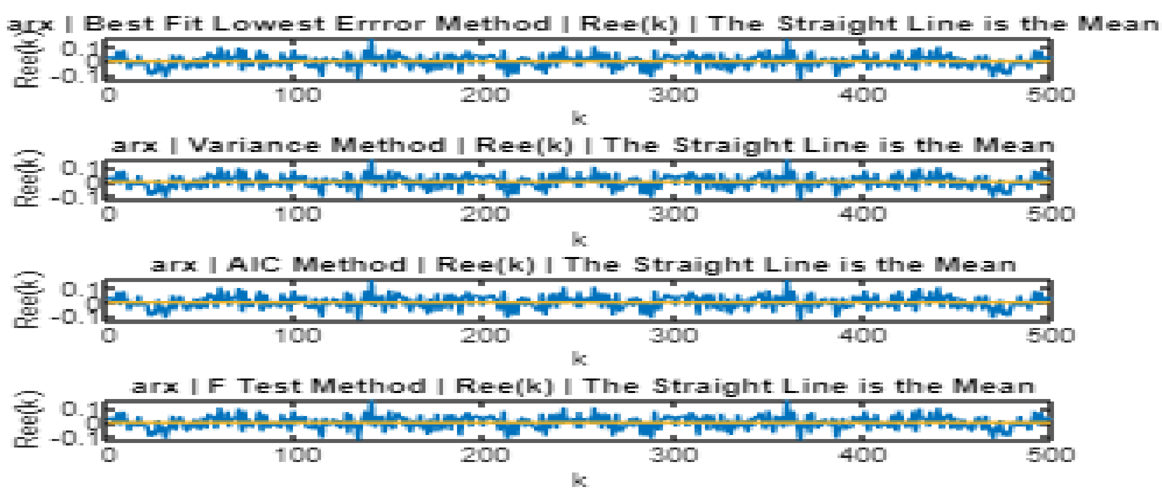


```
figure(4)
plot(t,y_val,t,FTest_y_hat)
legend('Real System','arx Model')
title(" arx | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



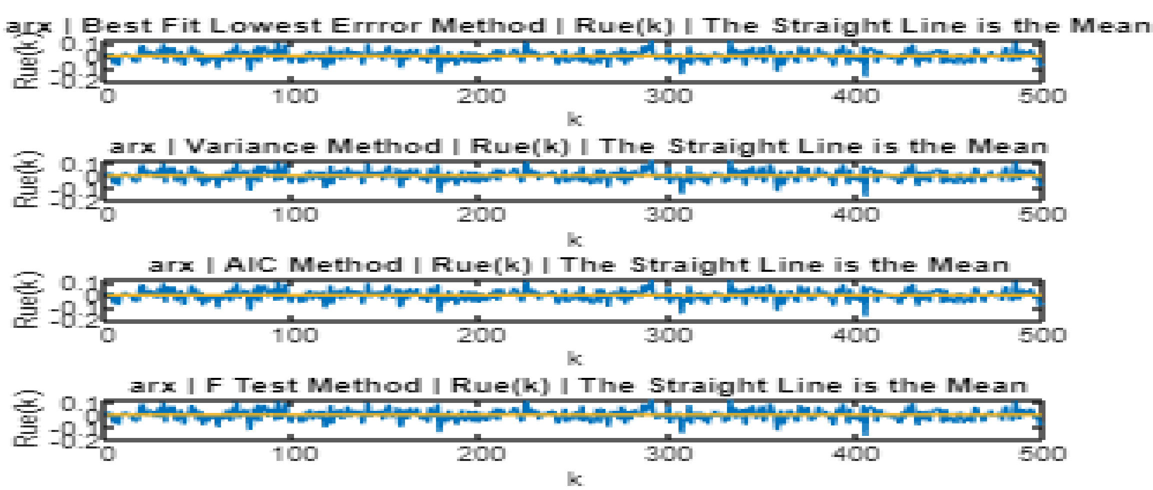

```
%%

figure(5)
subplot(4,1,1)
plot(1:N-1,arx_BestFit_Ree(2:end), 1:N-1,
mean(arx_BestFit_Ree(2:end))*ones(length(1:N-1)))
title(" arx | Best Fit Lowest Errror Method | Ree(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree(k)")

subplot(4,1,2)
plot(1:N-1,arx_Var_Ree(2:end), 1:N-1, mean(arx_Var_Ree(2:end))*ones(length(1:N-
1)))
title(" arx | Variance Method | Ree(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree(k)")
```

```
 subplot(4,1,3)
 plot(1:N-1,arx_AIC_Ree(2:end), 1:N-1, mean(arx_AIC_Ree(2:end))*ones(length(1:N-
1)))
 title(" arx | AIC Method | Ree(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree(k)")

 subplot(4,1,4)
 plot(1:N-1,arx_FTest_Ree(2:end), 1:N-1,
mean(arx_FTest_Ree(2:end))*ones(length(1:N-1)))
 title(" arx | F Test Method | Ree(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree(k)")
```




```
 %%

 figure(6)
 subplot(4,1,1)
 plot(1:N-1,arx_BestFit_Rue(2:end), 1:N-1,
mean(arx_BestFit_Rue(2:end))*ones(length(1:N-1)))
 title(" arx | Best Fit Lowest Errror Method | Rue(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue(k)")

 subplot(4,1,2)
 plot(1:N-1,arx_Var_Rue(2:end), 1:N-1, mean(arx_Var_Rue(2:end))*ones(length(1:N-
1)))
 title(" arx | Variance Method | Rue(k) | The Straight Line is the Mean")
```

```
xlabel("k")
ylabel("Rue(k)")

subplot(4,1,3)
plot(1:N-1,arx_AIC_Rue(2:end), 1:N-1, mean(arx_AIC_Rue(2:end))*ones(length(1:N-1)))
title(" arx | AIC Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")

subplot(4,1,4)
plot(1:N-1,arx_FTest_Rue(2:end), 1:N-1, mean(arx_FTest_Rue(2:end))*ones(length(1:N-1)))
title(" arx | F Test Method | Rue(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue(k)")
```




```
%%

figure(7)

denom = arx_AICModel(1:minAICIndex);
num = arx_AICModel(minAICIndex+1:minAICIndex*2);
G = tf(num', [1 denom'], 'Ts', Ts);
G_AIC_poles = pole(G);

fprintf(">>> We have used  u, y and v for model estimation under arx structure.\n")
```

```
>>> We have used  u, y and v for model estimation under arx structure.
```

```
fprintf("    Accordingly, the transfer function of the proper system degree is as follows:\n")
```

```
    Accordingly, the transfer function of the proper system degree is as follows:
```

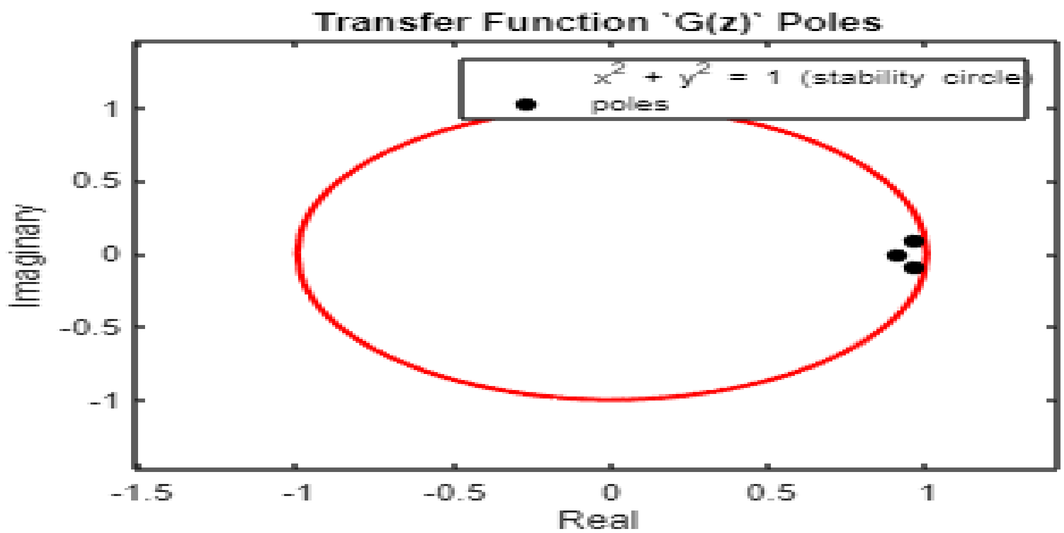
```
fprintf(" G(z) = \n")
```

```
G(z) =
```

```
disp(G)
```

```
  tf with properties:

      Numerator: {[0 0.0016 0.0062 0.0015]}
    Denominator: {[1 -2.8369 2.6900 -0.8521]}
       Variable: 'z'
        IODelay: 0
     InputDelay: 0
    OutputDelay: 0
             Ts: 0.1000
       TimeUnit: 'seconds'
      InputName: {''}
      InputUnit: {''}
     InputGroup: [1×1 struct]
     OutputName: {''}
     OutputUnit: {''}
    OutputGroup: [1×1 struct]
          Notes: [0×1 string]
       UserData: []
           Name: ''
   SamplingGrid: [1×1 struct]
```

```

xx = -1:0.01:+1;
yy = [sqrt(1-xx.^2), flip(-sqrt(1-xx.^2))];
xx = [xx flip(xx)];


plot(xx, yy, LineWidth=2, Color="red")
hold on
scatter(real(G_AIC_poles), imag(G_AIC_poles), 'fill', 'black')
legend("x^2 + y^2 = 1 (stability circle)", "poles")
title("Transfer Function `G(z)` Poles")
xlabel("Real")
ylabel("Imaginary")
```




```
fprintf(">>> It is shown in the figure that all the poles are in the stability circle area and \n")
```

```
>>> It is shown in the figure that all the poles are in the stability circle area and
```

```
fprintf("    hence, they are all stable. Therefre the estimated model in stable. \n")
```

```
    hence, they are all stable. Therefre the estimated model in stable.
```

Q1 - part d

```
clear; clc;
%%

load q1_402123100.mat

u_val = u(length(u)/2+1:end);
v_val = v(length(u)/2+1:end);
z_val = z(length(u)/2+1:end);
y_val = y(length(u)/2+1:end);

u = u(1:length(u)/2);
v = v(1:length(v)/2);
z = z(1:length(z)/2);
y = y(1:length(y)/2);


%%
Ts = 0.1;
t = 0:Ts:length(u)*Ts-Ts;
N = length(y);
data = iddata(z,y,Ts);
fprintf("========================================================================\n")
```

```
========================================================================
```

```
fprintf("========================================================================\n")
```

```
========================================================================
```

```


fprintf(">>> Taking the proper system degree and applying ARMAX model on the given data,\n")
```

```
>>> Taking the proper system degree and applying ARMAX model on the given data,
```

```
fprintf("    leads to the following estimated system:   \n")
```

```
    leads to the following estimated system:
```

```


%%


na = 1;
nb = 1;
```

```
nc = 1;

sys = armax(data, [na nb nc 1])
```

```
sys =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 0.9241 z^-1

  B(z) = 0.4168 z^-1

  C(z) = 1 - 0.04691 z^-1

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=1   nb=1   nc=1   nk=1
   Number of free coefficients: 3
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data".
Fit to estimation data: 93.48% (prediction focus)
FPE: 0.2473, MSE: 0.2443
```

```
armax_y_hat = lsim(sys, y_val, t);

[r2_armax, mse_armax] = rSQR(z_val, armax_y_hat);

error = y_val - armax_y_hat;
S_hat = 0;
for i=1:length(error)
    S_hat = S_hat + error(i)^2;
end

fprintf("=========================================================================\n")
```

```
========================================================================
```

```
fprintf(">>> The estimated model evaluation report is as the following: \n")
```

```
>>> The estimated model evaluation report is as the following:
```

```
fprintf("    R2 = %.4f  | mse = %.4f  | sse = %.4f  \n",r2_armax, mse_armax, S_hat)
```

```
    R2 = 0.9752  | mse = 1.7617  | sse = 26399.0567
```

```
fprintf("=========================================================================\n")
```

```
========================================================================
```

```
fprintf("========================================================================\n")
```

```
========================================================================
```

Q2 - part a | ARX

```
clc; clear
%%

load HW5_question2

u1 = Z1.u;
y1 = Z1.y;

u2 = Z2.u;
y2 = Z2.y;

u3 = Z3.u;
y3 = Z3.y;

u1_val = u1(601:end);
y1_val = y1(601:end);

u2_val = u2(601:end);
y2_val = y2(601:end);

u3_val = u1(601:end);
y3_val = y1(601:end);

u1 = u1(1:600);
y1 = y1(1:600);

u2 = u2(1:600);
y2 = y2(1:600);

u3 = u3(1:600);
y3 = y3(1:600);
```

System Z1

```
%%

% System Z1 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System I Identification Begins:------------------------------\n")
```

```
>>> System I Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);
%%

fprintf("===================Degree Extraction |
RUN==========================\n")
```

```
===================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    U = arx_U_builder_3(u1, y1, na, nb, nk);
    theta_hat_1 = inv(U'*U)*U'*y1;
    y_hat_1 = form_tf_lsim_2(theta_hat_1, u1, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y1, y_hat_1);

    error = y1 - y_hat_1;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    covariance = variance*inv(U'*U);
```

```
     cov = trace(covariance)/p;
     covs = [covs; cov];

     fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_arx, mse_arx, variance, S_hat)
     fprintf("------------------------------------------------------------\n")

     ps = [ps; p];
     R2s = [R2s; r2_arx];
     MSEs = [MSEs; mse_arx];
     vars = [vars; variance];
     S_hats = [S_hats; S_hat];
     AICs = [AICs; AIC];

 end
```

```
>>> Degree = 1 : R2=0.093926 | MSE=75.816369 | var=76.069935 | s_hat=45489.821374 |
------------------------------------------------------------
>>> Degree = 2 : R2=0.289808 | MSE=59.425779 | var=59.824610 | s_hat=35655.467584 |
------------------------------------------------------------
>>> Degree = 3 : R2=0.347672 | MSE=54.583989 | var=55.135342 | s_hat=32750.393402 |
------------------------------------------------------------
>>> Degree = 4 : R2=0.330092 | MSE=56.054978 | var=56.812477 | s_hat=33632.986514 |
------------------------------------------------------------
>>> Degree = 5 : R2=0.346183 | MSE=54.708603 | var=55.635867 | s_hat=32825.161577 |
------------------------------------------------------------
>>> Degree = 6 : R2=0.471854 | MSE=44.193019 | var=45.094918 | s_hat=26515.811643 |
------------------------------------------------------------
>>> Degree = 7 : R2=0.646119 | MSE=29.611268 | var=30.318704 | s_hat=17766.760571 |
------------------------------------------------------------
>>> Degree = 8 : R2=0.793634 | MSE=17.267848 | var=17.740940 | s_hat=10360.708996 |
------------------------------------------------------------
>>> Degree = 9 : R2=0.871345 | MSE=10.765263 | var=11.098209 | s_hat=6459.157794 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.898923 | MSE=8.457678 | var=8.749322 | s_hat=5074.607048 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.910834 | MSE=7.461007 | var=7.744990 | s_hat=4476.604262 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.916108 | MSE=7.019748 | var=7.312237 | s_hat=4211.848651 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.918121 | MSE=6.851283 | var=7.161620 | s_hat=4110.769836 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.919352 | MSE=6.748275 | var=7.078610 | s_hat=4048.964847 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.919832 | MSE=6.708127 | var=7.061186 | s_hat=4024.876049 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.919612 | MSE=6.726553 | var=7.105514 | s_hat=4035.932040 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.919029 | MSE=6.775321 | var=7.182319 | s_hat=4065.192792 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.918681 | MSE=6.804422 | var=7.238747 | s_hat=4082.653469 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.918954 | MSE=6.781570 | var=7.240110 | s_hat=4068.941875 |
```

```
-------------------------------------------------------------
>>> Degree = 20 : R2=0.919085 | MSE=6.770623 | var=7.254239 | s_hat=4062.373629 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.920108 | MSE=6.685008 | var=7.188181 | s_hat=4011.004949 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.920455 | MSE=6.655975 | var=7.182707 | s_hat=3993.585184 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.921769 | MSE=6.546063 | var=7.089599 | s_hat=3927.637913 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.922224 | MSE=6.507920 | var=7.073826 | s_hat=3904.752033 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.922725 | MSE=6.466009 | var=7.053828 | s_hat=3879.605344 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.923046 | MSE=6.439214 | var=7.050234 | s_hat=3863.528475 |
-------------------------------------------------------------
>>> Degree = 27 : R2=0.923093 | MSE=6.435243 | var=7.071695 | s_hat=3861.145694 |
-------------------------------------------------------------
>>> Degree = 28 : R2=0.923623 | MSE=6.390936 | var=7.048826 | s_hat=3834.561587 |
-------------------------------------------------------------
>>> Degree = 29 : R2=0.924193 | MSE=6.343210 | var=7.022004 | s_hat=3805.926120 |
-------------------------------------------------------------
>>> Degree = 30 : R2=0.923959 | MSE=6.362812 | var=7.069791 | s_hat=3817.686976 |
-------------------------------------------------------------
>>> Degree = 31 : R2=0.923962 | MSE=6.362516 | var=7.095742 | s_hat=3817.509377 |
-------------------------------------------------------------
>>> Degree = 32 : R2=0.923815 | MSE=6.374858 | var=7.136035 | s_hat=3824.914791 |
-------------------------------------------------------------
>>> Degree = 33 : R2=0.923923 | MSE=6.365763 | var=7.152543 | s_hat=3819.458009 |
-------------------------------------------------------------
>>> Degree = 34 : R2=0.924527 | MSE=6.315225 | var=7.122434 | s_hat=3789.135118 |
-------------------------------------------------------------
>>> Degree = 35 : R2=0.924718 | MSE=6.299253 | var=7.131230 | s_hat=3779.551963 |
-------------------------------------------------------------
>>> Degree = 36 : R2=0.925586 | MSE=6.226656 | var=7.075745 | s_hat=3735.993427 |
-------------------------------------------------------------
>>> Degree = 37 : R2=0.925741 | MSE=6.213635 | var=7.087797 | s_hat=3728.181058 |
-------------------------------------------------------------
>>> Degree = 38 : R2=0.925689 | MSE=6.218064 | var=7.119921 | s_hat=3730.838654 |
-------------------------------------------------------------
>>> Degree = 39 : R2=0.925931 | MSE=6.197805 | var=7.123913 | s_hat=3718.682840 |
-------------------------------------------------------------
>>> Degree = 40 : R2=0.926073 | MSE=6.185893 | var=7.137569 | s_hat=3711.536096 |
-------------------------------------------------------------
>>> Degree = 41 : R2=0.925783 | MSE=6.210189 | var=7.193269 | s_hat=3726.113493 |
-------------------------------------------------------------
>>> Degree = 42 : R2=0.925773 | MSE=6.210969 | var=7.222057 | s_hat=3726.581411 |
-------------------------------------------------------------
>>> Degree = 43 : R2=0.926281 | MSE=6.168512 | var=7.200598 | s_hat=3701.107322 |
-------------------------------------------------------------
>>> Degree = 44 : R2=0.926282 | MSE=6.168421 | var=7.228618 | s_hat=3701.052397 |
-------------------------------------------------------------
>>> Degree = 45 : R2=0.926534 | MSE=6.147345 | var=7.232171 | s_hat=3688.406965 |
-------------------------------------------------------------
>>> Degree = 46 : R2=0.926574 | MSE=6.143965 | var=7.256651 | s_hat=3686.378907 |
-------------------------------------------------------------
>>> Degree = 47 : R2=0.926672 | MSE=6.135785 | var=7.275634 | s_hat=3681.470842 |
-------------------------------------------------------------
```

```
>>> Degree = 48 : R2=0.926693 | MSE=6.134042 | var=7.302431 | s_hat=3680.424983 |
-------------------------------------------------------------
>>> Degree = 49 : R2=0.927089 | MSE=6.100851 | var=7.291854 | s_hat=3660.510771 |
-------------------------------------------------------------
>>> Degree = 50 : R2=0.927059 | MSE=6.103419 | var=7.324103 | s_hat=3662.051305 |
-------------------------------------------------------------
>>> Degree = 51 : R2=0.927157 | MSE=6.095219 | var=7.343637 | s_hat=3657.131291 |
-------------------------------------------------------------
>>> Degree = 52 : R2=0.926952 | MSE=6.112340 | var=7.393960 | s_hat=3667.404266 |
-------------------------------------------------------------
>>> Degree = 53 : R2=0.926903 | MSE=6.116471 | var=7.428912 | s_hat=3669.882380 |
-------------------------------------------------------------
>>> Degree = 54 : R2=0.926999 | MSE=6.108414 | var=7.449286 | s_hat=3665.048627 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.927077 | MSE=6.101908 | var=7.471724 | s_hat=3661.144618 |
-------------------------------------------------------------
>>> Degree = 56 : R2=0.927382 | MSE=6.076376 | var=7.470954 | s_hat=3645.825632 |
-------------------------------------------------------------
>>> Degree = 57 : R2=0.927550 | MSE=6.062339 | var=7.484369 | s_hat=3637.403324 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.927543 | MSE=6.062880 | var=7.515967 | s_hat=3637.727986 |
-------------------------------------------------------------
>>> Degree = 59 : R2=0.927631 | MSE=6.055560 | var=7.538041 | s_hat=3633.335936 |
-------------------------------------------------------------
>>> Degree = 60 : R2=0.927621 | MSE=6.056369 | var=7.570462 | s_hat=3633.821577 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.927471 | MSE=6.068881 | var=7.617842 | s_hat=3641.328606 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.927753 | MSE=6.045284 | var=7.620106 | s_hat=3627.170277 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.927885 | MSE=6.034254 | var=7.638297 | s_hat=3620.552575 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.927886 | MSE=6.034166 | var=7.670549 | s_hat=3620.499355 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.927986 | MSE=6.025855 | var=7.692580 | s_hat=3615.512798 |
-------------------------------------------------------------
>>> Degree = 66 : R2=0.927979 | MSE=6.026407 | var=7.726163 | s_hat=3615.844321 |
-------------------------------------------------------------
>>> Degree = 67 : R2=0.928016 | MSE=6.023313 | var=7.755338 | s_hat=3613.987591 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.927981 | MSE=6.026217 | var=7.792522 | s_hat=3615.730249 |
-------------------------------------------------------------
>>> Degree = 69 : R2=0.928349 | MSE=5.995428 | var=7.786269 | s_hat=3597.256505 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.928340 | MSE=5.996215 | var=7.821150 | s_hat=3597.728811 |
-------------------------------------------------------------
>>> Degree = 71 : R2=0.928499 | MSE=5.982901 | var=7.837862 | s_hat=3589.740822 |
-------------------------------------------------------------
>>> Degree = 72 : R2=0.928543 | MSE=5.979236 | var=7.867415 | s_hat=3587.541359 |
-------------------------------------------------------------
>>> Degree = 73 : R2=0.928334 | MSE=5.996705 | var=7.925160 | s_hat=3598.022757 |
-------------------------------------------------------------
>>> Degree = 74 : R2=0.928342 | MSE=5.996054 | var=7.959364 | s_hat=3597.632695 |
-------------------------------------------------------------
>>> Degree = 75 : R2=0.928442 | MSE=5.987691 | var=7.983588 | s_hat=3592.614751 |
-------------------------------------------------------------
>>> Degree = 76 : R2=0.928910 | MSE=5.948473 | var=7.966705 | s_hat=3569.083757 |
```

```
--------------------------------------------------------------
>>> Degree = 77 : R2=0.928845 | MSE=5.953912 | var=8.009747 | s_hat=3572.347154 |
--------------------------------------------------------------
>>> Degree = 78 : R2=0.928888 | MSE=5.950320 | var=8.040973 | s_hat=3570.191908 |
--------------------------------------------------------------
>>> Degree = 79 : R2=0.928701 | MSE=5.965982 | var=8.098618 | s_hat=3579.588973 |
--------------------------------------------------------------
>>> Degree = 80 : R2=0.928807 | MSE=5.957125 | var=8.123352 | s_hat=3574.274857 |
--------------------------------------------------------------
>>> Degree = 81 : R2=0.928918 | MSE=5.947836 | var=8.147720 | s_hat=3568.701400 |
--------------------------------------------------------------
>>> Degree = 82 : R2=0.929712 | MSE=5.881378 | var=8.093640 | s_hat=3528.827051 |
--------------------------------------------------------------
>>> Degree = 83 : R2=0.929554 | MSE=5.894578 | var=8.149187 | s_hat=3536.746943 |
--------------------------------------------------------------
>>> Degree = 84 : R2=0.931689 | MSE=5.716006 | var=7.938897 | s_hat=3429.603511 |
--------------------------------------------------------------
>>> Degree = 85 : R2=0.931693 | MSE=5.715647 | var=7.975321 | s_hat=3429.387954 |
--------------------------------------------------------------
>>> Degree = 86 : R2=0.931635 | MSE=5.720502 | var=8.019395 | s_hat=3432.301273 |
--------------------------------------------------------------
>>> Degree = 87 : R2=0.931742 | MSE=5.711497 | var=8.044362 | s_hat=3426.898097 |
--------------------------------------------------------------
>>> Degree = 88 : R2=0.931743 | MSE=5.711436 | var=8.082221 | s_hat=3426.861513 |
--------------------------------------------------------------
>>> Degree = 89 : R2=0.931848 | MSE=5.702645 | var=8.108027 | s_hat=3421.587251 |
--------------------------------------------------------------
>>> Degree = 90 : R2=0.932011 | MSE=5.689025 | var=8.127179 | s_hat=3413.415068 |
--------------------------------------------------------------
>>> Degree = 91 : R2=0.931955 | MSE=5.693676 | var=8.172741 | s_hat=3416.205757 |
--------------------------------------------------------------
>>> Degree = 92 : R2=0.931974 | MSE=5.692110 | var=8.209774 | s_hat=3415.266140 |
--------------------------------------------------------------
>>> Degree = 93 : R2=0.931838 | MSE=5.703529 | var=8.265984 | s_hat=3422.117360 |
--------------------------------------------------------------
>>> Degree = 94 : R2=0.931850 | MSE=5.702479 | var=8.304582 | s_hat=3421.487665 |
--------------------------------------------------------------
>>> Degree = 95 : R2=0.932442 | MSE=5.652928 | var=8.272577 | s_hat=3391.756572 |
--------------------------------------------------------------
>>> Degree = 96 : R2=0.933055 | MSE=5.601707 | var=8.237805 | s_hat=3361.024405 |
--------------------------------------------------------------
>>> Degree = 97 : R2=0.933086 | MSE=5.599090 | var=8.274517 | s_hat=3359.453769 |
--------------------------------------------------------------
>>> Degree = 98 : R2=0.933285 | MSE=5.582403 | var=8.290697 | s_hat=3349.441735 |
--------------------------------------------------------------
>>> Degree = 99 : R2=0.933291 | MSE=5.581959 | var=8.331281 | s_hat=3349.175149 |
--------------------------------------------------------------
>>> Degree = 100 : R2=0.933489 | MSE=5.565335 | var=8.348002 | s_hat=3339.200954 |
--------------------------------------------------------------
```

```
fprintf("==============================================================\n")
```

```
==============================================================
```

```

```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 100
```

```
na = bestFitDegree;
nb = bestFitDegree;
p = na+nb;

BestFitU = arx_U_builder_3(u1, y1, na, nb, nk);
BestFitModel_1 = inv(BestFitU'*BestFitU)*BestFitU'*y1;
BestFit_y_hat_1 = form_tf_lsim_2(BestFitModel_1, u1_val, t_val, na, Ts);
```

```
G =


    0.94 z^-1 + 1.568 z^-2 + 2.597 z^-3 + 3.04 z^-4 + 2.671 z^-5 + 1.966 z^-6 + 0.966 z^-7 +
0.2992 z^-8

            - 0.4131 z^-9 - 0.5001 z^-10 - 0.6905 z^-11 - 0.268 z^-12 + 0.2823 z^-13 + 1.149
z^-14

            + 1.293 z^-15 + 1.618 z^-16 + 1.706 z^-17 + 1.452 z^-18 + 0.9768 z^-19 + 0.4067
z^-20

            - 0.03759 z^-21 - 0.4725 z^-22 - 0.7541 z^-23 - 0.3856 z^-24 - 0.106 z^-25 +
0.3588 z^-26

            + 0.8493 z^-27 + 1.123 z^-28 + 1.601 z^-29 + 1.3 z^-30 + 0.9205 z^-31 + 0.3807 z^

            -32 - 0.03072 z^-33 - 0.1438 z^-34 - 0.01266 z^-35 + 0.03153 z^-36 + 0.1131 z^-37

            + 0.2729 z^-38 + 0.4989 z^-39 + 0.7472 z^-40 + 0.851 z^-41 + 0.9306 z^-42 + 0.7891
z^-43

            + 0.8114 z^-44 + 0.454 z^-45 + 0.4713 z^-46 + 0.1362 z^-47 + 0.02659 z^-48 +
0.03371 z^-49

            + 0.3674 z^-50 + 0.138 z^-51 + 0.2825 z^-52 + 0.3897 z^-53 + 0.5174 z^-54 + 0.3517
z^-55
```

```
           + 0.3324 z^-56 + 0.2536 z^-57 + 0.2346 z^-58 + 0.1829 z^-59 + 0.1045 z^-60 +
0.1062 z^-61

           + 0.0506 z^-62 + 0.01309 z^-63 - 0.2856 z^-64 - 0.1684 z^-65 - 0.2471 z^-66 +
0.04224 z^-67

           + 0.3482 z^-68 + 0.3196 z^-69 + 0.522 z^-70 + 0.2485 z^-71 + 0.07725 z^-72 +
0.06661 z^-73

           + 0.005874 z^-74 - 0.1495 z^-75 + 0.04349 z^-76 - 0.2177 z^-77 - 0.1402 z^-78 -
0.2076 z^-79

           + 0.1776 z^-80 + 0.1383 z^-81 + 0.6933 z^-82 + 0.5439 z^-83 + 1.129 z^-84 + 0.6287
z^-85

           + 0.402 z^-86 + 0.3177 z^-87 + 0.199 z^-88 + 0.3411 z^-89 + 0.1928 z^-90 -
0.005171 z^-91

           + 0.0265 z^-92 - 0.02472 z^-93 - 0.06465 z^-94 - 0.2622 z^-95 - 0.3607 z^-96 -
0.1151 z^-97

                                                        - 0.3546 z^-98 + 0.09927 z^-99 +
0.08102 z^-100

  ---------------------------------------------------------------------------------------------
-------------

  1 - 0.06311 z^-1 - 0.02543 z^-2 + 0.01002 z^-3 - 0.09978 z^-4 + 0.01837 z^-5 + 0.04042 z^

          -6 - 0.009701 z^-7 - 0.02255 z^-8 - 0.05689 z^-9 - 0.03167 z^-10 + 0.0232 z^-11 -
0.04709 z^-12

          + 0.000582 z^-13 + 0.0557 z^-14 - 0.05256 z^-15 + 0.03889 z^-16 + 0.02455 z^-17 -
0.1323 z^-18

          - 0.07003 z^-19 - 0.009537 z^-20 - 0.05316 z^-21 + 0.03556 z^-22 - 0.04249 z^-23

          + 0.03666 z^-24 + 0.0185 z^-25 + 0.02833 z^-26 + 0.03233 z^-27 - 0.07104 z^-28 +
0.02414 z^-29

          + 0.0009001 z^-30 - 0.02458 z^-31 + 0.06785 z^-32 + 0.004769 z^-33 - 0.049 z^-34

          + 0.0182 z^-35 - 0.009848 z^-36 + 0.005082 z^-37 + 0.08173 z^-38 - 0.04469 z^-39

          - 0.01113 z^-40 + 0.08218 z^-41 - 0.01772 z^-42 + 0.06545 z^-43 - 0.02169 z^-44 -
0.05914 z^-45

          + 0.005515 z^-46 - 0.01955 z^-47 + 0.03617 z^-48 - 0.02444 z^-49 - 0.1273 z^-50 +
0.1109 z^-51

          - 0.11 z^-52 + 0.05062 z^-53 - 0.01956 z^-54 + 0.006906 z^-55 - 0.03579 z^-56 +
0.02591 z^-57

          + 0.02944 z^-58 - 0.02158 z^-59 + 0.01294 z^-60 - 0.09403 z^-61 - 0.09322 z^-62 -
0.02506 z^-63
```

```
          + 0.06338 z^-64 + 0.04705 z^-65 + 0.01249 z^-66 + 0.04194 z^-67 - 0.09699 z^-68 -
0.05719 z^-69

          + 0.02829 z^-70 + 0.01783 z^-71 + 0.04196 z^-72 - 0.05303 z^-73 - 0.01487 z^-74 -
0.005255 z^-75

          - 0.04786 z^-76 - 0.05136 z^-77 + 0.1137 z^-78 - 0.02766 z^-79 + 0.1027 z^-80 +
0.05717 z^-81

          - 0.046 z^-82 + 0.08842 z^-83 - 0.01659 z^-84 + 0.003666 z^-85 + 0.1213 z^-86 +
0.02938 z^-87

          - 0.07891 z^-88 - 0.01712 z^-89 - 0.03439 z^-90 + 0.0335 z^-91 - 0.009517 z^-92 -
0.1504 z^-93

          + 0.01574 z^-94 - 0.05125 z^-95 + 0.0189 z^-96 + 0.05031 z^-97 - 0.06184 z^-98 -
0.08013 z^-99

                                                                               + 0.03978
z^-100


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 29 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 29
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb;
```

```
VarU = arx_U_builder_3(u1, y1, na, nb, nk);
VarModel_1 = inv(VarU'*VarU)*VarU'*y1;
Var_y_hat_1 = form_tf_lsim_2(VarModel_1, u1_val, t_val, na, Ts);
```

```
G =


    0.8255 z^-1 + 1.592 z^-2 + 2.477 z^-3 + 2.828 z^-4 + 2.448 z^-5 + 1.764 z^-6 + 0.8116 z^-7

            + 0.2123 z^-8 - 0.3953 z^-9 - 0.52 z^-10 - 0.647 z^-11 - 0.2579 z^-12 + 0.275 z^-
13

            + 0.9874 z^-14 + 0.9923 z^-15 + 1.12 z^-16 + 1.268 z^-17 + 0.9681 z^-18 + 0.5871
z^-19

            + 0.1328 z^-20 + 0.111 z^-21 - 0.2979 z^-22 - 0.5182 z^-23 - 0.1875 z^-24 - 0.1129
z^-25

                                         - 0.09271 z^-26 + 0.07795 z^-27 - 0.06464 z^-28 +
0.4046 z^-29

  ---------------------------------------------------------------------------------------------
------------

  1 - 0.08692 z^-1 - 0.07726 z^-2 - 0.0006861 z^-3 - 0.06111 z^-4 + 0.02097 z^-5 + 0.01372 z^-
6

          + 0.02483 z^-7 - 0.02023 z^-8 - 0.0746 z^-9 - 0.02921 z^-10 - 0.003711 z^-11 -
0.07011 z^-12

          - 0.03507 z^-13 + 0.01708 z^-14 - 0.05828 z^-15 + 0.01111 z^-16 + 0.04315 z^-17 -
0.07129 z^-18

          - 0.06256 z^-19 + 0.01995 z^-20 - 0.05952 z^-21 + 0.00522 z^-22 - 0.08864 z^-23 -
0.03467 z^-24

                          - 0.05783 z^-25 - 0.04314 z^-26 - 0.01856 z^-27 - 0.03964 z^-28 +
0.02934 z^-29


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | CoVariance
Method================\n")
```

===============Degree Extraction | CoVariance Method=================

```
maxCovIndex = find(covs == min(covs));
fprintf(">>> Since the minimum CovMatrix trace occurs in iteration %d ;\n", maxCovIndex)
```

```
>>> Since the minimum CovMatrix trace occurs in iteration 11 ;
```

```
fprintf("    Degree = %d \n", maxCovIndex)
```

```
    Degree = 11
```

```
na = maxCovIndex;
nb = maxCovIndex;
p = na+nb;

CovU_1 = arx_U_builder_3(u1,y1,na,nb,1);
CovModel_1 = inv(CovU_1'*CovU_1)*CovU_1'*y1;

Cov_y_hat_1 = form_tf_lsim_2(CovModel_1, u1_val, t_val, na, Ts);
```

```
G =

    0.7172 z^-1 + 1.499 z^-2 + 2.178 z^-3 + 2.183 z^-4 + 1.503 z^-5 + 0.5843 z^-6 - 0.2748 z^
                                               -7 - 0.5856 z^-8 - 0.7757 z^-9 - 0.4399 z^-10 -
0.3997 z^-11

  ------------------------------------------------------------------------------------------
-----------

  1 - 0.2167 z^-1 - 0.1972 z^-2 - 0.1151 z^-3 - 0.1244 z^-4 + 0.04059 z^-5 + 0.08103 z^-6 +
0.08924 z^-7

                                               - 0.01349 z^-8 - 0.1021 z^-9 - 0.1507 z^-10 -
0.2102 z^-11


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
```

```
fprintf("===============Degree Extraction | AIC Method===================\n")
```

```
===============Degree Extraction | AIC Method===================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n", k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 15 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 15
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb;

AICU_1 = arx_U_builder_3(u1, y1, na, nb, nk);
AICModel_1 = inv(AICU_1'*AICU_1)*AICU_1'*y1;
AIC_y_hat_1 = form_tf_lsim_2(AICModel_1, u1_val, t_val, na, Ts);
```

```
G =

  0.7302 z^-1 + 1.531 z^-2 + 2.424 z^-3 + 2.708 z^-4 + 2.254 z^-5 + 1.565 z^-6 + 0.6697 z^-7

          + 0.1183 z^-8 - 0.4081 z^-9 - 0.5127 z^-10 - 0.6912 z^-11 - 0.4428 z^-12 - 0.1676 z^-13

                                                                        + 0.229 z^-14 - 0.002193 z^-15

  --------------------------------------------------------------------------------------------------

  1 - 0.1098 z^-1 - 0.1039 z^-2 - 0.02947 z^-3 - 0.06636 z^-4 + 0.04095 z^-5 + 0.04625 z^-6

          + 0.02382 z^-7 - 0.04502 z^-8 - 0.1033 z^-9 - 0.0914 z^-10 - 0.09036 z^-11 - 0.1606 z^-12

                                                        - 0.1038 z^-13 - 0.01427 z^-14 - 0.07528 z^-15


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", winScore)
```

```
>>> The F test is suggesting the best model with the m=1.22 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 14
```

```
na = winner;
nb = winner;
p = na+nb;

FTestU_1 = arx_U_builder_3(u1, y1, na, nb, nk);
FTestModel_1 = inv(FTestU_1'*FTestU_1)*FTestU_1'*y1;
FTest_y_hat_1 = form_tf_lsim_2(FTestModel_1, u1_val, t_val, na, Ts);
```

```
G =

  0.7264 z^-1 + 1.522 z^-2 + 2.405 z^-3 + 2.671 z^-4 + 2.177 z^-5 + 1.428 z^-6 + 0.5016 z^-7

        - 0.05858 z^-8 - 0.5852 z^-9 - 0.6322 z^-10 - 0.7477 z^-11 - 0.4356 z^-12 - 0.0874 z^-13
```

```
                                                                                          +
0.312 z^-14

  -------------------------------------------------------------------------------------------
-------

  1 - 0.1144 z^-1 - 0.1159 z^-2 - 0.04627 z^-3 - 0.07898 z^-4 + 0.02844 z^-5 + 0.03464 z^-6

          + 0.02004 z^-7 - 0.03901 z^-8 - 0.09249 z^-9 - 0.07923 z^-10 - 0.0901 z^-11 - 0.1615
z^-12

                                                                       - 0.1184 z^-13 -
0.03741 z^-14


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[BestFit_r2, BestFit_mse] = rSQR(y1_val, BestFit_y_hat_1);
[Var_r2, Var_mse] = rSQR(y1_val, Var_y_hat_1);
[AIC_r2, AIC_mse] = rSQR(y1_val, AIC_y_hat_1);
[Cov_r2, Cov_mse] = rSQR(y1_val, Cov_y_hat_1);
[FTest_r2, FTest_mse] = rSQR(y1_val, FTest_y_hat_1);

fprintf("===========================System III===========================\n")
```

```
===========================System III===========================
```

```
fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.6424    | MSE : 26.4444
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.6718    | MSE : 24.2699
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Covariance Method:\n")
```

```
>>> Covariance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Cov_r2, Cov_mse)
```

```
    R2 value : 0.7094    | MSE : 21.4933
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.6904    | MSE : 22.8949
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.6946    | MSE : 22.5857
```

```
fprintf("---------------------------------------------------------------\n")
```

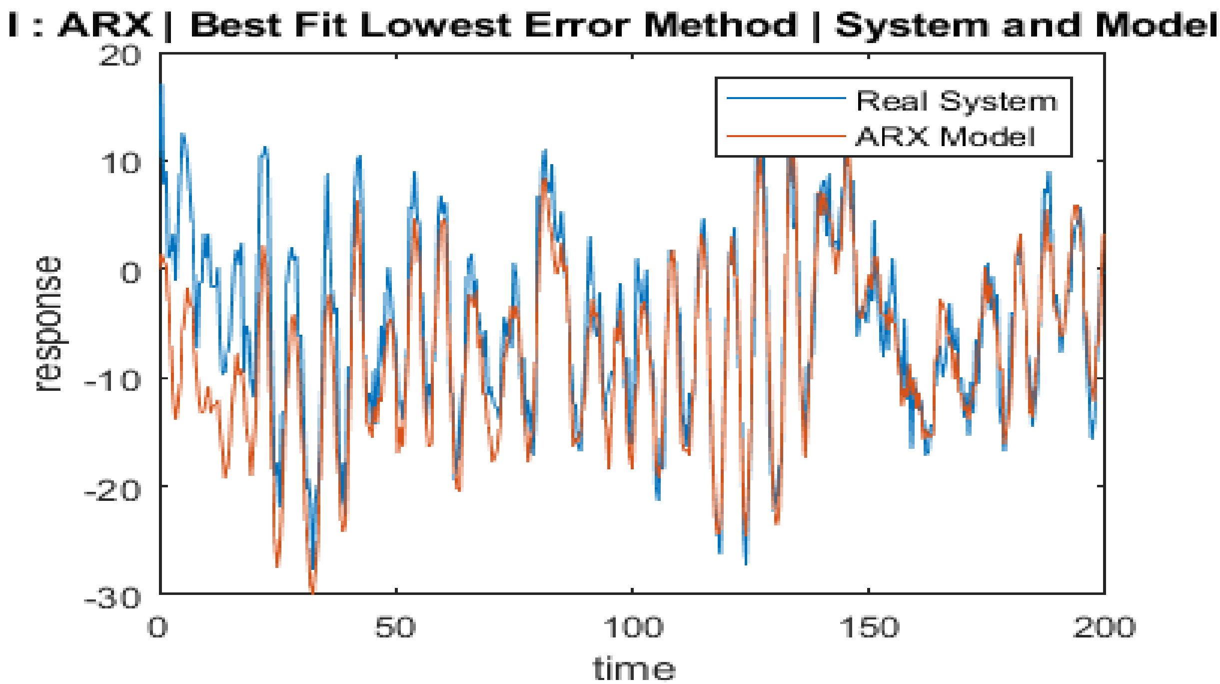
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
```

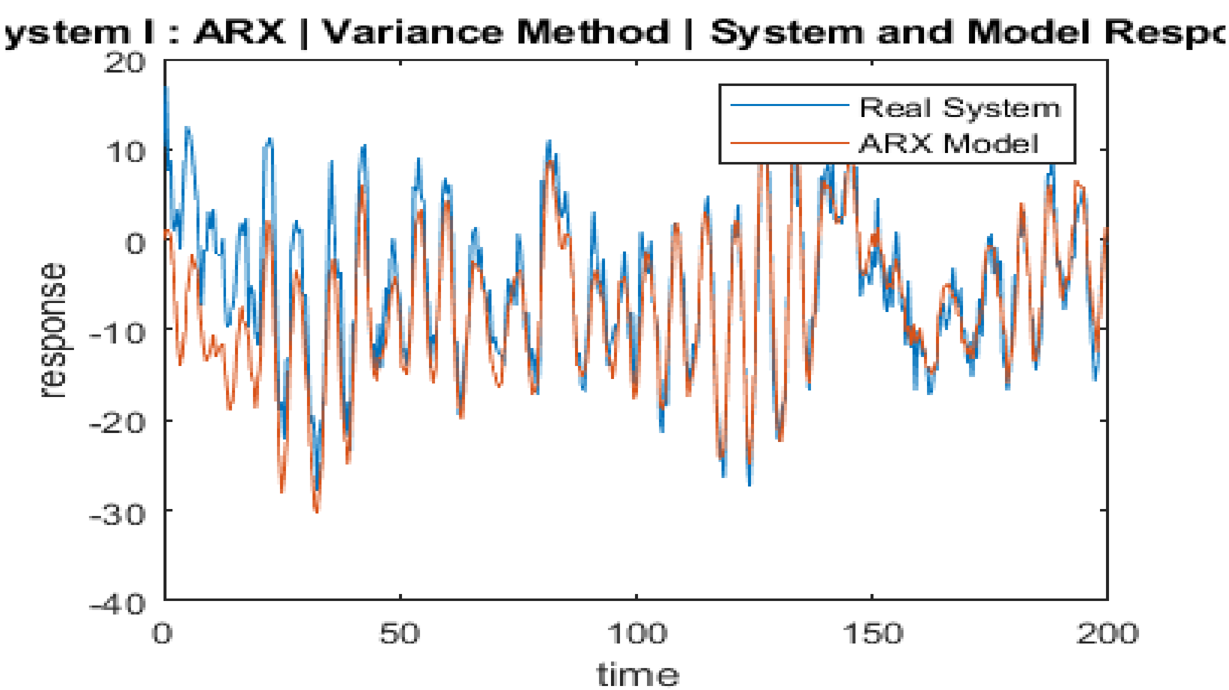
```
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

```
===============================================================
```

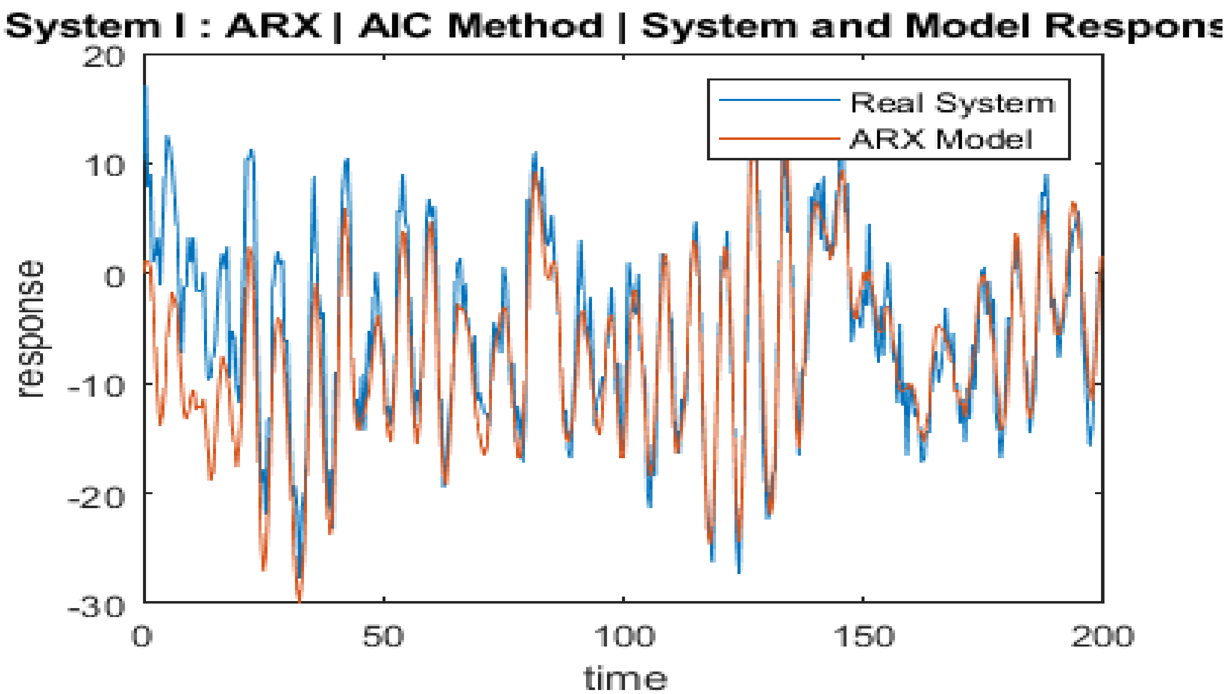
```



%%

BestFitError_1 = y1_val - BestFit_y_hat_1;
VarError_1 = y1_val - Var_y_hat_1;
CovError_1 = y1_val - Cov_y_hat_1;
AICError_1 = y1_val - AIC_y_hat_1;
FTestError_1 = y1_val - FTest_y_hat_1;

for k=0:N_val-1
    BestFit_Ree_1(k+1,1) = AutoCorrelate(BestFitError_1, k);
    Var_Ree_1(k+1,1) = AutoCorrelate(VarError_1, k);
    Cov_Ree_1(k+1,1) = AutoCorrelate(CovError_1, k);
    AIC_Ree_1(k+1,1) = AutoCorrelate(AICError_1, k);
    FTest_Ree_1(k+1,1) = AutoCorrelate(FTestError_1, k);
end

for k=0:N_val-1
    BestFit_Rue_1(k+1,1) = CrossCorrelate(u1_val, BestFitError_1, k);
    Var_Rue_1(k+1,1) = CrossCorrelate(u1_val, VarError_1, k);
    Cov_Rue_1(k+1,1) = CrossCorrelate(u1_val, CovError_1, k);
    AIC_Rue_1(k+1,1) = CrossCorrelate(u1_val, AICError_1, k);
    FTest_Rue_1(k+1,1) = CrossCorrelate(u1_val, FTestError_1, k);
end


%%
figure()  % figure(1)
plot(t_val,y1_val,t_val,BestFit_y_hat_1)
legend('Real System','ARX Model')
title(" System I : ARX | Best Fit Lowest Error Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



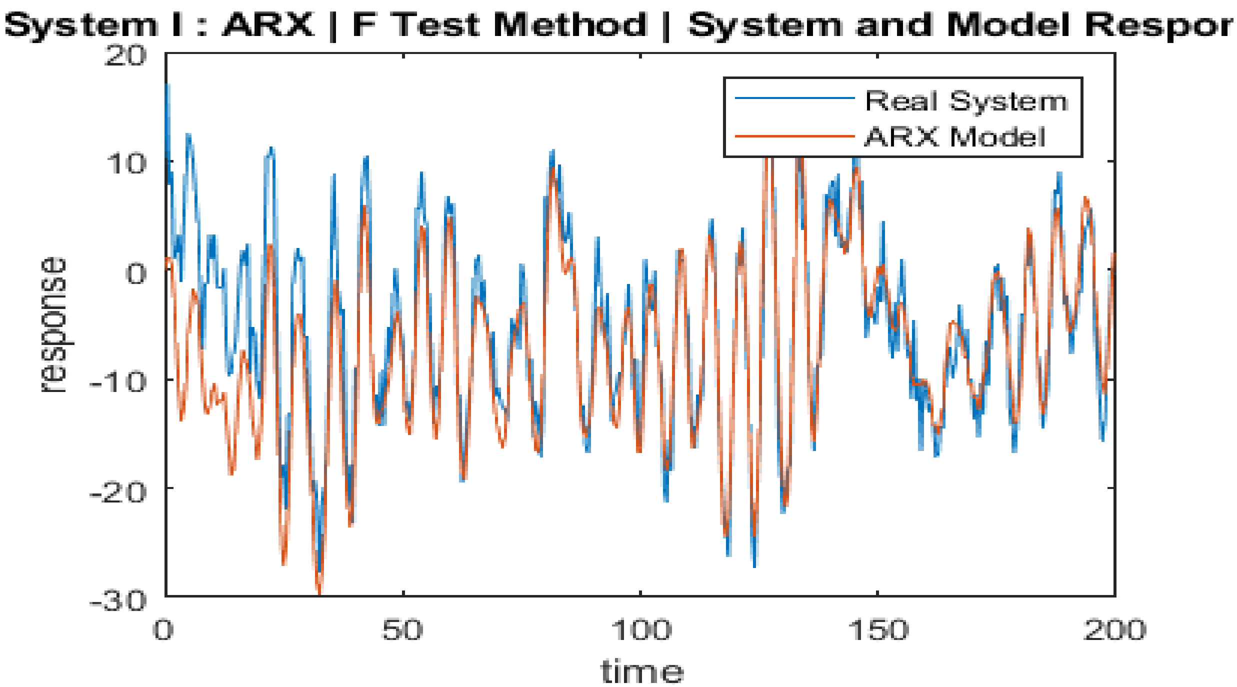

```
figure()  % figure(2)
plot(t_val,y1_val,t_val,Var_y_hat_1)
legend('Real System','ARX Model')
title(" System I : ARX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```



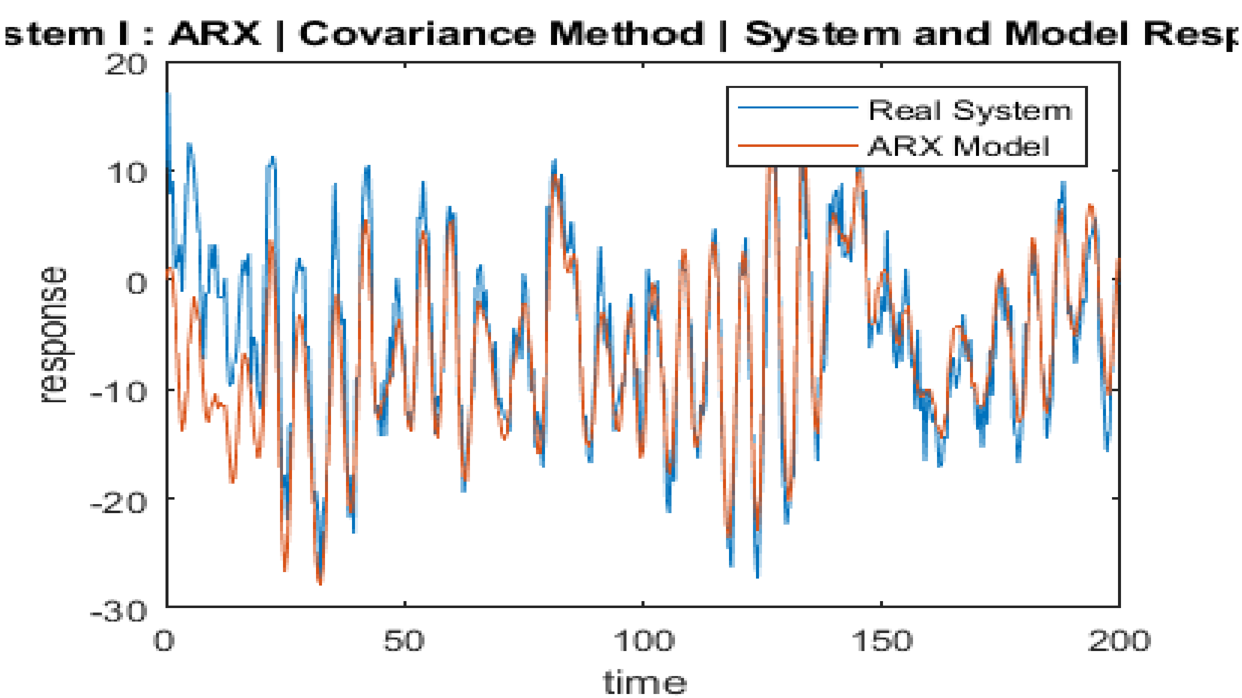
```
figure()  % figure(3)
plot(t_val,y1_val,t_val,AIC_y_hat_1)
legend('Real System','ARX Model')
title(" System I : ARX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```



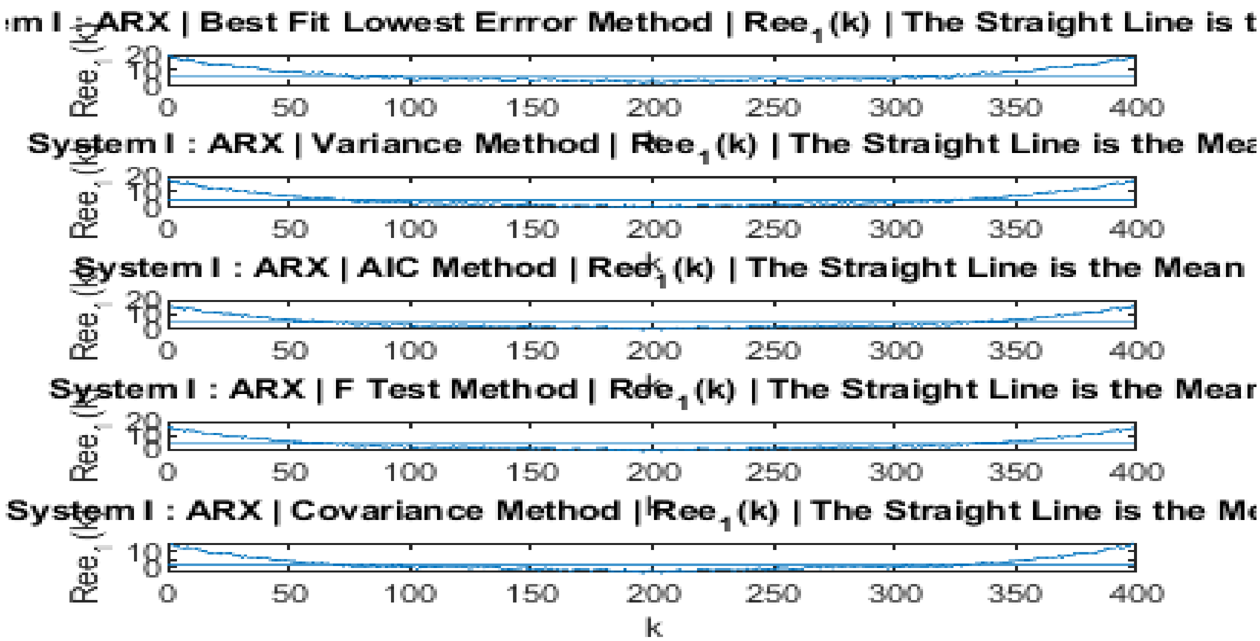

```
figure()  % figure(4)
plot(t_val,y1_val,t_val,FTest_y_hat_1)
legend('Real System','ARX Model')
title(" System I : ARX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure()  % figure(7)
plot(t_val,y1_val,t_val,Cov_y_hat_1)
legend('Real System','ARX Model')
title(" System I : ARX | Covariance Method | System and Model Response")
xlabel("time")
ylabel("response")
```

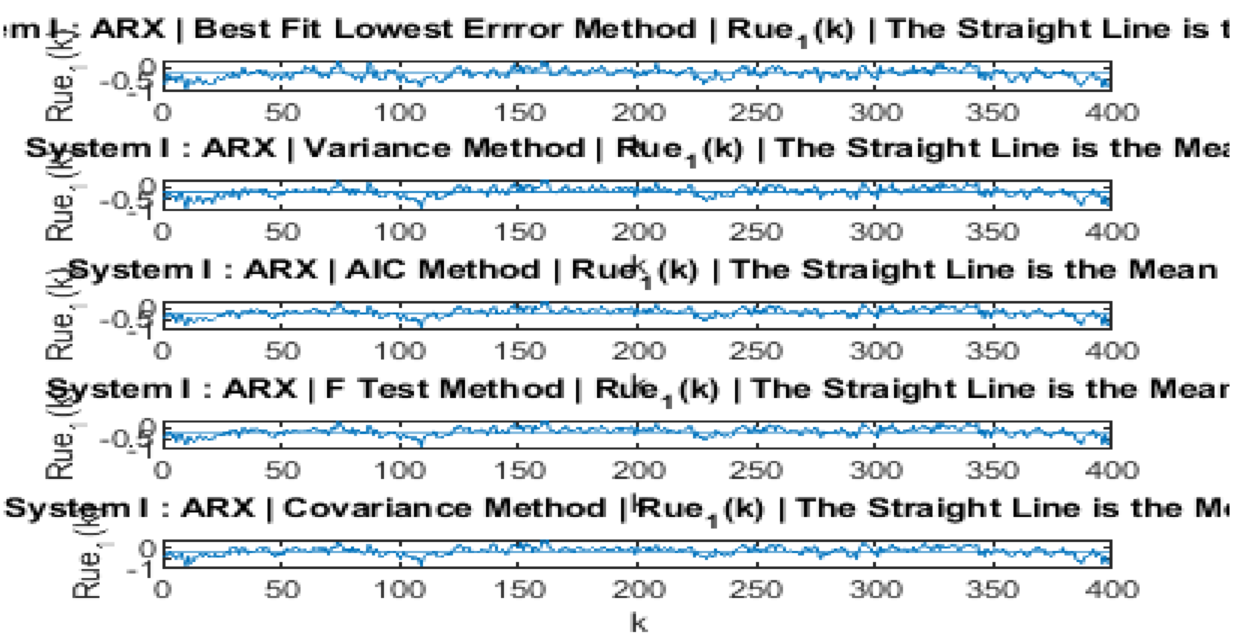


```
%%

figure()  % figure(5)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Ree_1(2:end), 1:N_val-1,
mean(BestFit_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | Best Fit Lowest Errror Method | Ree_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,2)
plot(1:N_val-1,Var_Ree_1(2:end), 1:N_val-1,
mean(Var_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | Variance Method | Ree_1(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,3)
plot(1:N_val-1,AIC_Ree_1(2:end), 1:N_val-1,
mean(AIC_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | AIC Method | Ree_1(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,4)
plot(1:N_val-1,FTest_Ree_1(2:end), 1:N_val-1,
mean(FTest_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | F Test Method | Ree_1(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,5)
plot(1:N_val-1,Cov_Ree_1(2:end), 1:N_val-1,
mean(Cov_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | Covariance Method | Ree_1(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_1(k)")
```



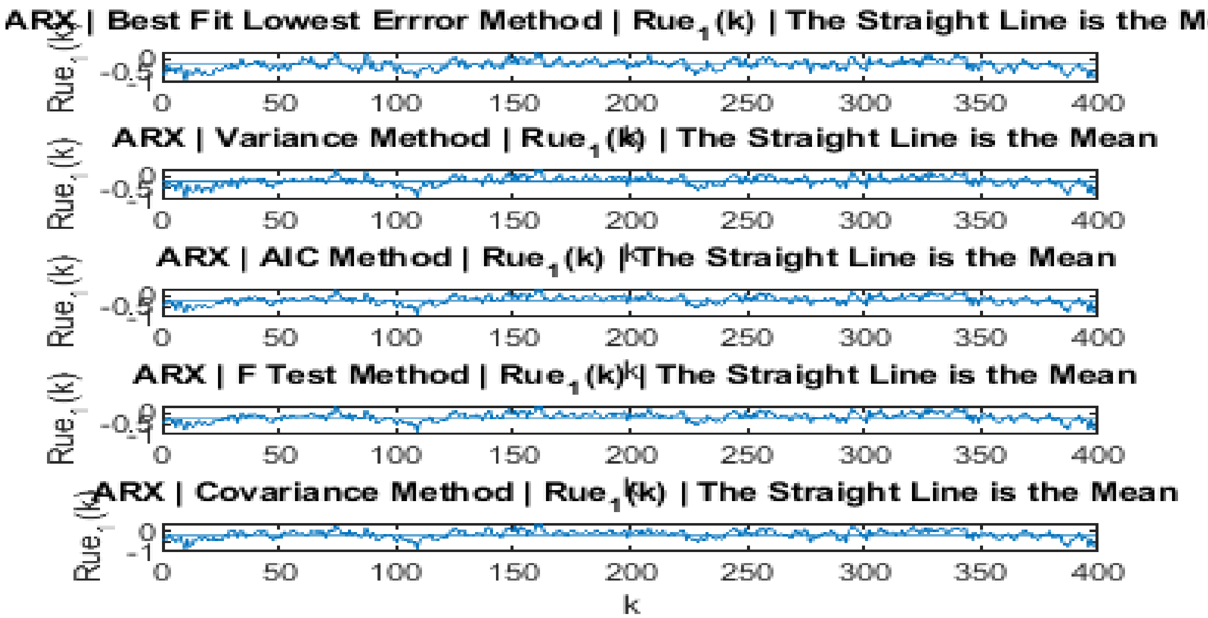

```
%%

figure()  % figure(6)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Rue_1(2:end), 1:N_val-1,
mean(BestFit_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | Best Fit Lowest Errror Method | Rue_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,2)
plot(1:N_val-1,Var_Rue_1(2:end), 1:N_val-1,
mean(Var_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | Variance Method | Rue_1(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,3)
plot(1:N_val-1,AIC_Rue_1(2:end), 1:N_val-1,
mean(AIC_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARX | AIC Method | Rue_1(k) | The Straight Line is the
Mean")
```

```
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Rue_1(2:end), 1:N_val-1,
mean(FTest_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : ARX | F Test Method | Rue_1(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_1(k)")


 subplot(5,1,5)
 plot(1:N_val-1,Cov_Rue_1(2:end), 1:N_val-1,
mean(Cov_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : ARX | Covariance Method | Rue_1(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")
```




```


 %%

 figure()  % figure(6)
```

```
 subplot(5,1,1)
 plot(1:N_val-1,BestFit_Rue_1(2:end), 1:N_val-1,
mean(BestFit_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" ARX | Best Fit Lowest Errror Method | Rue_1(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(5,1,2)
 plot(1:N_val-1,Var_Rue_1(2:end), 1:N_val-1,
mean(Var_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" ARX | Variance Method | Rue_1(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(5,1,3)
 plot(1:N_val-1,AIC_Rue_1(2:end), 1:N_val-1,
mean(AIC_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" ARX | AIC Method | Rue_1(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Rue_1(2:end), 1:N_val-1,
mean(FTest_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" ARX | F Test Method | Rue_1(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")


 subplot(5,1,5)
 plot(1:N_val-1,Cov_Rue_1(2:end), 1:N_val-1,
mean(Cov_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" ARX | Covariance Method | Rue_1(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")
```




```
% ***********************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

System II

```
%%
% System Z2 *************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System II Identification Begins:-----------------------------\n")
```

```
>>> System II Identification Begins:-----------------------------
```

```
%%
Ts = 0.5;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
```

```
N = length(y2);
N_val = length(y2_val);
%%

fprintf("===================Degree Extraction |
RUN==========================\n")
```

```
===================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    U = arx_U_builder_3(u2, y2, na, nb, nk);
    theta_hat_2 = inv(U'*U)*U'*y2;
    y_hat_2 = form_tf_lsim_2(theta_hat_2, u2, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y2, y_hat_2);

    error = y2 - y_hat_2;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    covariance = variance*inv(U'*U);
    cov = trace(covariance)/p;
    covs = [covs; cov];

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_arx, mse_arx, variance, S_hat)
```

```
    fprintf("--------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_arx];
    MSEs = [MSEs; mse_arx];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.031862 | MSE=72.612186 | var=72.855036 | s_hat=43567.311417 |
-----------------------------------------------------------------
>>> Degree = 2 : R2=0.486066 | MSE=38.546050 | var=38.804748 | s_hat=23127.629821 |
-----------------------------------------------------------------
>>> Degree = 3 : R2=0.579322 | MSE=31.551626 | var=31.870329 | s_hat=18930.975512 |
-----------------------------------------------------------------
>>> Degree = 4 : R2=0.613721 | MSE=28.971637 | var=29.363145 | s_hat=17382.982122 |
-----------------------------------------------------------------
>>> Degree = 5 : R2=0.623674 | MSE=28.225164 | var=28.703556 | s_hat=16935.098221 |
-----------------------------------------------------------------
>>> Degree = 6 : R2=0.634707 | MSE=27.397693 | var=27.956829 | s_hat=16438.615740 |
-----------------------------------------------------------------
>>> Degree = 7 : R2=0.639819 | MSE=27.014286 | var=27.659678 | s_hat=16208.571595 |
-----------------------------------------------------------------
>>> Degree = 8 : R2=0.644257 | MSE=26.681402 | var=27.412400 | s_hat=16008.841380 |
-----------------------------------------------------------------
>>> Degree = 9 : R2=0.650711 | MSE=26.197365 | var=27.007593 | s_hat=15718.419203 |
-----------------------------------------------------------------
>>> Degree = 10 : R2=0.654653 | MSE=25.901662 | var=26.794823 | s_hat=15540.997257 |
-----------------------------------------------------------------
>>> Degree = 11 : R2=0.662428 | MSE=25.318571 | var=26.282253 | s_hat=15191.142470 |
-----------------------------------------------------------------
>>> Degree = 12 : R2=0.664225 | MSE=25.183790 | var=26.233114 | s_hat=15110.273846 |
-----------------------------------------------------------------
>>> Degree = 13 : R2=0.665462 | MSE=25.091014 | var=26.227541 | s_hat=15054.608645 |
-----------------------------------------------------------------
>>> Degree = 14 : R2=0.665907 | MSE=25.057612 | var=26.284208 | s_hat=15034.567163 |
-----------------------------------------------------------------
>>> Degree = 15 : R2=0.666068 | MSE=25.045524 | var=26.363710 | s_hat=15027.314630 |
-----------------------------------------------------------------
>>> Degree = 16 : R2=0.668452 | MSE=24.866728 | var=26.267671 | s_hat=14920.036971 |
-----------------------------------------------------------------
>>> Degree = 17 : R2=0.674250 | MSE=24.431846 | var=25.899484 | s_hat=14659.107755 |
-----------------------------------------------------------------
>>> Degree = 18 : R2=0.675612 | MSE=24.329697 | var=25.882657 | s_hat=14597.818318 |
-----------------------------------------------------------------
>>> Degree = 19 : R2=0.674158 | MSE=24.438787 | var=26.091232 | s_hat=14663.272206 |
-----------------------------------------------------------------
>>> Degree = 20 : R2=0.675610 | MSE=24.329867 | var=26.067715 | s_hat=14597.920212 |
-----------------------------------------------------------------
>>> Degree = 21 : R2=0.675988 | MSE=24.301531 | var=26.130679 | s_hat=14580.918839 |
-----------------------------------------------------------------
>>> Degree = 22 : R2=0.677439 | MSE=24.192685 | var=26.107214 | s_hat=14515.611146 |
```

```
-------------------------------------------------------------
>>> Degree = 23 : R2=0.678675 | MSE=24.100015 | var=26.101099 | s_hat=14460.009092 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.680043 | MSE=23.997349 | var=26.084075 | s_hat=14398.409297 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.680383 | MSE=23.971879 | var=26.151140 | s_hat=14383.127152 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.684519 | MSE=23.661698 | var=25.906968 | s_hat=14197.018608 |
-------------------------------------------------------------
>>> Degree = 27 : R2=0.685284 | MSE=23.604289 | var=25.938780 | s_hat=14162.573649 |
-------------------------------------------------------------
>>> Degree = 28 : R2=0.686112 | MSE=23.542226 | var=25.965691 | s_hat=14125.335663 |
-------------------------------------------------------------
>>> Degree = 29 : R2=0.687051 | MSE=23.471773 | var=25.983512 | s_hat=14083.063565 |
-------------------------------------------------------------
>>> Degree = 30 : R2=0.690508 | MSE=23.212459 | var=25.791621 | s_hat=13927.475412 |
-------------------------------------------------------------
>>> Degree = 31 : R2=0.691122 | MSE=23.166471 | var=25.836213 | s_hat=13899.882653 |
-------------------------------------------------------------
>>> Degree = 32 : R2=0.690935 | MSE=23.180452 | var=25.948267 | s_hat=13908.270982 |
-------------------------------------------------------------
>>> Degree = 33 : R2=0.691337 | MSE=23.150295 | var=26.011568 | s_hat=13890.177173 |
-------------------------------------------------------------
>>> Degree = 34 : R2=0.691815 | MSE=23.114473 | var=26.068954 | s_hat=13868.683516 |
-------------------------------------------------------------
>>> Degree = 35 : R2=0.691992 | MSE=23.101187 | var=26.152288 | s_hat=13860.712449 |
-------------------------------------------------------------
>>> Degree = 36 : R2=0.692444 | MSE=23.067314 | var=26.212857 | s_hat=13840.388236 |
-------------------------------------------------------------
>>> Degree = 37 : R2=0.692441 | MSE=23.067490 | var=26.312726 | s_hat=13840.493848 |
-------------------------------------------------------------
>>> Degree = 38 : R2=0.693857 | MSE=22.961302 | var=26.291567 | s_hat=13776.781072 |
-------------------------------------------------------------
>>> Degree = 39 : R2=0.693808 | MSE=22.964975 | var=26.396523 | s_hat=13778.985013 |
-------------------------------------------------------------
>>> Degree = 40 : R2=0.694182 | MSE=22.936957 | var=26.465719 | s_hat=13762.173927 |
-------------------------------------------------------------
>>> Degree = 41 : R2=0.694530 | MSE=22.910854 | var=26.537669 | s_hat=13746.512384 |
-------------------------------------------------------------
>>> Degree = 42 : R2=0.694604 | MSE=22.905312 | var=26.634083 | s_hat=13743.186985 |
-------------------------------------------------------------
>>> Degree = 43 : R2=0.694544 | MSE=22.909763 | var=26.742914 | s_hat=13745.857563 |
-------------------------------------------------------------
>>> Degree = 44 : R2=0.696221 | MSE=22.784006 | var=26.700007 | s_hat=13670.403362 |
-------------------------------------------------------------
>>> Degree = 45 : R2=0.696207 | MSE=22.785044 | var=26.805934 | s_hat=13671.026136 |
-------------------------------------------------------------
>>> Degree = 46 : R2=0.697497 | MSE=22.688290 | var=26.797193 | s_hat=13612.973795 |
-------------------------------------------------------------
>>> Degree = 47 : R2=0.698469 | MSE=22.615384 | var=26.816661 | s_hat=13569.230650 |
-------------------------------------------------------------
>>> Degree = 48 : R2=0.699775 | MSE=22.517472 | var=26.806515 | s_hat=13510.483343 |
-------------------------------------------------------------
>>> Degree = 49 : R2=0.699792 | MSE=22.516203 | var=26.911796 | s_hat=13509.721520 |
-------------------------------------------------------------
>>> Degree = 50 : R2=0.700740 | MSE=22.445086 | var=26.934103 | s_hat=13467.051543 |
-------------------------------------------------------------
```

```
>>> Degree = 51 : R2=0.701477 | MSE=22.389789 | var=26.975649 | s_hat=13433.873334 |
-------------------------------------------------------------
>>> Degree = 52 : R2=0.701618 | MSE=22.379195 | var=27.071607 | s_hat=13427.517202 |
-------------------------------------------------------------
>>> Degree = 53 : R2=0.702000 | MSE=22.350543 | var=27.146409 | s_hat=13410.325844 |
-------------------------------------------------------------
>>> Degree = 54 : R2=0.702653 | MSE=22.301569 | var=27.197035 | s_hat=13380.941406 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.704029 | MSE=22.198400 | var=27.181714 | s_hat=13319.039782 |
-------------------------------------------------------------
>>> Degree = 56 : R2=0.705521 | MSE=22.086467 | var=27.155492 | s_hat=13251.880246 |
-------------------------------------------------------------
>>> Degree = 57 : R2=0.705108 | MSE=22.117470 | var=27.305518 | s_hat=13270.481872 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.705501 | MSE=22.087981 | var=27.381795 | s_hat=13252.788845 |
-------------------------------------------------------------
>>> Degree = 59 : R2=0.706821 | MSE=21.989007 | var=27.372208 | s_hat=13193.404160 |
-------------------------------------------------------------
>>> Degree = 60 : R2=0.707415 | MSE=21.944414 | var=27.430517 | s_hat=13166.648385 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.709602 | MSE=21.780402 | var=27.339417 | s_hat=13068.241137 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.709419 | MSE=21.794161 | var=27.471631 | s_hat=13076.496559 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.709619 | MSE=21.779101 | var=27.568482 | s_hat=13067.460302 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.710135 | MSE=21.740422 | var=27.636130 | s_hat=13044.253318 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.710396 | MSE=21.720881 | var=27.728784 | s_hat=13032.528564 |
-------------------------------------------------------------
>>> Degree = 66 : R2=0.710459 | MSE=21.716132 | var=27.841194 | s_hat=13029.678903 |
-------------------------------------------------------------
>>> Degree = 67 : R2=0.710231 | MSE=21.733222 | var=27.982689 | s_hat=13039.933161 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.710656 | MSE=21.701322 | var=28.062054 | s_hat=13020.793027 |
-------------------------------------------------------------
>>> Degree = 69 : R2=0.710657 | MSE=21.701256 | var=28.183449 | s_hat=13020.753357 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.713086 | MSE=21.519116 | var=28.068412 | s_hat=12911.469629 |
-------------------------------------------------------------
>>> Degree = 71 : R2=0.714607 | MSE=21.405019 | var=28.041510 | s_hat=12843.011366 |
-------------------------------------------------------------
>>> Degree = 72 : R2=0.715004 | MSE=21.375209 | var=28.125275 | s_hat=12825.125336 |
-------------------------------------------------------------
>>> Degree = 73 : R2=0.715067 | MSE=21.370529 | var=28.242989 | s_hat=12822.317206 |
-------------------------------------------------------------
>>> Degree = 74 : R2=0.717426 | MSE=21.193602 | var=28.133100 | s_hat=12716.161141 |
-------------------------------------------------------------
>>> Degree = 75 : R2=0.716840 | MSE=21.237560 | var=28.316747 | s_hat=12742.535974 |
-------------------------------------------------------------
>>> Degree = 76 : R2=0.716840 | MSE=21.237522 | var=28.443110 | s_hat=12742.513075 |
-------------------------------------------------------------
>>> Degree = 77 : R2=0.719169 | MSE=21.062842 | var=28.335661 | s_hat=12637.704936 |
-------------------------------------------------------------
>>> Degree = 78 : R2=0.719458 | MSE=21.041162 | var=28.434003 | s_hat=12624.697135 |
-------------------------------------------------------------
>>> Degree = 79 : R2=0.720179 | MSE=20.987097 | var=28.489272 | s_hat=12592.258215 |
```

```
-------------------------------------------------------------
>>> Degree = 80 : R2=0.720880 | MSE=20.934568 | var=28.547138 | s_hat=12560.740504 |
-------------------------------------------------------------
>>> Degree = 81 : R2=0.720941 | MSE=20.929963 | var=28.671182 | s_hat=12557.977639 |
-------------------------------------------------------------
>>> Degree = 82 : R2=0.725805 | MSE=20.565154 | var=28.300671 | s_hat=12339.092474 |
-------------------------------------------------------------
>>> Degree = 83 : R2=0.728613 | MSE=20.354521 | var=28.139891 | s_hat=12212.712824 |
-------------------------------------------------------------
>>> Degree = 84 : R2=0.728774 | MSE=20.342439 | var=28.253388 | s_hat=12205.463582 |
-------------------------------------------------------------
>>> Degree = 85 : R2=0.729040 | MSE=20.322486 | var=28.356958 | s_hat=12193.491752 |
-------------------------------------------------------------
>>> Degree = 86 : R2=0.729453 | MSE=20.291557 | var=28.446107 | s_hat=12174.933919 |
-------------------------------------------------------------
>>> Degree = 87 : R2=0.729284 | MSE=20.304184 | var=28.597442 | s_hat=12182.510409 |
-------------------------------------------------------------
>>> Degree = 88 : R2=0.729488 | MSE=20.288953 | var=28.710782 | s_hat=12173.371758 |
-------------------------------------------------------------
>>> Degree = 89 : R2=0.729629 | MSE=20.278330 | var=28.831748 | s_hat=12166.997717 |
-------------------------------------------------------------
>>> Degree = 90 : R2=0.729952 | MSE=20.254131 | var=28.934473 | s_hat=12152.478773 |
-------------------------------------------------------------
>>> Degree = 91 : R2=0.730417 | MSE=20.219221 | var=29.022806 | s_hat=12131.532831 |
-------------------------------------------------------------
>>> Degree = 92 : R2=0.733067 | MSE=20.020507 | var=28.875732 | s_hat=12012.304430 |
-------------------------------------------------------------
>>> Degree = 93 : R2=0.733380 | MSE=19.996988 | var=28.981142 | s_hat=11998.192835 |
-------------------------------------------------------------
>>> Degree = 94 : R2=0.734759 | MSE=19.893584 | var=28.971239 | s_hat=11936.150618 |
-------------------------------------------------------------
>>> Degree = 95 : R2=0.734243 | MSE=19.932293 | var=29.169209 | s_hat=11959.375709 |
-------------------------------------------------------------
>>> Degree = 96 : R2=0.734607 | MSE=19.904999 | var=29.272057 | s_hat=11942.999139 |
-------------------------------------------------------------
>>> Degree = 97 : R2=0.735571 | MSE=19.832695 | var=29.309401 | s_hat=11899.616916 |
-------------------------------------------------------------
>>> Degree = 98 : R2=0.737037 | MSE=19.722699 | var=29.291137 | s_hat=11833.619223 |
-------------------------------------------------------------
>>> Degree = 99 : R2=0.737140 | MSE=19.715029 | var=29.425416 | s_hat=11829.017119 |
-------------------------------------------------------------
>>> Degree = 100 : R2=0.737560 | MSE=19.683485 | var=29.525227 | s_hat=11810.090703 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("==============Degree Extraction | BestFit Method================\n")
```

```
==============Degree Extraction | BestFit Method================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 100
```

```
na = bestFitDegree;
nb = bestFitDegree;
p = na+nb;

BestFitU = arx_U_builder_3(u2, y2, na, nb, nk);
BestFitModel_2 = inv(BestFitU'*BestFitU)*BestFitU'*y2;
BestFit_y_hat_2 = form_tf_lsim_2(BestFitModel_2, u2_val, t_val, na, Ts);
```

```
G =


   -6.771 z^-1 - 0.1819 z^-2 + 1.514 z^-3 + 16.19 z^-4 + 10.97 z^-5 - 36.61 z^-6 + 33.01 z^-7

           + 13.78 z^-8 - 19.45 z^-9 + 5.383 z^-10 + 13.39 z^-11 - 0.4511 z^-12 + 17 z^-13 +
24.03 z^-14

           - 9.561 z^-15 + 1.32 z^-16 + 43.49 z^-17 + 11.77 z^-18 - 18.25 z^-19 + 43.74 z^-20

           + 13.16 z^-21 + 7.488 z^-22 + 28.82 z^-23 - 4.357 z^-24 - 15.58 z^-25 + 53.59 z^-26

           - 6.113 z^-27 + 8.534 z^-28 - 14.97 z^-29 + 5.761 z^-30 + 13.5 z^-31 - 8.857 z^-32

           - 27.89 z^-33 + 6.185 z^-34 - 6.139 z^-35 - 25.47 z^-36 + 13.24 z^-37 - 31.02 z^-38

           + 16.1 z^-39 + 4.861 z^-40 - 1.18 z^-41 + 0.002377 z^-42 - 8.095 z^-43 + 23.46 z^

           -44 - 13.49 z^-45 + 11.9 z^-46 + 21.45 z^-47 - 7.956 z^-48 - 12.91 z^-49 + 20.38
z^-50

           + 6.197 z^-51 - 9.135 z^-52 + 2.675 z^-53 - 17.9 z^-54 + 22.73 z^-55 - 7.541 z^-56

           + 2.044 z^-57 + 21.29 z^-58 - 34.85 z^-59 - 8.286 z^-60 + 17.86 z^-61 - 4.441 z^-62

           + 11.16 z^-63 - 0.1818 z^-64 - 16.2 z^-65 + 10.78 z^-66 + 5.359 z^-67 + 3.483 z^-68

           - 24.97 z^-69 + 6.332 z^-70 - 15.58 z^-71 + 11.37 z^-72 - 9.485 z^-73 - 47.91 z^-74

           + 0.4369 z^-75 + 10.95 z^-76 - 64.1 z^-77 + 8.368 z^-78 - 18.93 z^-79 - 27.04 z^-80

           + 40.27 z^-81 - 41.65 z^-82 + 10.62 z^-83 - 11.73 z^-84 + 22.26 z^-85 - 0.1232 z^
```

-86 + 18.85 z^-87 + 24.45 z^-88 + 17.63 z^-89 + 3.208 z^-90 + 5.547 z^-91 + 22.42 z^-92

+ 13.5 z^-93 + 17.27 z^-94 - 20.21 z^-95 + 0.4019 z^-96 + 33.53 z^-97 - 31.74 z^-98

+ 13.9 z^-99 - 10.49 z^-100

----

1 - 0.005877 z^-1 - 0.02741 z^-2 + 0.02911 z^-3 + 0.006188 z^-4 - 0.04976 z^-5 + 0.008231 z^-6

- 0.08701 z^-7 - 0.009138 z^-8 - 0.002757 z^-9 + 0.01034 z^-10 - 0.06959 z^-11 + 0.08683 z^-12

+ 0.06326 z^-13 + 0.001572 z^-14 - 0.03013 z^-15 - 0.03629 z^-16 + 0.01831 z^-17

+ 0.03054 z^-18 - 0.1561 z^-19 - 0.08828 z^-20 + 0.005671 z^-21 + 0.03994 z^-22 - 0.005378 z^-23

+ 0.02373 z^-24 + 0.004685 z^-25 + 0.02624 z^-26 - 0.02302 z^-27 - 0.06565 z^-28

- 0.02283 z^-29 - 0.07176 z^-30 - 0.0364 z^-31 - 0.03089 z^-32 - 0.0004725 z^-33

- 0.01111 z^-34 - 0.01168 z^-35 + 0.01173 z^-36 + 0.00371 z^-37 + 0.07496 z^-38 + 0.05896 z^-39

+ 0.01861 z^-40 - 0.0144 z^-41 - 0.03118 z^-42 - 0.02125 z^-43 + 0.03886 z^-44 - 0.009199 z^-45

- 0.01904 z^-46 - 0.007165 z^-47 - 0.006413 z^-48 + 0.02927 z^-49 + 0.02162 z^-50

+ 0.03067 z^-51 - 0.01472 z^-52 + 0.009604 z^-53 + 0.00917 z^-54 + 0.001957 z^-55

+ 0.03261 z^-56 - 0.06368 z^-57 - 0.04628 z^-58 + 0.003551 z^-59 - 0.002263 z^-60

+ 0.09086 z^-61 - 0.005461 z^-62 - 0.02028 z^-63 + 0.06856 z^-64 - 0.00747 z^-65

- 0.002697 z^-66 + 0.05456 z^-67 - 0.01591 z^-68 - 0.01983 z^-69 - 0.04107 z^-70

- 0.06069 z^-71 - 0.04807 z^-72 + 0.03355 z^-73 - 0.0811 z^-74 - 0.05443 z^-75 + 0.06991 z^-76

+ 0.04532 z^-77 - 0.005322 z^-78 - 0.1017 z^-79 + 0.01355 z^-80 - 0.04705 z^-81 - 0.06127 z^-82

+ 0.1185 z^-83 + 0.01365 z^-84 + 0.06086 z^-85 + 0.07423 z^-86 - 0.03171 z^-87 + 0.002778 z^-88

+ 0.01093 z^-89 + 0.02405 z^-90 - 0.01506 z^-91 - 0.04782 z^-92 - 0.0445 z^-93 - 0.0556 z^-94

+ 0.03084 z^-95 - 0.01171 z^-96 - 0.04327 z^-97 + 0.04489 z^-98 - 0.01557 z^-99 + 0.01813 z^-100

```
Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
%%

fprintf("===============Degree Extraction | Variance Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n", minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 30 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 30
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb;

VarU = arx_U_builder_3(u2, y2, na, nb, nk);
VarModel_2 = inv(VarU'*VarU)*VarU'*y2;
Var_y_hat_2 = form_tf_lsim_2(VarModel_2, u2_val, t_val, na, Ts);
```

```
G =

    -2.579 z^-1 - 0.8101 z^-2 + 0.05998 z^-3 + 18.08 z^-4 + 9.197 z^-5 - 34.25 z^-6 + 35.18
z^-7

            + 14.24 z^-8 - 17.89 z^-9 + 3.292 z^-10 + 16.56 z^-11 + 0.2597 z^-12 + 16.46 z^-13

            + 27.42 z^-14 - 8.051 z^-15 + 2.633 z^-16 + 47.1 z^-17 + 13.38 z^-18 - 17.45 z^-19

            + 44.26 z^-20 + 15.47 z^-21 + 10.42 z^-22 + 30.55 z^-23 + 1.885 z^-24 - 11.82 z^-
25

                                  + 59.02 z^-26 - 0.8562 z^-27 + 10.7 z^-28 - 16.67 z^-29 +
39.53 z^-30

  ----------------------------------------------------------------------------------------------
-----------

  1 - 0.03428 z^-1 - 0.03069 z^-2 - 0.0008298 z^-3 + 0.05458 z^-4 - 0.08377 z^-5 + 0.02854 z^-
6
```

```
          - 0.05059 z^-7 + 0.01999 z^-8 + 0.02922 z^-9 + 0.04133 z^-10 - 0.06665 z^-11 +
0.1088 z^-12

          + 0.06411 z^-13 + 0.02141 z^-14 - 0.03181 z^-15 - 0.0459 z^-16 + 0.01826 z^-17 +
0.04818 z^-18

          - 0.1102 z^-19 - 0.0577 z^-20 + 0.003737 z^-21 + 0.02863 z^-22 - 0.03923 z^-23 -
0.03075 z^-24

          - 0.03579 z^-25 - 0.004167 z^-26 - 0.05067 z^-27 - 0.03537 z^-28 - 0.001501 z^-29

                                                                                        -
0.03796 z^-30


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | CoVariance
Method=================\n")
```

```
===============Degree Extraction | CoVariance Method=================
```

```
maxCovIndex = find(covs == min(covs));
fprintf(">>> Since the minimum CovMatrix trace occurs in iteration %d ;\n",
maxCovIndex)
```

```
>>> Since the minimum CovMatrix trace occurs in iteration 1 ;
```

```
fprintf("    Degree = %d \n", maxCovIndex)
```

```
    Degree = 1
```

```
na = maxCovIndex;
nb = maxCovIndex;
p = na+nb;

CovU_2 = arx_U_builder_3(u2,y2,na,nb,1);
CovModel_2 = inv(CovU_2'*CovU_2)*CovU_2'*y2;
Cov_y_hat_2 = form_tf_lsim_2(CovModel_2, u2_val, t_val, na, Ts);
```

```
G =
```

```
    1.884 z^-1
  ---------------
  1 - 0.6361 z^-1

Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=============================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n", 
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 17 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 17
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb;

AICU_2 = arx_U_builder_3(u2, y2, na, nb, nk);
AICModel_2 = inv(AICU_2'*AICU_2)*AICU_2'*y2;
AIC_y_hat_2 = form_tf_lsim_2(AICModel_2, u2_val, t_val, na, Ts);
```

```
G =


    -14.78 z^-1 + 3.619 z^-2 + 0.5736 z^-3 + 15.79 z^-4 + 4.257 z^-5 - 41.52 z^-6 + 29.68 z^-7

            + 6.409 z^-8 - 22.24 z^-9 + 1.371 z^-10 + 14.21 z^-11 - 3.426 z^-12 + 10.42 z^-13

                                                 + 18.32 z^-14 - 21.49 z^-15 - 15.7 z^-16 + 
45.14 z^-17

  ---------------------------------------------------------------------------------------------
-----------
```

```
  1 - 0.06877 z^-1 - 0.04002 z^-2 - 0.01462 z^-3 + 0.0493 z^-4 - 0.05657 z^-5 + 0.05561 z^-6

          - 0.009907 z^-7 + 0.02052 z^-8 + 0.008042 z^-9 + 0.01305 z^-10 - 0.08173 z^-11 +
0.07044 z^-12

                       + 0.03566 z^-13 + 0.008449 z^-14 - 0.01041 z^-15 - 0.02567 z^-16 +
0.07158 z^-17


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("============================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test 
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", 
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.21 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 11
```

```
na = winner;
nb = winner;
p = na+nb;
```

```
FTestU_2 = arx_U_builder_3(u2, y2, na, nb, nk);
FTestModel_2 = inv(FTestU_2'*FTestU_2)*FTestU_2'*y2;
FTest_y_hat_2 = form_tf_lsim_2(FTestModel_2, u2_val, t_val, na, Ts);
```

```
G =

  -9.183 z^-1 + 6.441 z^-2 + 1.047 z^-3 + 14.29 z^-4 + 0.8618 z^-5 - 45.18 z^-6 + 28.77 z^-7

                                              + 7.506 z^-8 - 16.75 z^-9 + 12.11 z^-10 -
21.54 z^-11

  -------------------------------------------------------------------------------------------
-------

  1 - 0.0947 z^-1 - 0.04546 z^-2 - 0.0163 z^-3 + 0.05989 z^-4 - 0.02999 z^-5 + 0.08976 z^-6

                        + 0.01901 z^-7 + 0.04593 z^-8 + 0.003442 z^-9 - 0.008947 z^-10 -
0.1171 z^-11


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[BestFit_r2, BestFit_mse] = rSQR(y2_val, BestFit_y_hat_2);
[Var_r2, Var_mse] = rSQR(y2_val, Var_y_hat_2);
[AIC_r2, AIC_mse] = rSQR(y2_val, AIC_y_hat_2);
[Cov_r2, Cov_mse] = rSQR(y2_val, Cov_y_hat_2);
[FTest_r2, FTest_mse] = rSQR(y2_val, FTest_y_hat_2);

fprintf("===========================System III==========================\n")
```

```
===========================System III==========================
```

```
fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.4403    | MSE : 49.4593
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.6080    | MSE : 34.6387
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Covariance Method:\n")
```

```
>>> Covariance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Cov_r2, Cov_mse)
```

```
    R2 value : 0.0418    | MSE : 84.6691
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.6918    | MSE : 27.2343
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
R2 value : 0.6918    | MSE : 27.2300
```

```matlab
fprintf("--------------------------------------------------------------\n")
```

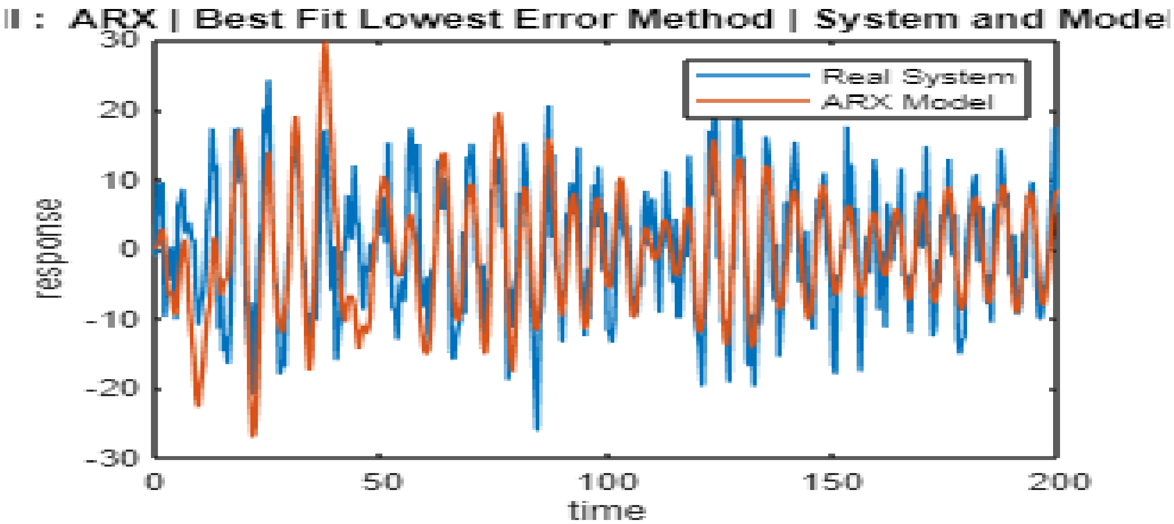
```
--------------------------------------------------------------
```

```matlab
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("==============================================================\n")
```

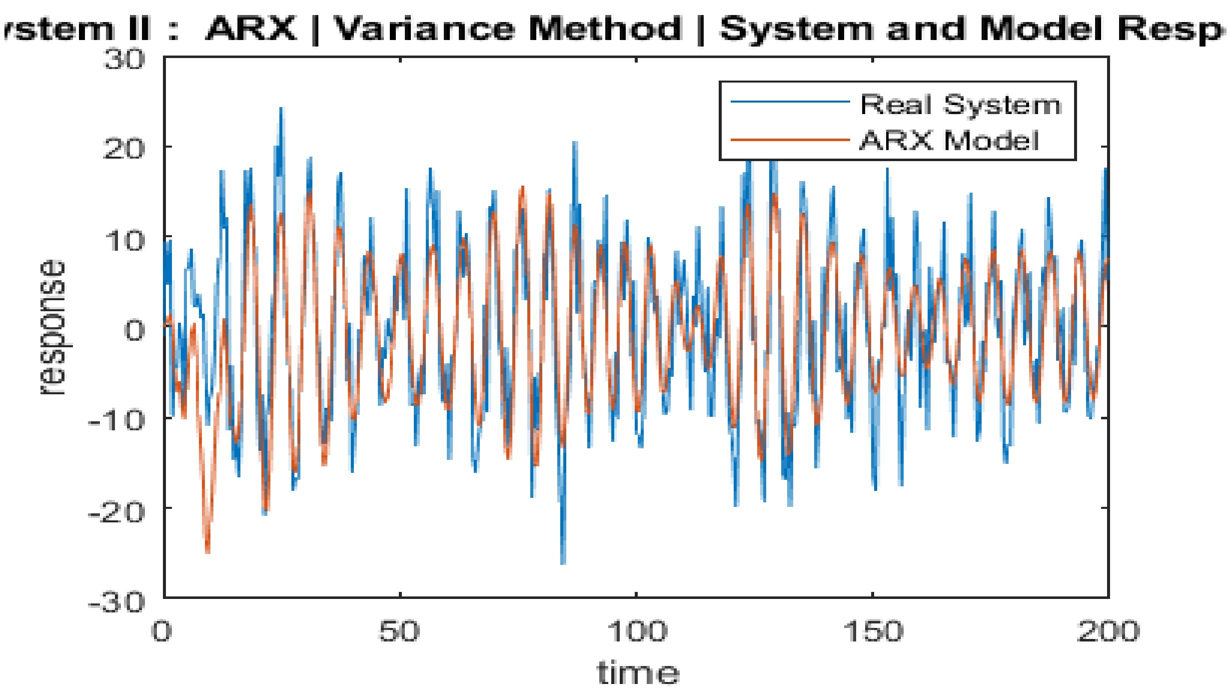
```
==============================================================
```

```matlab
%%

BestFitError_2 = y2_val - BestFit_y_hat_2;
VarError_2 = y2_val - Var_y_hat_2;
CovError_2 = y2_val - Cov_y_hat_2;
AICError_2 = y2_val - AIC_y_hat_2;
FTestError_2 = y2_val - FTest_y_hat_2;

for k=0:N_val-1
    BestFit_Ree_2(k+1,1) = AutoCorrelate(BestFitError_2, k);
    Var_Ree_2(k+1,1) = AutoCorrelate(VarError_2, k);
    Cov_Ree_2(k+1,1) = AutoCorrelate(CovError_2, k);
    AIC_Ree_2(k+1,1) = AutoCorrelate(AICError_2, k);
    FTest_Ree_2(k+1,1) = AutoCorrelate(FTestError_2, k);
end

for k=0:N_val-1
    BestFit_Rue_2(k+1,1) = CrossCorrelate(u2_val, BestFitError_2, k);
    Var_Rue_2(k+1,1) = CrossCorrelate(u2_val, VarError_2, k);
    Cov_Rue_2(k+1,1) = CrossCorrelate(u2_val, CovError_2, k);
    AIC_Rue_2(k+1,1) = CrossCorrelate(u2_val, AICError_2, k);
    FTest_Rue_2(k+1,1) = CrossCorrelate(u2_val, FTestError_2, k);
end


%%
figure()  % figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hat_2)
legend('Real System','ARX Model')
title(" System II :  ARX | Best Fit Lowest Error Method | System and Model Response")
xlabel("time")
```

```
ylabel("response")
```

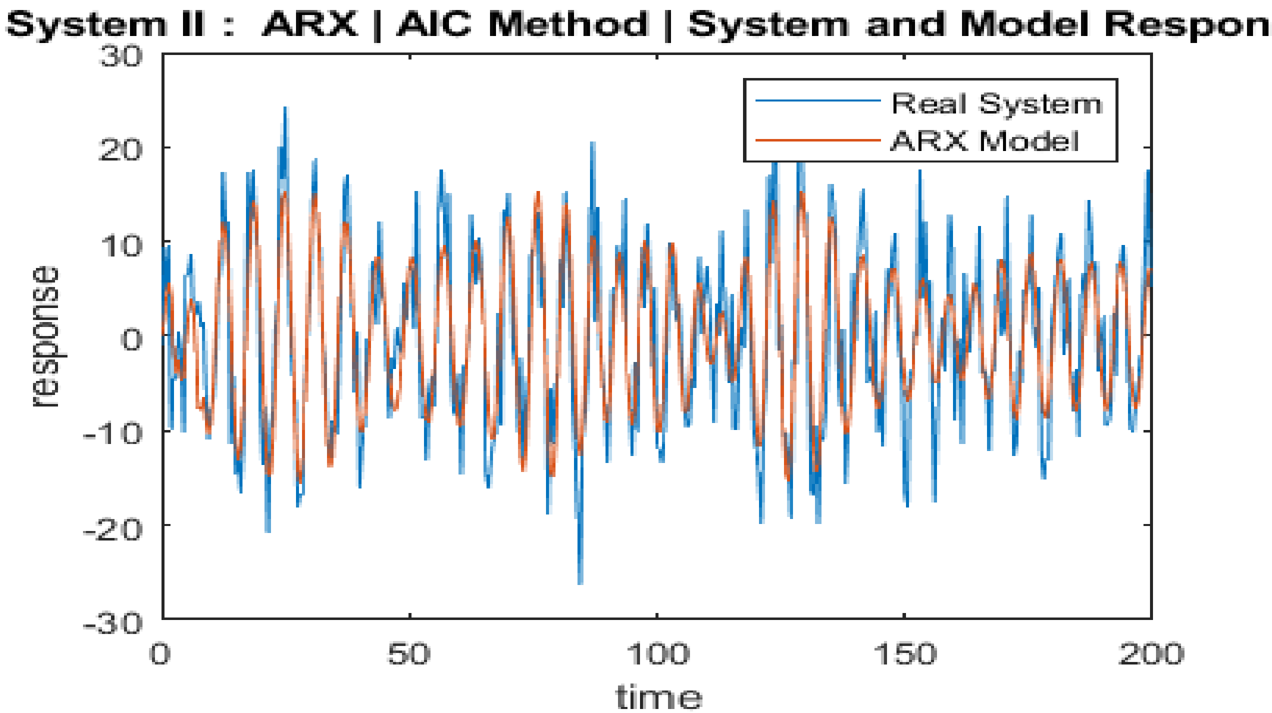



```
figure()  % figure(2)
plot(t_val,y2_val,t_val,Var_y_hat_2)
legend('Real System','ARX Model')
title(" System II :  ARX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```

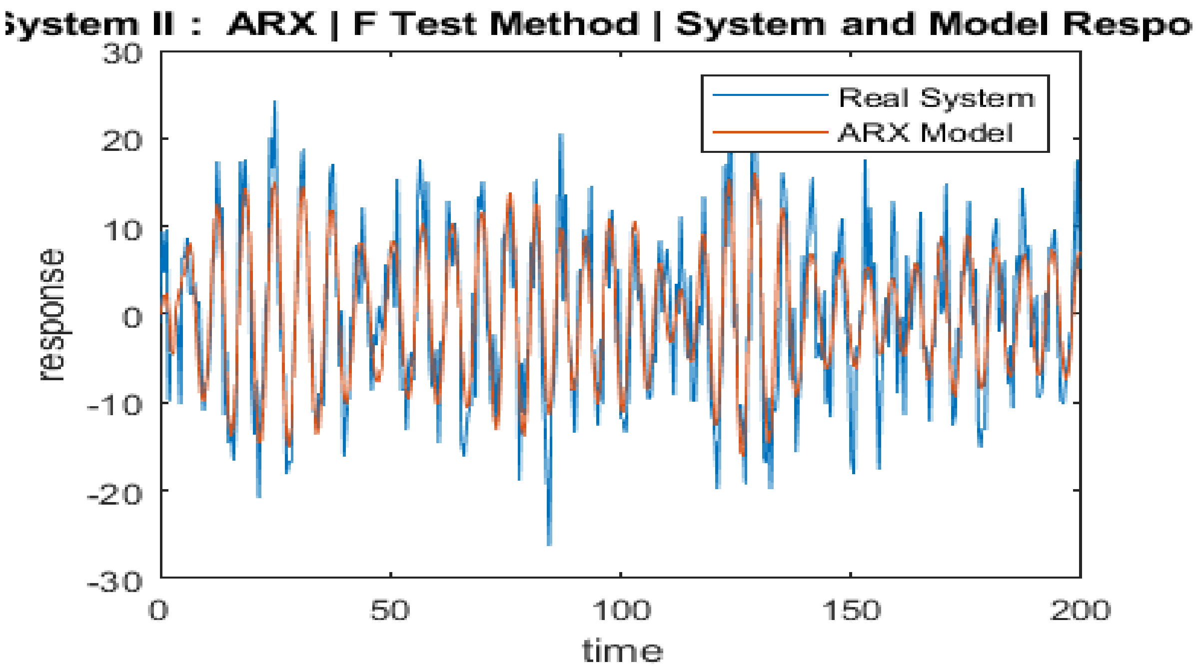



```
figure()  % figure(3)
```

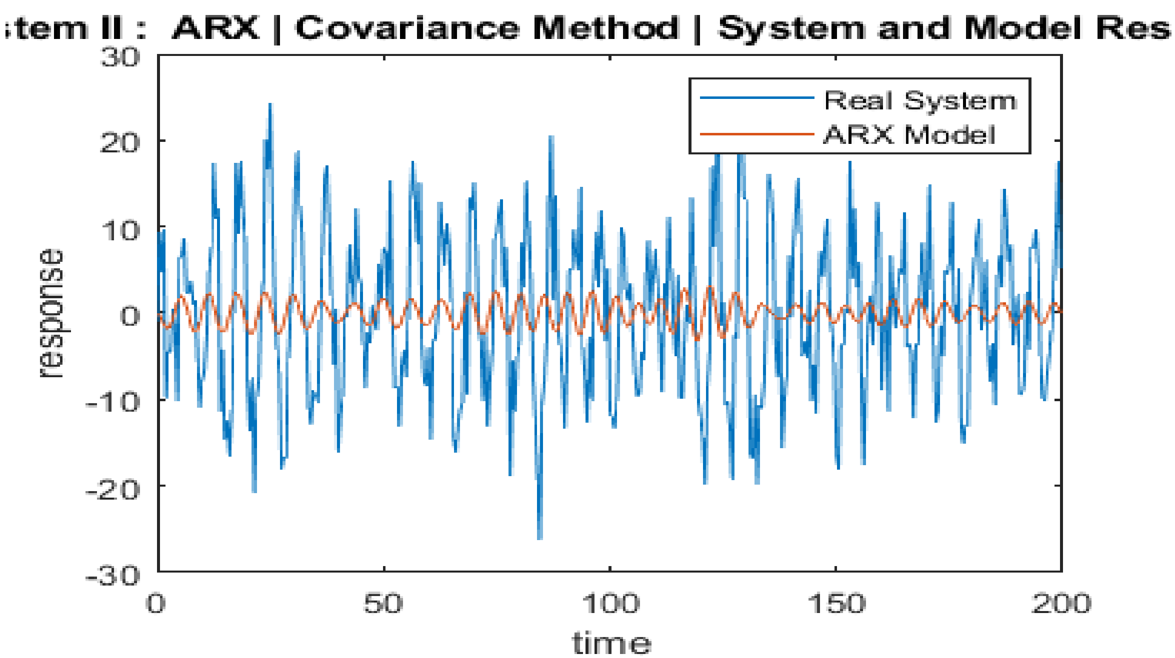
```
plot(t_val,y2_val,t_val,AIC_y_hat_2)
legend('Real System','ARX Model')
title(" System II :  ARX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure()  % figure(4)
plot(t_val,y2_val,t_val,FTest_y_hat_2)
legend('Real System','ARX Model')
title(" System II :  ARX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```

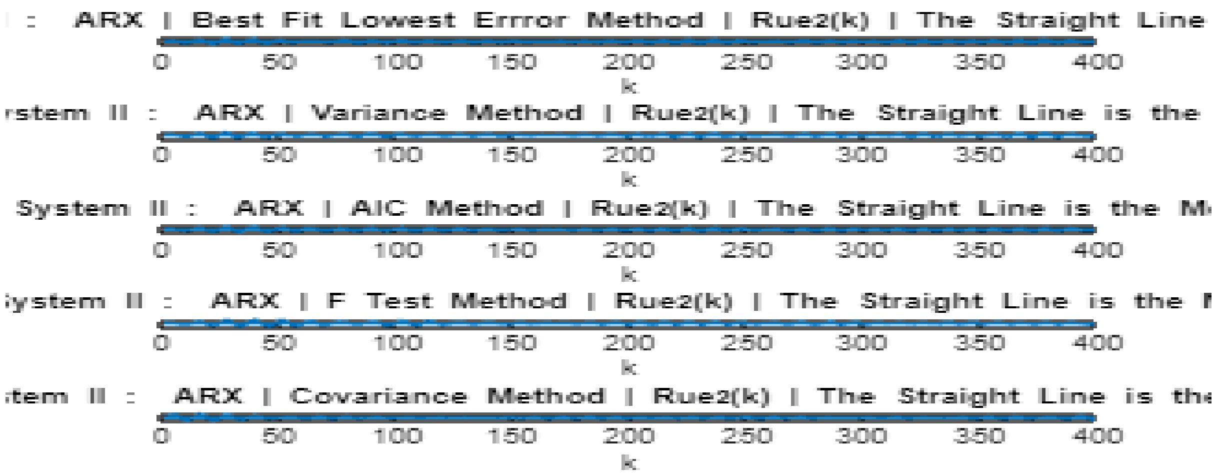



```
figure()  % figure(7)
plot(t_val,y2_val,t_val,Cov_y_hat_2)
legend('Real System','ARX Model')
title(" System II :  ARX | Covariance Method | System and Model Response")
xlabel("time")
ylabel("response")
```



```
%%

figure()  % figure(5)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Ree_2(2:end), 1:N_val-1,
mean(BestFit_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II :  ARX | Best Fit Lowest Errror Method | Ree_2(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,2)
plot(1:N_val-1,Var_Ree_2(2:end), 1:N_val-1,
mean(Var_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II :  ARX | Variance Method | Ree_2(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,3)
plot(1:N_val-1,AIC_Ree_2(2:end), 1:N_val-1,
mean(AIC_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II :  ARX | AIC Method | Ree_2(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,4)
plot(1:N_val-1,FTest_Ree_2(2:end), 1:N_val-1,
mean(FTest_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II :  ARX | F Test Method | Ree_2(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,5)
plot(1:N_val-1,Cov_Ree_2(2:end), 1:N_val-1,
mean(Cov_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II :  ARX | Covariance Method | Ree_1(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_2(k)")



%%
```

```
 figure()  % figure(6)
 subplot(5,1,1)
 plot(1:N_val-1,BestFit_Rue_2(2:end), 1:N_val-1,
mean(BestFit_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II :  ARX | Best Fit Lowest Errror Method | Rue_2(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(5,1,2)
 plot(1:N_val-1,Var_Rue_2(2:end), 1:N_val-1,
mean(Var_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II :  ARX | Variance Method | Rue_2(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(5,1,3)
 plot(1:N_val-1,AIC_Rue_2(2:end), 1:N_val-1,
mean(AIC_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II :  ARX | AIC Method | Rue_2(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Rue_2(2:end), 1:N_val-1,
mean(FTest_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II :  ARX | F Test Method | Rue_2(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_2(k)")


 subplot(5,1,5)
 plot(1:N_val-1,Cov_Rue_2(2:end), 1:N_val-1,
mean(Cov_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II :  ARX | Covariance Method | Rue_2(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")
```




```
fprintf("****************************************************************\n")
```

```
****************************************************************
```

```
fprintf("****************************************************************\n")
```

```
****************************************************************
```

Q2 - part c | ARMAX

```
clc; clear;
%%

load HW5_question2

u1 = Z1.u;
y1 = Z1.y;

u2 = Z2.u;
y2 = Z2.y;

u3 = Z3.u;
y3 = Z3.y;

u1_val = u1(601:end);
y1_val = y1(601:end);

u2_val = u2(601:end);
y2_val = y2(601:end);

u3_val = u1(601:end);
y3_val = y1(601:end);

u1 = u1(1:600);
y1 = y1(1:600);

u2 = u2(1:600);
y2 = y2(1:600);

u3 = u3(1:600);
y3 = y3(1:600);
```

System I

```
%%

% System Z1 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System I Identification Begins:------------------------------\n")
```

```
>>> System I Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data1 = iddata(y1,u1,Ts);


%%


fprintf("===================Degree Extraction |
RUN===========================\n")
```

```
===================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nk = 1;
    p = na+nb+nc;

    try
        sys = armax(data1, [na nb nc nk]);
        armax_y_hat_1 = lsim(sys, u1, t);
    catch
        break
    end

    [r2_armax, mse_armax] = rSQR(y1, armax_y_hat_1);

    error = y1 - armax_y_hat_1;
```

```
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

%     theta = [sys.A sys.B sys.C];
%     covs = [covs; cov(theta)];

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_armax, mse_armax, variance, S_hat)
    fprintf("--------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_armax];
    MSEs = [MSEs; mse_armax];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.080649 | MSE=76.927306 | var=77.313876 | s_hat=46156.383758 |
--------------------------------------------------------------
>>> Degree = 2 : R2=0.283547 | MSE=59.949681 | var=60.555233 | s_hat=35969.808577 |
--------------------------------------------------------------
>>> Degree = 3 : R2=0.920082 | MSE=6.687226 | var=6.789062 | s_hat=4012.335678 |
--------------------------------------------------------------
>>> Degree = 4 : R2=0.920432 | MSE=6.657940 | var=6.793816 | s_hat=3994.763805 |
--------------------------------------------------------------
>>> Degree = 5 : R2=0.920747 | MSE=6.631580 | var=6.801620 | s_hat=3978.947882 |
--------------------------------------------------------------
>>> Degree = 6 : R2=0.921126 | MSE=6.599870 | var=6.803990 | s_hat=3959.922044 |
--------------------------------------------------------------
>>> Degree = 7 : R2=0.922010 | MSE=6.525887 | var=6.762577 | s_hat=3915.531915 |
--------------------------------------------------------------
>>> Degree = 8 : R2=0.921277 | MSE=6.587202 | var=6.861669 | s_hat=3952.321167 |
--------------------------------------------------------------
>>> Degree = 9 : R2=0.922627 | MSE=6.474204 | var=6.779271 | s_hat=3884.522515 |
--------------------------------------------------------------
>>> Degree = 10 : R2=0.923640 | MSE=6.389483 | var=6.725772 | s_hat=3833.689899 |
--------------------------------------------------------------
>>> Degree = 11 : R2=0.922661 | MSE=6.471381 | var=6.848023 | s_hat=3882.828821 |
--------------------------------------------------------------
>>> Degree = 12 : R2=0.921376 | MSE=6.578956 | var=6.998889 | s_hat=3947.373605 |
--------------------------------------------------------------
>>> Degree = 13 : R2=0.922723 | MSE=6.466181 | var=6.915702 | s_hat=3879.708628 |
--------------------------------------------------------------
>>> Degree = 14 : R2=0.924222 | MSE=6.340771 | var=6.818033 | s_hat=3804.462682 |
```

```
--------------------------------------------------------------
>>> Degree = 15 : R2=0.925949 | MSE=6.196300 | var=6.698702 | s_hat=3717.779886 |
--------------------------------------------------------------
>>> Degree = 16 : R2=0.923542 | MSE=6.397703 | var=6.954025 | s_hat=3838.621805 |
--------------------------------------------------------------
>>> Degree = 17 : R2=0.925115 | MSE=6.266048 | var=6.848140 | s_hat=3759.628619 |
--------------------------------------------------------------
>>> Degree = 18 : R2=0.925110 | MSE=6.266484 | var=6.886247 | s_hat=3759.890603 |
--------------------------------------------------------------
>>> Degree = 19 : R2=0.924153 | MSE=6.346533 | var=7.012744 | s_hat=3807.919819 |
--------------------------------------------------------------
>>> Degree = 20 : R2=0.924061 | MSE=6.354210 | var=7.060233 | s_hat=3812.525879 |
--------------------------------------------------------------
>>> Degree = 21 : R2=0.927137 | MSE=6.096884 | var=6.812161 | s_hat=3658.130515 |
--------------------------------------------------------------
>>> Degree = 22 : R2=0.924725 | MSE=6.298685 | var=7.077174 | s_hat=3779.210880 |
--------------------------------------------------------------
>>> Degree = 23 : R2=0.925794 | MSE=6.209204 | var=7.016050 | s_hat=3725.522446 |
--------------------------------------------------------------
>>> Degree = 24 : R2=0.926549 | MSE=6.146046 | var=6.984143 | s_hat=3687.627719 |
--------------------------------------------------------------
>>> Degree = 25 : R2=0.926929 | MSE=6.114302 | var=6.987774 | s_hat=3668.581177 |
--------------------------------------------------------------
>>> Degree = 26 : R2=0.926950 | MSE=6.112539 | var=7.025907 | s_hat=3667.523245 |
--------------------------------------------------------------
>>> Degree = 27 : R2=0.925969 | MSE=6.194561 | var=7.161342 | s_hat=3716.736650 |
--------------------------------------------------------------
>>> Degree = 28 : R2=0.925885 | MSE=6.201612 | var=7.211177 | s_hat=3720.967253 |
--------------------------------------------------------------
>>> Degree = 29 : R2=0.925805 | MSE=6.208288 | var=7.261155 | s_hat=3724.972753 |
--------------------------------------------------------------
>>> Degree = 30 : R2=NaN | MSE=NaN | var=NaN | s_hat=NaN |
--------------------------------------------------------------
>>> Degree = 31 : R2=0.925887 | MSE=6.201456 | var=7.339002 | s_hat=3720.873762 |
--------------------------------------------------------------
>>> Degree = 32 : R2=0.925871 | MSE=6.202758 | var=7.384236 | s_hat=3721.654884 |
--------------------------------------------------------------
>>> Degree = 33 : R2=0.926118 | MSE=6.182165 | var=7.403791 | s_hat=3709.299121 |
--------------------------------------------------------------
>>> Degree = 34 : R2=0.925499 | MSE=6.233889 | var=7.510709 | s_hat=3740.333185 |
--------------------------------------------------------------
>>> Degree = 35 : R2=0.924962 | MSE=6.278868 | var=7.610749 | s_hat=3767.320596 |
--------------------------------------------------------------
>>> Degree = 36 : R2=0.927080 | MSE=6.101667 | var=7.441057 | s_hat=3660.999989 |
--------------------------------------------------------------
>>> Degree = 37 : R2=0.927660 | MSE=6.053132 | var=7.427155 | s_hat=3631.878958 |
--------------------------------------------------------------
>>> Degree = 38 : R2=0.928244 | MSE=6.004211 | var=7.412606 | s_hat=3602.526624 |
--------------------------------------------------------------
>>> Degree = 39 : R2=0.924555 | MSE=6.312925 | var=7.842143 | s_hat=3787.754969 |
--------------------------------------------------------------
>>> Degree = 40 : R2=0.923598 | MSE=6.393008 | var=7.991260 | s_hat=3835.804858 |
--------------------------------------------------------------
>>> Degree = 41 : R2=0.926416 | MSE=6.157193 | var=7.744897 | s_hat=3694.315682 |
--------------------------------------------------------------
>>> Degree = 42 : R2=0.925513 | MSE=6.232763 | var=7.889573 | s_hat=3739.657720 |
--------------------------------------------------------------
```

```
>>> Degree = 43 : R2=0.924533 | MSE=6.314779 | var=8.044304 | s_hat=3788.867346 |
-------------------------------------------------------------
>>> Degree = 44 : R2=0.849272 | MSE=12.612235 | var=16.169532 | s_hat=7567.340771 |
-------------------------------------------------------------
>>> Degree = 45 : R2=0.927621 | MSE=6.056367 | var=7.814667 | s_hat=3633.820006 |
-------------------------------------------------------------
>>> Degree = 46 : R2=0.923527 | MSE=6.398909 | var=8.310272 | s_hat=3839.345476 |
-------------------------------------------------------------
>>> Degree = 47 : R2=0.928522 | MSE=5.980981 | var=7.818276 | s_hat=3588.588599 |
-------------------------------------------------------------
>>> Degree = 48 : R2=-27.283888 | MSE=2366.673784 | var=3114.044453 | s_hat=1420004.270589 |
-------------------------------------------------------------
>>> Degree = 49 : R2=0.926954 | MSE=6.112141 | var=8.095551 | s_hat=3667.284457 |
-------------------------------------------------------------
>>> Degree = 50 : R2=0.914470 | MSE=7.156823 | var=9.542431 | s_hat=4294.094091 |
-------------------------------------------------------------
>>> Degree = 51 : R2=0.928935 | MSE=5.946416 | var=7.981766 | s_hat=3567.849364 |
-------------------------------------------------------------
>>> Degree = 52 : R2=0.916994 | MSE=6.945614 | var=9.385965 | s_hat=4167.368571 |
-------------------------------------------------------------
>>> Degree = 53 : R2=-234772.835614 | MSE=19644862.518363 | var=26727704.106616 |
s_hat=11786917511.017809 |
-------------------------------------------------------------
>>> Degree = 54 : R2=0.925944 | MSE=6.196665 | var=8.488583 | s_hat=3717.999187 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.928178 | MSE=6.009741 | var=8.289297 | s_hat=3605.844301 |
-------------------------------------------------------------
>>> Degree = 56 : R2=0.926862 | MSE=6.119893 | var=8.499852 | s_hat=3671.936056 |
-------------------------------------------------------------
>>> Degree = 57 : R2=0.926755 | MSE=6.128811 | var=8.571764 | s_hat=3677.286871 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.927379 | MSE=6.076618 | var=8.558617 | s_hat=3645.970749 |
-------------------------------------------------------------
>>> Degree = 59 : R2=-2238.386061 | MSE=187382.172233 | var=265790.315225 |
s_hat=112429303.340044 |
-------------------------------------------------------------
>>> Degree = 60 : R2=0.922385 | MSE=6.494488 | var=9.277840 | s_hat=3896.692693 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.928318 | MSE=5.998063 | var=8.630306 | s_hat=3598.837766 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.923548 | MSE=6.397215 | var=9.271326 | s_hat=3838.328803 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.924783 | MSE=6.293832 | var=9.188076 | s_hat=3776.299325 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.921746 | MSE=6.547962 | var=9.629356 | s_hat=3928.777198 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.924458 | MSE=6.321023 | var=9.364478 | s_hat=3792.613731 |
-------------------------------------------------------------
>>> Degree = 66 : R2=0.923136 | MSE=6.431689 | var=9.599536 | s_hat=3859.013517 |
-------------------------------------------------------------
>>> Degree = 67 : R2=0.925707 | MSE=6.216527 | var=9.348161 | s_hat=3729.916368 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.911162 | MSE=7.433542 | var=11.262943 | s_hat=4460.125316 |
-------------------------------------------------------------
>>> Degree = 69 : R2=-0.983424 | MSE=165.964334 | var=253.380663 | s_hat=99578.600385 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.776832 | MSE=18.673715 | var=28.728792 | s_hat=11204.228818 |
```

```
------------------------------------------------------------
>>> Degree = 71 : R2=0.892530 | MSE=8.992666 | var=13.942119 | s_hat=5395.599898 |
------------------------------------------------------------
>>> Degree = 72 : R2=0.927069 | MSE=6.102585 | var=9.535289 | s_hat=3661.550898 |
------------------------------------------------------------
Warning: The "C" polynomial is unstable and cannot be automatically stabilized to meet the
prescribed constraints. This can cause the estimation to fail. Make sure that the starting
polynomial value is stable and within the desired constraints.
```

```
fprintf("================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 51
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
p = na+nb+nc;

BestFitModel_1 = armax(data1, [na nb nc 1])
```

```
BestFitModel_1 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

  A(z) = 1 + 0.2833 z^-1 + 0.02414 z^-2 - 0.06437 z^-3 - 0.329 z^-4 + 0.07484 z^-5 - 0.2868
z^-6
          + 0.2099 z^-7 + 0.1113 z^-8 + 0.1013 z^-9 - 0.2052 z^-10 - 0.02271 z^-11 + 0.07545
z^-12
          - 0.08059 z^-13 + 0.0368 z^-14 - 0.2083 z^-15 + 0.008896 z^-16 - 0.06353 z^-17 -
0.3018 z^-18
          + 0.029 z^-19 + 0.1839 z^-20 + 0.03226 z^-21 + 0.05514 z^-22 - 0.08423 z^-23 +
0.02432 z^-24
          + 0.09121 z^-25 - 0.06995 z^-26 + 0.1317 z^-27 - 0.03288 z^-28 - 0.07743 z^-29 -
0.1287 z^-30
```

```
          + 0.02587 z^-31 + 0.1666 z^-32 + 0.04988 z^-33 + 0.05126 z^-34 - 0.05232 z^-35 +
0.03721 z^-36
          + 0.1048 z^-37 + 0.08769 z^-38 + 0.06335 z^-39 - 0.1915 z^-40 - 0.09423 z^-41 +
0.009659 z^-42
          - 0.04501 z^-43 + 0.02115 z^-44 + 0.007338 z^-45 - 0.0215 z^-46 - 0.1751 z^-47 -
0.1072 z^-48
                                                        - 0.1681 z^-49 - 0.02186 z^-50 +
0.1007 z^-51


  B(z) = 0.8334 z^-1 + 1.926 z^-2 + 3.218 z^-3 + 4.056 z^-4 + 3.546 z^-5 + 2.593 z^-6 + 1.003
z^-7
          - 0.3851 z^-8 - 1.217 z^-9 - 1.217 z^-10 - 0.8978 z^-11 - 0.1434 z^-12 + 0.8496 z^-
13
          + 1.834 z^-14 + 2.17 z^-15 + 2.039 z^-16 + 1.845 z^-17 + 0.8677 z^-18 - 0.05605 z^-
19
          - 1.09 z^-20 - 1.24 z^-21 - 1.042 z^-22 - 0.7463 z^-23 + 0.2065 z^-24 + 1.175 z^-25
          + 1.718 z^-26 + 1.874 z^-27 + 1.71 z^-28 + 1.799 z^-29 + 1.115 z^-30 - 0.003041 z^-
31
          - 0.6427 z^-32 - 0.8916 z^-33 - 0.6135 z^-34 - 0.09029 z^-35 + 0.7621 z^-36 + 1.398
z^-37
          + 1.776 z^-38 + 1.607 z^-39 + 1.773 z^-40 + 1.783 z^-41 + 1.024 z^-42 + 0.07944 z^-
43
          - 0.4013 z^-44 - 0.9407 z^-45 - 0.8701 z^-46 - 0.7584 z^-47 - 0.3623 z^-48 + 0.2459
z^-49
                                                                        + 0.1991 z^-50 +
0.1622 z^-51


  C(z) = 1 + 0.3664 z^-1 + 0.06127 z^-2 - 0.02041 z^-3 - 0.2839 z^-4 + 0.0479 z^-5 - 0.3311
z^-6
          + 0.166 z^-7 + 0.1774 z^-8 + 0.1543 z^-9 - 0.1711 z^-10 - 0.05377 z^-11 + 0.06951
z^-12
          - 0.1365 z^-13 - 0.01865 z^-14 - 0.161 z^-15 - 0.0752 z^-16 - 0.1593 z^-17 - 0.2574
z^-18
          + 0.1197 z^-19 + 0.2445 z^-20 + 0.1155 z^-21 + 0.009768 z^-22 - 0.0483 z^-23 -
0.1191 z^-24
          + 0.08813 z^-25 + 0.01009 z^-26 + 0.06269 z^-27 + 0.1061 z^-28 - 0.09221 z^-29 -
0.04284 z^-30
          + 0.0117 z^-31 + 0.1633 z^-32 + 0.04538 z^-33 + 0.04972 z^-34 + 0.01593 z^-35 -
0.07374 z^-36
          + 0.165 z^-37 - 0.02583 z^-38 + 0.1792 z^-39 - 0.1177 z^-40 - 0.1974 z^-41 + 0.02309
z^-42
          - 0.2787 z^-43 - 0.04661 z^-44 - 0.0189 z^-45 + 0.03768 z^-46 - 0.1389 z^-47 -
0.2299 z^-48
                                                     - 0.1594 z^-49 + 0.0008167 z^-50 +
0.08274 z^-51

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=51   nb=51   nc=51   nk=1
   Number of free coefficients: 153
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data1".
```

```
Fit to estimation data: 76.41% (prediction focus)
FPE: 9.455, MSE: 4.657
```

```
BestFit_y_hat_1 = lsim(BestFitModel_1, u1_val, t_val);
% [armax_BestFit_r2, armax_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```

minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 15 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 15
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
p = na+nb+nc;

armax_VarModel_1 = armax(data1, [na nb nc nk]);
Var_y_hat_1 = lsim(armax_VarModel_1, u1_val, t_val);
% [armax_Var_r2, armax_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```

%%
%
% fprintf("===============Degree Extraction | CoVariance
Method=================\n")
%
% maxCovIndex = find(covs == max(covs),1);
% fprintf(">>> Since the maximum accuracy occurs in iteration %d ;\n",
maxCovIndex)
% fprintf("    Degree = %d \n", maxCovIndex)
% na = maxCovIndex;
% nb = maxCovIndex;
```

```
% nc = maxCovIndex;
% p = na+nb+nc+1;
%
% armax_CovModel_1 = armax(data1, [na nb nc 1]);
% Cov_y_hat_1 = lsim(armax_CovModel_1, u1_val, t_val);
%
%
fprintf("=================================================================\n")


%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 3
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
p = na+nb+nc;

armax_AICModel_1 = armax(data1, [na nb nc nk])
```

```
armax_AICModel_1 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 2.637 z^-1 + 2.528 z^-2 - 0.8876 z^-3

  B(z) = 0.6649 z^-1 + 0.1202 z^-2 - 0.4849 z^-3

  C(z) = 1 - 2.658 z^-1 + 2.567 z^-2 - 0.9095 z^-3

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nc=3   nk=1
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
```

```
Status:
Estimated using ARMAX on time domain data "data1".
Fit to estimation data: 71.8% (prediction focus)
FPE: 6.856, MSE: 6.653
```

```
AIC_y_hat_1 = lsim(armax_AICModel_1, u1_val, t_val);
% [armax_AIC_r2, armax_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test 
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", 
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.20 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 3
```

```
na = winner;
nb = winner;
nc = winner;
p = na+nb+nc;

armax_FTestModel_1 = armax(data1, [na nb nc nk])
```

```
armax_FTestModel_1 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 2.637 z^-1 + 2.528 z^-2 - 0.8876 z^-3

  B(z) = 0.6649 z^-1 + 0.1202 z^-2 - 0.4849 z^-3

  C(z) = 1 - 2.658 z^-1 + 2.567 z^-2 - 0.9095 z^-3

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nc=3   nk=1
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data1".
Fit to estimation data: 71.8% (prediction focus)
FPE: 6.856, MSE: 6.653
```

```
FTest_y_hat_1 = lsim(armax_FTestModel_1, u1_val, t_val);
% [armax_FTest_r2, armax_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[armax_BestFit_r2_1, armax_BestFit_mse_1] = rSQR(y1_val, BestFit_y_hat_1);
[armax_Var_r2_1, armax_Var_mse_1] = rSQR(y1_val, Var_y_hat_1);
[armax_AIC_r2_1, armax_AIC_mse_1] = rSQR(y1_val, AIC_y_hat_1);
[armax_FTest_r2_1, armax_FTest_mse_1] = rSQR(y1_val, FTest_y_hat_1);

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_BestFit_r2_1,
armax_BestFit_mse_1)
```

```
    R2 value : 0.6559    | MSE : 25.4468
```

```
fprintf("----------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Var_r2_1, armax_Var_mse_1)
```

```
    R2 value : 0.6574    | MSE : 25.3383
```

```
% fprintf("----------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Cov_r2, armax_Cov_mse)
fprintf("----------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_AIC_r2_1, armax_AIC_mse_1)
```

```
    R2 value : 0.6627    | MSE : 24.9455
```

```
fprintf("----------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_FTest_r2_1, armax_FTest_mse_1)
```

```
    R2 value : 0.6627    | MSE : 24.9455
```

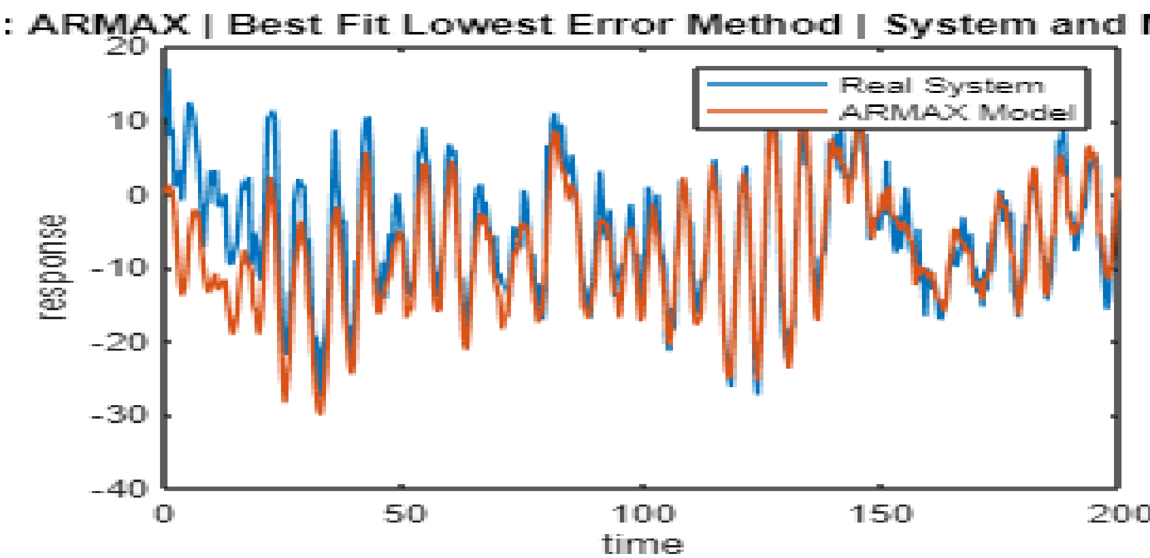
```
fprintf("----------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

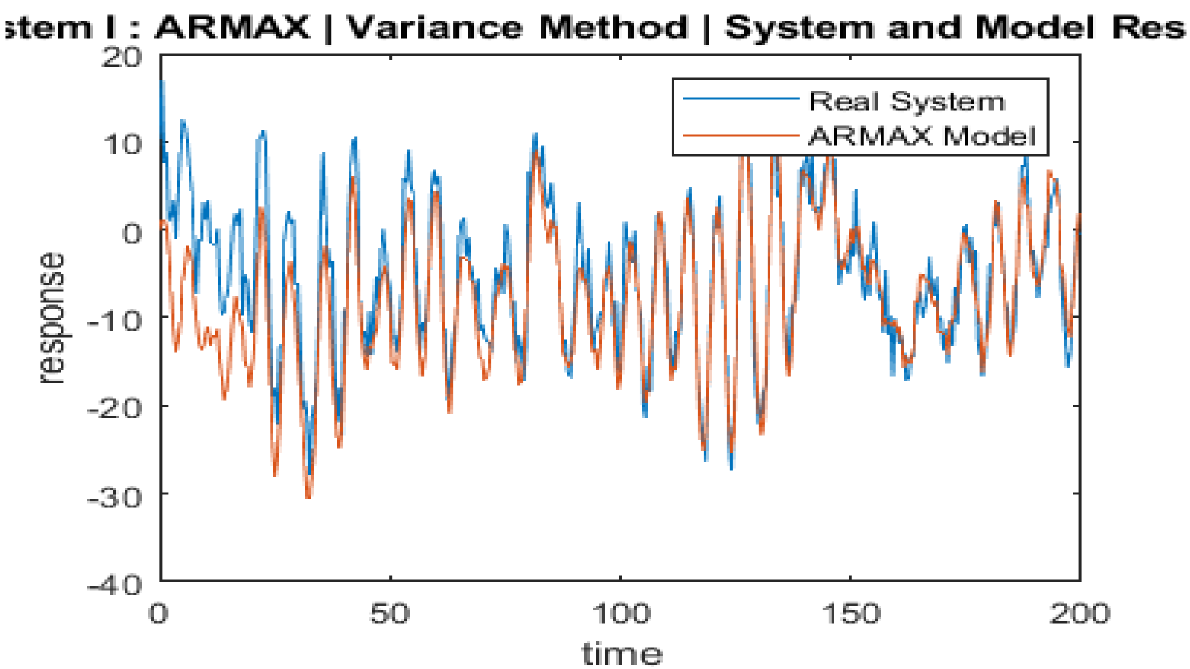
```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("================================================================\n")
```

```
=============================================================
```

```
%%

armax_BestFitError_1 = y1_val - BestFit_y_hat_1;
armax_VarError_1 = y1_val - Var_y_hat_1;
armax_AICError_1 = y1_val - AIC_y_hat_1;
armax_FTestError_1 = y1_val - FTest_y_hat_1;

for k=0:N_val-1
    armax_BestFit_Ree_1(k+1,1) = AutoCorrelate(armax_BestFitError_1, k);
    armax_Var_Ree_1(k+1,1) = AutoCorrelate(armax_VarError_1, k);
    armax_AIC_Ree_1(k+1,1) = AutoCorrelate(armax_AICError_1, k);
    armax_FTest_Ree_1(k+1,1) = AutoCorrelate(armax_FTestError_1, k);
end

for k=0:N_val-1
    armax_BestFit_Rue_1(k+1,1) = CrossCorrelate(u1_val, armax_BestFitError_1,
k);
    armax_Var_Rue_1(k+1,1) = CrossCorrelate(u1_val, armax_VarError_1, k);
    armax_AIC_Rue_1(k+1,1) = CrossCorrelate(u1_val, armax_AICError_1, k);
    armax_FTest_Rue_1(k+1,1) = CrossCorrelate(u1_val, armax_FTestError_1, k);
end


%%
figure(1)
plot(t_val,y1_val,t_val,BestFit_y_hat_1)
legend('Real System','ARMAX Model')
title(" System I : ARMAX | Best Fit Lowest Error Method | System and Model
Response")
xlabel("time")
ylabel("response")
```

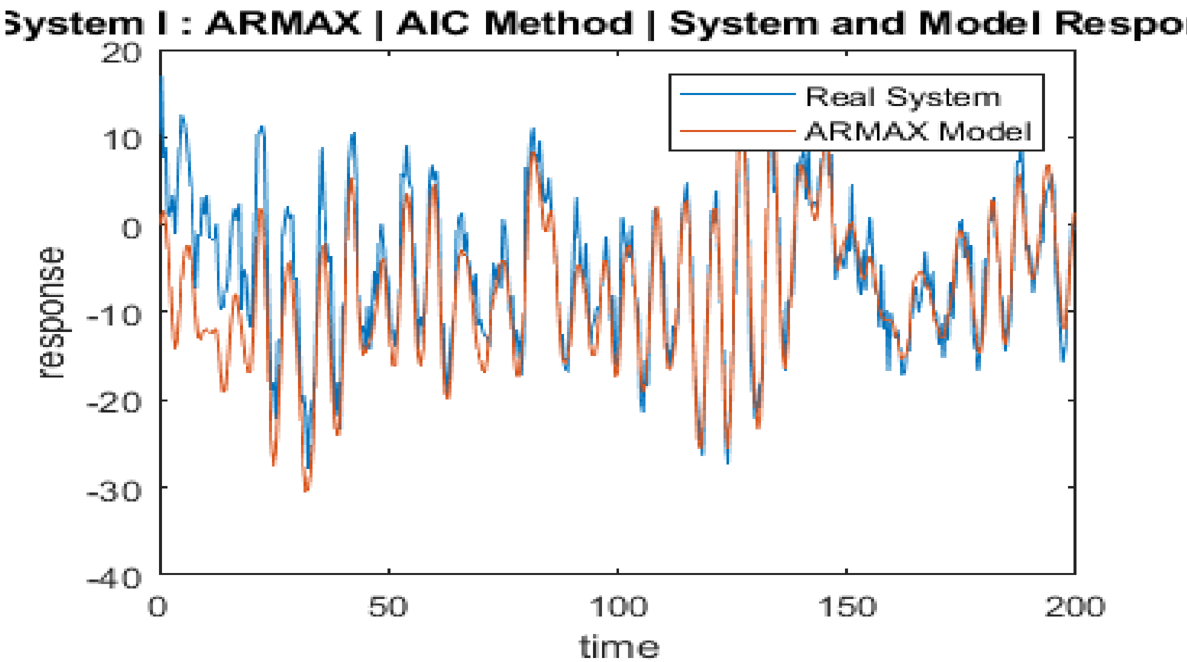



```
figure(2)
plot(t_val,y1_val,t_val,Var_y_hat_1)
legend('Real System','ARMAX Model')
title(" System I : ARMAX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```

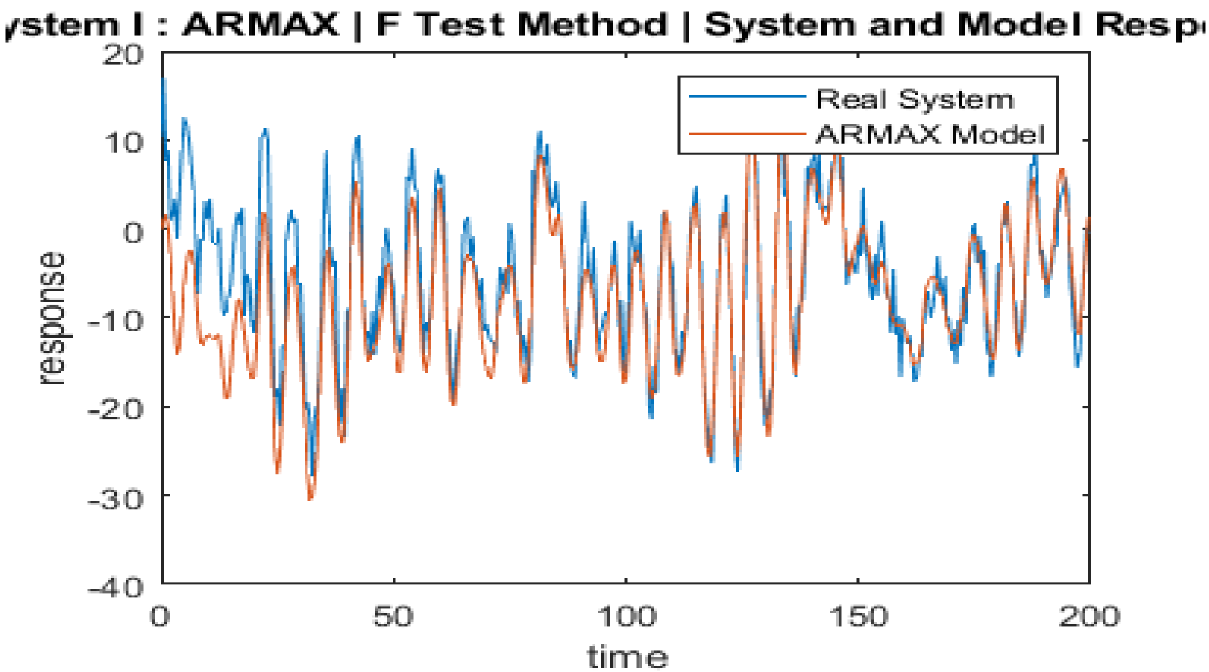



```
figure(3)
plot(t_val,y1_val,t_val,AIC_y_hat_1)
```

```
legend('Real System','ARMAX Model')
title(" System I : ARMAX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

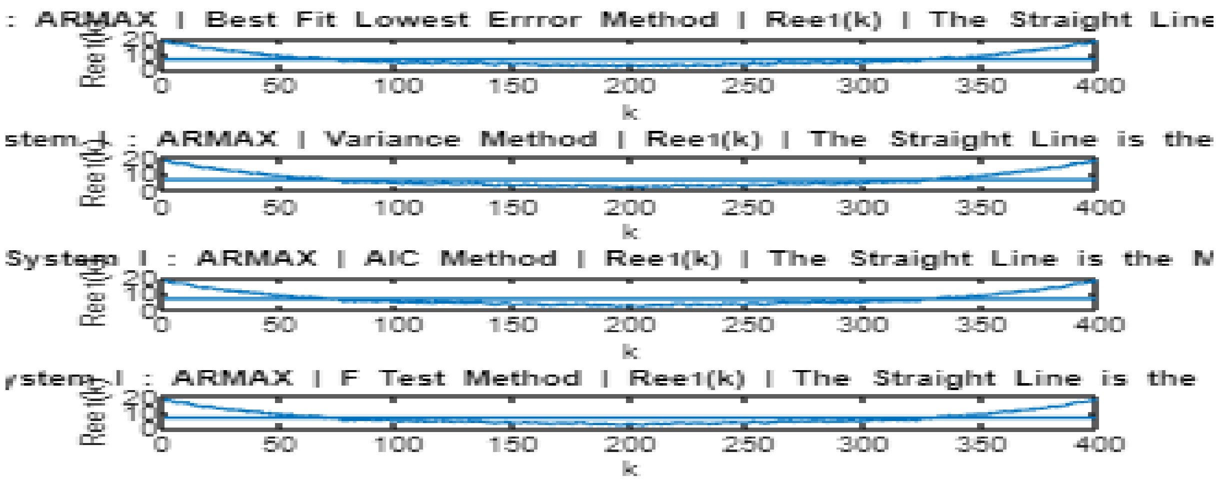



```
figure(4)
plot(t_val,y1_val,t_val,FTest_y_hat_1)
legend('Real System','ARMAX Model')
title(" System I : ARMAX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



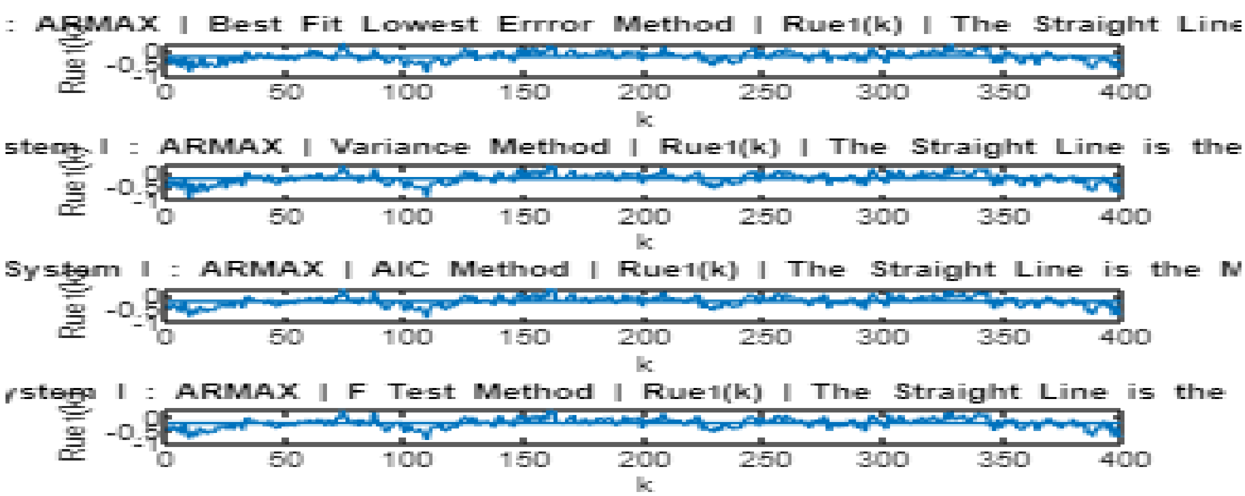

```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Ree_1(2:end), 1:N_val-1,
mean(armax_BestFit_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARMAX | Best Fit Lowest Errror Method | Ree_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(4,1,2)
plot(1:N_val-1,armax_Var_Ree_1(2:end), 1:N_val-1,
mean(armax_Var_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARMAX | Variance Method | Ree_1(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(4,1,3)
plot(1:N_val-1,armax_AIC_Ree_1(2:end), 1:N_val-1,
mean(armax_AIC_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARMAX | AIC Method | Ree_1(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_1(k)")
```

```
subplot(4,1,4)
plot(1:N_val-1,armax_FTest_Ree_1(2:end), 1:N_val-1,
mean(armax_FTest_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARMAX | F Test Method | Ree_1(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_1(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Rue_1(2:end), 1:N_val-1,
mean(armax_BestFit_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARMAX | Best Fit Lowest Errror Method | Rue_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(4,1,2)
plot(1:N_val-1,armax_Var_Rue_1(2:end), 1:N_val-1,
mean(armax_Var_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : ARMAX | Variance Method | Rue_1(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(4,1,3)
```

```
 plot(1:N_val-1,armax_AIC_Rue_1(2:end), 1:N_val-1,
mean(armax_AIC_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : ARMAX | AIC Method | Rue_1(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(4,1,4)
 plot(1:N_val-1,armax_FTest_Rue_1(2:end), 1:N_val-1,
mean(armax_FTest_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : ARMAX | F Test Method | Rue_1(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_1(k)")
```




```

```

System II

```
 %%

 % System Z2 **************************************************************
 fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
 fprintf(">>> System II Identification Begins:------------------------------\n")
```

```
>>> System II Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);

data2 = iddata(y2,u2,Ts);


%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nk = 1;
    p = na+nb+nc;

    try
        sys = armax(data2, [na nb nc nk]);
        armax_y_hat_2 = lsim(sys, u2, t);
    catch
        break
    end

    [r2_armax, mse_armax] = rSQR(y2, armax_y_hat_2);

    error = y2 - armax_y_hat_2;
```

```
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_armax, mse_armax, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_armax];
    MSEs = [MSEs; mse_armax];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.076105 | MSE=69.293914 | var=69.642125 | s_hat=41576.348374 |
------------------------------------------------------------
>>> Degree = 2 : R2=0.670105 | MSE=24.742754 | var=24.992681 | s_hat=14845.652283 |
------------------------------------------------------------
>>> Degree = 3 : R2=0.669880 | MSE=24.759631 | var=25.136681 | s_hat=14855.778603 |
------------------------------------------------------------
>>> Degree = 4 : R2=-30.247620 | MSE=2343.631100 | var=2391.460306 | s_hat=1406178.659745 |
------------------------------------------------------------
>>> Degree = 5 : R2=-402388866955710983831552OO.000000 |
MSE=30179932300060342085400657920.000000 | var=30953776718010605434558218240.000000 |
s_hat=18107959380036203887845976309760.000000 |
------------------------------------------------------------
>>> Degree = 6 : R2=-24225.169220 | MSE=1817008.886138 | var=1873205.037256 |
s_hat=1090205331.683068 |
------------------------------------------------------------
>>> Degree = 7 : R2=-739086.490314 | MSE=55432971.071652 | var=57443493.338499 |
s_hat=33259782642.991188 |
------------------------------------------------------------
>>> Degree = 8 : R2=-69809.682214 | MSE=5235934.281636 | var=5454098.210037 |
s_hat=3141560568.981592 |
------------------------------------------------------------
>>> Degree = 9 : R2=-256.249162 | MSE=19294.177681 | var=20203.327414 | s_hat=11576506.608390
|
------------------------------------------------------------
>>> Degree = 10 : R2=-166733.899968 | MSE=12505435.429084 | var=13163616.241141 |
s_hat=7503261257.450581 |
------------------------------------------------------------
>>> Degree = 11 : R2=-66230.650401 | MSE=4967500.071155 | var=5256613.831911 |
s_hat=2980500042.693299 |
------------------------------------------------------------
```

```
>>> Degree = 12 : R2=-7774946496.723517 | MSE=583135812660.824707 | var=620357247511.514526 |
s_hat=349881487596494.187500 |
------------------------------------------------------------
>>> Degree = 13 : R2=-12583011.268630 | MSE=943749913.537949 | var=1009358196.297272 |
s_hat=566249948122.769409 |
------------------------------------------------------------
>>> Degree = 14 : R2=-996418.652626 | MSE=74733373.928118 | var=80358466.589374 |
s_hat=44840024356.870613 |
------------------------------------------------------------
>>> Degree = 15 : R2=-3335875.305350 | MSE=250197083.777431 | var=270483333.813439 |
s_hat=150118250266.458801 |
------------------------------------------------------------
>>> Degree = 16 : R2=-58275248.362466 | MSE=4370754821.906986 | var=4750820458.594545 |
s_hat=2622452893144.188965 |
------------------------------------------------------------
>>> Degree = 17 : R2=-1695447573.266020 | MSE=127161800963.250519 | var=138974645861.476440 |
s_hat=76297080577950.562500 |
------------------------------------------------------------
>>> Degree = 18 : R2=-30110056688.674244 | MSE=2258311665827.886719 | var=2481661171239.435547
| s_hat=1354986999496731.750000 |
------------------------------------------------------------
>>> Degree = 19 : R2=-6086304.781440 | MSE=456484539.025645 | var=504402805.553198 |
s_hat=273890723415.386719 |
------------------------------------------------------------
>>> Degree = 20 : R2=-1111868769.854708 | MSE=83392277934.567581 | var=92658086593.964066 |
s_hat=50035366760740.593750 |
------------------------------------------------------------
>>> Degree = 21 : R2=-73664.534067 | MSE=5525055.521062 | var=6173246.392248 |
s_hat=3315033312.637136 |
------------------------------------------------------------
>>> Degree = 22 : R2=-72382735120842832.000000 | MSE=5428843154054445056.000000 |
var=6099823768600512512.000000 | s_hat=3257305892432673505280.000000 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.607638 | MSE=29.427870 | var=33.251830 | s_hat=17656.721960 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.609993 | MSE=29.251246 | var=33.240053 | s_hat=17550.747781 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.336169 | MSE=49.788613 | var=56.901272 | s_hat=29873.167575 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.520487 | MSE=35.964394 | var=41.338383 | s_hat=21578.636110 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.666492 | MSE=25.013737 | var=28.917615 | s_hat=15008.242382 |
------------------------------------------------------------
>>> Degree = 28 : R2=-2.790878 | MSE=284.323077 | var=330.608229 | s_hat=170593.846241 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.359757 | MSE=48.019470 | var=56.163123 | s_hat=28811.682216 |
------------------------------------------------------------
>>> Degree = 30 : R2=-652034.423248 | MSE=48903900.050079 | var=57534000.058917 |
s_hat=29342340030.047668 |
------------------------------------------------------------
>>> Degree = 31 : R2=-1250066.156720 | MSE=93757420.392267 | var=110955527.091440 |
s_hat=56254452235.360199 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.673498 | MSE=24.488294 | var=29.152731 | s_hat=14692.976349 |
------------------------------------------------------------
>>> Degree = 33 : R2=-204379467.875462 | MSE=15328849878.102179 | var=18357904045.631359 |
s_hat=9197309926861.310547 |
```

```
------------------------------------------------------------
>>> Degree = 34 : R2=-542674.516853 | MSE=40701698.542077 | var=49038191.014551 |
s_hat=24421019125.246372 |
------------------------------------------------------------
>>> Degree = 35 : R2=-938440967.417897 | MSE=70384862058.254852 | var=85314984313.036194 |
s_hat=42230917234952.914062 |
------------------------------------------------------------
>>> Degree = 36 : R2=-537856234.310637 | MSE=40340243236.965546 | var=49195418581.665367 |
s_hat=24204145942179.359375 |
------------------------------------------------------------
>>> Degree = 37 : R2=-1119561.407924 | MSE=83969315.385071 | var=103029834.828308 |
s_hat=50381589231.042549 |
------------------------------------------------------------
>>> Degree = 38 : R2=-1691968658238720164220146548736.000000 |
MSE=126900875627577497992812382126080.000000 | var=156667747688367245443810860204032.000000 |
s_hat=76140525376546485392974938221051904.000000 |
------------------------------------------------------------
>>> Degree = 39 : R2=-19247650813552774946488320.000000 |
MSE=1443610512594425407504121856.000000 | var=1793304984589348847912747008.000000 |
s_hat=866166307556655446812612624384.000000 |
------------------------------------------------------------
>>> Degree = 40 : R2=-603.648722 | MSE=45349.807062 | var=56687.258828 | s_hat=27209884.237478
|
------------------------------------------------------------
>>> Degree = 41 : R2=-402967996282568508001851 8016.000000 |
MSE=30223368103863189575222637 3632.000000 | var=38016815224985146996486569 9840.000000 |
s_hat=18134020862317914111051052 1417728.000000 |
------------------------------------------------------------
>>> Degree = 42 : R2=-
4975039200915920097467937533522445256665639148563428260996070150951921581969236819 9680.000000
|
MSE=3731374265141253136974569187563242039405543891645341555769721405360196647560196741136384.0
00000 |
var=4723258563469940109403250214019288602107191395191291164254845192502990286772930900656128.0
00000 |
s_hat=2238824559084751793842388273618728744478451297841279142087637999435170082455807980050841
600.000000 |
------------------------------------------------------------
>>> Degree = 43 : R2=-11154311.703088 | MSE=836594721.866295 | var=1065725760.339230 |
s_hat=501956833119.777222 |
------------------------------------------------------------
>>> Degree = 44 : R2=-815165.123898 | MSE=61139013.657762 | var=78383350.843284 |
s_hat=36683408194.656975 |
------------------------------------------------------------
>>> Degree = 45 : R2=-1119380.012590 | MSE=83955710.389085 | var=108329948.889142 |
s_hat=50373426233.451019 |
------------------------------------------------------------
>>> Degree = 46 : R2=-254477.184236 | MSE=19086349.058812 | var=24787466.310146 |
s_hat=11451809435.287413 |
------------------------------------------------------------
>>> Degree = 47 : R2=-144994195.953928 | MSE=10874841247.664993 | var=14215478755.117641 |
s_hat=6524904748598.997070 |
------------------------------------------------------------
>>> Degree = 48 : R2=-1903934744.459510 | MSE=142798736347.805145 | var=187893074141.848785 |
s_hat=85679241808683.046875 |
------------------------------------------------------------
```

```
>>> Degree = 49 : R2=-1153309387.328254 | MSE=86500403264.391846 | var=114570070548.863342 |
s_hat=51900241958635.093750 |
-------------------------------------------------------------
>>> Degree = 50 : R2=-114016272.406835 | MSE=8551437912.675125 | var=11401917216.900164 |
s_hat=5130862747605.073242 |
-------------------------------------------------------------
>>> Degree = 51 : R2=-85531811.481076 | MSE=6415049028.840205 | var=8610804065.557327 |
s_hat=3849029417304.125000 |
-------------------------------------------------------------
>>> Degree = 52 : R2=-44899280.119394 | MSE=3367531698.270926 | var=4550718511.176935 |
s_hat=2020519018962.559326 |
-------------------------------------------------------------
>>> Degree = 53 : R2=-45575221.605664 | MSE=3418228598.631987 | var=4650651154.601343 |
s_hat=2050937159179.192383 |
-------------------------------------------------------------
>>> Degree = 54 : R2=-89739659.638364 | MSE=6730645664.196054 | var=9220062553.693235 |
s_hat=4038387398517.637207 |
-------------------------------------------------------------
>>> Degree = 55 : R2=-23936003.063205 | MSE=1795245946.108655 | var=2476201304.977458 |
s_hat=1077147567665.194458 |
-------------------------------------------------------------
>>> Degree = 56 : R2=-694713185.150772 | MSE=52104813646.092186 | var=72367796730.683533 |
s_hat=31262888187655.289062 |
-------------------------------------------------------------
>>> Degree = 57 : R2=-128324089.066744 | MSE=9624551444.430111 | var=13460911111.091053 |
s_hat=5774730866658.061523 |
-------------------------------------------------------------
>>> Degree = 58 : R2=-7095215745.156698 | MSE=532154710177.157715 | var=749513676305.855957 |
s_hat=319292826106294.625000 |
-------------------------------------------------------------
>>> Degree = 59 : R2=-63327901.317026 | MSE=4749713427.938927 | var=6737182167.289258 |
s_hat=2849828056763.355957 |
-------------------------------------------------------------
>>> Degree = 60 : R2=-251427185203.018890 | MSE=18857518313714.191406 |
var=26939311876734.558594 | s_hat=11314510988228514.000000 |
-------------------------------------------------------------
>>> Degree = 61 : R2=-87191673229951991461129421517292134304815102558208.000000 |
MSE=6539541750041916629459360820665882931410495961300992.000000 |
var=9409412589988370962760312299518833352975273254977536.000000 |
s_hat=3923725050025150955987421390097612216048293943114203136.000000 |
-------------------------------------------------------------
>>> Degree = 62 : R2=-625174278.917282 | MSE=46889263081.225380 | var=67955453740.906319 |
s_hat=28133557848735.214844 |
-------------------------------------------------------------
>>> Degree = 63 : R2=-2801756.222029 | MSE=210137134.065747 | var=306769538.782113 |
s_hat=126082280439.448288 |
-------------------------------------------------------------
>>> Degree = 64 : R2=-7366754.262731 | MSE=552520691.694018 | var=812530428.961792 |
s_hat=331512415016.411133 |
-------------------------------------------------------------
>>> Degree = 65 : R2=-36219187.170655 | MSE=2716508175.843311 | var=4024456556.804911 |
s_hat=1629904905505.988770 |
-------------------------------------------------------------
>>> Degree = 66 : R2=-6616499.359666 | MSE=496250143.371941 | var=740671855.779017 |
s_hat=297750086023.164734 |
-------------------------------------------------------------
```

```
>>> Degree = 67 : R2=-3035060.495868 | MSE=227635399.470209 | var=342308871.383772 |
s_hat=136581239682.125122 |
-------------------------------------------------------------
>>> Degree = 68 : R2=-11950124791867392.000000 | MSE=896282145985728896.000000 |
var=1358003251493528832.000000 | s_hat=537769287591437402112.000000 |
-------------------------------------------------------------
>>> Degree = 69 : R2=-104996697673.003769 | MSE=7874952534112.009766 |
var=12022828296354.212891 | s_hat=4724971520467206.000000 |
-------------------------------------------------------------
>>> Degree = 70 : R2=-8547258.687897 | MSE=641060774.576959 | var=986247345.503014 |
s_hat=384636464746.175598 |
-------------------------------------------------------------
>>> Degree = 71 : R2=-1213561024237.416504 | MSE=91019390845966.046875 |
var=141115334644908.625000 | s_hat=54611634507579640.000000 |
-------------------------------------------------------------
>>> Degree = 72 : R2=-103138073.433910 | MSE=7735552247.066959 | var=12086800386.042124 |
s_hat=4641331348240.175781 |
-------------------------------------------------------------
>>> Degree = 73 : R2=-27359.254468 | MSE=2052071.255887 | var=3231608.276988 |
s_hat=1231242753.532333 |
-------------------------------------------------------------
>>> Degree = 74 : R2=-909778170.741116 | MSE=68235097653.041222 | var=108309678814.351105 |
s_hat=40941058591824.718750 |
-------------------------------------------------------------
Warning: The "C" polynomial is unstable and cannot be automatically stabilized to meet the
prescribed constraints. This can cause the estimation to fail. Make sure that the starting
polynomial value is stable and within the desired constraints.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("==============Degree Extraction | BestFit Method=================\n")
```

```
==============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 32
```

```
na = bestFitDegree;
```

```
nb = bestFitDegree;
nc = bestFitDegree;
p = na+nb+nc;

BestFitModel_2 = armax(data2, [na nb nc 1])
```

```
BestFitModel_2 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

  A(z) = 1 - 0.1298 z^-1 - 0.5515 z^-2 + 0.5311 z^-3 + 0.2281 z^-4 - 0.8008 z^-5 + 0.1028 z^-6
          + 0.4015 z^-7 - 0.473 z^-8 - 0.3776 z^-9 + 0.4806 z^-10 - 0.07343 z^-11 - 0.2202 z^
          -12 + 0.375 z^-13 - 0.07283 z^-14 - 0.08965 z^-15 + 0.2112 z^-16 - 0.02517 z^-17 -
0.2289 z^-18
          + 0.1477 z^-19 - 0.05333 z^-20 + 0.003588 z^-21 + 0.2317 z^-22 - 0.2229 z^-23 -
0.288 z^-24
          + 0.2889 z^-25 - 0.0665 z^-26 - 0.2729 z^-27 + 0.281 z^-28 + 0.07074 z^-29 - 0.1327
z^-30
                                                                      - 0.09726 z^-31 +
0.148 z^-32


  B(z) = -6.221 z^-1 + 3.725 z^-2 + 24.05 z^-3 - 36.96 z^-4 + 16.65 z^-5 + 27.26 z^-6 - 25.9
z^-7
          - 1.845 z^-8 + 15.01 z^-9 - 13.85 z^-10 - 0.9133 z^-11 - 3.604 z^-12 + 4.311 z^-13
          - 8.403 z^-14 + 3.302 z^-15 - 0.4167 z^-16 - 6.653 z^-17 + 8.791 z^-18 - 12.22 z^-19
          - 3.492 z^-20 + 16.33 z^-21 - 14.24 z^-22 + 1.307 z^-23 + 7.225 z^-24 - 3.351 z^-25
          + 0.01708 z^-26 + 2.378 z^-27 + 10.17 z^-28 - 2.896 z^-29 + 3.001 z^-30 + 8.21 z^-31
                                                                                      -
0.689 z^-32


  C(z) = 1 - 0.09709 z^-1 - 0.5356 z^-2 + 0.4812 z^-3 + 0.2152 z^-4 - 0.6637 z^-5 + 0.05455
z^-6
          + 0.3758 z^-7 - 0.3593 z^-8 - 0.4348 z^-9 + 0.3596 z^-10 + 0.07266 z^-11 - 0.3295
z^-12
          + 0.1402 z^-13 + 0.02017 z^-14 - 0.009084 z^-15 + 0.07767 z^-16 + 0.02579 z^-17 -
0.1179 z^-18
          + 0.1941 z^-19 - 0.0006442 z^-20 + 0.06738 z^-21 + 0.2331 z^-22 - 0.1551 z^-23 -
0.2879 z^-24
          + 0.1701 z^-25 - 0.038 z^-26 - 0.1822 z^-27 + 0.1808 z^-28 + 0.01789 z^-29 +
0.008163 z^-30
                                                                      - 0.1147 z^-31 +
0.1005 z^-32

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=32   nb=32   nc=32   nk=1
   Number of free coefficients: 96
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data2".
Fit to estimation data: 44.88% (prediction focus)
FPE: 31.47, MSE: 22.79
```

```
BestFit_y_hat_2 = lsim(BestFitModel_2, u2_val, t_val);
% [armax_BestFit_r2, armax_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("==============Degree Extraction | Variance
Method===================\n")
```

```
==============Degree Extraction | Variance Method===================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 2
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
p = na+nb+nc;

armax_VarModel_2 = armax(data2, [na nb nc nk])
```

```
armax_VarModel_2 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.638 z^-1 + 0.884 z^-2

  B(z) = -0.4923 z^-1 + 1.759 z^-2

  C(z) = 1 - 1.683 z^-1 + 0.9285 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data2".
Fit to estimation data: 42.73% (prediction focus)
FPE: 25.1, MSE: 24.6
```

```
Var_y_hat_2 = lsim(armax_VarModel_2, u2_val, t_val);
% [armax_Var_r2, armax_Var_mse] = rSQR(y_val, Var_y_hat);
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
p = na+nb+nc;

armax_AICModel_2 = armax(data2, [na nb nc nk])
```

```
armax_AICModel_2 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.638 z^-1 + 0.884 z^-2

  B(z) = -0.4923 z^-1 + 1.759 z^-2

  C(z) = 1 - 1.683 z^-1 + 0.9285 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data2".
Fit to estimation data: 42.73% (prediction focus)
FPE: 25.1, MSE: 24.6
```

```
AIC_y_hat_2 = lsim(armax_AICModel_2, u2_val, t_val);
% [armax_AIC_r2, armax_AIC_mse] = rSQR(y_val, AIC_y_hat);
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test 
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", 
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.20 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 2
```

```
na = winner;
nb = winner;
nc = winner;
p = na+nb+nc;

armax_FTestModel_2 = armax(data2, [na nb nc nk])
```

```
armax_FTestModel_2 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.638 z^-1 + 0.884 z^-2

  B(z) = -0.4923 z^-1 + 1.759 z^-2
```

```
  C(z) = 1 - 1.683 z^-1 + 0.9285 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data2".
Fit to estimation data: 42.73% (prediction focus)
FPE: 25.1, MSE: 24.6
```

```
FTest_y_hat_2 = lsim(armax_FTestModel_2, u2_val, t_val);
% [armax_FTest_r2, armax_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[armax_BestFit_r2_2, armax_BestFit_mse_2] = rSQR(y2_val, BestFit_y_hat_2);
[armax_Var_r2_2, armax_Var_mse_2] = rSQR(y2_val, Var_y_hat_2);
[armax_AIC_r2_2, armax_AIC_mse_2] = rSQR(y2_val, AIC_y_hat_2);
[armax_FTest_r2_2, armax_FTest_mse_2] = rSQR(y2_val, FTest_y_hat_2);

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_BestFit_r2_2,
armax_BestFit_mse_2)
```

```
    R2 value : 0.6946    | MSE : 26.9895
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_Var_r2_2, armax_Var_mse_2)
```

```
    R2 value : 0.7020   | MSE : 26.3329
```

```
% fprintf("--------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_Cov_r2, armax_Cov_mse)
fprintf("--------------------------------------------------------------\n")
```

```
--------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_AIC_r2_2, armax_AIC_mse_2)
```

```
    R2 value : 0.7020   | MSE : 26.3329
```

```
fprintf("--------------------------------------------------------------\n")
```

```
--------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_FTest_r2_2, armax_FTest_mse_2)
```

```
    R2 value : 0.7020   | MSE : 26.3329
```

```
fprintf("--------------------------------------------------------------\n")
```

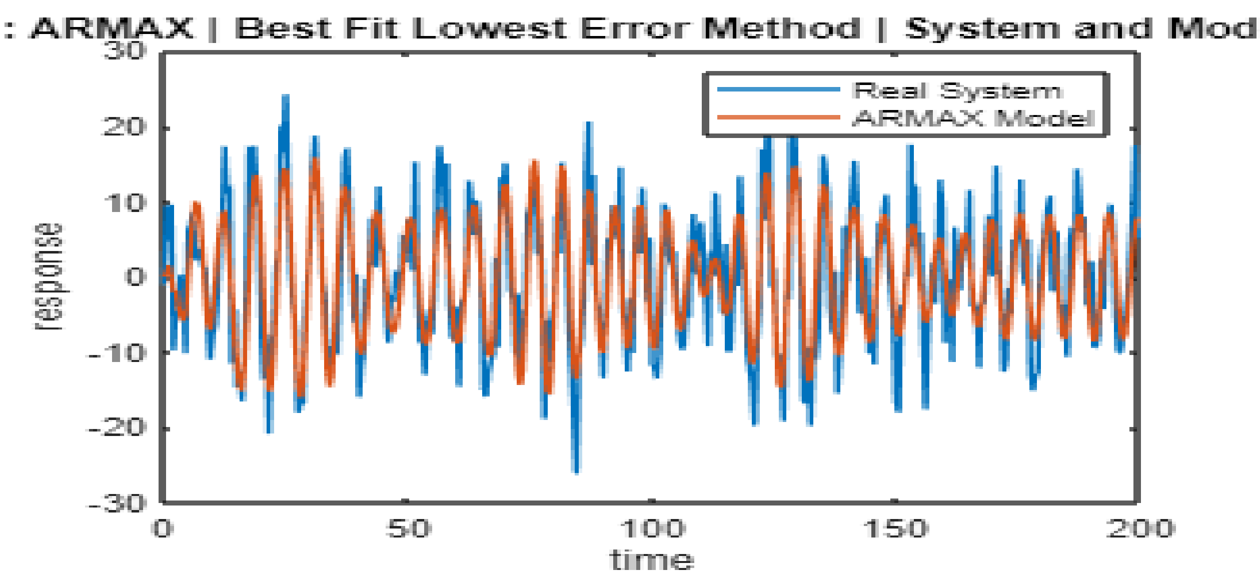
```
--------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("==============================================================\n")
```

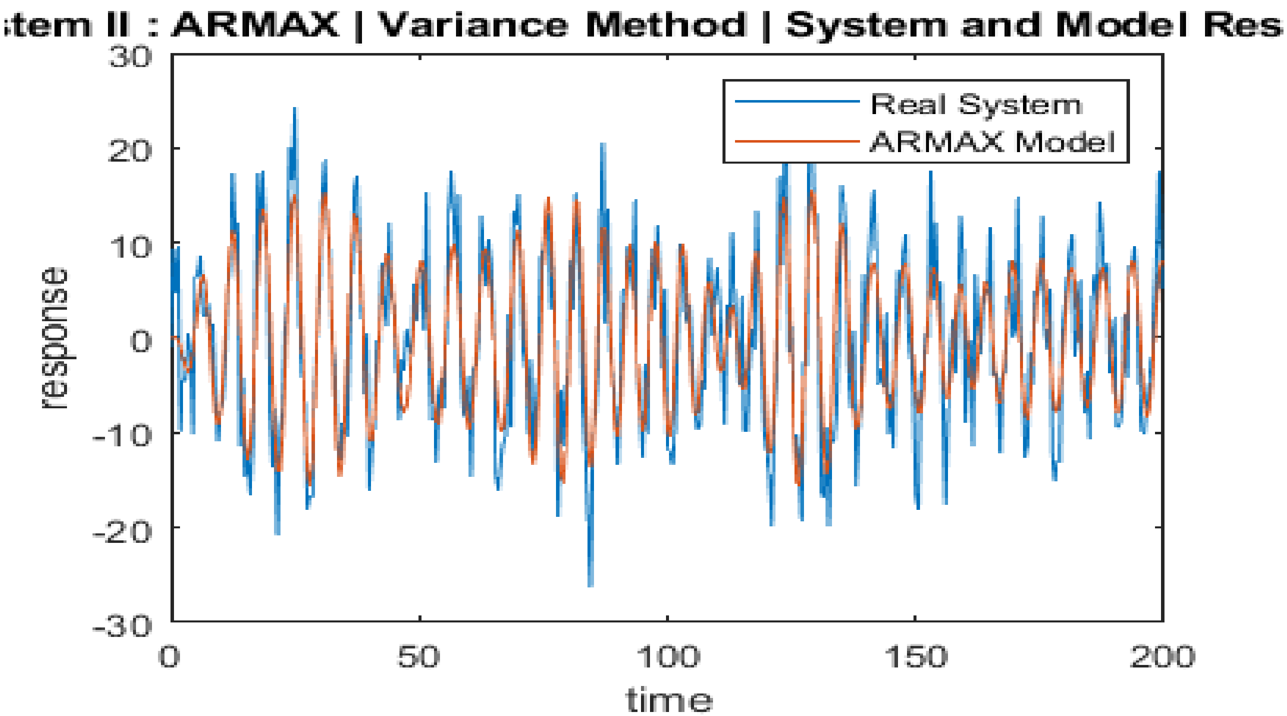
```
==============================================================
```

```
%%

armax_BestFitError_2 = y2_val - BestFit_y_hat_2;
```

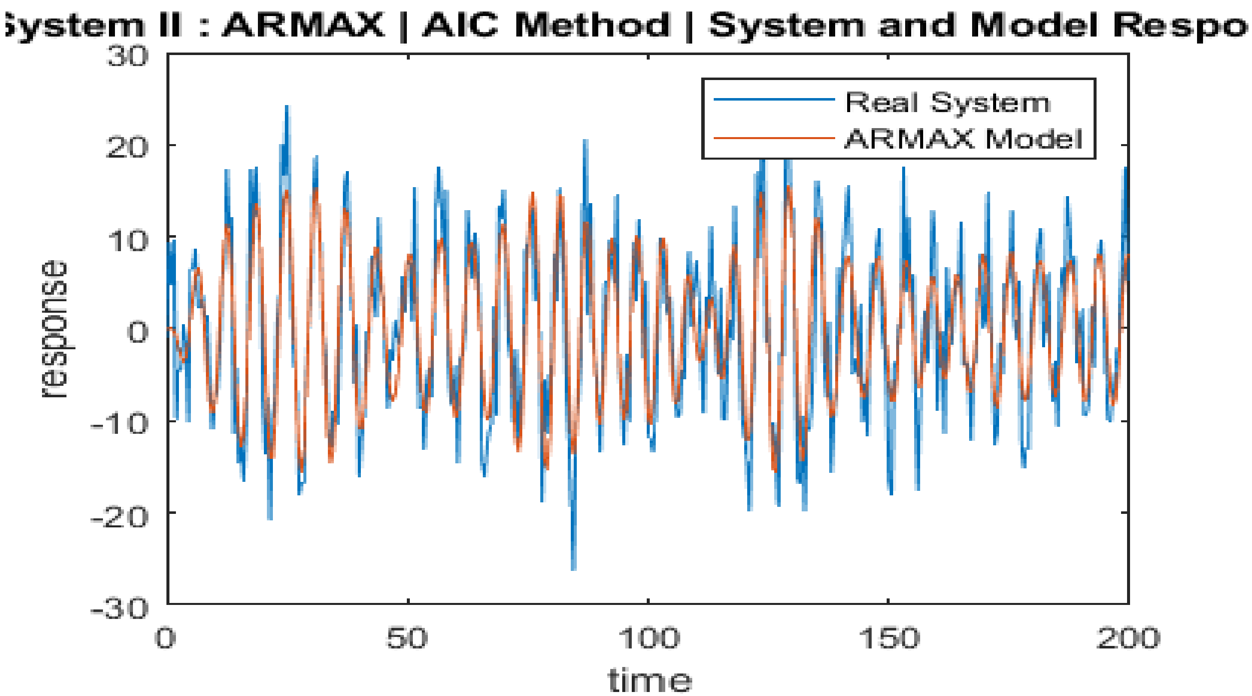
```
armax_VarError_2 = y2_val - Var_y_hat_2;
armax_AICError_2 = y2_val - AIC_y_hat_2;
armax_FTestError_2 = y2_val - FTest_y_hat_2;

for k=0:N_val-1
    armax_BestFit_Ree_2(k+1,1) = AutoCorrelate(armax_BestFitError_2, k);
    armax_Var_Ree_2(k+1,1) = AutoCorrelate(armax_VarError_2, k);
    armax_AIC_Ree_2(k+1,1) = AutoCorrelate(armax_AICError_2, k);
    armax_FTest_Ree_2(k+1,1) = AutoCorrelate(armax_FTestError_2, k);
end

for k=0:N_val-1
    armax_BestFit_Rue_2(k+1,1) = CrossCorrelate(u2_val, armax_BestFitError_2,
k);
    armax_Var_Rue_2(k+1,1) = CrossCorrelate(u2_val, armax_VarError_2, k);
    armax_AIC_Rue_2(k+1,1) = CrossCorrelate(u2_val, armax_AICError_2, k);
    armax_FTest_Rue_2(k+1,1) = CrossCorrelate(u2_val, armax_FTestError_2, k);
end



%%
figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hat_2)
legend('Real System','ARMAX Model')
title(" System II : ARMAX | Best Fit Lowest Error Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



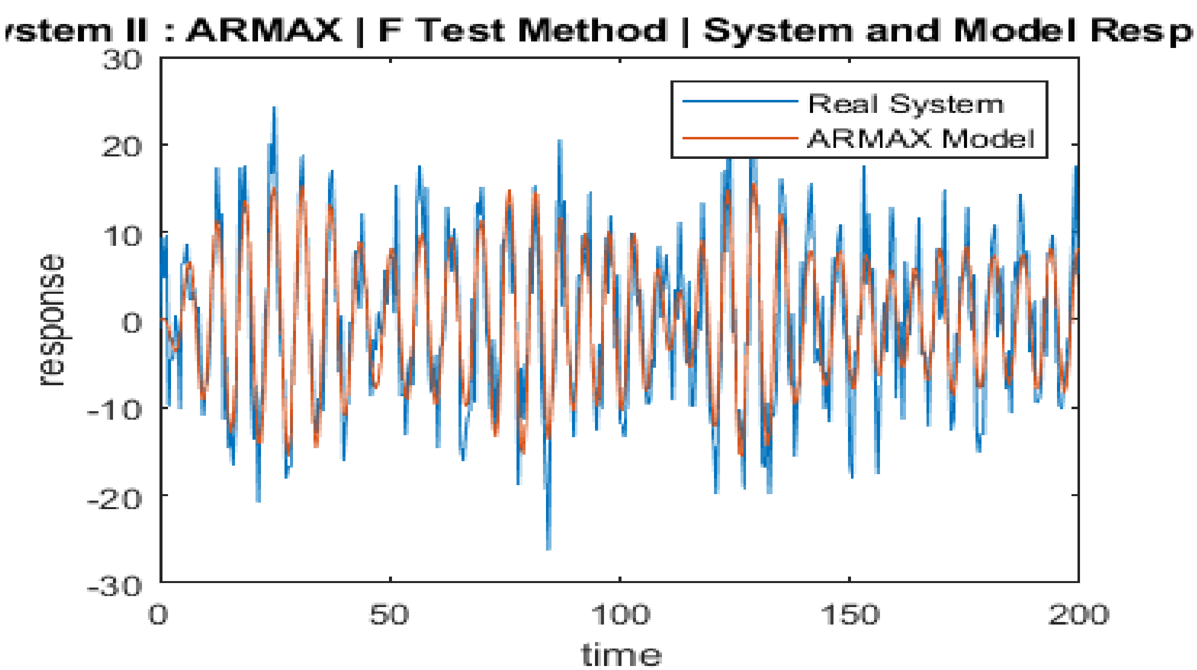
```
figure(2)
plot(t_val,y2_val,t_val,Var_y_hat_2)
legend('Real System','ARMAX Model')
title(" System II : ARMAX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```



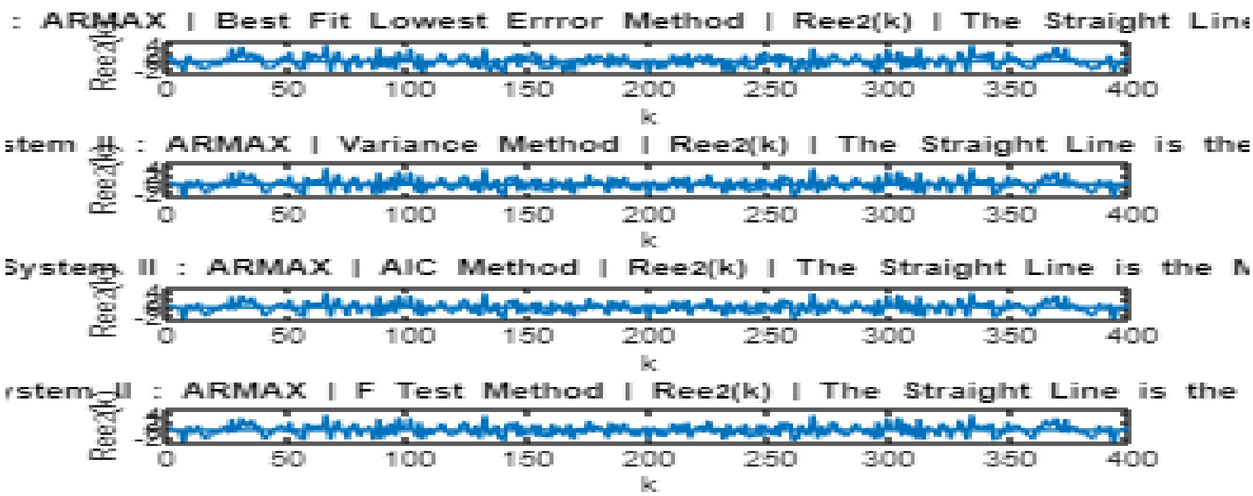

```
figure(3)
plot(t_val,y2_val,t_val,AIC_y_hat_2)
legend('Real System','ARMAX Model')
title(" System II : ARMAX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y2_val,t_val,FTest_y_hat_2)
legend('Real System','ARMAX Model')
title(" System II : ARMAX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Ree_2(2:end), 1:N_val-1,
mean(armax_BestFit_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : ARMAX | Best Fit Lowest Errror Method | Ree_2(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,2)
plot(1:N_val-1,armax_Var_Ree_2(2:end), 1:N_val-1,
mean(armax_Var_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : ARMAX | Variance Method | Ree_2(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,3)
plot(1:N_val-1,armax_AIC_Ree_2(2:end), 1:N_val-1,
mean(armax_AIC_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : ARMAX | AIC Method | Ree_2(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,4)
plot(1:N_val-1,armax_FTest_Ree_2(2:end), 1:N_val-1,
mean(armax_FTest_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : ARMAX | F Test Method | Ree_2(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_2(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Rue_2(2:end), 1:N_val-1,
mean(armax_BestFit_Rue_2(2:end))*ones(length(1:N_val-1)))
title(" System II : ARMAX | Best Fit Lowest Errror Method | Rue_2(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_2(k)")

subplot(4,1,2)
plot(1:N_val-1,armax_Var_Rue_2(2:end), 1:N_val-1,
mean(armax_Var_Rue_2(2:end))*ones(length(1:N_val-1)))
title(" System II : ARMAX | Variance Method | Rue_2(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Rue_2(k)")

subplot(4,1,3)
plot(1:N_val-1,armax_AIC_Rue_2(2:end), 1:N_val-1,
mean(armax_AIC_Rue_2(2:end))*ones(length(1:N_val-1)))
title(" System II : ARMAX | AIC Method | Rue_2(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Rue_2(k)")

subplot(4,1,4)
```

```
 plot(1:N_val-1,armax_FTest_Rue_2(2:end), 1:N_val-1,
mean(armax_FTest_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II : ARMAX | F Test Method | Rue_2(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_2(k)")
```

Q2 - part c | Box-Jenkins

```
clc; clear;
%%

load HW5_question2

u1 = Z1.u;
y1 = Z1.y;

u2 = Z2.u;
y2 = Z2.y;

u3 = Z3.u;
y3 = Z3.y;

u1_val = u1(601:end);
y1_val = y1(601:end);

u2_val = u2(601:end);
y2_val = y2(601:end);

u3_val = u1(601:end);
y3_val = y1(601:end);

u1 = u1(1:600);
y1 = y1(1:600);

u2 = u2(1:600);
y2 = y2(1:600);

u3 = u3(1:600);
y3 = y3(1:600);
```

System I

```
%%

% System Z1 ***************************************************************
fprintf("******************************************************************\n")
```

```
******************************************************************
```

```
fprintf(">>> System I Identification Begins:------------------------------\n")
```

```
>>> System I Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data1 = iddata(y1,u1,Ts);

%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
===================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nd = degree;
    nk = 1;
    p = na+nb+nc+nd;

    try
        sys = bj(data1, [na nb nc nd nk]);
        bj_y_hat_1 = lsim(sys, u1, t);
    catch
        break
    end

    [r2_bj, mse_bj] = rSQR(y1, bj_y_hat_1);
```

```
    error = y1 - bj_y_hat_1;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_bj, mse_bj, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_bj];
    MSEs = [MSEs; mse_bj];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.591082 | MSE=34.216505 | var=34.446146 | s_hat=20529.903249 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.341848 | MSE=55.071329 | var=55.815536 | s_hat=33042.797441 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.920162 | MSE=6.680476 | var=6.816813 | s_hat=4008.285760 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.920367 | MSE=6.663308 | var=6.845864 | s_hat=3997.984774 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.921021 | MSE=6.608597 | var=6.836480 | s_hat=3965.158188 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.920993 | MSE=6.610926 | var=6.886381 | s_hat=3966.555733 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.923192 | MSE=6.426930 | var=6.741535 | s_hat=3856.158046 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.601874 | MSE=33.313431 | var=35.190244 | s_hat=19988.058631 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.924425 | MSE=6.323756 | var=6.727400 | s_hat=3794.253827 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.923373 | MSE=6.411778 | var=6.869762 | s_hat=3847.066502 |
-------------------------------------------------------------
>>> Degree = 11 : R2=-34.614850 | MSE=2980.097149 | var=3215.932175 | s_hat=1788058.289177 |
-------------------------------------------------------------
>>> Degree = 12 : R2=0.911788 | MSE=7.381227 | var=8.023073 | s_hat=4428.736316 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.918027 | MSE=6.859154 | var=7.510022 | s_hat=4115.492329 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.924936 | MSE=6.281045 | var=6.927623 | s_hat=3768.626930 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.925088 | MSE=6.268296 | var=6.964774 | s_hat=3760.977888 |
```

```
------------------------------------------------------------
>>> Degree = 16 : R2=0.747527 | MSE=21.125842 | var=23.648330 | s_hat=12675.505042 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.876018 | MSE=10.374245 | var=11.700276 | s_hat=6224.547088 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.858171 | MSE=11.867664 | var=13.485982 | s_hat=7120.598498 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.917350 | MSE=6.915835 | var=7.918895 | s_hat=4149.501235 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.923411 | MSE=6.408676 | var=7.394626 | s_hat=3845.205347 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.924720 | MSE=6.299132 | var=7.324572 | s_hat=3779.478952 |
------------------------------------------------------------
>>> Degree = 22 : R2=-1.426182 | MSE=203.012469 | var=237.905237 | s_hat=121807.481461 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.757789 | MSE=20.267145 | var=23.937573 | s_hat=12160.287013 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.925039 | MSE=6.272410 | var=7.467155 | s_hat=3763.446211 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.915767 | MSE=7.048262 | var=8.457914 | s_hat=4228.957179 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.917542 | MSE=6.899756 | var=8.346479 | s_hat=4139.853609 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.924338 | MSE=6.331056 | var=7.720801 | s_hat=3798.633886 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.925810 | MSE=6.207940 | var=7.632713 | s_hat=3724.763711 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.921637 | MSE=6.557047 | var=8.128571 | s_hat=3934.228395 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.921055 | MSE=6.605759 | var=8.257199 | s_hat=3963.455331 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.917145 | MSE=6.932962 | var=8.739027 | s_hat=4159.776994 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.881709 | MSE=9.898088 | var=12.582316 | s_hat=5938.853081 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.903091 | MSE=8.108909 | var=10.396037 | s_hat=4865.345523 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.903813 | MSE=8.048521 | var=10.407570 | s_hat=4829.112340 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.874366 | MSE=10.512479 | var=13.711929 | s_hat=6307.487473 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.909091 | MSE=7.606864 | var=10.009032 | s_hat=4564.118516 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.921978 | MSE=6.528582 | var=8.666259 | s_hat=3917.149077 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.907109 | MSE=7.772677 | var=10.409835 | s_hat=4663.606113 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.921893 | MSE=6.535669 | var=8.831985 | s_hat=3921.401408 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.880371 | MSE=10.010045 | var=13.650061 | s_hat=6006.026860 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.879101 | MSE=10.116279 | var=13.921485 | s_hat=6069.767251 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.865477 | MSE=11.256324 | var=15.633784 | s_hat=6753.794618 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.673272 | MSE=27.339216 | var=38.326004 | s_hat=16403.529525 |
------------------------------------------------------------
```

```
>>> Degree = 44 : R2=0.923333 | MSE=6.415199 | var=9.078111 | s_hat=3849.119139 |
-------------------------------------------------------------
>>> Degree = 45 : R2=0.922452 | MSE=6.488917 | var=9.269881 | s_hat=3893.349987 |
-------------------------------------------------------------
>>> Degree = 46 : R2=0.908099 | MSE=7.689903 | var=11.091206 | s_hat=4613.941779 |
-------------------------------------------------------------
>>> Degree = 47 : R2=0.809762 | MSE=15.918259 | var=23.181931 | s_hat=9550.955572 |
-------------------------------------------------------------
>>> Degree = 48 : R2=0.689468 | MSE=25.983961 | var=38.211707 | s_hat=15590.376428 |
-------------------------------------------------------------
>>> Degree = 49 : R2=0.887206 | MSE=9.438097 | var=14.016976 | s_hat=5662.858405 |
-------------------------------------------------------------
>>> Degree = 50 : R2=0.872083 | MSE=10.703549 | var=16.055323 | s_hat=6422.129112 |
-------------------------------------------------------------
>>> Degree = 51 : R2=0.823022 | MSE=14.808775 | var=22.437538 | s_hat=8885.265152 |
-------------------------------------------------------------
>>> Degree = 52 : R2=0.886998 | MSE=9.455534 | var=14.472756 | s_hat=5673.320453 |
-------------------------------------------------------------
>>> Degree = 53 : R2=0.817106 | MSE=15.303784 | var=23.665646 | s_hat=9182.270514 |
-------------------------------------------------------------
>>> Degree = 54 : R2=0.893953 | MSE=8.873569 | var=13.864951 | s_hat=5324.141359 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.865465 | MSE=11.257305 | var=17.774692 | s_hat=6754.383072 |
-------------------------------------------------------------
>>> Degree = 56 : R2=0.858479 | MSE=11.841872 | var=18.896604 | s_hat=7105.123070 |
-------------------------------------------------------------
>>> Degree = 57 : R2=0.746200 | MSE=21.236852 | var=34.252987 | s_hat=12742.111209 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.422229 | MSE=48.345411 | var=78.824039 | s_hat=29007.246433 |
-------------------------------------------------------------
>>> Degree = 59 : R2=0.875470 | MSE=10.420152 | var=17.176075 | s_hat=6252.091204 |
-------------------------------------------------------------
>>> Degree = 60 : R2=0.837116 | MSE=13.629470 | var=22.715784 | s_hat=8177.682256 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.827458 | MSE=14.437551 | var=24.332951 | s_hat=8662.530583 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.905898 | MSE=7.874089 | var=13.421743 | s_hat=4724.453642 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.278433 | MSE=60.377646 | var=104.099389 | s_hat=36226.587454 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.729076 | MSE=22.669711 | var=39.540193 | s_hat=13601.826432 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.179707 | MSE=68.638605 | var=121.126951 | s_hat=41183.163227 |
-------------------------------------------------------------
>>> Degree = 66 : R2=0.851215 | MSE=12.449676 | var=22.231565 | s_hat=7469.805818 |
-------------------------------------------------------------
```

```
fprintf("============================================================\n")
```

```
=================================================================
```

```
%%
```

```
fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 28
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
nd = bestFitDegree;
nk = 1;
p = na+nb+nc+nd;

BestFitModel_1 = bj(data1, [na nb nc nd nk])
```

```
BestFitModel_1 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)

  B(z) = 0.7158 z^-1 + 2.263 z^-2 + 4.049 z^-3 + 5.004 z^-4 + 4.873 z^-5 + 4.06 z^-6 + 2.409
z^-7
          - 0.1253 z^-8 - 3.374 z^-9 - 5.961 z^-10 - 7.762 z^-11 - 8.053 z^-12 - 6.631 z^-13
          - 3.734 z^-14 - 0.9776 z^-15 + 1.37 z^-16 + 3.829 z^-17 + 4.659 z^-18 + 3.806 z^-19
          + 2.637 z^-20 + 2.309 z^-21 + 0.8774 z^-22 - 1.03 z^-23 - 1.495 z^-24 - 0.8161 z^-25
                                                        - 0.4307 z^-26 - 0.3146 z^-27 -
0.02747 z^-28


  C(z) = 1 - 0.4715 z^-1 - 0.5464 z^-2 - 0.2329 z^-3 - 0.01142 z^-4 + 0.5975 z^-5 - 0.03636
z^-6
          - 0.3124 z^-7 + 0.2979 z^-8 - 0.02955 z^-9 - 0.2872 z^-10 - 0.1756 z^-11 + 0.004577
z^-12
          + 0.1979 z^-13 + 0.2683 z^-14 + 0.06547 z^-15 - 0.1539 z^-16 + 0.04189 z^-17 -
0.1343 z^-18
          - 0.04254 z^-19 + 0.259 z^-20 - 0.3579 z^-21 - 0.02807 z^-22 + 0.3754 z^-23 -
0.02891 z^-24
                                         - 0.05351 z^-25 - 0.3091 z^-26 - 0.5289 z^-27 +
0.6325 z^-28


  D(z) = 1 - 0.5703 z^-1 - 0.5543 z^-2 - 0.07331 z^-3 - 0.06857 z^-4 + 0.647 z^-5 - 0.05293
z^-6
```

```
          - 0.3408 z^-7 + 0.3179 z^-8 - 0.1944 z^-9 - 0.2443 z^-10 + 0.03053 z^-11 - 0.06146
z^-12
          + 0.2111 z^-13 + 0.2904 z^-14 - 0.02341 z^-15 - 0.1174 z^-16 + 0.004066 z^-17 -
0.2893 z^-18
          + 0.1034 z^-19 + 0.3363 z^-20 - 0.3401 z^-21 + 0.09468 z^-22 + 0.09607 z^-23 -
0.02486 z^-24
                                         + 0.00158 z^-25 - 0.2811 z^-26 - 0.3178 z^-27 +
0.4704 z^-28


  F(z) = 1 + 0.8347 z^-1 - 0.1741 z^-2 - 0.4156 z^-3 + 0.5639 z^-4 + 0.00319 z^-5 - 0.9879 z^
          -6 - 0.6558 z^-7 - 0.4246 z^-8 - 0.5956 z^-9 - 0.4085 z^-10 + 0.09888 z^-11 - 0.2673
z^-12
          + 0.05383 z^-13 + 0.5076 z^-14 - 0.08605 z^-15 - 0.2502 z^-16 + 0.4139 z^-17 + 0.752
z^-18
          - 0.004593 z^-19 + 0.1406 z^-20 + 0.185 z^-21 + 0.1887 z^-22 + 0.1648 z^-23 + 0.1489
z^-24
                                        - 0.2911 z^-25 - 0.3883 z^-26 - 0.05903 z^-27 -
0.02474 z^-28

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=28   nc=28   nd=28   nf=28   nk=1
   Number of free coefficients: 112
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data1".
Fit to estimation data: 76.96% (prediction focus)
FPE: 7.896, MSE: 4.442
```

```
BestFit_y_hat_1 = lsim(BestFitModel_1, u1_val, t_val);
% [bj_BestFit_r2, bj_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 9 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 9
```

```
na = minVarIndex;
```

```
nb = minVarIndex;
nc = minVarIndex;
nd = minVarIndex;
nk = 1;
p = na+nb+nc+nd;

bj_VarModel_1 = bj(data1, [na nb nc nd nk])
```

```
bj_VarModel_1 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)

  B(z) = 0.7606 z^-1 + 1.281 z^-2 + 1.829 z^-3 + 1.858 z^-4 + 0.7429 z^-5 + 0.03923 z^-6 -
0.7025 z^-7
                                                                              - 0.8477 z^-8 -
0.36 z^-9


  C(z) = 1 + 0.2544 z^-1 + 0.5877 z^-2 - 0.2172 z^-3 + 0.08048 z^-4 + 0.6841 z^-5 + 0.1994 z^
                                                         -6 + 0.4749 z^-7 - 0.2642 z^-8 -
0.1075 z^-9


  D(z) = 1 + 0.2104 z^-1 + 0.5148 z^-2 - 0.2609 z^-3 + 0.01583 z^-4 + 0.7006 z^-5 + 0.1782 z^
                                                         -6 + 0.4618 z^-7 - 0.3214 z^-8 -
0.1955 z^-9


  F(z) = 1 - 0.6296 z^-1 + 0.1451 z^-2 + 0.1776 z^-3 - 1.216 z^-4 + z^-5 - 0.07293 z^-6 -
0.1845 z^-7
                                                                             + 0.6527 z^-8 -
0.823 z^-9

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=9   nc=9   nd=9   nf=9   nk=1
   Number of free coefficients: 36
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data1".
Fit to estimation data: 73.01% (prediction focus)
FPE: 6.874, MSE: 6.095
```

```
Var_y_hat_1 = lsim(bj_VarModel_1, u1_val, t_val);
% [bj_Var_r2, bj_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```


%%
```

```
fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 3
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
nd = minAICIndex;
nk = 1;
p = na+nb+nc+nd;

bj_AICModel_1 = bj(data1, [na nb nc nd nk])
```

```
bj_AICModel_1 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.6717 z^-1 + 0.1097 z^-2 - 0.4801 z^-3

  C(z) = 1 - 0.6714 z^-1 - 0.184 z^-2 + 0.803 z^-3

  D(z) = 1 - 0.6548 z^-1 - 0.2228 z^-2 + 0.8117 z^-3

  F(z) = 1 - 2.637 z^-1 + 2.528 z^-2 - 0.8878 z^-3

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=3   nc=3   nd=3   nf=3   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data1".
Fit to estimation data: 71.99% (prediction focus)
FPE: 6.834, MSE: 6.566
```

```
AIC_y_hat_1 = lsim(bj_AICModel_1, u1_val, t_val);
% [bj_AIC_r2, bj_AIC_mse] = rSQR(y_val, AIC_y_hat);
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
==============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.19 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 3
```

```
na = winner;
nb = winner;
nc = winner;
nd = winner;
nk = 1;
p = na+nb+nc+nd;

bj_FTestModel_1 = bj(data1, [na nb nc nd nk])
```

```
bj_FTestModel_1 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.6717 z^-1 + 0.1097 z^-2 - 0.4801 z^-3

  C(z) = 1 - 0.6714 z^-1 - 0.184 z^-2 + 0.803 z^-3
```

```
  D(z) = 1 - 0.6548 z^-1 - 0.2228 z^-2 + 0.8117 z^-3

  F(z) = 1 - 2.637 z^-1 + 2.528 z^-2 - 0.8878 z^-3

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=3   nc=3   nd=3   nf=3   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data1".
Fit to estimation data: 71.99% (prediction focus)
FPE: 6.834, MSE: 6.566
```

```
FTest_y_hat_1 = lsim(bj_FTestModel_1, u1_val, t_val);
% [bj_FTest_r2, bj_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[bj_BestFit_r2_1, bj_BestFit_mse_1] = rSQR(y1_val, BestFit_y_hat_1);
[bj_Var_r2_1, bj_Var_mse_1] = rSQR(y1_val, Var_y_hat_1);
[bj_AIC_r2_1, bj_AIC_mse_1] = rSQR(y1_val, AIC_y_hat_1);
[bj_FTest_r2_1, bj_FTest_mse_1] = rSQR(y1_val, FTest_y_hat_1);

%%

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_BestFit_r2_1,
bj_BestFit_mse_1)
```

```
    R2 value : 0.6488    | MSE : 25.9760
```

```
fprintf("-------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Var_r2_1, bj_Var_mse_1)
```

```
    R2 value : 0.6512    | MSE : 25.7960
```

```
% fprintf("-------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Cov_r2, bj_Cov_mse)
fprintf("-------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_AIC_r2_1, bj_AIC_mse_1)
```

```
    R2 value : 0.6541    | MSE : 25.5820
```

```
fprintf("-------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_FTest_r2_1, bj_FTest_mse_1)
```

```
    R2 value : 0.6541    | MSE : 25.5820
```

```
fprintf("-------------------------------------------------------------\n")
```

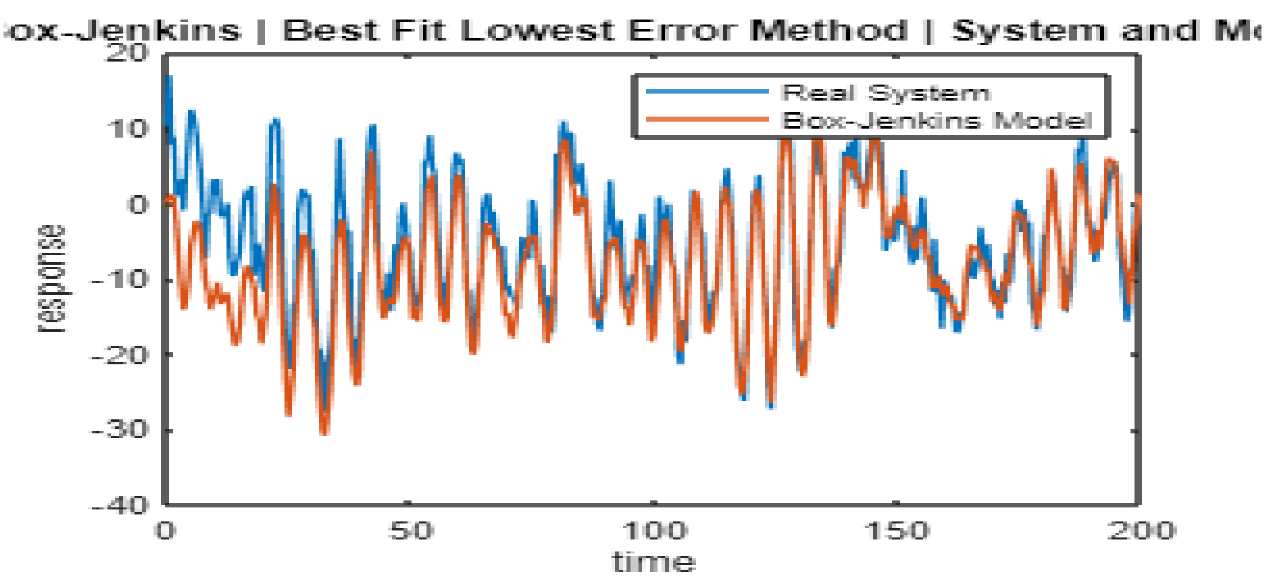
```
-------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("=============================================================\n")
```

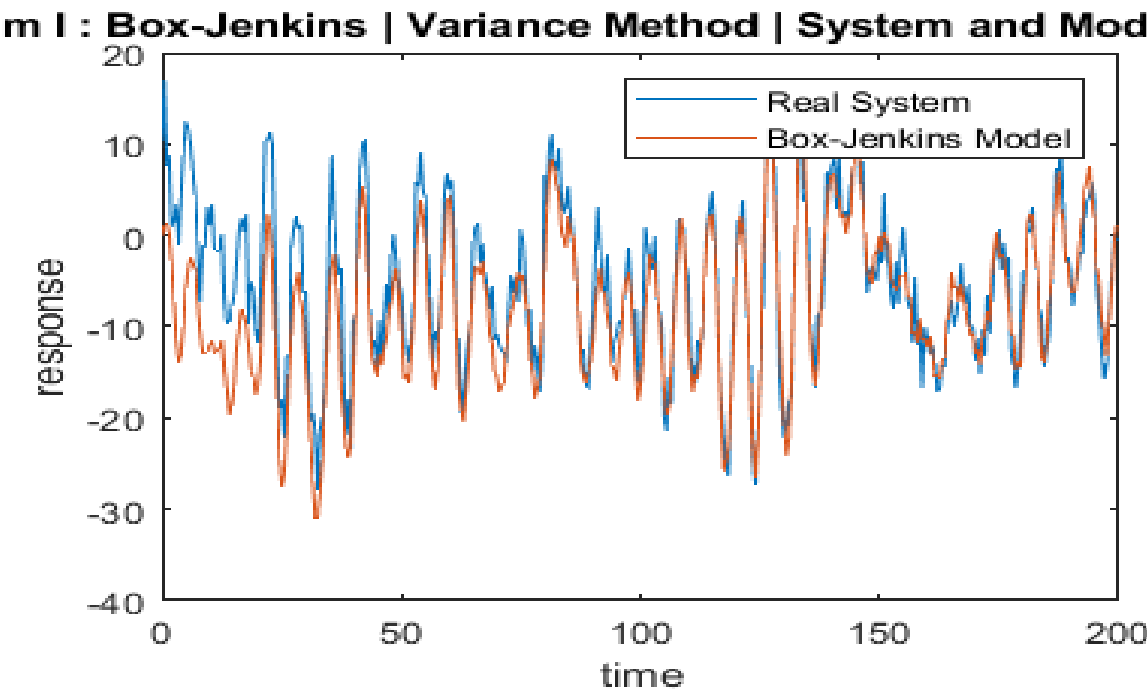
```
=============================================================
```

```



%%
```

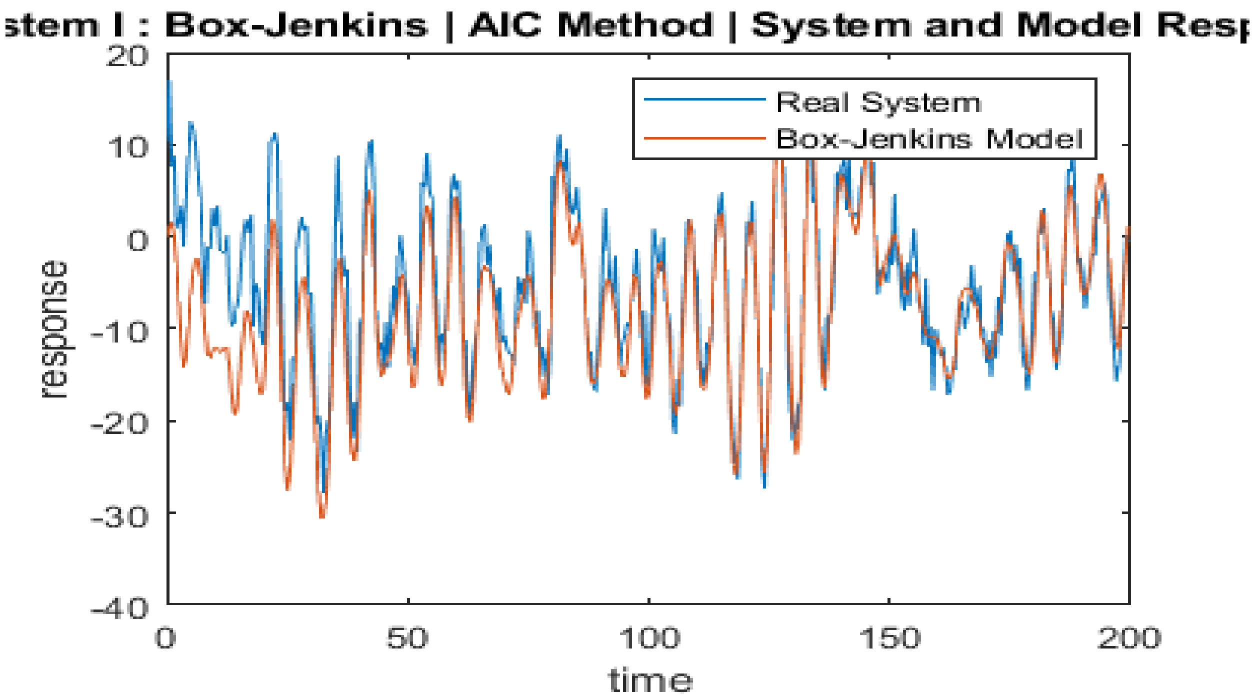
```
bj_BestFitError_1 = y1_val - BestFit_y_hat_1;
bj_VarError_1 = y1_val - Var_y_hat_1;
bj_AICError_1 = y1_val - AIC_y_hat_1;
bj_FTestError_1 = y1_val - FTest_y_hat_1;

for k=0:N_val-1
    bj_BestFit_Ree_1(k+1,1) = AutoCorrelate(bj_BestFitError_1, k);
    bj_Var_Ree_1(k+1,1) = AutoCorrelate(bj_VarError_1, k);
    bj_AIC_Ree_1(k+1,1) = AutoCorrelate(bj_AICError_1, k);
    bj_FTest_Ree_1(k+1,1) = AutoCorrelate(bj_FTestError_1, k);
end

for k=0:N_val-1
    bj_BestFit_Rue_1(k+1,1) = CrossCorrelate(u1_val, bj_BestFitError_1, k);
    bj_Var_Rue_1(k+1,1) = CrossCorrelate(u1_val, bj_VarError_1, k);
    bj_AIC_Rue_1(k+1,1) = CrossCorrelate(u1_val, bj_AICError_1, k);
    bj_FTest_Rue_1(k+1,1) = CrossCorrelate(u1_val, bj_FTestError_1, k);
end


%%
figure(1)
plot(t_val,y1_val,t_val,BestFit_y_hat_1)
legend('Real System','Box-Jenkins Model')
title(" System I : Box-Jenkins | Best Fit Lowest Error Method | System and
Model Response")
xlabel("time")
ylabel("response")
```



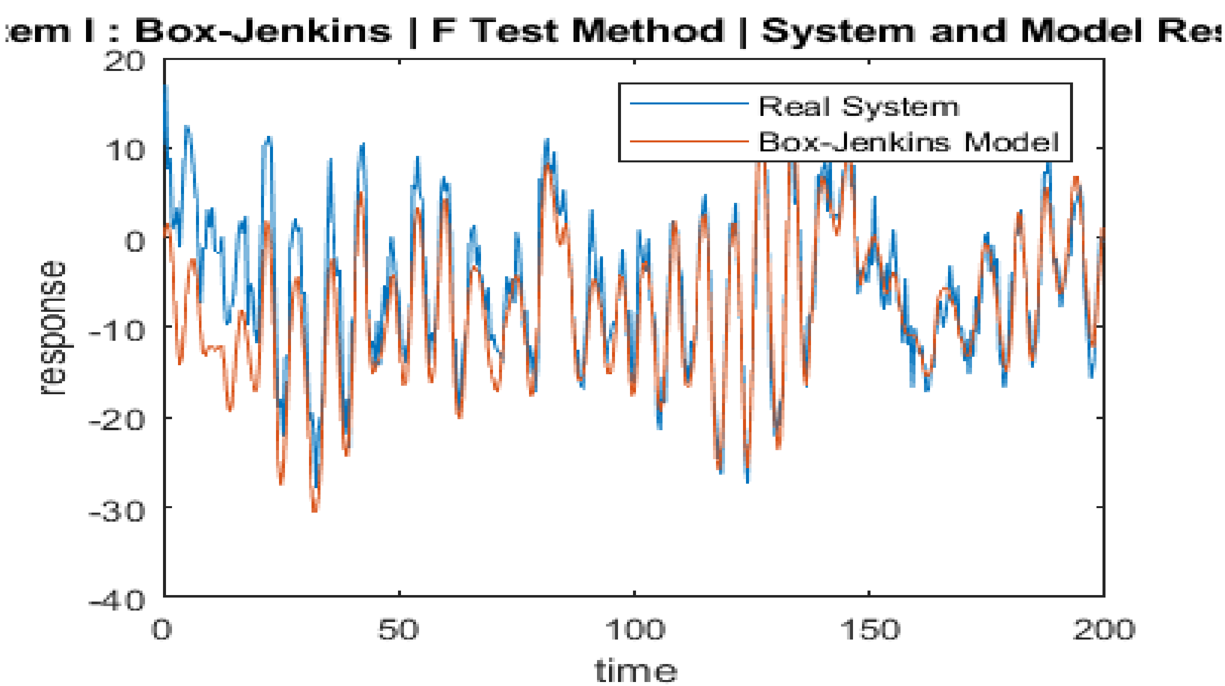
```
figure(2)
plot(t_val,y1_val,t_val,Var_y_hat_1)
legend('Real System','Box-Jenkins Model')
title(" System I : Box-Jenkins | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```



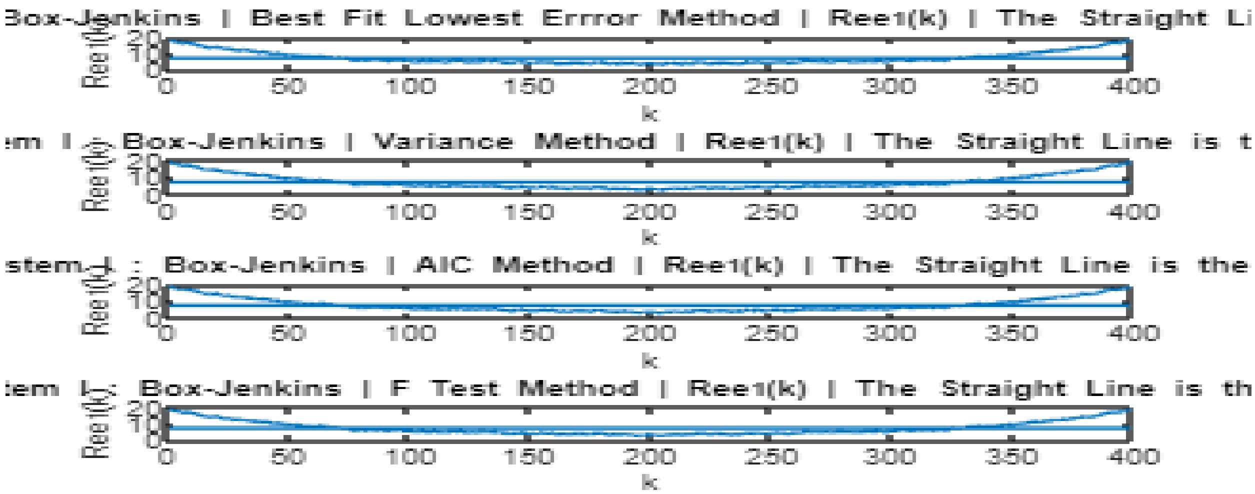

```
figure(3)
plot(t_val,y1_val,t_val,AIC_y_hat_1)
legend('Real System','Box-Jenkins Model')
title(" System I : Box-Jenkins | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y1_val,t_val,FTest_y_hat_1)
legend('Real System','Box-Jenkins Model')
title(" System I : Box-Jenkins | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```

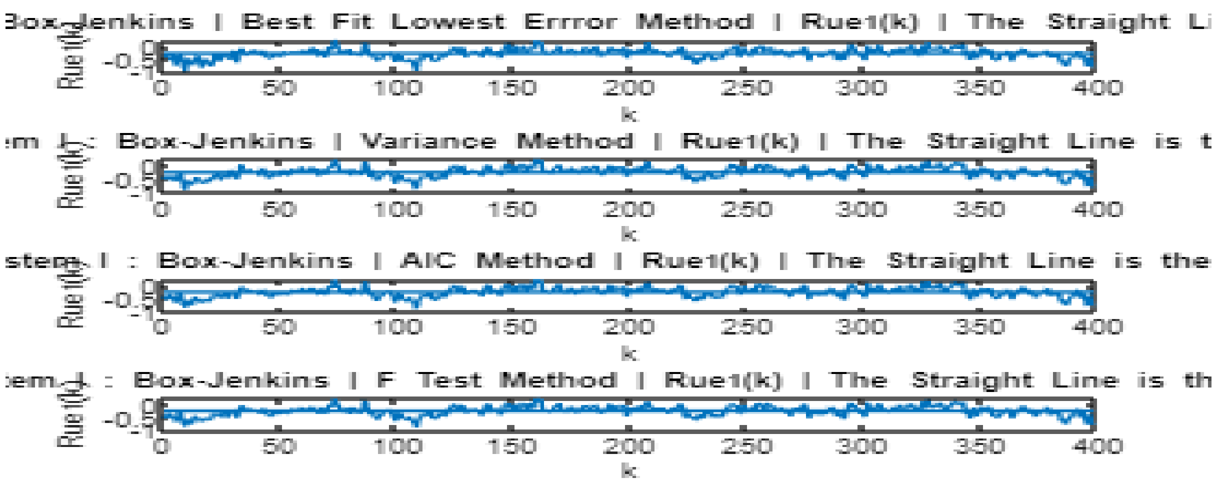


```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,bj_BestFit_Ree_1(2:end), 1:N_val-1,
mean(bj_BestFit_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Box-Jenkins | Best Fit Lowest Errror Method | Ree_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(4,1,2)
plot(1:N_val-1,bj_Var_Ree_1(2:end), 1:N_val-1,
mean(bj_Var_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Box-Jenkins | Variance Method | Ree_1(k) | The Straight Line
is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(4,1,3)
plot(1:N_val-1,bj_AIC_Ree_1(2:end), 1:N_val-1,
mean(bj_AIC_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Box-Jenkins | AIC Method | Ree_1(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(4,1,4)
plot(1:N_val-1,bj_FTest_Ree_1(2:end), 1:N_val-1,
mean(bj_FTest_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Box-Jenkins | F Test Method | Ree_1(k) | The Straight Line
is the Mean")
xlabel("k")
ylabel("Ree_1(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,bj_BestFit_Rue_1(2:end), 1:N_val-1,
mean(bj_BestFit_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Box-Jenkins | Best Fit Lowest Errror Method | Rue_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(4,1,2)
plot(1:N_val-1,bj_Var_Rue_1(2:end), 1:N_val-1,
mean(bj_Var_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Box-Jenkins | Variance Method | Rue_1(k) | The Straight Line
is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(4,1,3)
plot(1:N_val-1,bj_AIC_Rue_1(2:end), 1:N_val-1,
mean(bj_AIC_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Box-Jenkins | AIC Method | Rue_1(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(4,1,4)
```

```
 plot(1:N_val-1,bj_FTest_Rue_1(2:end), 1:N_val-1,
mean(bj_FTest_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : Box-Jenkins | F Test Method | Rue_1(k) | The Straight Line
is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")
```



System II

```
 %%

 % System Z2 ***************************************************************
 fprintf("*****************************************************************\n")
```

****

```
 fprintf(">>> System II Identification Begins:------------------------------\n")
```

>>> System II Identification Begins:------------------------------

```
 %%

 Ts = 0.5;
 t = 0:Ts:length(u2)*Ts-Ts;
 t_val = 0:Ts:length(u2_val)*Ts-Ts;
 N = length(y2);
 N_val = length(y2_val);
```

```
data2 = iddata(y2,u2,Ts);


%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
===================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nd = degree;
    nk = 1;
    p = na+nb+nc+nd;

    try
        sys = bj(data2, [na nb nc nd nk]);
        bj_y_hat_2 = lsim(sys, u2, t);
    catch
        break
    end

    [r2_bj, mse_bj] = rSQR(y2, bj_y_hat_2);

    error = y2 - bj_y_hat_2;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);
```

```
    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_bj, mse_bj, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_bj];
    MSEs = [MSEs; mse_bj];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.316984 | MSE=51.227536 | var=51.571345 | s_hat=30736.521790 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.670196 | MSE=24.735937 | var=25.070206 | s_hat=14841.561905 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.670189 | MSE=24.736482 | var=25.241309 | s_hat=14841.889456 |
-------------------------------------------------------------
>>> Degree = 4 : R2=-2.964854 | MSE=297.371626 | var=305.518794 | s_hat=178422.975463 |
-------------------------------------------------------------
>>> Degree = 5 : R2=-2404087.135744 | MSE=180311194.315684 | var=186528821.705880 |
s_hat=108186716589.410446 |
-------------------------------------------------------------
>>> Degree = 6 : R2=-134144.819268 | MSE=10061192.235438 | var=10480408.578581 |
s_hat=6036715341.262638 |
-------------------------------------------------------------
>>> Degree = 7 : R2=-10206457.069827 | MSE=765503816.994763 | var=802976031.812690 |
s_hat=459302290196.858459 |
-------------------------------------------------------------
>>> Degree = 8 : R2=-45442419.284294 | MSE=3408268171.300644 | var=3600283279.542933 |
s_hat=2044960902780.386230 |
-------------------------------------------------------------
>>> Degree = 9 : R2=-2657627.411681 | MSE=199327198.463634 | var=212050211.131525 |
s_hat=119596319078.180252 |
-------------------------------------------------------------
>>> Degree = 10 : R2=-27049849.524714 | MSE=2028790368.229715 | var=2173703965.960406 |
s_hat=1217274220937.827393 |
-------------------------------------------------------------
>>> Degree = 11 : R2=-22150322.810308 | MSE=1661316522.191645 | var=1792787613.875875 |
s_hat=996789913314.986816 |
-------------------------------------------------------------
>>> Degree = 12 : R2=-7529824.670283 | MSE=564751283.204409 | var=613860090.439575 |
s_hat=338850769922.645447 |
-------------------------------------------------------------
>>> Degree = 13 : R2=-166775936.024440 | MSE=12508513286.553482 | var=13695452503.525702 |
s_hat=7505107971932.083984 |
-------------------------------------------------------------
>>> Degree = 14 : R2=-20085999.115661 | MSE=1506488308.823869 | var=1661567987.673385 |
s_hat=903892985294.321655 |
-------------------------------------------------------------
```

```
>>> Degree = 15 : R2=-40502182.412845 | MSE=3037740985.857914 | var=3375267762.064350 |
s_hat=1822644591514.748779 |
-------------------------------------------------------------
>>> Degree = 16 : R2=-6840.385899 | MSE=513116.987671 | var=574384.687691 |
s_hat=307870192.602618 |
-------------------------------------------------------------
>>> Degree = 17 : R2=-14061208.116257 | MSE=1054617495.747789 | var=1189418228.286980 |
s_hat=632770497448.673218 |
-------------------------------------------------------------
>>> Degree = 18 : R2=-366232190.241741 | MSE=27468112677.667683 | var=31213764406.440552 |
s_hat=16480867606600.611328 |
-------------------------------------------------------------
>>> Degree = 19 : R2=-421987756.729301 | MSE=31649886479.404797 | var=36240328029.852821 |
s_hat=18989931887642.878906 |
-------------------------------------------------------------
>>> Degree = 20 : R2=-3624661.252297 | MSE=271856580.457921 | var=313680669.759140 |
s_hat=163113948274.752960 |
-------------------------------------------------------------
>>> Degree = 21 : R2=-21763574207.956158 | MSE=1632309564632.983398 | var=1898034377480.210938
| s_hat=979385738779788.875000 |
-------------------------------------------------------------
>>> Degree = 22 : R2=-23387245.862201 | MSE=1754088109.658652 | var=2055572003.506239 |
s_hat=1052452865795.194214 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.623785 | MSE=28.216859 | var=33.326998 | s_hat=16930.115158 |
-------------------------------------------------------------
>>> Degree = 24 : R2=-2593714.639835 | MSE=194533618.705853 | var=231587641.316492 |
s_hat=116720171223.512039 |
-------------------------------------------------------------
>>> Degree = 25 : R2=-12871812.625668 | MSE=965410566.004599 | var=1158492679.205521 |
s_hat=579246339602.760498 |
-------------------------------------------------------------
>>> Degree = 26 : R2=-424554134.236771 | MSE=31842369686.054630 | var=38518995587.969269 |
s_hat=19105421811632.757812 |
-------------------------------------------------------------
>>> Degree = 27 : R2=-7692975.690453 | MSE=576987920.814818 | var=703643805.871730 |
s_hat=346192752488.891052 |
-------------------------------------------------------------
>>> Degree = 28 : R2=-23279.726999 | MSE=1746098.916788 | var=2146842.930477 |
s_hat=1047659350.072673 |
-------------------------------------------------------------
>>> Degree = 29 : R2=-2698299.129616 | MSE=202377654.861963 | var=250881390.324747 |
s_hat=121426592917.177521 |
-------------------------------------------------------------
>>> Degree = 30 : R2=-962312691.729610 | MSE=72175286900.432465 | var=90219108625.540771 |
s_hat=43305172140259.570312 |
-------------------------------------------------------------
>>> Degree = 31 : R2=-104964153.893176 | MSE=7872511763.497084 | var=9923334155.668591 |
s_hat=4723507058098.249023 |
-------------------------------------------------------------
>>> Degree = 32 : R2=0.443474 | MSE=41.740527 | var=53.059992 | s_hat=25044.316108 |
-------------------------------------------------------------
>>> Degree = 33 : R2=-15204554.634723 | MSE=1140370664.775239 | var=1462013672.788769 |
s_hat=684222398865.144043 |
-------------------------------------------------------------
>>> Degree = 34 : R2=-6402088.486479 | MSE=480168919.042541 | var=620908084.968803 |
s_hat=288101351425.524536 |
```

```
------------------------------------------------------------
>>> Degree = 35 : R2=-1735552812.821838 | MSE=130169770402.935593 | var=169786657047.306885 |
s_hat=78101862241761.171875 |
------------------------------------------------------------
>>> Degree = 36 : R2=-61623112.870496 | MSE=4621851043.744657 | var=6081382952.295606 |
s_hat=2773110626246.796387 |
------------------------------------------------------------
>>> Degree = 37 : R2=-33914237.169555 | MSE=2543632530.663309 | var=3376503359.287581 |
s_hat=1526179518397.986816 |
------------------------------------------------------------
>>> Degree = 38 : R2=-13359549.401789 | MSE=1001991754.234143 | var=1341953242.277871 |
s_hat=601195052540.486328 |
------------------------------------------------------------
>>> Degree = 39 : R2=-6804.446995 | MSE=510421.501323 | var=689758.785572 |
s_hat=306252900.793943 |
------------------------------------------------------------
>>> Degree = 40 : R2=-2467622.734707 | MSE=185076485.388156 | var=252377025.529304 |
s_hat=111045891232.893875 |
------------------------------------------------------------
>>> Degree = 41 : R2=-10230189.785375 | MSE=767283815.914534 | var=1055895159.515412 |
s_hat=460370289548.719604 |
------------------------------------------------------------
>>> Degree = 42 : R2=-502932.331958 | MSE=37720958.894221 | var=52390220.686418 |
s_hat=22632575336.532646 |
------------------------------------------------------------
>>> Degree = 43 : R2=-3656788.435578 | MSE=274266180.464344 | var=384485299.716370 |
s_hat=164559708278.606232 |
------------------------------------------------------------
>>> Degree = 44 : R2=-7322629.024973 | MSE=549211214.723805 | var=777185681.212932 |
s_hat=329526728834.283020 |
------------------------------------------------------------
>>> Degree = 45 : R2=-407603.256928 | MSE=30571096.492712 | var=43672994.989588 |
s_hat=18342657895.627014 |
------------------------------------------------------------
>>> Degree = 46 : R2=-2879246.054555 | MSE=215949019.263421 | var=311464931.629934 |
s_hat=129569411558.052597 |
------------------------------------------------------------
>>> Degree = 47 : R2=-1622266.274091 | MSE=121673138.909303 | var=177193891.615490 |
s_hat=73003883345.582047 |
------------------------------------------------------------
>>> Degree = 48 : R2=-12840933.331805 | MSE=963094560.083949 | var=1416315529.535217 |
s_hat=577856736050.368652 |
------------------------------------------------------------
>>> Degree = 49 : R2=-4896530.194347 | MSE=367249176.321765 | var=545419568.794700 |
s_hat=220349505793.058777 |
------------------------------------------------------------
>>> Degree = 50 : R2=-19942.388250 | MSE=1495792.146934 | var=2243688.220401 |
s_hat=897475288.160488 |
------------------------------------------------------------
>>> Degree = 51 : R2=-517453.297313 | MSE=38810058.984590 | var=58803119.673621 |
s_hat=23286035390.753868 |
------------------------------------------------------------
>>> Degree = 52 : R2=-126818.843302 | MSE=9511730.068792 | var=14558770.513457 |
s_hat=5707038041.275229 |
------------------------------------------------------------
>>> Degree = 53 : R2=-620051.986916 | MSE=46505156.340826 | var=71915190.217773 |
s_hat=27903093804.495842 |
```

```
-------------------------------------------------------------
>>> Degree = 54 : R2=-8738.746470 | MSE=655497.650271 | var=1024215.078549 |
s_hat=393298590.162778 |
-------------------------------------------------------------
>>> Degree = 55 : R2=-382798.063865 | MSE=28710659.714245 | var=45332620.601440 |
s_hat=17226395828.547054 |
-------------------------------------------------------------
>>> Degree = 56 : R2=-4012990.550389 | MSE=300982018.284271 | var=480290454.708943 |
s_hat=180589210970.562683 |
-------------------------------------------------------------
>>> Degree = 57 : R2=-771117.532600 | MSE=57835360.320113 | var=93282839.225989 |
s_hat=34701216192.068069 |
-------------------------------------------------------------
>>> Degree = 58 : R2=-15742380.753726 | MSE=1180708649.231739 | var=1925068449.834358 |
s_hat=708425189539.043579 |
-------------------------------------------------------------
>>> Degree = 59 : R2=-1242562.183432 | MSE=93194608.087067 | var=153617485.857803 |
s_hat=55916764852.240349 |
-------------------------------------------------------------
>>> Degree = 60 : R2=-5824616.123262 | MSE=436857390.672224 | var=728095651.120372 |
s_hat=262114434403.333862 |
-------------------------------------------------------------
>>> Degree = 61 : R2=-15913.570364 | MSE=1193623.123319 | var=2011724.365145 |
s_hat=716173873.991554 |
-------------------------------------------------------------
>>> Degree = 62 : R2=-1053344.519552 | MSE=79002922.494223 | var=134664072.433334 |
s_hat=47401753496.533524 |
-------------------------------------------------------------
>>> Degree = 63 : R2=-11394203.154396 | MSE=854587038.141365 | var=1473425927.829938 |
s_hat=512752222884.818298 |
-------------------------------------------------------------
>>> Degree = 64 : R2=-258008.145218 | MSE=19351177.865294 | var=33752054.416211 |
s_hat=11610706719.176493 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("==============Degree Extraction | BestFit Method================\n")
```

```
==============Degree Extraction | BestFit Method================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 2
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
nd = bestFitDegree;
nk = 1;
p = na+nb+nc+nd;

BestFitModel_2 = bj(data2, [na nb nc nd nk])
```

```
BestFitModel_2 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.4335 z^-1 + 1.698 z^-2

  C(z) = 1 + 0.4345 z^-1 - 0.5655 z^-2

  D(z) = 1 + 0.3866 z^-1 - 0.5945 z^-2

  F(z) = 1 - 1.64 z^-1 + 0.8852 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data2".
Fit to estimation data: 42.82% (prediction focus)
FPE: 25.19, MSE: 24.53
```

```
BestFit_y_hat_2 = lsim(BestFitModel_2, u2_val, t_val);
% [bj_BestFit_r2, bj_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 2
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
nd = minVarIndex;
nk = 1;
p = na+nb+nc+nd;

bj_VarModel_2 = bj(data2, [na nb nc nd nk])
```

```
bj_VarModel_2 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.4335 z^-1 + 1.698 z^-2

  C(z) = 1 + 0.4345 z^-1 - 0.5655 z^-2

  D(z) = 1 + 0.3866 z^-1 - 0.5945 z^-2

  F(z) = 1 - 1.64 z^-1 + 0.8852 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data2".
Fit to estimation data: 42.82% (prediction focus)
FPE: 25.19, MSE: 24.53
```

```
Var_y_hat_2 = lsim(bj_VarModel_2, u2_val, t_val);
% [bj_Var_r2, bj_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
```

```
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```


na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
nd = minAICIndex;
nk = 1;
p = na+nb+nc+nd;




bj_AICModel_2 = bj(data2, [na nb nc nd nk])
```

```
bj_AICModel_2 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.4335 z^-1 + 1.698 z^-2

  C(z) = 1 + 0.4345 z^-1 - 0.5655 z^-2

  D(z) = 1 + 0.3866 z^-1 - 0.5945 z^-2

  F(z) = 1 - 1.64 z^-1 + 0.8852 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data2".
Fit to estimation data: 42.82% (prediction focus)
FPE: 25.19, MSE: 24.53
```

```
AIC_y_hat_2 = lsim(bj_AICModel_2, u2_val, t_val);
% [bj_AIC_r2, bj_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```

```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.19 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 2
```

```
na = winner;
nb = winner;
nc = winner;
nd = winner;
nk = 1;
p = na+nb+nc+nd;

bj_FTestModel_2 = bj(data2, [na nb nc nd nk])
```

```
bj_FTestModel_2 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -0.4335 z^-1 + 1.698 z^-2

  C(z) = 1 + 0.4345 z^-1 - 0.5655 z^-2

  D(z) = 1 + 0.3866 z^-1 - 0.5945 z^-2

  F(z) = 1 - 1.64 z^-1 + 0.8852 z^-2

Sample time: 0.5 seconds
```

```
Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data2".
Fit to estimation data: 42.82% (prediction focus)
FPE: 25.19, MSE: 24.53
```

```
FTest_y_hat_2 = lsim(bj_FTestModel_2, u2_val, t_val);
% [bj_FTest_r2, bj_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[bj_BestFit_r2_2, bj_BestFit_mse_2] = rSQR(y2_val, BestFit_y_hat_2);
[bj_Var_r2_2, bj_Var_mse_2] = rSQR(y2_val, Var_y_hat_2);
[bj_AIC_r2_2, bj_AIC_mse_2] = rSQR(y2_val, AIC_y_hat_2);
[bj_FTest_r2_2, bj_FTest_mse_2] = rSQR(y2_val, FTest_y_hat_2);

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", bj_BestFit_r2_2,
bj_BestFit_mse_2)
```

```
    R2 value : 0.7021   | MSE : 26.3189
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Var_r2_2, bj_Var_mse_2)
```

```
    R2 value : 0.7021    | MSE : 26.3189
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Cov_r2, bj_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_AIC_r2_2, bj_AIC_mse_2)
```

```
    R2 value : 0.7021    | MSE : 26.3189
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_FTest_r2_2, bj_FTest_mse_2)
```

```
    R2 value : 0.7021    | MSE : 26.3189
```

```
fprintf("---------------------------------------------------------------\n")
```

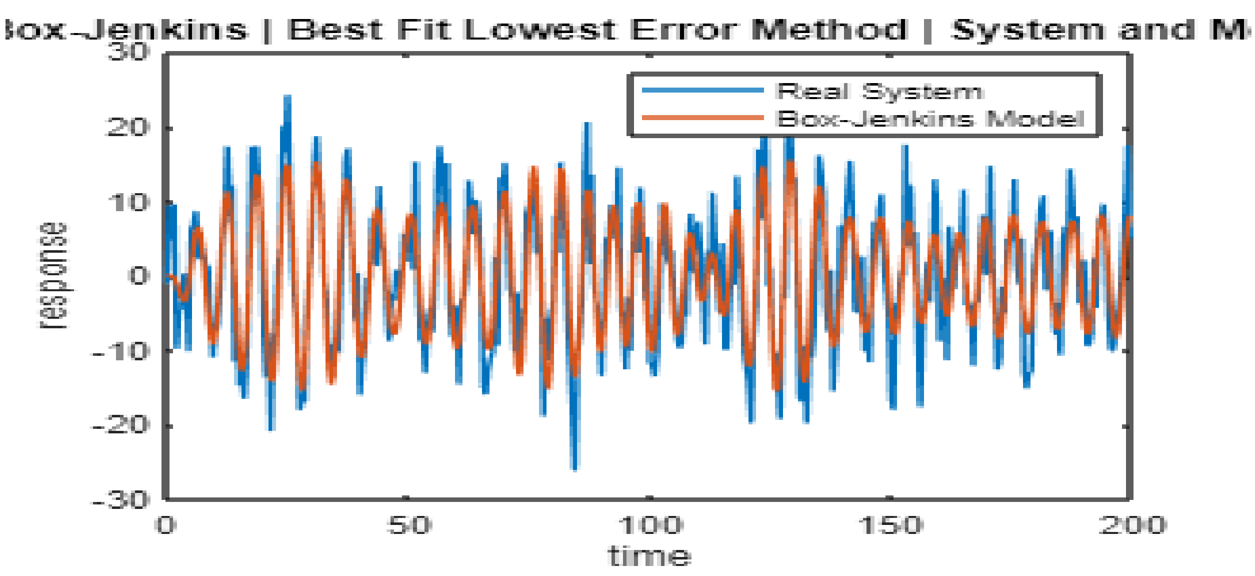
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

```
===============================================================
```

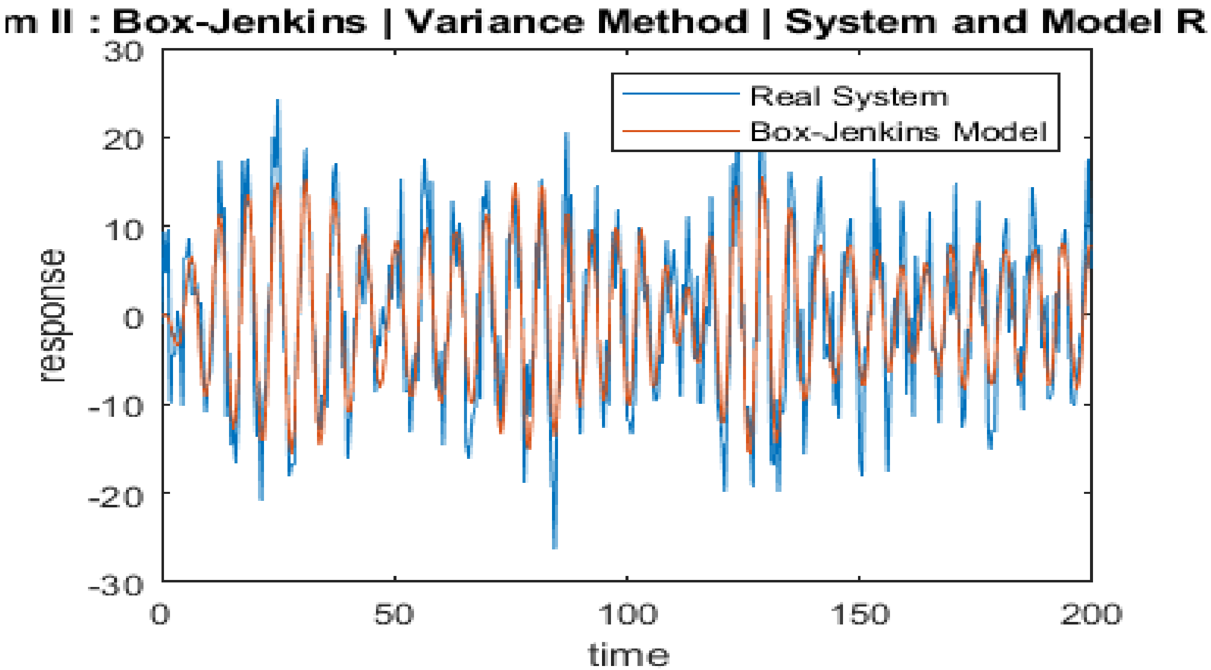
```



%%

bj_BestFitError_2 = y2_val - BestFit_y_hat_2;
bj_VarError_2 = y2_val - Var_y_hat_2;
bj_AICError_2 = y2_val - AIC_y_hat_2;
bj_FTestError_2 = y2_val - FTest_y_hat_2;

for k=0:N_val-1
```

```
    bj_BestFit_Ree_2(k+1,1) = AutoCorrelate(bj_BestFitError_2, k);
    bj_Var_Ree_2(k+1,1) = AutoCorrelate(bj_VarError_2, k);
    bj_AIC_Ree_2(k+1,1) = AutoCorrelate(bj_AICError_2, k);
    bj_FTest_Ree_2(k+1,1) = AutoCorrelate(bj_FTestError_2, k);
end

for k=0:N_val-1
    bj_BestFit_Rue_2(k+1,1) = CrossCorrelate(u2_val, bj_BestFitError_2, k);
    bj_Var_Rue_2(k+1,1) = CrossCorrelate(u2_val, bj_VarError_2, k);
    bj_AIC_Rue_2(k+1,1) = CrossCorrelate(u2_val, bj_AICError_2, k);
    bj_FTest_Rue_2(k+1,1) = CrossCorrelate(u2_val, bj_FTestError_2, k);
end



%%
figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hat_2)
legend('Real System','Box-Jenkins Model')
title(" System II : Box-Jenkins | Best Fit Lowest Error Method | System and
Model Response")
xlabel("time")
ylabel("response")
```

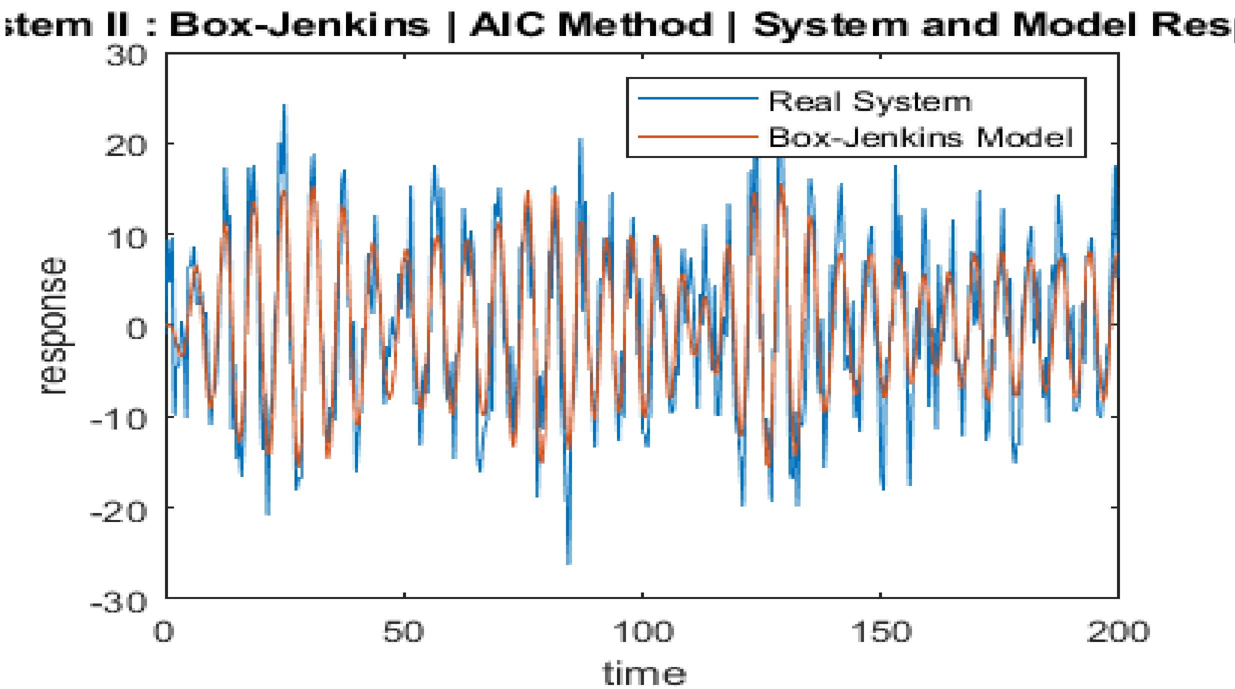



```

figure(2)
plot(t_val,y2_val,t_val,Var_y_hat_2)
legend('Real System','Box-Jenkins Model')
title(" System II : Box-Jenkins | Variance Method | System and Model Response")
xlabel("time")
```

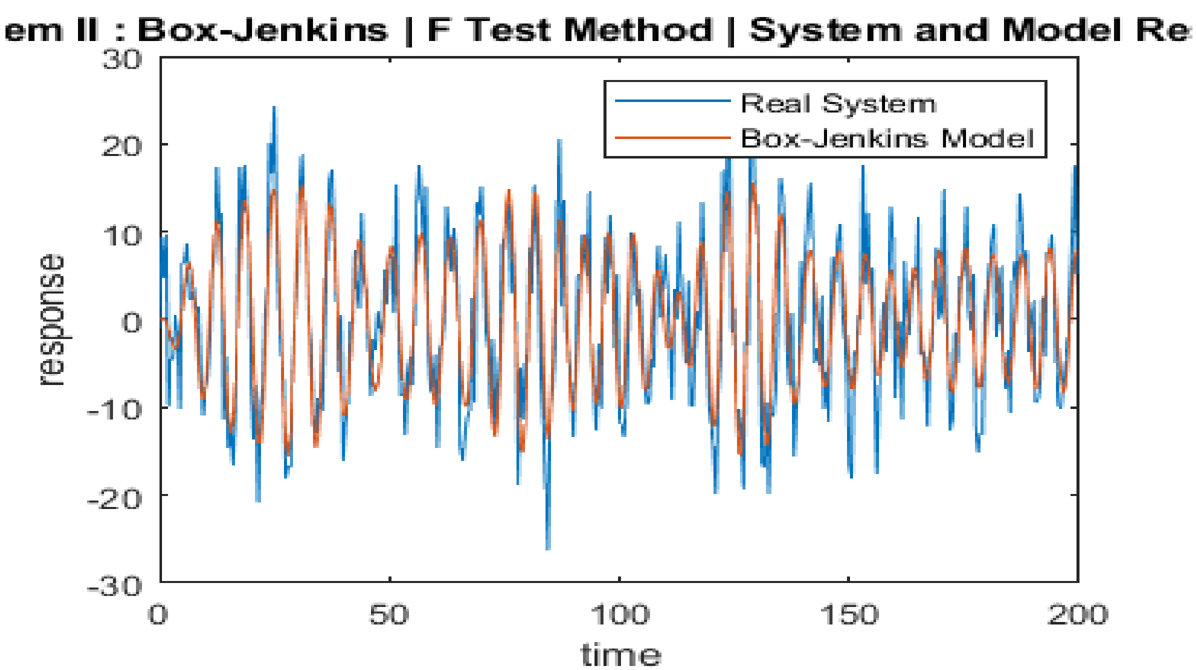
```
ylabel("response")
```



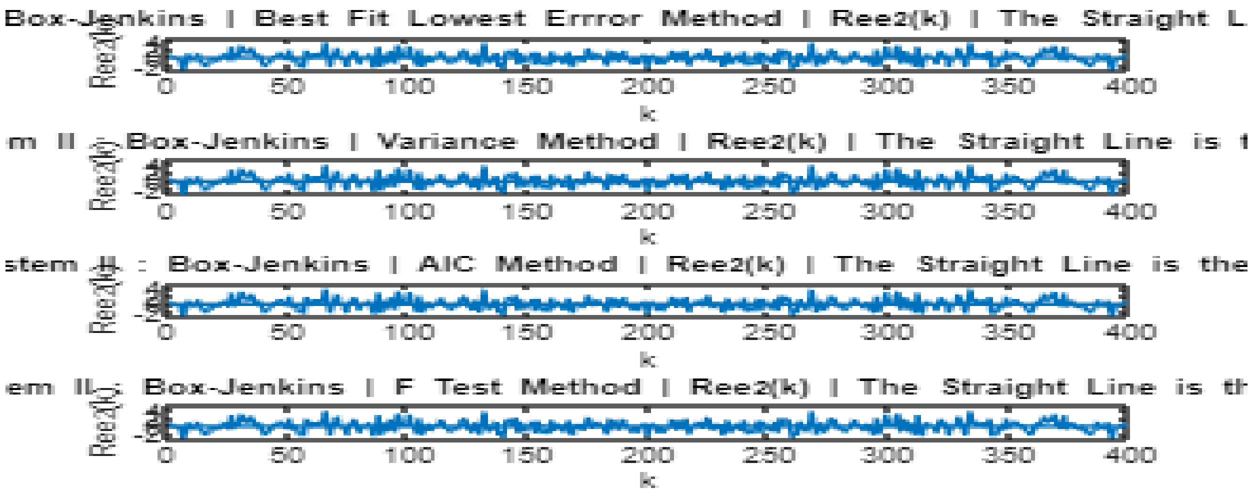

```
figure(3)
plot(t_val,y2_val,t_val,AIC_y_hat_2)
legend('Real System','Box-Jenkins Model')
title(" System II : Box-Jenkins | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y2_val,t_val,FTest_y_hat_2)
legend('Real System','Box-Jenkins Model')
title(" System II : Box-Jenkins | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```

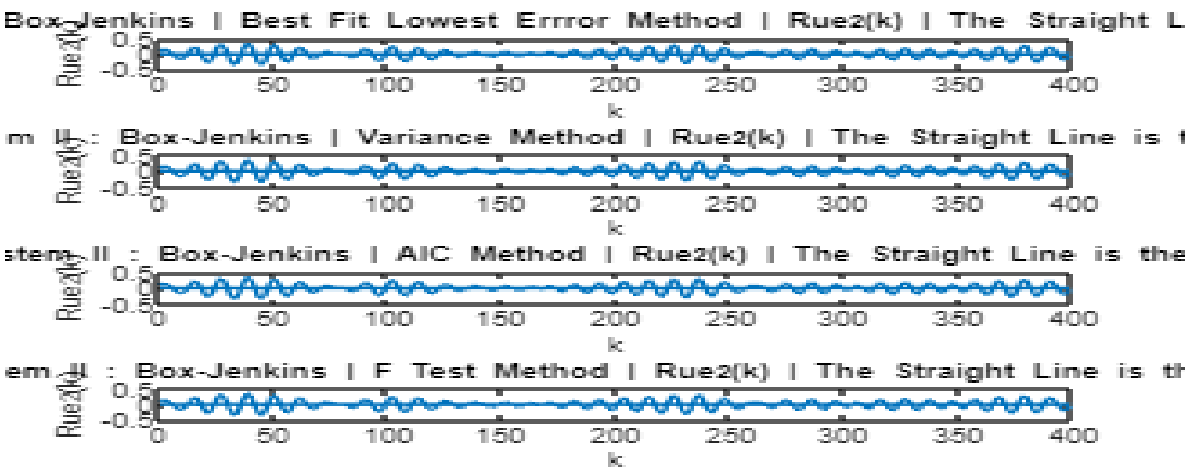


```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,bj_BestFit_Ree_2(2:end), 1:N_val-1,
mean(bj_BestFit_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Box-Jenkins | Best Fit Lowest Errror Method | Ree_2(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,2)
plot(1:N_val-1,bj_Var_Ree_2(2:end), 1:N_val-1,
mean(bj_Var_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Box-Jenkins | Variance Method | Ree_2(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,3)
plot(1:N_val-1,bj_AIC_Ree_2(2:end), 1:N_val-1,
mean(bj_AIC_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Box-Jenkins | AIC Method | Ree_2(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,4)
plot(1:N_val-1,bj_FTest_Ree_2(2:end), 1:N_val-1,
mean(bj_FTest_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Box-Jenkins | F Test Method | Ree_2(k) | The Straight Line
is the Mean")
xlabel("k")
ylabel("Ree_2(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,bj_BestFit_Rue_2(2:end), 1:N_val-1,
mean(bj_BestFit_Rue_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Box-Jenkins | Best Fit Lowest Errror Method | Rue_2(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_2(k)")

subplot(4,1,2)
plot(1:N_val-1,bj_Var_Rue_2(2:end), 1:N_val-1,
mean(bj_Var_Rue_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Box-Jenkins | Variance Method | Rue_2(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Rue_2(k)")

subplot(4,1,3)
plot(1:N_val-1,bj_AIC_Rue_2(2:end), 1:N_val-1,
mean(bj_AIC_Rue_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Box-Jenkins | AIC Method | Rue_2(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Rue_2(k)")

subplot(4,1,4)
```

```
 plot(1:N_val-1,bj_FTest_Rue_2(2:end), 1:N_val-1,
mean(bj_FTest_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II : Box-Jenkins | F Test Method | Rue_2(k) | The Straight Line
is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")
```




```

```

Q2 - part c | Output Error

```
clc; clear;
%%

load HW5_question2

u1 = Z1.u;
y1 = Z1.y;

u2 = Z2.u;
y2 = Z2.y;

u3 = Z3.u;
y3 = Z3.y;

u1_val = u1(601:end);
y1_val = y1(601:end);

u2_val = u2(601:end);
y2_val = y2(601:end);

u3_val = u1(601:end);
y3_val = y1(601:end);

u1 = u1(1:600);
y1 = y1(1:600);

u2 = u2(1:600);
y2 = y2(1:600);

u3 = u3(1:600);
y3 = y3(1:600);
```

System I

```
%%

% System Z1 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System I Identification Begins:------------------------------\n")
```

```
>>> System I Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data1 = iddata(y1,u1,Ts);

%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    try
        sys = oe(data1, [na nb nk]);
        oe_y_hat_1 = lsim(sys, u1, t);
    catch
        break
    end

    [r2_oe, mse_oe] = rSQR(y1, oe_y_hat_1);

    error = y1 - oe_y_hat_1;
    S_hat = 0;
```

```
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_oe, mse_oe, variance, S_hat)
    fprintf("--------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_oe];
    MSEs = [MSEs; mse_oe];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.599900 | MSE=33.478625 | var=33.590593 | s_hat=20087.174802 |
--------------------------------------------------------------
>>> Degree = 2 : R2=0.605320 | MSE=33.025154 | var=33.246800 | s_hat=19815.092682 |
--------------------------------------------------------------
>>> Degree = 3 : R2=0.920165 | MSE=6.680286 | var=6.747763 | s_hat=4008.171478 |
--------------------------------------------------------------
>>> Degree = 4 : R2=0.920558 | MSE=6.647359 | var=6.737189 | s_hat=3988.415641 |
--------------------------------------------------------------
>>> Degree = 5 : R2=0.921038 | MSE=6.607203 | var=6.719189 | s_hat=3964.321786 |
--------------------------------------------------------------
>>> Degree = 6 : R2=0.539485 | MSE=38.533901 | var=39.320307 | s_hat=23120.340427 |
--------------------------------------------------------------
>>> Degree = 7 : R2=0.921487 | MSE=6.569654 | var=6.726608 | s_hat=3941.792349 |
--------------------------------------------------------------
>>> Degree = 8 : R2=0.921499 | MSE=6.568660 | var=6.748623 | s_hat=3941.196079 |
--------------------------------------------------------------
>>> Degree = 9 : R2=0.923587 | MSE=6.393946 | var=6.591697 | s_hat=3836.367592 |
--------------------------------------------------------------
>>> Degree = 10 : R2=0.922592 | MSE=6.477194 | var=6.700545 | s_hat=3886.316323 |
--------------------------------------------------------------
>>> Degree = 11 : R2=0.925306 | MSE=6.250052 | var=6.487943 | s_hat=3750.031169 |
--------------------------------------------------------------
>>> Degree = 12 : R2=0.924866 | MSE=6.286893 | var=6.548846 | s_hat=3772.135543 |
--------------------------------------------------------------
>>> Degree = 13 : R2=0.921562 | MSE=6.563391 | var=6.860687 | s_hat=3938.034444 |
--------------------------------------------------------------
>>> Degree = 14 : R2=0.925055 | MSE=6.271090 | var=6.578066 | s_hat=3762.653991 |
--------------------------------------------------------------
>>> Degree = 15 : R2=0.921360 | MSE=6.580254 | var=6.926583 | s_hat=3948.152211 |
--------------------------------------------------------------
>>> Degree = 16 : R2=0.924669 | MSE=6.303376 | var=6.658495 | s_hat=3782.025419 |
--------------------------------------------------------------
```

```
>>> Degree = 17 : R2=0.922509 | MSE=6.484115 | var=6.873620 | s_hat=3890.468986 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.915164 | MSE=7.098740 | var=7.551851 | s_hat=4259.243979 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.926875 | MSE=6.118769 | var=6.532493 | s_hat=3671.261108 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.925904 | MSE=6.200043 | var=6.642903 | s_hat=3720.025920 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.928902 | MSE=5.949174 | var=6.396961 | s_hat=3569.504220 |
------------------------------------------------------------
>>> Degree = 22 : R2=-2.585662 | MSE=300.032734 | var=323.776331 | s_hat=180019.640245 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.928887 | MSE=5.950470 | var=6.444553 | s_hat=3570.282170 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.923298 | MSE=6.418112 | var=6.976208 | s_hat=3850.867018 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.926228 | MSE=6.172925 | var=6.734100 | s_hat=3703.755228 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.928232 | MSE=6.005197 | var=6.575033 | s_hat=3603.118137 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.928168 | MSE=6.010551 | var=6.605001 | s_hat=3606.330685 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.924585 | MSE=6.310441 | var=6.960045 | s_hat=3786.264485 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.928960 | MSE=5.944306 | var=6.580413 | s_hat=3566.583838 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.916095 | MSE=7.020774 | var=7.800860 | s_hat=4212.464563 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.922652 | MSE=6.472133 | var=7.217992 | s_hat=3883.279587 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.864667 | MSE=11.324071 | var=12.676199 | s_hat=6794.442430 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.913570 | MSE=7.232086 | var=8.125939 | s_hat=4339.251498 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.928909 | MSE=5.948581 | var=6.708926 | s_hat=3569.148504 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.841637 | MSE=13.251135 | var=15.001285 | s_hat=7950.681130 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.862850 | MSE=11.476091 | var=13.041012 | s_hat=6885.654337 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.923538 | MSE=6.397995 | var=7.298093 | s_hat=3838.796872 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.928346 | MSE=5.995668 | var=6.865269 | s_hat=3597.400911 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.927716 | MSE=6.048410 | var=6.952196 | s_hat=3629.046284 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.627259 | MSE=31.189343 | var=35.987703 | s_hat=18713.605571 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.851248 | MSE=12.446945 | var=14.417311 | s_hat=7468.167071 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.874388 | MSE=10.510631 | var=12.221664 | s_hat=6306.378685 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.861442 | MSE=11.593907 | var=13.533743 | s_hat=6956.344073 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.312501 | MSE=57.526957 | var=67.414402 | s_hat=34516.173905 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.920909 | MSE=6.617968 | var=7.785844 | s_hat=3970.780510 |
```

```
------------------------------------------------------------
>>> Degree = 46 : R2=-1.554671 | MSE=213.763860 | var=252.477000 | s_hat=128258.315823 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.931480 | MSE=5.733457 | var=6.798565 | s_hat=3440.073924 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.900337 | MSE=8.339373 | var=9.927826 | s_hat=5003.624062 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.878801 | MSE=10.141416 | var=12.121214 | s_hat=6084.849617 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.729371 | MSE=22.645048 | var=27.174058 | s_hat=13587.028918 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.857815 | MSE=11.897459 | var=14.334288 | s_hat=7138.475530 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.695742 | MSE=25.459040 | var=30.797225 | s_hat=15275.423764 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.927481 | MSE=6.068079 | var=7.370137 | s_hat=3640.847571 |
------------------------------------------------------------
>>> Degree = 54 : R2=-0.019523 | MSE=85.309306 | var=104.035738 | s_hat=51185.583309 |
------------------------------------------------------------
>>> Degree = 55 : R2=0.596457 | MSE=33.766733 | var=41.347019 | s_hat=20260.039540 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.721054 | MSE=23.341011 | var=28.697964 | s_hat=14004.606635 |
------------------------------------------------------------
>>> Degree = 57 : R2=-7.578715 | MSE=717.829879 | var=886.209727 | s_hat=430697.927361 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.823337 | MSE=14.782375 | var=18.325258 | s_hat=8869.424827 |
------------------------------------------------------------
>>> Degree = 59 : R2=0.750096 | MSE=20.910876 | var=26.030136 | s_hat=12546.525471 |
------------------------------------------------------------
>>> Degree = 60 : R2=-0.333085 | MSE=111.546816 | var=139.433520 | s_hat=66928.089406 |
------------------------------------------------------------
>>> Degree = 61 : R2=0.930581 | MSE=5.808663 | var=7.291209 | s_hat=3485.197966 |
------------------------------------------------------------
>>> Degree = 62 : R2=0.604787 | MSE=33.069702 | var=41.684499 | s_hat=19841.821448 |
------------------------------------------------------------
>>> Degree = 63 : R2=0.663082 | MSE=28.191877 | var=35.685921 | s_hat=16915.126490 |
------------------------------------------------------------
>>> Degree = 64 : R2=-1.963966 | MSE=248.011876 | var=315.269335 | s_hat=148807.125889 |
------------------------------------------------------------
>>> Degree = 65 : R2=-0.198751 | MSE=100.306342 | var=128.050650 | s_hat=60183.805299 |
------------------------------------------------------------
>>> Degree = 66 : R2=0.920530 | MSE=6.649689 | var=8.525242 | s_hat=3989.813319 |
------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
```

```
fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 47
```

```
na = bestFitDegree;
nb = bestFitDegree;
nk = 1;
p = na+nb;

BestFitModel_1 = oe(data1, [na nb nk])
```

```
BestFitModel_1 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.8075 z^-1 + 4.147 z^-2 + 10.42 z^-3 + 17.22 z^-4 + 20.91 z^-5 + 20.26 z^-6 + 17.78
z^-7
          + 16.73 z^-8 + 17.49 z^-9 + 17.78 z^-10 + 15.62 z^-11 + 11.54 z^-12 + 8.545 z^-13 +
9.437 z^-14
          + 13.41 z^-15 + 16.42 z^-16 + 16.08 z^-17 + 13.48 z^-18 + 9.947 z^-19 + 4.811 z^-20
          - 1.777 z^-21 - 7.106 z^-22 - 9.829 z^-23 - 11.07 z^-24 - 11.04 z^-25 - 9.525 z^-26
          - 8.202 z^-27 - 8.301 z^-28 - 7.886 z^-29 - 5.856 z^-30 - 5.689 z^-31 - 9.848 z^-32
          - 14.57 z^-33 - 15.29 z^-34 - 12.52 z^-35 - 9.078 z^-36 - 6.106 z^-37 - 3.974 z^-38
          - 2.912 z^-39 - 1.179 z^-40 + 2.514 z^-41 + 5.273 z^-42 + 3.634 z^-43 - 0.6296 z^-44
                                                 - 2.831 z^-45 - 1.926 z^-46 -
0.4765 z^-47


  F(z) = 1 + 3.015 z^-1 + 3.123 z^-2 + 0.4187 z^-3 - 1.431 z^-4 + 0.2407 z^-5 + 2.862 z^-6 +
2.903 z^-7
          + 0.9603 z^-8 - 0.5072 z^-9 - 0.9707 z^-10 - 0.4379 z^-11 + 1.271 z^-12 + 2.176 z^-
13
          + 0.5863 z^-14 - 1.526 z^-15 - 1.554 z^-16 - 0.6882 z^-17 - 1.186 z^-18 - 2.094 z^-
19
          - 1.652 z^-20 - 0.6234 z^-21 - 0.7062 z^-22 - 1.535 z^-23 - 1.935 z^-24 - 1.86 z^-25
          - 1.368 z^-26 - 0.3249 z^-27 + 0.1362 z^-28 - 0.8954 z^-29 - 2.06 z^-30 - 1.688 z^-
31
          + 0.1025 z^-32 + 1.528 z^-33 + 1.09 z^-34 - 0.3959 z^-35 - 0.863 z^-36 + 0.2723 z^-
37
          + 1.622 z^-38 + 1.615 z^-39 + 0.3731 z^-40 - 0.4466 z^-41 + 0.06246 z^-42 + 1.015
z^-43
                                          + 1.197 z^-44 + 0.5477 z^-45 - 0.06279 z^-46 -
0.1129 z^-47
```

```
Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=47   nf=47   nk=1
   Number of free coefficients: 94
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data1".
Fit to estimation data: 73.82%
FPE: 7.864, MSE: 5.733
```

```
BestFit_y_hat_1 = lsim(BestFitModel_1, u1_val, t_val);
% [oe_BestFit_r2, oe_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 21 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 21
```

```
na = minVarIndex;
nb = minVarIndex;
nk = 1;
p = na+nb;

oe_VarModel_1 = oe(data1, [na nb nk])
```

```
oe_VarModel_1 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.7847 z^-1 + 2.471 z^-2 + 5.289 z^-3 + 8.651 z^-4 + 11.58 z^-5 + 13.41 z^-6 + 13.68
z^-7
          + 12.27 z^-8 + 9.094 z^-9 + 5.284 z^-10 + 1.126 z^-11 - 1.927 z^-12 - 3.44 z^-13 -
3.392 z^-14
          - 2.523 z^-15 - 1.318 z^-16 - 0.07994 z^-17 + 0.5093 z^-18 + 0.686 z^-19 + 0.226 z^
                                                                                  -20 +
0.1801 z^-21
```

```
  F(z) = 1 + 1.011 z^-1 + 1.141 z^-2 + 0.9198 z^-3 + 0.8267 z^-4 + 0.5179 z^-5 + 0.3715 z^-6
          - 0.03825 z^-7 - 0.3694 z^-8 - 0.2333 z^-9 - 0.3928 z^-10 - 0.192 z^-11 - 0.5093 z^
          -12 - 0.6103 z^-13 - 0.5032 z^-14 - 0.7441 z^-15 - 0.7018 z^-16 - 0.5501 z^-17 -
0.1941 z^-18
                                                            - 0.1686 z^-19 + 0.3086 z^-20 -
0.1058 z^-21

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=21   nf=21   nk=1
   Number of free coefficients: 42
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data1".
Fit to estimation data: 73.34%
FPE: 6.845, MSE: 5.949
```

```
Var_y_hat_1 = lsim(oe_VarModel_1, u1_val, t_val);
% [oe_Var_r2, oe_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 11 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 11
```

```
na = minAICIndex;
nb = minAICIndex;
nk = 1;
p = na+nb;

oe_AICModel_1 = oe(data1, [na nb nk])
```

```
oe_AICModel_1 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.7977 z^-1 + 0.9157 z^-2 + 0.7573 z^-3 + 0.5313 z^-4 - 0.4986 z^-5 - 0.2844 z^-6 -
0.622 z^-7
                                          - 0.4993 z^-8 - 0.03505 z^-9 + 0.01915 z^-10 +
0.3905 z^-11


  F(z) = 1 - 0.9572 z^-1 - 0.4556 z^-2 + 0.7533 z^-3 - 0.3937 z^-4 + 0.3872 z^-5 - 0.6666 z^-6
                               + 0.5674 z^-7 + 0.4987 z^-8 - 1.165 z^-9 + 0.3204 z^-10 +
0.1274 z^-11

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=11   nf=11   nk=1
   Number of free coefficients: 22
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data1".
Fit to estimation data: 72.67%
FPE: 6.726, MSE: 6.25
```

```
AIC_y_hat_1 = lsim(oe_AICModel_1, u1_val, t_val);
% [oe_AIC_r2, oe_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test 
Method===================\n")
```

```
===============Degree Extraction | F test Method===================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
```

```
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.26 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 11
```

```


na = winner;
nb = winner;
nk = 1;
p = na+nb;

oe_FTestModel_1 = oe(data1, [na nb nk])
```

```
oe_FTestModel_1 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.7977 z^-1 + 0.9157 z^-2 + 0.7573 z^-3 + 0.5313 z^-4 - 0.4986 z^-5 - 0.2844 z^-6 -
0.622 z^-7
                                          - 0.4993 z^-8 - 0.03505 z^-9 + 0.01915 z^-10 +
0.3905 z^-11


  F(z) = 1 - 0.9572 z^-1 - 0.4556 z^-2 + 0.7533 z^-3 - 0.3937 z^-4 + 0.3872 z^-5 - 0.6666 z^-6
                               + 0.5674 z^-7 + 0.4987 z^-8 - 1.165 z^-9 + 0.3204 z^-10 +
0.1274 z^-11

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=11   nf=11   nk=1
   Number of free coefficients: 22
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data1".
Fit to estimation data: 72.67%
FPE: 6.726, MSE: 6.25
```

```
FTest_y_hat_1 = lsim(oe_FTestModel_1, u1_val, t_val);
% [oe_FTest_r2, oe_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[oe_BestFit_r2_1, oe_BestFit_mse_1] = rSQR(y1_val, BestFit_y_hat_1);
[oe_Var_r2_1, oe_Var_mse_1] = rSQR(y1_val, Var_y_hat_1);
[oe_AIC_r2_1, oe_AIC_mse_1] = rSQR(y1_val, AIC_y_hat_1);
[oe_FTest_r2_1, oe_FTest_mse_1] = rSQR(y1_val, FTest_y_hat_1);

%%

fprintf("==================Evaluation | R2 Metric=====================\n")
```

```
==================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_BestFit_r2_1,
oe_BestFit_mse_1)
```

```
    R2 value : 0.6738    | MSE : 24.1226
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Var_r2_1, oe_Var_mse_1)
```

```
    R2 value : 0.6483    | MSE : 26.0135
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Cov_r2, oe_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_AIC_r2_1, oe_AIC_mse_1)
```

```
    R2 value : 0.6519    | MSE : 25.7445
```

```matlab
fprintf("--------------------------------------------------------------\n")
```

```
--------------------------------------------------------------
```

```matlab
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```matlab
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_FTest_r2_1, oe_FTest_mse_1)
```

```
    R2 value : 0.6519    | MSE : 25.7445
```

```matlab
fprintf("--------------------------------------------------------------\n")
```

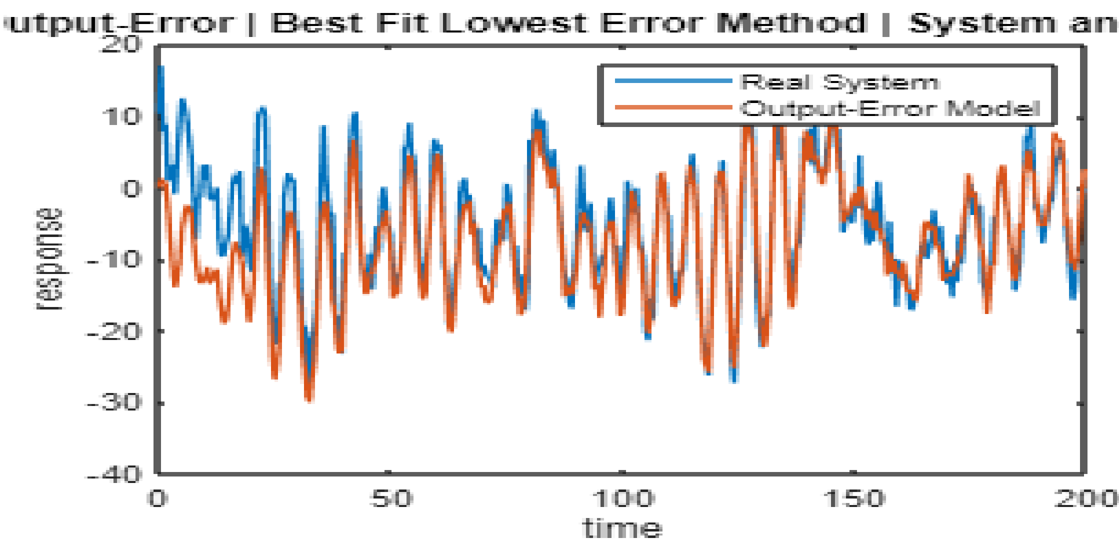
```
--------------------------------------------------------------
```

```matlab
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("==============================================================\n")
```

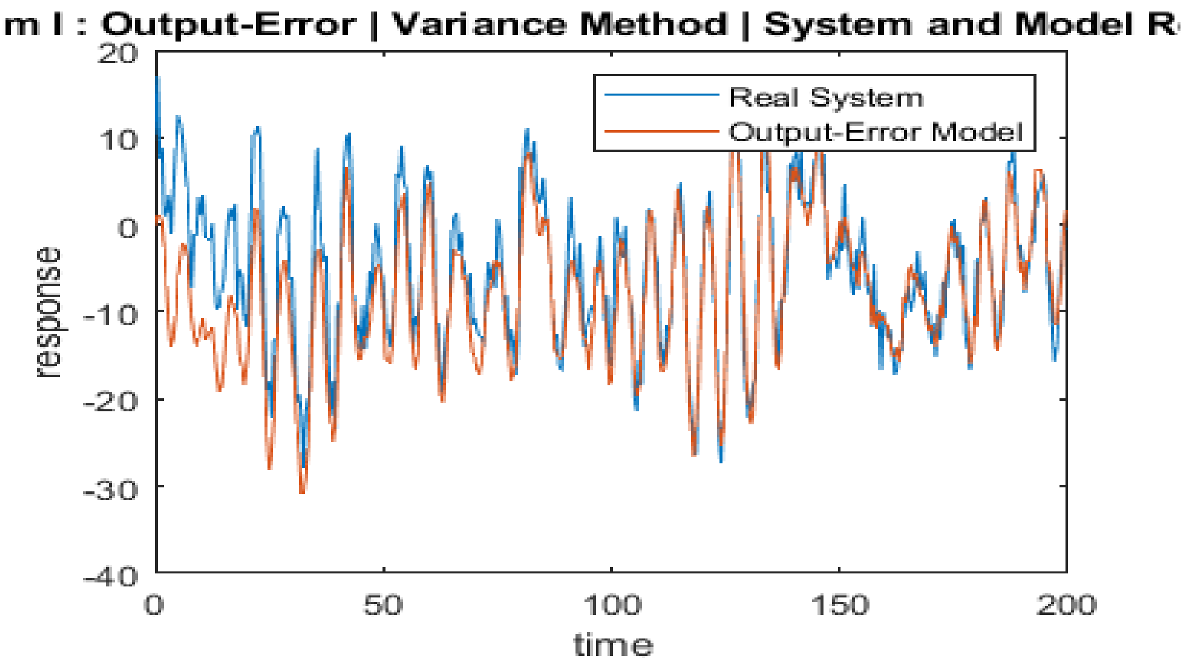
```
==============================================================
```

```matlab


%%

oe_BestFitError_1 = y1_val - BestFit_y_hat_1;
oe_VarError_1 = y1_val - Var_y_hat_1;
oe_AICError_1 = y1_val - AIC_y_hat_1;
oe_FTestError_1 = y1_val - FTest_y_hat_1;

for k=0:N_val-1
    oe_BestFit_Ree_1(k+1,1) = AutoCorrelate(oe_BestFitError_1, k);
    oe_Var_Ree_1(k+1,1) = AutoCorrelate(oe_VarError_1, k);
    oe_AIC_Ree_1(k+1,1) = AutoCorrelate(oe_AICError_1, k);
    oe_FTest_Ree_1(k+1,1) = AutoCorrelate(oe_FTestError_1, k);
end

for k=0:N_val-1
    oe_BestFit_Rue_1(k+1,1) = CrossCorrelate(u1_val, oe_BestFitError_1, k);
    oe_Var_Rue_1(k+1,1) = CrossCorrelate(u1_val, oe_VarError_1, k);
    oe_AIC_Rue_1(k+1,1) = CrossCorrelate(u1_val, oe_AICError_1, k);
    oe_FTest_Rue_1(k+1,1) = CrossCorrelate(u1_val, oe_FTestError_1, k);
end


%%
```

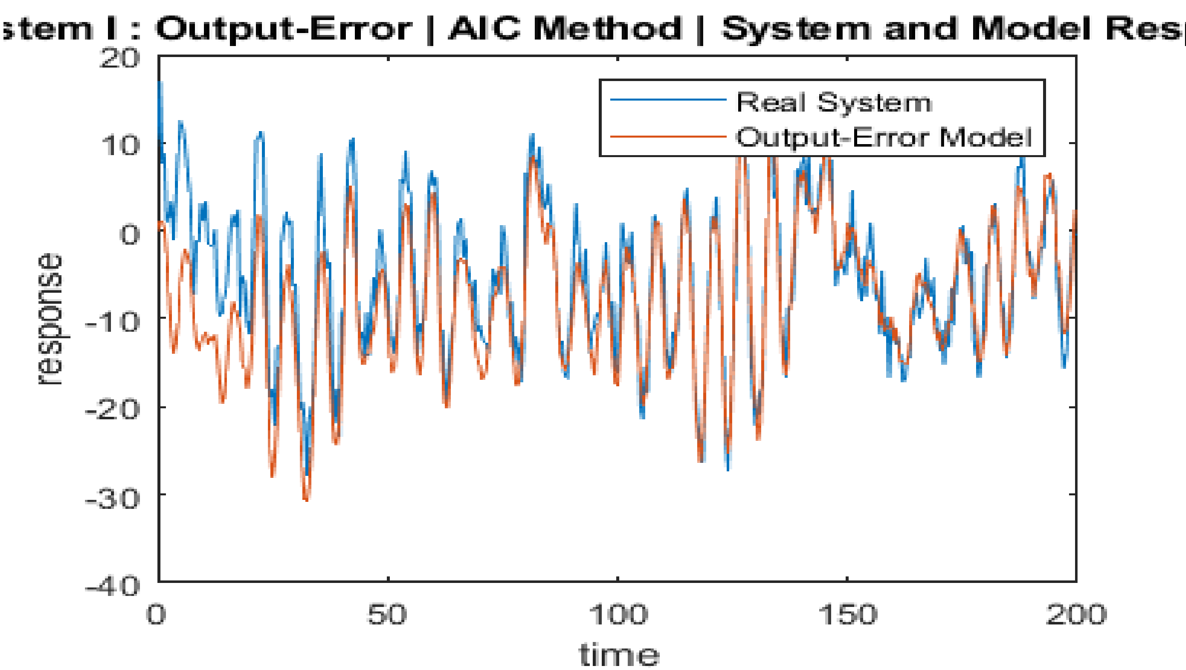
```
figure(1)
plot(t_val,y1_val,t_val,BestFit_y_hat_1)
legend('Real System','Output-Error Model')
title(" System I : Output-Error | Best Fit Lowest Error Method | System and
Model Response")
xlabel("time")
ylabel("response")
```



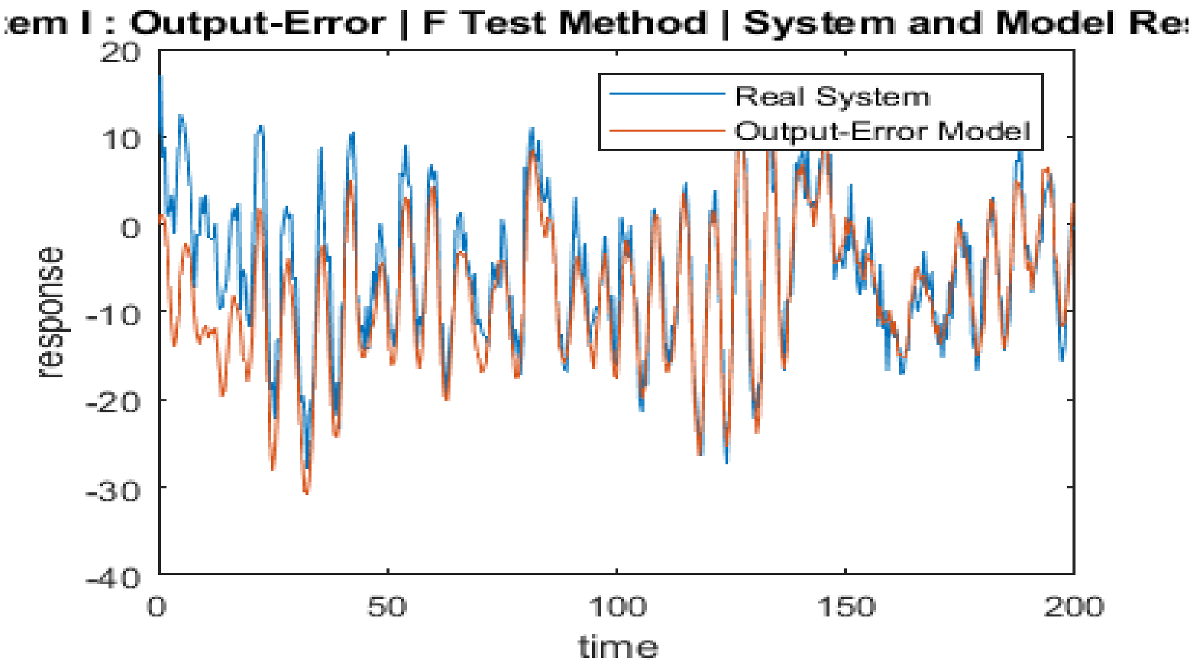

```
figure(2)
plot(t_val,y1_val,t_val,Var_y_hat_1)
legend('Real System','Output-Error Model')
title(" System I : Output-Error | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(3)
plot(t_val,y1_val,t_val,AIC_y_hat_1)
legend('Real System','Output-Error Model')
title(" System I : Output-Error | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

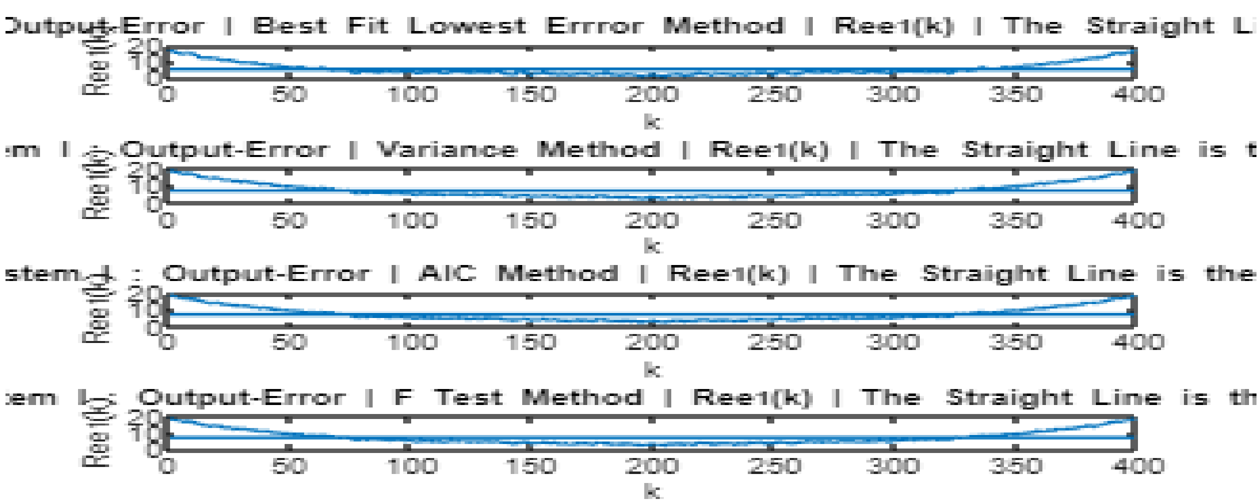


```
figure(4)
plot(t_val,y1_val,t_val,FTest_y_hat_1)
legend('Real System','Output-Error Model')
title(" System I : Output-Error | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,oe_BestFit_Ree_1(2:end), 1:N_val-1,
mean(oe_BestFit_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Output-Error | Best Fit Lowest Errror Method | Ree_1(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(4,1,2)
plot(1:N_val-1,oe_Var_Ree_1(2:end), 1:N_val-1,
mean(oe_Var_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" System I : Output-Error | Variance Method | Ree_1(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")
```

```
 subplot(4,1,3)
 plot(1:N_val-1,oe_AIC_Ree_1(2:end), 1:N_val-1,
mean(oe_AIC_Ree_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : Output-Error | AIC Method | Ree_1(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Ree_1(k)")

 subplot(4,1,4)
 plot(1:N_val-1,oe_FTest_Ree_1(2:end), 1:N_val-1,
mean(oe_FTest_Ree_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : Output-Error | F Test Method | Ree_1(k) | The Straight Line
is the Mean")
 xlabel("k")
 ylabel("Ree_1(k)")
```




```
 %%

 figure(6)
 subplot(4,1,1)
 plot(1:N_val-1,oe_BestFit_Rue_1(2:end), 1:N_val-1,
mean(oe_BestFit_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : Output-Error | Best Fit Lowest Errror Method | Rue_1(k) |
The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(4,1,2)
```

```
 plot(1:N_val-1,oe_Var_Rue_1(2:end), 1:N_val-1, mean(oe_Var_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : Output-Error | Variance Method | Rue_1(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(4,1,3)
 plot(1:N_val-1,oe_AIC_Rue_1(2:end), 1:N_val-1, mean(oe_AIC_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : Output-Error | AIC Method | Rue_1(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")

 subplot(4,1,4)
 plot(1:N_val-1,oe_FTest_Rue_1(2:end), 1:N_val-1, mean(oe_FTest_Rue_1(2:end))*ones(length(1:N_val-1)))
 title(" System I : Output-Error | F Test Method | Rue_1(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_1(k)")
```

System II

```
%%

% System Z2 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System II Identification Begins:------------------------------\n")
```

```
>>> System II Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);
```

```
data2 = iddata(y2,u2,Ts);


%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    try
        sys = oe(data2, [na nb nk]);
        oe_y_hat_2 = lsim(sys, u2, t);
    catch
        break
    end

    [r2_oe, mse_oe] = rSQR(y2, oe_y_hat_2);

    error = y2 - oe_y_hat_2;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);
```

```
    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_oe, mse_oe, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_oe];
    MSEs = [MSEs; mse_oe];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

 end
```

```
>>> Degree = 1 : R2=0.317572 | MSE=51.183366 | var=51.354548 | s_hat=30710.019542 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.670197 | MSE=24.735878 | var=24.901890 | s_hat=14841.526682 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.670196 | MSE=24.735894 | var=24.985752 | s_hat=14841.536471 |
-------------------------------------------------------------
>>> Degree = 4 : R2=-13.777812 | MSE=1108.364103 | var=1123.341996 | s_hat=665018.461700 |
-------------------------------------------------------------
>>> Degree = 5 : R2=-1940048505243.498291 | MSE=145507337193691.750000 |
var=147973563247822.093750 | s_hat=87304402316215040.000000 |
-------------------------------------------------------------
>>> Degree = 6 : R2=-56827.906184 | MSE=4262276.325629 | var=4349261.556764 |
s_hat=2557365795.377142 |
-------------------------------------------------------------
>>> Degree = 7 : R2=-30365186.495276 | MSE=2277446962.733490 | var=2331856958.430194 |
s_hat=1366468177640.093506 |
-------------------------------------------------------------
>>> Degree = 8 : R2=-2979222.720253 | MSE=223447459.827155 | var=229569308.041597 |
s_hat=134068475896.292648 |
-------------------------------------------------------------
>>> Degree = 9 : R2=-1095138.552643 | MSE=82137554.669262 | var=84677891.411610 |
s_hat=49282532801.557053 |
-------------------------------------------------------------
>>> Degree = 10 : R2=-2218321776.944307 | MSE=166378363259.929382 | var=172115548199.926910 |
s_hat=99827017955957.609375 |
-------------------------------------------------------------
>>> Degree = 11 : R2=-1788329461.544101 | MSE=134128119692.050644 | var=139233342240.883301 |
s_hat=80476871815230.546875 |
-------------------------------------------------------------
>>> Degree = 12 : R2=-326212089.943712 | MSE=24466528844.660946 | var=25485967546.521835 |
s_hat=14679917306796.576172 |
-------------------------------------------------------------
>>> Degree = 13 : R2=-1920873895155.508545 | MSE=144069204874789.218750 |
var=150594987673995.750000 | s_hat=86441522924873552.000000 |
-------------------------------------------------------------
>>> Degree = 14 : R2=-12790038.697357 | MSE=959277365.454097 | var=1006234998.728073 |
s_hat=575566419272.457764 |
-------------------------------------------------------------
>>> Degree = 15 : R2=-6059030020000415.000000 | MSE=454438804908020608.000000 |
var=478356636745285184.000000 | s_hat=272663282944812548096.000000 |
```

```
-------------------------------------------------------------
>>> Degree = 16 : R2=0.025954 | MSE=73.055291 | var=77.171082 | s_hat=43833.174444 |
-------------------------------------------------------------
>>> Degree = 17 : R2=-2526799739.846607 | MSE=189514798685.904053 | var=200899079879.050476 |
s_hat=113708879211542.562500 |
-------------------------------------------------------------
>>> Degree = 18 : R2=-1424689784.522457 | MSE=106854450524.317627 | var=113674947366.295288 |
s_hat=64112670314590.539062 |
-------------------------------------------------------------
>>> Degree = 19 : R2=-19117432.783214 | MSE=1433843987.020127 | var=1530794292.192310 |
s_hat=860306392212.078003 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.037563 | MSE=72.184639 | var=77.340685 | s_hat=43310.783423 |
-------------------------------------------------------------
>>> Degree = 21 : R2=-393564735.318411 | MSE=29518105676.338379 | var=31739898576.707920 |
s_hat=17710863405803.019531 |
-------------------------------------------------------------
>>> Degree = 22 : R2=-28160598.193307 | MSE=2112098636.358818 | var=2279243132.761312 |
s_hat=1267259181815.289551 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.678625 | MSE=24.103769 | var=26.105165 | s_hat=14462.261337 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.586645 | MSE=31.002393 | var=33.698253 | s_hat=18601.435838 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.073821 | MSE=69.465189 | var=75.780206 | s_hat=41679.113360 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.516430 | MSE=36.268658 | var=39.710210 | s_hat=21761.194966 |
-------------------------------------------------------------
>>> Degree = 27 : R2=-3.162287 | MSE=312.179480 | var=343.054374 | s_hat=187307.688286 |
-------------------------------------------------------------
>>> Degree = 28 : R2=-1.063848 | MSE=154.792501 | var=170.727023 | s_hat=92875.500445 |
-------------------------------------------------------------
>>> Degree = 29 : R2=-889056.950938 | MSE=66681041.583355 | var=73816651.199286 |
s_hat=40008624950.012978 |
-------------------------------------------------------------
>>> Degree = 30 : R2=-0.472224 | MSE=110.419572 | var=122.688413 | s_hat=66251.743085 |
-------------------------------------------------------------
>>> Degree = 31 : R2=-6034552.783992 | MSE=452603040.535917 | var=504761755.244517 |
s_hat=271561824321.550385 |
-------------------------------------------------------------
>>> Degree = 32 : R2=0.630419 | MSE=27.719270 | var=31.029034 | s_hat=16631.562020 |
-------------------------------------------------------------
>>> Degree = 33 : R2=-1.213716 | MSE=166.032888 | var=186.553806 | s_hat=99619.732578 |
-------------------------------------------------------------
>>> Degree = 34 : R2=0.659749 | MSE=25.519485 | var=28.781374 | s_hat=15311.690946 |
-------------------------------------------------------------
>>> Degree = 35 : R2=0.645153 | MSE=26.614181 | var=30.129261 | s_hat=15968.508365 |
-------------------------------------------------------------
>>> Degree = 36 : R2=-640846799.559899 | MSE=48064732013.912224 | var=54619013652.172958 |
s_hat=28838839208347.320312 |
-------------------------------------------------------------
>>> Degree = 37 : R2=-228787541.875956 | MSE=17159501969.654324 | var=19573576391.240665 |
s_hat=10295701181792.589844 |
-------------------------------------------------------------
>>> Degree = 38 : R2=0.276408 | MSE=54.270802 | var=62.142139 | s_hat=32562.480971 |
-------------------------------------------------------------
```

```
>>> Degree = 39 : R2=-49783445.349860 | MSE=3733853403.722093 | var=4291785521.519641 |
s_hat=2240312042233.252441 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.591450 | MSE=30.642062 | var=35.356225 | s_hat=18385.237123 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.541623 | MSE=34.379178 | var=39.821442 | s_hat=20627.506744 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.640974 | MSE=26.927649 | var=31.311220 | s_hat=16156.589631 |
------------------------------------------------------------
>>> Degree = 43 : R2=-1627230150.487433 | MSE=122045364177.316849 | var=142465405654.455292 |
s_hat=73227218506390.015625 |
------------------------------------------------------------
>>> Degree = 44 : R2=-90076928.252680 | MSE=6755941453.376260 | var=7917118890.675298 |
s_hat=4053564872025.752441 |
------------------------------------------------------------
>>> Degree = 45 : R2=-180240.538064 | MSE=13518459.040814 | var=15904069.459781 |
s_hat=8111075424.488367 |
------------------------------------------------------------
>>> Degree = 46 : R2=-113514527.203090 | MSE=8513806065.662397 | var=10055676455.506765 |
s_hat=5108283639397.436523 |
------------------------------------------------------------
>>> Degree = 47 : R2=-17330373.641059 | MSE=1299811143.779516 | var=1541278036.102192 |
s_hat=779886686267.709229 |
------------------------------------------------------------
>>> Degree = 48 : R2=-8005956906.307234 | MSE=600462033872.141479 | var=714835754609.692505 |
s_hat=360277220323285.000000 |
------------------------------------------------------------
>>> Degree = 49 : R2=-481974.451511 | MSE=36149077.898088 | var=43206069.200902 |
s_hat=21689446738.852951 |
------------------------------------------------------------
>>> Degree = 50 : R2=-7922204.716636 | MSE=594180534.874360 | var=713016641.849233 |
s_hat=356508320924.616333 |
------------------------------------------------------------
>>> Degree = 51 : R2=-286641843.986604 | MSE=21498684945.009323 | var=25902030054.228050 |
s_hat=12899210967005.568359 |
------------------------------------------------------------
>>> Degree = 52 : R2=-81616177.213821 | MSE=6121368992.432579 | var=7404881845.684588 |
s_hat=3672821395459.555664 |
------------------------------------------------------------
>>> Degree = 53 : R2=-1080988826.763225 | MSE=81076223320.578323 | var=98473145733.495972 |
s_hat=48645733992347.007812 |
------------------------------------------------------------
>>> Degree = 54 : R2=-9608799.696085 | MSE=720678374.346155 | var=878876066.275797 |
s_hat=432407024607.692017 |
------------------------------------------------------------
>>> Degree = 55 : R2=-468810769.945194 | MSE=35161701753.101250 | var=43055145003.797455 |
s_hat=21097021051860.753906 |
------------------------------------------------------------
>>> Degree = 56 : R2=-3283779.600339 | MSE=246289806.565106 | var=302815335.940704 |
s_hat=147773883939.063416 |
------------------------------------------------------------
>>> Degree = 57 : R2=-1877716207.435246 | MSE=140832296077.264587 | var=173867032194.153595 |
s_hat=84499377646358.640625 |
------------------------------------------------------------
>>> Degree = 58 : R2=-3000391455.404192 | MSE=225035080401.278961 | var=278969107935.470093 |
s_hat=135021048240767.515625 |
------------------------------------------------------------
```

```
>>> Degree = 59 : R2=-226163921.374842 | MSE=16962725429.329741 | var=21115425845.638695 |
s_hat=10177635257597.851562 |
-------------------------------------------------------------
>>> Degree = 60 : R2=-90208493.547610 | MSE=6765809101.365790 | var=8457261376.707239 |
s_hat=4059485460819.474609 |
-------------------------------------------------------------
>>> Degree = 61 : R2=-40137899.711452 | MSE=3010419088.636194 | var=3778768730.505678 |
s_hat=1806251453181.714111 |
-------------------------------------------------------------
>>> Degree = 62 : R2=-24545773.619024 | MSE=1840979900.510586 | var=2320562899.803261 |
s_hat=1104587940306.352051 |
-------------------------------------------------------------
>>> Degree = 63 : R2=-4673590.712116 | MSE=350528290.051577 | var=443706696.267819 |
s_hat=210316974030.946381 |
-------------------------------------------------------------
>>> Degree = 64 : R2=-1037270.398624 | MSE=77797332.774368 | var=98894914.543688 |
s_hat=46678399664.620651 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 23
```

```
na = bestFitDegree;
nb = bestFitDegree;
nk = 1;
p = na+nb;

BestFitModel_2 = oe(data2, [na nb nk])
```

```
BestFitModel_2 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
```

```
  B(z) = -20.91 z^-1 + 48.76 z^-2 - 87.54 z^-3 + 130.1 z^-4 - 145.6 z^-5 + 144.6 z^-6 - 115 
z^-7
          + 51.89 z^-8 + 15.66 z^-9 - 84.18 z^-10 + 136.6 z^-11 - 147.4 z^-12 + 134.4 z^-13 - 
86.74 z^-14
          + 14.94 z^-15 + 49.54 z^-16 - 112.1 z^-17 + 140.3 z^-18 - 139.9 z^-19 + 118.4 z^-20
                                                            - 88.33 z^-21 + 40.16 z^-22 - 
22.71 z^-23


  F(z) = 1 - 0.9615 z^-1 + 1.453 z^-2 - 1.843 z^-3 + 1.909 z^-4 - 1.273 z^-5 + 0.6805 z^-6 + 
0.1689 z^-7
          - 0.6841 z^-8 + 1.559 z^-9 - 1.638 z^-10 + 1.528 z^-11 - 0.8783 z^-12 + 0.292 z^-13
          + 0.548 z^-14 - 1.165 z^-15 + 1.693 z^-16 - 1.529 z^-17 + 1.226 z^-18 - 1.218 z^-19
                                          + 0.7092 z^-20 - 0.2412 z^-21 + 0.2391 z^-22 - 
0.2305 z^-23

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=23   nf=23   nk=1
   Number of free coefficients: 46
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data2".
Fit to estimation data: 43.31%
FPE: 28.11, MSE: 24.1
```

```
BestFit_y_hat_2 = lsim(BestFitModel_2, u2_val, t_val);
% [oe_BestFit_r2, oe_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance 
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n", 
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 2
```

```
na = minVarIndex;
nb = minVarIndex;
nk = 1;
p = na+nb;
```

```
oe_VarModel_2 = oe(data2, [na nb nk])
```

```
oe_VarModel_2 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -0.4374 z^-1 + 1.703 z^-2

  F(z) = 1 - 1.639 z^-1 + 0.8849 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=2   nf=2   nk=1
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data2".
Fit to estimation data: 42.57%
FPE: 25.07, MSE: 24.74
```

```
Var_y_hat_2 = lsim(oe_VarModel_2, u2_val, t_val);
% [oe_Var_r2, oe_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method===================\n")
```

```
===============Degree Extraction | AIC Method===================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```
na = minAICIndex;
nb = minAICIndex;
nk = 1;
p = na+nb;
```

```
oe_AICModel_2 = oe(data2, [na nb nk])
```

```
oe_AICModel_2 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -0.4374 z^-1 + 1.703 z^-2

  F(z) = 1 - 1.639 z^-1 + 0.8849 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=2   nf=2   nk=1
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data2".
Fit to estimation data: 42.57%
FPE: 25.07, MSE: 24.74
```

```
AIC_y_hat_2 = lsim(oe_AICModel_2, u2_val, t_val);
% [oe_AIC_r2, oe_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", winScore)
```

```
>>> The F test is suggesting the best model with the m=1.25 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 2
```

```


na = winner;
nb = winner;
nk = 1;
p = na+nb;

oe_FTestModel_2 = oe(data2, [na nb nk])
```

```
oe_FTestModel_2 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -0.4374 z^-1 + 1.703 z^-2

  F(z) = 1 - 1.639 z^-1 + 0.8849 z^-2

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=2   nf=2   nk=1
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data2".
Fit to estimation data: 42.57%
FPE: 25.07, MSE: 24.74
```

```
FTest_y_hat_2 = lsim(oe_FTestModel_2, u2_val, t_val);
% [oe_FTest_r2, oe_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```



%%

[oe_BestFit_r2_2, oe_BestFit_mse_2] = rSQR(y2_val, BestFit_y_hat_2);
[oe_Var_r2_2, oe_Var_mse_2] = rSQR(y2_val, Var_y_hat_2);
[oe_AIC_r2_2, oe_AIC_mse_2] = rSQR(y2_val, AIC_y_hat_2);
[oe_FTest_r2_2, oe_FTest_mse_2] = rSQR(y2_val, FTest_y_hat_2);

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_BestFit_r2_2,
oe_BestFit_mse_2)
```

```
    R2 value : 0.6609    | MSE : 29.9673
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Var_r2_2, oe_Var_mse_2)
```

```
    R2 value : 0.7021    | MSE : 26.3236
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Cov_r2, oe_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_AIC_r2_2, oe_AIC_mse_2)
```

```
    R2 value : 0.7021    | MSE : 26.3236
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_FTest_r2_2, oe_FTest_mse_2)
```

```
    R2 value : 0.7021    | MSE : 26.3236
```

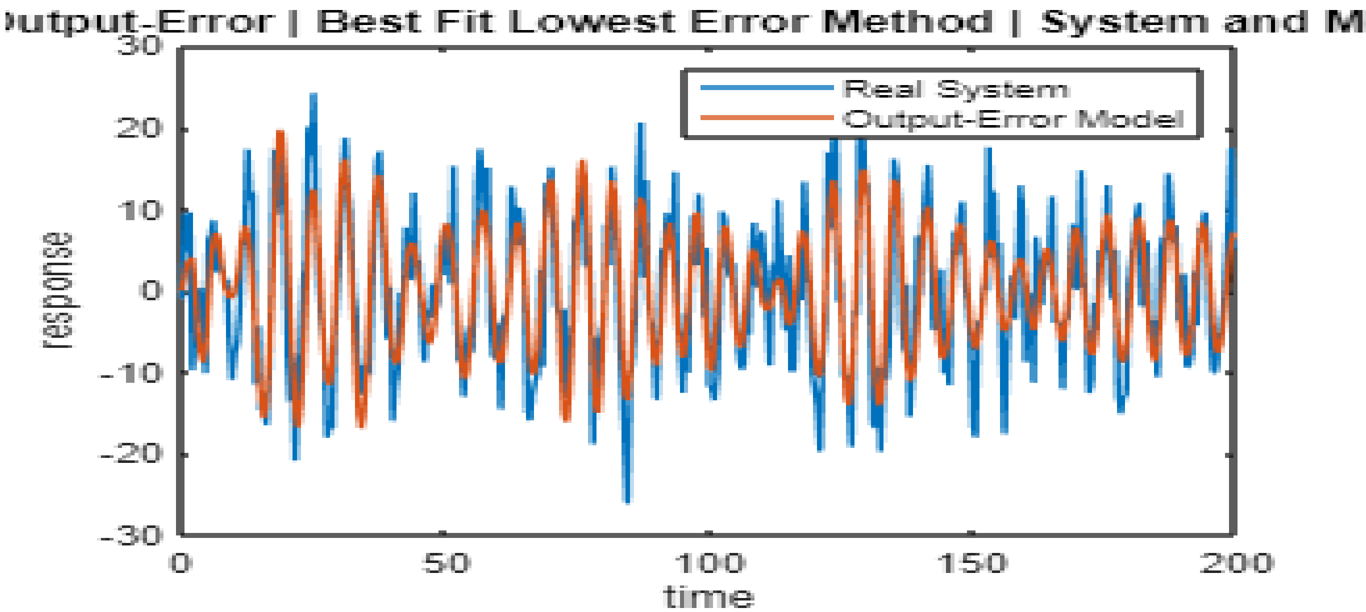
```
fprintf("---------------------------------------------------------------\n")
```

```
----------------------------------------------------------------------
```

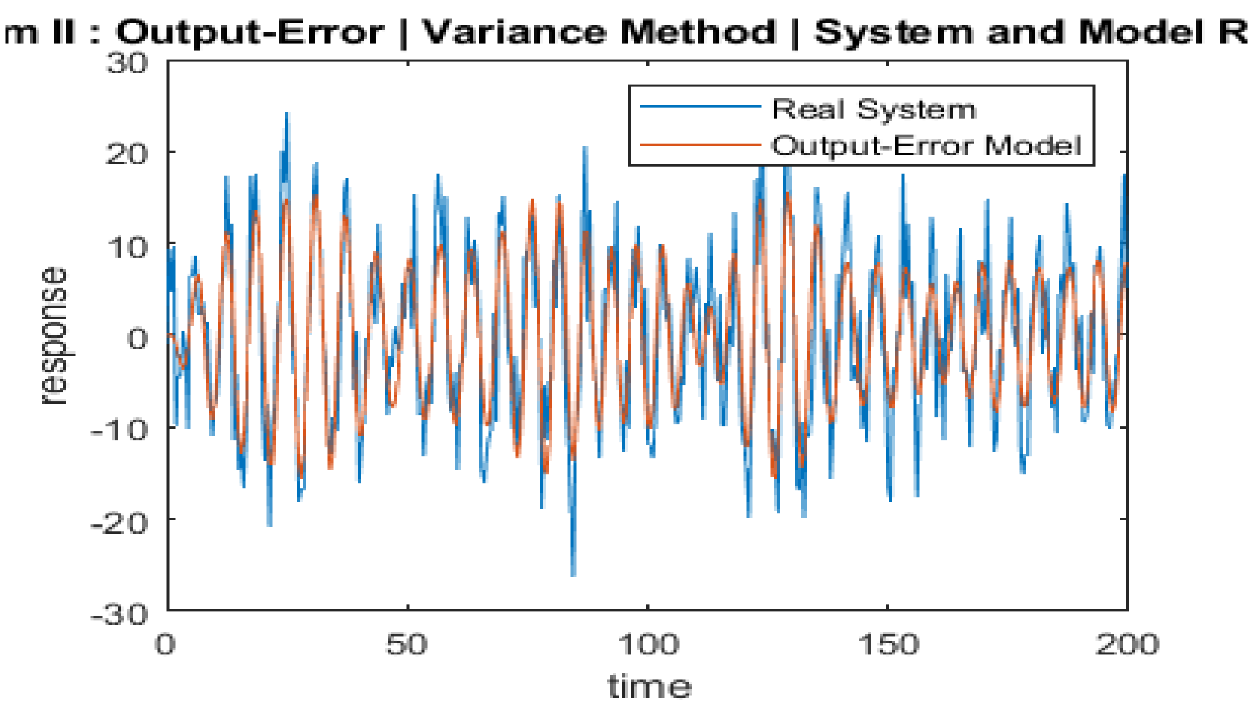
```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

```
======================================================================
```

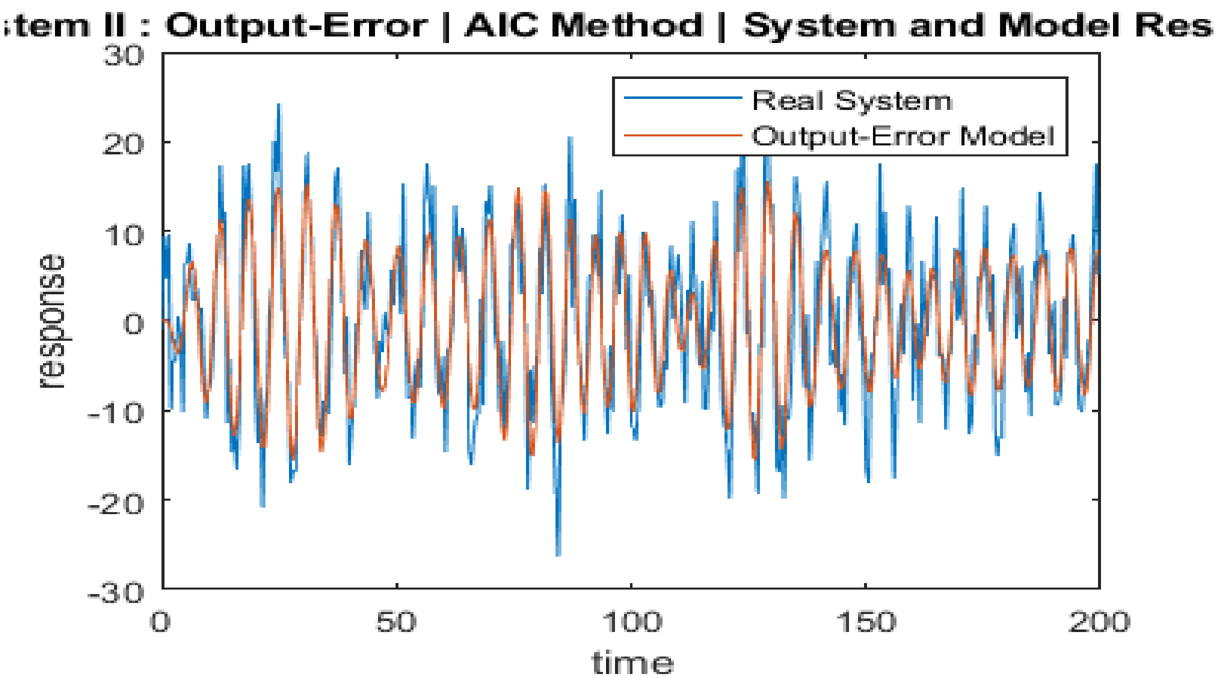
```
%%

oe_BestFitError_2 = y2_val - BestFit_y_hat_2;
oe_VarError_2 = y2_val - Var_y_hat_2;
oe_AICError_2 = y2_val - AIC_y_hat_2;
oe_FTestError_2 = y2_val - FTest_y_hat_2;

for k=0:N_val-1
    oe_BestFit_Ree_2(k+1,1) = AutoCorrelate(oe_BestFitError_2, k);
    oe_Var_Ree_2(k+1,1) = AutoCorrelate(oe_VarError_2, k);
    oe_AIC_Ree_2(k+1,1) = AutoCorrelate(oe_AICError_2, k);
    oe_FTest_Ree_2(k+1,1) = AutoCorrelate(oe_FTestError_2, k);
end

for k=0:N_val-1
    oe_BestFit_Rue_2(k+1,1) = CrossCorrelate(u2_val, oe_BestFitError_2, k);
    oe_Var_Rue_2(k+1,1) = CrossCorrelate(u2_val, oe_VarError_2, k);
    oe_AIC_Rue_2(k+1,1) = CrossCorrelate(u2_val, oe_AICError_2, k);
    oe_FTest_Rue_2(k+1,1) = CrossCorrelate(u2_val, oe_FTestError_2, k);
end


%%
figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hat_2)
legend('Real System','Output-Error Model')
title(" System II : Output-Error | Best Fit Lowest Error Method | System and
Model Response")
xlabel("time")
ylabel("response")
```



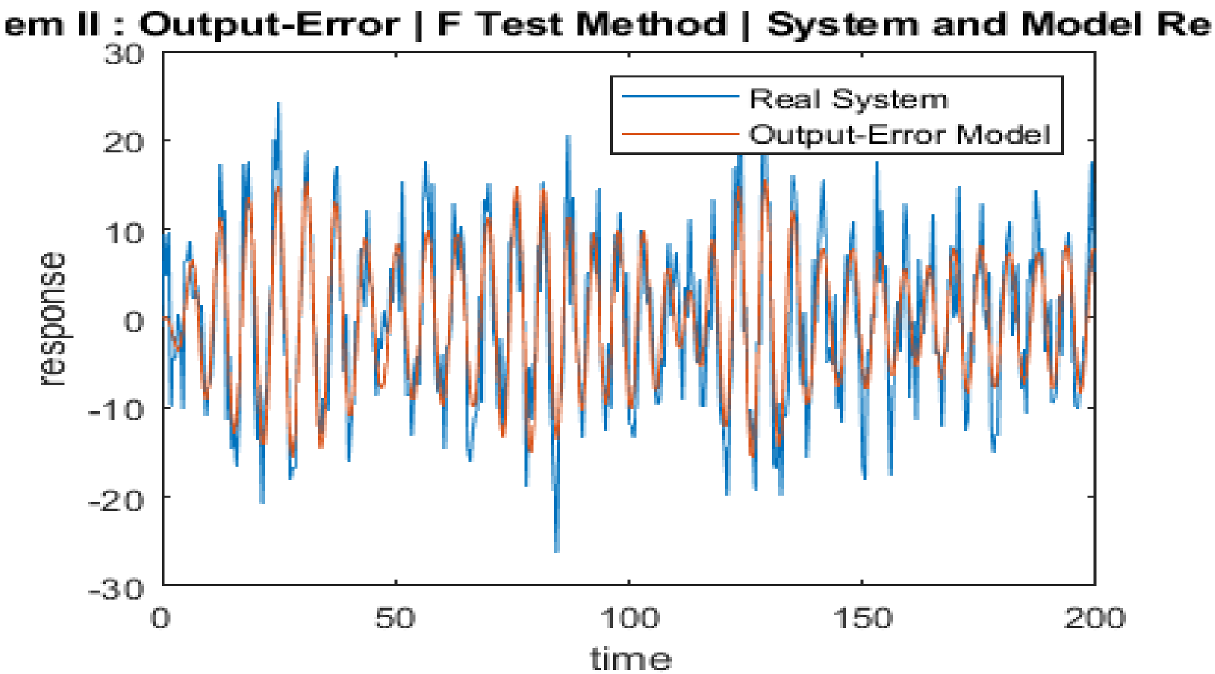

```
figure(2)
plot(t_val,y2_val,t_val,Var_y_hat_2)
legend('Real System','Output-Error Model')
title(" System II : Output-Error | Variance Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



```
figure(3)
plot(t_val,y2_val,t_val,AIC_y_hat_2)
legend('Real System','Output-Error Model')
title(" System II : Output-Error | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

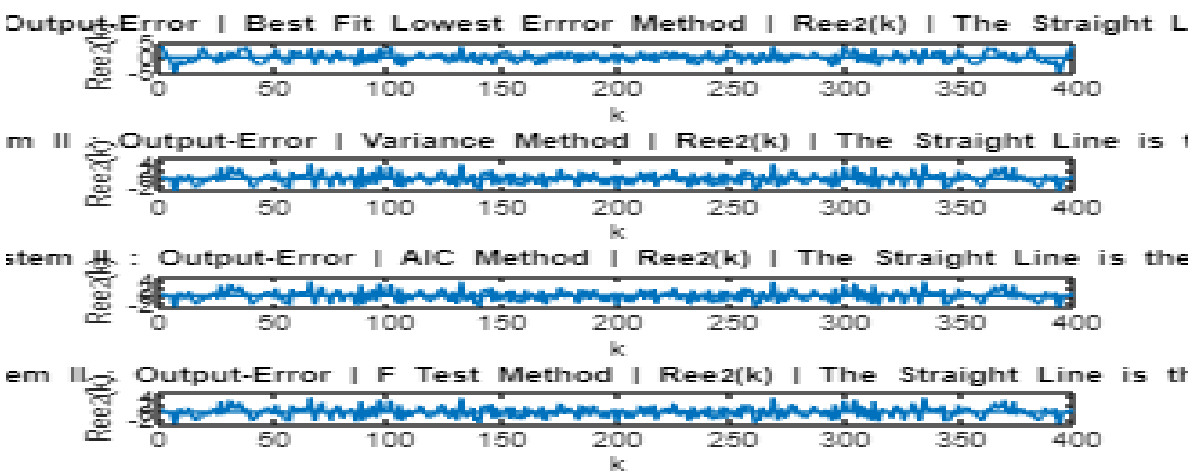



```
figure(4)
plot(t_val,y2_val,t_val,FTest_y_hat_2)
legend('Real System','Output-Error Model')
title(" System II : Output-Error | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



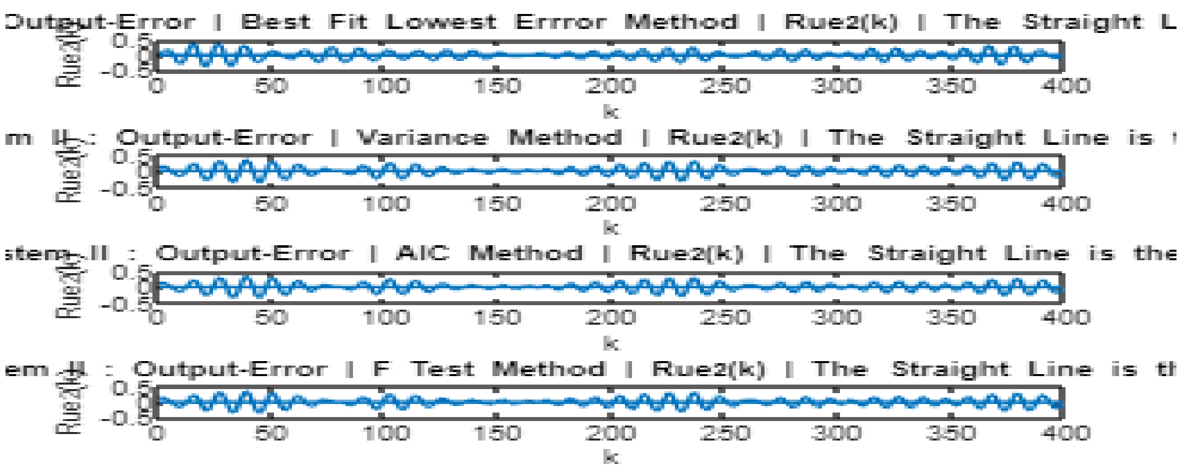

```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,oe_BestFit_Ree_2(2:end), 1:N_val-1,
mean(oe_BestFit_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Output-Error | Best Fit Lowest Errror Method | Ree_2(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,2)
plot(1:N_val-1,oe_Var_Ree_2(2:end), 1:N_val-1,
mean(oe_Var_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Output-Error | Variance Method | Ree_2(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree_2(k)")

subplot(4,1,3)
plot(1:N_val-1,oe_AIC_Ree_2(2:end), 1:N_val-1,
mean(oe_AIC_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" System II : Output-Error | AIC Method | Ree_2(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_2(k)")
```

```
 subplot(4,1,4)
 plot(1:N_val-1,oe_FTest_Ree_2(2:end), 1:N_val-1,
mean(oe_FTest_Ree_2(2:end))*ones(length(1:N_val-1)))
 title(" System II : Output-Error | F Test Method | Ree_2(k) | The Straight Line
is the Mean")
 xlabel("k")
 ylabel("Ree_2(k)")
```




```
 %%

 figure(6)
 subplot(4,1,1)
 plot(1:N_val-1,oe_BestFit_Rue_2(2:end), 1:N_val-1,
mean(oe_BestFit_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II : Output-Error | Best Fit Lowest Errror Method | Rue_2(k) |
The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(4,1,2)
 plot(1:N_val-1,oe_Var_Rue_2(2:end), 1:N_val-1,
mean(oe_Var_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II : Output-Error | Variance Method | Rue_2(k) | The Straight
Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(4,1,3)
```

```
 plot(1:N_val-1,oe_AIC_Rue_2(2:end), 1:N_val-1,
mean(oe_AIC_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II : Output-Error | AIC Method | Rue_2(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(4,1,4)
 plot(1:N_val-1,oe_FTest_Rue_2(2:end), 1:N_val-1,
mean(oe_FTest_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" System II : Output-Error | F Test Method | Rue_2(k) | The Straight Line
is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")
```



Q2 - part d | ARMAX

```
clc; clear;
%%

load HW5_question2


u3 = Z3.u;
y3 = Z3.y;

u3_val = u3;%(601:end);
y3_val = y3;%(601:end);

u3 = u3(1:600);
y3 = y3(1:600);

%%

% System Z3 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System III Identification Begins:------------------------------
\n")
```

```
>>> System III Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u3)*Ts-Ts;
t_val = 0:Ts:length(u3_val)*Ts-Ts;
N = length(y3);
N_val = length(y3_val);

data3 = iddata(y3,u3,Ts);


%%

fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
```

```
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nk = 1;
    p = na+nb+nc;

    try
        sys = armax(data3, [na nb nc nk]);
        armax_y_hat_3 = lsim(sys, u3, t);
    catch
        break
    end

    [r2_armax, mse_armax] = rSQR(y3, armax_y_hat_3);

    error = y3 - armax_y_hat_3;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_armax, mse_armax, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_armax];
    MSEs = [MSEs; mse_armax];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
```

```
end
```

```
>>> Degree = 1 : R2=-0.018452 | MSE=142.112996 | var=142.827132 | s_hat=85267.797605 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.618455 | MSE=53.240136 | var=53.777915 | s_hat=31944.081791 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.997544 | MSE=0.342692 | var=0.347911 | s_hat=205.615249 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.997525 | MSE=0.345340 | var=0.352388 | s_hat=207.204135 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.997550 | MSE=0.341931 | var=0.350699 | s_hat=205.158819 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.997525 | MSE=0.345310 | var=0.355989 | s_hat=207.185720 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.997562 | MSE=0.340202 | var=0.352541 | s_hat=204.121001 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.997547 | MSE=0.342297 | var=0.356559 | s_hat=205.377944 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.997559 | MSE=0.340617 | var=0.356667 | s_hat=204.370013 |
-------------------------------------------------------------
>>> Degree = 10 : R2=-
125826009174572493536768118134755543170640556739539718462485297386516870056911155938693256083981303081395638203469706268001320304640.000000 |
MSE=17557543286192878303977126696753971652407381690920112331606633980560099215254815793885422099465405215025172511575615599875553945976832.000000 |
var=18481624511781976498468450057307688690466042198549780474032427003276792134823285606267235532115826770944118350965226681020532737966080.000000 |
s_hat=10534525971715726356997633668879208934151504545072629084738358687508490015477576564143157302475516304647869509411988309042532905436315648.000000 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.997579 | MSE=0.337828 | var=0.357489 | s_hat=202.696513 |
-------------------------------------------------------------
>>> Degree = 12 : R2=0.997573 | MSE=0.338651 | var=0.360267 | s_hat=203.190541 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.997609 | MSE=0.333672 | var=0.356869 | s_hat=200.203345 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.997562 | MSE=0.340247 | var=0.365857 | s_hat=204.148079 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.997587 | MSE=0.336699 | var=0.363998 | s_hat=202.019156 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.997573 | MSE=0.338704 | var=0.368157 | s_hat=203.222405 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.997380 | MSE=0.365555 | var=0.399513 | s_hat=219.332787 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.997587 | MSE=0.336664 | var=0.369960 | s_hat=201.998141 |
-------------------------------------------------------------
>>> Degree = 19 : R2=-1648593344102371115963756052480.000000 |
MSE=230041858517882968689305202982912.000000 | var=254189898914787847332373989425152.000000 |
s_hat=138025115110729804560243590078398464.000000 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.997657 | MSE=0.326885 | var=0.363205 | s_hat=196.130925 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.997632 | MSE=0.330448 | var=0.369215 | s_hat=198.268632 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.997591 | MSE=0.336155 | var=0.377702 | s_hat=201.692817 |
-------------------------------------------------------------
```

```
>>> Degree = 23 : R2=0.997681 | MSE=0.323529 | var=0.365569 | s_hat=194.117158 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.997635 | MSE=0.330062 | var=0.375070 | s_hat=198.037106 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.997566 | MSE=0.339607 | var=0.388122 | s_hat=203.764018 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.997559 | MSE=0.340676 | var=0.391581 | s_hat=204.405480 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.997616 | MSE=0.332600 | var=0.384509 | s_hat=199.559918 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.997619 | MSE=0.332271 | var=0.386361 | s_hat=199.362358 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.997649 | MSE=0.328021 | var=0.383650 | s_hat=196.812558 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.997669 | MSE=0.325252 | var=0.382650 | s_hat=195.151366 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.997638 | MSE=0.329624 | var=0.390088 | s_hat=197.774532 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.997675 | MSE=0.324456 | var=0.386258 | s_hat=194.673836 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.997702 | MSE=0.320615 | var=0.383970 | s_hat=192.369125 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.997581 | MSE=0.337493 | var=0.406618 | s_hat=202.495879 |
------------------------------------------------------------
>>> Degree = 35 : R2=-1.550069 | MSE=355.832210 | var=431.311770 | s_hat=213499.325965 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.997551 | MSE=0.341680 | var=0.416683 | s_hat=205.008258 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.997603 | MSE=0.334407 | var=0.410316 | s_hat=200.644403 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.997849 | MSE=0.300121 | var=0.370519 | s_hat=180.072353 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.997732 | MSE=0.316442 | var=0.393096 | s_hat=189.865234 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.997716 | MSE=0.318701 | var=0.398376 | s_hat=191.220378 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.997692 | MSE=0.322026 | var=0.405064 | s_hat=193.215528 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.997999 | MSE=0.279161 | var=0.353368 | s_hat=167.496322 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.997757 | MSE=0.313006 | var=0.398733 | s_hat=187.803468 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.997663 | MSE=0.326079 | var=0.418050 | s_hat=195.647216 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.997479 | MSE=0.351783 | var=0.453914 | s_hat=211.069819 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.997682 | MSE=0.323393 | var=0.419991 | s_hat=194.036053 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.997703 | MSE=0.320487 | var=0.418937 | s_hat=192.292055 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.997697 | MSE=0.321325 | var=0.422796 | s_hat=192.795169 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.997728 | MSE=0.317034 | var=0.419912 | s_hat=190.220108 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.997869 | MSE=0.297308 | var=0.396411 | s_hat=178.384992 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.997976 | MSE=0.282473 | var=0.379158 | s_hat=169.483588 |
```

```
-------------------------------------------------------------
>>> Degree = 52 : R2=0.997886 | MSE=0.294981 | var=0.398623 | s_hat=176.988718 |
-------------------------------------------------------------
>>> Degree = 53 : R2=0.998073 | MSE=0.268944 | var=0.365910 | s_hat=161.366418 |
-------------------------------------------------------------
>>> Degree = 54 : R2=-36862778430977984262815048794112OO.000000 |
MSE=5143768225639841015689469786010419 20.000000 |
var=7046257843342248694459861792939048 96.000000 |
s_hat=308626093538390468024917911465092972544.000000 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.996363 | MSE=0.507536 | var=0.700050 | s_hat=304.521774 |
-------------------------------------------------------------
>>> Degree = 56 : R2=-580458794.734296 | MSE=80996214525.222488 | var=112494742396.142349 |
s_hat=48597728715133.492188 |
-------------------------------------------------------------
>>> Degree = 57 : R2=-
562708398723934844133847444008261684793633165809202048881321811835294803375104295787830033897 3
78651235209217653597374481109269065858176516096 0.000000 |
MSE=785193548846681373598818242748894109636261970346387969001817639447049896934495660703666041
90442456425790891620385752852764549490248531978695475 2.000000 |
var=109817279558976437094991907689644299696613659534444475619855971896785853366447234344862770
98245815723867262238097772108094088732227176981594112 00.000000 |
s_hat=471116129308008917043725698696634261658632351001136846521732608758085206371763347012013 4
366311131103938879542109903833791928641485108481428071383 04.000000 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.997909 | MSE=0.291763 | var=0.410934 | s_hat=175.058053 |
-------------------------------------------------------------
>>> Degree = 59 : R2=-283758802.210482 | MSE=39595211696.606354 | var=56163420846.250130 |
s_hat=23757127017963.804688 |
-------------------------------------------------------------
>>> Degree = 60 : R2=-29.955011 | MSE=4319.408661 | var=6170.583801 | s_hat=2591645.196465 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.998127 | MSE=0.261402 | var=0.376118 | s_hat=156.841228 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.997952 | MSE=0.285832 | var=0.414249 | s_hat=171.498993 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.997982 | MSE=0.281581 | var=0.411067 | s_hat=168.948393 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.997964 | MSE=0.284077 | var=0.417761 | s_hat=170.446293 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.998124 | MSE=0.261766 | var=0.387801 | s_hat=157.059576 |
-------------------------------------------------------------
>>> Degree = 66 : R2=0.998143 | MSE=0.259171 | var=0.386822 | s_hat=155.502580 |
-------------------------------------------------------------
>>> Degree = 67 : R2=0.998289 | MSE=0.238795 | var=0.359090 | s_hat=143.276737 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.997888 | MSE=0.294761 | var=0.446607 | s_hat=176.856450 |
-------------------------------------------------------------
>>> Degree = 69 : R2=0.997994 | MSE=0.279905 | var=0.427336 | s_hat=167.943087 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.998023 | MSE=0.275797 | var=0.424304 | s_hat=165.478490 |
-------------------------------------------------------------
>>> Degree = 71 : R2=-97565175149816270282569082208256.000000 |
MSE=13614075477362121066502125218955264.000000 |
var=21107093763352129116451804750544896.000000 |
s_hat=8168445286417273912726616217332219904.000000 |
-------------------------------------------------------------
```

```
>>> Degree = 72 : R2=-131601823498516.015625 | MSE=18363490408500260.000000 |
var=28692953763281652.000000 | s_hat=11018094245100154880.000000 |
-------------------------------------------------------------
>>> Degree = 73 : R2=0.994995 | MSE=0.698335 | var=1.099740 | s_hat=419.001073 |
-------------------------------------------------------------
>>> Degree = 74 : R2=0.998163 | MSE=0.256295 | var=0.406817 | s_hat=153.776940 |
-------------------------------------------------------------
Warning: The "C" polynomial is unstable and cannot be automatically stabilized to meet the
prescribed constraints. This can cause the estimation to fail. Make sure that the starting
polynomial value is stable and within the desired constraints.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 67
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
p = na+nb+nc;

BestFitModel_3 = armax(data3, [na nb nc 1])
```

```
BestFitModel_3 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

  A(z) = 1 - 0.2286 z^-1 + 0.1664 z^-2 + 0.4388 z^-3 - 0.263 z^-4 + 0.275 z^-5 + 0.1243 z^-6
          - 0.1869 z^-7 + 0.4421 z^-8 - 0.04054 z^-9 - 0.1755 z^-10 - 0.08172 z^-11 - 0.103
z^-12
          - 0.04915 z^-13 + 0.2309 z^-14 + 0.04809 z^-15 + 0.1812 z^-16 + 0.1346 z^-17 -
0.1456 z^-18
          + 0.1887 z^-19 + 0.1068 z^-20 - 0.2874 z^-21 + 0.2651 z^-22 - 0.0965 z^-23 + 0.01195
z^-24
```

+ 0.2021 z^-25 - 0.1148 z^-26 - 0.06295 z^-27 + 0.2156 z^-28 - 0.1203 z^-29 - 0.1921 z^-30
+ 0.1513 z^-31 - 0.1333 z^-32 - 0.03483 z^-33 - 0.03314 z^-34 - 0.2254 z^-35 - 0.01575 z^-36
- 0.08288 z^-37 - 0.0388 z^-38 + 0.1459 z^-39 + 0.0771 z^-40 + 0.1289 z^-41 + 0.08523 z^-42
- 0.09342 z^-43 + 0.1107 z^-44 + 0.008864 z^-45 - 0.07437 z^-46 + 0.08728 z^-47 + 0.1011 z^-48
- 0.1691 z^-49 + 0.05281 z^-50 + 0.02465 z^-51 - 0.1092 z^-52 + 0.03157 z^-53 - 0.02118 z^-54
+ 0.01324 z^-55 + 0.04688 z^-56 + 0.1288 z^-57 - 0.01644 z^-58 - 0.1166 z^-59 + 0.02446 z^-60
+ 0.05346 z^-61 - 0.1598 z^-62 + 0.04636 z^-63 + 0.05164 z^-64 - 0.03959 z^-65 + 0.08125 z^-66
- 0.09914 z^-67

B(z) = 0.618 z^-1 + 1.709 z^-2 + 2.288 z^-3 + 2.714 z^-4 + 2.531 z^-5 + 1.791 z^-6 + 0.9082 z^-7
- 0.1203 z^-8 - 0.7244 z^-9 - 0.5661 z^-10 - 0.1863 z^-11 + 0.3619 z^-12 + 0.7674 z^-13
+ 1.002 z^-14 + 1.124 z^-15 + 1.124 z^-16 + 1.051 z^-17 + 1.092 z^-18 + 0.8141 z^-19
+ 0.5423 z^-20 + 0.4331 z^-21 + 0.2189 z^-22 + 0.1188 z^-23 + 0.1483 z^-24 + 0.1677 z^-25
+ 0.519 z^-26 + 0.7996 z^-27 + 0.851 z^-28 + 0.8538 z^-29 + 0.7178 z^-30 + 0.2603 z^-31
- 0.1654 z^-32 - 0.5667 z^-33 - 0.7702 z^-34 - 0.6472 z^-35 - 0.5861 z^-36 - 0.3344 z^-37
- 0.1109 z^-38 - 0.05547 z^-39 + 0.1895 z^-40 + 0.4225 z^-41 + 0.5797 z^-42 + 0.8695 z^-43
+ 0.7274 z^-44 + 0.4878 z^-45 + 0.2576 z^-46 - 0.2036 z^-47 - 0.3742 z^-48 - 0.424 z^-49
- 0.3104 z^-50 + 0.0566 z^-51 + 0.2053 z^-52 + 0.5099 z^-53 + 0.4794 z^-54 + 0.2978 z^-55
+ 0.1742 z^-56 - 0.08027 z^-57 - 0.2132 z^-58 - 0.07702 z^-59 + 0.05008 z^-60 + 0.1283 z^-61
+ 0.08687 z^-62 + 0.1843 z^-63 - 0.05277 z^-64 - 0.03164 z^-65 - 0.1458 z^-66 - 0.1628 z^-67

C(z) = 1 - 0.1944 z^-1 + 0.1924 z^-2 + 0.1434 z^-3 - 0.4211 z^-4 + 0.07554 z^-5 - 0.03551 z^-6
- 0.186 z^-7 + 0.465 z^-8 + 0.1149 z^-9 - 0.1647 z^-10 - 0.04936 z^-11 - 0.2229 z^-12
- 0.3861 z^-13 + 0.3499 z^-14 - 0.09052 z^-15 + 0.09786 z^-16 + 0.3491 z^-17 - 0.318 z^-18
+ 0.1593 z^-19 + 0.01389 z^-20 - 0.4244 z^-21 + 0.2948 z^-22 - 0.2037 z^-23 + 0.07776 z^-24
+ 0.2202 z^-25 + 0.001188 z^-26 - 0.1963 z^-27 + 0.256 z^-28 - 0.2252 z^-29 - 0.3833 z^-30
+ 0.2549 z^-31 - 0.2174 z^-32 - 0.08739 z^-33 + 0.1861 z^-34 - 0.4903 z^-35 + 0.1082 z^-36
+ 0.06649 z^-37 - 0.1634 z^-38 + 0.2511 z^-39 + 0.05456 z^-40 - 0.03569 z^-41 + 0.2754 z^-42
- 0.2601 z^-43 - 0.07806 z^-44 + 0.2046 z^-45 - 0.0286 z^-46 + 0.09895 z^-47 + 0.3171 z^-48

```
          - 0.2588 z^-49 + 0.1334 z^-50 + 0.02328 z^-51 - 0.2703 z^-52 + 0.1624 z^-53 -
0.02987 z^-54
          - 0.02872 z^-55 + 0.2813 z^-56 + 0.1585 z^-57 - 0.107 z^-58 + 0.007821 z^-59 -
0.06414 z^-60
          + 0.1828 z^-61 - 0.1482 z^-62 + 0.06889 z^-63 - 0.004881 z^-64 + 0.1041 z^-65 +
0.0747 z^-66
                                                                                    -
0.2592 z^-67

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=67   nb=67   nc=67   nk=1
   Number of free coefficients: 201
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data3".
Fit to estimation data: 97.2% (prediction focus)
FPE: 0.2852, MSE: 0.1091
```

```
BestFit_y_hat_3 = lsim(BestFitModel_3, u3_val, t_val);
% [armax_BestFit_r2, armax_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 3
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
p = na+nb+nc;
```

```
armax_VarModel_3 = armax(data3, [na nb nc nk])
```

```
armax_VarModel_3 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 2.595 z^-1 + 2.542 z^-2 - 0.924 z^-3

  B(z) = 0.6315 z^-1 + 0.188 z^-2 - 0.5226 z^-3

  C(z) = 1 - 2.657 z^-1 + 2.575 z^-2 - 0.9148 z^-3

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nc=3   nk=1
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data3".
Fit to estimation data: 95.83% (prediction focus)
FPE: 0.2497, MSE: 0.2423
```

```
Var_y_hat_3 = lsim(armax_VarModel_3, u3_val, t_val);
% [armax_Var_r2, armax_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 3 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 3
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
```

```
p = na+nb+nc;

armax_AICModel_3 = armax(data3, [na nb nc nk])
```

```
armax_AICModel_3 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 2.595 z^-1 + 2.542 z^-2 - 0.924 z^-3

  B(z) = 0.6315 z^-1 + 0.188 z^-2 - 0.5226 z^-3

  C(z) = 1 - 2.657 z^-1 + 2.575 z^-2 - 0.9148 z^-3

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nc=3   nk=1
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data3".
Fit to estimation data: 95.83% (prediction focus)
FPE: 0.2497, MSE: 0.2423
```

```
AIC_y_hat_3 = lsim(armax_AICModel_3, u3_val, t_val);
% [armax_AIC_r2, armax_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method===================\n")
```

```
===============Degree Extraction | F test Method===================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
```

```
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.20 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 3
```

```

na = winner;
nb = winner;
nc = winner;
p = na+nb+nc;

armax_FTestModel_3 = armax(data3, [na nb nc nk])
```

```
armax_FTestModel_3 =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 2.595 z^-1 + 2.542 z^-2 - 0.924 z^-3

  B(z) = 0.6315 z^-1 + 0.188 z^-2 - 0.5226 z^-3

  C(z) = 1 - 2.657 z^-1 + 2.575 z^-2 - 0.9148 z^-3

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nc=3   nk=1
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data3".
Fit to estimation data: 95.83% (prediction focus)
FPE: 0.2497, MSE: 0.2423
```

```
FTest_y_hat_3 = lsim(armax_FTestModel_3, u3_val, t_val);
% [armax_FTest_r2, armax_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```


%%

[armax_BestFit_r2_3, armax_BestFit_mse_3] = rSQR(y3_val, BestFit_y_hat_3);
[armax_Var_r2_3, armax_Var_mse_3] = rSQR(y3_val, Var_y_hat_3);
```

```
[armax_AIC_r2_3, armax_AIC_mse_3] = rSQR(y3_val, AIC_y_hat_3);
[armax_FTest_r2_3, armax_FTest_mse_3] = rSQR(y3_val, FTest_y_hat_3);

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_BestFit_r2_3,
armax_BestFit_mse_3)
```

```
    R2 value : 0.9967    | MSE : 0.4220
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Var_r2_3,
armax_Var_mse_3)
```

```
    R2 value : 0.9964    | MSE : 0.4684
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Cov_r2, armax_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_AIC_r2_3,
armax_AIC_mse_3)
```

```
    R2 value : 0.9964    | MSE : 0.4684
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_FTest_r2_3,
armax_FTest_mse_3)
```

```
    R2 value : 0.9964    | MSE : 0.4684
```

```
fprintf("---------------------------------------------------------------\n")
```

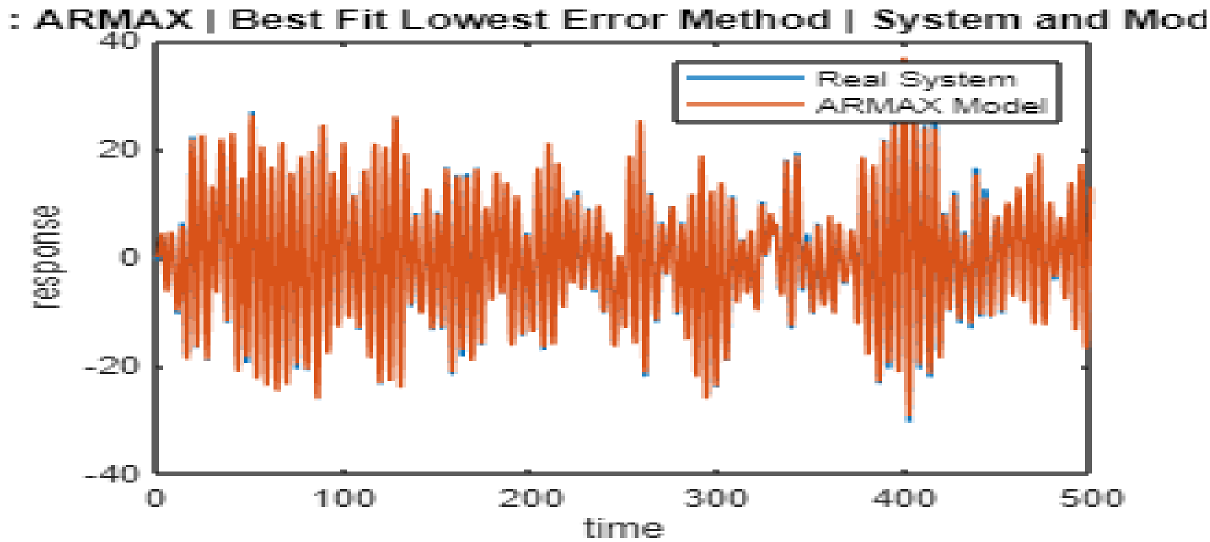
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

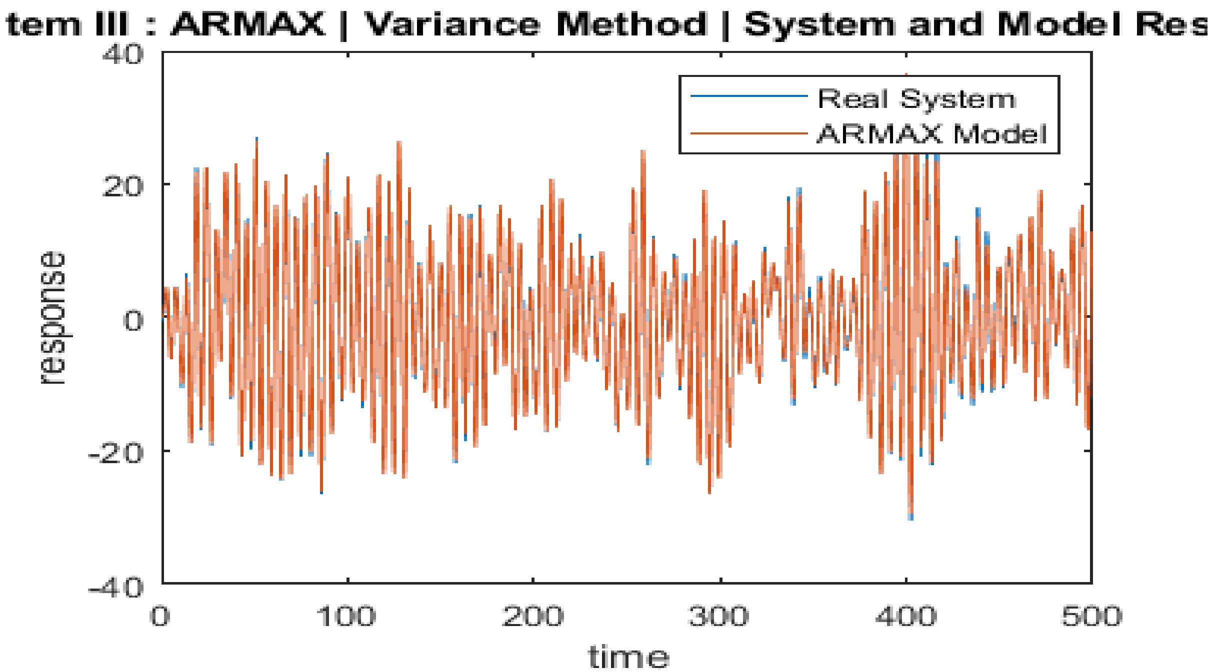
```
===============================================================
```

```


%%

armax_BestFitError_3 = y3_val - BestFit_y_hat_3;
armax_VarError_3 = y3_val - Var_y_hat_3;
armax_AICError_3 = y3_val - AIC_y_hat_3;
armax_FTestError_3 = y3_val - FTest_y_hat_3;

for k=0:N_val-1
    armax_BestFit_Ree_3(k+1,1) = AutoCorrelate(armax_BestFitError_3, k);
    armax_Var_Ree_3(k+1,1) = AutoCorrelate(armax_VarError_3, k);
    armax_AIC_Ree_3(k+1,1) = AutoCorrelate(armax_AICError_3, k);
    armax_FTest_Ree_3(k+1,1) = AutoCorrelate(armax_FTestError_3, k);
end

for k=0:N_val-1
    armax_BestFit_Rue_3(k+1,1) = CrossCorrelate(u3_val, armax_BestFitError_3,
k);
    armax_Var_Rue_3(k+1,1) = CrossCorrelate(u3_val, armax_VarError_3, k);
    armax_AIC_Rue_3(k+1,1) = CrossCorrelate(u3_val, armax_AICError_3, k);
    armax_FTest_Rue_3(k+1,1) = CrossCorrelate(u3_val, armax_FTestError_3, k);
end



%%
figure(1)
plot(t_val,y3_val,t_val,BestFit_y_hat_3)
```

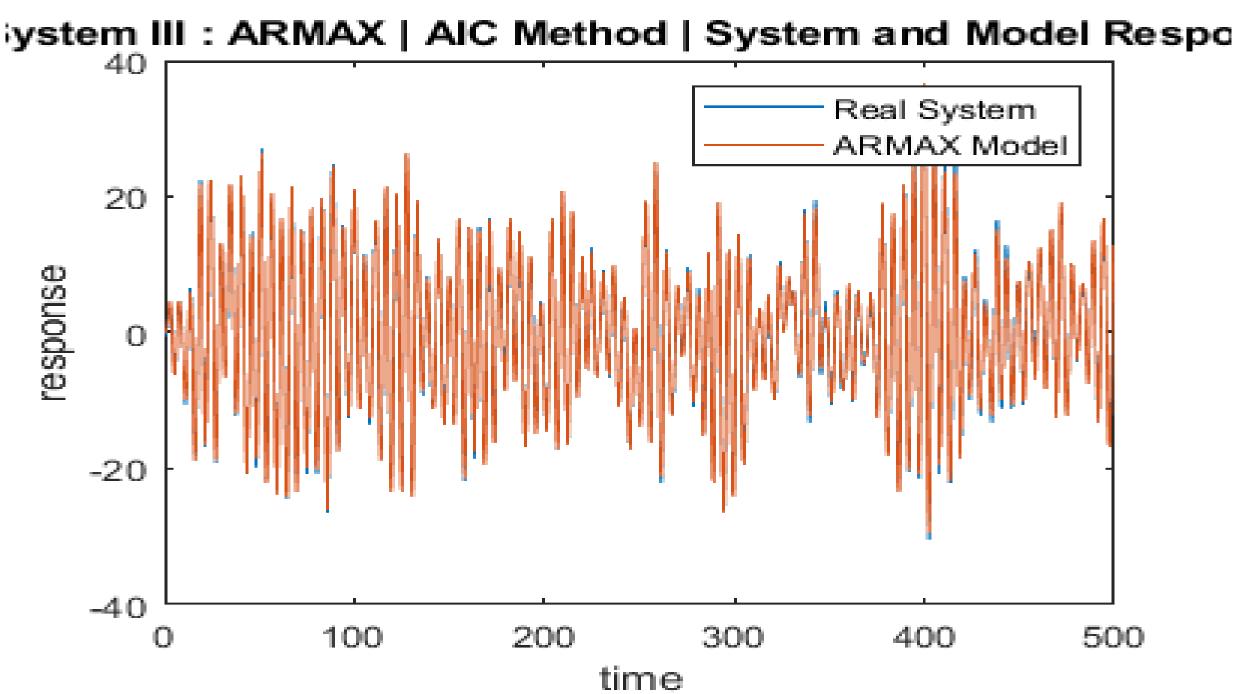
```
legend('Real System','ARMAX Model')
title(" System III : ARMAX | Best Fit Lowest Error Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



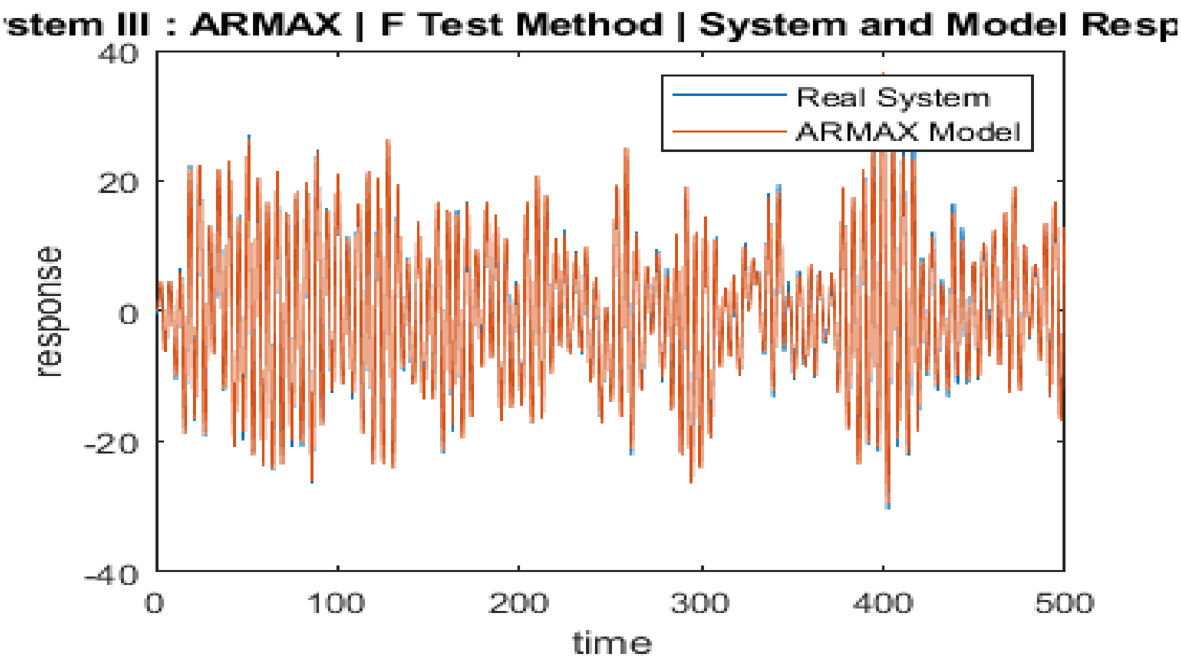

```
figure(2)
plot(t_val,y3_val,t_val,Var_y_hat_3)
legend('Real System','ARMAX Model')
title(" System III : ARMAX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(3)
plot(t_val,y3_val,t_val,AIC_y_hat_3)
legend('Real System','ARMAX Model')
title(" System III : ARMAX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

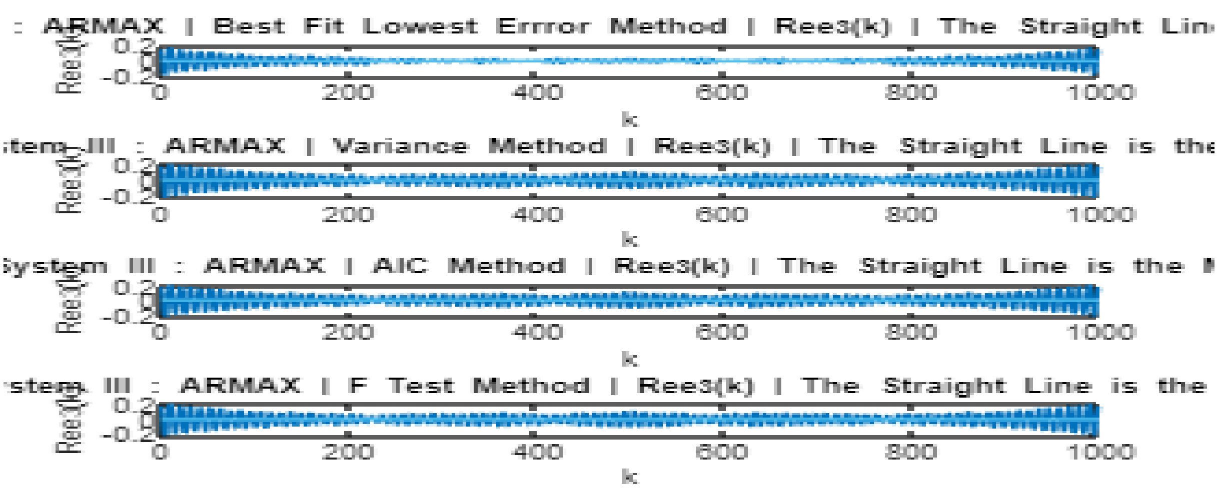


```
figure(4)
plot(t_val,y3_val,t_val,FTest_y_hat_3)
legend('Real System','ARMAX Model')
title(" System III : ARMAX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



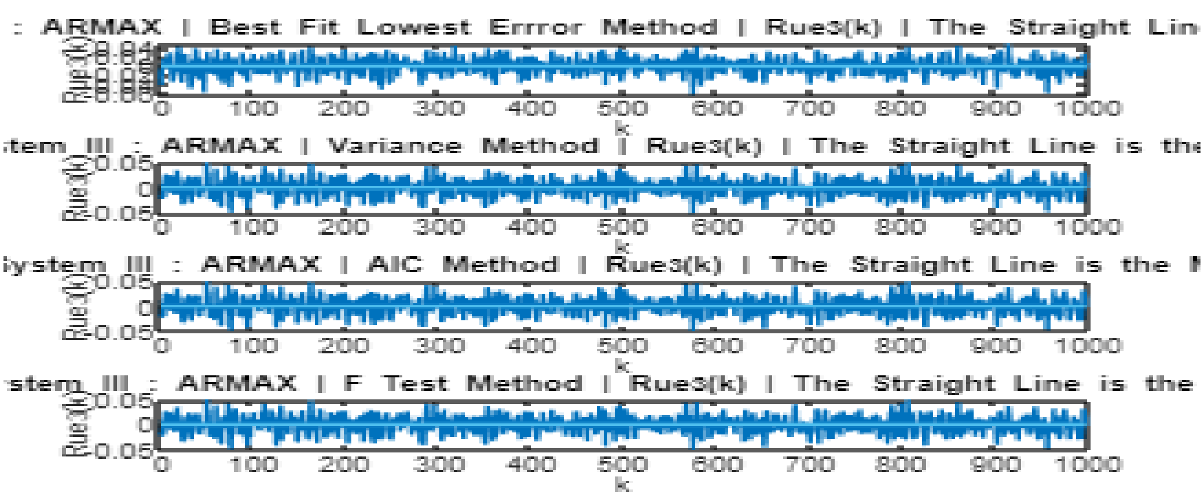

```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Ree_3(2:end), 1:N_val-1,
mean(armax_BestFit_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : ARMAX | Best Fit Lowest Errror Method | Ree_3(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_3(k)")

subplot(4,1,2)
plot(1:N_val-1,armax_Var_Ree_3(2:end), 1:N_val-1,
mean(armax_Var_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : ARMAX | Variance Method | Ree_3(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_3(k)")
```

```
subplot(4,1,3)
plot(1:N_val-1,armax_AIC_Ree_3(2:end), 1:N_val-1,
mean(armax_AIC_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : ARMAX | AIC Method | Ree_3(k) | The Straight Line is the
Mean")
xlabel("k")
ylabel("Ree_3(k)")

subplot(4,1,4)
plot(1:N_val-1,armax_FTest_Ree_3(2:end), 1:N_val-1,
mean(armax_FTest_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : ARMAX | F Test Method | Ree_3(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_3(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Rue_3(2:end), 1:N_val-1,
mean(armax_BestFit_Rue_3(2:end))*ones(length(1:N_val-1)))
title(" System III : ARMAX | Best Fit Lowest Errror Method | Rue_3(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_3(k)")

subplot(4,1,2)
```

```
 plot(1:N_val-1,armax_Var_Rue_3(2:end), 1:N_val-1,
mean(armax_Var_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARMAX | Variance Method | Rue_3(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(4,1,3)
 plot(1:N_val-1,armax_AIC_Rue_3(2:end), 1:N_val-1,
mean(armax_AIC_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARMAX | AIC Method | Rue_3(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(4,1,4)
 plot(1:N_val-1,armax_FTest_Rue_3(2:end), 1:N_val-1,
mean(armax_FTest_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARMAX | F Test Method | Rue_3(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")
```



Q2 - part d | ARX

```
clc; clear
%%

load HW5_question2

u3 = Z3.u;
y3 = Z3.y;

u3_val = u3;%(601:end);
y3_val = y3;%(601:end);

u3 = u3(1:600);
y3 = y3(1:600);

%%

% System Z3 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System III Identification Begins:------------------------------
\n")
```

```
>>> System III Identification Begins:------------------------------
```

```
%%

Ts = 0.5;
t = 0:Ts:length(u3)*Ts-Ts;
t_val = 0:Ts:length(u3_val)*Ts-Ts;
N = length(y3);
N_val = length(y3_val);
%%

fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
```

```
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    U = arx_U_builder_3(u3, y3, na, nb, nk);
    theta_hat_3 = inv(U'*U)*U'*y3;
    y_hat_3 = form_tf_lsim_2(theta_hat_3, u3, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y3, y_hat_3);

    error = y3 - y_hat_3;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_arx, mse_arx, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_arx];
    MSEs = [MSEs; mse_arx];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.037676 | MSE=134.281067 | var=134.730167 | s_hat=80568.640040 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.540160 | MSE=64.165287 | var=64.595926 | s_hat=38499.171934 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.476586 | MSE=73.036351 | var=73.774092 | s_hat=43821.810427 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.713941 | MSE=39.916142 | var=40.455549 | s_hat=23949.685132 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.893466 | MSE=14.865532 | var=15.117490 | s_hat=8919.319271 |
-------------------------------------------------------------
```

```
>>> Degree = 6 : R2=0.967564 | MSE=4.526087 | var=4.618456 | s_hat=2715.652231 |
--------------------------------------------------------------
>>> Degree = 7 : R2=0.991868 | MSE=1.134722 | var=1.161831 | s_hat=680.832959 |
--------------------------------------------------------------
>>> Degree = 8 : R2=0.996783 | MSE=0.448886 | var=0.461185 | s_hat=269.331754 |
--------------------------------------------------------------
>>> Degree = 9 : R2=0.997651 | MSE=0.327802 | var=0.337940 | s_hat=196.681234 |
--------------------------------------------------------------
>>> Degree = 10 : R2=0.997778 | MSE=0.310065 | var=0.320757 | s_hat=186.038869 |
--------------------------------------------------------------
>>> Degree = 11 : R2=0.997790 | MSE=0.308351 | var=0.320088 | s_hat=185.010861 |
--------------------------------------------------------------
>>> Degree = 12 : R2=0.997740 | MSE=0.315409 | var=0.328551 | s_hat=189.245295 |
--------------------------------------------------------------
>>> Degree = 13 : R2=0.997671 | MSE=0.324989 | var=0.339709 | s_hat=194.993197 |
--------------------------------------------------------------
>>> Degree = 14 : R2=0.997397 | MSE=0.363212 | var=0.380992 | s_hat=217.927334 |
--------------------------------------------------------------
>>> Degree = 15 : R2=0.997035 | MSE=0.413730 | var=0.435505 | s_hat=248.237757 |
--------------------------------------------------------------
>>> Degree = 16 : R2=0.997138 | MSE=0.399362 | var=0.421862 | s_hat=239.617339 |
--------------------------------------------------------------
>>> Degree = 17 : R2=0.997159 | MSE=0.396445 | var=0.420260 | s_hat=237.866893 |
--------------------------------------------------------------
>>> Degree = 18 : R2=0.997293 | MSE=0.377778 | var=0.401892 | s_hat=226.666897 |
--------------------------------------------------------------
>>> Degree = 19 : R2=0.997476 | MSE=0.352199 | var=0.376013 | s_hat=211.319345 |
--------------------------------------------------------------
>>> Degree = 20 : R2=0.997515 | MSE=0.346708 | var=0.371473 | s_hat=208.025016 |
--------------------------------------------------------------
>>> Degree = 21 : R2=0.997582 | MSE=0.337452 | var=0.362852 | s_hat=202.471239 |
--------------------------------------------------------------
>>> Degree = 22 : R2=0.997631 | MSE=0.330511 | var=0.356666 | s_hat=198.306328 |
--------------------------------------------------------------
>>> Degree = 23 : R2=0.997581 | MSE=0.337533 | var=0.365560 | s_hat=202.519980 |
--------------------------------------------------------------
>>> Degree = 24 : R2=0.997551 | MSE=0.341713 | var=0.371427 | s_hat=205.027945 |
--------------------------------------------------------------
>>> Degree = 25 : R2=0.997706 | MSE=0.320058 | var=0.349154 | s_hat=192.034698 |
--------------------------------------------------------------
>>> Degree = 26 : R2=0.997743 | MSE=0.314997 | var=0.344888 | s_hat=188.998399 |
--------------------------------------------------------------
>>> Degree = 27 : R2=0.997724 | MSE=0.317628 | var=0.349042 | s_hat=190.576809 |
--------------------------------------------------------------
>>> Degree = 28 : R2=0.997750 | MSE=0.313997 | var=0.346320 | s_hat=188.397969 |
--------------------------------------------------------------
>>> Degree = 29 : R2=0.997759 | MSE=0.312685 | var=0.346146 | s_hat=187.611233 |
--------------------------------------------------------------
>>> Degree = 30 : R2=0.997790 | MSE=0.308343 | var=0.342603 | s_hat=185.005600 |
--------------------------------------------------------------
>>> Degree = 31 : R2=0.997775 | MSE=0.310408 | var=0.346180 | s_hat=186.244879 |
--------------------------------------------------------------
>>> Degree = 32 : R2=0.997765 | MSE=0.311821 | var=0.349053 | s_hat=187.092591 |
--------------------------------------------------------------
>>> Degree = 33 : R2=0.997734 | MSE=0.316128 | var=0.355200 | s_hat=189.676936 |
--------------------------------------------------------------
>>> Degree = 34 : R2=0.997684 | MSE=0.323122 | var=0.364424 | s_hat=193.873499 |
```

```
---------------------------------------------------------------
>>> Degree = 35 : R2=0.997640 | MSE=0.329295 | var=0.372786 | s_hat=197.576820 |
---------------------------------------------------------------
>>> Degree = 36 : R2=0.997680 | MSE=0.323744 | var=0.367891 | s_hat=194.246219 |
---------------------------------------------------------------
>>> Degree = 37 : R2=0.997699 | MSE=0.321147 | var=0.366327 | s_hat=192.688172 |
---------------------------------------------------------------
>>> Degree = 38 : R2=0.997712 | MSE=0.319331 | var=0.365646 | s_hat=191.598683 |
---------------------------------------------------------------
>>> Degree = 39 : R2=0.997691 | MSE=0.322224 | var=0.370373 | s_hat=193.334604 |
---------------------------------------------------------------
>>> Degree = 40 : R2=0.997642 | MSE=0.329000 | var=0.379615 | s_hat=197.399757 |
---------------------------------------------------------------
>>> Degree = 41 : R2=0.997697 | MSE=0.321321 | var=0.372187 | s_hat=192.792744 |
---------------------------------------------------------------
>>> Degree = 42 : R2=0.997682 | MSE=0.323459 | var=0.376115 | s_hat=194.075191 |
---------------------------------------------------------------
>>> Degree = 43 : R2=0.997683 | MSE=0.323274 | var=0.377363 | s_hat=193.964556 |
---------------------------------------------------------------
>>> Degree = 44 : R2=0.997710 | MSE=0.319541 | var=0.374462 | s_hat=191.724425 |
---------------------------------------------------------------
>>> Degree = 45 : R2=0.997704 | MSE=0.320318 | var=0.376844 | s_hat=192.190693 |
---------------------------------------------------------------
>>> Degree = 46 : R2=0.997714 | MSE=0.318950 | var=0.376712 | s_hat=191.369856 |
---------------------------------------------------------------
>>> Degree = 47 : R2=0.997734 | MSE=0.316174 | var=0.374910 | s_hat=189.704334 |
---------------------------------------------------------------
>>> Degree = 48 : R2=0.997711 | MSE=0.319439 | var=0.380285 | s_hat=191.663534 |
---------------------------------------------------------------
>>> Degree = 49 : R2=0.997706 | MSE=0.320118 | var=0.382612 | s_hat=192.071052 |
---------------------------------------------------------------
>>> Degree = 50 : R2=0.997690 | MSE=0.322340 | var=0.386809 | s_hat=193.404252 |
---------------------------------------------------------------
>>> Degree = 51 : R2=0.997782 | MSE=0.309542 | var=0.372942 | s_hat=185.725116 |
---------------------------------------------------------------
>>> Degree = 52 : R2=0.997716 | MSE=0.318775 | var=0.385614 | s_hat=191.264761 |
---------------------------------------------------------------
>>> Degree = 53 : R2=0.997823 | MSE=0.303798 | var=0.368985 | s_hat=182.278733 |
---------------------------------------------------------------
>>> Degree = 54 : R2=0.997822 | MSE=0.303920 | var=0.370634 | s_hat=182.352156 |
---------------------------------------------------------------
>>> Degree = 55 : R2=0.997835 | MSE=0.302087 | var=0.369902 | s_hat=181.251995 |
---------------------------------------------------------------
>>> Degree = 56 : R2=0.997793 | MSE=0.307900 | var=0.378565 | s_hat=184.739957 |
---------------------------------------------------------------
>>> Degree = 57 : R2=0.997736 | MSE=0.315880 | var=0.389976 | s_hat=189.528268 |
---------------------------------------------------------------
>>> Degree = 58 : R2=0.997734 | MSE=0.316140 | var=0.391909 | s_hat=189.683809 |
---------------------------------------------------------------
>>> Degree = 59 : R2=0.997766 | MSE=0.311723 | var=0.388037 | s_hat=187.033784 |
---------------------------------------------------------------
>>> Degree = 60 : R2=0.997779 | MSE=0.309949 | var=0.387436 | s_hat=185.969292 |
---------------------------------------------------------------
>>> Degree = 61 : R2=0.997895 | MSE=0.293780 | var=0.368762 | s_hat=176.268239 |
---------------------------------------------------------------
>>> Degree = 62 : R2=0.997836 | MSE=0.301975 | var=0.380641 | s_hat=181.184997 |
---------------------------------------------------------------
```

```
>>> Degree = 63 : R2=0.997874 | MSE=0.296670 | var=0.375531 | s_hat=178.001929 |
---------------------------------------------------------------
>>> Degree = 64 : R2=0.997862 | MSE=0.298335 | var=0.379239 | s_hat=179.000954 |
---------------------------------------------------------------
>>> Degree = 65 : R2=0.997872 | MSE=0.296878 | var=0.378993 | s_hat=178.126711 |
---------------------------------------------------------------
>>> Degree = 66 : R2=0.997895 | MSE=0.293782 | var=0.376644 | s_hat=176.269385 |
---------------------------------------------------------------
>>> Degree = 67 : R2=0.997967 | MSE=0.283622 | var=0.365179 | s_hat=170.173254 |
---------------------------------------------------------------
>>> Degree = 68 : R2=0.997979 | MSE=0.281996 | var=0.364650 | s_hat=169.197672 |
---------------------------------------------------------------
>>> Degree = 69 : R2=0.997944 | MSE=0.286912 | var=0.372613 | s_hat=172.147173 |
---------------------------------------------------------------
>>> Degree = 70 : R2=0.997965 | MSE=0.283981 | var=0.370410 | s_hat=170.388495 |
---------------------------------------------------------------
>>> Degree = 71 : R2=0.997965 | MSE=0.284017 | var=0.372075 | s_hat=170.410209 |
---------------------------------------------------------------
>>> Degree = 72 : R2=0.997983 | MSE=0.281512 | var=0.370410 | s_hat=168.907174 |
---------------------------------------------------------------
>>> Degree = 73 : R2=0.997977 | MSE=0.282352 | var=0.373153 | s_hat=169.411395 |
---------------------------------------------------------------
>>> Degree = 74 : R2=0.998027 | MSE=0.275371 | var=0.365537 | s_hat=165.222601 |
---------------------------------------------------------------
>>> Degree = 75 : R2=0.998043 | MSE=0.273123 | var=0.364164 | s_hat=163.873715 |
---------------------------------------------------------------
>>> Degree = 76 : R2=0.998036 | MSE=0.274002 | var=0.366967 | s_hat=164.401187 |
---------------------------------------------------------------
>>> Degree = 77 : R2=0.998062 | MSE=0.270373 | var=0.363730 | s_hat=162.223776 |
---------------------------------------------------------------
>>> Degree = 78 : R2=0.998051 | MSE=0.272011 | var=0.367583 | s_hat=163.206739 |
---------------------------------------------------------------
>>> Degree = 79 : R2=0.998068 | MSE=0.269633 | var=0.366017 | s_hat=161.779721 |
---------------------------------------------------------------
>>> Degree = 80 : R2=0.998092 | MSE=0.266291 | var=0.363123 | s_hat=159.774333 |
---------------------------------------------------------------
>>> Degree = 81 : R2=0.998098 | MSE=0.265418 | var=0.363586 | s_hat=159.250706 |
---------------------------------------------------------------
>>> Degree = 82 : R2=0.998091 | MSE=0.266392 | var=0.366595 | s_hat=159.835215 |
---------------------------------------------------------------
>>> Degree = 83 : R2=0.998097 | MSE=0.265598 | var=0.367186 | s_hat=159.358933 |
---------------------------------------------------------------
>>> Degree = 84 : R2=0.998115 | MSE=0.263005 | var=0.365285 | s_hat=157.803160 |
---------------------------------------------------------------
>>> Degree = 85 : R2=0.998125 | MSE=0.261684 | var=0.365140 | s_hat=157.010275 |
---------------------------------------------------------------
>>> Degree = 86 : R2=0.998156 | MSE=0.257270 | var=0.360658 | s_hat=154.361823 |
---------------------------------------------------------------
>>> Degree = 87 : R2=0.998153 | MSE=0.257711 | var=0.362973 | s_hat=154.626413 |
---------------------------------------------------------------
>>> Degree = 88 : R2=0.998139 | MSE=0.259715 | var=0.367521 | s_hat=155.828814 |
---------------------------------------------------------------
>>> Degree = 89 : R2=0.998183 | MSE=0.253520 | var=0.360455 | s_hat=152.111982 |
---------------------------------------------------------------
>>> Degree = 90 : R2=0.998189 | MSE=0.252748 | var=0.361069 | s_hat=151.648920 |
---------------------------------------------------------------
>>> Degree = 91 : R2=0.998196 | MSE=0.251781 | var=0.361408 | s_hat=151.068345 |
```

```
------------------------------------------------------------
>>> Degree = 92 : R2=0.998187 | MSE=0.252955 | var=0.364838 | s_hat=151.772804 |
------------------------------------------------------------
>>> Degree = 93 : R2=0.998193 | MSE=0.252101 | var=0.365364 | s_hat=151.260641 |
------------------------------------------------------------
>>> Degree = 94 : R2=0.998177 | MSE=0.254381 | var=0.370458 | s_hat=152.628537 |
------------------------------------------------------------
>>> Degree = 95 : R2=0.998138 | MSE=0.259881 | var=0.380314 | s_hat=155.928879 |
------------------------------------------------------------
>>> Degree = 96 : R2=0.998133 | MSE=0.260451 | var=0.383017 | s_hat=156.270861 |
------------------------------------------------------------
>>> Degree = 97 : R2=0.998145 | MSE=0.258903 | var=0.382616 | s_hat=155.342083 |
------------------------------------------------------------
>>> Degree = 98 : R2=0.998164 | MSE=0.256245 | var=0.380563 | s_hat=153.747277 |
------------------------------------------------------------
>>> Degree = 99 : R2=0.998151 | MSE=0.257965 | var=0.385022 | s_hat=154.778924 |
------------------------------------------------------------
>>> Degree = 100 : R2=0.998206 | MSE=0.250343 | var=0.375514 | s_hat=150.205615 |
------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 100
```

```
na = bestFitDegree;
nb = bestFitDegree;
p = na+nb;

BestFitU = arx_U_builder_3(u3, y3, na, nb, nk);
BestFitModel_3 = inv(BestFitU'*BestFitU)*BestFitU'*y3;
BestFit_y_hat_3 = form_tf_lsim_2(BestFitModel_3, u3_val, t_val, na, Ts);
```

```
G =
```

```
     0.6068 z^-1 + 1.827 z^-2 + 2.583 z^-3 + 2.919 z^-4 + 2.768 z^-5 + 2.247 z^-6 + 1.449 z^-7

             + 0.6307 z^-8 - 0.02605 z^-9 - 0.4093 z^-10 - 0.4203 z^-11 - 0.05112 z^-12 +
0.5525 z^-13

             + 1.381 z^-14 + 2.126 z^-15 + 2.472 z^-16 + 2.478 z^-17 + 2.146 z^-18 + 1.479 z^-
19

             + 0.7155 z^-20 + 0.1303 z^-21 - 0.147 z^-22 - 0.06341 z^-23 + 0.2501 z^-24 +
0.7939 z^-25

             + 1.412 z^-26 + 1.904 z^-27 + 2.099 z^-28 + 1.981 z^-29 + 1.588 z^-30 + 0.9844 z^

             -31 + 0.4803 z^-32 + 0.09289 z^-33 - 0.1097 z^-34 + 0.03826 z^-35 + 0.2785 z^-36

             + 0.7435 z^-37 + 1.063 z^-38 + 1.228 z^-39 + 1.198 z^-40 + 1.067 z^-41 + 0.8362
z^-42

             + 0.6481 z^-43 + 0.4012 z^-44 + 0.379 z^-45 + 0.4088 z^-46 + 0.4805 z^-47 +
0.6543 z^-48

             + 0.6434 z^-49 + 0.6225 z^-50 + 0.5791 z^-51 + 0.4142 z^-52 + 0.5133 z^-53 +
0.3889 z^-54

             + 0.3614 z^-55 + 0.4293 z^-56 + 0.4172 z^-57 + 0.4285 z^-58 + 0.4588 z^-59 +
0.5052 z^-60

             + 0.4109 z^-61 - 0.02349 z^-62 - 0.2494 z^-63 - 0.5483 z^-64 - 0.6198 z^-65 -
0.5798 z^-66

             - 0.4217 z^-67 - 0.1524 z^-68 + 0.2194 z^-69 + 0.4523 z^-70 + 0.6359 z^-71 +
0.608 z^-72

             + 0.4812 z^-73 + 0.258 z^-74 + 0.00496 z^-75 - 0.2148 z^-76 - 0.3892 z^-77 -
0.4615 z^-78

             - 0.4549 z^-79 - 0.3205 z^-80 + 0.02942 z^-81 + 0.1035 z^-82 + 0.2491 z^-83 +
0.2181 z^-84

             + 0.1139 z^-85 + 0.01672 z^-86 - 0.1248 z^-87 - 0.2171 z^-88 - 0.3084 z^-89 -
0.3297 z^-90

             - 0.2284 z^-91 - 0.2999 z^-92 - 0.1053 z^-93 + 0.02004 z^-94 + 0.08949 z^-95 +
0.1502 z^-96

                                        + 0.1023 z^-97 - 0.07166 z^-98 - 0.03612 z^-99 -
0.07509 z^-100

  ---------------------------------------------------------------------------------------------
--------------

  1 - 0.03175 z^-1 + 0.06354 z^-2 + 0.2633 z^-3 + 0.2005 z^-4 + 0.1745 z^-5 + 0.1834 z^-6 +
0.09826 z^-7

          + 0.01323 z^-8 - 0.07709 z^-9 + 0.06733 z^-10 - 0.06744 z^-11 + 0.07051 z^-12 +
0.2335 z^-13
```

```
           + 0.02301 z^-14 + 0.1094 z^-15 + 0.2325 z^-16 + 0.05759 z^-17 + 0.1589 z^-18 +
0.05427 z^-19

           + 0.1164 z^-20 + 0.008299 z^-21 + 0.01002 z^-22 + 0.05719 z^-23 + 0.05153 z^-24 +
0.08678 z^-25

           + 0.06629 z^-26 + 0.1034 z^-27 + 0.1613 z^-28 + 0.02881 z^-29 + 0.1208 z^-30 +
0.07342 z^-31

           - 0.04145 z^-32 + 0.08681 z^-33 - 0.04911 z^-34 + 0.01429 z^-35 + 0.03968 z^-36 -
0.06344 z^-37

           + 0.07559 z^-38 - 0.006706 z^-39 + 0.1736 z^-40 + 0.09676 z^-41 - 0.01625 z^-42 +
0.1109 z^-43

           + 0.04474 z^-44 + 0.008541 z^-45 + 0.06005 z^-46 - 0.05169 z^-47 + 0.05252 z^-48

           - 0.04626 z^-49 + 0.005351 z^-50 + 0.1502 z^-51 - 0.02768 z^-52 + 0.02221 z^-53 +
0.03277 z^-54

           + 0.0009871 z^-55 + 0.02285 z^-56 + 0.05491 z^-57 + 0.05353 z^-58 - 0.06568 z^-59

           - 0.1134 z^-60 - 0.02589 z^-61 - 0.09054 z^-62 - 0.03131 z^-63 - 0.01416 z^-64 -
0.05589 z^-65

           - 0.1083 z^-66 + 0.02599 z^-67 + 0.05999 z^-68 + 0.02399 z^-69 - 0.03097 z^-70 +
0.1076 z^-71

           - 0.01535 z^-72 + 0.06582 z^-73 + 0.01182 z^-74 + 0.01114 z^-75 - 0.09208 z^-76 -
0.05773 z^-77

           - 0.009111 z^-78 + 0.06227 z^-79 - 0.1421 z^-80 + 0.06227 z^-81 - 0.06936 z^-82 -
0.0009541 z^-83

           + 0.01127 z^-84 + 0.0382 z^-85 - 0.009547 z^-86 + 0.01467 z^-87 - 0.07241 z^-88 -
0.003348 z^-89

           - 0.1111 z^-90 + 0.1006 z^-91 - 0.002315 z^-92 - 0.02006 z^-93 - 0.04031 z^-94 +
0.01264 z^-95

                     - 0.006385 z^-96 + 0.02454 z^-97 + 0.09584 z^-98 - 0.02278 z^-99 - 0.02535
z^-100


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 11 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 11
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb;

VarU = arx_U_builder_3(u3, y3, na, nb, nk);
VarModel_3 = inv(VarU'*VarU)*VarU'*y3;
Var_y_hat_3 = form_tf_lsim_2(VarModel_3, u3_val, t_val, na, Ts);
```

```
G =


   0.6147 z^-1 + 1.681 z^-2 + 2.089 z^-3 + 1.993 z^-4 + 1.377 z^-5 + 0.597 z^-6 - 0.1374 z^-7

                                             - 0.5972 z^-8 - 0.6874 z^-9 - 0.4626 z^-10 -
0.1979 z^-11

  -------------------------------------------------------------------------------------------
----------

  1 - 0.2284 z^-1 - 0.1256 z^-2 + 0.1043 z^-3 + 0.0761 z^-4 + 0.05185 z^-5 + 0.1282 z^-6 +
0.05207 z^-7

                                          - 0.04543 z^-8 - 0.08214 z^-9 - 0.1639 z^-10 -
0.2819 z^-11


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method===================\n")
```

```
===============Degree Extraction | AIC Method===================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n", k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 10 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 10
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb;

AICU_3 = arx_U_builder_3(u3, y3, na, nb, nk);
AICModel_3 = inv(AICU_3'*AICU_3)*AICU_3'*y3;
AIC_y_hat_3 = form_tf_lsim_2(AICModel_3, u3_val, t_val, na, Ts);
```

```
G =


   0.6038 z^-1 + 1.614 z^-2 + 1.888 z^-3 + 1.601 z^-4 + 0.8604 z^-5 + 0.1175 z^-6 - 0.4492 z^-
7

                                                             - 0.6673 z^-8 - 0.5011 z^-9 -
0.1655 z^-10

  ------------------------------------------------------------------------------------------
---------

  1 - 0.3235 z^-1 - 0.193 z^-2 + 0.0683 z^-3 + 0.0702 z^-4 + 0.06747 z^-5 + 0.1316 z^-6 +
0.06712 z^-7

                                                           - 0.04011 z^-8 - 0.1653 z^-9 -
0.2961 z^-10


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("============================================================================\n")
```

```
=================================================================
```

```
%%
```

```
fprintf("===============Degree Extraction | F test Method===================\n")
```

```
===============Degree Extraction | F test Method===================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", winScore)
```

```
>>> The F test is suggesting the best model with the m=1.21 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 10
```

```

na = winner;
nb = winner;
p = na+nb;

FTestU_3 = arx_U_builder_3(u3, y3, na, nb, nk);
FTestModel_3 = inv(FTestU_3'*FTestU_3)*FTestU_3'*y3;
FTest_y_hat_3 = form_tf_lsim_2(FTestModel_3, u3_val, t_val, na, Ts);
```

```
G =


   0.6038 z^-1 + 1.614 z^-2 + 1.888 z^-3 + 1.601 z^-4 + 0.8604 z^-5 + 0.1175 z^-6 - 0.4492 z^-7

                                                              - 0.6673 z^-8 - 0.5011 z^-9 - 0.1655 z^-10

  ------------------------------------------------------------------------------------------

  1 - 0.3235 z^-1 - 0.193 z^-2 + 0.0683 z^-3 + 0.0702 z^-4 + 0.06747 z^-5 + 0.1316 z^-6 + 0.06712 z^-7
```

```
                                                          - 0.04011 z^-8 - 0.1653 z^-9 -
0.2961 z^-10


Sample time: 0.5 seconds
Discrete-time transfer function.
```

```
fprintf("==================================================================\n")
```

```
==================================================================
```

```
%%

[BestFit_r2, BestFit_mse] = rSQR(y3_val, BestFit_y_hat_3);
[Var_r2, Var_mse] = rSQR(y3_val, Var_y_hat_3);
[AIC_r2, AIC_mse] = rSQR(y3_val, AIC_y_hat_3);
[FTest_r2, FTest_mse] = rSQR(y3_val, FTest_y_hat_3);
fprintf("===========================System III===========================\n")
```

```
===========================System III===========================
```

```
fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.9966    | MSE : 0.4355
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.9965    | MSE : 0.4472
```

```
fprintf("----------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", Cov_r2, Cov_mse)
% fprintf("---------------------------------------------------------------\n")
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.9965    | MSE : 0.4527
```

```
fprintf("---------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.9965    | MSE : 0.4527
```

```
fprintf("---------------------------------------------------------------\n")
```

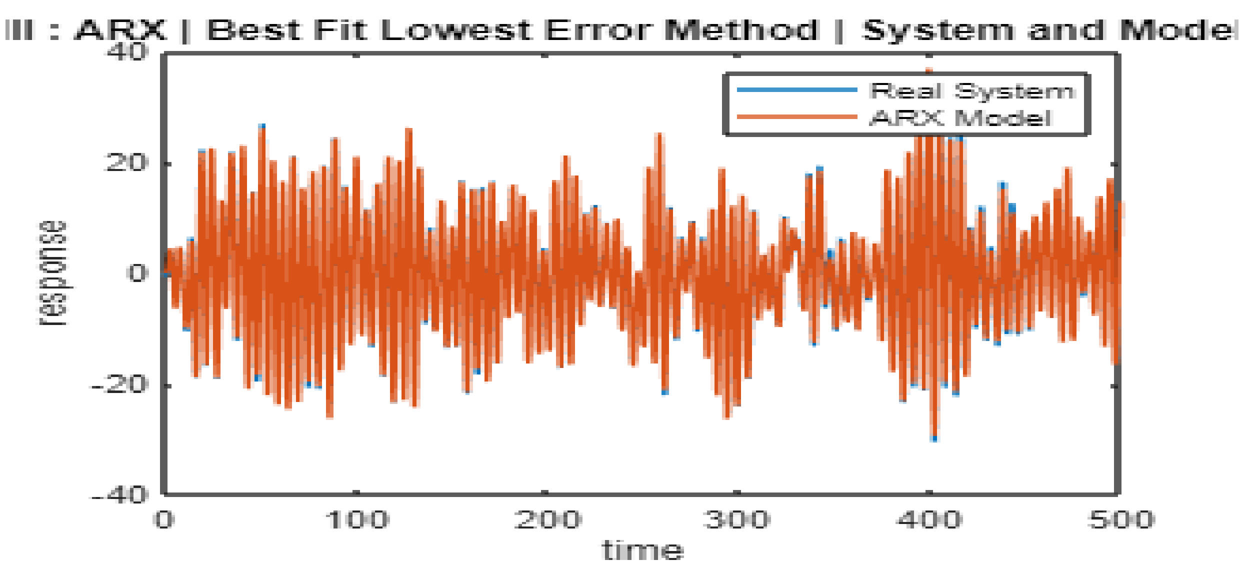
```
------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

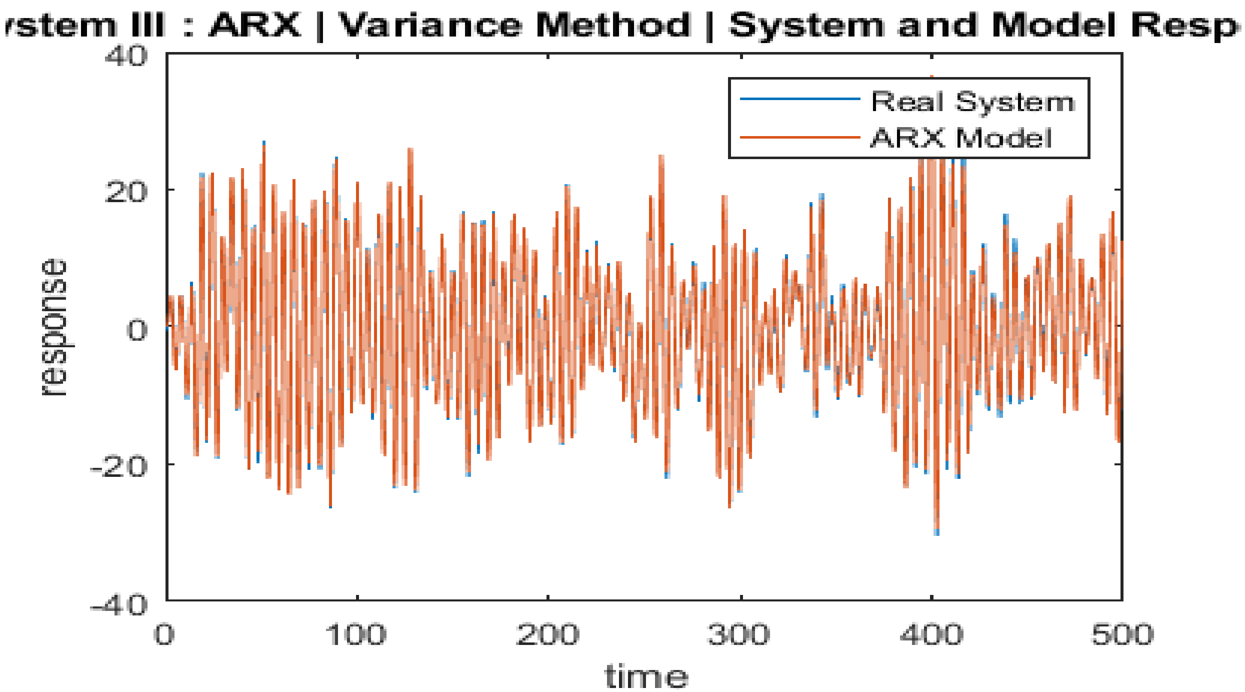
```
============================================================
```

```




%%

BestFitError_3 = y3_val - BestFit_y_hat_3;
VarError_3 = y3_val - Var_y_hat_3;
% CovError_3 = y_val - Cov_y_hat_3;
AICError_3 = y3_val - AIC_y_hat_3;
FTestError_3 = y3_val - FTest_y_hat_3;

for k=0:N_val-1
    BestFit_Ree_3(k+1,1) = AutoCorrelate(BestFitError_3, k);
    Var_Ree_3(k+1,1) = AutoCorrelate(VarError_3, k);
%     Cov_Ree_3(k+1,1) = AutoCorrelate(CovError_3, k);
```

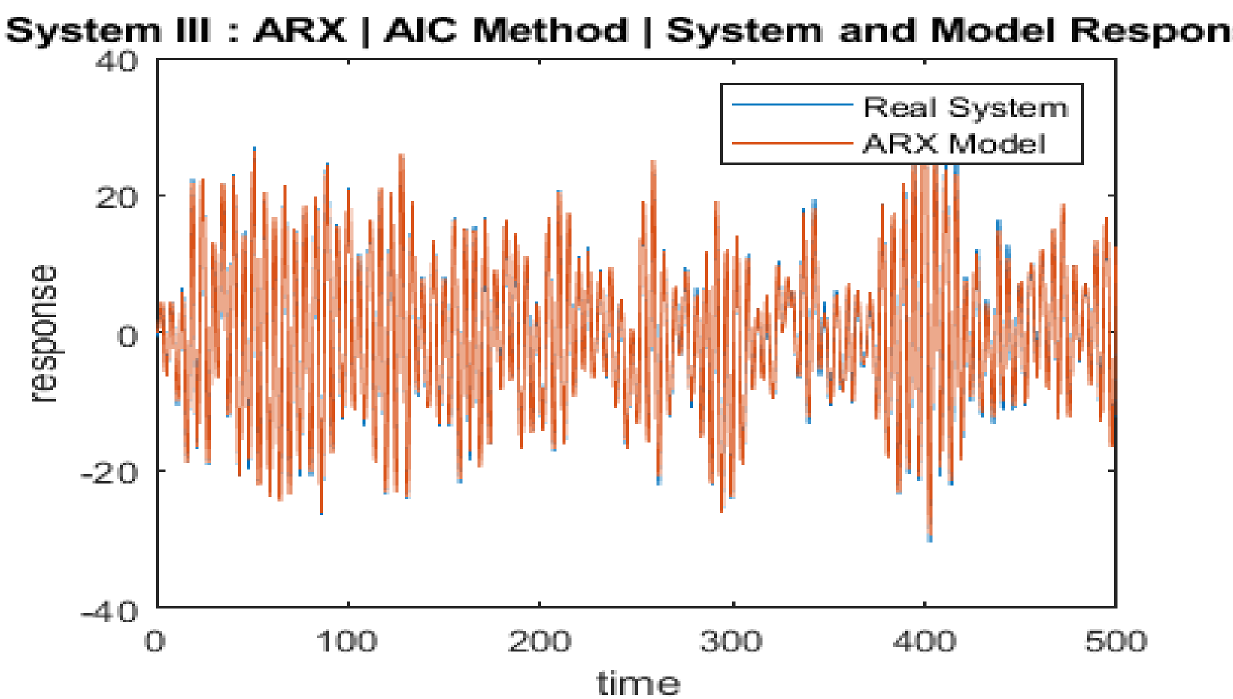
```
    AIC_Ree_3(k+1,1) = AutoCorrelate(AICError_3, k);
    FTest_Ree_3(k+1,1) = AutoCorrelate(FTestError_3, k);
end

for k=0:N_val-1
    BestFit_Rue_3(k+1,1) = CrossCorrelate(u3_val, BestFitError_3, k);
    Var_Rue_3(k+1,1) = CrossCorrelate(u3_val, VarError_3, k);
%     Cov_Rue_3(k+1,1) = CrossCorrelate(u3_val, CovError_3, k);
    AIC_Rue_3(k+1,1) = CrossCorrelate(u3_val, AICError_3, k);
    FTest_Rue_3(k+1,1) = CrossCorrelate(u3_val, FTestError_3, k);
end



%%
figure()  % figure(1)
plot(t_val,y3_val,t_val,BestFit_y_hat_3)
legend('Real System','ARX Model')
title(" System III : ARX | Best Fit Lowest Error Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



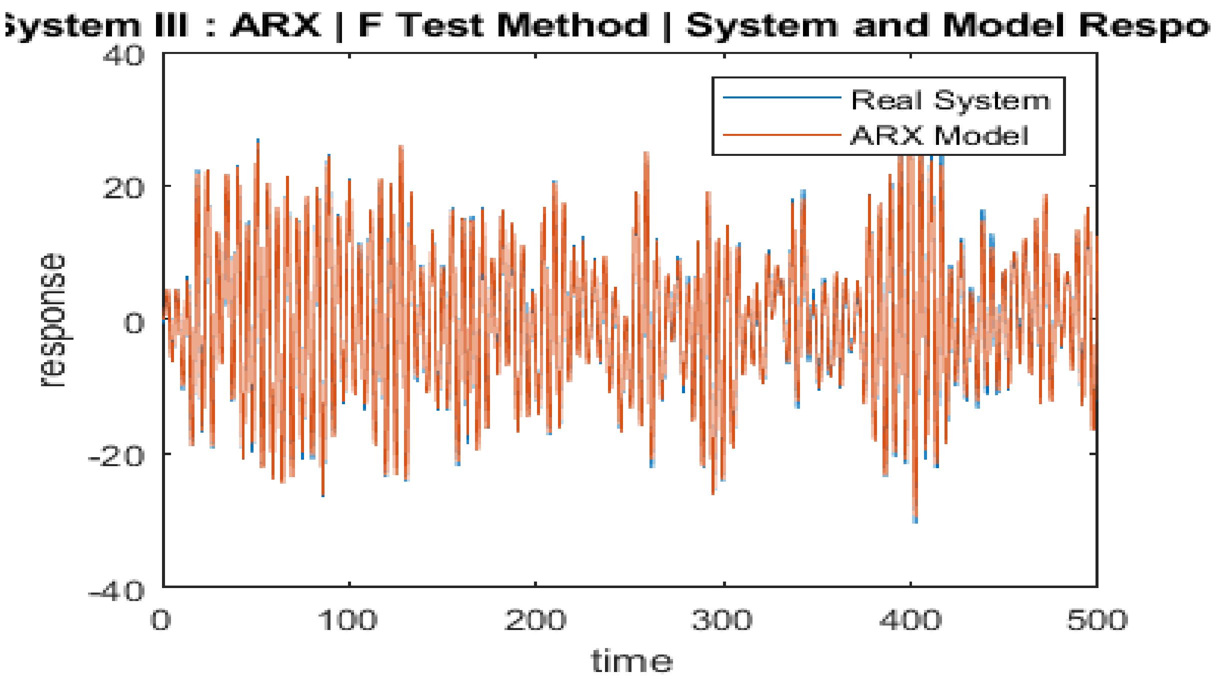

```
figure()  % figure(2)
plot(t_val,y3_val,t_val,Var_y_hat_3)
legend('Real System','ARX Model')
title(" System III : ARX | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure()  % figure(3)
plot(t_val,y3_val,t_val,AIC_y_hat_3)
legend('Real System','ARX Model')
title(" System III : ARX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

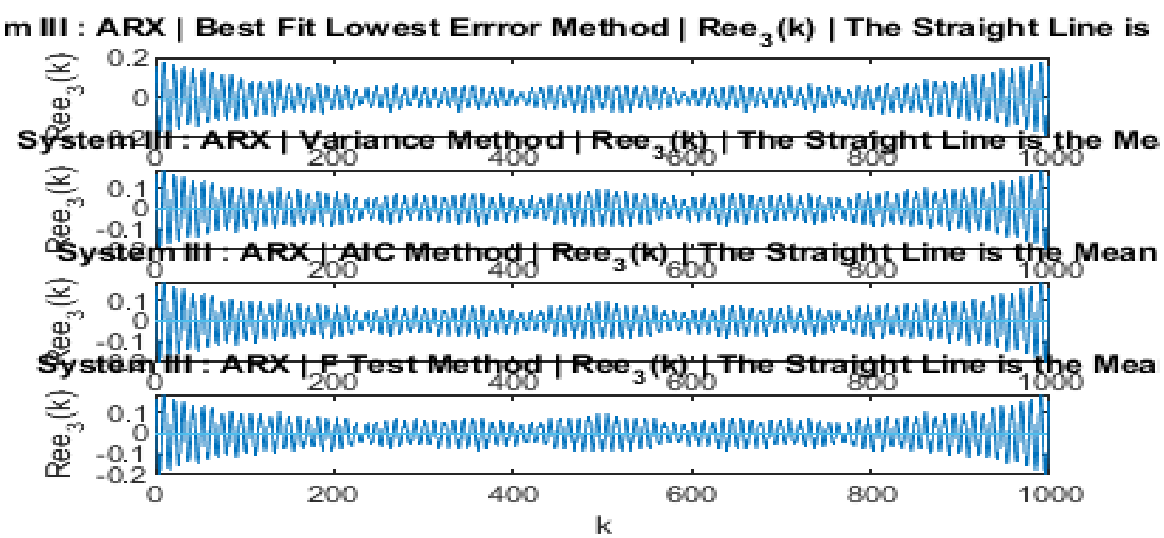


```
figure()  % figure(4)
plot(t_val,y3_val,t_val,FTest_y_hat_3)
legend('Real System','ARX Model')
title(" System III : ARX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



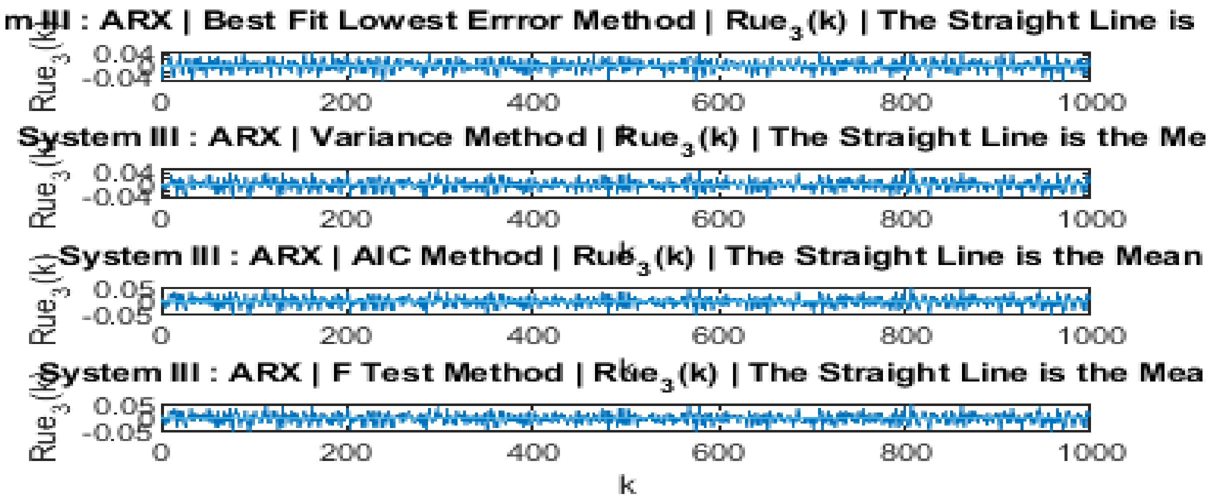

```
% figure()  % figure(7)
% plot(t,y_val,t,Cov_y_hat)
% legend('Real System','ARX Model')
% title(" System III : ARX | Covariance Method | System and Model Response")
% xlabel("time")
% ylabel("response")

%%

figure()  % figure(5)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Ree_3(2:end), 1:N_val-1,
mean(BestFit_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : ARX | Best Fit Lowest Errror Method | Ree_3(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_3(k)")
```

```
 subplot(5,1,2)
 plot(1:N_val-1,Var_Ree_3(2:end), 1:N_val-1,
mean(Var_Ree_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARX | Variance Method | Ree_3(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Ree_3(k)")

 subplot(5,1,3)
 plot(1:N_val-1,AIC_Ree_3(2:end), 1:N_val-1,
mean(AIC_Ree_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARX | AIC Method | Ree_3(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Ree_3(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Ree_3(2:end), 1:N_val-1,
mean(FTest_Ree_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARX | F Test Method | Ree_3(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Ree_3(k)")
```



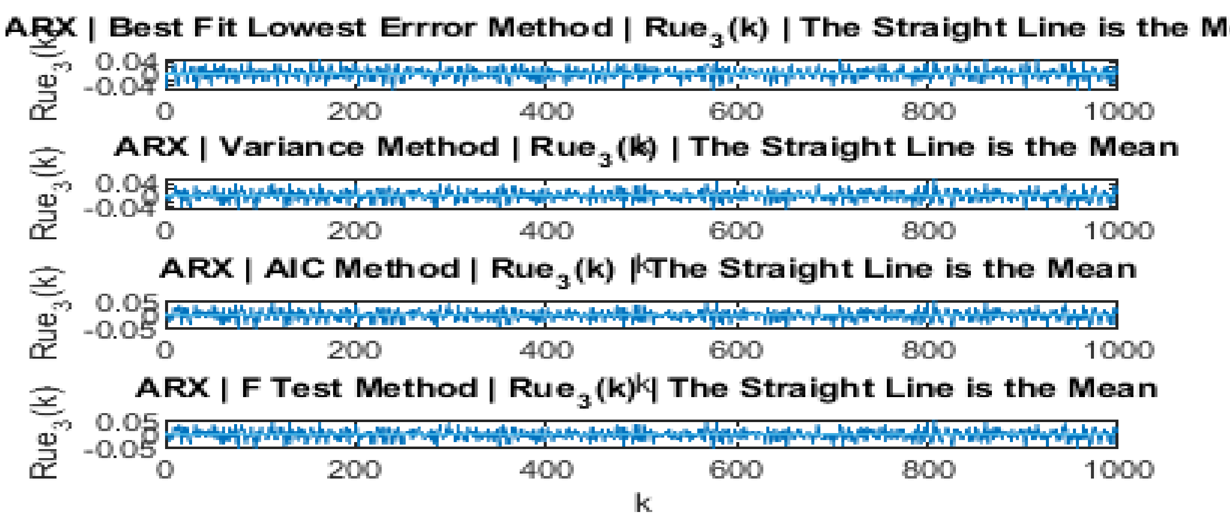

```
 % subplot(5,1,5)
```

```
 % plot(1:N_val-1,Cov_Ree_3(2:end), 1:N_val-1,
mean(Cov_Ree_3(2:end))*ones(length(1:N_val-1)))
 % title(" System III : ARX | Covariance Method | Ree_3(k) | The Straight Line
is the Mean")
 % xlabel("k")
 % ylabel("Ree_3(k)")


 %%

 figure()  % figure(6)
 subplot(5,1,1)
 plot(1:N_val-1,BestFit_Rue_3(2:end), 1:N_val-1,
mean(BestFit_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARX | Best Fit Lowest Errror Method | Rue_3(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(5,1,2)
 plot(1:N_val-1,Var_Rue_3(2:end), 1:N_val-1,
mean(Var_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARX | Variance Method | Rue_3(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(5,1,3)
 plot(1:N_val-1,AIC_Rue_3(2:end), 1:N_val-1,
mean(AIC_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARX | AIC Method | Rue_3(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Rue_3(2:end), 1:N_val-1,
mean(FTest_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : ARX | F Test Method | Rue_3(k) | The Straight Line is the
Mean")
 xlabel("k")
 ylabel("Rue_3(k)")
```




```
 % subplot(5,1,5)
 % plot(1:N_val-1,Cov_Rue_3(2:end), 1:N_val-1,
mean(Cov_Rue_3(2:end))*ones(length(1:N_val-1)))
 % title(" System III : ARX | Covariance Method | Rue_3(k) | The Straight Line
is the Mean")
 % xlabel("k")
 % ylabel("Rue_3(k)")



 %%

 figure()  % figure(6)
 subplot(5,1,1)
 plot(1:N_val-1,BestFit_Rue_3(2:end), 1:N_val-1,
mean(BestFit_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" ARX | Best Fit Lowest Errror Method | Rue_3(k) | The Straight Line is
the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(5,1,2)
 plot(1:N_val-1,Var_Rue_3(2:end), 1:N_val-1,
mean(Var_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" ARX | Variance Method | Rue_3(k) | The Straight Line is the Mean")
```

```
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(5,1,3)
 plot(1:N_val-1,AIC_Rue_3(2:end), 1:N_val-1,
mean(AIC_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" ARX | AIC Method | Rue_3(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Rue_3(2:end), 1:N_val-1,
mean(FTest_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" ARX | F Test Method | Rue_3(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")
```




```
 % subplot(5,1,5)
 % plot(1:N_val-1,Cov_Rue_3(2:end), 1:N_val-1,
mean(Cov_Rue_3(2:end))*ones(length(1:N_val-1)))
 % title(" ARX | Covariance Method | Rue_3(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_3(k)")
```

```
% *************************************************************************

fprintf("******************************************************************\n")
```

****

```
fprintf("******************************************************************\n")
```

****

Q2 - part d - Box-Jenkins

```
clc; clear;
%%

load HW5_question2

u3 = Z3.u;
y3 = Z3.y;

u3_val = u3;%(601:end);
y3_val = y3;%(601:end);

u3 = u3(1:600);
y3 = y3(1:600);


%%

% System Z3 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System III Identification Begins:------------------------------
\n")
```

```
>>> System III Identification Begins:------------------------------
```

```

%%

Ts = 0.5;
t = 0:Ts:length(u3)*Ts-Ts;
t_val = 0:Ts:length(u3_val)*Ts-Ts;
N = length(y3);
N_val = length(y3_val);

data3 = iddata(y3,u3,Ts);

%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nd = degree;
    nk = 1;
    p = na+nb+nc+nd;

    try
        sys = bj(data3, [na nb nc nd nk]);
        bj_y_hat_3 = lsim(sys, u3, t);
    catch
        break
    end

    [r2_bj, mse_bj] = rSQR(y3, bj_y_hat_3);

    error = y3 - bj_y_hat_3;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_bj, mse_bj, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_bj];
    MSEs = [MSEs; mse_bj];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
```

```
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=-0.006472 | MSE=140.441342 | var=141.383901 | s_hat=84264.804977 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.972050 | MSE=3.900046 | var=3.952749 | s_hat=2340.027667 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.970744 | MSE=4.082400 | var=4.165714 | s_hat=2449.439781 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.997734 | MSE=0.316203 | var=0.324866 | s_hat=189.721601 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.997768 | MSE=0.311455 | var=0.322195 | s_hat=186.872970 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.997793 | MSE=0.307950 | var=0.320781 | s_hat=184.769820 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.997784 | MSE=0.309178 | var=0.324312 | s_hat=185.506656 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.997814 | MSE=0.305089 | var=0.322277 | s_hat=183.053601 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.932754 | MSE=9.383429 | var=9.982371 | s_hat=5630.057487 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.997707 | MSE=0.319992 | var=0.342848 | s_hat=191.995091 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.997427 | MSE=0.359086 | var=0.387503 | s_hat=215.451394 |
-------------------------------------------------------------
>>> Degree = 12 : R2=-2.755093 | MSE=523.979107 | var=569.542508 | s_hat=314387.464498 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.994961 | MSE=0.703201 | var=0.769928 | s_hat=421.920516 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.997575 | MSE=0.338398 | var=0.373233 | s_hat=203.038680 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.996556 | MSE=0.480636 | var=0.534040 | s_hat=288.381424 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.997672 | MSE=0.324884 | var=0.363676 | s_hat=194.930328 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.997546 | MSE=0.342454 | var=0.386226 | s_hat=205.472194 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.968357 | MSE=4.415401 | var=5.017501 | s_hat=2649.240369 |
-------------------------------------------------------------
>>> Degree = 19 : R2=0.997319 | MSE=0.374082 | var=0.428338 | s_hat=224.449055 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.997907 | MSE=0.291998 | var=0.336921 | s_hat=175.198743 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.963669 | MSE=5.069549 | var=5.894825 | s_hat=3041.729590 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.797318 | MSE=28.281844 | var=33.142786 | s_hat=16969.106447 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.675810 | MSE=45.236945 | var=53.429462 | s_hat=27142.166863 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.995749 | MSE=0.593188 | var=0.706177 | s_hat=355.912985 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.450376 | MSE=76.693629 | var=92.032355 | s_hat=46016.177652 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.898308 | MSE=14.189908 | var=17.165212 | s_hat=8513.944915 |
```

```
------------------------------------------------------------
>>> Degree = 27 : R2=0.835568 | MSE=22.944616 | var=27.981240 | s_hat=13766.769884 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.560186 | MSE=61.370946 | var=75.456081 | s_hat=36822.567706 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.948144 | MSE=7.235902 | var=8.970127 | s_hat=4341.541395 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.991494 | MSE=1.186848 | var=1.483559 | s_hat=712.108554 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.954219 | MSE=6.388214 | var=8.052371 | s_hat=3832.928634 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.301123 | MSE=97.520121 | var=123.966255 | s_hat=58512.072349 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.988332 | MSE=1.628174 | var=2.087403 | s_hat=976.904462 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.991548 | MSE=1.179364 | var=1.525040 | s_hat=707.618499 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.997807 | MSE=0.305992 | var=0.399120 | s_hat=183.595064 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.984413 | MSE=2.174978 | var=2.861813 | s_hat=1304.986577 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.978059 | MSE=3.061577 | var=4.064041 | s_hat=1836.946455 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.987243 | MSE=1.780036 | var=2.383977 | s_hat=1068.021822 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.997503 | MSE=0.348421 | var=0.470839 | s_hat=209.052462 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.986861 | MSE=1.833435 | var=2.500139 | s_hat=1100.060946 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.997764 | MSE=0.311962 | var=0.429305 | s_hat=187.177176 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.997531 | MSE=0.344542 | var=0.478530 | s_hat=206.725110 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.997436 | MSE=0.357820 | var=0.501617 | s_hat=214.692016 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.948751 | MSE=7.151157 | var=10.119562 | s_hat=4290.694221 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.994530 | MSE=0.763338 | var=1.090482 | s_hat=458.002511 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.988004 | MSE=1.673887 | var=2.414260 | s_hat=1004.332218 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.994910 | MSE=0.710311 | var=1.034434 | s_hat=426.186808 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.988653 | MSE=1.583358 | var=2.328468 | s_hat=950.014770 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.973025 | MSE=3.764033 | var=5.590147 | s_hat=2258.419513 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.891589 | MSE=15.127512 | var=22.691268 | s_hat=9076.507323 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.986171 | MSE=1.929668 | var=2.923739 | s_hat=1157.800508 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.989965 | MSE=1.400214 | var=2.143185 | s_hat=840.128670 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.984684 | MSE=2.137193 | var=3.304937 | s_hat=1282.315726 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.985164 | MSE=2.070145 | var=3.234602 | s_hat=1242.087214 |
------------------------------------------------------------
```

```
>>> Degree = 55 : R2=0.985086 | MSE=2.081011 | var=3.285807 | s_hat=1248.606568 |
-------------------------------------------------------------
>>> Degree = 56 : R2=0.994162 | MSE=0.814672 | var=1.300008 | s_hat=488.803093 |
-------------------------------------------------------------
>>> Degree = 57 : R2=0.992262 | MSE=1.079756 | var=1.741541 | s_hat=647.853385 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.959463 | MSE=5.656398 | var=9.222388 | s_hat=3393.838908 |
-------------------------------------------------------------
>>> Degree = 59 : R2=0.995370 | MSE=0.646119 | var=1.065031 | s_hat=387.671162 |
-------------------------------------------------------------
>>> Degree = 60 : R2=0.996840 | MSE=0.440894 | var=0.734824 | s_hat=264.536602 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.983705 | MSE=2.273789 | var=3.832228 | s_hat=1364.273275 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.994614 | MSE=0.751589 | var=1.281118 | s_hat=450.953695 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 20
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
nd = bestFitDegree;
nk = 1;
p = na+nb+nc+nd;

BestFitModel_3 = bj(data3, [na nb nc nd nk])
```

```
BestFitModel_3 =
```

```
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)

  B(z) = 0.6145 z^-1 + 1.488 z^-2 + 0.83 z^-3 - 0.4696 z^-4 - 0.3811 z^-5 + 0.5697 z^-6 +
0.3122 z^-7
          - 0.3657 z^-8 + 0.1322 z^-9 + 0.6618 z^-10 - 0.3601 z^-11 - 0.5141 z^-12 + 0.7082
z^-13
          + 1.113 z^-14 + 0.06209 z^-15 - 0.4 z^-16 + 0.09941 z^-17 + 0.2256 z^-18 - 0.279 z^
                                                                                  -19 -
0.2058 z^-20


  C(z) = 1 - 1.235 z^-1 + 0.2556 z^-2 + 0.1398 z^-3 - 0.1573 z^-4 + 0.3126 z^-5 + 0.03903 z^-6
          - 0.2334 z^-7 + 0.339 z^-8 - 0.1934 z^-9 + 0.155 z^-10 - 0.4133 z^-11 + 0.5466 z^-12
          - 0.6696 z^-13 + 0.2641 z^-14 + 0.04616 z^-15 - 0.3344 z^-16 + 0.8224 z^-17 - 0.1894
z^-18
                                                                         - 0.7233 z^-19 +
0.3558 z^-20


  D(z) = 1 - 1.189 z^-1 + 0.267 z^-2 + 0.3438 z^-3 - 0.2705 z^-4 + 0.2806 z^-5 + 0.1205 z^-6
          - 0.3371 z^-7 + 0.3479 z^-8 - 0.06611 z^-9 + 0.03858 z^-10 - 0.2688 z^-11 + 0.6008
z^-12
          - 0.5214 z^-13 + 0.1012 z^-14 + 0.1632 z^-15 - 0.3574 z^-16 + 0.6133 z^-17 - 0.07104
z^-18
                                                                         - 0.6588 z^-19 +
0.4697 z^-20


  F(z) = 1 - 0.5758 z^-1 - 1.145 z^-2 + 0.6335 z^-3 + 1.494 z^-4 - 0.7703 z^-5 - 0.9963 z^-6
          + 0.6169 z^-7 + 1.065 z^-8 - 1.47 z^-9 - 0.3631 z^-10 + 1.532 z^-11 + 0.309 z^-12 -
1.544 z^-13
          - 0.06295 z^-14 + 1.011 z^-15 + 0.1355 z^-16 - 0.8079 z^-17 + 0.2399 z^-18 + 0.2619
z^-19
                                                                                      -
0.2679 z^-20

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=20   nc=20   nd=20   nf=20   nk=1
   Number of free coefficients: 80
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data3".
Fit to estimation data: 96.44% (prediction focus)
FPE: 0.2653, MSE: 0.1769
```

```
BestFit_y_hat_3 = lsim(BestFitModel_3, u3_val, t_val);
% [bj_BestFit_r2, bj_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method===================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 6 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 6
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
nd = minVarIndex;
nk = 1;
p = na+nb+nc+nd;

bj_VarModel_3 = bj(data3, [na nb nc nd nk])
```

```
bj_VarModel_3 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.6122 z^-1 + 1.056 z^-2 + 0.3816 z^-3 - 0.1856 z^-4 - 0.5302 z^-5 - 0.2009 z^-6

  C(z) = 1 + 0.5371 z^-1 - 0.219 z^-2 - 0.2607 z^-3 - 0.2039 z^-4 + 0.6426 z^-5 + 0.3419 z^-6

  D(z) = 1 + 0.4986 z^-1 - 0.2165 z^-2 - 0.02158 z^-3 + 0.06623 z^-4 + 0.7615 z^-5 + 0.3904
z^-6

  F(z) = 1 - 1.249 z^-1 + 0.1076 z^-2 + 0.1692 z^-3 + 0.3635 z^-4 + 0.08166 z^-5 - 0.3849 z^-6

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=6   nc=6   nd=6   nf=6   nk=1
   Number of free coefficients: 24
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data3".
Fit to estimation data: 95.84% (prediction focus)
FPE: 0.2614, MSE: 0.2413
```

```
Var_y_hat_3 = lsim(bj_VarModel_3, u3_val, t_val);
% [bj_Var_r2, bj_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 5 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 5
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
nd = minAICIndex;
nk = 1;
p = na+nb+nc+nd;

bj_AICModel_3 = bj(data3, [na nb nc nd nk])
```

```
bj_AICModel_3 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.6311 z^-1 + 1.008 z^-2 + 0.2968 z^-3 - 0.4581 z^-4 - 0.5216 z^-5

  C(z) = 1 - 0.06572 z^-1 - 0.174 z^-2 - 0.32 z^-3 + 0.0816 z^-4 + 0.6824 z^-5

  D(z) = 1 - 0.07638 z^-1 - 0.1269 z^-2 - 0.09488 z^-3 + 0.1981 z^-4 + 0.6785 z^-5

  F(z) = 1 - 1.306 z^-1 + 0.1415 z^-2 - 0.1006 z^-3 + 1.214 z^-4 - 0.8751 z^-5

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=5   nc=5   nd=5   nf=5   nk=1
   Number of free coefficients: 20
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data3".
Fit to estimation data: 95.87% (prediction focus)
FPE: 0.2547, MSE: 0.2383
```

```
AIC_y_hat_3 = lsim(bj_AICModel_3, u3_val, t_val);
% [bj_AIC_r2, bj_AIC_mse] = rSQR(y_val, AIC_y_hat);
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.19 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 4
```

```
na = winner;
nb = winner;
nc = winner;
nd = winner;
nk = 1;
p = na+nb+nc+nd;

bj_FTestModel_3 = bj(data3, [na nb nc nd nk])
```

```
bj_FTestModel_3 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.6156 z^-1 + 0.7605 z^-2 - 0.4021 z^-3 - 0.429 z^-4
```

```
  C(z) = 1 - 0.9074 z^-1 + 0.6011 z^-2 - 0.851 z^-3 + 0.8152 z^-4

  D(z) = 1 - 0.9173 z^-1 + 0.6585 z^-2 - 0.6833 z^-3 + 0.802 z^-4

  F(z) = 1 - 1.758 z^-1 + 0.3698 z^-2 + 1.204 z^-3 - 0.774 z^-4

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=4   nc=4   nd=4   nf=4   nk=1
   Number of free coefficients: 16
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data3".
Fit to estimation data: 95.85% (prediction focus)
FPE: 0.2532, MSE: 0.2401
```

```
FTest_y_hat_3 = lsim(bj_FTestModel_3, u3_val, t_val);
% [bj_FTest_r2, bj_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```



%%


[bj_BestFit_r2_3, bj_BestFit_mse_3] = rSQR(y3_val, BestFit_y_hat_3);
[bj_Var_r2_3, bj_Var_mse_3] = rSQR(y3_val, Var_y_hat_3);
[bj_AIC_r2_3, bj_AIC_mse_3] = rSQR(y3_val, AIC_y_hat_3);
[bj_FTest_r2_3, bj_FTest_mse_3] = rSQR(y3_val, FTest_y_hat_3);

%%

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_BestFit_r2_3, 
bj_BestFit_mse_3)
```

```
    R2 value : 0.9965    | MSE : 0.4529
```

```
fprintf("------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", bj_Var_r2_3, bj_Var_mse_3)
```

```
    R2 value : 0.9965   | MSE : 0.4502
```

```
% fprintf("------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f   | MSE : %.4f \n", bj_Cov_r2, bj_Cov_mse)
fprintf("------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", bj_AIC_r2_3, bj_AIC_mse_3)
```

```
    R2 value : 0.9965   | MSE : 0.4554
```

```
fprintf("------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", bj_FTest_r2_3, bj_FTest_mse_3)
```

```
    R2 value : 0.9964   | MSE : 0.4581
```

```
fprintf("------------------------------------------------------------\n")
```

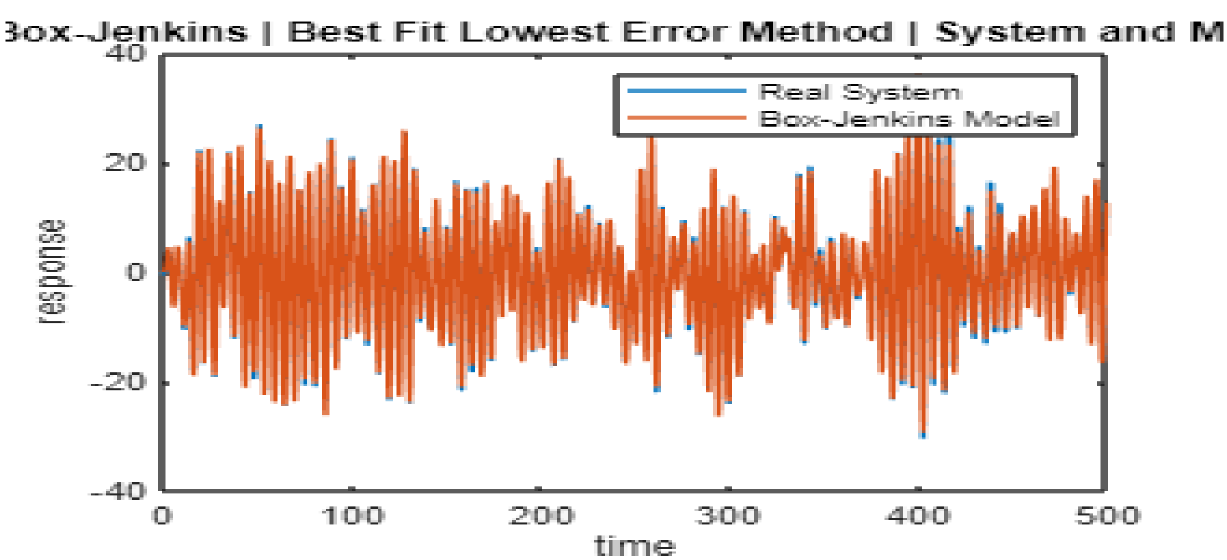
```
------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("============================================================\n")
```

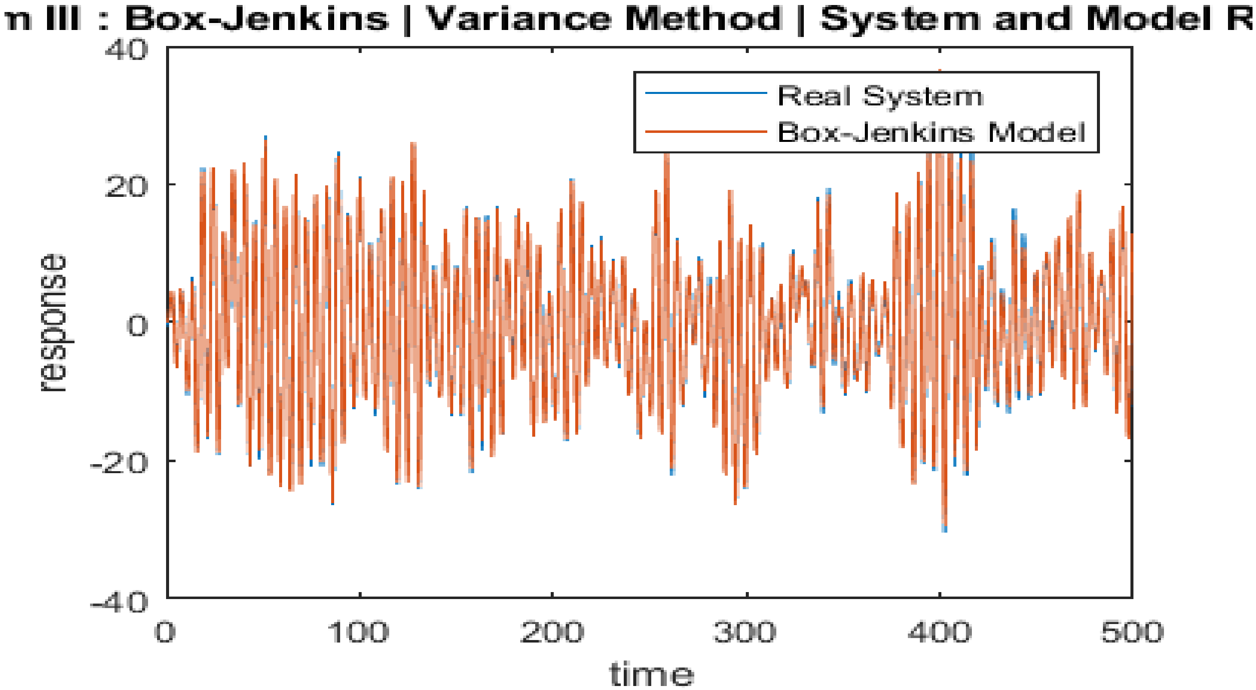
```
============================================================
```

```



%%
```

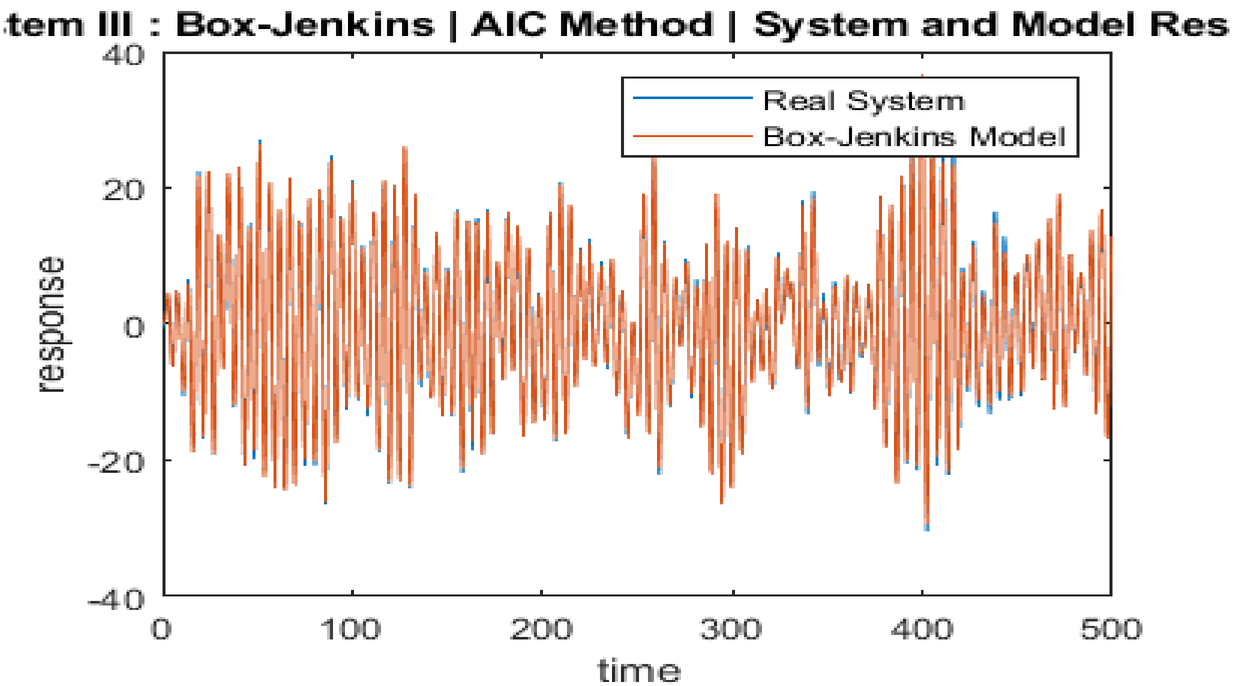
```
bj_BestFitError_3 = y3_val - BestFit_y_hat_3;
bj_VarError_3 = y3_val - Var_y_hat_3;
bj_AICError_3 = y3_val - AIC_y_hat_3;
bj_FTestError_3 = y3_val - FTest_y_hat_3;

for k=0:N_val-1
    bj_BestFit_Ree_3(k+1,1) = AutoCorrelate(bj_BestFitError_3, k);
    bj_Var_Ree_3(k+1,1) = AutoCorrelate(bj_VarError_3, k);
    bj_AIC_Ree_3(k+1,1) = AutoCorrelate(bj_AICError_3, k);
    bj_FTest_Ree_3(k+1,1) = AutoCorrelate(bj_FTestError_3, k);
end

for k=0:N_val-1
    bj_BestFit_Rue_3(k+1,1) = CrossCorrelate(u3_val, bj_BestFitError_3, k);
    bj_Var_Rue_3(k+1,1) = CrossCorrelate(u3_val, bj_VarError_3, k);
    bj_AIC_Rue_3(k+1,1) = CrossCorrelate(u3_val, bj_AICError_3, k);
    bj_FTest_Rue_3(k+1,1) = CrossCorrelate(u3_val, bj_FTestError_3, k);
end


%%
figure(1)
plot(t_val,y3_val,t_val,BestFit_y_hat_3)
legend('Real System','Box-Jenkins Model')
title(" System III : Box-Jenkins | Best Fit Lowest Error Method | System and
Model Response")
xlabel("time")
ylabel("response")
```



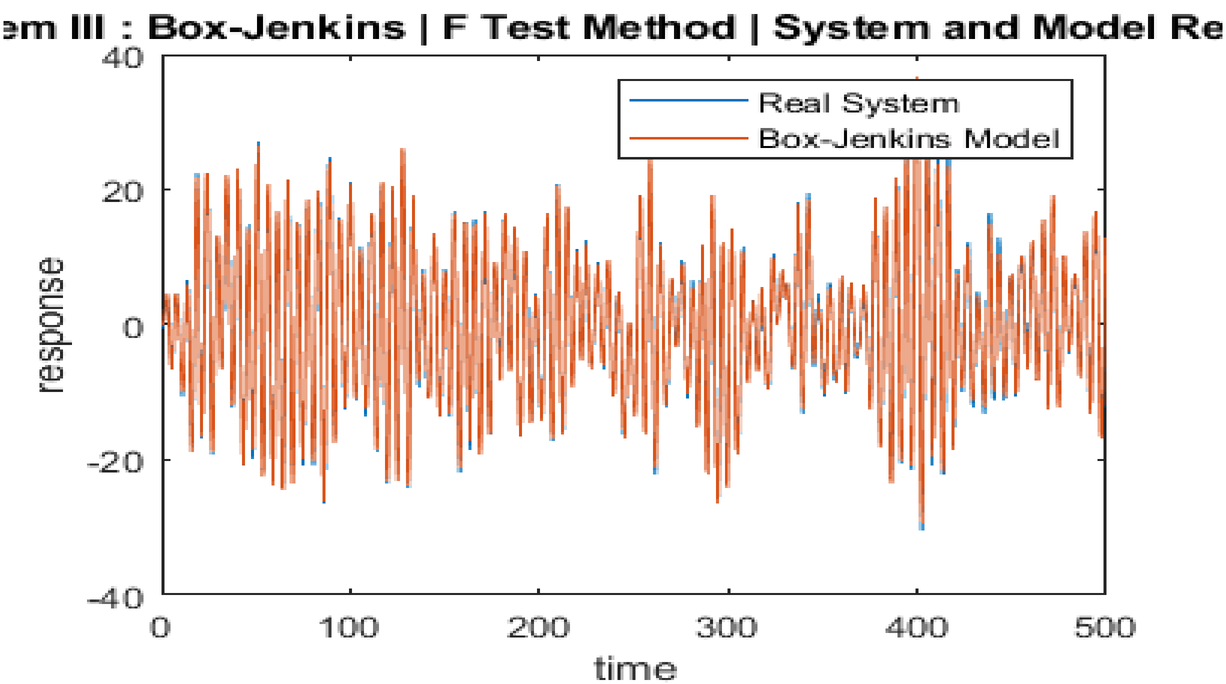
```
figure(2)
plot(t_val,y3_val,t_val,Var_y_hat_3)
legend('Real System','Box-Jenkins Model')
title(" System III : Box-Jenkins | Variance Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



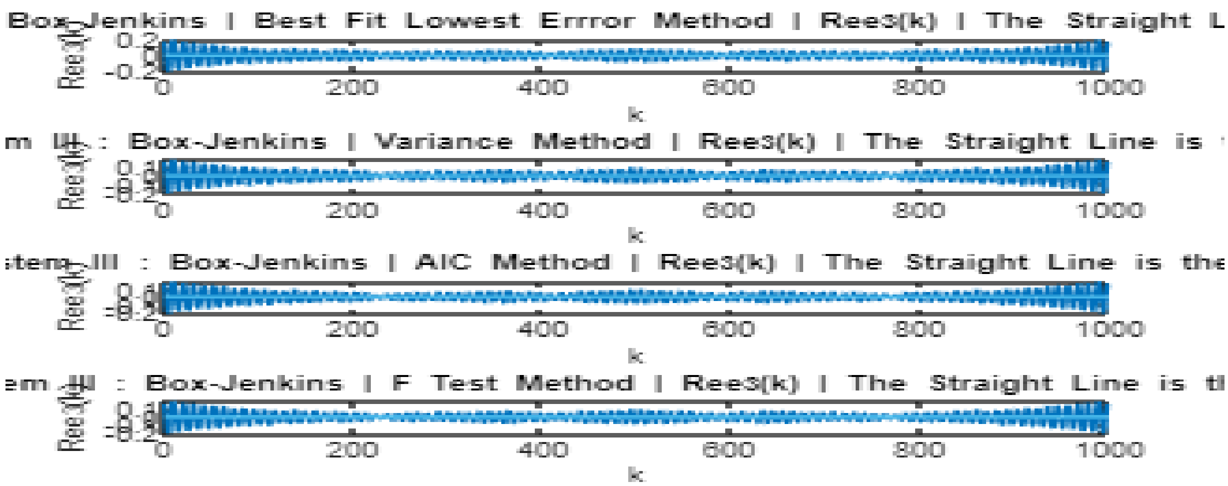

```
figure(3)
plot(t_val,y3_val,t_val,AIC_y_hat_3)
legend('Real System','Box-Jenkins Model')
title(" System III : Box-Jenkins | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y3_val,t_val,FTest_y_hat_3)
legend('Real System','Box-Jenkins Model')
title(" System III : Box-Jenkins | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```

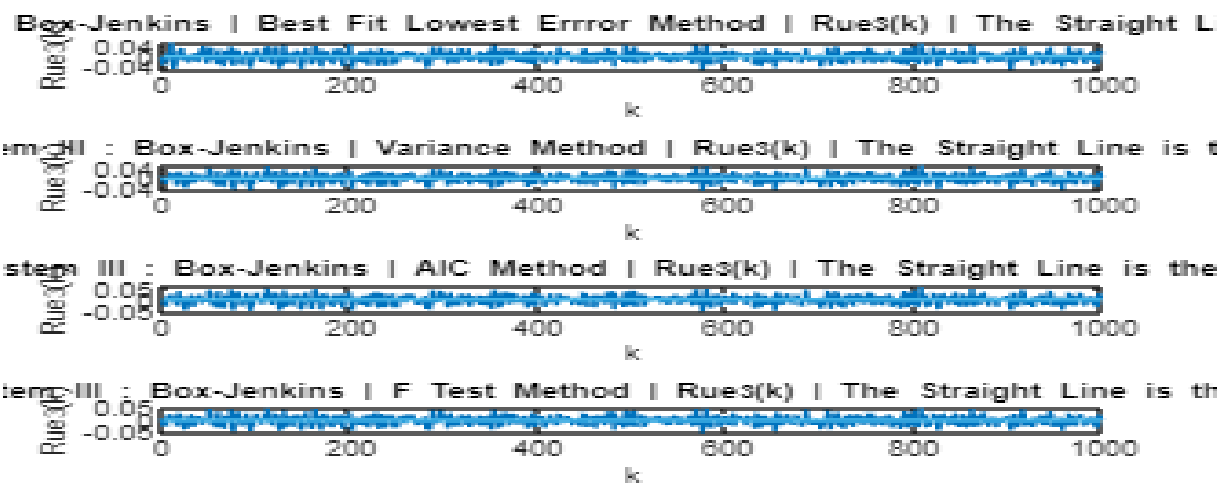


```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,bj_BestFit_Ree_3(2:end), 1:N_val-1,
mean(bj_BestFit_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Box-Jenkins | Best Fit Lowest Errror Method | Ree_3(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_3(k)")

subplot(4,1,2)
plot(1:N_val-1,bj_Var_Ree_3(2:end), 1:N_val-1,
mean(bj_Var_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Box-Jenkins | Variance Method | Ree_3(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree_3(k)")

subplot(4,1,3)
plot(1:N_val-1,bj_AIC_Ree_3(2:end), 1:N_val-1,
mean(bj_AIC_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Box-Jenkins | AIC Method | Ree_3(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Ree_3(k)")

subplot(4,1,4)
plot(1:N_val-1,bj_FTest_Ree_3(2:end), 1:N_val-1,
mean(bj_FTest_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Box-Jenkins | F Test Method | Ree_3(k) | The Straight Line
is the Mean")
xlabel("k")
ylabel("Ree_3(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,bj_BestFit_Rue_3(2:end), 1:N_val-1,
mean(bj_BestFit_Rue_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Box-Jenkins | Best Fit Lowest Errror Method | Rue_3(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_3(k)")

subplot(4,1,2)
plot(1:N_val-1,bj_Var_Rue_3(2:end), 1:N_val-1,
mean(bj_Var_Rue_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Box-Jenkins | Variance Method | Rue_3(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Rue_3(k)")

subplot(4,1,3)
plot(1:N_val-1,bj_AIC_Rue_3(2:end), 1:N_val-1,
mean(bj_AIC_Rue_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Box-Jenkins | AIC Method | Rue_3(k) | The Straight Line is
the Mean")
xlabel("k")
ylabel("Rue_3(k)")

subplot(4,1,4)
```

```
 plot(1:N_val-1,bj_FTest_Rue_3(2:end), 1:N_val-1,
mean(bj_FTest_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : Box-Jenkins | F Test Method | Rue_3(k) | The Straight Line
is the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")
```



Q2 - part d | Output Error

```
clc; clear;
%%

load HW5_question2

u3 = Z3.u;
y3 = Z3.y;

u3_val = u3;%(601:end);
y3_val = y3;%(601:end);

u3 = u3(1:600);
y3 = y3(1:600);

%%

% System Z3 **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> System III Identification Begins:------------------------------
\n")
```

```
>>> System III Identification Begins:----------------------------
```

```

%%

Ts = 0.5;
t = 0:Ts:length(u3)*Ts-Ts;
t_val = 0:Ts:length(u3_val)*Ts-Ts;
N = length(y3);
N_val = length(y3_val);

data3 = iddata(y3,u3,Ts);

%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
```

```
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    try
        sys = oe(data3, [na nb nk]);
        oe_y_hat_3 = lsim(sys, u3, t);
    catch
        break
    end

    [r2_oe, mse_oe] = rSQR(y3, oe_y_hat_3);

    error = y3 - oe_y_hat_3;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_oe, mse_oe, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_oe];
    MSEs = [MSEs; mse_oe];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.081631 | MSE=128.147675 | var=128.576263 | s_hat=76888.605097 |
------------------------------------------------------------
>>> Degree = 2 : R2=0.973406 | MSE=3.710940 | var=3.735846 | s_hat=2226.564102 |
------------------------------------------------------------
>>> Degree = 3 : R2=0.997642 | MSE=0.329021 | var=0.332345 | s_hat=197.412768 |
------------------------------------------------------------
>>> Degree = 4 : R2=0.997730 | MSE=0.316735 | var=0.321015 | s_hat=190.040824 |
------------------------------------------------------------
>>> Degree = 5 : R2=0.997767 | MSE=0.311633 | var=0.316915 | s_hat=186.979795 |
------------------------------------------------------------
>>> Degree = 6 : R2=0.997771 | MSE=0.311054 | var=0.317402 | s_hat=186.632100 |
------------------------------------------------------------
>>> Degree = 7 : R2=0.997789 | MSE=0.308561 | var=0.315932 | s_hat=185.136375 |
------------------------------------------------------------
>>> Degree = 8 : R2=0.997857 | MSE=0.299073 | var=0.307267 | s_hat=179.444081 |
------------------------------------------------------------
>>> Degree = 9 : R2=0.997792 | MSE=0.308169 | var=0.317700 | s_hat=184.901608 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.997095 | MSE=0.405294 | var=0.419270 | s_hat=243.176546 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.997781 | MSE=0.309570 | var=0.321352 | s_hat=185.741712 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.997802 | MSE=0.306657 | var=0.319435 | s_hat=183.994403 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.997793 | MSE=0.307974 | var=0.321924 | s_hat=184.784356 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.997815 | MSE=0.304908 | var=0.319833 | s_hat=182.944574 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.997831 | MSE=0.302638 | var=0.318566 | s_hat=181.582673 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.997829 | MSE=0.302923 | var=0.319989 | s_hat=181.753669 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.997776 | MSE=0.310368 | var=0.329012 | s_hat=186.220608 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.997818 | MSE=0.304528 | var=0.323966 | s_hat=182.716833 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.994546 | MSE=0.761032 | var=0.812490 | s_hat=456.619354 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.997121 | MSE=0.401713 | var=0.430407 | s_hat=241.027976 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.997854 | MSE=0.299473 | var=0.322014 | s_hat=179.683730 |
------------------------------------------------------------
>>> Degree = 22 : R2=0.997742 | MSE=0.315138 | var=0.340077 | s_hat=189.083019 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.997813 | MSE=0.305132 | var=0.330468 | s_hat=183.079029 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.997774 | MSE=0.310575 | var=0.337582 | s_hat=186.345120 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.997906 | MSE=0.292186 | var=0.318748 | s_hat=175.311344 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.997786 | MSE=0.308901 | var=0.338212 | s_hat=185.340303 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.997783 | MSE=0.309320 | var=0.339912 | s_hat=185.591819 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.997911 | MSE=0.291499 | var=0.321506 | s_hat=174.899115 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.997901 | MSE=0.292824 | var=0.324160 | s_hat=175.694599 |
```

```
------------------------------------------------------------
>>> Degree = 30 : R2=0.997943 | MSE=0.287040 | var=0.318933 | s_hat=172.224061 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.997952 | MSE=0.285744 | var=0.318674 | s_hat=171.446675 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.997982 | MSE=0.281628 | var=0.315256 | s_hat=168.977048 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.997959 | MSE=0.284741 | var=0.319933 | s_hat=170.844462 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.997969 | MSE=0.283391 | var=0.319614 | s_hat=170.034555 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.997942 | MSE=0.287194 | var=0.325126 | s_hat=172.316669 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.997981 | MSE=0.281722 | var=0.320139 | s_hat=169.033156 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.997969 | MSE=0.283439 | var=0.323314 | s_hat=170.063309 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.997951 | MSE=0.285896 | var=0.327362 | s_hat=171.537584 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.997954 | MSE=0.285473 | var=0.328130 | s_hat=171.283966 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.997887 | MSE=0.294880 | var=0.340246 | s_hat=176.927782 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.998038 | MSE=0.273731 | var=0.317063 | s_hat=164.238584 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.997822 | MSE=0.303927 | var=0.353404 | s_hat=182.356273 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.998034 | MSE=0.274381 | var=0.320290 | s_hat=164.628863 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.948373 | MSE=7.203984 | var=8.442169 | s_hat=4322.390326 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.997791 | MSE=0.308188 | var=0.362575 | s_hat=184.913031 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.994322 | MSE=0.792235 | var=0.935711 | s_hat=475.341044 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.997930 | MSE=0.288841 | var=0.342499 | s_hat=173.304628 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.997933 | MSE=0.288421 | var=0.343358 | s_hat=173.052592 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.998168 | MSE=0.255587 | var=0.305483 | s_hat=153.352461 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.996945 | MSE=0.426330 | var=0.511596 | s_hat=255.797805 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.997769 | MSE=0.311276 | var=0.375031 | s_hat=186.765564 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.997592 | MSE=0.335947 | var=0.406387 | s_hat=201.568022 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.997430 | MSE=0.358554 | var=0.435491 | s_hat=215.132340 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.998174 | MSE=0.254759 | var=0.310681 | s_hat=152.855117 |
------------------------------------------------------------
>>> Degree = 55 : R2=0.998222 | MSE=0.248088 | var=0.303781 | s_hat=148.852838 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.997100 | MSE=0.404713 | var=0.497598 | s_hat=242.827703 |
------------------------------------------------------------
>>> Degree = 57 : R2=0.998064 | MSE=0.270175 | var=0.333550 | s_hat=162.105196 |
------------------------------------------------------------
```

```
>>> Degree = 58 : R2=0.998011 | MSE=0.277552 | var=0.344073 | s_hat=166.531452 |
------------------------------------------------------------
>>> Degree = 59 : R2=0.995958 | MSE=0.564063 | var=0.702153 | s_hat=338.437940 |
------------------------------------------------------------
>>> Degree = 60 : R2=0.997864 | MSE=0.298010 | var=0.372513 | s_hat=178.806038 |
------------------------------------------------------------
>>> Degree = 61 : R2=0.996142 | MSE=0.538350 | var=0.675754 | s_hat=323.010273 |
------------------------------------------------------------
>>> Degree = 62 : R2=0.998183 | MSE=0.253586 | var=0.319646 | s_hat=152.151557 |
------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 55
```

```
na = bestFitDegree;
nb = bestFitDegree;
nk = 1;
p = na+nb;

BestFitModel_3 = oe(data3, [na nb nk])
```

```
BestFitModel_3 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.6374 z^-1 + 2.627 z^-2 + 5.379 z^-3 + 7.678 z^-4 + 8.304 z^-5 + 6.716 z^-6 + 3.693
z^-7
          + 0.6321 z^-8 - 1.437 z^-9 - 2.376 z^-10 - 2.391 z^-11 - 1.477 z^-12 + 0.3626 z^-13
          + 2.895 z^-14 + 5.592 z^-15 + 7.636 z^-16 + 8.599 z^-17 + 8.402 z^-18 + 7.087 z^-19
          + 5.156 z^-20 + 3.25 z^-21 + 1.891 z^-22 + 1.335 z^-23 + 1.379 z^-24 + 1.436 z^-25
```

```
           + 1.29 z^-26 + 1.242 z^-27 + 1.607 z^-28 + 2.47 z^-29 + 3.629 z^-30 + 4.677 z^-31 +
5.217 z^-32
           + 4.958 z^-33 + 3.659 z^-34 + 1.635 z^-35 - 0.5891 z^-36 - 2.054 z^-37 - 2.151 z^-38
           - 1.24 z^-39 + 0.1281 z^-40 + 1.197 z^-41 + 1.75 z^-42 + 2.203 z^-43 + 2.656 z^-44
           + 2.779 z^-45 + 2.328 z^-46 + 1.184 z^-47 - 0.2511 z^-48 - 1.235 z^-49 - 1.446 z^-50
                            - 0.9425 z^-51 - 0.345 z^-52 + 0.1304 z^-53 + 0.3016 z^-54 +
0.1119 z^-55


  F(z) = 1 + 1.259 z^-1 + 0.7318 z^-2 + 0.5336 z^-3 - 0.09561 z^-4 - 0.07937 z^-5 + 0.3406 z^
           -6 + 0.6459 z^-7 + 0.268 z^-8 - 0.2937 z^-9 - 0.3935 z^-10 - 0.1995 z^-11 + 0.2142
z^-12
           + 0.4767 z^-13 + 0.6344 z^-14 + 0.5722 z^-15 + 0.7 z^-16 + 0.7049 z^-17 + 0.6234 z^
           -18 + 0.6579 z^-19 + 0.5793 z^-20 + 0.4069 z^-21 + 0.1254 z^-22 - 0.1824 z^-23 -
0.5156 z^-24
           - 0.2694 z^-25 + 0.09218 z^-26 + 0.2206 z^-27 + 0.3677 z^-28 + 0.4156 z^-29 + 0.3263
z^-30
           + 0.2343 z^-31 + 0.02254 z^-32 - 0.147 z^-33 - 0.1576 z^-34 - 0.1061 z^-35 + 0.2561
z^-36
           + 0.03435 z^-37 - 0.2324 z^-38 - 0.3907 z^-39 - 0.4602 z^-40 + 0.01625 z^-41 +
0.4734 z^-42
           + 0.432 z^-43 + 0.05776 z^-44 - 0.004339 z^-45 - 0.3228 z^-46 - 0.04826 z^-47 +
0.3278 z^-48
           + 0.1065 z^-49 - 0.1782 z^-50 - 0.3873 z^-51 - 0.2657 z^-52 - 0.3056 z^-53 -
0.002488 z^-54
                                                                                            +
0.2142 z^-55

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=55   nf=55   nk=1
   Number of free coefficients: 110
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data3".
Fit to estimation data: 96.24%
FPE: 0.3476, MSE: 0.1977
```

```
BestFit_y_hat_3 = lsim(BestFitModel_3, u3_val, t_val);
% [oe_BestFit_r2, oe_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 55 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 55
```

```
na = minVarIndex;
nb = minVarIndex;
nk = 1;
p = na+nb;

oe_VarModel_3 = oe(data3, [na nb nk])
```

```
oe_VarModel_3 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.6374 z^-1 + 2.627 z^-2 + 5.379 z^-3 + 7.678 z^-4 + 8.304 z^-5 + 6.716 z^-6 + 3.693
z^-7
          + 0.6321 z^-8 - 1.437 z^-9 - 2.376 z^-10 - 2.391 z^-11 - 1.477 z^-12 + 0.3626 z^-13
          + 2.895 z^-14 + 5.592 z^-15 + 7.636 z^-16 + 8.599 z^-17 + 8.402 z^-18 + 7.087 z^-19
          + 5.156 z^-20 + 3.25 z^-21 + 1.891 z^-22 + 1.335 z^-23 + 1.379 z^-24 + 1.436 z^-25
          + 1.29 z^-26 + 1.242 z^-27 + 1.607 z^-28 + 2.47 z^-29 + 3.629 z^-30 + 4.677 z^-31 +
5.217 z^-32
          + 4.958 z^-33 + 3.659 z^-34 + 1.635 z^-35 - 0.5891 z^-36 - 2.054 z^-37 - 2.151 z^-38
          - 1.24 z^-39 + 0.1281 z^-40 + 1.197 z^-41 + 1.75 z^-42 + 2.203 z^-43 + 2.656 z^-44
          + 2.779 z^-45 + 2.328 z^-46 + 1.184 z^-47 - 0.2511 z^-48 - 1.235 z^-49 - 1.446 z^-50
                            - 0.9425 z^-51 - 0.345 z^-52 + 0.1304 z^-53 + 0.3016 z^-54 +
0.1119 z^-55


  F(z) = 1 + 1.259 z^-1 + 0.7318 z^-2 + 0.5336 z^-3 - 0.09561 z^-4 - 0.07937 z^-5 + 0.3406 z^
          -6 + 0.6459 z^-7 + 0.268 z^-8 - 0.2937 z^-9 - 0.3935 z^-10 - 0.1995 z^-11 + 0.2142
z^-12
          + 0.4767 z^-13 + 0.6344 z^-14 + 0.5722 z^-15 + 0.7 z^-16 + 0.7049 z^-17 + 0.6234 z^
          -18 + 0.6579 z^-19 + 0.5793 z^-20 + 0.4069 z^-21 + 0.1254 z^-22 - 0.1824 z^-23 -
0.5156 z^-24
          - 0.2694 z^-25 + 0.09218 z^-26 + 0.2206 z^-27 + 0.3677 z^-28 + 0.4156 z^-29 + 0.3263
z^-30
          + 0.2343 z^-31 + 0.02254 z^-32 - 0.147 z^-33 - 0.1576 z^-34 - 0.1061 z^-35 + 0.2561
z^-36
          + 0.03435 z^-37 - 0.2324 z^-38 - 0.3907 z^-39 - 0.4602 z^-40 + 0.01625 z^-41 +
0.4734 z^-42
          + 0.432 z^-43 + 0.05776 z^-44 - 0.004339 z^-45 - 0.3228 z^-46 - 0.04826 z^-47 +
0.3278 z^-48
          + 0.1065 z^-49 - 0.1782 z^-50 - 0.3873 z^-51 - 0.2657 z^-52 - 0.3056 z^-53 -
0.002488 z^-54
                                                                                          +
0.2142 z^-55

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=55   nf=55   nk=1
   Number of free coefficients: 110
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
```

```
Status:
Estimated using OE on time domain data "data3".
Fit to estimation data: 96.24%
FPE: 0.3476, MSE: 0.1977
```

```
Var_y_hat_3 = lsim(oe_VarModel_3, u3_val, t_val);
% [oe_Var_r2, oe_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("==================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 8 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 8
```

```
na = minAICIndex;
nb = minAICIndex;
nk = 1;
p = na+nb;

oe_AICModel_3 = oe(data3, [na nb nk])
```

```
oe_AICModel_3 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.6027 z^-1 + 1.577 z^-2 + 1.886 z^-3 + 1.45 z^-4 + 0.1744 z^-5 - 0.6966 z^-6 -
0.9147 z^-7
                                                                                         -
0.4228 z^-8


  F(z) = 1 - 0.4023 z^-1 + 0.04693 z^-2 - 0.5746 z^-3 + 0.1772 z^-4 + 0.3222 z^-5 + 0.3537 z^
                                                                      -6 + 0.06771 z^-7 -
0.7082 z^-8
```

```
Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=8   nf=8   nk=1
   Number of free coefficients: 16
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data3".
Fit to estimation data: 95.37%
FPE: 0.3155, MSE: 0.2991
```

```
AIC_y_hat_3 = lsim(oe_AICModel_3, u3_val, t_val);
% [oe_AIC_r2, oe_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.26 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 8
```

```
na = winner;
nb = winner;
nk = 1;
p = na+nb;

oe_FTestModel_3 = oe(data3, [na nb nk])
```

```
oe_FTestModel_3 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.6027 z^-1 + 1.577 z^-2 + 1.886 z^-3 + 1.45 z^-4 + 0.1744 z^-5 - 0.6966 z^-6 -
0.9147 z^-7
                                                                                         -
0.4228 z^-8


  F(z) = 1 - 0.4023 z^-1 + 0.04693 z^-2 - 0.5746 z^-3 + 0.1772 z^-4 + 0.3222 z^-5 + 0.3537 z^
                                                                      -6 + 0.06771 z^-7 -
0.7082 z^-8

Sample time: 0.5 seconds

Parameterization:
   Polynomial orders:   nb=8   nf=8   nk=1
   Number of free coefficients: 16
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data3".
Fit to estimation data: 95.37%
FPE: 0.3155, MSE: 0.2991
```

```
FTest_y_hat_3 = lsim(oe_FTestModel_3, u3_val, t_val);
% [oe_FTest_r2, oe_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[oe_BestFit_r2_3, oe_BestFit_mse_3] = rSQR(y3_val, BestFit_y_hat_3);
[oe_Var_r2_3, oe_Var_mse_3] = rSQR(y3_val, Var_y_hat_3);
[oe_AIC_r2_3, oe_AIC_mse_3] = rSQR(y3_val, AIC_y_hat_3);
[oe_FTest_r2_3, oe_FTest_mse_3] = rSQR(y3_val, FTest_y_hat_3);

%%

fprintf("==================Evaluation | R2 Metric=====================\n")
```

```
==================Evaluation | R2 Metric=====================
```

```
fprintf("--------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_BestFit_r2_3,
oe_BestFit_mse_3)
```

```
    R2 value : 0.9967    | MSE : 0.4250
```

```
fprintf("--------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Var_r2_3, oe_Var_mse_3)
```

```
    R2 value : 0.9967    | MSE : 0.4250
```

```
% fprintf("--------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Cov_r2, oe_Cov_mse)
fprintf("--------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_AIC_r2_3, oe_AIC_mse_3)
```

```
    R2 value : 0.9965    | MSE : 0.4541
```

```
fprintf("--------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_FTest_r2_3, oe_FTest_mse_3)
```

```
    R2 value : 0.9965    | MSE : 0.4541
```

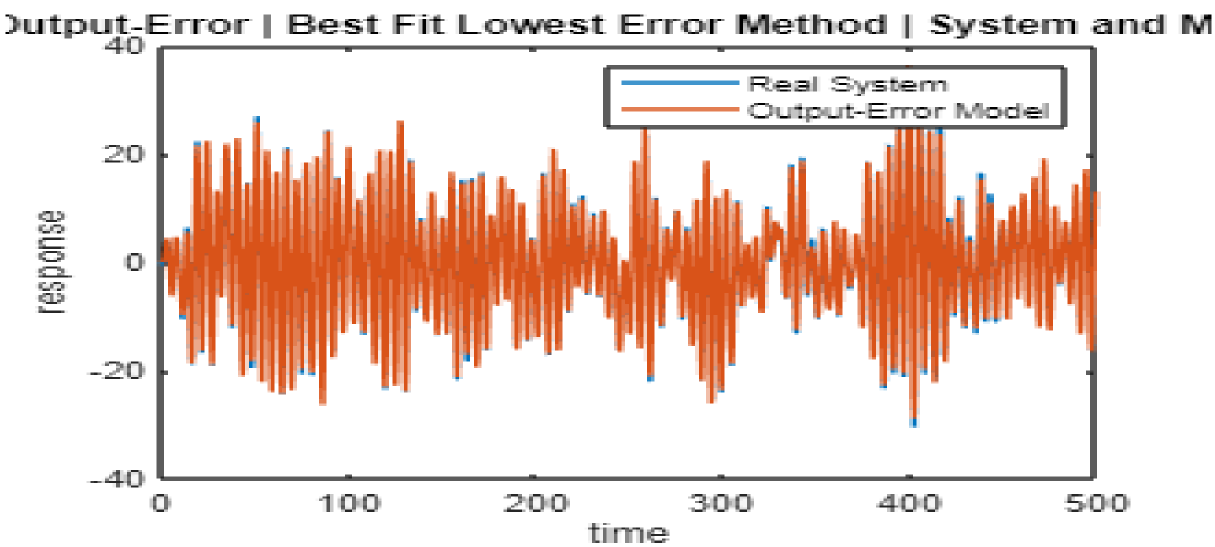
```
fprintf("--------------------------------------------------------------\n")
```

```
--------------------------------------------------------------
```

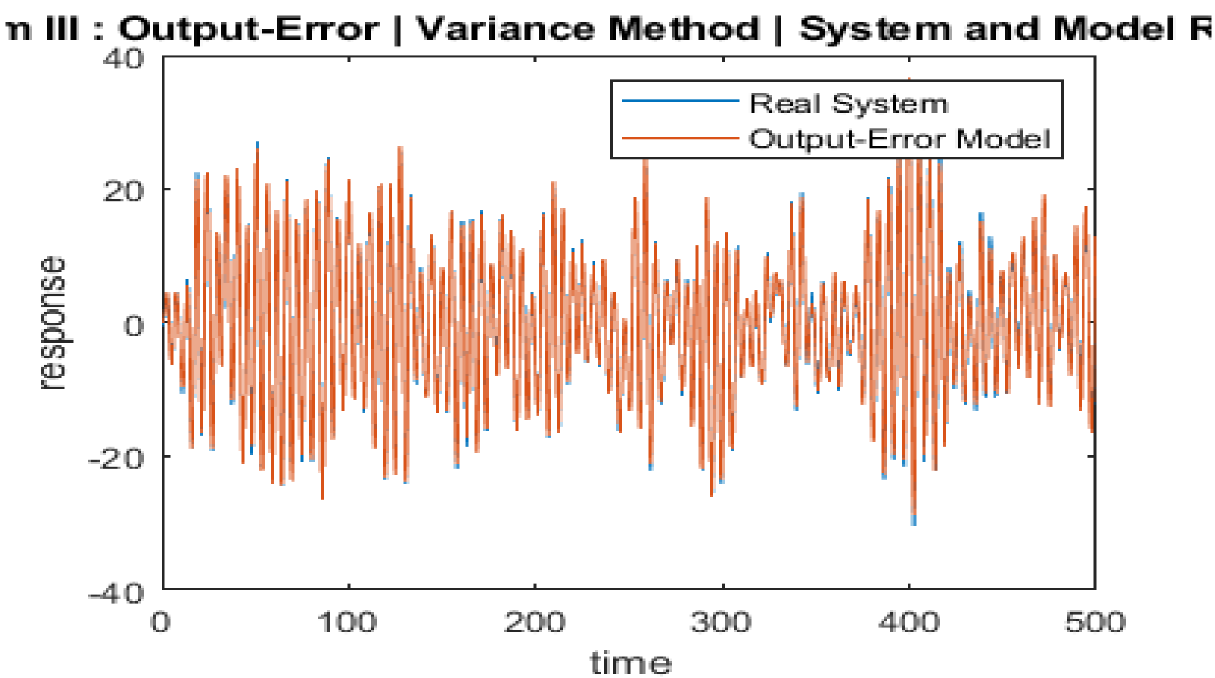
```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("==============================================================\n")
```

```
==============================================================
```

```



%%

oe_BestFitError_3 = y3_val - BestFit_y_hat_3;
oe_VarError_3 = y3_val - Var_y_hat_3;
oe_AICError_3 = y3_val - AIC_y_hat_3;
oe_FTestError_3 = y3_val - FTest_y_hat_3;

for k=0:N_val-1
    oe_BestFit_Ree_3(k+1,1) = AutoCorrelate(oe_BestFitError_3, k);
    oe_Var_Ree_3(k+1,1) = AutoCorrelate(oe_VarError_3, k);
    oe_AIC_Ree_3(k+1,1) = AutoCorrelate(oe_AICError_3, k);
    oe_FTest_Ree_3(k+1,1) = AutoCorrelate(oe_FTestError_3, k);
end

for k=0:N_val-1
    oe_BestFit_Rue_3(k+1,1) = CrossCorrelate(u3_val, oe_BestFitError_3, k);
    oe_Var_Rue_3(k+1,1) = CrossCorrelate(u3_val, oe_VarError_3, k);
    oe_AIC_Rue_3(k+1,1) = CrossCorrelate(u3_val, oe_AICError_3, k);
    oe_FTest_Rue_3(k+1,1) = CrossCorrelate(u3_val, oe_FTestError_3, k);
end


%%
figure(1)
plot(t_val,y3_val,t_val,BestFit_y_hat_3)
legend('Real System','Output-Error Model')
title(" System III : Output-Error | Best Fit Lowest Error Method | System and
Model Response")
xlabel("time")
ylabel("response")
```

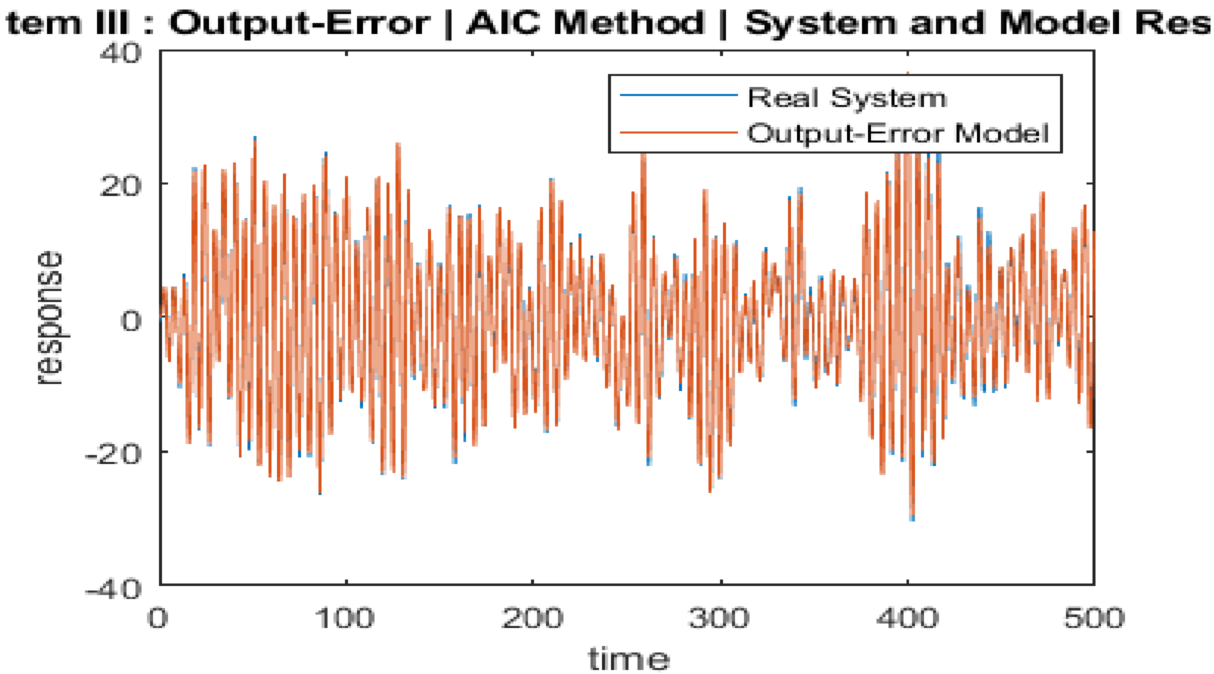



```
figure(2)
plot(t_val,y3_val,t_val,Var_y_hat_3)
legend('Real System','Output-Error Model')
title(" System III : Output-Error | Variance Method | System and Model
Response")
xlabel("time")
ylabel("response")
```

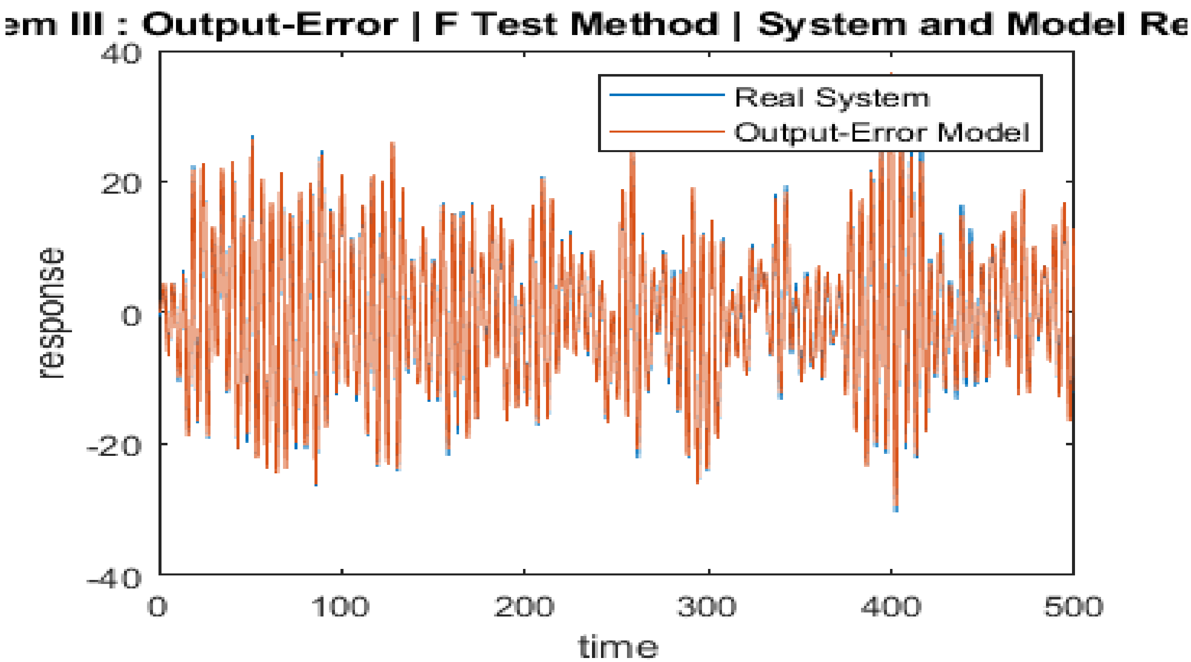



```
figure(3)
```

```
plot(t_val,y3_val,t_val,AIC_y_hat_3)
legend('Real System','Output-Error Model')
title(" System III : Output-Error | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```

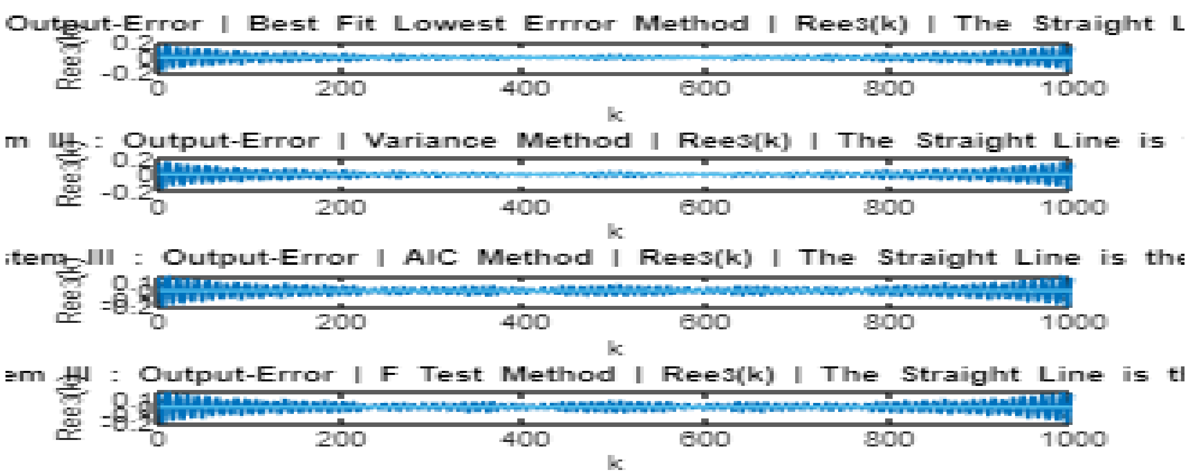



```
figure(4)
plot(t_val,y3_val,t_val,FTest_y_hat_3)
legend('Real System','Output-Error Model')
title(" System III : Output-Error | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```



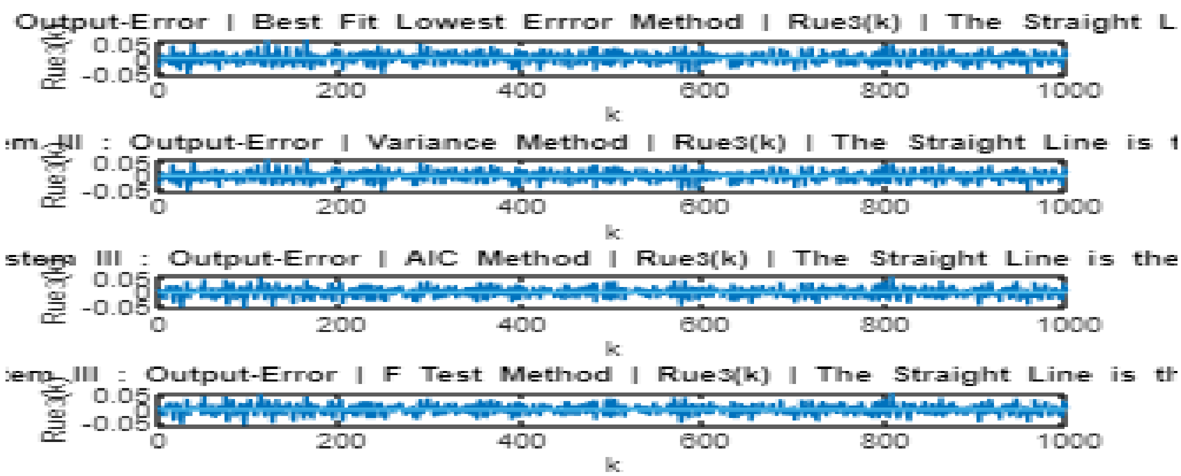

```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,oe_BestFit_Ree_3(2:end), 1:N_val-1,
mean(oe_BestFit_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Output-Error | Best Fit Lowest Errror Method | Ree_3(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_3(k)")

subplot(4,1,2)
plot(1:N_val-1,oe_Var_Ree_3(2:end), 1:N_val-1,
mean(oe_Var_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Output-Error | Variance Method | Ree_3(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree_3(k)")

subplot(4,1,3)
plot(1:N_val-1,oe_AIC_Ree_3(2:end), 1:N_val-1,
mean(oe_AIC_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Output-Error | AIC Method | Ree_3(k) | The Straight Line
is the Mean")
xlabel("k")
ylabel("Ree_3(k)")
```

```
subplot(4,1,4)
plot(1:N_val-1,oe_FTest_Ree_3(2:end), 1:N_val-1,
mean(oe_FTest_Ree_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Output-Error | F Test Method | Ree_3(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Ree_3(k)")
```




```
%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,oe_BestFit_Rue_3(2:end), 1:N_val-1,
mean(oe_BestFit_Rue_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Output-Error | Best Fit Lowest Errror Method | Rue_3(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_3(k)")

subplot(4,1,2)
plot(1:N_val-1,oe_Var_Rue_3(2:end), 1:N_val-1,
mean(oe_Var_Rue_3(2:end))*ones(length(1:N_val-1)))
title(" System III : Output-Error | Variance Method | Rue_3(k) | The Straight
Line is the Mean")
xlabel("k")
ylabel("Rue_3(k)")

subplot(4,1,3)
```

```
 plot(1:N_val-1,oe_AIC_Rue_3(2:end), 1:N_val-1,
mean(oe_AIC_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : Output-Error | AIC Method | Rue_3(k) | The Straight Line
is the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")

 subplot(4,1,4)
 plot(1:N_val-1,oe_FTest_Rue_3(2:end), 1:N_val-1,
mean(oe_FTest_Rue_3(2:end))*ones(length(1:N_val-1)))
 title(" System III : Output-Error | F Test Method | Rue_3(k) | The Straight
Line is the Mean")
 xlabel("k")
 ylabel("Rue_3(k)")
```



Q3 - part a | ARX

```
clc; clear;

%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;
```

Guassian Input - PRBS Validation

```
%%
% Guassian Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> Guassian Input Identification Begins:-----------------------------
-\n")
```

```
>>> Guassian Input Identification Begins:------------------------------
```

```
%%
Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

%%

fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
```

```
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    U = arx_U_builder_3(u1, y1, na, nb, nk);
    theta_hat_1 = inv(U'*U)*U'*y1;
    y_hat_1 = form_tf_lsim_2(theta_hat_1, u1, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y1, y_hat_1);

    error = y1 - y_hat_1;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    covariance = variance*inv(U'*U);
    cov = trace(covariance)/p;
    covs = [covs; cov];

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_arx, mse_arx, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_arx];
    MSEs = [MSEs; mse_arx];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.893369 | MSE=0.028697 | var=0.028711 | s_hat=117.541609 |
-----------------------------------------------------------
```

```
>>> Degree = 2 : R2=0.921083 | MSE=0.021238 | var=0.021259 | s_hat=86.991301 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.935419 | MSE=0.017380 | var=0.017406 | s_hat=71.189426 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.940151 | MSE=0.016107 | var=0.016138 | s_hat=65.973187 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.940996 | MSE=0.015879 | var=0.015918 | s_hat=65.041472 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.940908 | MSE=0.015903 | var=0.015950 | s_hat=65.138384 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.940825 | MSE=0.015925 | var=0.015980 | s_hat=65.229326 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.940875 | MSE=0.015912 | var=0.015974 | s_hat=65.174834 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.941038 | MSE=0.015868 | var=0.015938 | s_hat=64.994673 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.941086 | MSE=0.015855 | var=0.015933 | s_hat=64.941520 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.941092 | MSE=0.015853 | var=0.015939 | s_hat=64.935054 |
-------------------------------------------------------------
>>> Degree = 12 : R2=0.941090 | MSE=0.015854 | var=0.015947 | s_hat=64.937948 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.941094 | MSE=0.015853 | var=0.015954 | s_hat=64.932930 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.941106 | MSE=0.015850 | var=0.015959 | s_hat=64.919906 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.941135 | MSE=0.015842 | var=0.015959 | s_hat=64.888210 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.941138 | MSE=0.015841 | var=0.015966 | s_hat=64.884185 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.941140 | MSE=0.015840 | var=0.015973 | s_hat=64.882604 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.941148 | MSE=0.015838 | var=0.015979 | s_hat=64.873412 |
-------------------------------------------------------------
>>> Degree = 19 : R2=0.941198 | MSE=0.015825 | var=0.015973 | s_hat=64.818534 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.941197 | MSE=0.015825 | var=0.015981 | s_hat=64.819543 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.941199 | MSE=0.015825 | var=0.015989 | s_hat=64.817919 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.941218 | MSE=0.015819 | var=0.015991 | s_hat=64.796573 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.941197 | MSE=0.015825 | var=0.016005 | s_hat=64.819753 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.941259 | MSE=0.015808 | var=0.015996 | s_hat=64.750954 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.941274 | MSE=0.015804 | var=0.016000 | s_hat=64.735108 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.941273 | MSE=0.015805 | var=0.016008 | s_hat=64.735598 |
-------------------------------------------------------------
>>> Degree = 27 : R2=0.941288 | MSE=0.015801 | var=0.016012 | s_hat=64.719315 |
-------------------------------------------------------------
>>> Degree = 28 : R2=0.941342 | MSE=0.015786 | var=0.016005 | s_hat=64.660046 |
-------------------------------------------------------------
>>> Degree = 29 : R2=0.941490 | MSE=0.015746 | var=0.015972 | s_hat=64.496433 |
-------------------------------------------------------------
>>> Degree = 30 : R2=0.941492 | MSE=0.015746 | var=0.015980 | s_hat=64.494680 |
```

```
------------------------------------------------------------
>>> Degree = 31 : R2=0.941495 | MSE=0.015745 | var=0.015987 | s_hat=64.490803 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.941498 | MSE=0.015744 | var=0.015994 | s_hat=64.488439 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.941493 | MSE=0.015745 | var=0.016003 | s_hat=64.493340 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.941526 | MSE=0.015736 | var=0.016002 | s_hat=64.456559 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.941518 | MSE=0.015739 | var=0.016012 | s_hat=64.465609 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.941524 | MSE=0.015737 | var=0.016019 | s_hat=64.459198 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.941517 | MSE=0.015739 | var=0.016029 | s_hat=64.467224 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.941565 | MSE=0.015726 | var=0.016023 | s_hat=64.414385 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.941648 | MSE=0.015704 | var=0.016009 | s_hat=64.322692 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.941671 | MSE=0.015697 | var=0.016010 | s_hat=64.296677 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.941672 | MSE=0.015697 | var=0.016018 | s_hat=64.295667 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.941672 | MSE=0.015697 | var=0.016026 | s_hat=64.295835 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.941671 | MSE=0.015698 | var=0.016034 | s_hat=64.297409 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.941678 | MSE=0.015696 | var=0.016040 | s_hat=64.289505 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.941701 | MSE=0.015690 | var=0.016042 | s_hat=64.264299 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.941702 | MSE=0.015689 | var=0.016050 | s_hat=64.263342 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.941701 | MSE=0.015689 | var=0.016058 | s_hat=64.263776 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.941724 | MSE=0.015683 | var=0.016060 | s_hat=64.239120 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.941752 | MSE=0.015676 | var=0.016060 | s_hat=64.207365 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.941756 | MSE=0.015675 | var=0.016067 | s_hat=64.203447 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.941777 | MSE=0.015669 | var=0.016069 | s_hat=64.180813 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.941790 | MSE=0.015666 | var=0.016074 | s_hat=64.166528 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.941795 | MSE=0.015664 | var=0.016080 | s_hat=64.160593 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.941834 | MSE=0.015654 | var=0.016077 | s_hat=64.116976 |
------------------------------------------------------------
>>> Degree = 55 : R2=0.941868 | MSE=0.015645 | var=0.016076 | s_hat=64.080445 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.941888 | MSE=0.015639 | var=0.016079 | s_hat=64.057756 |
------------------------------------------------------------
>>> Degree = 57 : R2=0.941913 | MSE=0.015632 | var=0.016080 | s_hat=64.030009 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.941961 | MSE=0.015620 | var=0.016075 | s_hat=63.977504 |
------------------------------------------------------------
```

```
>>> Degree = 59 : R2=0.942054 | MSE=0.015594 | var=0.016057 | s_hat=63.874931 |
-------------------------------------------------------------
>>> Degree = 60 : R2=0.942057 | MSE=0.015594 | var=0.016064 | s_hat=63.871439 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.942151 | MSE=0.015568 | var=0.016046 | s_hat=63.767683 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.942163 | MSE=0.015565 | var=0.016051 | s_hat=63.755093 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.942164 | MSE=0.015565 | var=0.016059 | s_hat=63.754258 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.942154 | MSE=0.015567 | var=0.016070 | s_hat=63.764408 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.942182 | MSE=0.015560 | var=0.016070 | s_hat=63.734375 |
-------------------------------------------------------------
>>> Degree = 66 : R2=0.942203 | MSE=0.015554 | var=0.016072 | s_hat=63.710548 |
-------------------------------------------------------------
>>> Degree = 67 : R2=0.942195 | MSE=0.015557 | var=0.016083 | s_hat=63.719656 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.942201 | MSE=0.015555 | var=0.016089 | s_hat=63.713365 |
-------------------------------------------------------------
>>> Degree = 69 : R2=0.942215 | MSE=0.015551 | var=0.016093 | s_hat=63.697869 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.942217 | MSE=0.015550 | var=0.016101 | s_hat=63.694837 |
-------------------------------------------------------------
>>> Degree = 71 : R2=0.942214 | MSE=0.015551 | var=0.016110 | s_hat=63.698685 |
-------------------------------------------------------------
>>> Degree = 72 : R2=0.942185 | MSE=0.015559 | var=0.016126 | s_hat=63.730056 |
-------------------------------------------------------------
>>> Degree = 73 : R2=0.942222 | MSE=0.015549 | var=0.016124 | s_hat=63.689508 |
-------------------------------------------------------------
>>> Degree = 74 : R2=0.942225 | MSE=0.015549 | var=0.016131 | s_hat=63.686915 |
-------------------------------------------------------------
>>> Degree = 75 : R2=0.942253 | MSE=0.015541 | var=0.016132 | s_hat=63.656097 |
-------------------------------------------------------------
>>> Degree = 76 : R2=0.942272 | MSE=0.015536 | var=0.016134 | s_hat=63.634316 |
-------------------------------------------------------------
>>> Degree = 77 : R2=0.942278 | MSE=0.015534 | var=0.016141 | s_hat=63.628294 |
-------------------------------------------------------------
>>> Degree = 78 : R2=0.942289 | MSE=0.015531 | var=0.016146 | s_hat=63.616145 |
-------------------------------------------------------------
>>> Degree = 79 : R2=0.942284 | MSE=0.015533 | var=0.016156 | s_hat=63.621317 |
-------------------------------------------------------------
>>> Degree = 80 : R2=0.942299 | MSE=0.015529 | var=0.016160 | s_hat=63.604835 |
-------------------------------------------------------------
>>> Degree = 81 : R2=0.942347 | MSE=0.015516 | var=0.016155 | s_hat=63.552315 |
-------------------------------------------------------------
>>> Degree = 82 : R2=0.942369 | MSE=0.015510 | var=0.016157 | s_hat=63.527831 |
-------------------------------------------------------------
>>> Degree = 83 : R2=0.942384 | MSE=0.015506 | var=0.016161 | s_hat=63.511142 |
-------------------------------------------------------------
>>> Degree = 84 : R2=0.942416 | MSE=0.015497 | var=0.016160 | s_hat=63.475808 |
-------------------------------------------------------------
>>> Degree = 85 : R2=0.942431 | MSE=0.015493 | var=0.016164 | s_hat=63.459004 |
-------------------------------------------------------------
>>> Degree = 86 : R2=0.942432 | MSE=0.015493 | var=0.016172 | s_hat=63.458308 |
-------------------------------------------------------------
>>> Degree = 87 : R2=0.942430 | MSE=0.015493 | var=0.016181 | s_hat=63.460029 |
```

```
------------------------------------------------------------
>>> Degree = 88 : R2=0.942431 | MSE=0.015493 | var=0.016189 | s_hat=63.458969 |
------------------------------------------------------------
>>> Degree = 89 : R2=0.942441 | MSE=0.015490 | var=0.016194 | s_hat=63.448653 |
------------------------------------------------------------
>>> Degree = 90 : R2=0.942464 | MSE=0.015484 | var=0.016196 | s_hat=63.422611 |
------------------------------------------------------------
>>> Degree = 91 : R2=0.942504 | MSE=0.015473 | var=0.016193 | s_hat=63.379223 |
------------------------------------------------------------
>>> Degree = 92 : R2=0.942508 | MSE=0.015472 | var=0.016200 | s_hat=63.374970 |
------------------------------------------------------------
>>> Degree = 93 : R2=0.942531 | MSE=0.015466 | var=0.016202 | s_hat=63.349616 |
------------------------------------------------------------
>>> Degree = 94 : R2=0.942526 | MSE=0.015467 | var=0.016211 | s_hat=63.354408 |
------------------------------------------------------------
>>> Degree = 95 : R2=0.942551 | MSE=0.015461 | var=0.016213 | s_hat=63.327613 |
------------------------------------------------------------
>>> Degree = 96 : R2=0.942572 | MSE=0.015455 | var=0.016215 | s_hat=63.303681 |
------------------------------------------------------------
>>> Degree = 97 : R2=0.942578 | MSE=0.015454 | var=0.016222 | s_hat=63.297899 |
------------------------------------------------------------
>>> Degree = 98 : R2=0.942582 | MSE=0.015452 | var=0.016229 | s_hat=63.293225 |
------------------------------------------------------------
>>> Degree = 99 : R2=0.942589 | MSE=0.015451 | var=0.016235 | s_hat=63.285413 |
------------------------------------------------------------
>>> Degree = 100 : R2=0.942601 | MSE=0.015447 | var=0.016240 | s_hat=63.272015 |
------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("==============Degree Extraction | BestFit Method================\n")
```

```
==============Degree Extraction | BestFit Method================
```

```
bestFitDegree = find(S_hats == min(S_hats));
fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 100
```

```
na = bestFitDegree;
```

```
nb = bestFitDegree;
p = na+nb;

BestFitU = arx_U_builder_3(u1, y1, na, nb, nk);
BestFitModel_1 = inv(BestFitU'*BestFitU)*BestFitU'*y1;
BestFit_y_hat_1 = form_tf_lsim_2(BestFitModel_1, u1_val, t_val, na, Ts);
```

```
G =


  0.3952 z^-1 + 0.2013 z^-2 + 0.05884 z^-3 - 0.01008 z^-4 - 0.04394 z^-5 - 0.04718 z^-6 - 
0.03742 z^-7

          - 0.02346 z^-8 - 0.002027 z^-9 + 0.005351 z^-10 + 0.02352 z^-11 + 0.01018 z^-12 + 
0.001935 z^-13

          + 0.002625 z^-14 - 0.005436 z^-15 - 0.01269 z^-16 - 0.00866 z^-17 - 0.002842 z^-18

          + 0.007692 z^-19 + 0.01169 z^-20 + 0.007725 z^-21 + 0.005266 z^-22 - 0.01339 z^-23

          - 0.001967 z^-24 - 0.0107 z^-25 - 0.01187 z^-26 - 0.009003 z^-27 - 0.001271 z^-28

          - 0.01018 z^-29 + 0.00474 z^-30 + 0.005581 z^-31 + 0.005109 z^-32 + 0.003274 z^-33

          - 0.005808 z^-34 - 0.003475 z^-35 - 0.00721 z^-36 - 0.001599 z^-37 - 0.008035 z^-38

          - 0.00861 z^-39 - 0.005435 z^-40 - 0.000616 z^-41 - 0.005684 z^-42 + 0.002887 z^-43

          - 0.006308 z^-44 - 0.01445 z^-45 - 0.006217 z^-46 - 0.003075 z^-47 - 0.00297 z^-48

          - 0.002429 z^-49 + 0.00218 z^-50 - 0.002983 z^-51 - 0.002373 z^-52 + 0.0001564 z^

          -53 - 0.009219 z^-54 - 0.01336 z^-55 - 0.002562 z^-56 + 0.001818 z^-57 - 0.0003685 
z^-58

          + 0.0006183 z^-59 + 0.011 z^-60 + 0.001036 z^-61 - 0.008957 z^-62 - 0.007354 z^-63

          - 0.01195 z^-64 - 0.002321 z^-65 + 0.004528 z^-66 + 0.01045 z^-67 + 0.003609 z^-68

          + 0.003437 z^-69 - 0.01007 z^-70 - 0.01464 z^-71 - 0.01431 z^-72 + 0.001798 z^-73

          + 0.0005956 z^-74 + 0.001274 z^-75 + 0.003096 z^-76 + 0.008544 z^-77 + 0.009464 z^-
78

          + 0.008679 z^-79 + 0.0002531 z^-80 - 0.005127 z^-81 - 0.003588 z^-82 - 0.0009237 z^-
83

          - 0.004912 z^-84 + 0.002198 z^-85 + 0.002379 z^-86 + 0.003106 z^-87 + 0.0004893 z^-
88

          - 0.009167 z^-89 - 0.009135 z^-90 - 0.01281 z^-91 - 0.003873 z^-92 - 0.002348 z^-93

          - 0.007815 z^-94 + 0.003071 z^-95 + 0.004305 z^-96 + 0.004672 z^-97 + 0.01379 z^-98
```

```
                                                                      + 0.01358 z^-99 - 0.001956
z^-100

  ----------------------------------------------------------------------------------------------
-------------

  1 - 0.1698 z^-1 - 0.07406 z^-2 - 0.0201 z^-3 - 0.01776 z^-4 - 0.01578 z^-5 - 0.01214 z^-6

          - 0.009367 z^-7 + 0.01639 z^-8 - 0.002319 z^-9 + 0.03685 z^-10 - 0.01465 z^-11 -
0.005572 z^-12

          + 0.009622 z^-13 - 0.02265 z^-14 - 0.01382 z^-15 - 0.0006951 z^-16 + 0.009371 z^-17

          + 0.02258 z^-18 + 0.003618 z^-19 + 0.001623 z^-20 + 0.002933 z^-21 - 0.03647 z^-22

          + 0.02081 z^-23 - 0.01966 z^-24 - 0.02675 z^-25 - 0.00429 z^-26 + 0.003592 z^-27

          - 0.01939 z^-28 + 0.005834 z^-29 + 0.00534 z^-30 + 0.005536 z^-31 + 0.004395 z^-32

          - 0.0224 z^-33 + 0.01244 z^-34 - 0.01973 z^-35 + 0.007261 z^-36 - 0.01669 z^-37 -
0.01196 z^-38

          - 0.008406 z^-39 + 0.01337 z^-40 - 0.01415 z^-41 + 0.01748 z^-42 - 0.02048 z^-43

          - 0.02723 z^-44 + 0.00312 z^-45 + 0.001622 z^-46 - 0.001528 z^-47 - 0.005533 z^-48

          + 0.004076 z^-49 - 0.01613 z^-50 + 0.008857 z^-51 + 0.003969 z^-52 - 0.03025 z^-53

          - 0.01175 z^-54 + 0.01751 z^-55 + 0.004236 z^-56 - 0.008606 z^-57 + 0.006453 z^-58

          + 0.02501 z^-59 - 0.01605 z^-60 - 0.007327 z^-61 - 0.0003832 z^-62 - 0.02015 z^-63

          + 0.01845 z^-64 + 0.006833 z^-65 + 0.01162 z^-66 - 0.008151 z^-67 + 0.001365 z^-68

          - 0.02299 z^-69 - 0.01432 z^-70 - 0.01938 z^-71 + 0.0269 z^-72 - 0.01064 z^-73 +
0.004234 z^-74

          - 0.0008938 z^-75 + 0.01052 z^-76 + 0.01527 z^-77 + 0.01315 z^-78 - 0.007356 z^-79

          - 0.01094 z^-80 - 0.0004854 z^-81 + 0.008973 z^-82 - 0.01211 z^-83 + 0.01161 z^-84

          + 0.007519 z^-85 + 0.004661 z^-86 - 0.004127 z^-87 - 0.01823 z^-88 - 0.01522 z^-89

          - 0.004363 z^-90 - 0.002628 z^-91 - 0.004715 z^-92 - 0.01285 z^-93 + 0.01705 z^-94

          - 0.004155 z^-95 + 0.007878 z^-96 + 0.02789 z^-97 + 0.006448 z^-98 - 0.02349 z^-99

                                                                                     + 0.01182
z^-100


Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
%%
```

```
fprintf("==============Degree Extraction | Variance
Method===================\n")
```

```
==============Degree Extraction | Variance Method===================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 5 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 5
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb;

VarU = arx_U_builder_3(u1, y1, na, nb, nk);
VarModel_1 = inv(VarU'*VarU)*VarU'*y1;
Var_y_hat_1 = form_tf_lsim_2(VarModel_1, u1_val, t_val, na, Ts);
```

```
G =

    0.3951 z^-1 + 0.1927 z^-2 + 0.04997 z^-3 - 0.01372 z^-4 - 0.02994 z^-5
  ---------------------------------------------------------------------------
  1 - 0.1897 z^-1 - 0.08256 z^-2 - 0.01541 z^-3 + 0.01903 z^-4 + 0.07503 z^-5

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("==============Degree Extraction | CoVariance
Method================\n")
```

```
==============Degree Extraction | CoVariance Method================
```

```
maxCovIndex = find(covs == min(covs));
```

```
fprintf(">>> Since the minimum CovMatrix trace occurs in iteration %d ;\n", maxCovIndex)
```

```
>>> Since the minimum CovMatrix trace occurs in iteration 1 ;
```

```
fprintf("    Degree = %d \n", maxCovIndex)
```

```
    Degree = 1
```

```
na = maxCovIndex;
nb = maxCovIndex;
p = na+nb;

CovU_1 = arx_U_builder_3(u1,y1,na,nb,1);
CovModel_1 = inv(CovU_1'*CovU_1)*CovU_1'*y1;

CovU_1_val = arx_U_builder_3(u1_val,y1_val,na,nb,1);
Cov_y_hat_1 = CovU_1_val*CovModel_1;

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n", k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 5 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 5
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb;

AICU_1 = arx_U_builder_3(u1, y1, na, nb, nk);
AICModel_1 = inv(AICU_1'*AICU_1)*AICU_1'*y1;
```

```
AIC_y_hat_1 = form_tf_lsim_2(AICModel_1, u1_val, t_val, na, Ts);
```

```
G =

    0.3951 z^-1 + 0.1927 z^-2 + 0.04997 z^-3 - 0.01372 z^-4 - 0.02994 z^-5
  ---------------------------------------------------------------------------
  1 - 0.1897 z^-1 - 0.08256 z^-2 - 0.01541 z^-3 + 0.01903 z^-4 + 0.07503 z^-5

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.18 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 5
```

```
na = winner;
nb = winner;
p = na+nb;
```

```
FTestU_1 = arx_U_builder_3(u1, y1, na, nb, nk);
FTestModel_1 = inv(FTestU_1'*FTestU_1)*FTestU_1'*y1;
FTest_y_hat_1 = form_tf_lsim_2(FTestModel_1, u1_val, t_val, na, Ts);
```

```
G =

    0.3951 z^-1 + 0.1927 z^-2 + 0.04997 z^-3 - 0.01372 z^-4 - 0.02994 z^-5
  ---------------------------------------------------------------------------
  1 - 0.1897 z^-1 - 0.08256 z^-2 - 0.01541 z^-3 + 0.01903 z^-4 + 0.07503 z^-5

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[BestFit_r2, BestFit_mse] = rSQR(y1_val, BestFit_y_hat_1);
[Var_r2, Var_mse] = rSQR(y1_val, Var_y_hat_1);
[AIC_r2, AIC_mse] = rSQR(y1_val, AIC_y_hat_1);
[Cov_r2, Cov_mse] = rSQR(y1_val, Cov_y_hat_1);
[FTest_r2, FTest_mse] = rSQR(y1_val, FTest_y_hat_1);
fprintf("===========================PRBS Ident - Guassian
ValidI===========================\n")
```

```
===========================PRBS Ident - Guassian ValidI===========================
```

```
fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.9419    | MSE : 0.0155
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.9429    | MSE : 0.0152
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Covariance Method:\n")
```

```
>>> Covariance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Cov_r2, Cov_mse)
```

```
    R2 value : 0.9167    | MSE : 0.0222
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.9429    | MSE : 0.0152
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.9429    | MSE : 0.0152
```

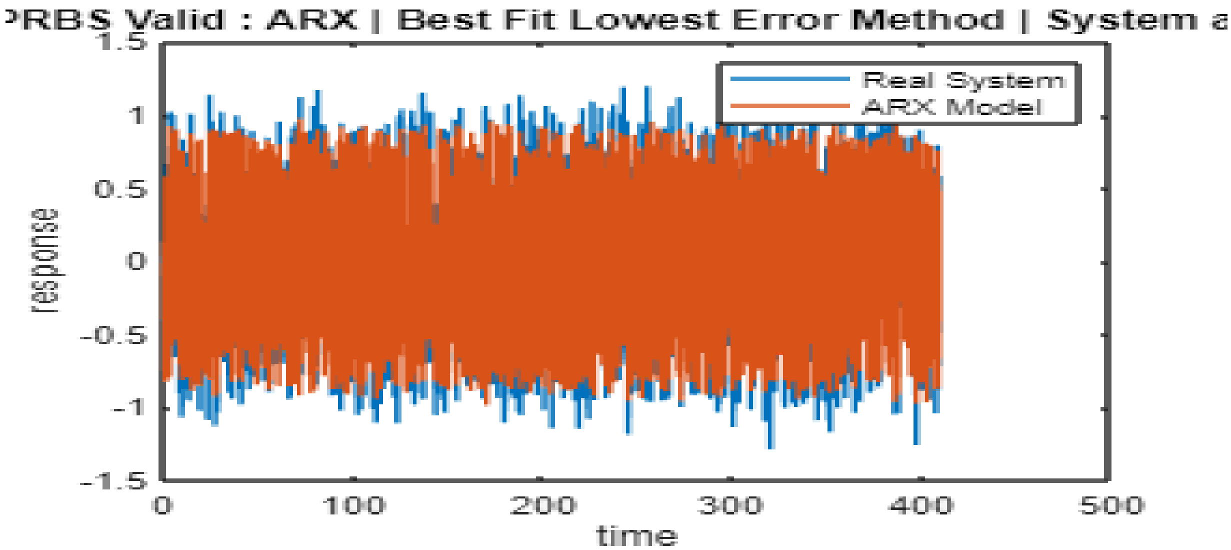
```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

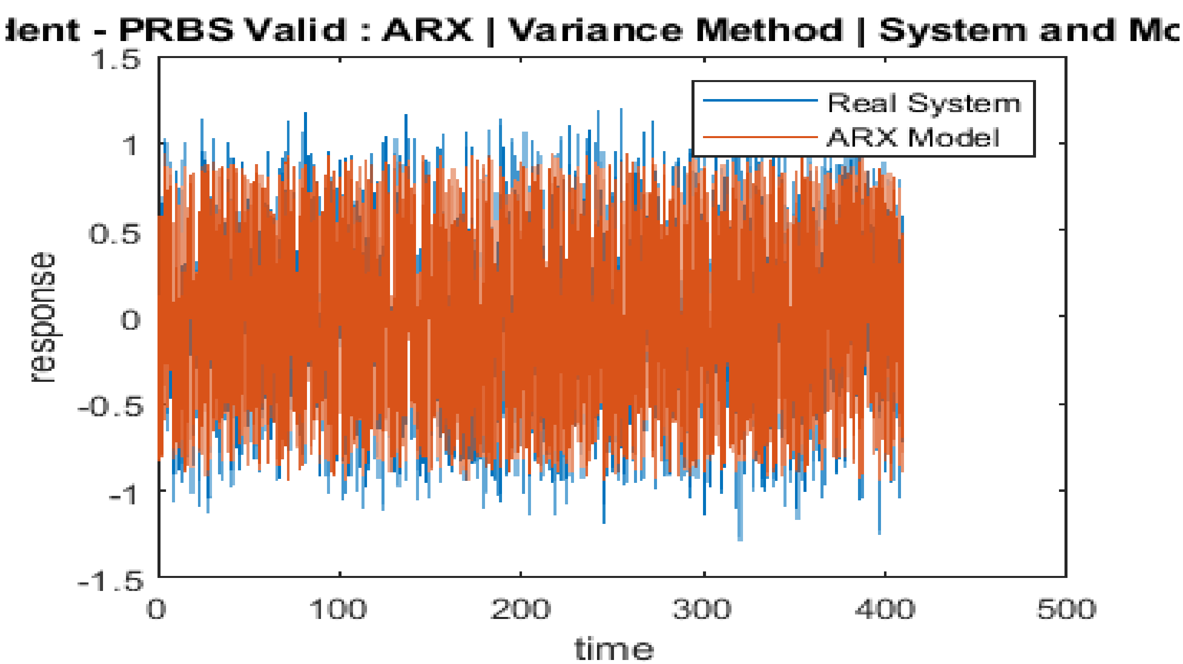
```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

```
===============================================================
```

```
%%

BestFitError_1 = y1_val - BestFit_y_hat_1;
VarError_1 = y1_val - Var_y_hat_1;
CovError_1 = y1_val - Cov_y_hat_1;
AICError_1 = y1_val - AIC_y_hat_1;
FTestError_1 = y1_val - FTest_y_hat_1;

for k=0:N_val-1
    BestFit_Ree_1(k+1,1) = AutoCorrelate(BestFitError_1, k);
    Var_Ree_1(k+1,1) = AutoCorrelate(VarError_1, k);
    Cov_Ree_1(k+1,1) = AutoCorrelate(CovError_1, k);
    AIC_Ree_1(k+1,1) = AutoCorrelate(AICError_1, k);
    FTest_Ree_1(k+1,1) = AutoCorrelate(FTestError_1, k);
end

for k=0:N_val-1
    BestFit_Rue_1(k+1,1) = CrossCorrelate(u1_val, BestFitError_1, k);
    Var_Rue_1(k+1,1) = CrossCorrelate(u1_val, VarError_1, k);
    Cov_Rue_1(k+1,1) = CrossCorrelate(u1_val, CovError_1, k);
    AIC_Rue_1(k+1,1) = CrossCorrelate(u1_val, AICError_1, k);
    FTest_Rue_1(k+1,1) = CrossCorrelate(u1_val, FTestError_1, k);
end


%%
figure()  % figure(1)
plot(t_val,y1_val,t_val,BestFit_y_hat_1)
legend('Real System','ARX Model')
title(" Guassian Ident - PRBS Valid : ARX | Best Fit Lowest Error Method | System and Model Response")
xlabel("time")
ylabel("response")
```

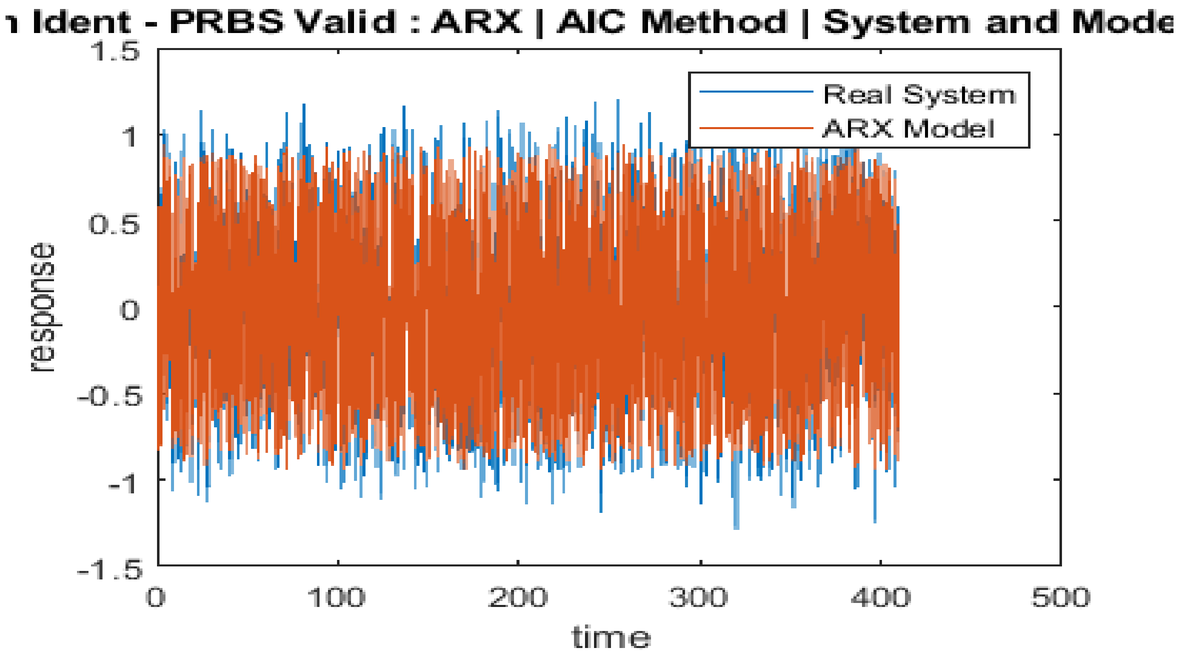



```
figure()  % figure(2)
plot(t_val,y1_val,t_val,Var_y_hat_1)
legend('Real System','ARX Model')
title(" Guassian Ident - PRBS Valid : ARX | Variance Method | System and Model
Response")
xlabel("time")
ylabel("response")
```

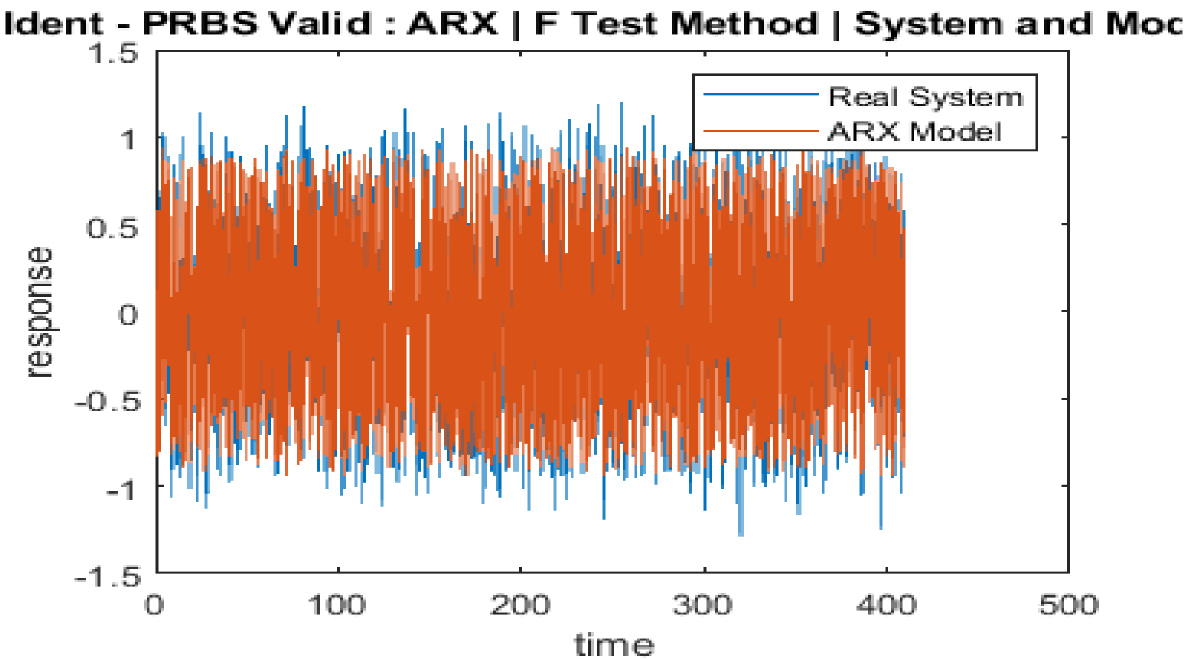



```
figure()  % figure(3)
plot(t_val,y1_val,t_val,AIC_y_hat_1)
```

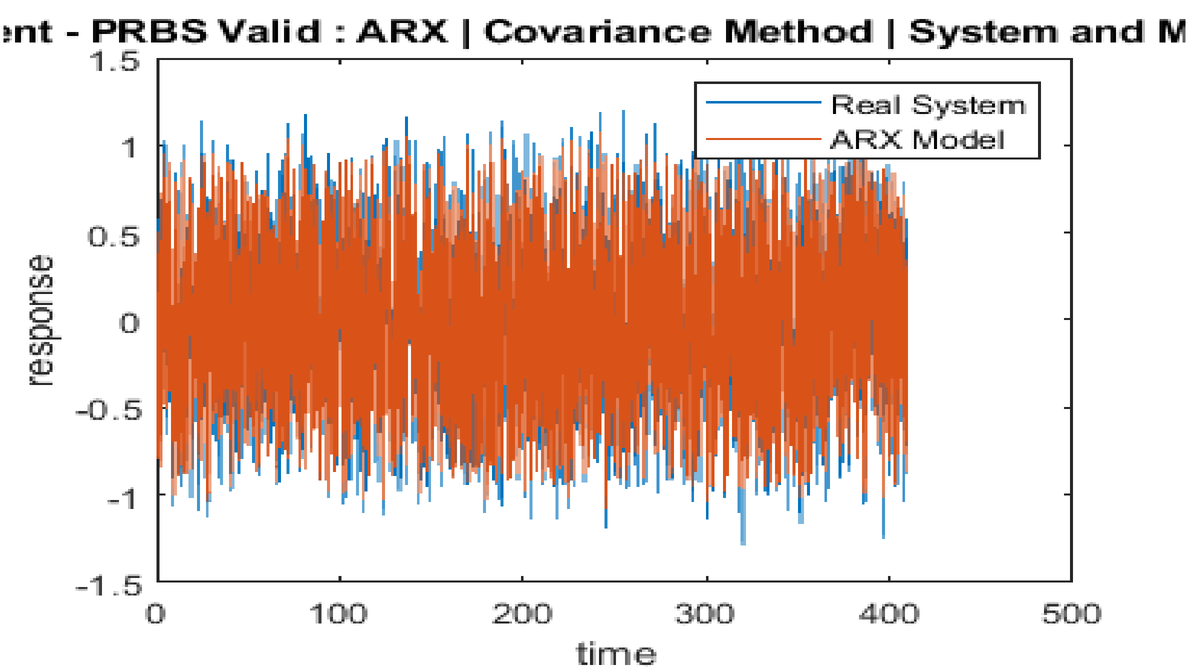
```
 legend('Real System','ARX Model')
 title(" Guassian Ident - PRBS Valid : ARX | AIC Method | System and Model
Response")
 xlabel("time")
 ylabel("response")
```




```
 figure()  % figure(4)
 plot(t_val,y1_val,t_val,FTest_y_hat_1)
 legend('Real System','ARX Model')
 title(" Guassian Ident - PRBS Valid : ARX | F Test Method | System and Model
Response")
 xlabel("time")
 ylabel("response")
```




```
figure()  % figure(7)
plot(t_val,y1_val,t_val,Cov_y_hat_1)
legend('Real System','ARX Model')
title(" Guassian Ident - PRBS Valid : ARX | Covariance Method | System and 
Model Response")
xlabel("time")
ylabel("response")
```



```
%%

figure()  % figure(5)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Ree_1(2:end), 1:N_val-1,
mean(BestFit_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | Best Fit Lowest Errror Method |
Ree_1(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,2)
plot(1:N_val-1,Var_Ree_1(2:end), 1:N_val-1,
mean(Var_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | Variance Method | Ree_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,3)
plot(1:N_val-1,AIC_Ree_1(2:end), 1:N_val-1,
mean(AIC_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | AIC Method | Ree_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,4)
plot(1:N_val-1,FTest_Ree_1(2:end), 1:N_val-1,
mean(FTest_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | F Test Method | Ree_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,5)
plot(1:N_val-1,Cov_Ree_1(2:end), 1:N_val-1,
mean(Cov_Ree_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | Covariance Method | Ree_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")
```

```
%%

figure()  % figure(6)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Rue_1(2:end), 1:N_val-1,
mean(BestFit_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | Best Fit Lowest Errror Method |
Rue_1(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,2)
plot(1:N_val-1,Var_Rue_1(2:end), 1:N_val-1,
mean(Var_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | Variance Method | Rue_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,3)
plot(1:N_val-1,AIC_Rue_1(2:end), 1:N_val-1,
mean(AIC_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | AIC Method | Rue_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,4)
plot(1:N_val-1,FTest_Rue_1(2:end), 1:N_val-1,
mean(FTest_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | F Test Method | Rue_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,5)
plot(1:N_val-1,Cov_Rue_1(2:end), 1:N_val-1,
mean(Cov_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARX | Covariance Method | Rue_1(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")
```

```
%%

figure()  % figure(6)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Rue_1(2:end), 1:N_val-1, mean(BestFit_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" ARX | Best Fit Lowest Errror Method | Rue_1(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,2)
plot(1:N_val-1,Var_Rue_1(2:end), 1:N_val-1, mean(Var_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" ARX | Variance Method | Rue_1(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,3)
plot(1:N_val-1,AIC_Rue_1(2:end), 1:N_val-1, mean(AIC_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" ARX | AIC Method | Rue_1(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")

subplot(5,1,4)
plot(1:N_val-1,FTest_Rue_1(2:end), 1:N_val-1, mean(FTest_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" ARX | F Test Method | Rue_1(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")


subplot(5,1,5)
plot(1:N_val-1,Cov_Rue_1(2:end), 1:N_val-1, mean(Cov_Rue_1(2:end))*ones(length(1:N_val-1)))
title(" ARX | Covariance Method | Rue_1(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_1(k)")


% ***********************************************************************

fprintf("****************************************************************\n")
fprintf("****************************************************************\n")
```

PRBS Input - Guassian Validation

```
%%
% PRBS Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> PRBS Input Identification Begins:------------------------------
\n")
```

```
>>> PRBS Input Identification Begins:------------------------------
```

```
%%

Ts = 0.1;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);

%%

fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;
```

```
    U = arx_U_builder_3(u2, y2, na, nb, nk);
    theta_hat_2 = inv(U'*U)*U'*y2;
    y_hat_2 = form_tf_lsim_2(theta_hat_2, u2, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y2, y_hat_2);

    error = y2 - y_hat_2;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    covariance = variance*inv(U'*U);
    cov = trace(covariance)/p;
    covs = [covs; cov];

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_arx, mse_arx, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_arx];
    MSEs = [MSEs; mse_arx];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.895788 | MSE=0.027773 | var=0.027787 | s_hat=113.760130 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.920829 | MSE=0.021100 | var=0.021120 | s_hat=86.424697 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.936579 | MSE=0.016902 | var=0.016927 | s_hat=69.231950 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.942226 | MSE=0.015397 | var=0.015427 | s_hat=63.067300 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.943224 | MSE=0.015131 | var=0.015168 | s_hat=61.977742 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.943283 | MSE=0.015116 | var=0.015160 | s_hat=61.913458 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.943220 | MSE=0.015132 | var=0.015184 | s_hat=61.981833 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.943242 | MSE=0.015127 | var=0.015186 | s_hat=61.958162 |
-------------------------------------------------------------
```

```
>>> Degree = 9 : R2=0.943319 | MSE=0.015106 | var=0.015173 | s_hat=61.873876 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.943355 | MSE=0.015096 | var=0.015170 | s_hat=61.834188 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.943382 | MSE=0.015089 | var=0.015171 | s_hat=61.805613 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.943384 | MSE=0.015089 | var=0.015178 | s_hat=61.803130 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.943432 | MSE=0.015076 | var=0.015172 | s_hat=61.750787 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.943431 | MSE=0.015076 | var=0.015180 | s_hat=61.751484 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.943439 | MSE=0.015074 | var=0.015185 | s_hat=61.742803 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.943426 | MSE=0.015077 | var=0.015196 | s_hat=61.756677 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.943468 | MSE=0.015066 | var=0.015192 | s_hat=61.711004 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.943478 | MSE=0.015064 | var=0.015197 | s_hat=61.700494 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.943483 | MSE=0.015062 | var=0.015203 | s_hat=61.695052 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.943481 | MSE=0.015063 | var=0.015211 | s_hat=61.697195 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.943520 | MSE=0.015052 | var=0.015208 | s_hat=61.654609 |
------------------------------------------------------------
>>> Degree = 22 : R2=0.943533 | MSE=0.015049 | var=0.015212 | s_hat=61.640598 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.943527 | MSE=0.015051 | var=0.015222 | s_hat=61.647380 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.943521 | MSE=0.015052 | var=0.015231 | s_hat=61.653905 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.943516 | MSE=0.015053 | var=0.015239 | s_hat=61.658789 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.943588 | MSE=0.015034 | var=0.015228 | s_hat=61.580760 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.943606 | MSE=0.015029 | var=0.015230 | s_hat=61.560655 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.943618 | MSE=0.015026 | var=0.015235 | s_hat=61.547405 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.943632 | MSE=0.015022 | var=0.015238 | s_hat=61.531792 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.943624 | MSE=0.015025 | var=0.015248 | s_hat=61.541138 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.943649 | MSE=0.015018 | var=0.015249 | s_hat=61.513934 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.943682 | MSE=0.015009 | var=0.015247 | s_hat=61.477563 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.943689 | MSE=0.015007 | var=0.015253 | s_hat=61.469767 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.943734 | MSE=0.014995 | var=0.015249 | s_hat=61.421365 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.943742 | MSE=0.014993 | var=0.015254 | s_hat=61.412691 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.943751 | MSE=0.014991 | var=0.015259 | s_hat=61.402503 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.943763 | MSE=0.014988 | var=0.015263 | s_hat=61.389789 |
```

```
-------------------------------------------------------------
>>> Degree = 38 : R2=0.943803 | MSE=0.014977 | var=0.015260 | s_hat=61.345190 |
-------------------------------------------------------------
>>> Degree = 39 : R2=0.943814 | MSE=0.014974 | var=0.015265 | s_hat=61.333789 |
-------------------------------------------------------------
>>> Degree = 40 : R2=0.943813 | MSE=0.014974 | var=0.015273 | s_hat=61.334951 |
-------------------------------------------------------------
>>> Degree = 41 : R2=0.943817 | MSE=0.014973 | var=0.015279 | s_hat=61.330813 |
-------------------------------------------------------------
>>> Degree = 42 : R2=0.943847 | MSE=0.014965 | var=0.015279 | s_hat=61.297754 |
-------------------------------------------------------------
>>> Degree = 43 : R2=0.943871 | MSE=0.014959 | var=0.015280 | s_hat=61.271156 |
-------------------------------------------------------------
>>> Degree = 44 : R2=0.943903 | MSE=0.014950 | var=0.015278 | s_hat=61.235979 |
-------------------------------------------------------------
>>> Degree = 45 : R2=0.943920 | MSE=0.014946 | var=0.015281 | s_hat=61.217425 |
-------------------------------------------------------------
>>> Degree = 46 : R2=0.943921 | MSE=0.014946 | var=0.015289 | s_hat=61.217020 |
-------------------------------------------------------------
>>> Degree = 47 : R2=0.943912 | MSE=0.014948 | var=0.015299 | s_hat=61.226699 |
-------------------------------------------------------------
>>> Degree = 48 : R2=0.943992 | MSE=0.014927 | var=0.015285 | s_hat=61.139389 |
-------------------------------------------------------------
>>> Degree = 49 : R2=0.943997 | MSE=0.014925 | var=0.015291 | s_hat=61.133808 |
-------------------------------------------------------------
>>> Degree = 50 : R2=0.943998 | MSE=0.014925 | var=0.015298 | s_hat=61.132466 |
-------------------------------------------------------------
>>> Degree = 51 : R2=0.943996 | MSE=0.014925 | var=0.015307 | s_hat=61.134796 |
-------------------------------------------------------------
>>> Degree = 52 : R2=0.944011 | MSE=0.014922 | var=0.015310 | s_hat=61.118903 |
-------------------------------------------------------------
>>> Degree = 53 : R2=0.944013 | MSE=0.014921 | var=0.015317 | s_hat=61.116384 |
-------------------------------------------------------------
>>> Degree = 54 : R2=0.944045 | MSE=0.014913 | var=0.015316 | s_hat=61.081918 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.944043 | MSE=0.014913 | var=0.015324 | s_hat=61.083310 |
-------------------------------------------------------------
>>> Degree = 56 : R2=0.944069 | MSE=0.014906 | var=0.015325 | s_hat=61.055272 |
-------------------------------------------------------------
>>> Degree = 57 : R2=0.944067 | MSE=0.014907 | var=0.015333 | s_hat=61.057452 |
-------------------------------------------------------------
>>> Degree = 58 : R2=0.944079 | MSE=0.014903 | var=0.015338 | s_hat=61.044717 |
-------------------------------------------------------------
>>> Degree = 59 : R2=0.944084 | MSE=0.014902 | var=0.015344 | s_hat=61.038898 |
-------------------------------------------------------------
>>> Degree = 60 : R2=0.944129 | MSE=0.014890 | var=0.015339 | s_hat=60.989475 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.944137 | MSE=0.014888 | var=0.015345 | s_hat=60.981405 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.944144 | MSE=0.014886 | var=0.015351 | s_hat=60.972849 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.944145 | MSE=0.014886 | var=0.015358 | s_hat=60.972506 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.944172 | MSE=0.014879 | var=0.015359 | s_hat=60.942783 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.944187 | MSE=0.014875 | var=0.015362 | s_hat=60.926025 |
-------------------------------------------------------------
```

```
>>> Degree = 66 : R2=0.944190 | MSE=0.014874 | var=0.015369 | s_hat=60.923636 |
-------------------------------------------------------------
>>> Degree = 67 : R2=0.944193 | MSE=0.014873 | var=0.015376 | s_hat=60.920197 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.944205 | MSE=0.014870 | var=0.015381 | s_hat=60.906787 |
-------------------------------------------------------------
>>> Degree = 69 : R2=0.944214 | MSE=0.014867 | var=0.015386 | s_hat=60.896643 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.944227 | MSE=0.014864 | var=0.015390 | s_hat=60.883273 |
-------------------------------------------------------------
>>> Degree = 71 : R2=0.944252 | MSE=0.014857 | var=0.015391 | s_hat=60.855260 |
-------------------------------------------------------------
>>> Degree = 72 : R2=0.944343 | MSE=0.014833 | var=0.015374 | s_hat=60.756220 |
-------------------------------------------------------------
>>> Degree = 73 : R2=0.944349 | MSE=0.014832 | var=0.015380 | s_hat=60.749933 |
-------------------------------------------------------------
>>> Degree = 74 : R2=0.944378 | MSE=0.014824 | var=0.015379 | s_hat=60.718099 |
-------------------------------------------------------------
>>> Degree = 75 : R2=0.944378 | MSE=0.014824 | var=0.015387 | s_hat=60.718076 |
-------------------------------------------------------------
>>> Degree = 76 : R2=0.944383 | MSE=0.014822 | var=0.015394 | s_hat=60.712430 |
-------------------------------------------------------------
>>> Degree = 77 : R2=0.944403 | MSE=0.014817 | var=0.015396 | s_hat=60.690187 |
-------------------------------------------------------------
>>> Degree = 78 : R2=0.944404 | MSE=0.014817 | var=0.015403 | s_hat=60.689026 |
-------------------------------------------------------------
>>> Degree = 79 : R2=0.944416 | MSE=0.014814 | var=0.015408 | s_hat=60.676209 |
-------------------------------------------------------------
>>> Degree = 80 : R2=0.944417 | MSE=0.014813 | var=0.015416 | s_hat=60.675471 |
-------------------------------------------------------------
>>> Degree = 81 : R2=0.944424 | MSE=0.014811 | var=0.015421 | s_hat=60.667759 |
-------------------------------------------------------------
>>> Degree = 82 : R2=0.944424 | MSE=0.014811 | var=0.015429 | s_hat=60.667177 |
-------------------------------------------------------------
>>> Degree = 83 : R2=0.944431 | MSE=0.014809 | var=0.015435 | s_hat=60.659666 |
-------------------------------------------------------------
>>> Degree = 84 : R2=0.944436 | MSE=0.014808 | var=0.015442 | s_hat=60.654380 |
-------------------------------------------------------------
>>> Degree = 85 : R2=0.944460 | MSE=0.014802 | var=0.015443 | s_hat=60.628688 |
-------------------------------------------------------------
>>> Degree = 86 : R2=0.944463 | MSE=0.014801 | var=0.015450 | s_hat=60.624816 |
-------------------------------------------------------------
>>> Degree = 87 : R2=0.944465 | MSE=0.014801 | var=0.015457 | s_hat=60.623061 |
-------------------------------------------------------------
>>> Degree = 88 : R2=0.944494 | MSE=0.014793 | var=0.015457 | s_hat=60.591070 |
-------------------------------------------------------------
>>> Degree = 89 : R2=0.944494 | MSE=0.014793 | var=0.015465 | s_hat=60.590986 |
-------------------------------------------------------------
>>> Degree = 90 : R2=0.944494 | MSE=0.014793 | var=0.015473 | s_hat=60.591642 |
-------------------------------------------------------------
>>> Degree = 91 : R2=0.944498 | MSE=0.014792 | var=0.015480 | s_hat=60.586778 |
-------------------------------------------------------------
>>> Degree = 92 : R2=0.944508 | MSE=0.014789 | var=0.015485 | s_hat=60.576230 |
-------------------------------------------------------------
>>> Degree = 93 : R2=0.944517 | MSE=0.014787 | var=0.015490 | s_hat=60.566474 |
-------------------------------------------------------------
>>> Degree = 94 : R2=0.944584 | MSE=0.014769 | var=0.015479 | s_hat=60.493159 |
```

```
-------------------------------------------------------------
>>> Degree = 95 : R2=0.944586 | MSE=0.014768 | var=0.015487 | s_hat=60.490652 |
-------------------------------------------------------------
>>> Degree = 96 : R2=0.944584 | MSE=0.014769 | var=0.015495 | s_hat=60.493047 |
-------------------------------------------------------------
>>> Degree = 97 : R2=0.944633 | MSE=0.014756 | var=0.015489 | s_hat=60.439948 |
-------------------------------------------------------------
>>> Degree = 98 : R2=0.944644 | MSE=0.014753 | var=0.015494 | s_hat=60.427409 |
-------------------------------------------------------------
>>> Degree = 99 : R2=0.944775 | MSE=0.014718 | var=0.015465 | s_hat=60.284284 |
-------------------------------------------------------------
>>> Degree = 100 : R2=0.944777 | MSE=0.014717 | var=0.015473 | s_hat=60.281976 |
-------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));
fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 100
```

```
na = bestFitDegree;
nb = bestFitDegree;
p = na+nb;

BestFitU = arx_U_builder_3(u2, y2, na, nb, nk);
BestFitModel_2 = inv(BestFitU'*BestFitU)*BestFitU'*y2;
BestFit_y_hat_2 = form_tf_lsim_2(BestFitModel_2, u2_val, t_val, na, Ts);
```

```
G =


   0.3995 z^-1 + 0.1987 z^-2 + 0.05854 z^-3 - 0.008633 z^-4 - 0.04029 z^-5 - 0.03948 z^-6 - 
0.03828 z^-7
```

- 0.01768 z^-8 - 0.01024 z^-9 - 0.01381 z^-10 - 0.0005987 z^-11 - 0.00609 z^-12 - 0.00927 z^-13

+ 0.001472 z^-14 + 0.00684 z^-15 + 0.01139 z^-16 + 0.001024 z^-17 - 0.001579 z^-18

- 0.004866 z^-19 + 0.001107 z^-20 - 0.007672 z^-21 + 0.0005517 z^-22 - 0.004231 z^-23

- 0.01919 z^-24 - 0.009131 z^-25 + 0.004207 z^-26 - 0.00646 z^-27 + 0.001066 z^-28

+ 0.001555 z^-29 + 0.004788 z^-30 - 0.009019 z^-31 - 0.005447 z^-32 + 0.005285 z^

-33 - 0.0007778 z^-34 - 0.00806 z^-35 - 0.0006539 z^-36 - 0.007494 z^-37 + 0.001146 z^-38

+ 0.003139 z^-39 + 0.003262 z^-40 - 0.004614 z^-41 + 0.00242 z^-42 + 0.004685 z^-43

- 0.00172 z^-44 - 0.0003774 z^-45 + 0.005199 z^-46 + 0.01611 z^-47 + 0.009276 z^-48

+ 0.001348 z^-49 + 0.01008 z^-50 + 0.007886 z^-51 + 0.006629 z^-52 - 0.001169 z^-53

- 0.0005031 z^-54 - 0.001532 z^-55 - 0.01609 z^-56 + 0.0027 z^-57 - 0.008219 z^-58

- 0.007545 z^-59 + 0.01278 z^-60 - 0.001571 z^-61 - 0.005708 z^-62 + 0.008001 z^-63

+ 0.02014 z^-64 + 0.005165 z^-65 - 0.005589 z^-66 - 0.001798 z^-67 + 0.001176 z^-68

+ 0.00266 z^-69 + 0.0005287 z^-70 + 0.008866 z^-71 - 0.001316 z^-72 - 0.003947 z^

-73 + 0.001887 z^-74 + 0.001505 z^-75 - 0.001738 z^-76 - 0.005157 z^-77 - 0.003578 z^-78

- 0.001217 z^-79 + 0.002554 z^-80 + 0.003297 z^-81 + 0.008239 z^-82 - 0.0008608 z^-83

+ 0.002026 z^-84 + 0.003797 z^-85 + 0.00693 z^-86 + 0.006608 z^-87 + 0.0001984 z^

-88 + 0.000524 z^-89 - 0.003584 z^-90 - 0.0003955 z^-91 - 0.01111 z^-92 - 0.007324 z^-93

- 0.02034 z^-94 - 0.005121 z^-95 - 0.003073 z^-96 + 0.001762 z^-97 - 0.007552 z^-98

- 0.01478 z^-99 + 0.0008169 z^-100

----

1 - 0.1694 z^-1 - 0.08117 z^-2 - 0.01847 z^-3 - 0.002561 z^-4 - 0.004717 z^-5 - 0.01923 z^-6

+ 0.006924 z^-7 - 0.006758 z^-8 - 0.03148 z^-9 + 0.01111 z^-10 - 0.02441 z^-11 - 0.008859 z^-12

+ 0.009785 z^-13 + 0.01756 z^-14 + 0.01387 z^-15 - 0.01618 z^-16 + 0.006497 z^-17

- 0.008601 z^-18 + 0.01238 z^-19 - 0.01654 z^-20 + 0.004609 z^-21 - 0.003874 z^-22

```
          - 0.04678 z^-23 + 0.01246 z^-24 + 0.02119 z^-25 - 0.03099 z^-26 + 0.002604 z^-27

          + 0.003918 z^-28 + 0.003518 z^-29 - 0.03021 z^-30 + 0.003231 z^-31 + 0.01181 z^-32

          - 0.008175 z^-33 - 0.008438 z^-34 + 0.003489 z^-35 - 0.01529 z^-36 + 0.01287 z^-37

          - 0.004138 z^-38 + 0.007186 z^-39 - 0.0164 z^-40 + 0.0135 z^-41 + 0.02016 z^-42 -
0.01964 z^-43

          - 0.002534 z^-44 + 0.01654 z^-45 + 0.03578 z^-46 - 0.01031 z^-47 + 0.005746 z^-48

          + 0.02538 z^-49 + 0.005684 z^-50 + 0.004263 z^-51 - 0.005722 z^-52 + 0.004654 z^-53

          - 0.01081 z^-54 - 0.03464 z^-55 + 0.03678 z^-56 - 0.02574 z^-57 - 0.006902 z^-58

          + 0.03431 z^-59 - 0.01914 z^-60 - 0.007296 z^-61 + 0.02444 z^-62 + 0.03135 z^-63

          - 0.01427 z^-64 - 0.01667 z^-65 + 0.0104 z^-66 + 0.004214 z^-67 - 6.788e-05 z^-68

          - 0.002314 z^-69 + 0.01816 z^-70 - 0.01623 z^-71 + 0.005838 z^-72 + 0.01387 z^-73

          - 0.009538 z^-74 - 0.008916 z^-75 - 0.014 z^-76 + 0.007082 z^-77 + 0.0027 z^-78 -
0.0007922 z^-79

          + 0.005298 z^-80 + 0.02053 z^-81 - 0.01937 z^-82 + 0.009326 z^-83 + 0.01021 z^-84

          + 0.008159 z^-85 + 0.01071 z^-86 - 0.01056 z^-87 + 0.01117 z^-88 - 0.009287 z^-89

          + 0.005571 z^-90 - 0.02213 z^-91 - 0.004161 z^-92 - 0.03901 z^-93 + 0.01002 z^-94

          - 0.002569 z^-95 + 0.00147 z^-96 - 0.01497 z^-97 - 0.015 z^-98 + 0.003629 z^-99 -
0.0007185 z^-100


Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 6 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 6
```

```
na = minVarIndex;
nb = minVarIndex;
p = na+nb;

VarU = arx_U_builder_3(u2, y2, na, nb, nk);
VarModel_2 = inv(VarU'*VarU)*VarU'*y2;
Var_y_hat_2 = form_tf_lsim_2(VarModel_2, u2_val, t_val, na, Ts);
```

```
G =
 
   0.3995 z^-1 + 0.1954 z^-2 + 0.05691 z^-3 - 0.008226 z^-4 - 0.03263 z^-5 - 0.01977 z^-6
  ---------------------------------------------------------------------------------------
  1 - 0.1763 z^-1 - 0.08033 z^-2 - 0.01654 z^-3 + 0.01585 z^-4 + 0.0318 z^-5 + 0.03665 z^-6
 
Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | CoVariance
Method=================\n")
```

```
===============Degree Extraction | CoVariance Method=================
```

```
maxCovIndex = find(covs == min(covs));
fprintf(">>> Since the minimum CovMatrix trace occurs in iteration %d ;\n",
maxCovIndex)
```

```
>>> Since the minimum CovMatrix trace occurs in iteration 1 ;
```

```
fprintf("    Degree = %d \n", maxCovIndex)
```

```
    Degree = 1
```

```
na = maxCovIndex;
nb = maxCovIndex;
p = na+nb;

CovU_2 = arx_U_builder_3(u2,y2,na,nb,1);
CovModel_2 = inv(CovU_2'*CovU_2)*CovU_2'*y2;
```

```
CovU_2_val = arx_U_builder_3(u2_val,y2_val,na,nb,1);
Cov_y_hat_2 = CovU_2_val*CovModel_2;

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 6 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 6
```

```
na = minAICIndex;
nb = minAICIndex;
p = na+nb;

AICU_2 = arx_U_builder_3(u2, y2, na, nb, nk);
AICModel_2 = inv(AICU_2'*AICU_2)*AICU_2'*y2;
AIC_y_hat_2 = form_tf_lsim_2(AICModel_2, u2_val, t_val, na, Ts);
```

```
G =

   0.3995 z^-1 + 0.1954 z^-2 + 0.05691 z^-3 - 0.008226 z^-4 - 0.03263 z^-5 - 0.01977 z^-6
  ----------------------------------------------------------------------------------------
  1 - 0.1763 z^-1 - 0.08033 z^-2 - 0.01654 z^-3 + 0.01585 z^-4 + 0.0318 z^-5 + 0.03665 z^-6

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test Method===================\n")
```

```
===============Degree Extraction | F test Method===================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", winScore)
```

```
>>> The F test is suggesting the best model with the m=1.18 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 5
```

```

na = winner;
nb = winner;
p = na+nb;

FTestU_2 = arx_U_builder_3(u2, y2, na, nb, nk);
FTestModel_2 = inv(FTestU_2'*FTestU_2)*FTestU_2'*y2;
FTest_y_hat_2 = form_tf_lsim_2(FTestModel_2, u2_val, t_val, na, Ts);
```

```
G =

    0.3995 z^-1 + 0.1917 z^-2 + 0.04961 z^-3 - 0.01635 z^-4 - 0.03413 z^-5
  ----------------------------------------------------------------------------
  1 - 0.1857 z^-1 - 0.09241 z^-2 - 0.02589 z^-3 + 0.02353 z^-4 + 0.07956 z^-5

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
[BestFit_r2, BestFit_mse] = rSQR(y2_val, BestFit_y_hat_2);
[Var_r2, Var_mse] = rSQR(y2_val, Var_y_hat_2);
[AIC_r2, AIC_mse] = rSQR(y2_val, AIC_y_hat_2);
[Cov_r2, Cov_mse] = rSQR(y2_val, Cov_y_hat_2);
[FTest_r2, FTest_mse] = rSQR(y2_val, FTest_y_hat_2);
fprintf("===========================PRBS Ident - Guassian ValidI===========================\n")
```

```
===========================PRBS Ident - Guassian ValidI===========================
```

```
fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", BestFit_r2, BestFit_mse)
```

```
    R2 value : 0.9398    | MSE : 0.0162
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Var_r2, Var_mse)
```

```
    R2 value : 0.9407    | MSE : 0.0160
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Covariance Method:\n")
```

```
>>> Covariance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", Cov_r2, Cov_mse)
```

```
    R2 value : 0.9132    | MSE : 0.0234
```

```
fprintf("----------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", AIC_r2, AIC_mse)
```

```
    R2 value : 0.9407    | MSE : 0.0160
```

```
fprintf("----------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", FTest_r2, FTest_mse)
```

```
    R2 value : 0.9409    | MSE : 0.0159
```

```
fprintf("----------------------------------------------------------------\n")
```

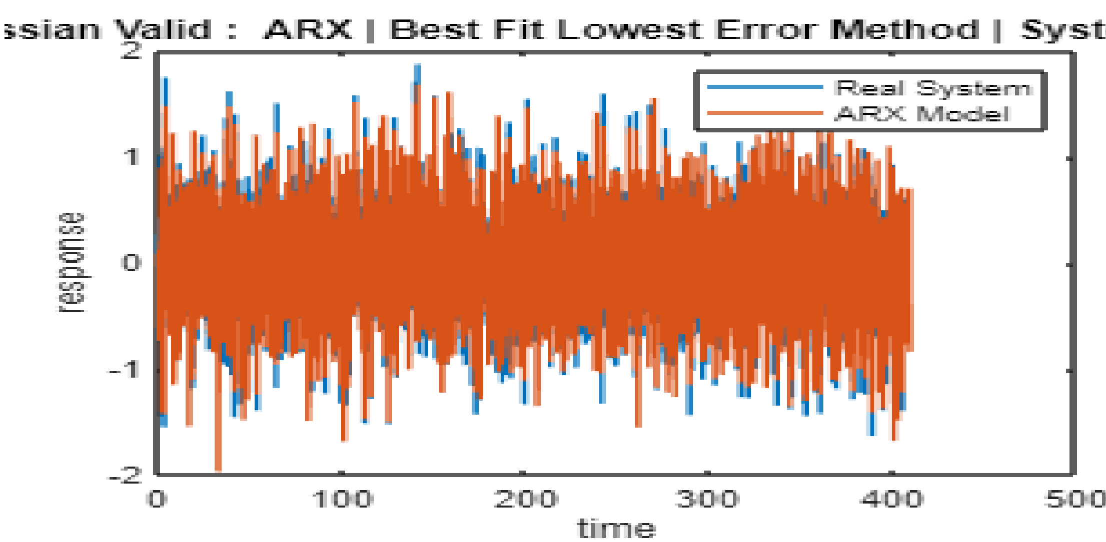
```
-------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("================================================================\n")
```

```
=============================================================
```

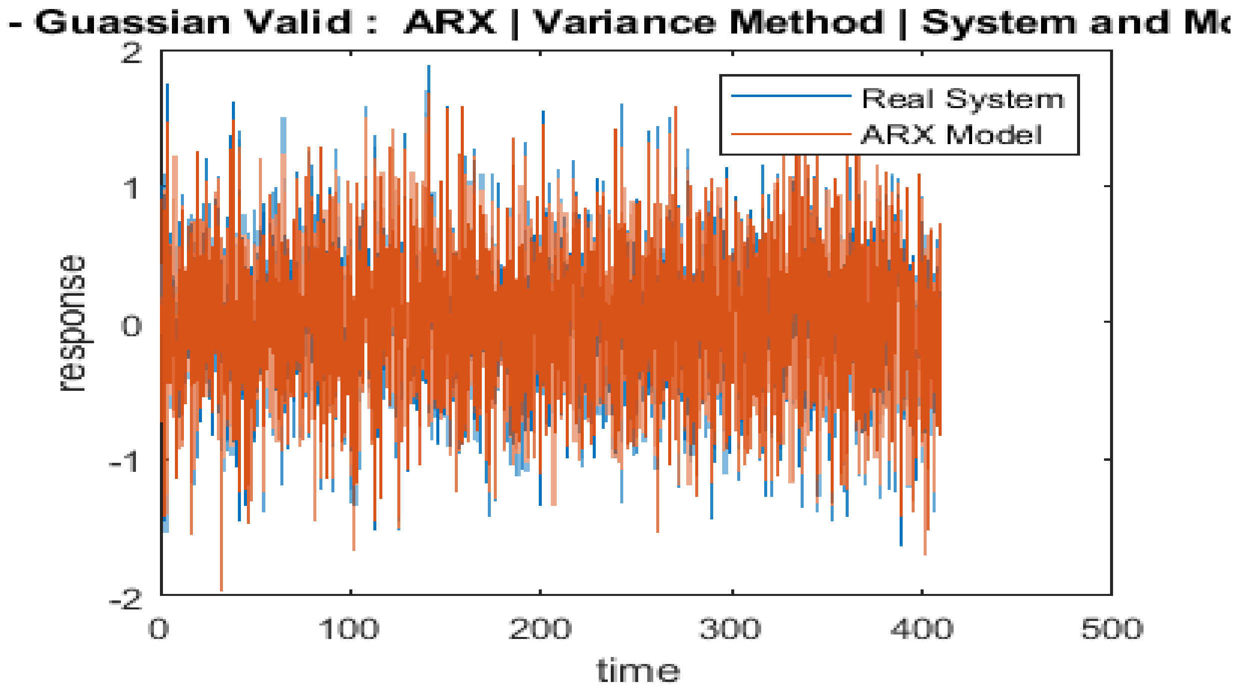
```
%%

BestFitError_2 = y2_val - BestFit_y_hat_2;
VarError_2 = y2_val - Var_y_hat_2;
CovError_2 = y2_val - Cov_y_hat_2;
AICError_2 = y2_val - AIC_y_hat_2;
FTestError_2 = y2_val - FTest_y_hat_2;

%%

for k=0:N_val-1
    BestFit_Ree_2(k+1,1) = AutoCorrelate(BestFitError_2, k);
```

```
    Var_Ree_2(k+1,1) = AutoCorrelate(VarError_2, k);
    Cov_Ree_2(k+1,1) = AutoCorrelate(CovError_2, k);
    AIC_Ree_2(k+1,1) = AutoCorrelate(AICError_2, k);
    FTest_Ree_2(k+1,1) = AutoCorrelate(FTestError_2, k);
end

for k=0:N_val-1
    BestFit_Rue_2(k+1,1) = CrossCorrelate(u2_val, BestFitError_2, k);
    Var_Rue_2(k+1,1) = CrossCorrelate(u2_val, VarError_2, k);
    Cov_Rue_2(k+1,1) = CrossCorrelate(u2_val, CovError_2, k);
    AIC_Rue_2(k+1,1) = CrossCorrelate(u2_val, AICError_2, k);
    FTest_Rue_2(k+1,1) = CrossCorrelate(u2_val, FTestError_2, k);
end


%%
figure()  % figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hat_2)
legend('Real System','ARX Model')
title(" PRBS Ident - Guassian Valid :  ARX | Best Fit Lowest Error Method |
System and Model Response")
xlabel("time")
ylabel("response")
```

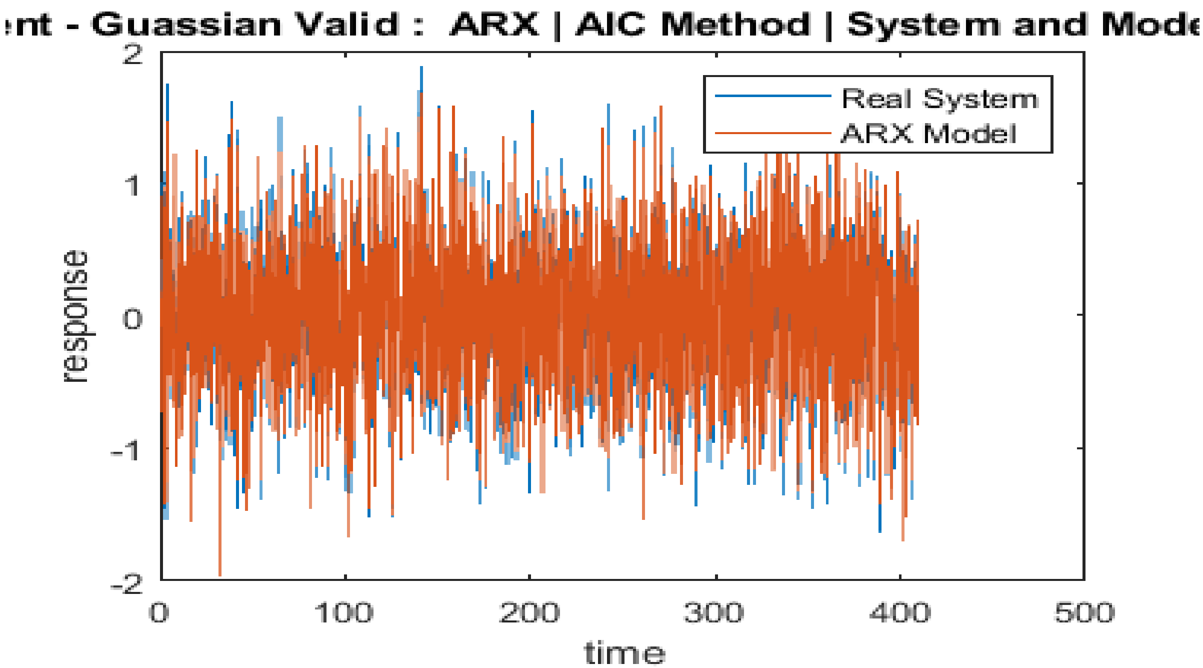



```
figure()  % figure(2)
plot(t_val,y2_val,t_val,Var_y_hat_2)
legend('Real System','ARX Model')
title(" PRBS Ident - Guassian Valid :  ARX | Variance Method | System and Model
Response")
xlabel("time")
```

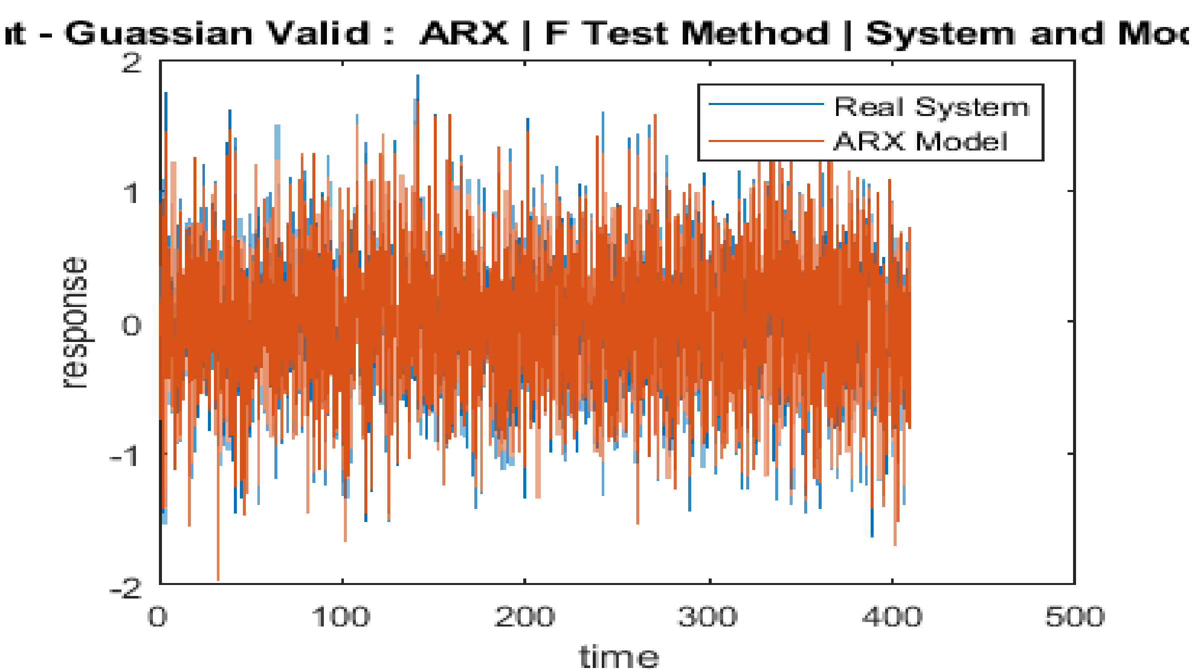
```
ylabel("response")
```



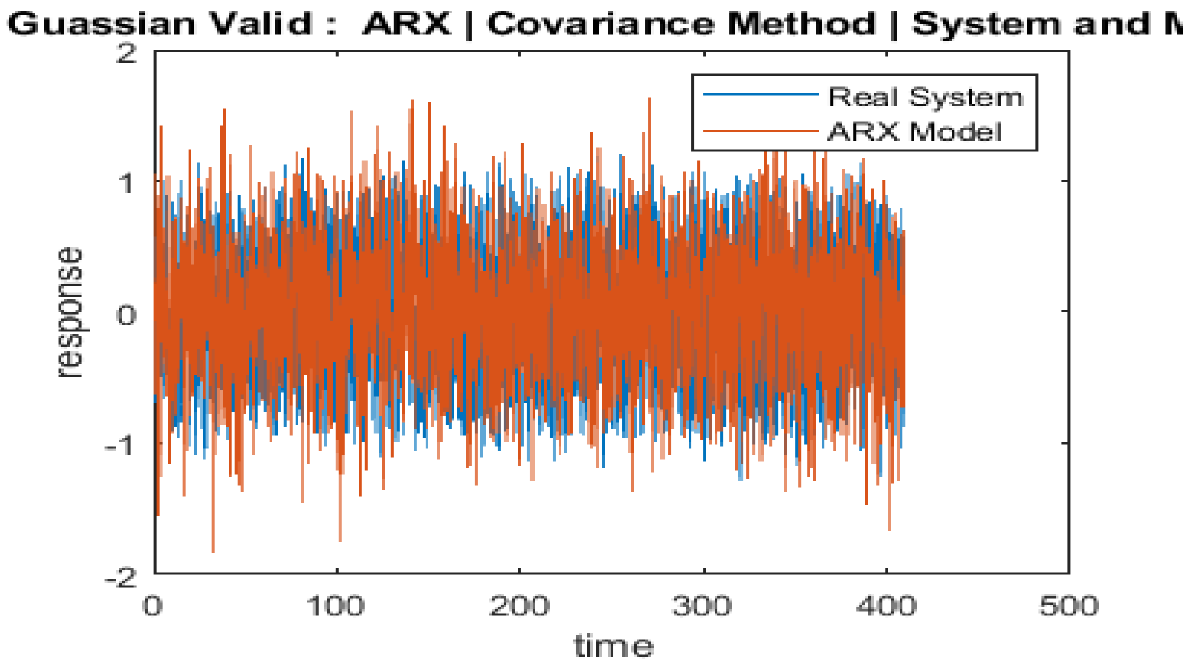

```
figure()  % figure(3)
plot(t_val,y2_val,t_val,AIC_y_hat_2)
legend('Real System','ARX Model')
title(" PRBS Ident - Guassian Valid :  ARX | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure()  % figure(4)
plot(t_val,y2_val,t_val,FTest_y_hat_2)
legend('Real System','ARX Model')
title(" PRBS Ident - Guassian Valid :  ARX | F Test Method | System and Model
Response")
xlabel("time")
ylabel("response")
```



```
figure()  % figure(7)
plot(t_val,y1_val,t_val,Cov_y_hat_2)
legend('Real System','ARX Model')
title(" PRBS Ident - Guassian Valid :  ARX | Covariance Method | System and
Model Response")
xlabel("time")
ylabel("response")
```




```
%%

figure()  % figure(5)
subplot(5,1,1)
plot(1:N_val-1,BestFit_Ree_2(2:end), 1:N_val-1,
mean(BestFit_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" PRBS Ident - Guassian Valid :  ARX | Best Fit Lowest Errror Method |
Ree_2(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_1(k)")

subplot(5,1,2)
plot(1:N_val-1,Var_Ree_2(2:end), 1:N_val-1,
mean(Var_Ree_2(2:end))*ones(length(1:N_val-1)))
title(" PRBS Ident - Guassian Valid :  ARX | Variance Method | Ree_2(k) | The
Straight Line is the Mean")
xlabel("k")
```

```
 ylabel("Ree_1(k)")

 subplot(5,1,3)
 plot(1:N_val-1,AIC_Ree_2(2:end), 1:N_val-1,
mean(AIC_Ree_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | AIC Method | Ree_2(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree_1(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Ree_2(2:end), 1:N_val-1,
mean(FTest_Ree_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | F Test Method | Ree_2(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree_1(k)")

 subplot(5,1,5)
 plot(1:N_val-1,Cov_Ree_2(2:end), 1:N_val-1,
mean(Cov_Ree_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | Covariance Method | Ree_1(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree_1(k)")


 %%

 figure()  % figure(6)
 subplot(5,1,1)
 plot(1:N_val-1,BestFit_Rue_2(2:end), 1:N_val-1,
mean(BestFit_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | Best Fit Lowest Errror Method |
Rue_2(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(5,1,2)
 plot(1:N_val-1,Var_Rue_2(2:end), 1:N_val-1,
mean(Var_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | Variance Method | Rue_2(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")
```

```
 subplot(5,1,3)
 plot(1:N_val-1,AIC_Rue_2(2:end), 1:N_val-1,
mean(AIC_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | AIC Method | Rue_2(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")

 subplot(5,1,4)
 plot(1:N_val-1,FTest_Rue_2(2:end), 1:N_val-1,
mean(FTest_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | F Test Method | Rue_2(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")


 subplot(5,1,5)
 plot(1:N_val-1,Cov_Rue_2(2:end), 1:N_val-1,
mean(Cov_Rue_2(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid :  ARX | Covariance Method | Rue_2(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_2(k)")


 fprintf("*****************************************************************\n")
 fprintf("*****************************************************************\n")
```

Q3 - part a | ARMAX

```
clc; clear;
%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;
```

Guassian Input - PRBS Validation

```
%%
% Guassian Input *************************************************************
fprintf("******************************************************************\n")
```

```
*******************************************************************
```

```
fprintf(">>> Guassian Input Identification Begins:----------------------------
-\n")
```

```
>>> Guassian Input Identification Begins:-----------------------------
```

```
%%

Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data_guassian = iddata(y1,u1,Ts);

%%

fprintf("====================Degree Extraction |
RUN==========================\n")
```

```
====================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
```

```
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nk = 1;
    p = na+nb+nc;

    try
        sys = armax(data_guassian, [na nb nc nk]);
        armax_y_hat_guassian = lsim(sys, u1, t);
    catch
        break
    end

    [r2_armax, mse_armax] = rSQR(y1, armax_y_hat_guassian);

    error = y1 - armax_y_hat_guassian;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_armax, mse_armax, variance, S_hat)
    fprintf("--------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_armax];
    MSEs = [MSEs; mse_armax];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.894175 | MSE=0.028480 | var=0.028501 | s_hat=116.653345 |
--------------------------------------------------------------
>>> Degree = 2 : R2=0.941007 | MSE=0.015876 | var=0.015900 | s_hat=65.029299 |
```

```
-------------------------------------------------------------
>>> Degree = 3 : R2=0.941070 | MSE=0.015859 | var=0.015894 | s_hat=64.960181 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.941097 | MSE=0.015852 | var=0.015899 | s_hat=64.929912 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.941096 | MSE=0.015852 | var=0.015911 | s_hat=64.931427 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.941174 | MSE=0.015831 | var=0.015901 | s_hat=64.845300 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.941183 | MSE=0.015829 | var=0.015911 | s_hat=64.835294 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.941202 | MSE=0.015824 | var=0.015917 | s_hat=64.814153 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.941275 | MSE=0.015804 | var=0.015909 | s_hat=64.733556 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.941202 | MSE=0.015824 | var=0.015941 | s_hat=64.814492 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.941333 | MSE=0.015789 | var=0.015917 | s_hat=64.670187 |
-------------------------------------------------------------
>>> Degree = 12 : R2=0.941324 | MSE=0.015791 | var=0.015931 | s_hat=64.679666 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.941346 | MSE=0.015785 | var=0.015937 | s_hat=64.655535 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.941371 | MSE=0.015778 | var=0.015942 | s_hat=64.628138 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.941412 | MSE=0.015767 | var=0.015943 | s_hat=64.583097 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.941381 | MSE=0.015776 | var=0.015963 | s_hat=64.617357 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.941471 | MSE=0.015751 | var=0.015950 | s_hat=64.517949 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.941598 | MSE=0.015717 | var=0.015927 | s_hat=64.378164 |
-------------------------------------------------------------
>>> Degree = 19 : R2=0.941511 | MSE=0.015741 | var=0.015963 | s_hat=64.474060 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.941534 | MSE=0.015734 | var=0.015968 | s_hat=64.448280 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.941619 | MSE=0.015712 | var=0.015957 | s_hat=64.354998 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.941581 | MSE=0.015722 | var=0.015979 | s_hat=64.396575 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.941585 | MSE=0.015721 | var=0.015990 | s_hat=64.391735 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.941613 | MSE=0.015713 | var=0.015994 | s_hat=64.361582 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.942027 | MSE=0.015602 | var=0.015893 | s_hat=63.904233 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.941623 | MSE=0.015710 | var=0.016015 | s_hat=64.349939 |
-------------------------------------------------------------
>>> Degree = 27 : R2=0.941812 | MSE=0.015660 | var=0.015975 | s_hat=64.141421 |
-------------------------------------------------------------
>>> Degree = 28 : R2=0.941808 | MSE=0.015661 | var=0.015989 | s_hat=64.146323 |
-------------------------------------------------------------
>>> Degree = 29 : R2=0.941926 | MSE=0.015629 | var=0.015968 | s_hat=64.016313 |
-------------------------------------------------------------
>>> Degree = 30 : R2=0.941929 | MSE=0.015628 | var=0.015979 | s_hat=64.013332 |
-------------------------------------------------------------
```

```
>>> Degree = 31 : R2=0.941910 | MSE=0.015633 | var=0.015996 | s_hat=64.033572 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.942119 | MSE=0.015577 | var=0.015951 | s_hat=63.803226 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.941095 | MSE=0.015853 | var=0.016245 | s_hat=64.932675 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.941991 | MSE=0.015612 | var=0.016010 | s_hat=63.944735 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.942003 | MSE=0.015608 | var=0.016019 | s_hat=63.931209 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.941928 | MSE=0.015628 | var=0.016052 | s_hat=64.013789 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.942157 | MSE=0.015567 | var=0.016000 | s_hat=63.761865 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.940982 | MSE=0.015883 | var=0.016338 | s_hat=65.056345 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.941900 | MSE=0.015636 | var=0.016096 | s_hat=64.045086 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.942097 | MSE=0.015583 | var=0.016053 | s_hat=63.828138 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.942064 | MSE=0.015592 | var=0.016075 | s_hat=63.864181 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.942074 | MSE=0.015589 | var=0.016084 | s_hat=63.852609 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.942367 | MSE=0.015510 | var=0.016015 | s_hat=63.530182 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.942295 | MSE=0.015530 | var=0.016047 | s_hat=63.609247 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.942375 | MSE=0.015508 | var=0.016037 | s_hat=63.521087 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.942169 | MSE=0.015564 | var=0.016106 | s_hat=63.748295 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.942357 | MSE=0.015513 | var=0.016066 | s_hat=63.540543 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.942316 | MSE=0.015524 | var=0.016090 | s_hat=63.586012 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.942309 | MSE=0.015526 | var=0.016104 | s_hat=63.594123 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.942035 | MSE=0.015600 | var=0.016193 | s_hat=63.895681 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.939694 | MSE=0.016230 | var=0.016859 | s_hat=66.476132 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.942389 | MSE=0.015504 | var=0.016118 | s_hat=63.506051 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.942313 | MSE=0.015525 | var=0.016152 | s_hat=63.589656 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.942590 | MSE=0.015450 | var=0.016086 | s_hat=63.284049 |
------------------------------------------------------------
>>> Degree = 55 : R2=0.942888 | MSE=0.015370 | var=0.016015 | s_hat=62.956097 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.942409 | MSE=0.015499 | var=0.016162 | s_hat=63.484084 |
------------------------------------------------------------
>>> Degree = 57 : R2=0.942516 | MSE=0.015470 | var=0.016144 | s_hat=63.365279 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.942830 | MSE=0.015386 | var=0.016068 | s_hat=63.019388 |
------------------------------------------------------------
>>> Degree = 59 : R2=0.942733 | MSE=0.015412 | var=0.016108 | s_hat=63.126481 |
```

```
-------------------------------------------------------------
>>> Degree = 60 : R2=0.942718 | MSE=0.015416 | var=0.016124 | s_hat=63.142838 |
-------------------------------------------------------------
>>> Degree = 61 : R2=0.942973 | MSE=0.015347 | var=0.016065 | s_hat=62.861478 |
-------------------------------------------------------------
>>> Degree = 62 : R2=0.942562 | MSE=0.015458 | var=0.016193 | s_hat=63.315081 |
-------------------------------------------------------------
>>> Degree = 63 : R2=0.942918 | MSE=0.015362 | var=0.016105 | s_hat=62.922800 |
-------------------------------------------------------------
>>> Degree = 64 : R2=0.943112 | MSE=0.015310 | var=0.016063 | s_hat=62.708731 |
-------------------------------------------------------------
>>> Degree = 65 : R2=0.943005 | MSE=0.015339 | var=0.016105 | s_hat=62.826962 |
-------------------------------------------------------------
>>> Degree = 66 : R2=0.943020 | MSE=0.015334 | var=0.016113 | s_hat=62.809698 |
-------------------------------------------------------------
>>> Degree = 67 : R2=0.942852 | MSE=0.015380 | var=0.016173 | s_hat=62.995737 |
-------------------------------------------------------------
>>> Degree = 68 : R2=0.943131 | MSE=0.015305 | var=0.016107 | s_hat=62.688248 |
-------------------------------------------------------------
>>> Degree = 69 : R2=0.942780 | MSE=0.015399 | var=0.016219 | s_hat=63.074734 |
-------------------------------------------------------------
>>> Degree = 70 : R2=0.942838 | MSE=0.015383 | var=0.016215 | s_hat=63.010616 |
-------------------------------------------------------------
>>> Degree = 71 : R2=0.943053 | MSE=0.015326 | var=0.016166 | s_hat=62.774326 |
-------------------------------------------------------------
>>> Degree = 72 : R2=0.942663 | MSE=0.015430 | var=0.016290 | s_hat=63.203292 |
-------------------------------------------------------------
>>> Degree = 73 : R2=0.942968 | MSE=0.015348 | var=0.016215 | s_hat=62.867328 |
-------------------------------------------------------------
>>> Degree = 74 : R2=0.943023 | MSE=0.015334 | var=0.016212 | s_hat=62.806536 |
-------------------------------------------------------------
>>> Degree = 75 : R2=0.943017 | MSE=0.015335 | var=0.016227 | s_hat=62.813507 |
-------------------------------------------------------------
>>> Degree = 76 : R2=0.943142 | MSE=0.015302 | var=0.016204 | s_hat=62.675695 |
-------------------------------------------------------------
>>> Degree = 77 : R2=0.942958 | MSE=0.015351 | var=0.016269 | s_hat=62.878139 |
-------------------------------------------------------------
>>> Degree = 78 : R2=0.943002 | MSE=0.015339 | var=0.016269 | s_hat=62.830238 |
-------------------------------------------------------------
>>> Degree = 79 : R2=0.943320 | MSE=0.015254 | var=0.016191 | s_hat=62.479499 |
-------------------------------------------------------------
>>> Degree = 80 : R2=0.943215 | MSE=0.015282 | var=0.016233 | s_hat=62.595227 |
-------------------------------------------------------------
>>> Degree = 81 : R2=0.942842 | MSE=0.015382 | var=0.016352 | s_hat=63.005901 |
-------------------------------------------------------------
>>> Degree = 82 : R2=-505736128319626158534306037776.000000 |
MSE=136104188670273747178466836648.000000 | var=144800716050244494711408558808.000000 |
s_hat=557482756793441268443000162222208.000000 |
-------------------------------------------------------------
>>> Degree = 83 : R2=0.942756 | MSE=0.015406 | var=0.016403 | s_hat=63.101642 |
-------------------------------------------------------------
>>> Degree = 84 : R2=0.943538 | MSE=0.015195 | var=0.016191 | s_hat=62.238820 |
-------------------------------------------------------------
>>> Degree = 85 : R2=0.943119 | MSE=0.015308 | var=0.016324 | s_hat=62.701124 |
-------------------------------------------------------------
>>> Degree = 86 : R2=0.943250 | MSE=0.015272 | var=0.016299 | s_hat=62.556142 |
-------------------------------------------------------------
```

```
Warning: The "C" polynomial is unstable and cannot be automatically stabilized to meet the
prescribed constraints. This can cause the estimation to fail. Make sure that the starting
polynomial value is stable and within the desired constraints.
```

```
fprintf("=============================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 84
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
p = na+nb+nc;

BestFitModel_guassian = armax(data_guassian, [na nb nc 1])
```

```
BestFitModel_guassian =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

  A(z) = 1 + 0.1794 z^-1 - 0.1271 z^-2 + 0.2046 z^-3 + 0.214 z^-4 - 0.2439 z^-5 + 0.5541 z^-6
          + 0.5669 z^-7 - 0.03662 z^-8 + 0.4561 z^-9 + 0.5168 z^-10 - 0.1479 z^-11 + 0.4463
z^-12
          + 0.5741 z^-13 - 0.01891 z^-14 - 0.1412 z^-15 + 0.5684 z^-16 + 0.08544 z^-17 +
0.2419 z^-18
          + 0.4736 z^-19 - 0.07487 z^-20 + 0.08958 z^-21 + 0.1805 z^-22 + 0.1611 z^-23 +
0.1673 z^-24
          + 0.2311 z^-25 + 0.1152 z^-26 - 0.2218 z^-27 - 0.03184 z^-28 - 0.2695 z^-29 + 0.155
z^-30
          + 0.2449 z^-31 - 0.1874 z^-32 - 0.1959 z^-33 - 0.04641 z^-34 - 0.1267 z^-35 - 0.3121
z^-36
          + 0.176 z^-37 - 0.2605 z^-38 - 0.5203 z^-39 + 0.08833 z^-40 - 0.004627 z^-41 -
0.1073 z^-42
          - 0.09401 z^-43 + 0.1329 z^-44 - 0.4491 z^-45 - 0.1893 z^-46 + 0.1345 z^-47 - 0.1488
z^-48
          + 0.03492 z^-49 - 0.03669 z^-50 - 0.2413 z^-51 - 0.15 z^-52 + 0.1723 z^-53 - 0.0159
z^-54
```

```
          + 0.07324 z^-55 + 0.1114 z^-56 - 0.0561 z^-57 + 0.01355 z^-58 + 0.1043 z^-59 -
0.2789 z^-60
          + 0.1383 z^-61 + 0.1251 z^-62 - 0.1394 z^-63 - 0.01704 z^-64 + 0.4196 z^-65 - 0.118
z^-66
          + 0.06828 z^-67 + 0.1976 z^-68 - 0.1991 z^-69 - 0.09553 z^-70 + 0.2193 z^-71 -
0.2716 z^-72
          - 0.195 z^-73 + 0.1827 z^-74 + 0.02614 z^-75 - 0.1567 z^-76 + 0.1281 z^-77 +
0.006808 z^-78
          - 0.06129 z^-79 - 0.1044 z^-80 - 0.01458 z^-81 - 0.02957 z^-82 + 0.05137 z^-83 -
0.02701 z^-84


  B(z) = 0.395 z^-1 + 0.3401 z^-2 + 0.132 z^-3 + 0.1115 z^-4 + 0.1158 z^-5 - 0.05219 z^-6 +
0.1544 z^-7
          + 0.3226 z^-8 + 0.182 z^-9 + 0.2578 z^-10 + 0.3396 z^-11 + 0.1197 z^-12 + 0.1939 z^-
13
          + 0.3168 z^-14 + 0.1665 z^-15 - 0.000853 z^-16 + 0.1802 z^-17 + 0.1342 z^-18 +
0.1529 z^-19
          + 0.2662 z^-20 + 0.1223 z^-21 + 0.07095 z^-22 + 0.08635 z^-23 + 0.09838 z^-24 +
0.105 z^-25
          + 0.1461 z^-26 + 0.1273 z^-27 - 0.02917 z^-28 - 0.06101 z^-29 - 0.147 z^-30 -
0.03942 z^-31
          + 0.08993 z^-32 + 0.004754 z^-33 - 0.07691 z^-34 - 0.07756 z^-35 - 0.08696 z^-36 -
0.1679 z^-37
          - 0.03114 z^-38 - 0.09042 z^-39 - 0.2454 z^-40 - 0.1191 z^-41 - 0.0457 z^-42 -
0.04073 z^-43
          - 0.04723 z^-44 + 0.03449 z^-45 - 0.1343 z^-46 - 0.1695 z^-47 - 0.04902 z^-48 -
0.063 z^-49
          - 0.01158 z^-50 - 0.009163 z^-51 - 0.09127 z^-52 - 0.1181 z^-53 - 0.002641 z^-54 +
0.00761 z^-55
          + 0.04024 z^-56 + 0.07448 z^-57 + 0.02387 z^-58 + 0.003332 z^-59 + 0.03006 z^-60 -
0.09426 z^-61
          - 0.02478 z^-62 + 0.03684 z^-63 - 0.02439 z^-64 - 0.03457 z^-65 + 0.1362 z^-66 +
0.05324 z^-67
          + 0.04155 z^-68 + 0.08477 z^-69 - 0.03023 z^-70 - 0.08116 z^-71 + 0.01935 z^-72 -
0.08789 z^-73
          - 0.1368 z^-74 - 0.01865 z^-75 + 0.02128 z^-76 - 0.03412 z^-77 + 0.02088 z^-78 +
0.01995 z^-79
                    - 0.01068 z^-80 - 0.05414 z^-81 - 0.04474 z^-82 - 0.03392 z^-83 + 0.001124
z^-84


  C(z) = 1 + 0.3541 z^-1 + 0.007259 z^-2 + 0.2531 z^-3 + 0.2793 z^-4 - 0.1605 z^-5 + 0.5758
z^-6
          + 0.6789 z^-7 + 0.1146 z^-8 + 0.538 z^-9 + 0.6165 z^-10 + 0.0129 z^-11 + 0.525 z^-12
          + 0.6956 z^-13 + 0.1675 z^-14 - 0.01262 z^-15 + 0.5985 z^-16 + 0.1878 z^-17 + 0.3199
z^-18
          + 0.5607 z^-19 + 0.04553 z^-20 + 0.1647 z^-21 + 0.2718 z^-22 + 0.229 z^-23 + 0.248
z^-24
          + 0.3445 z^-25 + 0.21 z^-26 - 0.175 z^-27 + 0.01685 z^-28 - 0.2634 z^-29 + 0.1024
z^-30
          + 0.2933 z^-31 - 0.09595 z^-32 - 0.1829 z^-33 - 0.06418 z^-34 - 0.1042 z^-35 -
0.3563 z^-36
          + 0.1512 z^-37 - 0.2068 z^-38 - 0.5409 z^-39 - 0.01896 z^-40 - 0.0009514 z^-41 -
0.117 z^-42
          - 0.1057 z^-43 + 0.1659 z^-44 - 0.3937 z^-45 - 0.2384 z^-46 + 0.1108 z^-47 - 0.1296
z^-48
```

```
          + 0.03997 z^-49 - 0.004434 z^-50 - 0.221 z^-51 - 0.1936 z^-52 + 0.1825 z^-53 +
0.0298 z^-54
          + 0.09943 z^-55 + 0.1776 z^-56 + 0.02789 z^-57 + 0.02378 z^-58 + 0.1188 z^-59 -
0.2348 z^-60
          + 0.1399 z^-61 + 0.177 z^-62 - 0.05172 z^-63 - 0.01735 z^-64 + 0.3946 z^-65 - 0.0379
z^-66
          + 0.1019 z^-67 + 0.2053 z^-68 - 0.1061 z^-69 - 0.02634 z^-70 + 0.2064 z^-71 - 0.2569
z^-72
          - 0.2008 z^-73 + 0.1302 z^-74 + 0.014 z^-75 - 0.1293 z^-76 + 0.09834 z^-77 - 0.01433
z^-78
          + 0.003744 z^-79 - 0.09113 z^-80 - 0.04175 z^-81 - 0.05318 z^-82 + 0.04414 z^-83 -
0.05913 z^-84

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=84   nb=84   nc=84   nk=1
   Number of free coefficients: 252
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_guassian".
Fit to estimation data: 77.15% (prediction focus)
FPE: 0.01589, MSE: 0.01405
```

```
BestFit_y_hat_guassian = lsim(BestFitModel_guassian, u1_val, t_val);
% [armax_BestFit_r2, armax_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 25 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 25
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
p = na+nb+nc;

armax_VarModel_guassian = armax(data_guassian, [na nb nc nk])
```

```
armax_VarModel_guassian =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

  A(z) = 1 - 0.6369 z^-1 - 0.1519 z^-2 - 0.4378 z^-3 - 0.4319 z^-4 + 0.2646 z^-5 + 0.2285 z^
          -6 + 0.8206 z^-7 - 0.7309 z^-8 + 0.3031 z^-9 - 0.6609 z^-10 - 0.02517 z^-11 + 1.028
z^-12
          - 0.2974 z^-13 + 0.5281 z^-14 - 0.7234 z^-15 - 0.3857 z^-16 - 0.2124 z^-17 + 0.1451
z^-18
          + 0.3051 z^-19 - 0.1103 z^-20 + 0.591 z^-21 - 0.4849 z^-22 - 0.05937 z^-23 + 0.1512
z^-24
                                                                                    - 0.003266
z^-25


  B(z) = 0.395 z^-1 + 0.01674 z^-2 - 0.0972 z^-3 - 0.2585 z^-4 - 0.3481 z^-5 - 0.1001 z^-6 +
0.07821 z^-7
          + 0.4119 z^-8 + 0.001841 z^-9 + 0.07549 z^-10 - 0.2262 z^-11 - 0.1874 z^-12 + 0.2964
z^-13
          + 0.1129 z^-14 + 0.2627 z^-15 - 0.1258 z^-16 - 0.2807 z^-17 - 0.2779 z^-18 - 0.1034
z^-19
          + 0.1001 z^-20 + 0.05857 z^-21 + 0.2794 z^-22 - 0.006255 z^-23 - 0.0649 z^-24 -
0.004054 z^-25


  C(z) = 1 - 0.4651 z^-1 - 0.1608 z^-2 - 0.4772 z^-3 - 0.5167 z^-4 + 0.1378 z^-5 + 0.2102 z^
          -6 + 0.8536 z^-7 - 0.6102 z^-8 + 0.2696 z^-9 - 0.6835 z^-10 - 0.0744 z^-11 + 0.97
z^-12
          - 0.1384 z^-13 + 0.5961 z^-14 - 0.658 z^-15 - 0.4238 z^-16 - 0.3546 z^-17 + 0.09136
z^-18
          + 0.2473 z^-19 - 0.0666 z^-20 + 0.5943 z^-21 - 0.4244 z^-22 - 0.05289 z^-23 + 0.1098
z^-24
                                                                                     + 0.04974
z^-25

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=25   nb=25   nc=25   nk=1
   Number of free coefficients: 75
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_guassian".
Fit to estimation data: 76.6% (prediction focus)
FPE: 0.01528, MSE: 0.01473
```

```
Var_y_hat_guassian = lsim(armax_VarModel_guassian, u1_val, t_val);
% [armax_Var_r2, armax_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
```

```
fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
p = na+nb+nc;

armax_AICModel_guassian = armax(data_guassian, [na nb nc nk])
```

```
armax_AICModel_guassian =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.14 z^-1 + 0.4313 z^-2

  B(z) = 0.395 z^-1 - 0.1828 z^-2

  C(z) = 1 - 0.977 z^-1 + 0.3611 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_guassian".
Fit to estimation data: 76.24% (prediction focus)
FPE: 0.01524, MSE: 0.0152
```

```
AIC_y_hat_guassian = lsim(armax_AICModel_guassian, u1_val, t_val);
% [armax_AIC_r2, armax_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method===================\n")
```

```
===============Degree Extraction | F test Method===================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.16 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 2
```

```
na = winner;
nb = winner;
nc = winner;
p = na+nb+nc;

armax_FTestModel_guassian = armax(data_guassian, [na nb nc nk])
```

```
armax_FTestModel_guassian =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.14 z^-1 + 0.4313 z^-2

  B(z) = 0.395 z^-1 - 0.1828 z^-2

  C(z) = 1 - 0.977 z^-1 + 0.3611 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
```

```
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_guassian".
Fit to estimation data: 76.24% (prediction focus)
FPE: 0.01524, MSE: 0.0152
```

```
FTest_y_hat_guassian = lsim(armax_FTestModel_guassian, u1_val, t_val);
% [armax_FTest_r2, armax_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[armax_BestFit_r2_guassian, armax_BestFit_mse_guassian] = rSQR(y1_val,
BestFit_y_hat_guassian);
[armax_Var_r2_guassian, armax_Var_mse_guassian] = rSQR(y1_val,
Var_y_hat_guassian);
[armax_AIC_r2_guassian, armax_AIC_mse_guassian] = rSQR(y1_val,
AIC_y_hat_guassian);
[armax_FTest_r2_guassian, armax_FTest_mse_guassian] = rSQR(y1_val,
FTest_y_hat_guassian);

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_BestFit_r2_guassian,
armax_BestFit_mse_guassian)
```

```
    R2 value : 0.9414    | MSE : 0.0156
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_Var_r2_guassian,
armax_Var_mse_guassian)
```

```
    R2 value : 0.9426   | MSE : 0.0153
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_Cov_r2, armax_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_AIC_r2_guassian,
armax_AIC_mse_guassian)
```

```
    R2 value : 0.9430   | MSE : 0.0152
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", armax_FTest_r2_guassian,
armax_FTest_mse_guassian)
```

```
    R2 value : 0.9430   | MSE : 0.0152
```

```
fprintf("---------------------------------------------------------------\n")
```

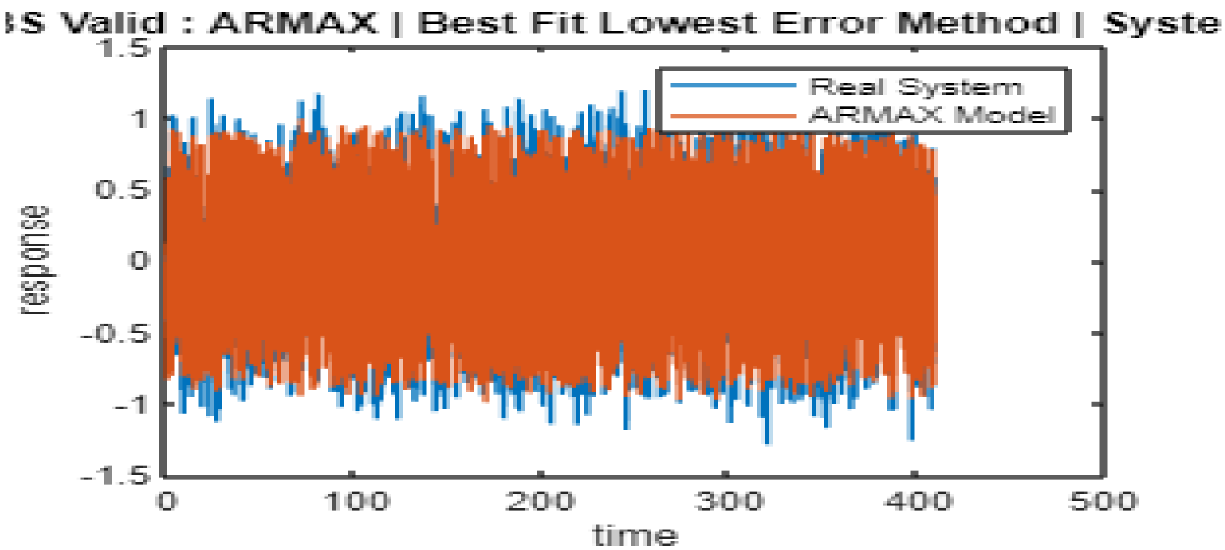
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

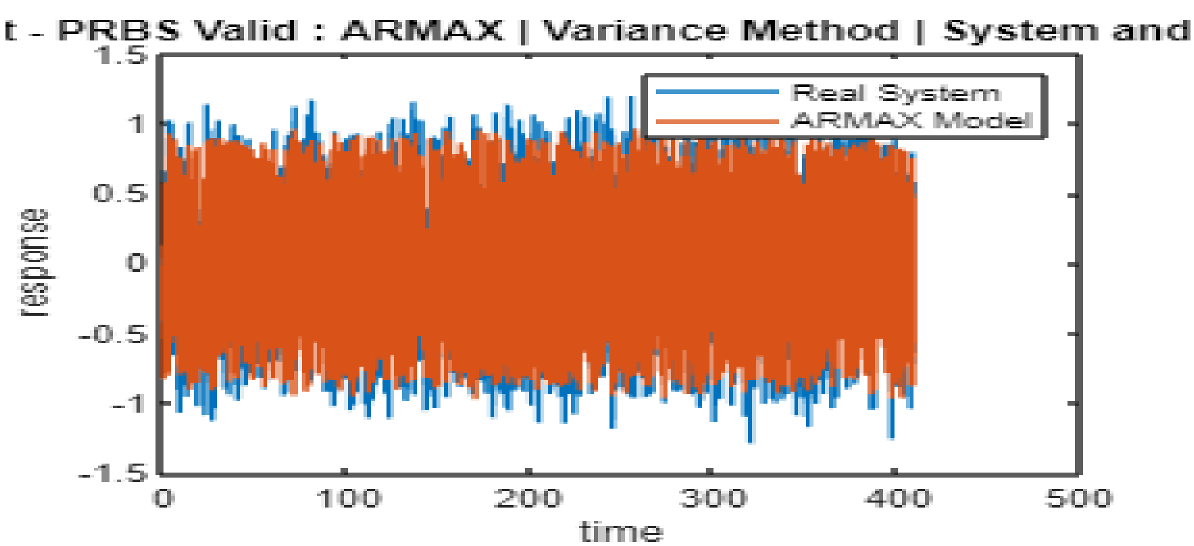
```
===============================================================
```

```
%%

armax_BestFitError_guassian = y1_val - BestFit_y_hat_guassian;
armax_VarError_guassian = y1_val - Var_y_hat_guassian;
armax_AICError_guassian = y1_val - AIC_y_hat_guassian;
```

```
armax_FTestError_guassian = y1_val - FTest_y_hat_guassian;

for k=0:N_val-1
     armax_BestFit_Ree_guassian(k+1,1) = 
AutoCorrelate(armax_BestFitError_guassian, k);
     armax_Var_Ree_guassian(k+1,1) = AutoCorrelate(armax_VarError_guassian, k);
     armax_AIC_Ree_guassian(k+1,1) = AutoCorrelate(armax_AICError_guassian, k);
     armax_FTest_Ree_guassian(k+1,1) = AutoCorrelate(armax_FTestError_guassian, 
k);
end

for k=0:N_val-1
     armax_BestFit_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, 
armax_BestFitError_guassian, k);
     armax_Var_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, 
armax_VarError_guassian, k);
     armax_AIC_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, 
armax_AICError_guassian, k);
     armax_FTest_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, 
armax_FTestError_guassian, k);
end


%%
figure(1)
plot(t_val,y1_val,t_val,BestFit_y_hat_guassian)
legend('Real System','ARMAX Model')
title(" Guassian Ident - PRBS Valid : ARMAX | Best Fit Lowest Error Method | 
System and Model Response")
xlabel("time")
ylabel("response")
```

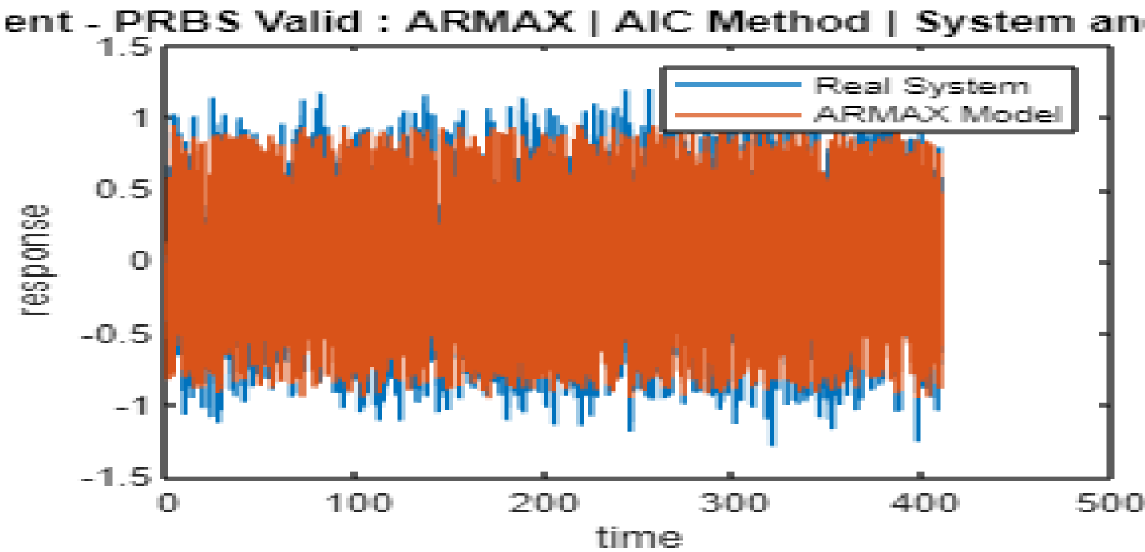



```
figure(2)
plot(t_val,y1_val,t_val,Var_y_hat_guassian)
legend('Real System','ARMAX Model')
title(" Guassian Ident - PRBS Valid : ARMAX | Variance Method | System and
Model Response")
xlabel("time")
ylabel("response")
```

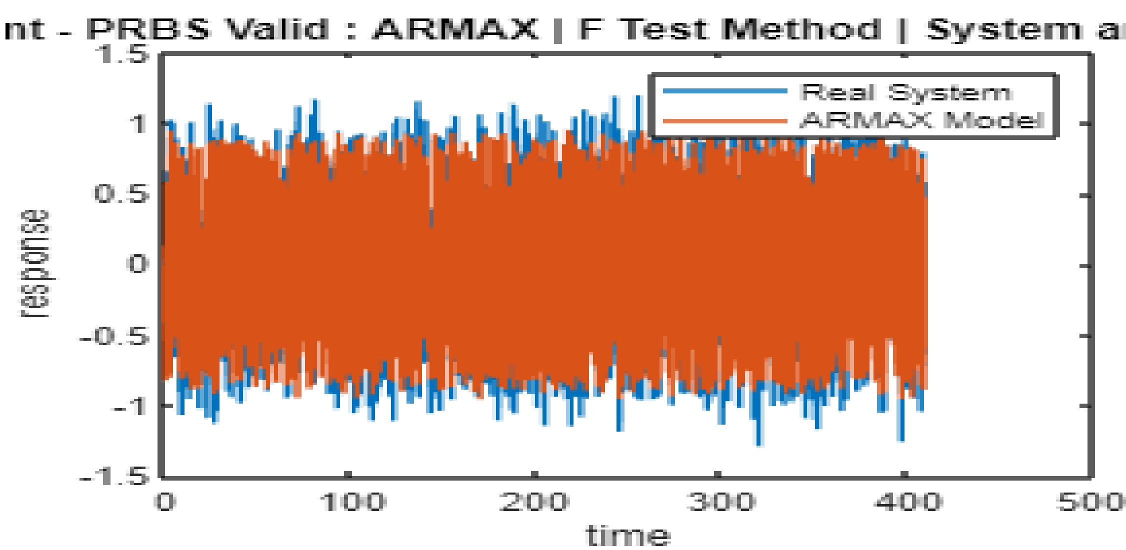



```
figure(3)
plot(t_val,y1_val,t_val,AIC_y_hat_guassian)
legend('Real System','ARMAX Model')
title(" Guassian Ident - PRBS Valid : ARMAX | AIC Method | System and Model
Response")
```

```
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y1_val,t_val,FTest_y_hat_guassian)
legend('Real System','ARMAX Model')
title(" Guassian Ident - PRBS Valid : ARMAX | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
%%
```

```
figure(5)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Ree_guassian(2:end), 1:N_val-1,
mean(armax_BestFit_Ree_guassian(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARMAX | Best Fit Lowest Errror Method |
Ree_guassian(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_guassian(k)")

subplot(4,1,2)
plot(1:N_val-1,armax_Var_Ree_guassian(2:end), 1:N_val-1,
mean(armax_Var_Ree_guassian(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARMAX | Variance Method | Ree_guassian(k)
| The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_guassian(k)")

subplot(4,1,3)
plot(1:N_val-1,armax_AIC_Ree_guassian(2:end), 1:N_val-1,
mean(armax_AIC_Ree_guassian(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARMAX | AIC Method | Ree_guassian(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_guassian(k)")

subplot(4,1,4)
plot(1:N_val-1,armax_FTest_Ree_guassian(2:end), 1:N_val-1,
mean(armax_FTest_Ree_guassian(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARMAX | F Test Method | Ree_guassian(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_guassian(k)")

%%

figure(6)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Rue_guassian(2:end), 1:N_val-1,
mean(armax_BestFit_Rue_guassian(2:end))*ones(length(1:N_val-1)))
title(" Guassian Ident - PRBS Valid : ARMAX | Best Fit Lowest Errror Method |
Rue_guassian(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Rue_guassian(k)")

subplot(4,1,2)
```

```
 plot(1:N_val-1,armax_Var_Rue_guassian(2:end), 1:N_val-1,
mean(armax_Var_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 title(" Guassian Ident - PRBS Valid : ARMAX | Variance Method | Rue_guassian(k)
| The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_guassian(k)")

 subplot(4,1,3)
 plot(1:N_val-1,armax_AIC_Rue_guassian(2:end), 1:N_val-1,
mean(armax_AIC_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 title(" Guassian Ident - PRBS Valid : ARMAX | AIC Method | Rue_guassian(k) |
The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_guassian(k)")

 subplot(4,1,4)
 plot(1:N_val-1,armax_FTest_Rue_guassian(2:end), 1:N_val-1,
mean(armax_FTest_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 title(" Guassian Ident - PRBS Valid : ARMAX | F Test Method | Rue_guassian(k) |
The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_guassian(k)")
```

PRBS Input - Guassian Validation

```
 %%
 % PRBS Input **************************************************************
 fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
 fprintf(">>> PRBS Input Identification Begins:------------------------------
\n")
```

```
>>> PRBS Input Identification Begins:------------------------------
```

```
 %%

 t = 0:Ts:length(u2)*Ts-Ts;
 t_val = 0:Ts:length(u2_val)*Ts-Ts;
 N = length(y2);
 N_val = length(y2_val);

 data_prbs = iddata(y2,u2,Ts);
```

```
%%

fprintf("===================Degree Extraction |
RUN==========================\n")
```

```
===================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nk = 1;
    p = na+nb+nc;

    try
        sys = armax(data_prbs, [na nb nc nk]);
        armax_y_hat_prbs = lsim(sys, u2, t);
    catch
        break
    end

    [r2_armax, mse_armax] = rSQR(y2, armax_y_hat_prbs);

    error = y2 - armax_y_hat_prbs;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_armax, mse_armax, variance, S_hat)
```

```
    fprintf("--------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_armax];
    MSEs = [MSEs; mse_armax];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.896281 | MSE=0.027642 | var=0.027662 | s_hat=113.220916 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.943231 | MSE=0.015129 | var=0.015151 | s_hat=61.969490 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.943233 | MSE=0.015129 | var=0.015162 | s_hat=61.968240 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.943306 | MSE=0.015109 | var=0.015154 | s_hat=61.888331 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.943425 | MSE=0.015078 | var=0.015133 | s_hat=61.758329 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.943380 | MSE=0.015090 | var=0.015156 | s_hat=61.807142 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.943385 | MSE=0.015088 | var=0.015166 | s_hat=61.802020 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.943466 | MSE=0.015067 | var=0.015155 | s_hat=61.712944 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.943615 | MSE=0.015027 | var=0.015127 | s_hat=61.550890 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.943589 | MSE=0.015034 | var=0.015145 | s_hat=61.579227 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.943662 | MSE=0.015015 | var=0.015137 | s_hat=61.499999 |
-------------------------------------------------------------
>>> Degree = 12 : R2=0.943600 | MSE=0.015031 | var=0.015164 | s_hat=61.566932 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.943733 | MSE=0.014996 | var=0.015140 | s_hat=61.422221 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.943865 | MSE=0.014961 | var=0.015116 | s_hat=61.278385 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.943755 | MSE=0.014990 | var=0.015156 | s_hat=61.398141 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.943753 | MSE=0.014990 | var=0.015168 | s_hat=61.400538 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.943876 | MSE=0.014958 | var=0.015146 | s_hat=61.266211 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.943980 | MSE=0.014930 | var=0.015129 | s_hat=61.152503 |
-------------------------------------------------------------
>>> Degree = 19 : R2=0.943829 | MSE=0.014970 | var=0.015181 | s_hat=61.317619 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.943896 | MSE=0.014952 | var=0.015174 | s_hat=61.243710 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.943774 | MSE=0.014985 | var=0.015219 | s_hat=61.377390 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.943881 | MSE=0.014956 | var=0.015201 | s_hat=61.260650 |
```

```
------------------------------------------------------------
>>> Degree = 23 : R2=0.944150 | MSE=0.014885 | var=0.015140 | s_hat=60.967056 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.943867 | MSE=0.014960 | var=0.015228 | s_hat=61.276071 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.944087 | MSE=0.014901 | var=0.015179 | s_hat=61.035899 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.943802 | MSE=0.014977 | var=0.015268 | s_hat=61.346962 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.944264 | MSE=0.014854 | var=0.015154 | s_hat=60.842730 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.944122 | MSE=0.014892 | var=0.015204 | s_hat=60.997871 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.943953 | MSE=0.014937 | var=0.015261 | s_hat=61.182139 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.944176 | MSE=0.014877 | var=0.015212 | s_hat=60.938036 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.944274 | MSE=0.014851 | var=0.015196 | s_hat=60.831441 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.944224 | MSE=0.014865 | var=0.015222 | s_hat=60.886215 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.944049 | MSE=0.014911 | var=0.015281 | s_hat=61.076654 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.944353 | MSE=0.014830 | var=0.015209 | s_hat=60.745488 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.944246 | MSE=0.014859 | var=0.015250 | s_hat=60.861959 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.943855 | MSE=0.014963 | var=0.015368 | s_hat=61.289197 |
------------------------------------------------------------
>>> Degree = 37 : R2=0.944155 | MSE=0.014883 | var=0.015298 | s_hat=60.960822 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.944574 | MSE=0.014771 | var=0.015194 | s_hat=60.503544 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.944561 | MSE=0.014775 | var=0.015209 | s_hat=60.517705 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.944569 | MSE=0.014773 | var=0.015219 | s_hat=60.509045 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.944551 | MSE=0.014778 | var=0.015235 | s_hat=60.529287 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.944334 | MSE=0.014835 | var=0.015306 | s_hat=60.765534 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.944609 | MSE=0.014762 | var=0.015242 | s_hat=60.466012 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.944410 | MSE=0.014815 | var=0.015308 | s_hat=60.682537 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.944759 | MSE=0.014722 | var=0.015224 | s_hat=60.301823 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.944628 | MSE=0.014757 | var=0.015272 | s_hat=60.444840 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.944682 | MSE=0.014743 | var=0.015268 | s_hat=60.385996 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.942543 | MSE=0.015313 | var=0.015871 | s_hat=62.720831 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.944680 | MSE=0.014743 | var=0.015292 | s_hat=60.387812 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.944664 | MSE=0.014748 | var=0.015308 | s_hat=60.406269 |
------------------------------------------------------------
```

```
>>> Degree = 51 : R2=0.944937 | MSE=0.014675 | var=0.015244 | s_hat=60.107804 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.944659 | MSE=0.014749 | var=0.015333 | s_hat=60.410868 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.939271 | MSE=0.016185 | var=0.016838 | s_hat=66.292438 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.944584 | MSE=0.014769 | var=0.015377 | s_hat=60.493508 |
------------------------------------------------------------
>>> Degree = 55 : R2=-743932476040666170045245685768.000000 |
MSE=198264033154873818458536017792.000000 | var=206585978072338591877724897728.000000 |
s_hat=812089479802362980262178434580048.000000 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.944717 | MSE=0.014733 | var=0.015363 | s_hat=60.347487 |
------------------------------------------------------------
>>> Degree = 57 : R2=0.944563 | MSE=0.014774 | var=0.015418 | s_hat=60.515929 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.944852 | MSE=0.014697 | var=0.015349 | s_hat=60.200303 |
------------------------------------------------------------
>>> Degree = 59 : R2=0.944756 | MSE=0.014723 | var=0.015388 | s_hat=60.305245 |
------------------------------------------------------------
>>> Degree = 60 : R2=0.944742 | MSE=0.014727 | var=0.015404 | s_hat=60.320748 |
------------------------------------------------------------
>>> Degree = 61 : R2=0.944915 | MSE=0.014681 | var=0.015367 | s_hat=60.131846 |
------------------------------------------------------------
>>> Degree = 62 : R2=0.944898 | MSE=0.014685 | var=0.015384 | s_hat=60.149898 |
------------------------------------------------------------
>>> Degree = 63 : R2=0.944885 | MSE=0.014689 | var=0.015399 | s_hat=60.164397 |
------------------------------------------------------------
>>> Degree = 64 : R2=0.945138 | MSE=0.014621 | var=0.015340 | s_hat=59.888268 |
------------------------------------------------------------
>>> Degree = 65 : R2=0.945336 | MSE=0.014568 | var=0.015297 | s_hat=59.671768 |
------------------------------------------------------------
>>> Degree = 66 : R2=0.945051 | MSE=0.014644 | var=0.015388 | s_hat=59.983540 |
------------------------------------------------------------
>>> Degree = 67 : R2=-
2746033392441135182203187402614249446331859181641917731114137133234418316242110343939341693435
981555516643437707264.000000 |
MSE=731840150951520992680279763780803027484848881564519865419322550339012697989672240287639245
581870145366926028701696.000000 |
var=769606484800367265766123082606652034246276195307867923513975485372103812143363457113670660
900293470641847288201216.000000 |
s_hat=2997617258297430545954611456897221839938511160999342899168919474850979735962972737166138
144943576460642302331463925760.000000 |
------------------------------------------------------------
>>> Degree = 68 : R2=0.944629 | MSE=0.014757 | var=0.015530 | s_hat=60.443443 |
------------------------------------------------------------
>>> Degree = 69 : R2=0.944933 | MSE=0.014676 | var=0.015457 | s_hat=60.111837 |
------------------------------------------------------------
>>> Degree = 70 : R2=0.944780 | MSE=0.014717 | var=0.015512 | s_hat=60.279145 |
------------------------------------------------------------
>>> Degree = 71 : R2=0.944794 | MSE=0.014713 | var=0.015520 | s_hat=60.263770 |
------------------------------------------------------------
>>> Degree = 72 : R2=0.944528 | MSE=0.014784 | var=0.015607 | s_hat=60.554210 |
------------------------------------------------------------
>>> Degree = 73 : R2=-1569606.064068 | MSE=418313.001533 | var=441942.237369 |
s_hat=1713410054.279447 |
------------------------------------------------------------
```

```
>>> Degree = 74 : R2=0.944803 | MSE=0.014711 | var=0.015554 | s_hat=60.254448 |
------------------------------------------------------------
>>> Degree = 75 : R2=0.945318 | MSE=0.014573 | var=0.015420 | s_hat=59.692101 |
------------------------------------------------------------
>>> Degree = 76 : R2=0.945103 | MSE=0.014630 | var=0.015493 | s_hat=59.925969 |
------------------------------------------------------------
>>> Degree = 77 : R2=0.929768 | MSE=0.018717 | var=0.019836 | s_hat=76.666505 |
------------------------------------------------------------
>>> Degree = 78 : R2=0.944743 | MSE=0.014726 | var=0.015619 | s_hat=60.319339 |
------------------------------------------------------------
>>> Degree = 79 : R2=0.945002 | MSE=0.014657 | var=0.015558 | s_hat=60.036526 |
------------------------------------------------------------
>>> Degree = 80 : R2=0.945228 | MSE=0.014597 | var=0.015506 | s_hat=59.790048 |
------------------------------------------------------------
>>> Degree = 81 : R2=0.945057 | MSE=0.014643 | var=0.015566 | s_hat=59.977124 |
------------------------------------------------------------
>>> Degree = 82 : R2=0.945228 | MSE=0.014597 | var=0.015530 | s_hat=59.790315 |
------------------------------------------------------------
>>> Degree = 83 : R2=-3.424753 | MSE=1.179232 | var=1.255559 | s_hat=4830.136103 |
------------------------------------------------------------
>>> Degree = 84 : R2=0.945126 | MSE=0.014624 | var=0.015583 | s_hat=59.901699 |
------------------------------------------------------------
>>> Degree = 85 : R2=0.945104 | MSE=0.014630 | var=0.015602 | s_hat=59.925522 |
------------------------------------------------------------
>>> Degree = 86 : R2=0.945154 | MSE=0.014617 | var=0.015599 | s_hat=59.870709 |
------------------------------------------------------------
>>> Degree = 87 : R2=0.944824 | MSE=0.014705 | var=0.015706 | s_hat=60.230912 |
------------------------------------------------------------
>>> Degree = 88 : R2=0.945019 | MSE=0.014653 | var=0.015662 | s_hat=60.018187 |
------------------------------------------------------------
>>> Degree = 89 : R2=0.944806 | MSE=0.014710 | var=0.015735 | s_hat=60.250266 |
------------------------------------------------------------
>>> Degree = 90 : R2=-57986043337651983210250240.000000 |
MSE=15453750426386463415336960.000000 | var=16544318281881583106392064.000000 |
s_hat=63298561746478936557034143744.000000 |
------------------------------------------------------------
>>> Degree = 91 : R2=0.945197 | MSE=0.014605 | var=0.015648 | s_hat=59.824038 |
------------------------------------------------------------
>>> Degree = 92 : R2=0.944911 | MSE=0.014682 | var=0.015742 | s_hat=60.135847 |
------------------------------------------------------------
Warning: The "C" polynomial is unstable and cannot be automatically stabilized to meet the
prescribed constraints. This can cause the estimation to fail. Make sure that the starting
polynomial value is stable and within the desired constraints.
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("==============Degree Extraction | BestFit Method================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 65
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
p = na+nb+nc;

BestFitModel_prbs = armax(data_prbs, [na nb nc 1])
```

```
BestFitModel_prbs =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

  A(z) = 1 - 2.411 z^-1 + 2.765 z^-2 - 2.004 z^-3 + 1.134 z^-4 - 0.1054 z^-5 - 0.5993 z^-6 +
0.581 z^-7
          - 0.3432 z^-8 - 0.3868 z^-9 + 1.733 z^-10 - 2.288 z^-11 + 1.553 z^-12 - 0.1342 z^-13
          - 1.564 z^-14 + 3.042 z^-15 - 3.579 z^-16 + 3.224 z^-17 - 1.775 z^-18 - 0.3266 z^-19
          + 1.737 z^-20 - 1.826 z^-21 + 1.089 z^-22 + 0.2407 z^-23 - 1.6 z^-24 + 2.13 z^-25
          - 1.962 z^-26 + 0.9412 z^-27 + 0.9814 z^-28 - 2.681 z^-29 + 3.008 z^-30 - 2.362 z^
          -31 + 1.361 z^-32 + 0.02278 z^-33 - 1.178 z^-34 + 1.573 z^-35 - 1.287 z^-36 + 0.6499
z^-37
          + 0.1055 z^-38 - 0.4087 z^-39 + 0.05172 z^-40 + 0.4327 z^-41 - 1.063 z^-42 + 1.714
z^-43
          - 1.988 z^-44 + 1.688 z^-45 - 1.156 z^-46 + 0.4965 z^-47 - 0.1941 z^-48 + 0.01254
z^-49
          + 0.4467 z^-50 - 0.6795 z^-51 + 0.3836 z^-52 - 0.22 z^-53 + 0.2446 z^-54 - 0.3604
z^-55
          + 0.7 z^-56 - 0.7808 z^-57 + 0.3299 z^-58 - 0.2746 z^-59 + 0.5305 z^-60 - 0.4905 z^-
61
                                        + 0.2431 z^-62 - 0.1869 z^-63 + 0.1379 z^-64 - 0.04506
z^-65


  B(z) = 0.3999 z^-1 - 0.6972 z^-2 + 0.599 z^-3 - 0.3497 z^-4 + 0.1759 z^-5 + 0.1117 z^-6 -
0.1985 z^-7
          + 0.1088 z^-8 - 0.07578 z^-9 - 0.1929 z^-10 + 0.5602 z^-11 - 0.5145 z^-12 + 0.2284
z^-13
          + 0.1382 z^-14 - 0.5646 z^-15 + 0.8526 z^-16 - 0.843 z^-17 + 0.6775 z^-18 - 0.1949
z^-19
          - 0.3289 z^-20 + 0.5097 z^-21 - 0.3894 z^-22 + 0.1525 z^-23 + 0.2334 z^-24 - 0.5189
z^-25
          + 0.5099 z^-26 - 0.4177 z^-27 + 0.05656 z^-28 + 0.4807 z^-29 - 0.7711 z^-30 + 0.6635
z^-31
```

```
          - 0.4459 z^-32 + 0.1863 z^-33 + 0.1924 z^-34 - 0.3876 z^-35 + 0.3808 z^-36 - 0.2501
z^-37
          + 0.07043 z^-38 + 0.1139 z^-39 - 0.1103 z^-40 - 0.0465 z^-41 + 0.142 z^-42 - 0.33
z^-43
          + 0.4737 z^-44 - 0.4695 z^-45 + 0.3302 z^-46 - 0.1983 z^-47 + 0.02744 z^-48 -
0.02476 z^-49
          - 0.02817 z^-50 + 0.1787 z^-51 - 0.161 z^-52 + 0.03594 z^-53 - 0.05147 z^-54 +
0.05963 z^-55
          - 0.09886 z^-56 + 0.2122 z^-57 - 0.1655 z^-58 + 0.008334 z^-59 - 0.09611 z^-60 +
0.1458 z^-61
                                            - 0.08014 z^-62 + 0.02989 z^-63 - 0.0518 z^-64 + 0.02422
z^-65


  C(z) = 1 - 2.25 z^-1 + 2.485 z^-2 - 1.767 z^-3 + 0.9981 z^-4 - 0.01915 z^-5 - 0.5576 z^-6
          + 0.4717 z^-7 - 0.2499 z^-8 - 0.4359 z^-9 + 1.624 z^-10 - 1.958 z^-11 + 1.221 z^-12
          + 0.04341 z^-13 - 1.556 z^-14 + 2.818 z^-15 - 3.168 z^-16 + 2.774 z^-17 - 1.399 z^
          -18 - 0.481 z^-19 + 1.642 z^-20 - 1.631 z^-21 + 0.9495 z^-22 + 0.3005 z^-23 - 1.495
z^-24
          + 1.822 z^-25 - 1.544 z^-26 + 0.5163 z^-27 + 1.235 z^-28 - 2.617 z^-29 + 2.727 z^-30
          - 2.063 z^-31 + 1.065 z^-32 + 0.2932 z^-33 - 1.266 z^-34 + 1.479 z^-35 - 1.101 z^-36
          + 0.4541 z^-37 + 0.2107 z^-38 - 0.3469 z^-39 - 0.09308 z^-40 + 0.622 z^-41 - 1.335
z^-42
          + 1.939 z^-43 - 2.054 z^-44 + 1.606 z^-45 - 0.9798 z^-46 + 0.2597 z^-47 - 0.01116
z^-48
          - 0.1313 z^-49 + 0.4778 z^-50 - 0.5289 z^-51 + 0.1639 z^-52 - 0.09579 z^-53 + 0.2044
z^-54
          - 0.4161 z^-55 + 0.8093 z^-56 - 0.8387 z^-57 + 0.3448 z^-58 - 0.3016 z^-59 + 0.4527
z^-60
                         - 0.323 z^-61 + 0.08519 z^-62 - 0.08445 z^-63 + 0.06587 z^-64 - 0.01852
z^-65

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=65   nb=65   nc=65   nk=1
   Number of free coefficients: 195
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_prbs".
Fit to estimation data: 77.37% (prediction focus)
FPE: 0.01501, MSE: 0.01365
```

```
BestFit_y_hat_prbs = lsim(BestFitModel_prbs, u2_val, t_val);
% [armax_BestFit_r2, armax_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 14 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 14
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
p = na+nb+nc;

armax_VarModel_prbs = armax(data_prbs, [na nb nc nk])
```

```
armax_VarModel_prbs =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)

  A(z) = 1 - 0.9201 z^-1 + 0.3584 z^-2 + 0.09735 z^-3 + 0.7608 z^-4 - 1.058 z^-5 - 0.5145 z^
          -6 + 1.345 z^-7 - 0.5387 z^-8 - 0.07999 z^-9 - 0.4112 z^-10 + 0.8942 z^-11 - 0.3314
z^-12
                                                                       - 0.2091 z^-13 + 0.1465
z^-14


  B(z) = 0.3998 z^-1 - 0.1015 z^-2 + 0.03471 z^-3 + 0.05203 z^-4 + 0.3241 z^-5 - 0.2111 z^-6
          - 0.3894 z^-7 + 0.2842 z^-8 + 0.008117 z^-9 - 0.02725 z^-10 - 0.1878 z^-11 + 0.23
z^-12
                                                                      + 0.0406 z^-13 - 0.06951
z^-14


  C(z) = 1 - 0.752 z^-1 + 0.3088 z^-2 + 0.1138 z^-3 + 0.793 z^-4 - 0.9129 z^-5 - 0.5861 z^-6
          + 1.174 z^-7 - 0.3793 z^-8 - 0.05455 z^-9 - 0.4267 z^-10 + 0.8133 z^-11 - 0.2278 z^-
12
                                                                      - 0.1776 z^-13 + 0.09029
z^-14

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=14   nb=14   nc=14   nk=1
   Number of free coefficients: 42
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_prbs".
Fit to estimation data: 76.85% (prediction focus)
FPE: 0.01458, MSE: 0.01428
```

```
Var_y_hat_prbs = lsim(armax_VarModel_prbs, u2_val, t_val);
```

```
% [armax_Var_r2, armax_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
p = na+nb+nc;

armax_AICModel_prbs = armax(data_prbs, [na nb nc nk])
```

```
armax_AICModel_prbs =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.179 z^-1 + 0.4484 z^-2

  B(z) = 0.399 z^-1 - 0.2033 z^-2

  C(z) = 1 - 1.017 z^-1 + 0.3672 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_prbs".
Fit to estimation data: 76.65% (prediction focus)
FPE: 0.01457, MSE: 0.01453
```

```
AIC_y_hat_prbs = lsim(armax_AICModel_prbs, u2_val, t_val);
% [armax_AIC_r2, armax_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.15 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 2
```

```
na = winner;
nb = winner;
nc = winner;
p = na+nb+nc;

armax_FTestModel_prbs = armax(data_prbs, [na nb nc nk])
```

```
armax_FTestModel_prbs =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.179 z^-1 + 0.4484 z^-2
```

```
  B(z) = 0.399 z^-1 - 0.2033 z^-2

  C(z) = 1 - 1.017 z^-1 + 0.3672 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_prbs".
Fit to estimation data: 76.65% (prediction focus)
FPE: 0.01457, MSE: 0.01453
```

```
FTest_y_hat_prbs = lsim(armax_FTestModel_prbs, u2_val, t_val);
% [armax_FTest_r2, armax_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[armax_BestFit_r2_prbs, armax_BestFit_mse_prbs] = rSQR(y2_val,
BestFit_y_hat_prbs);
[armax_Var_r2_prbs, armax_Var_mse_prbs] = rSQR(y2_val, Var_y_hat_prbs);
[armax_AIC_r2_prbs, armax_AIC_mse_prbs] = rSQR(y2_val, AIC_y_hat_prbs);
[armax_FTest_r2_prbs, armax_FTest_mse_prbs] = rSQR(y2_val, FTest_y_hat_prbs);

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_BestFit_r2_prbs,
armax_BestFit_mse_prbs)
```

```
    R2 value : 0.9388    | MSE : 0.0165
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Var_r2_prbs, armax_Var_mse_prbs)
```

```
    R2 value : 0.9403    | MSE : 0.0161
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_Cov_r2, armax_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_AIC_r2_prbs, armax_AIC_mse_prbs)
```

```
    R2 value : 0.9407    | MSE : 0.0159
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", armax_FTest_r2_prbs, armax_FTest_mse_prbs)
```

```
    R2 value : 0.9407    | MSE : 0.0159
```

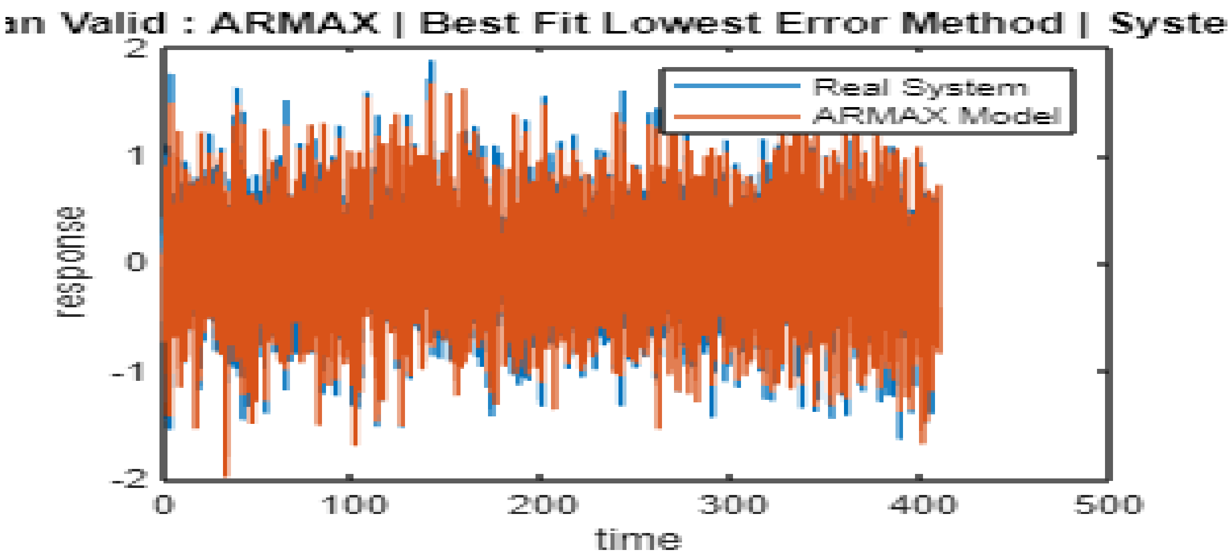
```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

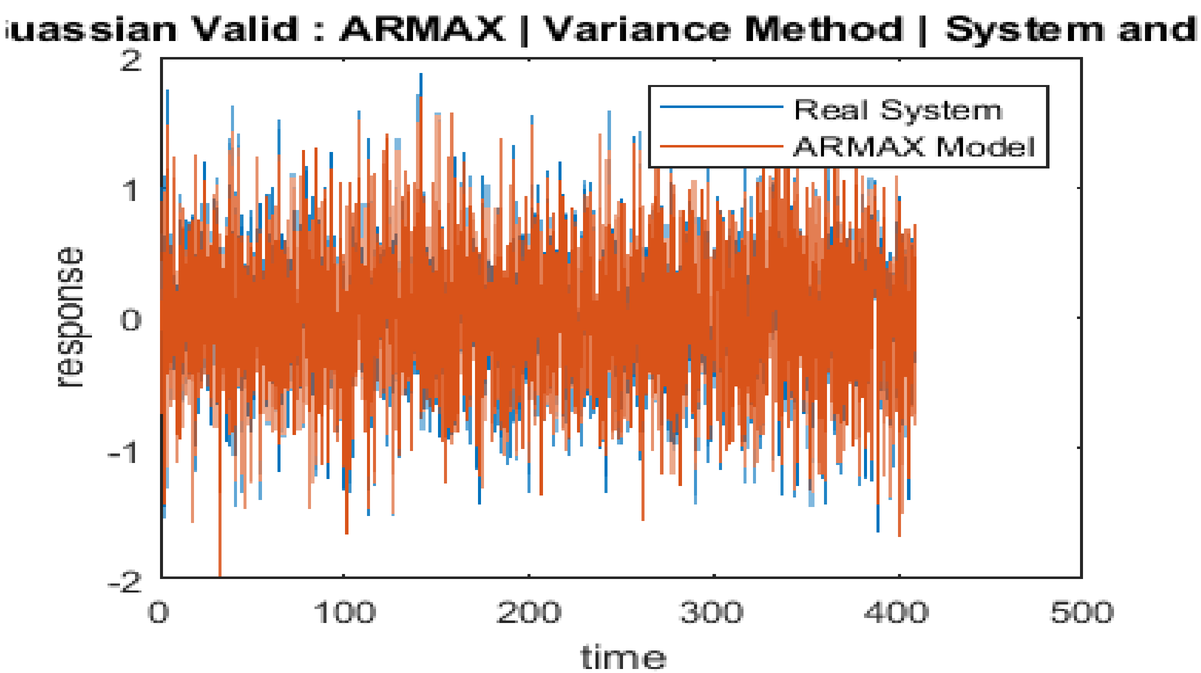
```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

```
===============================================================
```

```
%%

armax_BestFitError_prbs = y2_val - BestFit_y_hat_prbs;
armax_VarError_prbs = y2_val - Var_y_hat_prbs;
armax_AICError_prbs = y2_val - AIC_y_hat_prbs;
armax_FTestError_prbs = y2_val - FTest_y_hat_prbs;

for k=0:N_val-1
    armax_BestFit_Ree_prbs(k+1,1) = AutoCorrelate(armax_BestFitError_prbs, k);
    armax_Var_Ree_prbs(k+1,1) = AutoCorrelate(armax_VarError_prbs, k);
    armax_AIC_Ree_prbs(k+1,1) = AutoCorrelate(armax_AICError_prbs, k);
    armax_FTest_Ree_prbs(k+1,1) = AutoCorrelate(armax_FTestError_prbs, k);
end

for k=0:N_val-1
    armax_BestFit_Rue_prbs(k+1,1) = CrossCorrelate(u2_val,
armax_BestFitError_prbs, k);
    armax_Var_Rue_prbs(k+1,1) = CrossCorrelate(u2_val, armax_VarError_prbs, k);
    armax_AIC_Rue_prbs(k+1,1) = CrossCorrelate(u2_val, armax_AICError_prbs, k);
    armax_FTest_Rue_prbs(k+1,1) = CrossCorrelate(u2_val, armax_FTestError_prbs,
k);
end


%%
figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hat_prbs)
legend('Real System','ARMAX Model')
title(" PRBS Ident - Guassian Valid : ARMAX | Best Fit Lowest Error Method |
System and Model Response")
xlabel("time")
ylabel("response")
```

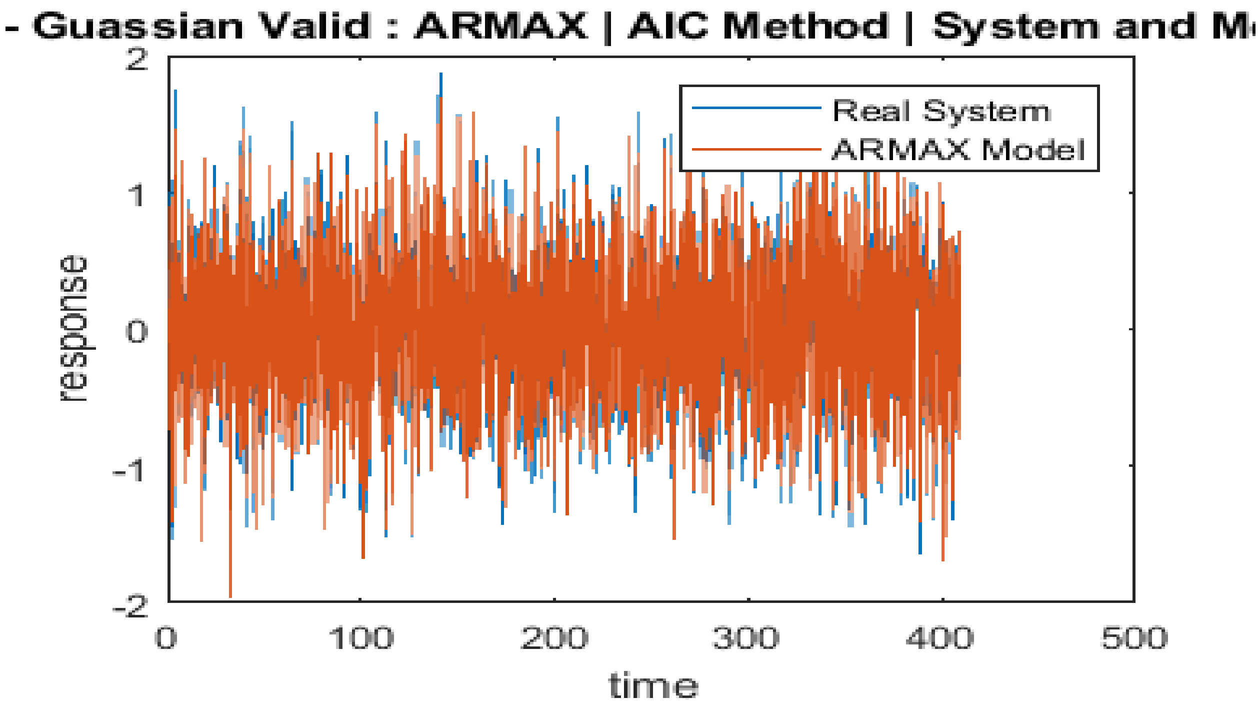



```
figure(2)
plot(t_val,y2_val,t_val,Var_y_hat_prbs)
legend('Real System','ARMAX Model')
title(" PRBS Ident - Guassian Valid : ARMAX | Variance Method | System and
Model Response")
xlabel("time")
ylabel("response")
```

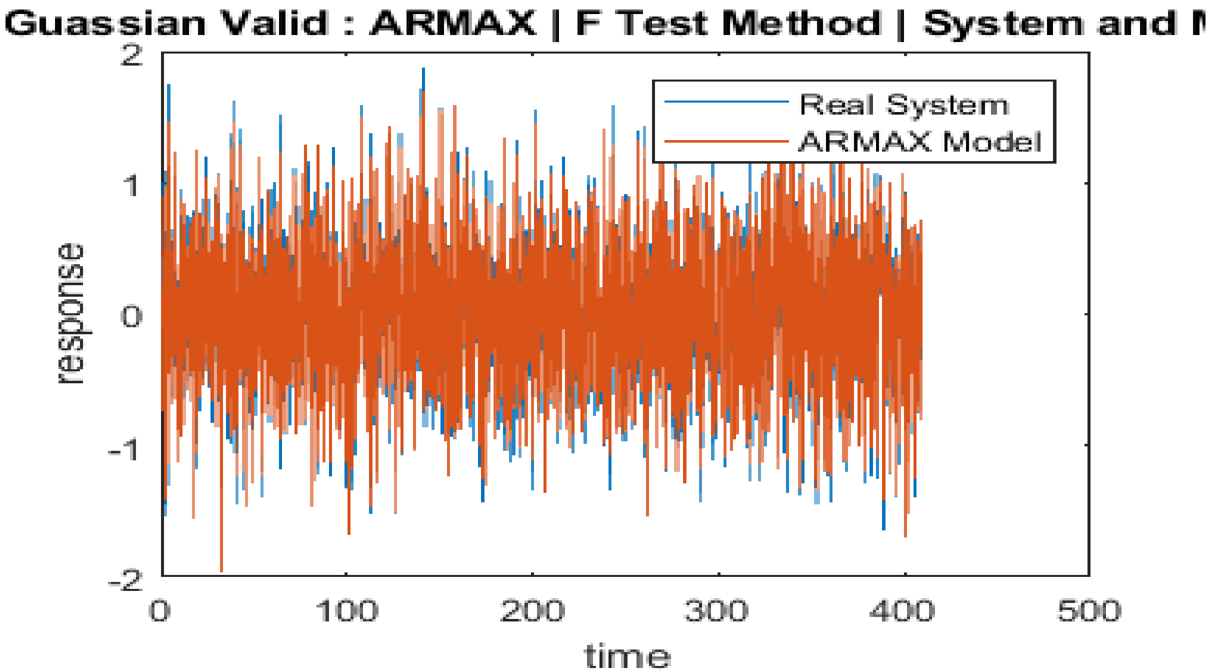



```
figure(3)
```

```
plot(t_val,y2_val,t_val,AIC_y_hat_prbs)
legend('Real System','ARMAX Model')
title(" PRBS Ident - Guassian Valid : ARMAX | AIC Method | System and Model
Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y2_val,t_val,FTest_y_hat_prbs)
legend('Real System','ARMAX Model')
title(" PRBS Ident - Guassian Valid : ARMAX | F Test Method | System and Model
Response")
xlabel("time")
ylabel("response")
```




```
%%

figure(5)
subplot(4,1,1)
plot(1:N_val-1,armax_BestFit_Ree_prbs(2:end), 1:N_val-1,
mean(armax_BestFit_Ree_prbs(2:end))*ones(length(1:N_val-1)))
title(" PRBS Ident - Guassian Valid : ARMAX | Best Fit Lowest Errror Method |
Ree_prbs(k) | The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_prbs(k)")

subplot(4,1,2)
plot(1:N_val-1,armax_Var_Ree_prbs(2:end), 1:N_val-1,
mean(armax_Var_Ree_prbs(2:end))*ones(length(1:N_val-1)))
title(" PRBS Ident - Guassian Valid : ARMAX | Variance Method | Ree_prbs(k) |
The Straight Line is the Mean")
xlabel("k")
ylabel("Ree_prbs(k)")

subplot(4,1,3)
plot(1:N_val-1,armax_AIC_Ree_prbs(2:end), 1:N_val-1,
mean(armax_AIC_Ree_prbs(2:end))*ones(length(1:N_val-1)))
title(" PRBS Ident - Guassian Valid : ARMAX | AIC Method | Ree_prbs(k) | The
Straight Line is the Mean")
xlabel("k")
ylabel("Ree_prbs(k)")
```

```
 subplot(4,1,4)
 plot(1:N_val-1,armax_FTest_Ree_prbs(2:end), 1:N_val-1,
mean(armax_FTest_Ree_prbs(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid : ARMAX | F Test Method | Ree_prbs(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Ree_prbs(k)")

 %%

 figure(6)
 subplot(4,1,1)
 plot(1:N_val-1,armax_BestFit_Rue_prbs(2:end), 1:N_val-1,
mean(armax_BestFit_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid : ARMAX | Best Fit Lowest Errror Method |
Rue_prbs(k) | The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_prbs(k)")

 subplot(4,1,2)
 plot(1:N_val-1,armax_Var_Rue_prbs(2:end), 1:N_val-1,
mean(armax_Var_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid : ARMAX | Variance Method | Rue_prbs(k) |
The Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_prbs(k)")

 subplot(4,1,3)
 plot(1:N_val-1,armax_AIC_Rue_prbs(2:end), 1:N_val-1,
mean(armax_AIC_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid : ARMAX | AIC Method | Rue_prbs(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_prbs(k)")

 subplot(4,1,4)
 plot(1:N_val-1,armax_FTest_Rue_prbs(2:end), 1:N_val-1,
mean(armax_FTest_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 title(" PRBS Ident - Guassian Valid : ARMAX | F Test Method | Rue_prbs(k) | The
Straight Line is the Mean")
 xlabel("k")
 ylabel("Rue_prbs(k)")
```

Q3 - part a | Box-Jenkins

```
clc; clear;

%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;
```

Guassian Input - PRBS Validation

```

%%
% Guassian Input ***************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> Guassian Input Identification Begins:-----------------------------
-\n")
```

```
>>> Guassian Input Identification Begins:-----------------------------
```

```

Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data_guassian = iddata(y1,u1,Ts);

%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
```

```
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nd = degree;
    nk = 1;
    p = na+nb+nc+nd;

    try
        sys = bj(data_guassian, [na nb nc nd nk]);
        bj_y_hat_guassian = lsim(sys, u1, t);
    catch
        break
    end

    [r2_bj, mse_bj] = rSQR(y1, bj_y_hat_guassian);

    error = y1 - bj_y_hat_guassian;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_bj, mse_bj, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_bj];
    MSEs = [MSEs; mse_bj];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
```

```
end
```

```
>>> Degree = 1 : R2=0.894672 | MSE=0.028346 | var=0.028374 | s_hat=116.104729 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.941005 | MSE=0.015877 | var=0.015908 | s_hat=65.031084 |
-------------------------------------------------------------
>>> Degree = 3 : R2=0.941010 | MSE=0.015875 | var=0.015922 | s_hat=65.025778 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.941099 | MSE=0.015852 | var=0.015914 | s_hat=64.928113 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.941110 | MSE=0.015848 | var=0.015926 | s_hat=64.915253 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.941118 | MSE=0.015846 | var=0.015940 | s_hat=64.907134 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.941177 | MSE=0.015831 | var=0.015940 | s_hat=64.842113 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.941097 | MSE=0.015852 | var=0.015977 | s_hat=64.930430 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.941133 | MSE=0.015842 | var=0.015983 | s_hat=64.889790 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.941158 | MSE=0.015836 | var=0.015992 | s_hat=64.863000 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.941375 | MSE=0.015777 | var=0.015949 | s_hat=64.623509 |
-------------------------------------------------------------
>>> Degree = 12 : R2=0.941307 | MSE=0.015795 | var=0.015983 | s_hat=64.698110 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.941359 | MSE=0.015782 | var=0.015984 | s_hat=64.641187 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.940769 | MSE=0.015940 | var=0.016161 | s_hat=65.291153 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.941451 | MSE=0.015757 | var=0.015991 | s_hat=64.539167 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.926402 | MSE=0.019807 | var=0.020121 | s_hat=81.128693 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.941221 | MSE=0.015819 | var=0.016086 | s_hat=64.793186 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.941588 | MSE=0.015720 | var=0.016001 | s_hat=64.388373 |
-------------------------------------------------------------
>>> Degree = 19 : R2=0.941515 | MSE=0.015740 | var=0.016037 | s_hat=64.469124 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.940124 | MSE=0.016114 | var=0.016435 | s_hat=66.002875 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.940079 | MSE=0.016126 | var=0.016464 | s_hat=66.051870 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.940512 | MSE=0.016009 | var=0.016361 | s_hat=65.574732 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.941676 | MSE=0.015696 | var=0.016057 | s_hat=64.291152 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.941848 | MSE=0.015650 | var=0.016025 | s_hat=64.101610 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.941848 | MSE=0.015650 | var=0.016041 | s_hat=64.101808 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.937915 | MSE=0.016708 | var=0.017144 | s_hat=68.436964 |
-------------------------------------------------------------
>>> Degree = 27 : R2=0.942045 | MSE=0.015597 | var=0.016019 | s_hat=63.884493 |
-------------------------------------------------------------
```

```
>>> Degree = 28 : R2=0.929190 | MSE=0.019056 | var=0.019592 | s_hat=78.054811 |
--------------------------------------------------------------
>>> Degree = 29 : R2=0.858947 | MSE=0.037960 | var=0.039067 | s_hat=155.485789 |
--------------------------------------------------------------
>>> Degree = 30 : R2=0.940473 | MSE=0.016020 | var=0.016503 | s_hat=65.617737 |
--------------------------------------------------------------
>>> Degree = 31 : R2=0.941115 | MSE=0.015847 | var=0.016342 | s_hat=64.910047 |
--------------------------------------------------------------
>>> Degree = 32 : R2=0.921220 | MSE=0.021201 | var=0.021885 | s_hat=86.840888 |
--------------------------------------------------------------
>>> Degree = 33 : R2=0.871403 | MSE=0.034608 | var=0.035761 | s_hat=141.755397 |
--------------------------------------------------------------
>>> Degree = 34 : R2=0.940141 | MSE=0.016109 | var=0.016663 | s_hat=65.983934 |
--------------------------------------------------------------
>>> Degree = 35 : R2=0.897716 | MSE=0.027527 | var=0.028501 | s_hat=112.749526 |
--------------------------------------------------------------
>>> Degree = 36 : R2=0.940826 | MSE=0.015925 | var=0.016505 | s_hat=65.229182 |
--------------------------------------------------------------
>>> Degree = 37 : R2=0.926601 | MSE=0.019753 | var=0.020494 | s_hat=80.908898 |
--------------------------------------------------------------
>>> Degree = 38 : R2=0.942005 | MSE=0.015608 | var=0.016209 | s_hat=63.928887 |
--------------------------------------------------------------
>>> Degree = 39 : R2=0.921887 | MSE=0.021022 | var=0.021854 | s_hat=86.105920 |
--------------------------------------------------------------
>>> Degree = 40 : R2=0.942010 | MSE=0.015606 | var=0.016241 | s_hat=63.923070 |
--------------------------------------------------------------
>>> Degree = 41 : R2=0.936417 | MSE=0.017111 | var=0.017825 | s_hat=70.088585 |
--------------------------------------------------------------
>>> Degree = 42 : R2=0.941881 | MSE=0.015641 | var=0.016310 | s_hat=64.065974 |
--------------------------------------------------------------
>>> Degree = 43 : R2=0.591534 | MSE=0.109927 | var=0.114745 | s_hat=450.260120 |
--------------------------------------------------------------
>>> Degree = 44 : R2=0.939003 | MSE=0.016415 | var=0.017152 | s_hat=67.237665 |
--------------------------------------------------------------
>>> Degree = 45 : R2=0.925574 | MSE=0.020030 | var=0.020950 | s_hat=82.041707 |
--------------------------------------------------------------
>>> Degree = 46 : R2=0.848802 | MSE=0.040691 | var=0.042604 | s_hat=166.668434 |
--------------------------------------------------------------
>>> Degree = 47 : R2=0.822998 | MSE=0.047635 | var=0.049926 | s_hat=195.112716 |
--------------------------------------------------------------
>>> Degree = 48 : R2=0.902232 | MSE=0.026311 | var=0.027605 | s_hat=107.771573 |
--------------------------------------------------------------
>>> Degree = 49 : R2=0.941560 | MSE=0.015727 | var=0.016518 | s_hat=64.419319 |
--------------------------------------------------------------
>>> Degree = 50 : R2=0.891222 | MSE=0.029274 | var=0.030777 | s_hat=119.908333 |
--------------------------------------------------------------
>>> Degree = 51 : R2=0.840460 | MSE=0.042936 | var=0.045186 | s_hat=175.864079 |
--------------------------------------------------------------
>>> Degree = 52 : R2=0.938161 | MSE=0.016642 | var=0.017533 | s_hat=68.166463 |
--------------------------------------------------------------
>>> Degree = 53 : R2=0.920739 | MSE=0.021331 | var=0.022495 | s_hat=87.370763 |
--------------------------------------------------------------
>>> Degree = 54 : R2=-5.004821 | MSE=1.616023 | var=1.705987 | s_hat=6619.231038 |
--------------------------------------------------------------
>>> Degree = 55 : R2=0.937095 | MSE=0.016929 | var=0.017890 | s_hat=69.341368 |
--------------------------------------------------------------
>>> Degree = 56 : R2=-0.259309 | MSE=0.338906 | var=0.358513 | s_hat=1388.160777 |
```

```
------------------------------------------------------------
>>> Degree = 57 : R2=0.891399 | MSE=0.029227 | var=0.030950 | s_hat=119.712781 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.872470 | MSE=0.034321 | var=0.036382 | s_hat=140.578469 |
------------------------------------------------------------
>>> Degree = 59 : R2=0.134289 | MSE=0.232981 | var=0.247225 | s_hat=954.290285 |
------------------------------------------------------------
>>> Degree = 60 : R2=0.663598 | MSE=0.090533 | var=0.096168 | s_hat=370.822839 |
------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 27
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
nd = bestFitDegree;
nk = 1;
p = na+nb+nc+nd;

BestFitModel_guassian = bj(data_guassian, [na nb nc nd nk])
```

```
BestFitModel_guassian =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)

  B(z) = 0.3951 z^-1 - 0.3933 z^-2 + 0.1126 z^-3 + 0.2073 z^-4 - 0.4613 z^-5 + 0.2168 z^-6 +
0.05889 z^-7
          + 4.516e-05 z^-8 - 0.003104 z^-9 + 0.123 z^-10 - 0.1812 z^-11 - 0.09866 z^-12 +
0.2483 z^-13
```

```
         - 0.2612 z^-14 + 0.2311 z^-15 + 0.02441 z^-16 - 0.1135 z^-17 - 0.01194 z^-18 +
0.01868 z^-19
         - 0.04686 z^-20 - 0.0823 z^-21 + 0.4258 z^-22 - 0.386 z^-23 + 0.04364 z^-24 + 0.1802
z^-25
                                                               - 0.3443 z^-26 +
0.1109 z^-27


  C(z) = 1 + 0.02486 z^-1 + 0.08647 z^-2 - 0.1307 z^-3 + 0.0675 z^-4 + 1.064 z^-5 - 0.7321 z^
         -6 + 0.5425 z^-7 - 0.4227 z^-8 + 0.08323 z^-9 - 0.02307 z^-10 - 0.9125 z^-11 +
0.7276 z^-12
         - 0.7345 z^-13 + 0.2704 z^-14 - 0.6439 z^-15 - 0.3039 z^-16 + 0.3284 z^-17 - 0.6996
z^-18
         + 0.3571 z^-19 - 0.1274 z^-20 + 0.4834 z^-21 - 0.2569 z^-22 - 0.2902 z^-23 + 0.3587
z^-24
                                                 + 0.3915 z^-25 + 0.3086 z^-26 -
0.3105 z^-27


  D(z) = 1 - 0.1492 z^-1 + 0.007362 z^-2 - 0.1753 z^-3 + 0.066 z^-4 + 1.05 z^-5 - 0.9292 z^-6
         + 0.5827 z^-7 - 0.4801 z^-8 + 0.106 z^-9 + 0.01919 z^-10 - 0.9302 z^-11 + 0.8938 z^
         -12 - 0.773 z^-13 + 0.3283 z^-14 - 0.5957 z^-15 - 0.2567 z^-16 + 0.4875 z^-17 -
0.7113 z^-18
         + 0.4435 z^-19 - 0.1121 z^-20 + 0.4231 z^-21 - 0.2799 z^-22 - 0.2939 z^-23 + 0.4139
z^-24
                                                 + 0.3334 z^-25 + 0.1814 z^-26 -
0.3759 z^-27


  F(z) = 1 - 1.674 z^-1 + 1.081 z^-2 + 0.2597 z^-3 - 1.507 z^-4 + 1.401 z^-5 - 0.3418 z^-6 -
0.06678 z^-7
         + 0.0006304 z^-8 + 0.325 z^-9 - 0.6526 z^-10 + 0.09284 z^-11 + 0.7619 z^-12 - 1.131
z^-13
         + 1.088 z^-14 - 0.3925 z^-15 - 0.2851 z^-16 + 0.1991 z^-17 + 0.03951 z^-18 - 0.1685
z^-19
         - 0.1107 z^-20 + 1.185 z^-21 - 1.733 z^-22 + 0.8983 z^-23 + 0.3037 z^-24 - 1.202 z^
                                                           -25 + 0.9404 z^-26 -
0.2925 z^-27

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=27   nc=27   nd=27   nf=27   nk=1
   Number of free coefficients: 108
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_guassian".
Fit to estimation data: 76.69% (prediction focus)
FPE: 0.01541, MSE: 0.01462
```

```
BestFit_y_hat_guassian = lsim(BestFitModel_guassian, u1_val, t_val);
% [bj_BestFit_r2, bj_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%
```

```
fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 2
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
nd = minVarIndex;
nk = 1;
p = na+nb+nc+nd;

bj_VarModel_guassian = bj(data_guassian, [na nb nc nd nk])
```

```
bj_VarModel_guassian =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.3949 z^-1 - 0.1838 z^-2

  C(z) = 1 + 0.1904 z^-1 - 0.263 z^-2

  D(z) = 1 + 0.017 z^-1 - 0.372 z^-2

  F(z) = 1 - 1.143 z^-1 + 0.4335 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_guassian".
Fit to estimation data: 76.27% (prediction focus)
FPE: 0.01521, MSE: 0.01515
```

```
Var_y_hat_guassian = lsim(bj_VarModel_guassian, u1_val, t_val);
% [bj_Var_r2, bj_Var_mse] = rSQR(y_val, Var_y_hat);
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%
fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
nd = minAICIndex;
nk = 1;
p = na+nb+nc+nd;

bj_AICModel_guassian = bj(data_guassian, [na nb nc nd nk])
```

```
bj_AICModel_guassian =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.3949 z^-1 - 0.1838 z^-2

  C(z) = 1 + 0.1904 z^-1 - 0.263 z^-2

  D(z) = 1 + 0.017 z^-1 - 0.372 z^-2

  F(z) = 1 - 1.143 z^-1 + 0.4335 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_guassian".
Fit to estimation data: 76.27% (prediction focus)
```

```
FPE: 0.01521, MSE: 0.01515
```

```
AIC_y_hat_guassian = lsim(bj_AICModel_guassian, u1_val, t_val);
% [bj_AIC_r2, bj_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test 
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n", 
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.16 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 2
```

```
na = winner;
nb = winner;
nc = winner;
nd = winner;
nk = 1;
p = na+nb+nc+nd;

bj_FTestModel_guassian = bj(data_guassian, [na nb nc nd nk])
```

```
bj_FTestModel_guassian =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.3949 z^-1 - 0.1838 z^-2

  C(z) = 1 + 0.1904 z^-1 - 0.263 z^-2

  D(z) = 1 + 0.017 z^-1 - 0.372 z^-2

  F(z) = 1 - 1.143 z^-1 + 0.4335 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_guassian".
Fit to estimation data: 76.27% (prediction focus)
FPE: 0.01521, MSE: 0.01515
```

```
FTest_y_hat_guassian = lsim(bj_FTestModel_guassian, u1_val, t_val);
% [bj_FTest_r2, bj_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[bj_BestFit_r2_guassian, bj_BestFit_mse_guassian] = rSQR(y1_val,
BestFit_y_hat_guassian);
[bj_Var_r2_guassian, bj_Var_mse_guassian] = rSQR(y1_val, Var_y_hat_guassian);
[bj_AIC_r2_guassian, bj_AIC_mse_guassian] = rSQR(y1_val, AIC_y_hat_guassian);
[bj_FTest_r2_guassian, bj_FTest_mse_guassian] = rSQR(y1_val,
FTest_y_hat_guassian);

%%

fprintf("===================Evaluation | R2 Metric=====================\n")
```

```
===================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_BestFit_r2_guassian, bj_BestFit_mse_guassian)
```

```
    R2 value : 0.9422    | MSE : 0.0154
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Var_r2_guassian, bj_Var_mse_guassian)
```

```
    R2 value : 0.9431    | MSE : 0.0152
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Cov_r2, bj_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_AIC_r2_guassian, bj_AIC_mse_guassian)
```

```
    R2 value : 0.9431    | MSE : 0.0152
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_FTest_r2_guassian, bj_FTest_mse_guassian)
```

```
    R2 value : 0.9431    | MSE : 0.0152
```

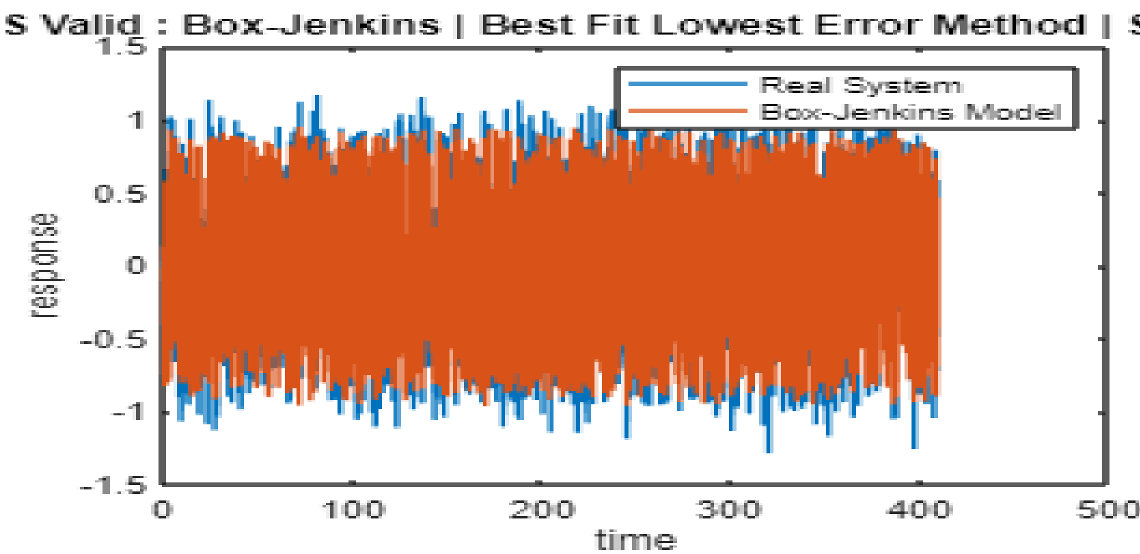
```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

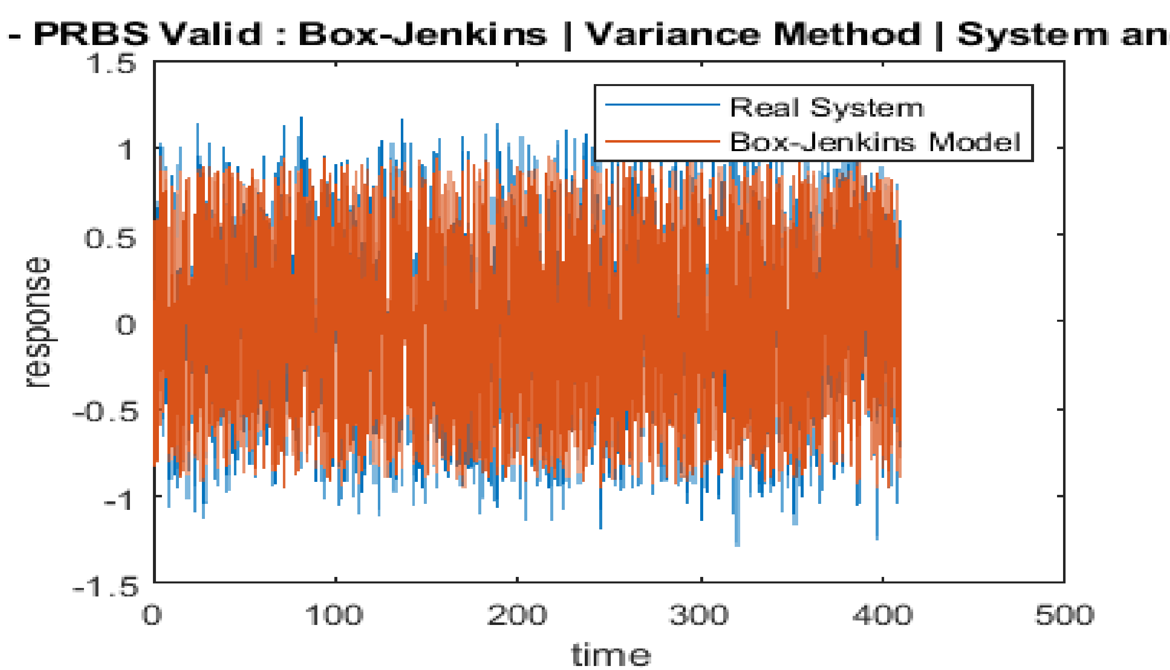
```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

```
===============================================================
```

```


%%

% bj_BestFitError_guassian = y1_val - BestFit_y_hat_guassian;
% bj_VarError_guassian = y1_val - Var_y_hat_guassian;
% bj_AICError_guassian = y1_val - AIC_y_hat_guassian;
% bj_FTestError_guassian = y1_val - FTest_y_hat_guassian;
%
% for k=0:N_val-1
%     bj_BestFit_Ree_guassian(k+1,1) = AutoCorrelate(bj_BestFitError_guassian, k);
%     bj_Var_Ree_guassian(k+1,1) = AutoCorrelate(bj_VarError_guassian, k);
%     bj_AIC_Ree_guassian(k+1,1) = AutoCorrelate(bj_AICError_guassian, k);
%     bj_FTest_Ree_guassian(k+1,1) = AutoCorrelate(bj_FTestError_guassian, k);
% end
%
% for k=0:N_val-1
%     bj_BestFit_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, bj_BestFitError_guassian, k);
%     bj_Var_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, bj_VarError_guassian, k);
%     bj_AIC_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, bj_AICError_guassian, k);
%     bj_FTest_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, bj_FTestError_guassian, k);
% end


%%
figure(1)
plot(t_val,y1_val,t_val,BestFit_y_hat_guassian)
legend('Real System','Box-Jenkins Model')
title(" Guassian Ident - PRBS Valid : Box-Jenkins | Best Fit Lowest Error Method | System and Model Response")
xlabel("time")
ylabel("response")
```

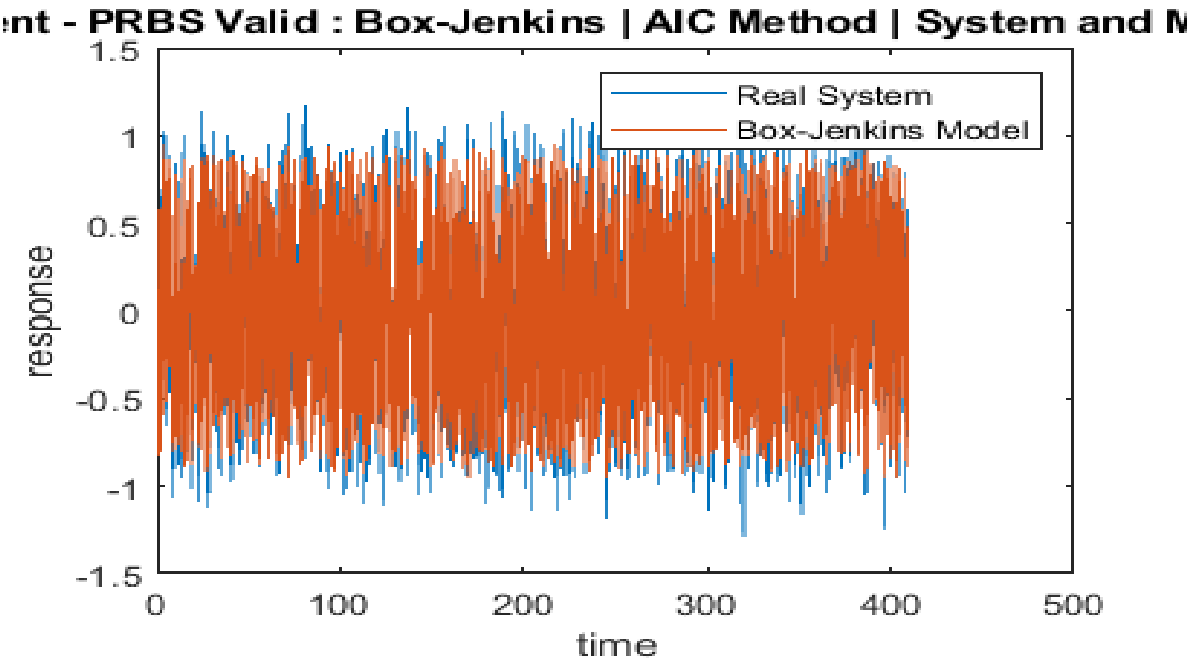



```
 figure(2)
 plot(t_val,y1_val,t_val,Var_y_hat_guassian)
 legend('Real System','Box-Jenkins Model')
 title(" Guassian Ident - PRBS Valid : Box-Jenkins | Variance Method | System
and Model Response")
 xlabel("time")
 ylabel("response")
```

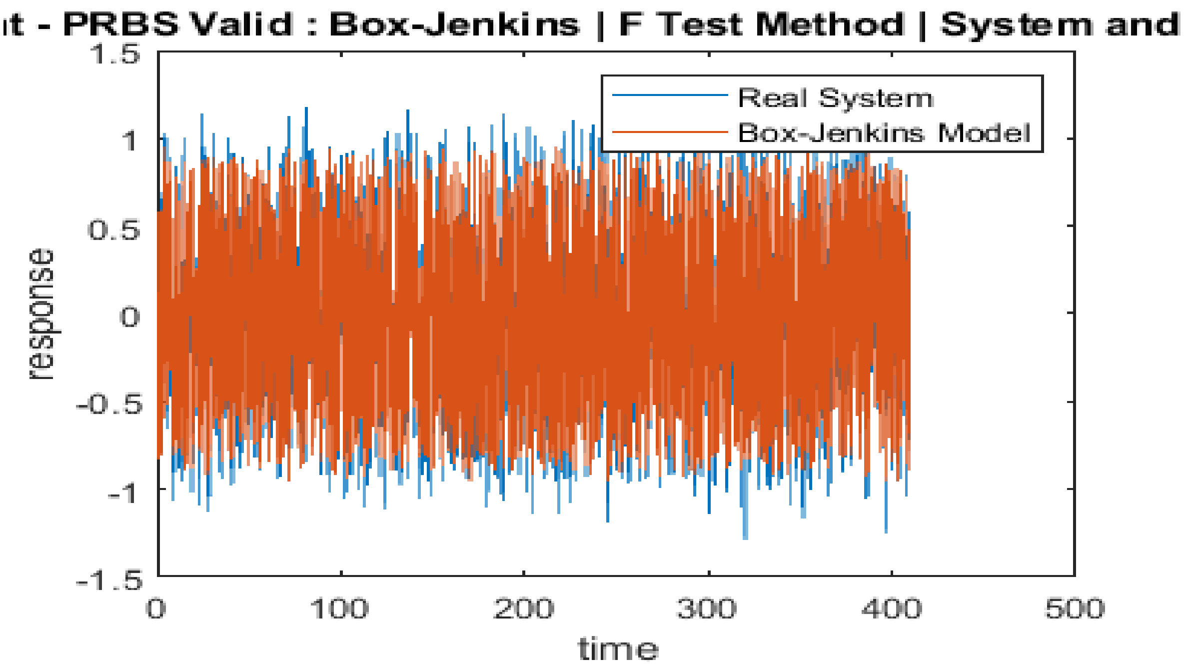



```
 figure(3)
 plot(t_val,y1_val,t_val,AIC_y_hat_guassian)
```

```
legend('Real System','Box-Jenkins Model')
title(" Guassian Ident - PRBS Valid : Box-Jenkins | AIC Method | System and
Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y1_val,t_val,FTest_y_hat_guassian)
legend('Real System','Box-Jenkins Model')
title(" Guassian Ident - PRBS Valid : Box-Jenkins | F Test Method | System and
Model Response")
xlabel("time")
ylabel("response")
```




```
%%

% figure(5)
% subplot(4,1,1)
% plot(1:N_val-1,bj_BestFit_Ree_guassian(2:end), 1:N_val-1,
mean(bj_BestFit_Ree_guassian(2:end))*ones(length(1:N_val-1)))
% title(" Guassian Ident - PRBS Valid : Box-Jenkins | Best Fit Lowest Errror
Method | Ree_guassian(k) | The Straight Line is the Mean")
% xlabel("k")
% ylabel("Ree_guassian(k)")
%
% subplot(4,1,2)
% plot(1:N_val-1,bj_Var_Ree_guassian(2:end), 1:N_val-1,
mean(bj_Var_Ree_guassian(2:end))*ones(length(1:N_val-1)))
% title(" Guassian Ident - PRBS Valid : Box-Jenkins | Variance Method |
Ree_guassian(k) | The Straight Line is the Mean")
% xlabel("k")
% ylabel("Ree_guassian(k)")
%
% subplot(4,1,3)
% plot(1:N_val-1,bj_AIC_Ree_guassian(2:end), 1:N_val-1,
mean(bj_AIC_Ree_guassian(2:end))*ones(length(1:N_val-1)))
% title(" Guassian Ident - PRBS Valid : Box-Jenkins | AIC Method |
Ree_guassian(k) | The Straight Line is the Mean")
% xlabel("k")
% ylabel("Ree_guassian(k)")
```

```
 %
 % subplot(4,1,4)
 % plot(1:N_val-1,bj_FTest_Ree_guassian(2:end), 1:N_val-1,
mean(bj_FTest_Ree_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Box-Jenkins | F Test Method |
Ree_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_guassian(k)")

 %%

 % figure(6)
 % subplot(4,1,1)
 % plot(1:N_val-1,bj_BestFit_Rue_guassian(2:end), 1:N_val-1,
mean(bj_BestFit_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Box-Jenkins | Best Fit Lowest Errror
Method | Rue_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_guassian(k)")
 %
 % subplot(4,1,2)
 % plot(1:N_val-1,bj_Var_Rue_guassian(2:end), 1:N_val-1,
mean(bj_Var_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Box-Jenkins | Variance Method |
Rue_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_guassian(k)")
 %
 % subplot(4,1,3)
 % plot(1:N_val-1,bj_AIC_Rue_guassian(2:end), 1:N_val-1,
mean(bj_AIC_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Box-Jenkins | AIC Method |
Rue_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_guassian(k)")
 %
 % subplot(4,1,4)
 % plot(1:N_val-1,bj_FTest_Rue_guassian(2:end), 1:N_val-1,
mean(bj_FTest_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Box-Jenkins | F Test Method |
Rue_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_guassian(k)")
```

PRBS Input - Guassian Validation

```
%%
% PRBS Input ***************************************************************
fprintf("*******************************************************************\n")
```

```
*******************************************************************
```

```
fprintf(">>> PRBS Input Identification Begins:------------------------------
\n")
```

```
>>> PRBS Input Identification Begins:----------------------------
```

```
%%

Ts = 0.1;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);

data_prbs = iddata(y2,u2,Ts);

%%


fprintf("===================Degree Extraction |
RUN==========================\n")
```

```
===================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nd = degree;
```

```
    nk = 1;
    p = na+nb+nc+nd;

    try
        sys = bj(data_prbs, [na nb nc nd nk]);
        bj_y_hatprbs = lsim(sys, u2, t);
    catch
        break
    end

    [r2_bj, mse_bj] = rSQR(y2, bj_y_hatprbs);

    error = y2 - bj_y_hatprbs;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_bj, mse_bj, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_bj];
    MSEs = [MSEs; mse_bj];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.896389 | MSE=0.027613 | var=0.027640 | s_hat=113.103267 |
--------------------------------------------------------------
>>> Degree = 2 : R2=0.943231 | MSE=0.015129 | var=0.015159 | s_hat=61.969486 |
--------------------------------------------------------------
>>> Degree = 3 : R2=0.943297 | MSE=0.015112 | var=0.015156 | s_hat=61.898301 |
--------------------------------------------------------------
>>> Degree = 4 : R2=0.943361 | MSE=0.015095 | var=0.015154 | s_hat=61.828175 |
--------------------------------------------------------------
>>> Degree = 5 : R2=0.943379 | MSE=0.015090 | var=0.015164 | s_hat=61.808049 |
--------------------------------------------------------------
>>> Degree = 6 : R2=0.943435 | MSE=0.015075 | var=0.015164 | s_hat=61.747231 |
--------------------------------------------------------------
>>> Degree = 7 : R2=0.943455 | MSE=0.015070 | var=0.015173 | s_hat=61.725250 |
--------------------------------------------------------------
>>> Degree = 8 : R2=0.943352 | MSE=0.015097 | var=0.015216 | s_hat=61.837477 |
```

```
------------------------------------------------------------
>>> Degree = 9 : R2=0.943539 | MSE=0.015047 | var=0.015181 | s_hat=61.634183 |
------------------------------------------------------------
>>> Degree = 10 : R2=0.905857 | MSE=0.025090 | var=0.025337 | s_hat=102.768328 |
------------------------------------------------------------
>>> Degree = 11 : R2=0.943664 | MSE=0.015014 | var=0.015177 | s_hat=61.496977 |
------------------------------------------------------------
>>> Degree = 12 : R2=0.943480 | MSE=0.015063 | var=0.015242 | s_hat=61.698586 |
------------------------------------------------------------
>>> Degree = 13 : R2=0.943712 | MSE=0.015001 | var=0.015194 | s_hat=61.445042 |
------------------------------------------------------------
>>> Degree = 14 : R2=0.943413 | MSE=0.015081 | var=0.015290 | s_hat=61.770971 |
------------------------------------------------------------
>>> Degree = 15 : R2=0.943776 | MSE=0.014984 | var=0.015207 | s_hat=61.374935 |
------------------------------------------------------------
>>> Degree = 16 : R2=0.934303 | MSE=0.017509 | var=0.017787 | s_hat=71.716109 |
------------------------------------------------------------
>>> Degree = 17 : R2=0.939729 | MSE=0.016063 | var=0.016334 | s_hat=65.792462 |
------------------------------------------------------------
>>> Degree = 18 : R2=0.943913 | MSE=0.014948 | var=0.015215 | s_hat=61.225653 |
------------------------------------------------------------
>>> Degree = 19 : R2=0.938134 | MSE=0.016488 | var=0.016800 | s_hat=67.534332 |
------------------------------------------------------------
>>> Degree = 20 : R2=0.943830 | MSE=0.014970 | var=0.015268 | s_hat=61.316324 |
------------------------------------------------------------
>>> Degree = 21 : R2=0.943978 | MSE=0.014930 | var=0.015243 | s_hat=61.155006 |
------------------------------------------------------------
>>> Degree = 22 : R2=0.832246 | MSE=0.044708 | var=0.045689 | s_hat=183.123154 |
------------------------------------------------------------
>>> Degree = 23 : R2=0.944084 | MSE=0.014902 | var=0.015245 | s_hat=61.039311 |
------------------------------------------------------------
>>> Degree = 24 : R2=0.943937 | MSE=0.014941 | var=0.015300 | s_hat=61.199052 |
------------------------------------------------------------
>>> Degree = 25 : R2=0.940315 | MSE=0.015906 | var=0.016305 | s_hat=65.152821 |
------------------------------------------------------------
>>> Degree = 26 : R2=0.942278 | MSE=0.015383 | var=0.015784 | s_hat=63.010340 |
------------------------------------------------------------
>>> Degree = 27 : R2=0.938508 | MSE=0.016388 | var=0.016832 | s_hat=67.125439 |
------------------------------------------------------------
>>> Degree = 28 : R2=0.790265 | MSE=0.055896 | var=0.057467 | s_hat=228.949826 |
------------------------------------------------------------
>>> Degree = 29 : R2=0.938130 | MSE=0.016489 | var=0.016969 | s_hat=67.538610 |
------------------------------------------------------------
>>> Degree = 30 : R2=0.646850 | MSE=0.094117 | var=0.096958 | s_hat=385.504304 |
------------------------------------------------------------
>>> Degree = 31 : R2=0.944309 | MSE=0.014842 | var=0.015306 | s_hat=60.793782 |
------------------------------------------------------------
>>> Degree = 32 : R2=0.944011 | MSE=0.014921 | var=0.015403 | s_hat=61.118165 |
------------------------------------------------------------
>>> Degree = 33 : R2=0.944117 | MSE=0.014893 | var=0.015389 | s_hat=61.002983 |
------------------------------------------------------------
>>> Degree = 34 : R2=0.937615 | MSE=0.016626 | var=0.017197 | s_hat=68.100933 |
------------------------------------------------------------
>>> Degree = 35 : R2=0.799565 | MSE=0.053418 | var=0.055308 | s_hat=218.798181 |
------------------------------------------------------------
>>> Degree = 36 : R2=0.930257 | MSE=0.018587 | var=0.019264 | s_hat=76.132524 |
------------------------------------------------------------
```

```
>>> Degree = 37 : R2=-1.862377 | MSE=0.762847 | var=0.791444 | s_hat=3124.619859 |
------------------------------------------------------------
>>> Degree = 38 : R2=0.942931 | MSE=0.015209 | var=0.015795 | s_hat=62.297265 |
------------------------------------------------------------
>>> Degree = 39 : R2=0.943058 | MSE=0.015176 | var=0.015776 | s_hat=62.159180 |
------------------------------------------------------------
>>> Degree = 40 : R2=0.936673 | MSE=0.016877 | var=0.017563 | s_hat=69.129093 |
------------------------------------------------------------
>>> Degree = 41 : R2=0.851150 | MSE=0.039670 | var=0.041324 | s_hat=162.486725 |
------------------------------------------------------------
>>> Degree = 42 : R2=0.943493 | MSE=0.015060 | var=0.015704 | s_hat=61.684318 |
------------------------------------------------------------
>>> Degree = 43 : R2=0.911842 | MSE=0.023495 | var=0.024525 | s_hat=96.234934 |
------------------------------------------------------------
>>> Degree = 44 : R2=0.937892 | MSE=0.016552 | var=0.017295 | s_hat=67.797928 |
------------------------------------------------------------
>>> Degree = 45 : R2=0.944896 | MSE=0.014686 | var=0.015361 | s_hat=60.152565 |
------------------------------------------------------------
>>> Degree = 46 : R2=0.898527 | MSE=0.027043 | var=0.028315 | s_hat=110.770162 |
------------------------------------------------------------
>>> Degree = 47 : R2=0.710046 | MSE=0.077275 | var=0.080992 | s_hat=316.518248 |
------------------------------------------------------------
>>> Degree = 48 : R2=0.943838 | MSE=0.014968 | var=0.015704 | s_hat=61.307690 |
------------------------------------------------------------
>>> Degree = 49 : R2=0.944256 | MSE=0.014856 | var=0.015603 | s_hat=60.851649 |
------------------------------------------------------------
>>> Degree = 50 : R2=0.917774 | MSE=0.021914 | var=0.023039 | s_hat=89.759494 |
------------------------------------------------------------
>>> Degree = 51 : R2=0.804015 | MSE=0.052232 | var=0.054969 | s_hat=213.940572 |
------------------------------------------------------------
>>> Degree = 52 : R2=0.926104 | MSE=0.019694 | var=0.020747 | s_hat=80.665725 |
------------------------------------------------------------
>>> Degree = 53 : R2=0.793352 | MSE=0.055073 | var=0.058079 | s_hat=225.580578 |
------------------------------------------------------------
>>> Degree = 54 : R2=0.936182 | MSE=0.017008 | var=0.017955 | s_hat=69.665024 |
------------------------------------------------------------
>>> Degree = 55 : R2=0.941128 | MSE=0.015690 | var=0.016580 | s_hat=64.265909 |
------------------------------------------------------------
>>> Degree = 56 : R2=0.687427 | MSE=0.083303 | var=0.088122 | s_hat=341.210039 |
------------------------------------------------------------
>>> Degree = 57 : R2=0.935527 | MSE=0.017183 | var=0.018196 | s_hat=70.380292 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.756895 | MSE=0.064789 | var=0.068679 | s_hat=265.377292 |
------------------------------------------------------------
>>> Degree = 59 : R2=0.877746 | MSE=0.032582 | var=0.034574 | s_hat=133.454515 |
------------------------------------------------------------
>>> Degree = 60 : R2=0.938995 | MSE=0.016258 | var=0.017270 | s_hat=66.594464 |
------------------------------------------------------------
>>> Degree = 61 : R2=-3.073234 | MSE=1.085550 | var=1.154313 | s_hat=4446.412472 |
------------------------------------------------------------
>>> Degree = 62 : R2=0.874036 | MSE=0.033570 | var=0.035734 | s_hat=137.504021 |
------------------------------------------------------------
>>> Degree = 63 : R2=0.910892 | MSE=0.023748 | var=0.025305 | s_hat=97.272339 |
------------------------------------------------------------
>>> Degree = 64 : R2=0.942735 | MSE=0.015261 | var=0.016279 | s_hat=62.511049 |
------------------------------------------------------------
>>> Degree = 65 : R2=0.852036 | MSE=0.039434 | var=0.042106 | s_hat=161.520433 |
```

```
------------------------------------------------------------
>>> Degree = 66 : R2=0.897313 | MSE=0.027367 | var=0.029252 | s_hat=112.094803 |
------------------------------------------------------------
```

```
fprintf("============================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 45
```

```
na = bestFitDegree;
nb = bestFitDegree;
nc = bestFitDegree;
nd = bestFitDegree;
nk = 1;
p = na+nb+nc+nd;

BestFitModelprbs = bj(data_prbs, [na nb nc nd nk])
```

```
BestFitModelprbs =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)

  B(z) = 0.3993 z^-1 + 0.3031 z^-2 + 0.1035 z^-3 - 0.2598 z^-4 - 0.297 z^-5 - 0.1771 z^-6 +
0.007883 z^-7
          + 0.106 z^-8 + 0.2921 z^-9 + 0.1383 z^-10 - 0.05151 z^-11 - 0.09066 z^-12 + 0.03352
z^-13
          + 0.2308 z^-14 + 0.03389 z^-15 + 0.01698 z^-16 - 0.06623 z^-17 + 0.08592 z^-18 -
0.06407 z^-19
          + 0.05337 z^-20 + 0.01212 z^-21 + 0.1517 z^-22 - 0.03357 z^-23 - 0.02582 z^-24 -
0.08473 z^-25
          + 0.07974 z^-26 - 0.006178 z^-27 + 0.06122 z^-28 + 0.1308 z^-29 + 0.2184 z^-30 +
0.0738 z^-31
          - 0.1632 z^-32 - 0.03119 z^-33 + 0.09311 z^-34 + 0.1756 z^-35 - 0.004259 z^-36 -
0.06738 z^-37
```

```
         - 0.1402 z^-38 - 0.1152 z^-39 - 0.06354 z^-40 + 0.2396 z^-41 + 0.2353 z^-42 + 0.2341
z^-43
                                                                       - 0.03587 z^-44 -
0.09681 z^-45


  C(z) = 1 + 1.248 z^-1 + 1.642 z^-2 + 0.8527 z^-3 + 0.7157 z^-4 - 0.04736 z^-5 - 0.9575 z^-6
         - 1.174 z^-7 - 1.913 z^-8 - 1.378 z^-9 - 1.482 z^-10 - 0.6374 z^-11 - 0.4932 z^-12
         - 0.1856 z^-13 - 0.4099 z^-14 - 0.5701 z^-15 - 0.5254 z^-16 - 0.3068 z^-17 + 0.05059
z^-18
         + 0.745 z^-19 + 1.777 z^-20 + 1.878 z^-21 + 2.264 z^-22 + 1.602 z^-23 + 1.079 z^-24
         - 0.2322 z^-25 - 0.6721 z^-26 - 0.9413 z^-27 - 0.8916 z^-28 - 0.6666 z^-29 - 0.4905
z^-30
         - 0.1528 z^-31 - 0.2686 z^-32 - 0.07717 z^-33 - 0.7256 z^-34 - 0.5913 z^-35 - 1.027
z^-36
         - 0.4863 z^-37 - 0.2318 z^-38 + 0.2515 z^-39 + 0.6661 z^-40 + 0.6157 z^-41 + 0.9236
z^-42
                                                          + 0.3213 z^-43 + 0.1723 z^-44 -
0.1981 z^-45


  D(z) = 1 + 1.079 z^-1 + 1.351 z^-2 + 0.4677 z^-3 + 0.43 z^-4 - 0.254 z^-5 - 1.03 z^-6 - 1.03
z^-7
         - 1.658 z^-8 - 0.9862 z^-9 - 1.104 z^-10 - 0.2963 z^-11 - 0.2708 z^-12 - 0.06472 z^
         -13 - 0.2918 z^-14 - 0.4074 z^-15 - 0.3286 z^-16 - 0.0824 z^-17 + 0.1925 z^-18 +
0.8362 z^-19
         + 1.637 z^-20 + 1.504 z^-21 + 1.748 z^-22 + 0.9797 z^-23 + 0.551 z^-24 - 0.6182 z^-
25
         - 0.7562 z^-26 - 0.8657 z^-27 - 0.7289 z^-28 - 0.4559 z^-29 - 0.3363 z^-30 - 0.03439
z^-31
         - 0.2008 z^-32 + 0.01094 z^-33 - 0.6036 z^-34 - 0.3741 z^-35 - 0.7381 z^-36 - 0.1513
z^-37
         + 0.0368 z^-38 + 0.393 z^-39 + 0.635 z^-40 + 0.4605 z^-41 + 0.7071 z^-42 + 0.03598
z^-43
                                                                        - 0.03797 z^-44 -
0.3112 z^-45


  F(z) = 1 + 0.08935 z^-1 - 0.1448 z^-2 - 0.6982 z^-3 - 0.1921 z^-4 + 0.03046 z^-5 + 0.2356
z^-6
         + 0.143 z^-7 + 0.5062 z^-8 - 0.1429 z^-9 - 0.2735 z^-10 - 0.06345 z^-11 + 0.279 z^-
12
         + 0.5004 z^-13 - 0.2985 z^-14 + 0.04227 z^-15 - 0.1546 z^-16 + 0.3664 z^-17 - 0.3185
z^-18
         + 0.268 z^-19 - 0.07684 z^-20 + 0.3745 z^-21 - 0.3345 z^-22 + 0.04976 z^-23 - 0.1594
z^-24
         + 0.3538 z^-25 - 0.1858 z^-26 + 0.1729 z^-27 + 0.2168 z^-28 + 0.3471 z^-29 - 0.1381
z^-30
         - 0.4391 z^-31 + 0.2527 z^-32 + 0.2773 z^-33 + 0.2696 z^-34 - 0.2927 z^-35 - 0.09997
z^-36
         - 0.217 z^-37 - 0.06072 z^-38 - 0.004539 z^-39 + 0.6498 z^-40 + 0.145 z^-41 + 0.2439
z^-42
                                                         - 0.3979 z^-43 - 0.07331 z^-44 +
0.1948 z^-45

Sample time: 0.1 seconds
```

```
Parameterization:
   Polynomial orders:   nb=45   nc=45   nd=45   nf=45   nk=1
   Number of free coefficients: 180
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_prbs".
Fit to estimation data: 77.27% (prediction focus)
FPE: 0.01503, MSE: 0.01377
```

```
BestFit_y_hatprbs = lsim(BestFitModelprbs, u2_val, t_val);
% [bj_BestFit_r2, bj_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 4 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 4
```

```
na = minVarIndex;
nb = minVarIndex;
nc = minVarIndex;
nd = minVarIndex;
nk = 1;
p = na+nb+nc+nd;

bj_VarModelprbs = bj(data_prbs, [na nb nc nd nk])
```

```
bj_VarModelprbs =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.3996 z^-1 + 0.1489 z^-2 - 0.2052 z^-3 + 0.0306 z^-4

  C(z) = 1 - 0.1317 z^-1 - 0.09652 z^-2 - 0.8553 z^-3 + 0.2689 z^-4

  D(z) = 1 - 0.3042 z^-1 - 0.1378 z^-2 - 0.864 z^-3 + 0.4254 z^-4

  F(z) = 1 - 0.2924 z^-1 - 0.6606 z^-2 + 0.509 z^-3 - 0.04483 z^-4

Sample time: 0.1 seconds
```

```
Parameterization:
   Polynomial orders:   nb=4   nc=4   nd=4   nf=4   nk=1
   Number of free coefficients: 16
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_prbs".
Fit to estimation data: 76.73% (prediction focus)
FPE: 0.01455, MSE: 0.01444
```

```
Var_y_hatprbs = lsim(bj_VarModelprbs, u2_val, t_val);
% [bj_Var_r2, bj_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 2
```

```
na = minAICIndex;
nb = minAICIndex;
nc = minAICIndex;
nd = minAICIndex;
nk = 1;
p = na+nb+nc+nd;



bj_AICModelprbs = bj(data_prbs, [na nb nc nd nk])
```

```
bj_AICModelprbs =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.399 z^-1 - 0.2035 z^-2
```

```
  C(z) = 1 - 0.04707 z^-1 - 0.1139 z^-2

  D(z) = 1 - 0.2156 z^-1 - 0.1815 z^-2

  F(z) = 1 - 1.179 z^-1 + 0.449 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_prbs".
Fit to estimation data: 76.67% (prediction focus)
FPE: 0.01456, MSE: 0.0145
```

```
AIC_y_hatprbs = lsim(bj_AICModelprbs, u2_val, t_val);
% [bj_AIC_r2, bj_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.16 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 2
```

```
na = winner;
nb = winner;
nc = winner;
nd = winner;
nk = 1;
p = na+nb+nc+nd;

bj_FTestModelprbs = bj(data_prbs, [na nb nc nd nk])
```

```
bj_FTestModelprbs =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.399 z^-1 - 0.2035 z^-2

  C(z) = 1 - 0.04707 z^-1 - 0.1139 z^-2

  D(z) = 1 - 0.2156 z^-1 - 0.1815 z^-2

  F(z) = 1 - 1.179 z^-1 + 0.449 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_prbs".
Fit to estimation data: 76.67% (prediction focus)
FPE: 0.01456, MSE: 0.0145
```

```
FTest_y_hatprbs = lsim(bj_FTestModelprbs, u2_val, t_val);
% [bj_FTest_r2, bj_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[bj_BestFit_r2prbs, bj_BestFit_mseprbs] = rSQR(y2_val, BestFit_y_hatprbs);
[bj_Var_r2prbs, bj_Var_mseprbs] = rSQR(y2_val, Var_y_hatprbs);
[bj_AIC_r2prbs, bj_AIC_mseprbs] = rSQR(y2_val, AIC_y_hatprbs);
[bj_FTest_r2prbs, bj_FTest_mseprbs] = rSQR(y2_val, FTest_y_hatprbs);
```

```
fprintf("==================Evaluation | R2 Metric=====================\n")
```

```
==================Evaluation | R2 Metric=====================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_BestFit_r2prbs,
bj_BestFit_mseprbs)
```

```
    R2 value : 0.9385    | MSE : 0.0165
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Var_r2prbs, bj_Var_mseprbs)
```

```
    R2 value : 0.9408    | MSE : 0.0159
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_Cov_r2, bj_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", bj_AIC_r2prbs, bj_AIC_mseprbs)
```

```
    R2 value : 0.9407    | MSE : 0.0159
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", bj_FTest_r2prbs,
bj_FTest_mseprbs)
```

```
    R2 value : 0.9407    | MSE : 0.0159
```

```
fprintf("---------------------------------------------------------------\n")
```

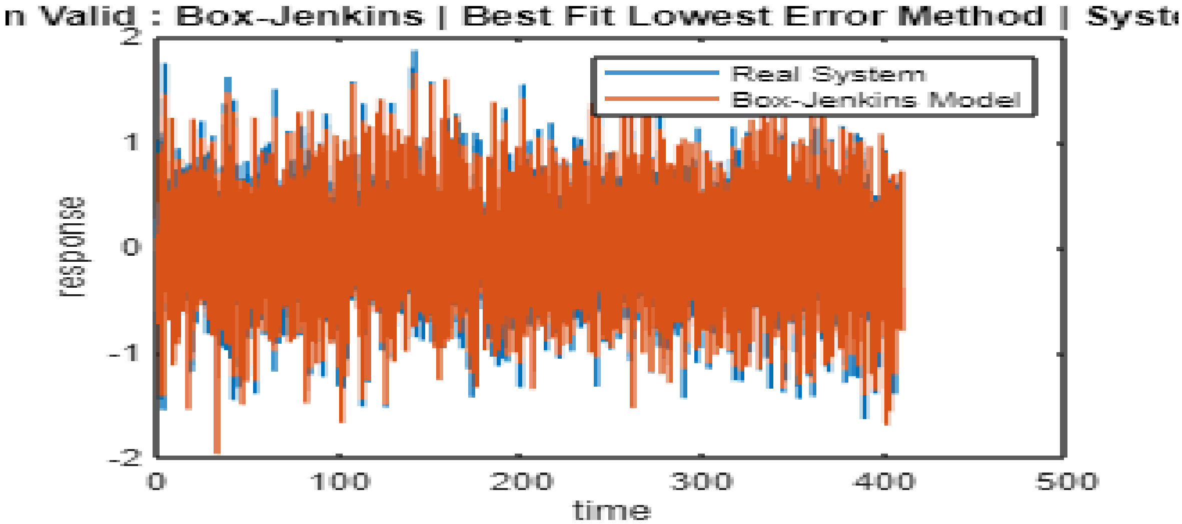
```
-------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

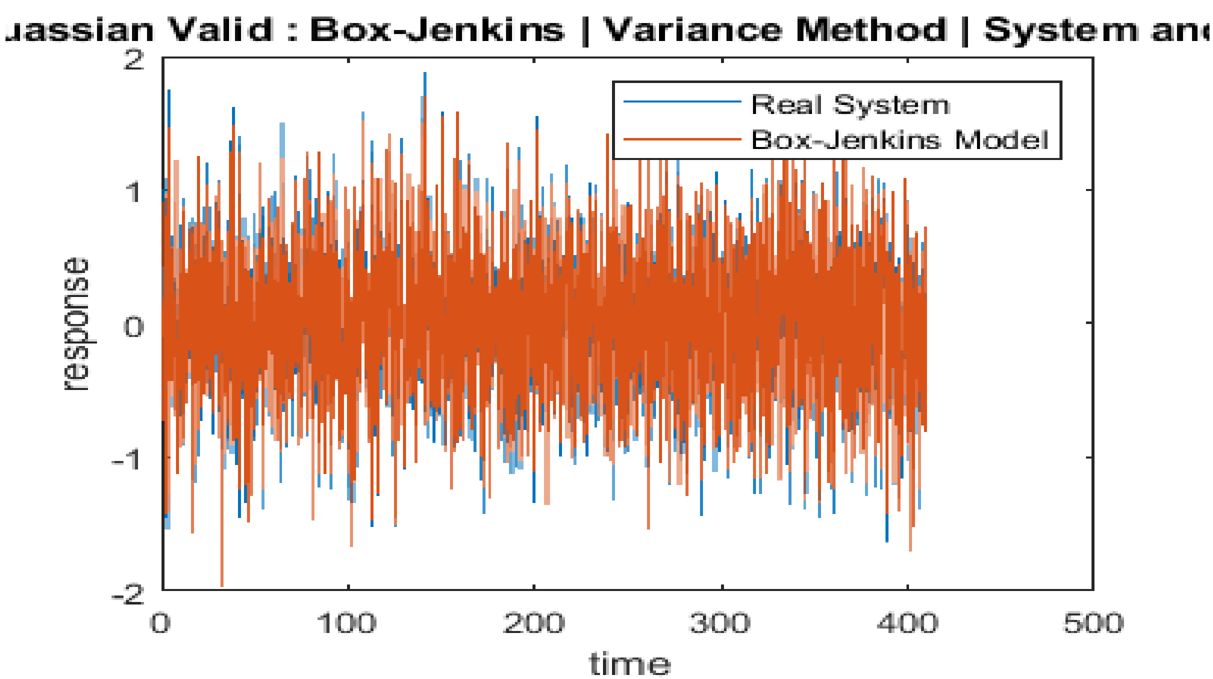
```
=============================================================
```

```
%%
%
% bj_BestFitErrorprbs = y2_val - BestFit_y_hatprbs;
% bj_VarErrorprbs = y2_val - Var_y_hatprbs;
% bj_AICErrorprbs = y2_val - AIC_y_hatprbs;
% bj_FTestErrorprbs = y2_val - FTest_y_hatprbs;
%
% for k=0:N_val-1
%     bj_BestFit_Reeprbs(k+1,1) = AutoCorrelate(bj_BestFitErrorprbs, k);
%     bj_Var_Reeprbs(k+1,1) = AutoCorrelate(bj_VarErrorprbs, k);
%     bj_AIC_Reeprbs(k+1,1) = AutoCorrelate(bj_AICErrorprbs, k);
%     bj_FTest_Reeprbs(k+1,1) = AutoCorrelate(bj_FTestErrorprbs, k);
% end
%
% for k=0:N_val-1
%     bj_BestFit_Rueprbs(k+1,1) = CrossCorrelate(u2_val, bj_BestFitErrorprbs,
k);
%     bj_Var_Rueprbs(k+1,1) = CrossCorrelate(u2_val, bj_VarErrorprbs, k);
%     bj_AIC_Rueprbs(k+1,1) = CrossCorrelate(u2_val, bj_AICErrorprbs, k);
%     bj_FTest_Rueprbs(k+1,1) = CrossCorrelate(u2_val, bj_FTestErrorprbs, k);
% end
%

%%
figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hatprbs)
legend('Real System','Box-Jenkins Model')
title(" PRBS Ident - Guassian Valid : Box-Jenkins | Best Fit Lowest Error
Method | System and Model Response")
xlabel("time")
```

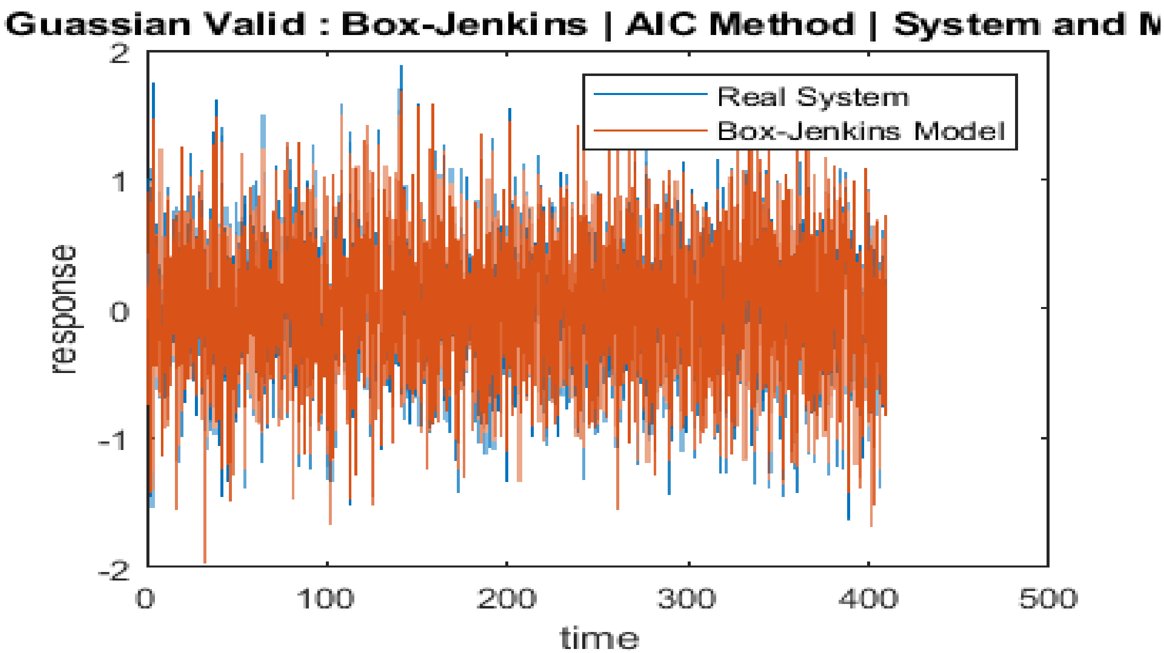
```
ylabel("response")
```



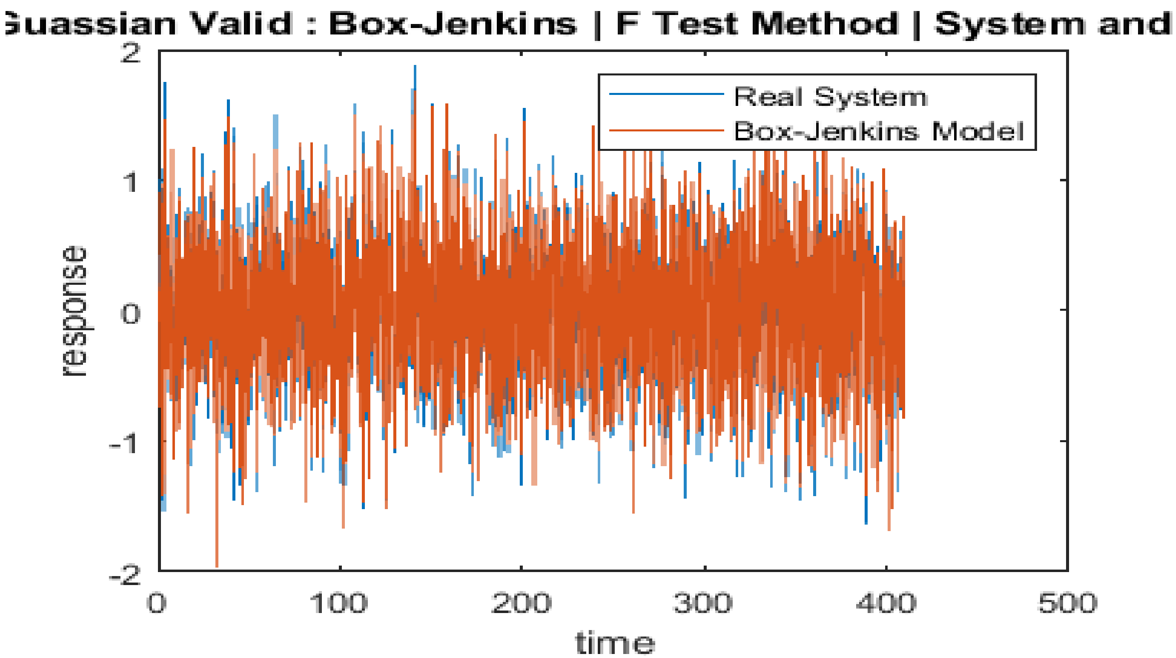

```
figure(2)
plot(t_val,y2_val,t_val,Var_y_hatprbs)
legend('Real System','Box-Jenkins Model')
title(" PRBS Ident - Guassian Valid : Box-Jenkins | Variance Method | System
and Model Response")
xlabel("time")
ylabel("response")
```



```
figure(3)
plot(t_val,y2_val,t_val,AIC_y_hatprbs)
legend('Real System','Box-Jenkins Model')
title(" PRBS Ident - Guassian Valid : Box-Jenkins | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y2_val,t_val,FTest_y_hatprbs)
legend('Real System','Box-Jenkins Model')
title(" PRBS Ident - Guassian Valid : Box-Jenkins | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
 %
 % %%
 %
 % figure(5)
 % subplot(4,1,1)
 % plot(1:N_val-1,bj_BestFit_Reeprbs(2:end), 1:N_val-1,
mean(bj_BestFit_Reeprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | Best Fit Lowest Errror
Method | Reeprbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Reeprbs(k)")
 %
 % subplot(4,1,2)
 % plot(1:N_val-1,bj_Var_Reeprbs(2:end), 1:N_val-1,
mean(bj_Var_Reeprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | Variance Method |
Reeprbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Reeprbs(k)")
 %
 % subplot(4,1,3)
 % plot(1:N_val-1,bj_AIC_Reeprbs(2:end), 1:N_val-1,
mean(bj_AIC_Reeprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | AIC Method | Reeprbs(k) |
The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Reeprbs(k)")
```

```
 %
 % subplot(4,1,4)
 % plot(1:N_val-1,bj_FTest_Reeprbs(2:end), 1:N_val-1,
mean(bj_FTest_Reeprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | F Test Method |
Reeprbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Reeprbs(k)")
 %
 % %%
 %
 % figure(6)
 % subplot(4,1,1)
 % plot(1:N_val-1,bj_BestFit_Rueprbs(2:end), 1:N_val-1,
mean(bj_BestFit_Rueprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | Best Fit Lowest Errror
Method | Rueprbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rueprbs(k)")
 %
 % subplot(4,1,2)
 % plot(1:N_val-1,bj_Var_Rueprbs(2:end), 1:N_val-1,
mean(bj_Var_Rueprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | Variance Method |
Rueprbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rueprbs(k)")
 %
 % subplot(4,1,3)
 % plot(1:N_val-1,bj_AIC_Rueprbs(2:end), 1:N_val-1,
mean(bj_AIC_Rueprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | AIC Method | Rueprbs(k) |
The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rueprbs(k)")
 %
 % subplot(4,1,4)
 % plot(1:N_val-1,bj_FTest_Rueprbs(2:end), 1:N_val-1,
mean(bj_FTest_Rueprbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Box-Jenkins | F Test Method |
Rueprbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rueprbs(k)")
```

Q3 - part a | Output Error

```
clc; clear;

%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;


%%
% Guassian Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> Guassian Input Identification Begins:-----------------------------
-\n")
```

```
>>> Guassian Input Identification Begins:-----------------------------
```

Guassian Input - PRBS Validation

```
%%

Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data_guassian = iddata(y1,u1,Ts);

%%


fprintf("====================Degree Extraction |
RUN===========================\n")
```

```
====================Degree Extraction | RUN===========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    try
        sys = oe(data_guassian, [na nb nk]);
        oe_y_hat_guassian = lsim(sys, u1, t);
    catch
        break
    end

    [r2_oe, mse_oe] = rSQR(y1, oe_y_hat_guassian);

    error = y1 - oe_y_hat_guassian;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_oe, mse_oe, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_oe];
    MSEs = [MSEs; mse_oe];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];
```

```
end
```

```
>>> Degree = 1 : R2=0.900419 | MSE=0.026799 | var=0.026812 | s_hat=109.770020 |
--------------------------------------------------------------
>>> Degree = 2 : R2=0.941010 | MSE=0.015875 | var=0.015891 | s_hat=65.025758 |
--------------------------------------------------------------
>>> Degree = 3 : R2=0.941031 | MSE=0.015870 | var=0.015893 | s_hat=65.003129 |
--------------------------------------------------------------
>>> Degree = 4 : R2=0.941099 | MSE=0.015852 | var=0.015883 | s_hat=64.928103 |
--------------------------------------------------------------
>>> Degree = 5 : R2=0.941100 | MSE=0.015851 | var=0.015890 | s_hat=64.927129 |
--------------------------------------------------------------
>>> Degree = 6 : R2=0.941163 | MSE=0.015834 | var=0.015881 | s_hat=64.857208 |
--------------------------------------------------------------
>>> Degree = 7 : R2=0.941201 | MSE=0.015824 | var=0.015878 | s_hat=64.814954 |
--------------------------------------------------------------
>>> Degree = 8 : R2=0.941159 | MSE=0.015835 | var=0.015897 | s_hat=64.861253 |
--------------------------------------------------------------
>>> Degree = 9 : R2=0.941196 | MSE=0.015825 | var=0.015895 | s_hat=64.820692 |
--------------------------------------------------------------
>>> Degree = 10 : R2=0.941233 | MSE=0.015815 | var=0.015893 | s_hat=64.779760 |
--------------------------------------------------------------
>>> Degree = 11 : R2=0.941293 | MSE=0.015799 | var=0.015885 | s_hat=64.713704 |
--------------------------------------------------------------
>>> Degree = 12 : R2=0.941317 | MSE=0.015793 | var=0.015886 | s_hat=64.687766 |
--------------------------------------------------------------
>>> Degree = 13 : R2=0.941522 | MSE=0.015738 | var=0.015838 | s_hat=64.461285 |
--------------------------------------------------------------
>>> Degree = 14 : R2=0.941509 | MSE=0.015741 | var=0.015850 | s_hat=64.476263 |
--------------------------------------------------------------
>>> Degree = 15 : R2=0.941597 | MSE=0.015718 | var=0.015833 | s_hat=64.378891 |
--------------------------------------------------------------
>>> Degree = 16 : R2=0.941732 | MSE=0.015681 | var=0.015805 | s_hat=64.229651 |
--------------------------------------------------------------
>>> Degree = 17 : R2=0.941658 | MSE=0.015701 | var=0.015832 | s_hat=64.311192 |
--------------------------------------------------------------
>>> Degree = 18 : R2=0.941659 | MSE=0.015701 | var=0.015840 | s_hat=64.310109 |
--------------------------------------------------------------
>>> Degree = 19 : R2=0.941602 | MSE=0.015716 | var=0.015863 | s_hat=64.373346 |
--------------------------------------------------------------
>>> Degree = 20 : R2=0.941967 | MSE=0.015618 | var=0.015772 | s_hat=63.971079 |
--------------------------------------------------------------
>>> Degree = 21 : R2=0.941032 | MSE=0.015869 | var=0.016034 | s_hat=65.001240 |
--------------------------------------------------------------
>>> Degree = 22 : R2=0.942004 | MSE=0.015608 | var=0.015777 | s_hat=63.930429 |
--------------------------------------------------------------
>>> Degree = 23 : R2=0.942116 | MSE=0.015578 | var=0.015755 | s_hat=63.806437 |
--------------------------------------------------------------
>>> Degree = 24 : R2=0.942084 | MSE=0.015586 | var=0.015771 | s_hat=63.841427 |
--------------------------------------------------------------
>>> Degree = 25 : R2=0.941490 | MSE=0.015746 | var=0.015941 | s_hat=64.497124 |
--------------------------------------------------------------
>>> Degree = 26 : R2=0.941571 | MSE=0.015725 | var=0.015927 | s_hat=64.407832 |
--------------------------------------------------------------
>>> Degree = 27 : R2=0.941957 | MSE=0.015621 | var=0.015829 | s_hat=63.982130 |
--------------------------------------------------------------
```

```
>>> Degree = 28 : R2=0.942048 | MSE=0.015596 | var=0.015812 | s_hat=63.881498 |
-------------------------------------------------------------
>>> Degree = 29 : R2=0.941512 | MSE=0.015740 | var=0.015966 | s_hat=64.472365 |
-------------------------------------------------------------
>>> Degree = 30 : R2=0.942093 | MSE=0.015584 | var=0.015816 | s_hat=63.832496 |
-------------------------------------------------------------
>>> Degree = 31 : R2=0.942216 | MSE=0.015551 | var=0.015790 | s_hat=63.696321 |
-------------------------------------------------------------
>>> Degree = 32 : R2=0.941742 | MSE=0.015679 | var=0.015927 | s_hat=64.219356 |
-------------------------------------------------------------
>>> Degree = 33 : R2=0.940597 | MSE=0.015987 | var=0.016248 | s_hat=65.481093 |
-------------------------------------------------------------
>>> Degree = 34 : R2=0.939936 | MSE=0.016165 | var=0.016437 | s_hat=66.209960 |
-------------------------------------------------------------
>>> Degree = 35 : R2=0.942253 | MSE=0.015541 | var=0.015811 | s_hat=63.655194 |
-------------------------------------------------------------
>>> Degree = 36 : R2=0.941893 | MSE=0.015638 | var=0.015918 | s_hat=64.053002 |
-------------------------------------------------------------
>>> Degree = 37 : R2=0.942539 | MSE=0.015464 | var=0.015749 | s_hat=63.340691 |
-------------------------------------------------------------
>>> Degree = 38 : R2=0.942183 | MSE=0.015560 | var=0.015854 | s_hat=63.733244 |
-------------------------------------------------------------
>>> Degree = 39 : R2=0.942477 | MSE=0.015481 | var=0.015781 | s_hat=63.408610 |
-------------------------------------------------------------
>>> Degree = 40 : R2=0.942322 | MSE=0.015522 | var=0.015832 | s_hat=63.579534 |
-------------------------------------------------------------
>>> Degree = 41 : R2=0.942660 | MSE=0.015432 | var=0.015747 | s_hat=63.207534 |
-------------------------------------------------------------
>>> Degree = 42 : R2=0.941308 | MSE=0.015795 | var=0.016126 | s_hat=64.697525 |
-------------------------------------------------------------
>>> Degree = 43 : R2=0.941098 | MSE=0.015852 | var=0.016192 | s_hat=64.928426 |
-------------------------------------------------------------
>>> Degree = 44 : R2=0.942326 | MSE=0.015521 | var=0.015862 | s_hat=63.575124 |
-------------------------------------------------------------
>>> Degree = 45 : R2=0.941502 | MSE=0.015743 | var=0.016097 | s_hat=64.483252 |
-------------------------------------------------------------
>>> Degree = 46 : R2=0.942023 | MSE=0.015603 | var=0.015961 | s_hat=63.908997 |
-------------------------------------------------------------
>>> Degree = 47 : R2=0.942236 | MSE=0.015545 | var=0.015911 | s_hat=63.673888 |
-------------------------------------------------------------
>>> Degree = 48 : R2=0.939579 | MSE=0.016260 | var=0.016651 | s_hat=66.602998 |
-------------------------------------------------------------
>>> Degree = 49 : R2=0.942683 | MSE=0.015425 | var=0.015803 | s_hat=63.181364 |
-------------------------------------------------------------
>>> Degree = 50 : R2=0.941831 | MSE=0.015654 | var=0.016046 | s_hat=64.120411 |
-------------------------------------------------------------
>>> Degree = 51 : R2=0.942836 | MSE=0.015384 | var=0.015777 | s_hat=63.012758 |
-------------------------------------------------------------
>>> Degree = 52 : R2=0.936155 | MSE=0.017182 | var=0.017630 | s_hat=70.377995 |
-------------------------------------------------------------
>>> Degree = 53 : R2=0.942367 | MSE=0.015510 | var=0.015922 | s_hat=63.529953 |
-------------------------------------------------------------
>>> Degree = 54 : R2=0.285658 | MSE=0.192244 | var=0.197451 | s_hat=787.433261 |
-------------------------------------------------------------
>>> Degree = 55 : R2=0.940230 | MSE=0.016085 | var=0.016529 | s_hat=65.885122 |
-------------------------------------------------------------
>>> Degree = 56 : R2=0.942310 | MSE=0.015526 | var=0.015962 | s_hat=63.593058 |
```

```
------------------------------------------------------------
>>> Degree = 57 : R2=-0.341654 | MSE=0.361067 | var=0.371404 | s_hat=1478.931634 |
------------------------------------------------------------
>>> Degree = 58 : R2=0.938748 | MSE=0.016484 | var=0.016965 | s_hat=67.519121 |
------------------------------------------------------------
>>> Degree = 59 : R2=0.935919 | MSE=0.017246 | var=0.017757 | s_hat=70.638194 |
------------------------------------------------------------
>>> Degree = 60 : R2=0.281113 | MSE=0.193467 | var=0.199307 | s_hat=792.442731 |
------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 51
```

```
na = bestFitDegree;
nb = bestFitDegree;
nk = 1;
p = na+nb;

BestFitModel_guassian = oe(data_guassian, [na nb nk])
```

```
BestFitModel_guassian =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.3946 z^-1 + 0.4077 z^-2 - 0.1489 z^-3 + 0.1777 z^-4 + 0.3182 z^-5 - 0.2529 z^-6 +
0.07911 z^-7
          + 0.2174 z^-8 - 0.2401 z^-9 - 0.08814 z^-10 + 0.0186 z^-11 - 0.1046 z^-12 - 0.009979
z^-13
          - 0.008735 z^-14 - 0.07688 z^-15 - 0.02146 z^-16 - 0.0354 z^-17 + 0.03094 z^-18 +
0.03416 z^-19
```

```
          - 0.09171 z^-20 + 0.05775 z^-21 - 0.1334 z^-22 - 0.3367 z^-23 + 0.0304 z^-24 -
0.1109 z^-25
          - 0.1628 z^-26 + 0.1466 z^-27 - 0.2092 z^-28 - 0.06924 z^-29 + 0.2999 z^-30 -
0.01549 z^-31
          + 0.06125 z^-32 + 0.2349 z^-33 - 0.00811 z^-34 - 0.01466 z^-35 + 0.1046 z^-36 +
0.07866 z^-37
          + 0.02255 z^-38 - 0.08935 z^-39 - 0.0974 z^-40 + 0.05583 z^-41 + 0.01593 z^-42 -
0.02448 z^-43
          + 0.03334 z^-44 - 0.06486 z^-45 - 0.05717 z^-46 - 0.04649 z^-47 - 0.05502 z^-48 +
0.06801 z^-49
                                                                + 0.02989 z^-50 -
0.03846 z^-51


  F(z) = 1 + 0.3546 z^-1 - 0.9562 z^-2 + 0.8785 z^-3 + 0.534 z^-4 - 1.107 z^-5 + 0.7589 z^-6
          + 0.392 z^-7 - 0.9579 z^-8 + 0.2518 z^-9 + 0.1507 z^-10 - 0.3341 z^-11 + 0.1378 z^-
12
          - 0.04603 z^-13 - 0.2061 z^-14 + 0.06581 z^-15 - 0.07157 z^-16 + 0.1158 z^-17 +
0.02018 z^-18
          - 0.2866 z^-19 + 0.3006 z^-20 - 0.4496 z^-21 - 0.6119 z^-22 + 0.6053 z^-23 - 0.4365
z^-24
          - 0.2693 z^-25 + 0.5746 z^-26 - 0.8396 z^-27 + 0.2071 z^-28 + 0.7818 z^-29 - 0.5974
z^-30
          + 0.2831 z^-31 + 0.4902 z^-32 - 0.3994 z^-33 + 0.09614 z^-34 + 0.3112 z^-35 + 0.016
z^-36
          - 0.01011 z^-37 - 0.231 z^-38 - 0.1004 z^-39 + 0.3195 z^-40 - 0.07497 z^-41 - 0.1201
z^-42
          + 0.1507 z^-43 - 0.2125 z^-44 - 0.06114 z^-45 - 0.01433 z^-46 - 0.05504 z^-47 +
0.2189 z^-48
                                                   - 0.06792 z^-49 - 0.1253 z^-50 +
0.07802 z^-51

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=51   nf=51   nk=1
   Number of free coefficients: 102
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_guassian".
Fit to estimation data: 76.09%
FPE: 0.01617, MSE: 0.01538
```

```
BestFit_y_hat_guassian = lsim(BestFitModel_guassian, u1_val, t_val);
% [oe_BestFit_r2, oe_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("===============Degree Extraction | Variance
Method====================\n")
```

```
===============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 41 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 41
```

```
na = minVarIndex;
nb = minVarIndex;
nk = 1;
p = na+nb;

oe_VarModel_guassian = oe(data_guassian, [na nb nk])
```

```
oe_VarModel_guassian =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.3944 z^-1 + 0.09471 z^-2 - 0.2451 z^-3 - 0.494 z^-4 - 0.4705 z^-5 + 0.4124 z^-6 +
0.6421 z^-7
          + 0.3704 z^-8 - 0.4233 z^-9 - 0.6109 z^-10 - 0.08606 z^-11 + 0.367 z^-12 + 0.528 z^
          -13 - 0.09654 z^-14 - 0.4645 z^-15 - 0.332 z^-16 - 0.01693 z^-17 + 0.4262 z^-18 +
0.3446 z^-19
          - 0.003276 z^-20 - 0.2231 z^-21 - 0.4415 z^-22 + 0.06982 z^-23 + 0.2581 z^-24 +
0.1974 z^-25
          + 0.08259 z^-26 - 0.2893 z^-27 - 0.09261 z^-28 - 0.04366 z^-29 + 0.05425 z^-30 +
0.05082 z^-31
          + 0.06938 z^-32 + 0.1776 z^-33 - 0.0468 z^-34 - 0.02084 z^-35 - 0.1673 z^-36 -
0.0707 z^-37
                                       + 0.1096 z^-38 + 0.02293 z^-39 + 0.01608 z^-40 -
0.03671 z^-41


  F(z) = 1 - 0.4399 z^-1 - 0.6604 z^-2 - 0.7561 z^-3 - 0.3758 z^-4 + 1.693 z^-5 + 0.7013 z^-6
          - 0.1344 z^-7 - 1.52 z^-8 - 0.6183 z^-9 + 0.7969 z^-10 + 0.893 z^-11 + 0.602 z^-12
          - 1.1 z^-13 - 0.8441 z^-14 + 0.00575 z^-15 + 0.4201 z^-16 + 0.9798 z^-17 + 0.07659
z^-18
          - 0.5387 z^-19 - 0.427 z^-20 - 0.6794 z^-21 + 0.9399 z^-22 + 0.4044 z^-23 + 0.005933
z^-24
          - 0.1162 z^-25 - 0.8172 z^-26 + 0.3568 z^-27 + 0.02021 z^-28 + 0.1658 z^-29 - 0.0299
z^-30
          + 0.06552 z^-31 + 0.3546 z^-32 - 0.3641 z^-33 + 0.0984 z^-34 - 0.3925 z^-35 +
0.07917 z^-36
                          + 0.3673 z^-37 - 0.1598 z^-38 + 0.02501 z^-39 - 0.1012 z^-40 +
0.0428 z^-41

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=41   nf=41   nk=1
   Number of free coefficients: 82
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
```

```
Status:
Estimated using OE on time domain data "data_guassian".
Fit to estimation data: 76.05%
FPE: 0.01606, MSE: 0.01543
```

```
Var_y_hat_guassian = lsim(oe_VarModel_guassian, u1_val, t_val);
% [oe_Var_r2, oe_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n", 
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=4095.00) occurs in iteration 23 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 23
```

```
na = minAICIndex;
nb = minAICIndex;
nk = 1;
p = na+nb;

oe_AICModel_guassian = oe(data_guassian, [na nb nk])
```

```
oe_AICModel_guassian =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.3948 z^-1 + 0.3591 z^-2 + 0.09581 z^-3 + 0.02834 z^-4 - 0.5452 z^-5 - 0.8486 z^-6
          - 0.6845 z^-7 - 0.3167 z^-8 + 0.4237 z^-9 + 0.8544 z^-10 + 1.026 z^-11 + 0.6478 z^-
12
          - 0.03535 z^-13 - 0.5195 z^-14 - 0.8214 z^-15 - 0.6014 z^-16 - 0.3342 z^-17 + 0.1284
z^-18
                           + 0.4344 z^-19 + 0.1425 z^-20 + 0.1968 z^-21 + 0.1189 z^-22 -
0.1317 z^-23


  F(z) = 1 + 0.2309 z^-1 - 0.2532 z^-2 + 0.06548 z^-3 - 1.326 z^-4 - 1.153 z^-5 - 0.4149 z^-6
```

```
          + 0.05697 z^-7 + 1.265 z^-8 + 1.176 z^-9 + 1.132 z^-10 + 0.1319 z^-11 - 0.7642 z^-12
          - 0.8583 z^-13 - 1.012 z^-14 - 0.1787 z^-15 - 0.09638 z^-16 + 0.5816 z^-17 + 0.6287
z^-18
                          - 0.4694 z^-19 + 0.3455 z^-20 + 0.04885 z^-21 - 0.4322 z^-22 +
0.3133 z^-23

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=23   nf=23   nk=1
   Number of free coefficients: 46
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_guassian".
Fit to estimation data: 75.94%
FPE: 0.01593, MSE: 0.01558
```

```
AIC_y_hat_guassian = lsim(oe_AICModel_guassian, u1_val, t_val);
% [oe_AIC_r2, oe_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.28 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 20
```

```
na = winner;
nb = winner;
nk = 1;
p = na+nb;

oe_FTestModel_guassian = oe(data_guassian, [na nb nk])
```

```
oe_FTestModel_guassian =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.3947 z^-1 + 0.7537 z^-2 + 0.3385 z^-3 - 0.1495 z^-4 - 0.2849 z^-5 - 0.2466 z^-6 -
0.1073 z^-7
          - 0.06362 z^-8 - 0.06199 z^-9 - 0.0145 z^-10 + 0.106 z^-11 + 0.2625 z^-12 + 0.4098
z^-13
          + 0.4324 z^-14 + 0.2564 z^-15 + 0.004619 z^-16 - 0.4 z^-17 - 0.4283 z^-18 + 0.03692
z^-19
                                                                                      +
0.1326 z^-20


  F(z) = 1 + 1.23 z^-1 - 0.3173 z^-2 - 0.6802 z^-3 - 0.2378 z^-4 - 0.07313 z^-5 + 0.0972 z^-6
          - 0.07461 z^-7 - 0.1282 z^-8 + 0.009477 z^-9 + 0.2382 z^-10 + 0.4483 z^-11 + 0.6168
z^-12
          + 0.4916 z^-13 + 0.06593 z^-14 - 0.2346 z^-15 - 0.8453 z^-16 - 0.3037 z^-17 + 0.7503
z^-18
                                                                        + 0.1012 z^-19 -
0.3017 z^-20

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=20   nf=20   nk=1
   Number of free coefficients: 40
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_guassian".
Fit to estimation data: 75.91%
FPE: 0.01593, MSE: 0.01562
```

```
FTest_y_hat_guassian = lsim(oe_FTestModel_guassian, u1_val, t_val);
% [oe_FTest_r2, oe_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

[oe_BestFit_r2_guassian, oe_BestFit_mse_guassian] = rSQR(y1_val,
BestFit_y_hat_guassian);
[oe_Var_r2_guassian, oe_Var_mse_guassian] = rSQR(y1_val, Var_y_hat_guassian);
[oe_AIC_r2_guassian, oe_AIC_mse_guassian] = rSQR(y1_val, AIC_y_hat_guassian);
[oe_FTest_r2_guassian, oe_FTest_mse_guassian] = rSQR(y1_val,
FTest_y_hat_guassian);

%%

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_BestFit_r2_guassian,
oe_BestFit_mse_guassian)
```

```
    R2 value : 0.9416    | MSE : 0.0156
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Var_r2_guassian,
oe_Var_mse_guassian)
```

```
    R2 value : 0.9419    | MSE : 0.0155
```

```
% fprintf("----------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("    R2 value : %.4f    | MSE : %.4f \n", oe_Cov_r2, oe_Cov_mse)
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", oe_AIC_r2_guassian,
oe_AIC_mse_guassian)
```

```
    R2 value : 0.9420    | MSE : 0.0155
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("    R2 value : %.4f   | MSE : %.4f \n", oe_FTest_r2_guassian,
oe_FTest_mse_guassian)
```

```
    R2 value : 0.9414    | MSE : 0.0156
```

```
fprintf("---------------------------------------------------------------\n")
```

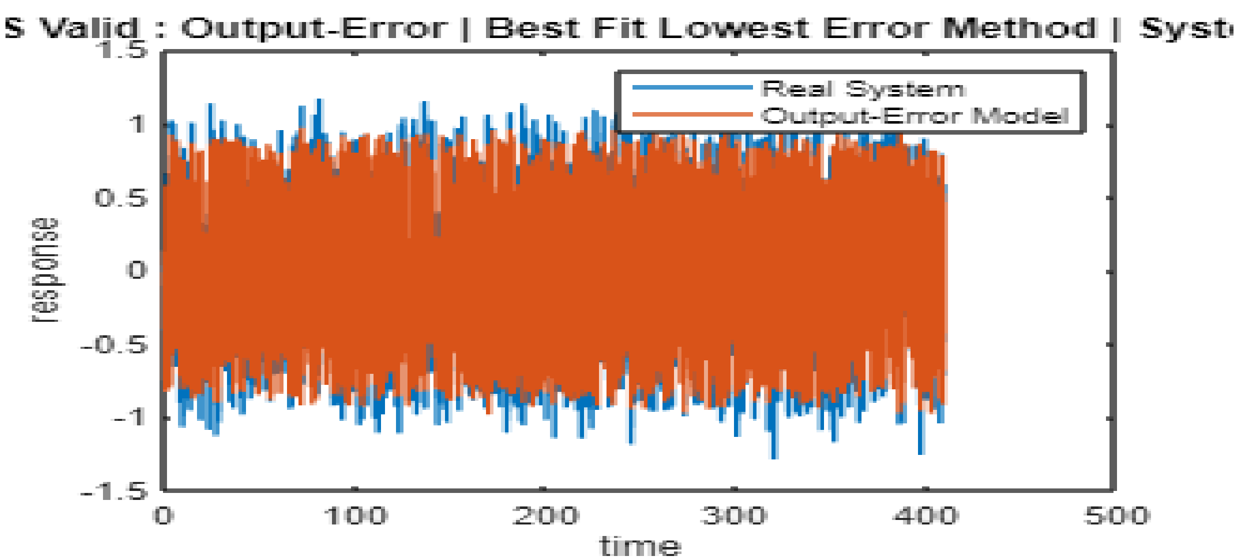
```
---------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

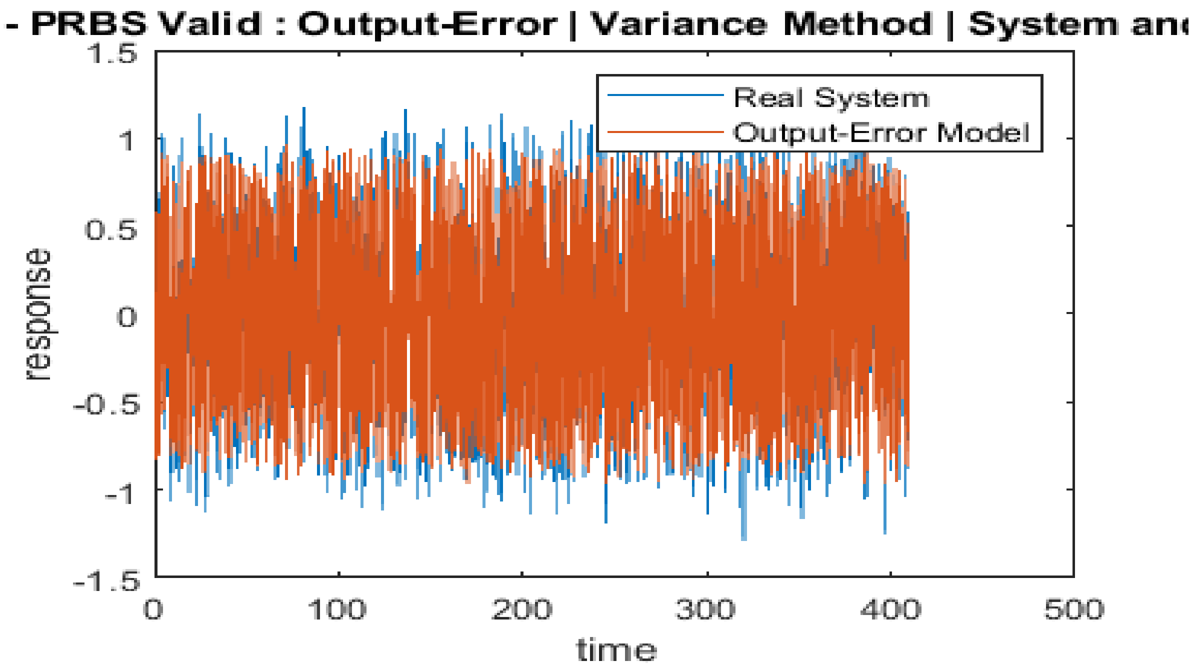
```
===============================================================
```

```



%%

oe_BestFitError_guassian = y1_val - BestFit_y_hat_guassian;
oe_VarError_guassian = y1_val - Var_y_hat_guassian;
oe_AICError_guassian = y1_val - AIC_y_hat_guassian;
oe_FTestError_guassian = y1_val - FTest_y_hat_guassian;

for k=0:N_val-1
    oe_BestFit_Ree_guassian(k+1,1) = AutoCorrelate(oe_BestFitError_guassian,
k);
    oe_Var_Ree_guassian(k+1,1) = AutoCorrelate(oe_VarError_guassian, k);
    oe_AIC_Ree_guassian(k+1,1) = AutoCorrelate(oe_AICError_guassian, k);
    oe_FTest_Ree_guassian(k+1,1) = AutoCorrelate(oe_FTestError_guassian, k);
end

for k=0:N_val-1
    oe_BestFit_Rue_guassian(k+1,1) = CrossCorrelate(u1_val,
oe_BestFitError_guassian, k);
```

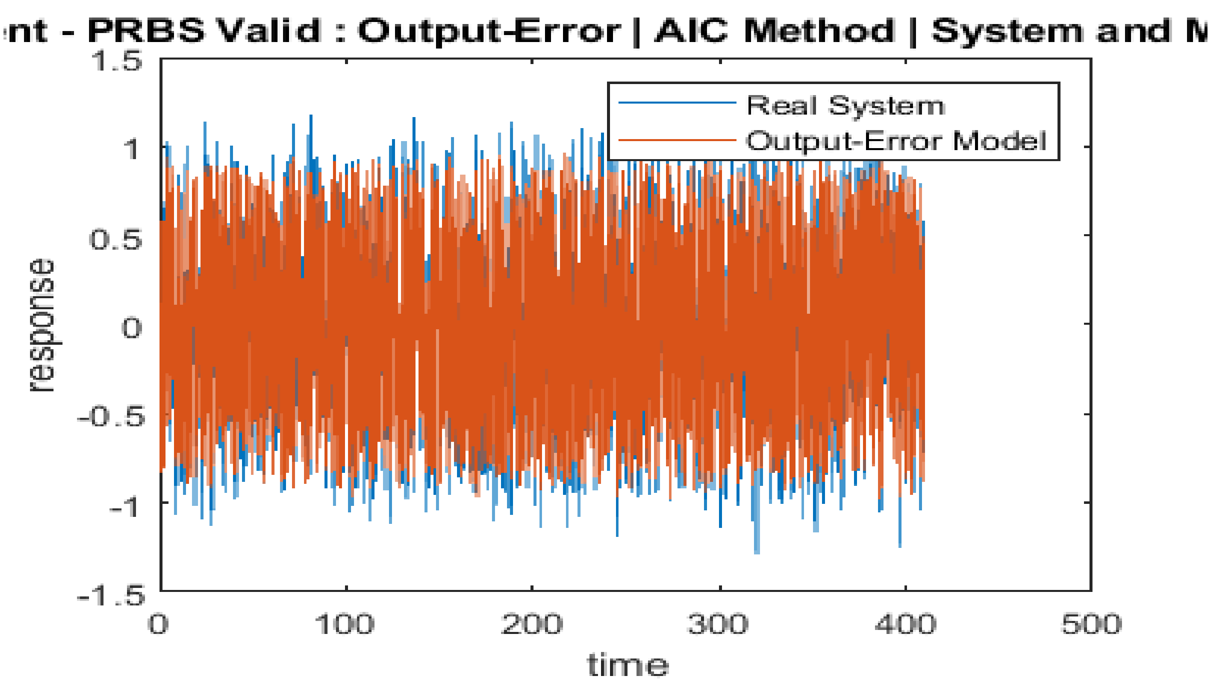
```
    oe_Var_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, oe_VarError_guassian,
k);
    oe_AIC_Rue_guassian(k+1,1) = CrossCorrelate(u1_val, oe_AICError_guassian,
k);
    oe_FTest_Rue_guassian(k+1,1) = CrossCorrelate(u1_val,
oe_FTestError_guassian, k);
 end


 %%
 figure(1)
 plot(t_val,y1_val,t_val,BestFit_y_hat_guassian)
 legend('Real System','Output-Error Model')
 title(" Guassian Ident - PRBS Valid : Output-Error | Best Fit Lowest Error
Method | System and Model Response")
 xlabel("time")
 ylabel("response")
```



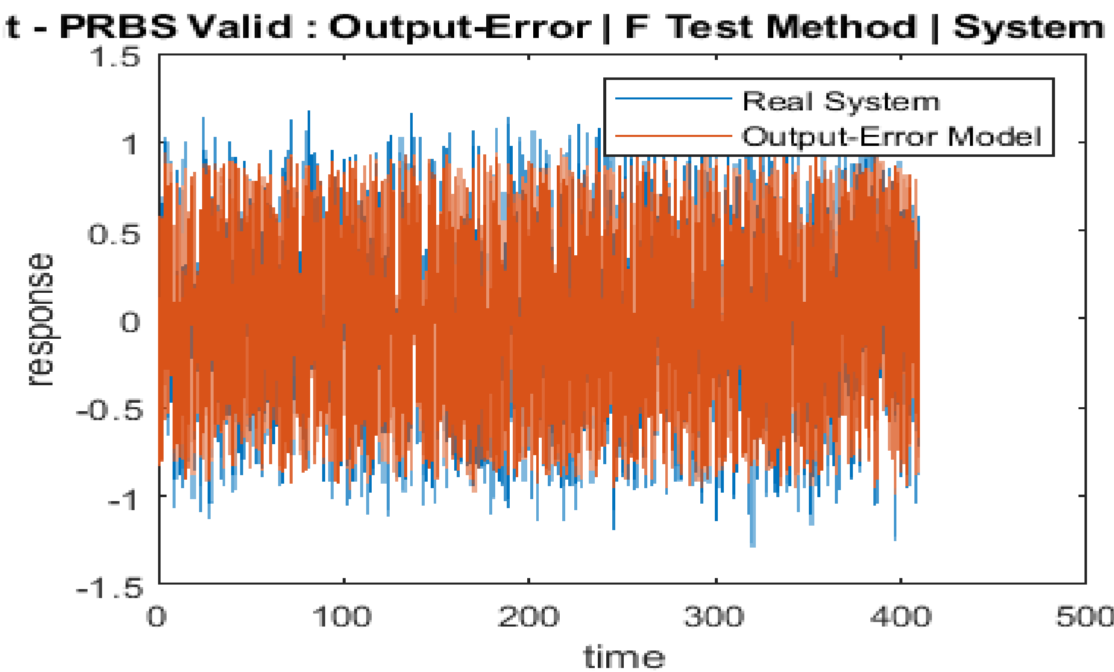

```
 figure(2)
 plot(t_val,y1_val,t_val,Var_y_hat_guassian)
 legend('Real System','Output-Error Model')
 title(" Guassian Ident - PRBS Valid : Output-Error | Variance Method | System
and Model Response")
 xlabel("time")
 ylabel("response")
```




```
figure(3)
plot(t_val,y1_val,t_val,AIC_y_hat_guassian)
legend('Real System','Output-Error Model')
title(" Guassian Ident - PRBS Valid : Output-Error | AIC Method | System and
Model Response")
xlabel("time")
ylabel("response")
```



```
 figure(4)
 plot(t_val,y1_val,t_val,FTest_y_hat_guassian)
 legend('Real System','Output-Error Model')
 title(" Guassian Ident - PRBS Valid : Output-Error | F Test Method | System and
Model Response")
 xlabel("time")
 ylabel("response")
```




```
 %
 % %%
 %
 % figure(5)
 % subplot(4,1,1)
 % plot(1:N_val-1,oe_BestFit_Ree_guassian(2:end), 1:N_val-1,
mean(oe_BestFit_Ree_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Output-Error | Best Fit Lowest Errror
Method | Ree_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_guassian(k)")
 %
 % subplot(4,1,2)
 % plot(1:N_val-1,oe_Var_Ree_guassian(2:end), 1:N_val-1,
mean(oe_Var_Ree_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Output-Error | Variance Method |
Ree_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
```

```
 % ylabel("Ree_guassian(k)")
 %
 % subplot(4,1,3)
 % plot(1:N_val-1,oe_AIC_Ree_guassian(2:end), 1:N_val-1,
mean(oe_AIC_Ree_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Output-Error | AIC Method |
Ree_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_guassian(k)")
 %
 % subplot(4,1,4)
 % plot(1:N_val-1,oe_FTest_Ree_guassian(2:end), 1:N_val-1,
mean(oe_FTest_Ree_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Output-Error | F Test Method |
Ree_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_guassian(k)")
 %
 % %%
 %
 % figure(6)
 % subplot(4,1,1)
 % plot(1:N_val-1,oe_BestFit_Rue_guassian(2:end), 1:N_val-1,
mean(oe_BestFit_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Output-Error | Best Fit Lowest Errror
Method | Rue_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_guassian(k)")
 %
 % subplot(4,1,2)
 % plot(1:N_val-1,oe_Var_Rue_guassian(2:end), 1:N_val-1,
mean(oe_Var_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Output-Error | Variance Method |
Rue_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_guassian(k)")
 %
 % subplot(4,1,3)
 % plot(1:N_val-1,oe_AIC_Rue_guassian(2:end), 1:N_val-1,
mean(oe_AIC_Rue_guassian(2:end))*ones(length(1:N_val-1)))
 % title(" Guassian Ident - PRBS Valid : Output-Error | AIC Method |
Rue_guassian(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_guassian(k)")
 %
```

```
% subplot(4,1,4)
% plot(1:N_val-1,oe_FTest_Rue_guassian(2:end), 1:N_val-1,
mean(oe_FTest_Rue_guassian(2:end))*ones(length(1:N_val-1)))
% title(" Guassian Ident - PRBS Valid : Output-Error | F Test Method |
Rue_guassian(k) | The Straight Line is the Mean")
% xlabel("k")
% ylabel("Rue_guassian(k)")
%
```

PRBS Input - Guassian Validation

```
%%

% PRBS Input ***************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> PRBS Input Identification Begins:-----------------------------
\n")
```

```
>>> PRBS Input Identification Begins:-----------------------------
```

```
%%

Ts = 0.1;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);

data_prbs = iddata(y2,u2,Ts);


%%


fprintf("====================Degree Extraction |
RUN==========================\n")
```

```
====================Degree Extraction | RUN==========================
```

```
R2s  = [];
MSEs = [];
dets = [];
vars = [];
```

```
covs = [];
S_hats = [];
AICs = [];
ps = [];
k = 0.75;

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    try
        sys = oe(data_prbs, [na nb nk]);
        oe_y_hat_prbs = lsim(sys, u2, t);
    catch
        break
    end

    [r2_oe, mse_oe] = rSQR(y2, oe_y_hat_prbs);

    error = y2 - oe_y_hat_prbs;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    AIC = AIC_criteria(S_hat, k, p, N);
    variance = Variance_criteria(S_hat, N, p);

    fprintf(">>> Degree = %d : R2=%f | MSE=%f | var=%f | s_hat=%f | \n",
degree, r2_oe, mse_oe, variance, S_hat)
    fprintf("-------------------------------------------------------------\n")

    ps = [ps; p];
    R2s = [R2s; r2_oe];
    MSEs = [MSEs; mse_oe];
    vars = [vars; variance];
    S_hats = [S_hats; S_hat];
    AICs = [AICs; AIC];

end
```

```
>>> Degree = 1 : R2=0.902599 | MSE=0.025958 | var=0.025971 | s_hat=106.325062 |
-------------------------------------------------------------
>>> Degree = 2 : R2=0.943233 | MSE=0.015129 | var=0.015144 | s_hat=61.967521 |
```

```
-------------------------------------------------------------
>>> Degree = 3 : R2=0.943272 | MSE=0.015118 | var=0.015141 | s_hat=61.924787 |
-------------------------------------------------------------
>>> Degree = 4 : R2=0.943346 | MSE=0.015099 | var=0.015128 | s_hat=61.844003 |
-------------------------------------------------------------
>>> Degree = 5 : R2=0.943374 | MSE=0.015091 | var=0.015128 | s_hat=61.813447 |
-------------------------------------------------------------
>>> Degree = 6 : R2=0.943377 | MSE=0.015091 | var=0.015135 | s_hat=61.810976 |
-------------------------------------------------------------
>>> Degree = 7 : R2=0.943455 | MSE=0.015070 | var=0.015121 | s_hat=61.725879 |
-------------------------------------------------------------
>>> Degree = 8 : R2=0.943514 | MSE=0.015054 | var=0.015113 | s_hat=61.660918 |
-------------------------------------------------------------
>>> Degree = 9 : R2=0.943676 | MSE=0.015011 | var=0.015077 | s_hat=61.483935 |
-------------------------------------------------------------
>>> Degree = 10 : R2=0.943632 | MSE=0.015022 | var=0.015096 | s_hat=61.531820 |
-------------------------------------------------------------
>>> Degree = 11 : R2=0.943682 | MSE=0.015009 | var=0.015090 | s_hat=61.477505 |
-------------------------------------------------------------
>>> Degree = 12 : R2=0.943714 | MSE=0.015001 | var=0.015089 | s_hat=61.442444 |
-------------------------------------------------------------
>>> Degree = 13 : R2=0.943801 | MSE=0.014977 | var=0.015073 | s_hat=61.347674 |
-------------------------------------------------------------
>>> Degree = 14 : R2=0.943696 | MSE=0.015005 | var=0.015109 | s_hat=61.462400 |
-------------------------------------------------------------
>>> Degree = 15 : R2=0.943704 | MSE=0.015003 | var=0.015114 | s_hat=61.453963 |
-------------------------------------------------------------
>>> Degree = 16 : R2=0.943929 | MSE=0.014943 | var=0.015061 | s_hat=61.208193 |
-------------------------------------------------------------
>>> Degree = 17 : R2=0.943025 | MSE=0.015184 | var=0.015311 | s_hat=62.194357 |
-------------------------------------------------------------
>>> Degree = 18 : R2=0.944077 | MSE=0.014904 | var=0.015036 | s_hat=61.047048 |
-------------------------------------------------------------
>>> Degree = 19 : R2=0.944113 | MSE=0.014894 | var=0.015034 | s_hat=61.006680 |
-------------------------------------------------------------
>>> Degree = 20 : R2=0.944221 | MSE=0.014866 | var=0.015012 | s_hat=60.889715 |
-------------------------------------------------------------
>>> Degree = 21 : R2=0.943920 | MSE=0.014946 | var=0.015101 | s_hat=61.217523 |
-------------------------------------------------------------
>>> Degree = 22 : R2=0.929689 | MSE=0.018739 | var=0.018942 | s_hat=76.753108 |
-------------------------------------------------------------
>>> Degree = 23 : R2=0.944069 | MSE=0.014906 | var=0.015075 | s_hat=61.055481 |
-------------------------------------------------------------
>>> Degree = 24 : R2=0.944080 | MSE=0.014903 | var=0.015080 | s_hat=61.042710 |
-------------------------------------------------------------
>>> Degree = 25 : R2=0.944267 | MSE=0.014853 | var=0.015037 | s_hat=60.839289 |
-------------------------------------------------------------
>>> Degree = 26 : R2=0.942470 | MSE=0.015332 | var=0.015529 | s_hat=62.801170 |
-------------------------------------------------------------
>>> Degree = 27 : R2=0.700504 | MSE=0.079818 | var=0.080884 | s_hat=326.934438 |
-------------------------------------------------------------
>>> Degree = 28 : R2=0.938939 | MSE=0.016273 | var=0.016499 | s_hat=66.655556 |
-------------------------------------------------------------
>>> Degree = 29 : R2=0.944080 | MSE=0.014903 | var=0.015117 | s_hat=61.043146 |
-------------------------------------------------------------
>>> Degree = 30 : R2=0.944409 | MSE=0.014815 | var=0.015036 | s_hat=60.683968 |
-------------------------------------------------------------
```

```
>>> Degree = 31 : R2=0.944543 | MSE=0.014780 | var=0.015007 | s_hat=60.537944 |
---------------------------------------------------------------
>>> Degree = 32 : R2=0.944044 | MSE=0.014913 | var=0.015149 | s_hat=61.082574 |
---------------------------------------------------------------
>>> Degree = 33 : R2=0.944219 | MSE=0.014866 | var=0.015110 | s_hat=60.891521 |
---------------------------------------------------------------
>>> Degree = 34 : R2=0.944264 | MSE=0.014854 | var=0.015105 | s_hat=60.842270 |
---------------------------------------------------------------
>>> Degree = 35 : R2=0.944264 | MSE=0.014854 | var=0.015112 | s_hat=60.842918 |
---------------------------------------------------------------
>>> Degree = 36 : R2=0.944418 | MSE=0.014813 | var=0.015078 | s_hat=60.674763 |
---------------------------------------------------------------
>>> Degree = 37 : R2=0.944585 | MSE=0.014768 | var=0.015040 | s_hat=60.491691 |
---------------------------------------------------------------
>>> Degree = 38 : R2=0.943844 | MSE=0.014966 | var=0.015249 | s_hat=61.301101 |
---------------------------------------------------------------
>>> Degree = 39 : R2=0.944949 | MSE=0.014672 | var=0.014956 | s_hat=60.094503 |
---------------------------------------------------------------
>>> Degree = 40 : R2=0.774757 | MSE=0.060029 | var=0.061225 | s_hat=245.879655 |
---------------------------------------------------------------
>>> Degree = 41 : R2=0.944830 | MSE=0.014703 | var=0.015004 | s_hat=60.224422 |
---------------------------------------------------------------
>>> Degree = 42 : R2=0.944776 | MSE=0.014718 | var=0.015026 | s_hat=60.283854 |
---------------------------------------------------------------
>>> Degree = 43 : R2=0.945332 | MSE=0.014570 | var=0.014882 | s_hat=59.676737 |
---------------------------------------------------------------
>>> Degree = 44 : R2=0.943826 | MSE=0.014971 | var=0.015299 | s_hat=61.320320 |
---------------------------------------------------------------
>>> Degree = 45 : R2=0.943816 | MSE=0.014974 | var=0.015310 | s_hat=61.331557 |
---------------------------------------------------------------
>>> Degree = 46 : R2=0.881952 | MSE=0.031461 | var=0.032184 | s_hat=128.863420 |
---------------------------------------------------------------
>>> Degree = 47 : R2=0.942191 | MSE=0.015406 | var=0.015768 | s_hat=63.105004 |
---------------------------------------------------------------
>>> Degree = 48 : R2=0.933224 | MSE=0.017796 | var=0.018223 | s_hat=72.893433 |
---------------------------------------------------------------
>>> Degree = 49 : R2=0.945196 | MSE=0.014606 | var=0.014964 | s_hat=59.825272 |
---------------------------------------------------------------
>>> Degree = 50 : R2=0.943774 | MSE=0.014985 | var=0.015360 | s_hat=61.377394 |
---------------------------------------------------------------
>>> Degree = 51 : R2=-0.022273 | MSE=0.272444 | var=0.279402 | s_hat=1115.930509 |
---------------------------------------------------------------
>>> Degree = 52 : R2=0.941309 | MSE=0.015642 | var=0.016049 | s_hat=64.067790 |
---------------------------------------------------------------
>>> Degree = 53 : R2=0.883015 | MSE=0.031177 | var=0.032006 | s_hat=127.702727 |
---------------------------------------------------------------
>>> Degree = 54 : R2=0.942396 | MSE=0.015352 | var=0.015768 | s_hat=62.881060 |
---------------------------------------------------------------
>>> Degree = 55 : R2=0.942028 | MSE=0.015450 | var=0.015876 | s_hat=63.283712 |
---------------------------------------------------------------
>>> Degree = 56 : R2=0.820368 | MSE=0.047873 | var=0.049219 | s_hat=196.089247 |
---------------------------------------------------------------
>>> Degree = 57 : R2=0.945294 | MSE=0.014580 | var=0.014997 | s_hat=59.717687 |
---------------------------------------------------------------
>>> Degree = 58 : R2=0.942350 | MSE=0.015364 | var=0.015812 | s_hat=62.931955 |
---------------------------------------------------------------
>>> Degree = 59 : R2=0.907415 | MSE=0.024675 | var=0.025407 | s_hat=101.067877 |
```

```
------------------------------------------------------------
>>> Degree = 60 : R2=0.890846 | MSE=0.029090 | var=0.029968 | s_hat=119.153839 |
------------------------------------------------------------
>>> Degree = 61 : R2=0.945075 | MSE=0.014638 | var=0.015087 | s_hat=59.957297 |
------------------------------------------------------------
>>> Degree = 62 : R2=0.935197 | MSE=0.017270 | var=0.017810 | s_hat=70.739945 |
------------------------------------------------------------
>>> Degree = 63 : R2=0.945256 | MSE=0.014590 | var=0.015053 | s_hat=59.759680 |
------------------------------------------------------------
>>> Degree = 64 : R2=0.945506 | MSE=0.014523 | var=0.014992 | s_hat=59.487049 |
------------------------------------------------------------
>>> Degree = 65 : R2=0.939330 | MSE=0.016169 | var=0.016699 | s_hat=66.228283 |
------------------------------------------------------------
>>> Degree = 66 : R2=0.936459 | MSE=0.016934 | var=0.017498 | s_hat=69.362596 |
------------------------------------------------------------
```

```
fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | BestFit Method=================\n")
```

```
===============Degree Extraction | BestFit Method=================
```

```
bestFitDegree = find(S_hats == min(S_hats));

fprintf(">>> Looking for the minimum SSE , leads to: \n")
```

```
>>> Looking for the minimum SSE , leads to:
```

```
fprintf("    Degree = %d \n", bestFitDegree)
```

```
    Degree = 64
```

```
na = bestFitDegree;
nb = bestFitDegree;
nk = 1;
p = na+nb;

BestFitModel_prbs = oe(data_prbs, [na nb nk])
```

```
BestFitModel_prbs =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.4 z^-1 + 1.667 z^-2 + 3.376 z^-3 + 4.824 z^-4 + 5.451 z^-5 + 4.889 z^-6 + 3.581 z^
          -7 + 2.242 z^-8 + 1.279 z^-9 + 0.804 z^-10 + 0.8973 z^-11 + 1.548 z^-12 + 2.307 z^-
13
```

```
           + 2.891 z^-14 + 2.918 z^-15 + 1.933 z^-16 + 0.356 z^-17 - 1.143 z^-18 - 2.03 z^-19
           - 1.995 z^-20 - 1.257 z^-21 - 0.3899 z^-22 - 0.01874 z^-23 - 0.2311 z^-24 - 0.8612
z^-25
           - 1.819 z^-26 - 2.906 z^-27 - 3.855 z^-28 - 4.265 z^-29 - 3.911 z^-30 - 2.931 z^-31
           - 1.766 z^-32 - 1.028 z^-33 - 1.013 z^-34 - 1.463 z^-35 - 1.968 z^-36 - 2.369 z^-37
           - 2.469 z^-38 - 2.104 z^-39 - 1.412 z^-40 - 0.4556 z^-41 + 0.5126 z^-42 + 1.086 z^-
43
           + 1.083 z^-44 + 0.6221 z^-45 + 0.1228 z^-46 - 0.05374 z^-47 + 0.4597 z^-48 + 1.514
z^-49
           + 2.475 z^-50 + 3.142 z^-51 + 3.426 z^-52 + 3.374 z^-53 + 3.17 z^-54 + 2.778 z^-55
           + 2.184 z^-56 + 1.467 z^-57 + 0.9207 z^-58 + 0.6438 z^-59 + 0.4211 z^-60 + 0.222 z^
                                                    -61 + 0.0182 z^-62 - 0.1067 z^-63 -
0.05507 z^-64


  F(z) = 1 + 3.502 z^-1 + 5.764 z^-2 + 6.91 z^-3 + 6.706 z^-4 + 4.98 z^-5 + 3.194 z^-6 + 2.165
z^-7
           + 1.71 z^-8 + 1.615 z^-9 + 2.112 z^-10 + 3.18 z^-11 + 3.814 z^-12 + 4.093 z^-13 +
3.44 z^-14
           + 1.239 z^-15 - 0.8881 z^-16 - 2.17 z^-17 - 2.381 z^-18 - 1.476 z^-19 - 0.2718 z^-20
           + 0.3676 z^-21 - 0.1161 z^-22 - 1.011 z^-23 - 1.988 z^-24 - 3.24 z^-25 - 4.475 z^-26
           - 5.361 z^-27 - 5.275 z^-28 - 4.221 z^-29 - 2.755 z^-30 - 1.582 z^-31 - 1.481 z^-32
           - 2.263 z^-33 - 2.997 z^-34 - 3.264 z^-35 - 3.399 z^-36 - 3.1 z^-37 - 2.217 z^-38 -
1.266 z^-39
           - 0.03744 z^-40 + 0.9783 z^-41 + 1.172 z^-42 + 0.6877 z^-43 - 0.04667 z^-44 - 0.3298
z^-45
           + 0.0335 z^-46 + 1.459 z^-47 + 3.104 z^-48 + 3.817 z^-49 + 4.195 z^-50 + 4.199 z^-51
           + 3.999 z^-52 + 3.885 z^-53 + 3.389 z^-54 + 2.555 z^-55 + 1.627 z^-56 + 1.215 z^-57
           + 1.135 z^-58 + 0.7764 z^-59 + 0.4514 z^-60 + 0.1107 z^-61 - 0.001191 z^-62 + 0.1925
z^-63
                                                                                            +
0.1288 z^-64

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=64   nf=64   nk=1
   Number of free coefficients: 128
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_prbs".
Fit to estimation data: 76.66%
FPE: 0.01546, MSE: 0.01452
```

```
BestFit_y_hat_prbs = lsim(BestFitModel_prbs, u2_val, t_val);
% [oe_BestFit_r2, oe_BestFit_mse] = rSQR(y_val, BestFit_y_hat);

%%

fprintf("==============Degree Extraction | Variance Method===================\n")
```

```
==============Degree Extraction | Variance Method====================
```

```
minVarIndex = find(vars == min(vars));
fprintf(">>> Since the minimum variance value occurs in iteration %d ;\n",
minVarIndex)
```

```
>>> Since the minimum variance value occurs in iteration 43 ;
```

```
fprintf("    Degree = %d \n", minVarIndex)
```

```
    Degree = 43
```

```
na = minVarIndex;
nb = minVarIndex;
nk = 1;
p = na+nb;

oe_VarModel_prbs = oe(data_prbs, [na nb nk])
```

```
oe_VarModel_prbs =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.4001 z^-1 + 0.4824 z^-2 + 0.3751 z^-3 + 0.1855 z^-4 + 0.2906 z^-5 + 0.4229 z^-6 +
0.2404 z^-7
          - 0.1565 z^-8 - 0.2863 z^-9 - 0.3451 z^-10 - 0.2725 z^-11 - 0.4533 z^-12 - 0.4559
z^-13
          - 0.2718 z^-14 + 0.0182 z^-15 + 0.02309 z^-16 + 0.06658 z^-17 + 0.1945 z^-18 +
0.3682 z^-19
          + 0.3773 z^-20 + 0.3763 z^-21 + 0.4828 z^-22 + 0.3193 z^-23 + 0.1602 z^-24 + 0.07245
z^-25
          + 0.08678 z^-26 - 0.1702 z^-27 - 0.3869 z^-28 - 0.4589 z^-29 - 0.3018 z^-30 - 0.3564
z^-31
          - 0.4103 z^-32 - 0.3389 z^-33 - 0.02951 z^-34 + 0.2336 z^-35 + 0.181 z^-36 - 0.03987
z^-37
         + 0.1234 z^-38 + 0.2308 z^-39 + 0.245 z^-40 + 0.002998 z^-41 - 0.06325 z^-42 +
0.02637 z^-43


  F(z) = 1 + 0.5376 z^-1 + 0.2362 z^-2 + 0.01188 z^-3 + 0.6191 z^-4 + 0.7181 z^-5 + 0.04949
z^-6
          - 0.6129 z^-7 - 0.2889 z^-8 - 0.3422 z^-9 - 0.1598 z^-10 - 0.81 z^-11 - 0.5112 z^-12
          - 0.1109 z^-13 + 0.292 z^-14 - 0.1646 z^-15 + 0.04215 z^-16 + 0.3427 z^-17 + 0.6101
z^-18
          + 0.3767 z^-19 + 0.4432 z^-20 + 0.7381 z^-21 + 0.1905 z^-22 + 0.08018 z^-23 + 0.1161
z^-24
          + 0.2442 z^-25 - 0.4808 z^-26 - 0.6264 z^-27 - 0.5155 z^-28 - 0.09652 z^-29 - 0.5632
z^-30
          - 0.6105 z^-31 - 0.3576 z^-32 + 0.2922 z^-33 + 0.452 z^-34 - 0.03791 z^-35 - 0.3908
z^-36
          + 0.429 z^-37 + 0.401 z^-38 + 0.2788 z^-39 - 0.3194 z^-40 - 0.05722 z^-41 + 0.2266
z^-42
                                                                                         -
0.03162 z^-43

Sample time: 0.1 seconds
```

```
Parameterization:
   Polynomial orders:   nb=43   nf=43   nk=1
   Number of free coefficients: 86
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_prbs".
Fit to estimation data: 76.62%
FPE: 0.01519, MSE: 0.01457
```

```
Var_y_hat_prbs = lsim(oe_VarModel_prbs, u2_val, t_val);
% [oe_Var_r2, oe_Var_mse] = rSQR(y_val, Var_y_hat);

fprintf("=================================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | AIC Method====================\n")
```

```
===============Degree Extraction | AIC Method====================
```

```
minAICIndex = find(AICs == min(AICs));
fprintf(">>> Since the minimum AIC value (k=%.2f) occurs in iteration %d ;\n",
k, minAICIndex)
```

```
>>> Since the minimum AIC value (k=0.75) occurs in iteration 43 ;
```

```
fprintf("    Degree = %d \n", minAICIndex)
```

```
    Degree = 43
```

```
na = minAICIndex;
nb = minAICIndex;
nk = 1;
p = na+nb;

oe_AICModel_prbs = oe(data_prbs, [na nb nk])
```

```
oe_AICModel_prbs =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.4001 z^-1 + 0.4824 z^-2 + 0.3751 z^-3 + 0.1855 z^-4 + 0.2906 z^-5 + 0.4229 z^-6 +
0.2404 z^-7
          - 0.1565 z^-8 - 0.2863 z^-9 - 0.3451 z^-10 - 0.2725 z^-11 - 0.4533 z^-12 - 0.4559
z^-13
```

```
         - 0.2718 z^-14 + 0.0182 z^-15 + 0.02309 z^-16 + 0.06658 z^-17 + 0.1945 z^-18 +
0.3682 z^-19
         + 0.3773 z^-20 + 0.3763 z^-21 + 0.4828 z^-22 + 0.3193 z^-23 + 0.1602 z^-24 + 0.07245
z^-25
         + 0.08678 z^-26 - 0.1702 z^-27 - 0.3869 z^-28 - 0.4589 z^-29 - 0.3018 z^-30 - 0.3564
z^-31
         - 0.4103 z^-32 - 0.3389 z^-33 - 0.02951 z^-34 + 0.2336 z^-35 + 0.181 z^-36 - 0.03987
z^-37
        + 0.1234 z^-38 + 0.2308 z^-39 + 0.245 z^-40 + 0.002998 z^-41 - 0.06325 z^-42 +
0.02637 z^-43


  F(z) = 1 + 0.5376 z^-1 + 0.2362 z^-2 + 0.01188 z^-3 + 0.6191 z^-4 + 0.7181 z^-5 + 0.04949
z^-6
         - 0.6129 z^-7 - 0.2889 z^-8 - 0.3422 z^-9 - 0.1598 z^-10 - 0.81 z^-11 - 0.5112 z^-12
         - 0.1109 z^-13 + 0.292 z^-14 - 0.1646 z^-15 + 0.04215 z^-16 + 0.3427 z^-17 + 0.6101
z^-18
         + 0.3767 z^-19 + 0.4432 z^-20 + 0.7381 z^-21 + 0.1905 z^-22 + 0.08018 z^-23 + 0.1161
z^-24
         + 0.2442 z^-25 - 0.4808 z^-26 - 0.6264 z^-27 - 0.5155 z^-28 - 0.09652 z^-29 - 0.5632
z^-30
         - 0.6105 z^-31 - 0.3576 z^-32 + 0.2922 z^-33 + 0.452 z^-34 - 0.03791 z^-35 - 0.3908
z^-36
         + 0.429 z^-37 + 0.401 z^-38 + 0.2788 z^-39 - 0.3194 z^-40 - 0.05722 z^-41 + 0.2266
z^-42
                                                                                     -
0.03162 z^-43

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=43   nf=43   nk=1
   Number of free coefficients: 86
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_prbs".
Fit to estimation data: 76.62%
FPE: 0.01519, MSE: 0.01457
```

```
AIC_y_hat_prbs = lsim(oe_AICModel_prbs, u2_val, t_val);
% [oe_AIC_r2, oe_AIC_mse] = rSQR(y_val, AIC_y_hat);

fprintf("=============================================================\n")
```

```
=================================================================
```

```
%%

fprintf("===============Degree Extraction | F test
Method====================\n")
```

```
===============Degree Extraction | F test Method====================
```

```
winScore = 0;
winner = 1;
for i=2:length(ps)
    first = winner;
    second = i;
    winScore = finv(0.95, ps(second)-ps(first), N-ps(first));
    score = ((S_hats(first)-S_hats(second))/(ps(second)-
ps(first)))/((S_hats(first))/(N-ps(first)));
    if score > winScore
        winner = i;
    end
end
fprintf(">>> The F test is suggesting the best model with the m=%.2f as\n",
winScore)
```

```
>>> The F test is suggesting the best model with the m=1.37 as
```

```
fprintf("    Degree = %d \n", winner)
```

```
    Degree = 43
```

```
na = winner;
nb = winner;
nk = 1;
p = na+nb;

oe_FTestModel_prbs = oe(data_prbs, [na nb nk])
```

```
oe_FTestModel_prbs =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.4001 z^-1 + 0.4824 z^-2 + 0.3751 z^-3 + 0.1855 z^-4 + 0.2906 z^-5 + 0.4229 z^-6 +
0.2404 z^-7
          - 0.1565 z^-8 - 0.2863 z^-9 - 0.3451 z^-10 - 0.2725 z^-11 - 0.4533 z^-12 - 0.4559
z^-13
          - 0.2718 z^-14 + 0.0182 z^-15 + 0.02309 z^-16 + 0.06658 z^-17 + 0.1945 z^-18 +
0.3682 z^-19
          + 0.3773 z^-20 + 0.3763 z^-21 + 0.4828 z^-22 + 0.3193 z^-23 + 0.1602 z^-24 + 0.07245
z^-25
          + 0.08678 z^-26 - 0.1702 z^-27 - 0.3869 z^-28 - 0.4589 z^-29 - 0.3018 z^-30 - 0.3564
z^-31
          - 0.4103 z^-32 - 0.3389 z^-33 - 0.02951 z^-34 + 0.2336 z^-35 + 0.181 z^-36 - 0.03987
z^-37
         + 0.1234 z^-38 + 0.2308 z^-39 + 0.245 z^-40 + 0.002998 z^-41 - 0.06325 z^-42 +
0.02637 z^-43


  F(z) = 1 + 0.5376 z^-1 + 0.2362 z^-2 + 0.01188 z^-3 + 0.6191 z^-4 + 0.7181 z^-5 + 0.04949
z^-6
          - 0.6129 z^-7 - 0.2889 z^-8 - 0.3422 z^-9 - 0.1598 z^-10 - 0.81 z^-11 - 0.5112 z^-12
```

```
          - 0.1109 z^-13 + 0.292 z^-14 - 0.1646 z^-15 + 0.04215 z^-16 + 0.3427 z^-17 + 0.6101
z^-18
          + 0.3767 z^-19 + 0.4432 z^-20 + 0.7381 z^-21 + 0.1905 z^-22 + 0.08018 z^-23 + 0.1161
z^-24
          + 0.2442 z^-25 - 0.4808 z^-26 - 0.6264 z^-27 - 0.5155 z^-28 - 0.09652 z^-29 - 0.5632
z^-30
          - 0.6105 z^-31 - 0.3576 z^-32 + 0.2922 z^-33 + 0.452 z^-34 - 0.03791 z^-35 - 0.3908
z^-36
          + 0.429 z^-37 + 0.401 z^-38 + 0.2788 z^-39 - 0.3194 z^-40 - 0.05722 z^-41 + 0.2266
z^-42
                                                                                     -
0.03162 z^-43

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=43   nf=43   nk=1
   Number of free coefficients: 86
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_prbs".
Fit to estimation data: 76.62%
FPE: 0.01519, MSE: 0.01457
```

```
FTest_y_hat_prbs = lsim(oe_FTestModel_prbs, u2_val, t_val);
% [oe_FTest_r2, oe_FTest_mse] = rSQR(y_val, FTest_y_hat);

fprintf("============================================================================\n")
```

```
=================================================================
```

```



%%


[oe_BestFit_r2_prbs, oe_BestFit_mse_prbs] = rSQR(y2_val, BestFit_y_hat_prbs);
[oe_Var_r2_prbs, oe_Var_mse_prbs] = rSQR(y2_val, Var_y_hat_prbs);
[oe_AIC_r2_prbs, oe_AIC_mse_prbs] = rSQR(y2_val, AIC_y_hat_prbs);
[oe_FTest_r2_prbs, oe_FTest_mse_prbs] = rSQR(y2_val, FTest_y_hat_prbs);

fprintf("===================Evaluation | R2 Metric======================\n")
```

```
===================Evaluation | R2 Metric======================
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf(">>> BestFit Lowest Error Method:\n")
```

```
>>> BestFit Lowest Error Method:
```

```
fprintf("     R2 value : %.4f    | MSE : %.4f \n", oe_BestFit_r2_prbs,
oe_BestFit_mse_prbs)
```

```
    R2 value : 0.9382    | MSE : 0.0166
```

```
fprintf("---------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> Variance Method:\n")
```

```
>>> Variance Method:
```

```
fprintf("     R2 value : %.4f    | MSE : %.4f \n", oe_Var_r2_prbs,
oe_Var_mse_prbs)
```

```
    R2 value : 0.9384    | MSE : 0.0166
```

```
% fprintf("---------------------------------------------------------------\n")
% fprintf(">>> Covariance Method:\n")
% fprintf("     R2 value : %.4f    | MSE : %.4f \n", oe_Cov_r2, oe_Cov_mse)
fprintf("---------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> AIC Method:\n")
```

```
>>> AIC Method:
```

```
fprintf("     R2 value : %.4f    | MSE : %.4f \n", oe_AIC_r2_prbs,
oe_AIC_mse_prbs)
```

```
    R2 value : 0.9384    | MSE : 0.0166
```

```
fprintf("---------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf(">>> FTest Method:\n")
```

```
>>> FTest Method:
```

```
fprintf("     R2 value : %.4f    | MSE : %.4f \n", oe_FTest_r2_prbs,
oe_FTest_mse_prbs)
```

```
    R2 value : 0.9384    | MSE : 0.0166
```

```
fprintf("---------------------------------------------------------------\n")
```

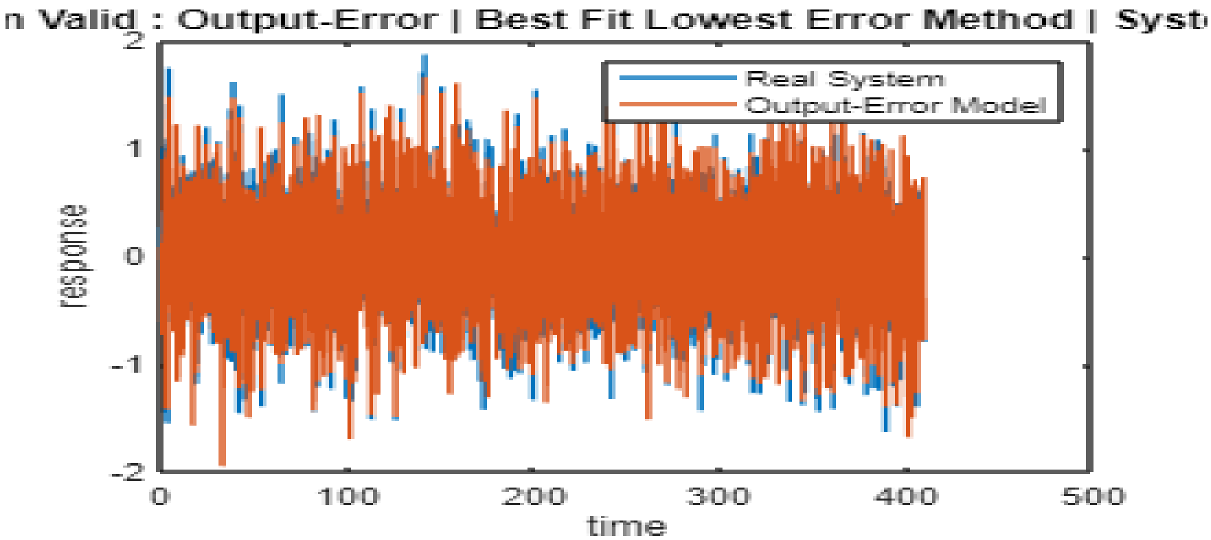
```
----------------------------------------------------------------
```

```
% fprintf(">>> Winner:\n")
```

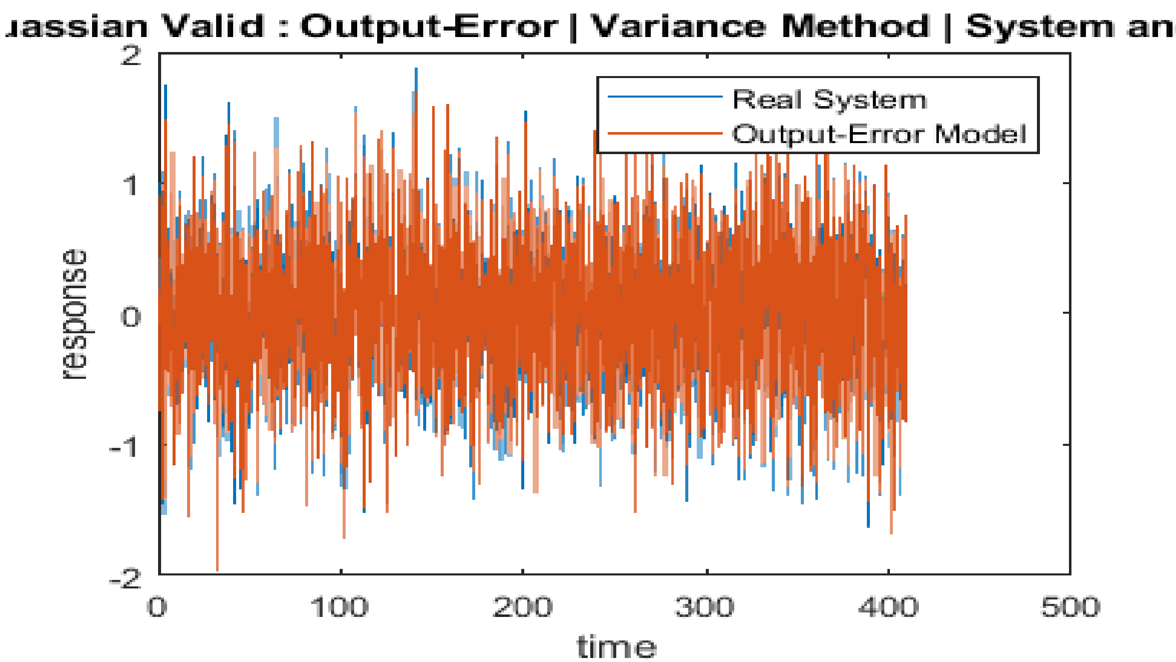
```
% fprintf("    The best R2 value is \n")
fprintf("===============================================================\n")
```

===

```
%%

oe_BestFitError_prbs = y2_val - BestFit_y_hat_prbs;
oe_VarError_prbs = y2_val - Var_y_hat_prbs;
oe_AICError_prbs = y2_val - AIC_y_hat_prbs;
oe_FTestError_prbs = y2_val - FTest_y_hat_prbs;

for k=0:N_val-1
    oe_BestFit_Ree_prbs(k+1,1) = AutoCorrelate(oe_BestFitError_prbs, k);
    oe_Var_Ree_prbs(k+1,1) = AutoCorrelate(oe_VarError_prbs, k);
    oe_AIC_Ree_prbs(k+1,1) = AutoCorrelate(oe_AICError_prbs, k);
    oe_FTest_Ree_prbs(k+1,1) = AutoCorrelate(oe_FTestError_prbs, k);
end

for k=0:N_val-1
    oe_BestFit_Rue_prbs(k+1,1) = CrossCorrelate(u2_val, oe_BestFitError_prbs,
k);
    oe_Var_Rue_prbs(k+1,1) = CrossCorrelate(u2_val, oe_VarError_prbs, k);
    oe_AIC_Rue_prbs(k+1,1) = CrossCorrelate(u2_val, oe_AICError_prbs, k);
    oe_FTest_Rue_prbs(k+1,1) = CrossCorrelate(u2_val, oe_FTestError_prbs, k);
end


%%
figure(1)
plot(t_val,y2_val,t_val,BestFit_y_hat_prbs)
legend('Real System','Output-Error Model')
title(" PRBS Ident - Guassian Valid : Output-Error | Best Fit Lowest Error
Method | System and Model Response")
xlabel("time")
ylabel("response")
```

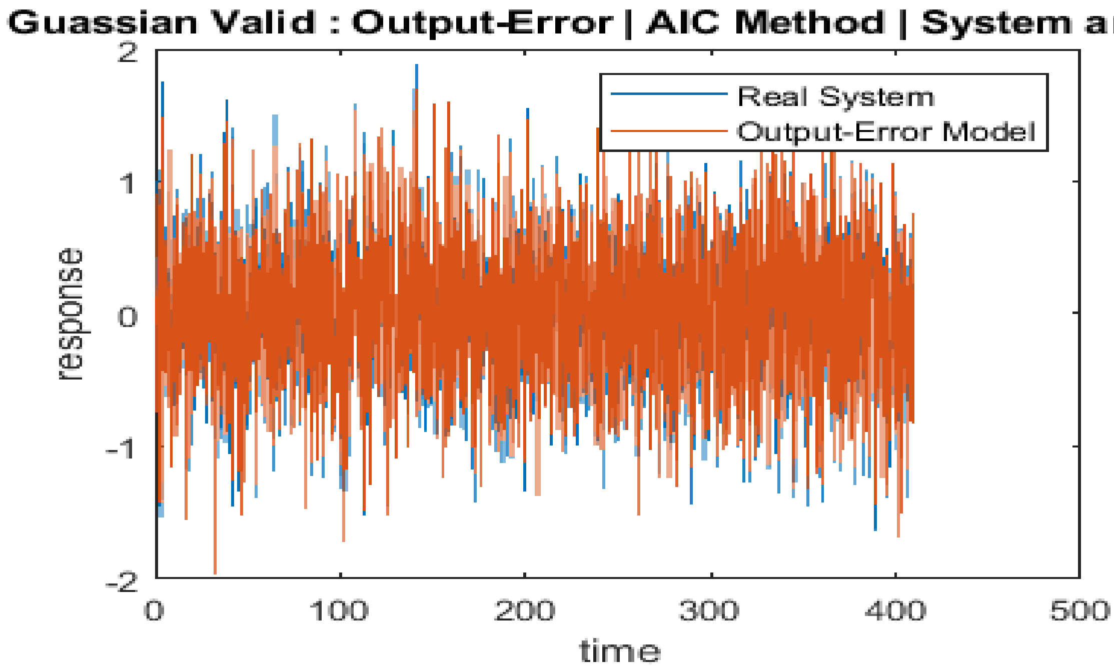



```
figure(2)
plot(t_val,y2_val,t_val,Var_y_hat_prbs)
legend('Real System','Output-Error Model')
title(" PRBS Ident - Guassian Valid : Output-Error | Variance Method | System and Model Response")
xlabel("time")
ylabel("response")
```

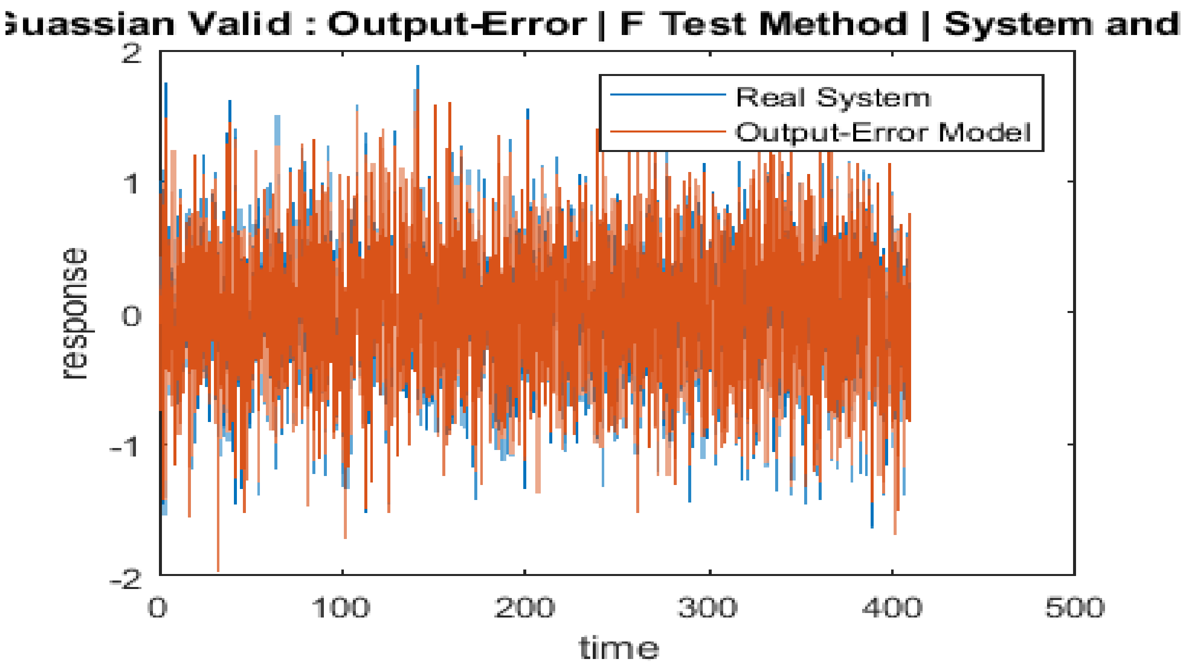



```
figure(3)
plot(t_val,y2_val,t_val,AIC_y_hat_prbs)
```

```
legend('Real System','Output-Error Model')
title(" PRBS Ident - Guassian Valid : Output-Error | AIC Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
figure(4)
plot(t_val,y2_val,t_val,FTest_y_hat_prbs)
legend('Real System','Output-Error Model')
title(" PRBS Ident - Guassian Valid : Output-Error | F Test Method | System and Model Response")
xlabel("time")
ylabel("response")
```




```
 % %%
 %
 % figure(5)
 % subplot(4,1,1)
 % plot(1:N_val-1,oe_BestFit_Ree_prbs(2:end), 1:N_val-1,
mean(oe_BestFit_Ree_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | Best Fit Lowest Errror
Method | Ree_prbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_prbs(k)")
 %
 % subplot(4,1,2)
 % plot(1:N_val-1,oe_Var_Ree_prbs(2:end), 1:N_val-1,
mean(oe_Var_Ree_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | Variance Method |
Ree_prbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_prbs(k)")
 %
 % subplot(4,1,3)
 % plot(1:N_val-1,oe_AIC_Ree_prbs(2:end), 1:N_val-1,
mean(oe_AIC_Ree_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | AIC Method | Ree_prbs(k)
| The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_prbs(k)")
```

```
 %
 % subplot(4,1,4)
 % plot(1:N_val-1,oe_FTest_Ree_prbs(2:end), 1:N_val-1,
mean(oe_FTest_Ree_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | F Test Method |
Ree_prbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Ree_prbs(k)")
 %
 % %%
 %
 % figure(6)
 % subplot(4,1,1)
 % plot(1:N_val-1,oe_BestFit_Rue_prbs(2:end), 1:N_val-1,
mean(oe_BestFit_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | Best Fit Lowest Errror
Method | Rue_prbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_prbs(k)")
 %
 % subplot(4,1,2)
 % plot(1:N_val-1,oe_Var_Rue_prbs(2:end), 1:N_val-1,
mean(oe_Var_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | Variance Method |
Rue_prbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_prbs(k)")
 %
 % subplot(4,1,3)
 % plot(1:N_val-1,oe_AIC_Rue_prbs(2:end), 1:N_val-1,
mean(oe_AIC_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | AIC Method | Rue_prbs(k)
| The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_prbs(k)")
 %
 % subplot(4,1,4)
 % plot(1:N_val-1,oe_FTest_Rue_prbs(2:end), 1:N_val-1,
mean(oe_FTest_Rue_prbs(2:end))*ones(length(1:N_val-1)))
 % title(" PRBS Ident - Guassian Valid : Output-Error | F Test Method |
Rue_prbs(k) | The Straight Line is the Mean")
 % xlabel("k")
 % ylabel("Rue_prbs(k)")
```

Q3 - part b | AutoCorrelate

```
clc; clear;

%%

load q3_402123100.mat

N = length(u1);

%%

fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("*******System Degree Ident. by Output AutoCorrelation**********\n")
```

```
*******System Degree Ident. by Output AutoCorrelation**********
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
Ry1 = [];
Ry2 = [];
for k=1:N
    Ry1(k) = AutoCorrelate(y1, k);
    Ry2(k) = AutoCorrelate(y2, k);
end

queue1 = Ry1(1:50);
queue2 = Ry2(1:50);

order_by_Ry1 = find(queue1==queue1(find(abs(diff(queue1)) ==
max(abs(diff(queue1))))))+1;
order_by_Ry2 = find(queue2==queue2(find(abs(diff(queue2)) ==
max(abs(diff(queue2))))))+1;

fprintf(">>> The system degree based on y1 data : %d \n", order_by_Ry1);
```

```
>>> The system degree based on y1 data : 2
```

```
fprintf(">>> The system degree based on y2 data : %d \n", order_by_Ry2);
```

```
>>> The system degree based on y2 data : 2
```

```
fprintf("------------------------------------------------------------\n")
```

```
------------------------------------------------------------
```

```
fprintf("************************************************************\n")
```

```
************************************************************
```

```
fprintf("************************************************************\n")
```

```
************************************************************
```

Q3 - part b | FPE ARX

```
clc; clear;

%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;
```

Guassian Input - PRBS Validation

```
%%
% Guassian Input **************************************************************
fprintf("******************************************************************\n")
```

```
*******************************************************************
```

```
fprintf(">>> Guassian Input Identification Begins:-----------------------------
-\n")
```

```
>>> Guassian Input Identification Begins:-----------------------------
```

```
%%

Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);


%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
```

```
    p = na+nb;

    U = arx_U_builder_3(u1, y1, na, nb, nk);
    theta_hat_guassian = inv(U'*U)*U'*y1;
    y_hat_guassian = form_tf_lsim_2(theta_hat_guassian, u1, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y1, y_hat_guassian);

    error = y1 - y_hat_guassian;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("-------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];

end
```

```
>>> Degree = 1 | FPE = 0.0144
-------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0106
-------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0087
-------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 7 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0080
```

```
-----------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 21 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 28 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 37 | FPE = 0.0082
-----------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0082
-----------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0082
-----------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0082
-----------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0082
-----------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0082
-----------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0082
-----------------------------------------------------------------
>>> Degree = 44 | FPE = 0.0082
-----------------------------------------------------------------
```

```
>>> Degree = 45 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 47 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 48 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 49 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 50 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 51 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 52 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 53 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 54 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 55 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 56 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 57 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 58 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 59 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 60 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 61 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 62 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 63 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 64 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 65 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 66 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 67 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 68 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 69 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 70 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 71 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 72 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 73 | FPE = 0.0083
```

```
-------------------------------------------------------------
>>> Degree = 74 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 75 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 76 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 77 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 78 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 79 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 80 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 81 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 82 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 83 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 84 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 85 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 86 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 87 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 88 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 89 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 90 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 91 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 92 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 93 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 94 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 95 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 96 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 97 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 98 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 99 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 100 | FPE = 0.0085
-------------------------------------------------------------
```

```
%%

fprintf("*************************************************************\n")
```

```
*************************************************************
```

```
fprintf("************System Degree Ident. by FPE Method***************\n")
```

```
************System Degree Ident. by FPE Method***************
```

```
fprintf("-------------------------------------------------------------\n")
```

```
-------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n",
minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 5 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 5
```

```
na = minFPEIndex;
nb = minFPEIndex;
nk = 1;
p = na+nb;

FPEU_guassian = arx_U_builder_3(u1, y1, na, nb, nk);
FPEModel_guassian = inv(FPEU_guassian'*FPEU_guassian)*FPEU_guassian'*y1;
FPE_y_hat_guassian = form_tf_lsim_2(FPEModel_guassian, u1_val, t_val, na, Ts);
```

```
G =

    0.3951 z^-1 + 0.1927 z^-2 + 0.04997 z^-3 - 0.01372 z^-4 - 0.02994 z^-5
  ---------------------------------------------------------------------------
  1 - 0.1897 z^-1 - 0.08256 z^-2 - 0.01541 z^-3 + 0.01903 z^-4 + 0.07503 z^-5

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
[FPE_r2, FPE_mse] = rSQR(y1_val, FPE_y_hat_guassian);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9429
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```

```

PRBS Input - Guassian Validation

```
%%

% PRBS Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> PRBS Input Identification Begins:------------------------------
\n")
```

```
>>> PRBS Input Identification Begins:------------------------------
```

```

%%

Ts = 0.1;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);


%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    U = arx_U_builder_3(u2, y2, na, nb, nk);
    theta_hat_prbs = inv(U'*U)*U'*y2;
```

```
    y_hat_prbs = form_tf_lsim_2(theta_hat_prbs, u2, t, na, Ts);

    [r2_arx, mse_arx] = rSQR(y2, y_hat_prbs);

    error = y2 - y_hat_prbs;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("-------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];

end
```

```
>>> Degree = 1 | FPE = 0.0139
-------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0106
-------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 7 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0077
```

```
-------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 21 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 28 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 37 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 44 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 45 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0078
-------------------------------------------------------------
```

```
>>> Degree = 47 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 48 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 49 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 50 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 51 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 52 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 53 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 54 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 55 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 56 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 57 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 58 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 59 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 60 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 61 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 62 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 63 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 64 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 65 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 66 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 67 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 68 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 69 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 70 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 71 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 72 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 73 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 74 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 75 | FPE = 0.0080
```

```
-----------------------------------------------------------------
>>> Degree = 76 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 77 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 78 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 79 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 80 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 81 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 82 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 83 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 84 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 85 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 86 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 87 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 88 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 89 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 90 | FPE = 0.0080
-----------------------------------------------------------------
>>> Degree = 91 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 92 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 93 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 94 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 95 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 96 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 97 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 98 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 99 | FPE = 0.0081
-----------------------------------------------------------------
>>> Degree = 100 | FPE = 0.0081
-----------------------------------------------------------------
```

```
%%
```

```
fprintf("****************************************************************\n")
```

```
****************************************************************
```

```
fprintf("************System Degree Ident. by FPE Method***************\n")
```

```
************System Degree Ident. by FPE Method***************
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n",
minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 6 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 6
```

```
na = minFPEIndex;
nb = minFPEIndex;
nk = 1;
p = na+nb;

FPEU_prbs = arx_U_builder_3(u2, y2, na, nb, nk);
FPEModel_prbs = inv(FPEU_prbs'*FPEU_prbs)*FPEU_prbs'*y2;
FPE_y_hat_prbs = form_tf_lsim_2(FPEModel_prbs, u2_val, t_val, na, Ts);
```

```
G =

   0.3995 z^-1 + 0.1954 z^-2 + 0.05691 z^-3 - 0.008226 z^-4 - 0.03263 z^-5 - 0.01977 z^-6
  ------------------------------------------------------------------------------------------
  1 - 0.1763 z^-1 - 0.08033 z^-2 - 0.01654 z^-3 + 0.01585 z^-4 + 0.0318 z^-5 + 0.03665 z^-6

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
[FPE_r2, FPE_mse] = rSQR(y2_val, FPE_y_hat_prbs);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9407
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

Q3 - part b | FPE ARMAX

```
clc; clear;

%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;
```

Guassian Input - PRBS Validation

```
%%
% Guassian Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> Guassian Input Identification Begins:-----------------------------
-\n")
```

```
>>> Guassian Input Identification Begins:------------------------------
```

```

%%

Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data_guassian = iddata(y1,u1,Ts);

%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nk = 1;
```

```
    p = na+nb+nc;

    try
        sys = armax(data_guassian, [na nb nc nk]);
        armax_y_hat_guassian = lsim(sys, u1, t);
    catch
        break
    end

    error = y1 - armax_y_hat_guassian;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("-------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];

end
```

```
>>> Degree = 1 | FPE = 0.0143
-------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 7 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0081
```

```
-------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 21 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 28 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 37 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0084
-------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0082
-------------------------------------------------------------
```

```
>>> Degree = 44 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 45 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 47 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 48 | FPE = 0.0083
-------------------------------------------------------------
```

```
%%
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("*************System Degree Ident. by FPE Method***************\n")
```

```
*************System Degree Ident. by FPE Method***************
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n",
minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 2
```

```
na = minFPEIndex;
nb = minFPEIndex;
nc = minFPEIndex;
nk = 1;
p = na+nb+nc;

armax_FPEModel_guassian = armax(data_guassian, [na nb nc nk])
```

```
armax_FPEModel_guassian =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.14 z^-1 + 0.4313 z^-2

  B(z) = 0.395 z^-1 - 0.1828 z^-2
```

```
  C(z) = 1 - 0.977 z^-1 + 0.3611 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_guassian".
Fit to estimation data: 76.24% (prediction focus)
FPE: 0.01524, MSE: 0.0152
```

```
FPE_y_hat_guassian = lsim(armax_FPEModel_guassian, u1_val, t_val);

[FPE_r2, FPE_mse] = rSQR(y1_val, FPE_y_hat_guassian);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9430
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

PRBS Input - Guassian Output

```
%%

% PRBS Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> PRBS Input Identification Begins:------------------------------
\n")
```

```
>>> PRBS Input Identification Begins:------------------------------
```

```


%%

Ts = 0.1;
t = 0:Ts:length(u2)*Ts-Ts;
```

```
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);

data_prbs = iddata(y2,u2,Ts);

%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nk = 1;
    p = na+nb+nc;

    try
        sys = armax(data_prbs, [na nb nc nk]);
        armax_y_hat_prbs = lsim(sys, u2, t);
    catch
        break
    end

    error = y2 - armax_y_hat_prbs;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("-------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];

end
```

```
>>> Degree = 1 | FPE = 0.0138
-------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0076
-------------------------------------------------------------
```

```
>>> Degree = 7 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 21 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 28 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0078
```

```
-------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 37 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 44 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 45 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 47 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 48 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 49 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 50 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 51 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 52 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 53 | FPE = 0.0087
-------------------------------------------------------------
>>> Degree = 54 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 55 | FPE = 10711872689739639414511894528.0000
-------------------------------------------------------------
>>> Degree = 56 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 57 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 58 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 59 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 60 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 61 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 62 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 63 | FPE = 0.0080
-------------------------------------------------------------
```

```
>>> Degree = 64 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 65 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 66 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 67 | FPE =
4018331297582936720417110977629559802122885362074579196418350624092152363305391201591888371009
88782894968779833344.0000
-------------------------------------------------------------
>>> Degree = 68 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 69 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 70 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 71 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 72 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 73 | FPE = 231522.3570
-------------------------------------------------------------
>>> Degree = 74 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 75 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 76 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 77 | FPE = 0.0104
-------------------------------------------------------------
>>> Degree = 78 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 79 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 80 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 81 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 82 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 83 | FPE = 0.6613
-------------------------------------------------------------
>>> Degree = 84 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 85 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 86 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 87 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 88 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 89 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 90 | FPE = 874555505087007131749508.0000
-------------------------------------------------------------
>>> Degree = 91 | FPE = 0.0083
```

```
------------------------------------------------------------
>>> Degree = 92 | FPE = 0.0083
------------------------------------------------------------
Warning: The "C" polynomial is unstable and cannot be automatically stabilized to meet the
prescribed constraints. This can cause the estimation to fail. Make sure that the starting
polynomial value is stable and within the desired constraints.
```

```
%%
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("*************System Degree Ident. by FPE Method***************\n")
```

```
*************System Degree Ident. by FPE Method***************
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n",
minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 2
```

```
na = minFPEIndex;
nb = minFPEIndex;
nc = minFPEIndex;
nk = 1;
p = na+nb+nc;

armax_FPEModel_prbs = armax(data_prbs, [na nb nc nk])
```

```
armax_FPEModel_prbs =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
  A(z) = 1 - 1.179 z^-1 + 0.4484 z^-2

  B(z) = 0.399 z^-1 - 0.2033 z^-2

  C(z) = 1 - 1.017 z^-1 + 0.3672 z^-2

Sample time: 0.1 seconds
```

```
Parameterization:
   Polynomial orders:   na=2   nb=2   nc=2   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARMAX on time domain data "data_prbs".
Fit to estimation data: 76.65% (prediction focus)
FPE: 0.01457, MSE: 0.01453
```

```
FPE_y_hat_prbs = lsim(armax_FPEModel_prbs, u2_val, t_val);

[FPE_r2, FPE_mse] = rSQR(y2_val, FPE_y_hat_prbs);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9407
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf("****************************************************************\n")
```

```
****************************************************************
```

```
fprintf("****************************************************************\n")
```

```
****************************************************************
```

Q3 - part b | FPE Box-Jenkins

```
clc; clear;

%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;
```

Guassian Input - PRBS Validation

```
%%
% Guassian Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> Guassian Input Identification Begins:-----------------------------
-\n")
```

```
>>> Guassian Input Identification Begins:------------------------------
```

```

%%

Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data_guassian = iddata(y1,u1,Ts);

%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nd = degree;
    nk = 1;
```

```
    p = na+nb+nc+nd;

    try
        sys = bj(data_guassian, [na nb nc nd nk]);
        bj_y_hat_guassian = lsim(sys, u1, t);
    catch
        break
    end

    error = y1 - bj_y_hat_guassian;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("--------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];

end
```

```
>>> Degree = 1 | FPE = 0.0142
------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 7 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0082
------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0081
```

```
--------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0102
--------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0082
--------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0081
--------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0082
--------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0084
--------------------------------------------------------------
>>> Degree = 21 | FPE = 0.0084
--------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0083
--------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0082
--------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0082
--------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0082
--------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0088
--------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0082
--------------------------------------------------------------
>>> Degree = 28 | FPE = 0.0100
--------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0201
--------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0085
--------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0084
--------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0113
--------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0184
--------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0086
--------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0147
--------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0085
--------------------------------------------------------------
>>> Degree = 37 | FPE = 0.0106
--------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0084
--------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0113
--------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0084
--------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0092
--------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0085
--------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0596
--------------------------------------------------------------
```

```
>>> Degree = 44 | FPE = 0.0089
-------------------------------------------------------------
>>> Degree = 45 | FPE = 0.0109
-------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0222
-------------------------------------------------------------
>>> Degree = 47 | FPE = 0.0260
-------------------------------------------------------------
>>> Degree = 48 | FPE = 0.0144
-------------------------------------------------------------
>>> Degree = 49 | FPE = 0.0086
-------------------------------------------------------------
>>> Degree = 50 | FPE = 0.0161
-------------------------------------------------------------
>>> Degree = 51 | FPE = 0.0236
-------------------------------------------------------------
>>> Degree = 52 | FPE = 0.0092
-------------------------------------------------------------
>>> Degree = 53 | FPE = 0.0118
-------------------------------------------------------------
>>> Degree = 54 | FPE = 0.8932
-------------------------------------------------------------
>>> Degree = 55 | FPE = 0.0094
-------------------------------------------------------------
>>> Degree = 56 | FPE = 0.1880
-------------------------------------------------------------
>>> Degree = 57 | FPE = 0.0162
-------------------------------------------------------------
>>> Degree = 58 | FPE = 0.0191
-------------------------------------------------------------
>>> Degree = 59 | FPE = 0.1299
-------------------------------------------------------------
>>> Degree = 60 | FPE = 0.0506
-------------------------------------------------------------
```

```
%%
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("*************System Degree Ident. by FPE Method****************\n")
```

```
*************System Degree Ident. by FPE Method****************
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n", minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 2
```

```

na = minFPEIndex;
nb = minFPEIndex;
nc = minFPEIndex;
nd = minFPEIndex;
nk = 1;
p = na+nb+nc+nd;

bj_FPEModel_guassian = bj(data_guassian, [na nb nc nd nk])
```

```
bj_FPEModel_guassian =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.3949 z^-1 - 0.1838 z^-2

  C(z) = 1 + 0.1904 z^-1 - 0.263 z^-2

  D(z) = 1 + 0.017 z^-1 - 0.372 z^-2

  F(z) = 1 - 1.143 z^-1 + 0.4335 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_guassian".
Fit to estimation data: 76.27% (prediction focus)
FPE: 0.01521, MSE: 0.01515
```

```
FPE_y_hat_guassian = lsim(bj_FPEModel_guassian, u1_val, t_val);
[FPE_r2, FPE_mse] = rSQR(y1_val, FPE_y_hat_guassian);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9431
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

PRBS Input - Guassian Validation

```
%%

% PRBS Input **************************************************************
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> PRBS Input Identification Begins:------------------------------
\n")
```

```
>>> PRBS Input Identification Begins:------------------------------
```

```
%%

Ts = 0.1;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);

data_prbs = iddata(y2,u2,Ts);

%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nc = degree;
    nd = degree;
    nk = 1;
    p = na+nb+nc+nd;

    try
        sys = bj(data_prbs, [na nb nc nd nk]);
        bj_y_hat_prbs = lsim(sys, u2, t);
    catch
        break
    end
```

```
    error = y2 - bj_y_hat_prbs;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("-------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];

end
```

```
>>> Degree = 1 | FPE = 0.0138
-------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 7 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0128
-------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0090
-------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0078
-------------------------------------------------------------
```

```
>>> Degree = 21 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0233
-------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0083
-------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0086
-------------------------------------------------------------
>>> Degree = 28 | FPE = 0.0295
-------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0087
-------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0498
-------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0089
-------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0285
-------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0099
-------------------------------------------------------------
>>> Degree = 37 | FPE = 0.4090
-------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0091
-------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0214
-------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0127
-------------------------------------------------------------
>>> Degree = 44 | FPE = 0.0090
-------------------------------------------------------------
>>> Degree = 45 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0147
-------------------------------------------------------------
>>> Degree = 47 | FPE = 0.0422
-------------------------------------------------------------
>>> Degree = 48 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 49 | FPE = 0.0081
```

```
-------------------------------------------------------------
>>> Degree = 50 | FPE = 0.0120
-------------------------------------------------------------
>>> Degree = 51 | FPE = 0.0287
-------------------------------------------------------------
>>> Degree = 52 | FPE = 0.0108
-------------------------------------------------------------
>>> Degree = 53 | FPE = 0.0304
-------------------------------------------------------------
>>> Degree = 54 | FPE = 0.0094
-------------------------------------------------------------
>>> Degree = 55 | FPE = 0.0087
-------------------------------------------------------------
>>> Degree = 56 | FPE = 0.0462
-------------------------------------------------------------
>>> Degree = 57 | FPE = 0.0095
-------------------------------------------------------------
>>> Degree = 58 | FPE = 0.0361
-------------------------------------------------------------
>>> Degree = 59 | FPE = 0.0182
-------------------------------------------------------------
>>> Degree = 60 | FPE = 0.0091
-------------------------------------------------------------
>>> Degree = 61 | FPE = 0.6074
-------------------------------------------------------------
>>> Degree = 62 | FPE = 0.0188
-------------------------------------------------------------
>>> Degree = 63 | FPE = 0.0133
-------------------------------------------------------------
>>> Degree = 64 | FPE = 0.0086
-------------------------------------------------------------
>>> Degree = 65 | FPE = 0.0222
-------------------------------------------------------------
>>> Degree = 66 | FPE = 0.0154
-------------------------------------------------------------
```

```
%%
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("*************System Degree Ident. by FPE Method****************\n")
```

```
*************System Degree Ident. by FPE Method****************
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
```

```
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n", minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 2
```

```
na = minFPEIndex;
nb = minFPEIndex;
nc = minFPEIndex;
nd = minFPEIndex;
nk = 1;
p = na+nb+nc+nd;

bj_FPEModel_prbs = bj(data_prbs, [na nb nc nd nk])
```

```
bj_FPEModel_prbs =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 0.399 z^-1 - 0.2035 z^-2

  C(z) = 1 - 0.04707 z^-1 - 0.1139 z^-2

  D(z) = 1 - 0.2156 z^-1 - 0.1815 z^-2

  F(z) = 1 - 1.179 z^-1 + 0.449 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using BJ on time domain data "data_prbs".
Fit to estimation data: 76.67% (prediction focus)
FPE: 0.01456, MSE: 0.0145
```

```
FPE_y_hat_prbs = lsim(bj_FPEModel_prbs, u2_val, t_val);
[FPE_r2, FPE_mse] = rSQR(y2_val, FPE_y_hat_prbs);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9407
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf("****************************************************************\n")
```

****

```
fprintf("***************************************************************\n")
```

****

Q3 - part b | FPE Output Error

```
clc; clear;

%%

load q3_402123100.mat

u1_val = u2;
y1_val = y2;

u2_val = u1;
y2_val = y1;
```

Guassian Input - PRBS Validation

```
%%
% Guassian Input ***********************************************************
fprintf("*****************************************************************\n")
fprintf(">>> Guassian Input Identification Begins:----------------------------
-\n")


%%

Ts = 0.1;
t = 0:Ts:length(u1)*Ts-Ts;
t_val = 0:Ts:length(u1_val)*Ts-Ts;
N = length(y1);
N_val = length(y1_val);

data_guassian = iddata(y1,u1,Ts);

%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    try
        sys = oe(data_guassian, [na nb nk]);
        oe_y_hat_guassian = lsim(sys, u1, t);
    catch
```

```
        break
    end

    error = y1 - oe_y_hat_guassian;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("-------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];

end
```

```
>>> Degree = 1 | FPE = 0.0134
-------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 7 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0080
-------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0080
```

```
------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 21 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 28 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0083
------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0084
------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 37 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0080
------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0082
------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0083
------------------------------------------------------------
>>> Degree = 44 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 45 | FPE = 0.0082
------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0082
------------------------------------------------------------
>>> Degree = 47 | FPE = 0.0081
------------------------------------------------------------
```

```
>>> Degree = 48 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 49 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 50 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 51 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 52 | FPE = 0.0090
-------------------------------------------------------------
>>> Degree = 53 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 54 | FPE = 0.1012
-------------------------------------------------------------
>>> Degree = 55 | FPE = 0.0085
-------------------------------------------------------------
>>> Degree = 56 | FPE = 0.0082
-------------------------------------------------------------
>>> Degree = 57 | FPE = 0.1906
-------------------------------------------------------------
>>> Degree = 58 | FPE = 0.0087
-------------------------------------------------------------
>>> Degree = 59 | FPE = 0.0091
-------------------------------------------------------------
>>> Degree = 60 | FPE = 0.1024
-------------------------------------------------------------
```

```
%%
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("*************System Degree Ident. by FPE Method***************\n")
```

```
*************System Degree Ident. by FPE Method***************
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n",
minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 2 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 2
```

```
na = minFPEIndex;
nb = minFPEIndex;
nk = 1;
p = na+nb;

oe_FPEModel_guassian = oe(data_guassian, [na nb nk])
```

```
oe_FPEModel_guassian =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = 0.3947 z^-1 - 0.1803 z^-2

  F(z) = 1 - 1.136 z^-1 + 0.43 z^-2

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=2   nf=2   nk=1
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_guassian".
Fit to estimation data: 75.71%
FPE: 0.01591, MSE: 0.01588
```

```
FPE_y_hat_guassian = lsim(oe_FPEModel_guassian, u1_val, t_val);

[FPE_r2, FPE_mse] = rSQR(y1_val, FPE_y_hat_guassian);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9430
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```
fprintf("****************************************************************\n")
```

```
****************************************************************
```

```
fprintf("****************************************************************\n")
```

```
****************************************************************
```

PRBS Input - Guassian Validation

```
%%

% PRBS Input ****************************************************************
```

```
fprintf("*****************************************************************\n")
```

```
*****************************************************************
```

```
fprintf(">>> PRBS Input Identification Begins:------------------------------
\n")
```

```
>>> PRBS Input Identification Begins:------------------------------
```

```
%%

Ts = 0.1;
t = 0:Ts:length(u2)*Ts-Ts;
t_val = 0:Ts:length(u2_val)*Ts-Ts;
N = length(y2);
N_val = length(y2_val);

data_prbs = iddata(y2,u2,Ts);
%%
FPEs = [];

for degree=1:100
    na = degree;
    nb = degree;
    nk = 1;
    p = na+nb;

    try
        sys = oe(data_prbs, [na nb nk]);
        oe_y_hat_prbs = lsim(sys, u2, t);
    catch
        break
    end

    error = y2 - oe_y_hat_prbs;
    S_hat = 0;
    for i=1:length(error)
        S_hat = S_hat + error(i)^2;
    end

    FPE = FPE_criteria(S_hat, p, N);
    fprintf(">>> Degree = %d | FPE = %.4f \n", degree, FPE)
    fprintf("-------------------------------------------------------------\n")
    FPEs = [FPEs; FPE];
```

```
end
```

```
>>> Degree = 1 | FPE = 0.0130
-------------------------------------------------------------
>>> Degree = 2 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 3 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 4 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 5 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 6 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 7 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 8 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 9 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 10 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 11 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 12 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 13 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 14 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 15 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 16 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 17 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 18 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 19 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 20 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 21 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 22 | FPE = 0.0096
-------------------------------------------------------------
>>> Degree = 23 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 24 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 25 | FPE = 0.0076
-------------------------------------------------------------
>>> Degree = 26 | FPE = 0.0079
-------------------------------------------------------------
>>> Degree = 27 | FPE = 0.0410
-------------------------------------------------------------
```

```
>>> Degree = 28 | FPE = 0.0084
------------------------------------------------------------
>>> Degree = 29 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 30 | FPE = 0.0076
------------------------------------------------------------
>>> Degree = 31 | FPE = 0.0076
------------------------------------------------------------
>>> Degree = 32 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 33 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 34 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 35 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 36 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 37 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 38 | FPE = 0.0078
------------------------------------------------------------
>>> Degree = 39 | FPE = 0.0076
------------------------------------------------------------
>>> Degree = 40 | FPE = 0.0312
------------------------------------------------------------
>>> Degree = 41 | FPE = 0.0076
------------------------------------------------------------
>>> Degree = 42 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 43 | FPE = 0.0076
------------------------------------------------------------
>>> Degree = 44 | FPE = 0.0078
------------------------------------------------------------
>>> Degree = 45 | FPE = 0.0078
------------------------------------------------------------
>>> Degree = 46 | FPE = 0.0164
------------------------------------------------------------
>>> Degree = 47 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 48 | FPE = 0.0093
------------------------------------------------------------
>>> Degree = 49 | FPE = 0.0077
------------------------------------------------------------
>>> Degree = 50 | FPE = 0.0079
------------------------------------------------------------
>>> Degree = 51 | FPE = 0.1430
------------------------------------------------------------
>>> Degree = 52 | FPE = 0.0082
------------------------------------------------------------
>>> Degree = 53 | FPE = 0.0164
------------------------------------------------------------
>>> Degree = 54 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 55 | FPE = 0.0081
------------------------------------------------------------
>>> Degree = 56 | FPE = 0.0252
```

```
-------------------------------------------------------------
>>> Degree = 57 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 58 | FPE = 0.0081
-------------------------------------------------------------
>>> Degree = 59 | FPE = 0.0130
-------------------------------------------------------------
>>> Degree = 60 | FPE = 0.0154
-------------------------------------------------------------
>>> Degree = 61 | FPE = 0.0078
-------------------------------------------------------------
>>> Degree = 62 | FPE = 0.0092
-------------------------------------------------------------
>>> Degree = 63 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 64 | FPE = 0.0077
-------------------------------------------------------------
>>> Degree = 65 | FPE = 0.0086
-------------------------------------------------------------
>>> Degree = 66 | FPE = 0.0090
-------------------------------------------------------------
```

```
%%
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("*************System Degree Ident. by FPE Method***************\n")
```

```
*************System Degree Ident. by FPE Method***************
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
minFPEIndex = find(FPEs == min(FPEs));
fprintf(">>> Since the minimum FPE value occurs in iteration %d ;\n", minFPEIndex)
```

```
>>> Since the minimum FPE value occurs in iteration 9 ;
```

```
fprintf("    Degree = %d \n", minFPEIndex)
```

```
    Degree = 9
```

```
na = minFPEIndex;
nb = minFPEIndex;
nk = 1;
p = na+nb;
```

```
oe_FPEModel_prbs = oe(data_prbs, [na nb nk])
```

```
oe_FPEModel_prbs =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)

  B(z) = 0.3996 z^-1 + 0.1145 z^-2 + 0.008003 z^-3 - 0.122 z^-4 - 0.2037 z^-5 - 0.2375 z^-6 -
0.13 z^-7
                                                                          + 0.2217 z^-8 -
0.04925 z^-9


  F(z) = 1 - 0.379 z^-1 - 0.07091 z^-2 - 0.2344 z^-3 - 0.2504 z^-4 - 0.2725 z^-5 + 0.02085 z^
                                                         -6 + 0.6719 z^-7 - 0.5721 z^-8 +
0.08866 z^-9

Sample time: 0.1 seconds

Parameterization:
   Polynomial orders:   nb=9   nf=9   nk=1
   Number of free coefficients: 18
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using OE on time domain data "data_prbs".
Fit to estimation data: 76.27%
FPE: 0.01514, MSE: 0.01501
```

```
FPE_y_hat_prbs = lsim(oe_FPEModel_prbs, u2_val, t_val);

[FPE_r2, FPE_mse] = rSQR(y2_val, FPE_y_hat_prbs);
fprintf("    R2 = %.4f \n", FPE_r2)
```

```
    R2 = 0.9405
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

Q3 - part c | Delay Input-Output CrossCorrelation

```
clc; clear;

%%

load q3_402123100.mat

N = length(u1);

%%

fprintf("****************************************************************\n")
```

```
****************************************************************
```

```
fprintf("******System Delay Ident. by Input-Output CrossCorrelation*****\n")
```

```
******System Delay Ident. by Input-Output CrossCorrelation*****
```

```
fprintf("----------------------------------------------------------------\n")
```

```
----------------------------------------------------------------
```

```

Ruy1 = [];
Ruy2 = [];
for k=1:N
    Ruy1(k) = CrossCorrelate(u1,y1,k);
    Ruy2(k) = CrossCorrelate(u2,y2,k);
end

queue1 = Ruy1(1:50);
queue2 = Ruy2(1:50);

delay_by_Ruy1 = find(queue1==queue1(find(abs(diff(queue1)) ==
max(abs(diff(queue1))))))+1;
delay_by_Ruy2 = find(queue2==queue2(find(abs(diff(queue2)) ==
max(abs(diff(queue2))))))+1;

fprintf(">>> The system delay based on y1 data : %d \n", delay_by_Ruy1);
```

```
>>> The system delay based on y1 data : 3
```

```
fprintf(">>> The system delay based on y2 data : %d \n", delay_by_Ruy2);
```

```
>>> The system delay based on y2 data : 2
```

```
fprintf("---------------------------------------------------------------\n")
```

```
---------------------------------------------------------------
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```

```
fprintf("***************************************************************\n")
```

```
***************************************************************
```