

Artificial Intelligence — Final Test

February 7th, 2025

1 Propositional Logic

Given the following formulas in propositional logic

- $\varphi_1: p \rightarrow (q \vee r)$
- $\varphi_2: (q \rightarrow (s \wedge t))$
- $\varphi_3: (r \rightarrow (s \wedge t))$
- $\varphi_4: (p \vee r) \wedge s$

show whether the formula $s \wedge t$ is a logical consequence of the theory $\Phi = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$. State your answer as a proof using either a deduction mechanism of your choice or a semantic argument. Truth-tables are not accepted as an answer.

2 First Order Logic

Consider the following theory in first order logic

1. $\forall x. \text{likes}(x, x)$
2. $\forall x. \exists y. \text{likes}(x, y)$
3. $\forall x. \forall y. (\exists z. \text{likes}(x, z) \wedge \text{likes}(z, y)) \rightarrow \text{likes}(x, y).$

and tell whether the following sentences are a logical consequence of the theory

1. $\exists x. \forall y. \text{likes}(x, y)$
2. $\exists y. \forall x. \text{likes}(x, y)$
3. $\forall x. \forall y. \text{likes}(x, y) \rightarrow \text{likes}(y, x)$
4. $\forall x. \forall y. \text{likes}(x, y) \wedge \text{likes}(y, x)$
5. $\exists x. \forall y. \neg \text{likes}(x, y)$

3 Description Logic

Show that it is possible to formalize the theory of exercise two as a TBox of a knowledge base Σ using \mathcal{ALC} . Then show the following:

1. All the sentences that are logical consequence of the theory in exercise two can be added to the TBox of Σ without making it inconsistent.
2. An ABox which is consistent with the TBox.
3. An ABox which is inconsistent with the TBox.