MCM Cheatsheet

Great Minds of Genova

January 2024

Figure 1: My attitude when I need to study

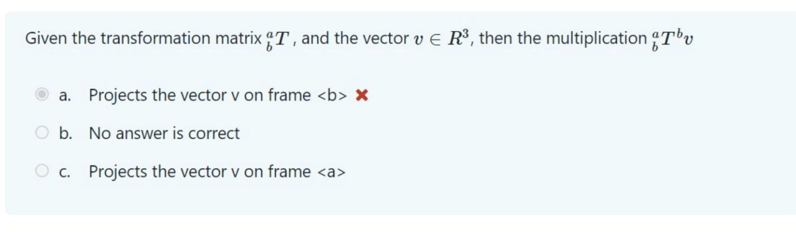
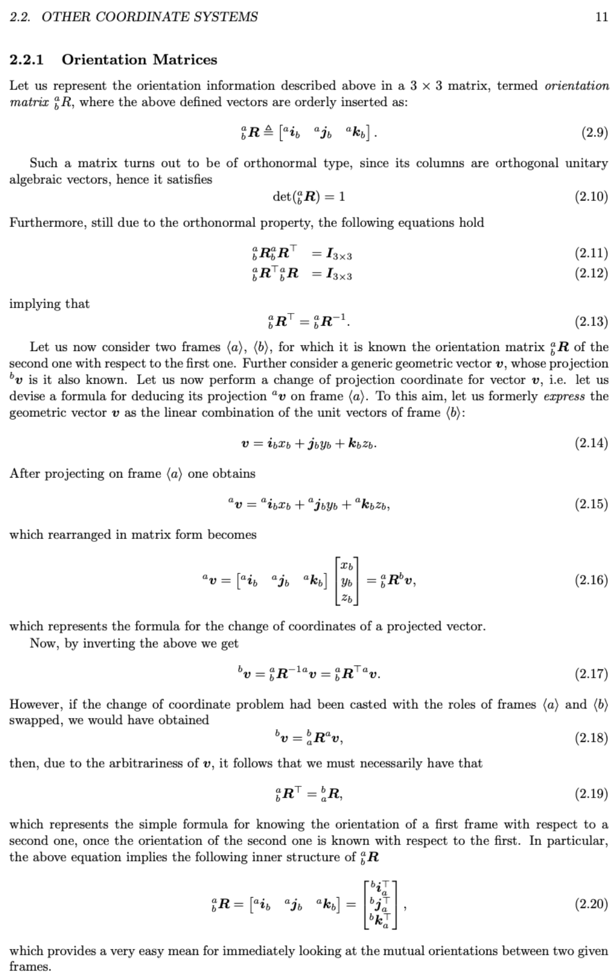


Figure 2: Enter Caption

Given the transformation ${}^a_bT$ and vector $v \in R^3$ then the multiplication ${}^a_bT^b v$ Answer: No answer is correct

Why? Simply, the v vector is a **3 x 1** and Matrix T is the **transformation matrix** meaning it's size is **4 x 4** From what I know such operation is incorrect.

However if we are dealing with 3 x 3 Orientation matrix this would project the vector onto frame $< a >$., This page from mcm notes provides nice explanations about it:



2.2. OTHER COORDINATE SYSTEMS 11

2.2.1 Orientation Matrices

Let us represent the orientation information described above in a 3×3 matrix, termed *orientation matrix* ${}^a_bR$, where the above defined vectors are orderly inserted as:

$$ {}^a_b\boldsymbol{R} \triangleq \begin{bmatrix} {}^a\boldsymbol{i}_b & {}^a\boldsymbol{j}_b & {}^a\boldsymbol{k}_b \end{bmatrix}. \tag{2.9}$$

Such a matrix turns out to be of orthonormal type, since its columns are orthogonal unitary algebraic vectors, hence it satisfies

$$ \det({}^a_b\boldsymbol{R}) = 1 \tag{2.10}$$

Furthermore, still due to the orthonormal property, the following equations hold

$$ {}^a_b\boldsymbol{R}\,{}^a_b\boldsymbol{R}^\top = \boldsymbol{I}_{3\times3} \tag{2.11}$$

$$ {}^a_b\boldsymbol{R}^\top\,{}^a_b\boldsymbol{R} = \boldsymbol{I}_{3\times3} \tag{2.12}$$

implying that

$$ {}^a_b\boldsymbol{R}^\top = {}^a_b\boldsymbol{R}^{-1}. \tag{2.13}$$

Let us now consider two frames $\langle a \rangle$, $\langle b \rangle$, for which it is known the orientation matrix ${}^a_b\boldsymbol{R}$ of the second one with respect to the first one. Further consider a generic geometric vector $\boldsymbol{v}$, whose projection ${}^b\boldsymbol{v}$ is it also known. Let us now perform a change of projection coordinate for vector $\boldsymbol{v}$, i.e. let us devise a formula for deducing its projection ${}^a\boldsymbol{v}$ on frame $\langle a \rangle$. To this aim, let us formerly *express* the geometric vector $\boldsymbol{v}$ as the linear combination of the unit vectors of frame $\langle b \rangle$:

$$ \boldsymbol{v} = \boldsymbol{i}_b x_b + \boldsymbol{j}_b y_b + \boldsymbol{k}_b z_b. \tag{2.14}$$

After projecting on frame $\langle a \rangle$ one obtains

$$ {}^a\boldsymbol{v} = {}^a\boldsymbol{i}_b x_b + {}^a\boldsymbol{j}_b y_b + {}^a\boldsymbol{k}_b z_b, \tag{2.15}$$

which rearranged in matrix form becomes

$$ {}^a\boldsymbol{v} = \begin{bmatrix} {}^a\boldsymbol{i}_b & {}^a\boldsymbol{j}_b & {}^a\boldsymbol{k}_b \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = {}^a_b\boldsymbol{R}\,{}^b\boldsymbol{v}, \tag{2.16}$$

which represents the formula for the change of coordinates of a projected vector.

Now, by inverting the above we get

$$ {}^b\boldsymbol{v} = {}^a_b\boldsymbol{R}^{-1}\,{}^a\boldsymbol{v} = {}^a_b\boldsymbol{R}^\top\,{}^a\boldsymbol{v}. \tag{2.17}$$

However, if the change of coordinate problem had been casted with the roles of frames $\langle a \rangle$ and $\langle b \rangle$ swapped, we would have obtained

$$ {}^b\boldsymbol{v} = {}^b_a\boldsymbol{R}\,{}^a\boldsymbol{v}, \tag{2.18}$$

then, due to the arbitrariness of $\boldsymbol{v}$, it follows that we must necessarily have that

$$ {}^a_b\boldsymbol{R}^\top = {}^b_a\boldsymbol{R}, \tag{2.19}$$

which represents the simple formula for knowing the orientation of a first frame with respect to a second one, once the orientation of the second one is known with respect to the first. In particular, the above equation implies the following inner structure of ${}^a_b\boldsymbol{R}$

$$ {}^a_b\boldsymbol{R} = \begin{bmatrix} {}^a\boldsymbol{i}_b & {}^a\boldsymbol{j}_b & {}^a\boldsymbol{k}_b \end{bmatrix} = \begin{bmatrix} {}^b\boldsymbol{i}_a^\top \\ {}^b\boldsymbol{j}_a^\top \\ {}^b\boldsymbol{k}_a^\top \end{bmatrix}, \tag{2.20}$$

which provides a very easy mean for immediately looking at the mutual orientations between two given frames.

Figure 3: Enter Caption

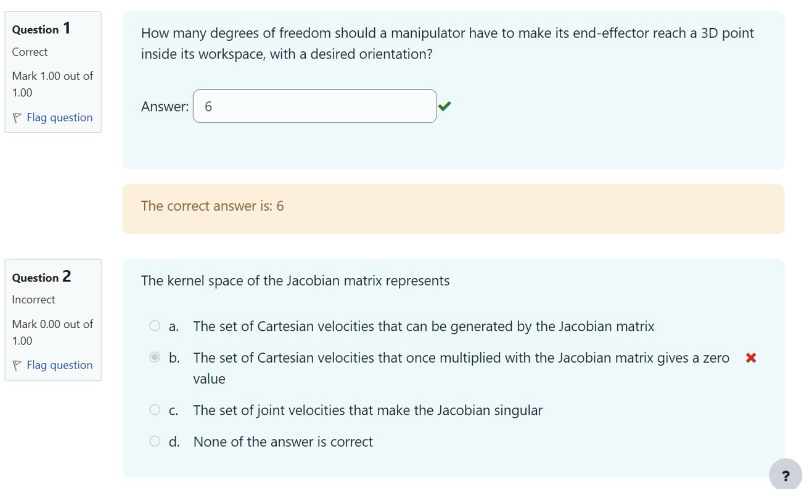


Figure 4: Enter Caption

2. **all of the answers are incorrect**

posture.

- The *null* space of $\boldsymbol{J}$ is the subspace $\mathcal{N}(\boldsymbol{J})$ in $\mathbb{R}^n$ of joint velocities that do not produce any end-effector velocity, in the given manipulator posture.

Figure 5: Enter Caption

a) Singularities represent configurations at which mobility of the structure is reduced, i.e., it is not possible to impose an arbitrary motion to the end-effector.
b) When the structure is at a singularity, infinite solutions to the inverse kinematics problem may exist.
c) In the neighbourhood of a singularity, small velocities in the operational space may cause large velocities in the joint space.

Figure 6: Enter Caption

In the distance zeroing task where it is necessary to track a goal frame <g>, the contribution $v_{g/0}$

- a. it represents a feedforward term, which allows tracking of the goal frame even when the distance is zero ✔
- b. it represents the velocity of the goal, hence it is not needed in the control loop
- c. it represents the velocity of the goal, and its use is not mandatory in the control loop
- d. it represents a feedforward term, which allows reducing the distance to the goal frame

Your answer is correct.

The correct answer is:
it represents a feedforward term, which allows tracking of the goal frame even when the distance is zero

Figure 7: Enter Caption

From this picture, we can see that $v_{g/0}$ is a feedforward term, of course this represents the veocity of the goal. $v_{g/0}$ is needed in control loop and its use is not mandatory only when $v_{g/0} = 0$. This doesn't reduce the distance to the goal frame.

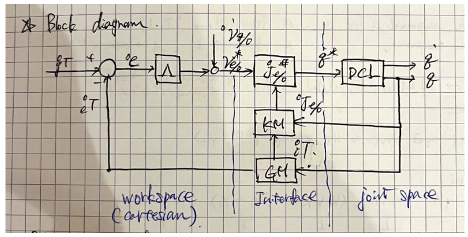


Figure 8: Enter Caption

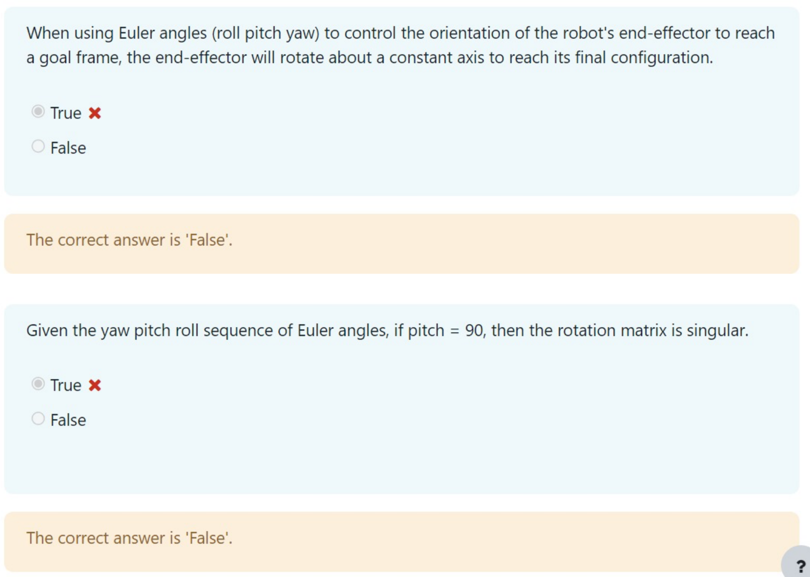


Figure 9: Enter Caption

1. False - I believe it is because the end effector will rotate about an axis with respect to certain frame, it **does not need to be constant**. 1. We need three axis (z, y', x"). The y' means the y-axis after rotating the object around z-axis.

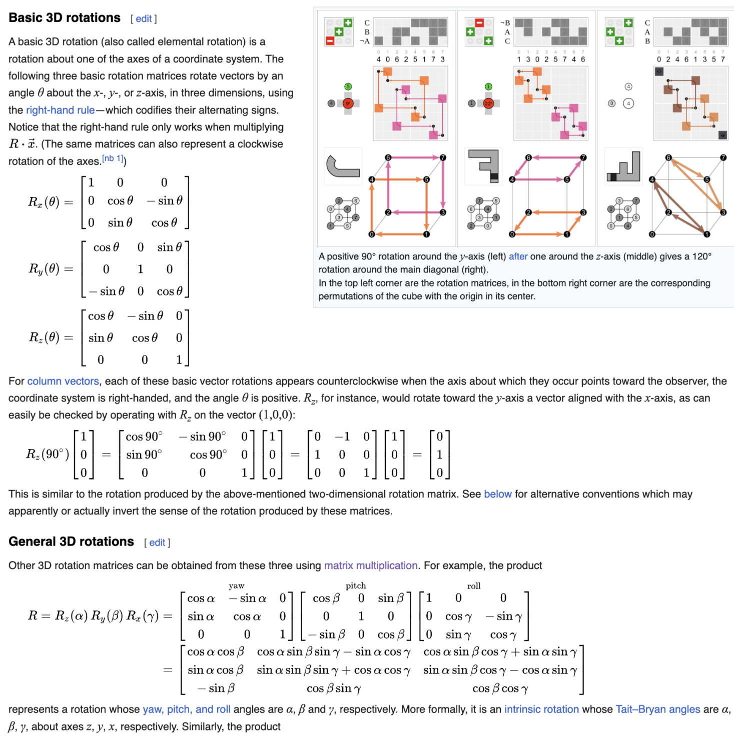
2. False, Why? **yaw - x pitch - y roll - z**

$$R_y(\frac{\pi}{2}) = cos(\frac{\pi}{2})0sin(\frac{\pi}{2})010 - sin(\frac{\pi}{2})0cos(\frac{\pi}{2}) = 001010 - 100 \tag{1}$$

and then singular means that $R = 0$ which is not the case here. Also the rule is that $det(R) = 0$

In this situation, this occurs in gimbal lock so this is singular if this is represented as yaw pitch roll form, but rotation matrix doesn't have any singularity itself. Neither does quaternion.

Some notes



Basic 3D rotations [edit]

A basic 3D rotation (also called elemental rotation) is a rotation about one of the axes of a coordinate system. The following three basic rotation matrices rotate vectors by an angle θ about the x-, y-, or z-axis, in three dimensions, using the right-hand rule—which codifies their alternating signs. Notice that the right-hand rule only works when multiplying $R \cdot \vec{x}$. (The same matrices can also represent a clockwise rotation of the axes.[nb 1])

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A positive 90° rotation around the y-axis (left) after one around the z-axis (middle) gives a 120° rotation around the main diagonal (right).
In the top left corner are the rotation matrices, in the bottom right corner are the corresponding permutations of the cube with the origin in its center.

For column vectors, each of these basic vector rotations appears counterclockwise when the axis about which they occur points toward the observer, the coordinate system is right-handed, and the angle θ is positive. R_z, for instance, would rotate toward the y-axis a vector aligned with the x-axis, as can easily be checked by operating with R_z on the vector $(1,0,0)$:

$$R_z(90^\circ)\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

This is similar to the rotation produced by the above-mentioned two-dimensional rotation matrix. See below for alternative conventions which may apparently or actually invert the sense of the rotation produced by these matrices.

General 3D rotations [edit]

Other 3D rotation matrices can be obtained from these three using matrix multiplication. For example, the product

$$\begin{aligned} R = R_z(\alpha)\,R_y(\beta)\,R_x(\gamma) &= \overbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\text{yaw}} \overbrace{\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}}^{\text{pitch}} \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}}^{\text{roll}} \\ &= \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix} \end{aligned}$$

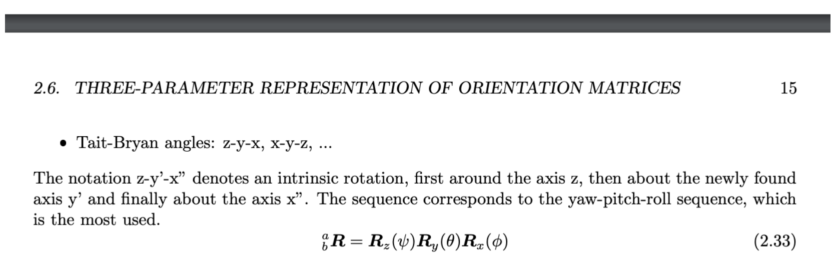
represents a rotation whose yaw, pitch, and roll angles are α, β and γ, respectively. More formally, it is an intrinsic rotation whose Tait–Bryan angles are α, β, γ, about axes z, y, x, respectively. Similarly, the product

Figure 10: Enter Caption

2.6.1 Euler angles

Any orientation can be achieved by composing three elementary rotations in sequence. These can occur either about the axes of a fixed coordinate system (extrinsic rotations), or about the axes of a rotating coordinate system (intrinsic rotations) initially aligned with the fixed one. Usually the rotating frame is assumed attached to a rigid body. We have the followinig distinction:

- Proper Euler angles: x-z-x, y-z-y, ...



2.6. THREE-PARAMETER REPRESENTATION OF ORIENTATION MATRICES 15

- Tait-Bryan angles: z-y-x, x-y-z, ...

The notation z-y'-x" denotes an intrinsic rotation, first around the axis z, then about the newly found axis y' and finally about the axis x". The sequence corresponds to the yaw-pitch-roll sequence, which is the most used.

$$^{a}_{b}\boldsymbol{R} = \boldsymbol{R}_z(\psi)\boldsymbol{R}_y(\theta)\boldsymbol{R}_x(\phi) \tag{2.33}$$

Figure 11: Enter Caption

Given a desired Cartesian velocity $\dot{\bar{x}}$, the pseudo-inverse ${}^0J^{\#}_{e/0}$ allows to

- a. find the joint velocity vector $\dot{q}$ that makes the actual Cartesian velocity $\dot{x} = \dot{\bar{x}}$
- b. find the joint velocity vector $\dot{q}$ that makes the actual Cartesian velocity $\dot{x}$ approximate at best, in the least square sense, the desired reference velocity.
- c. find the joint velocity vector $\dot{q}$ that makes the actual Cartesian velocity $\dot{x} = \dot{\bar{x}}$ if the Jacobian is not singular, otherwise it returns zero.

Your answer is partially correct.

The correct answer is:
find the joint velocity vector $\dot{q}$ that makes the actual Cartesian velocity $\dot{x}$ approximate at best, in the least square sense, the desired reference velocity.

Figure 12: Enter Caption

All of the answers are partially correct, but **B** is the most appropiate one, because we are talking here about pseudo-inverse Jacobian which is the already an approximation.

An angular velocity vector ω_a^b is calculated as follows:

$$^{a}\omega_{b/a} =^{a}_{b} \dot{R}^{a}_{b} R^T$$

I-th Column of the Jacobian matrix ${}^1J_{e/0}$ represents the set of velocities, both linear and angular. This will be the velocity of frame $< e >$ with respect to $< 0 >$ $\rightarrow$ thats why $< e/0 >$, but we project in onto the frame $< 1 >$

The vector ${}^av = {}^ai_bx_b + {}^aj_by_b + {}^ak_bz_b$ represents:

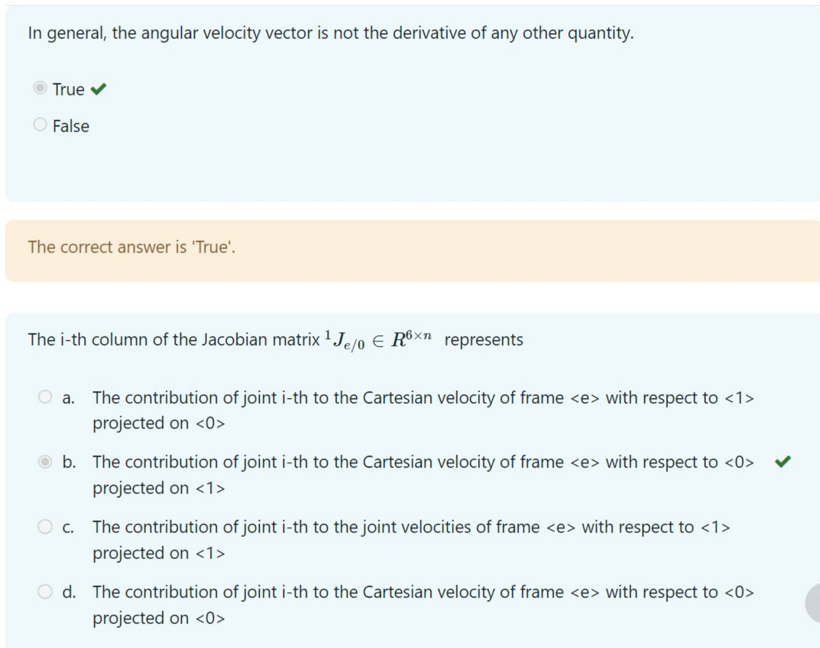
- a. The algebraic vector v projected on <b>
- b. The geometric vector v projected on <a>
- c. The geometric vector v projected on <b>
- d. The algebraic vector v projected on <a>

Your answer is incorrect.

The correct answer is:
The algebraic vector v projected on <a>

Figure 14: Enter Caption

Answer: **D** - The algebraic vector v projected on $< a >$



In general, the angular velocity vector is not the derivative of any other quantity.

- True ✔
- False

The correct answer is 'True'.

The i-th column of the Jacobian matrix ${}^{1}J_{e/0} \in R^{6\times n}$ represents

a. The contribution of joint i-th to the Cartesian velocity of frame <e> with respect to <1> projected on <0>

b. The contribution of joint i-th to the Cartesian velocity of frame <e> with respect to <0> projected on <1> ✔

c. The contribution of joint i-th to the joint velocities of frame <e> with respect to <1> projected on <1>

d. The contribution of joint i-th to the Cartesian velocity of frame <e> with respect to <0> projected on <0>

Figure 13: Enter Caption

Why? Someone else could explain it better, but the idea is that when you have

$$ {}^{a}v \tag{2} $$

means that $< a >$ is the observer, or idk the frame that our vector V is projected on, but if we look closesly we have some more information

$$ {}^{a}v =^{a} i_b x_b +^{a} j_b y_b +^{a} k_b z_b \tag{3} $$

Tells us that this vector has reference in frame $< b >$

$$ {}^{a}v_c^b \tag{4} $$

This means velocity of b, WRT c. And the a represents the orientation of the observer (c).

The equation below is the equivalent. And i,j,k is the rotational matrix, and x,y,z is to scale it up from unit vectors to the original size (The vector with respect to its own frame).

$$ {}^{a}v = [i_b, j_b, k_b][x_b, y_b, z_b]^T \tag{5} $$

2.2.1 Orientation Matrices

Let us represent the orientation information described above in a 3×3 matrix, termed *orientation matrix* ${}^a_bR$, where the above defined vectors are orderly inserted as:

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Such a matrix turns out to be of orthonormal type, since its columns are orthogonal unitary algebraic vectors, hence it satisfies

$$ \det({}^a_b\boldsymbol{R}) = 1 \tag{2.10} $$

Furthermore, still due to the orthonormal property, the following equations hold

$$ {}^a_b\boldsymbol{R}{}^a_b\boldsymbol{R}^\top = \boldsymbol{I}_{3\times3} \tag{2.11} $$
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implying that

$$ {}^a_b\boldsymbol{R}^\top = {}^a_b\boldsymbol{R}^{-1}. \tag{2.13} $$

Let us now consider two frames $\langle a \rangle$, $\langle b \rangle$, for which it is known the orientation matrix ${}^a_b\boldsymbol{R}$ of the second one with respect to the first one. Further consider a generic geometric vector $\boldsymbol{v}$, whose projection ${}^b\boldsymbol{v}$ is it also known. Let us now perform a change of projection coordinate for vector $\boldsymbol{v}$, i.e. let us devise a formula for deducing its projection ${}^a\boldsymbol{v}$ on frame $\langle a \rangle$. To this aim, let us formerly *express* the geometric vector $\boldsymbol{v}$ as the linear combination of the unit vectors of frame $\langle b \rangle$:

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After projecting on frame $\langle a \rangle$ one obtains

$$ {}^a\boldsymbol{v} = {}^a\boldsymbol{i}_b x_b + {}^a\boldsymbol{j}_b y_b + {}^a\boldsymbol{k}_b z_b, \tag{2.15} $$

which rearranged in matrix form becomes

$$ {}^a\boldsymbol{v} = \begin{bmatrix} {}^a\boldsymbol{i}_b & {}^a\boldsymbol{j}_b & {}^a\boldsymbol{k}_b \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = {}^a_b\boldsymbol{R}{}^b\boldsymbol{v}, \tag{2.16} $$

Figure 16: Enter Caption

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Question 3

Correct

Mark 1.00 out of 1.00

A Jacobian is singular if

- a. All the singular values are zero
- b. The determinant is zero and at least one singular value is zero ✔
- c. The determinant is zero and only one singular value is zero
- d. One singular value is zero but the determinant can be different from zero

Your answer is correct.

The correct answer is:
The determinant is zero and at least one singular value is zero

Figure 17: Enter Caption

$$ \boldsymbol{v} = \boldsymbol{i}_b x_b + \boldsymbol{j}_b y_b + \boldsymbol{k}_b z_b. \tag{2.14} $$

After projecting on frame $\langle a \rangle$ one obtains

$$ {}^a\boldsymbol{v} = {}^a\boldsymbol{i}_b x_b + {}^a\boldsymbol{j}_b y_b + {}^a\boldsymbol{k}_b z_b, \tag{2.15} $$

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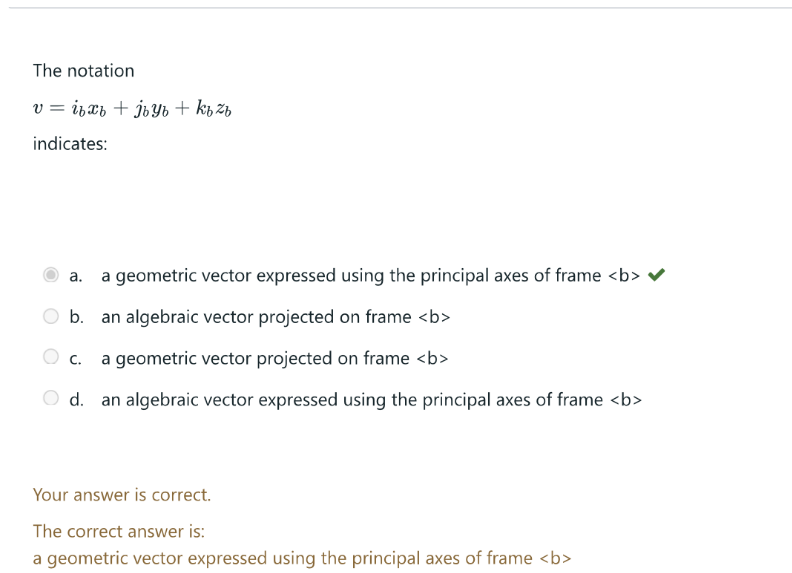
which represents the formula for the change of coordinates of a projected vector.

Figure 15:

More notes:

Question: In the distance zeroing problem, if one considers the derivative of the norm of the distance, then ...



The notation

$v = i_b x_b + j_b y_b + k_b z_b$

indicates:

a. a geometric vector expressed using the principal axes of frame <b> ✔

b. an algebraic vector projected on frame <b>

c. a geometric vector projected on frame <b>

d. an algebraic vector expressed using the principal axes of frame <b>

Your answer is correct.

The correct answer is:
a geometric vector expressed using the principal axes of frame <b>

Figure 18: Enter Caption

Correct answers:

1. The end-effector reaches the goal position reducing the norm to zero exponentially

2. The end-effector velocity might include components orthogonal to the current distance vector

Why? -

Given the transformation ${}^a_bT$ and vector $v \in R^3$ then the multiplication ${}^a_bT^bv$ Answer: No answer is correct

Why? Simply, the v vector is a **3 x 1** and Matrix T is the **transformation matrix** meaning it's size is **4 x 4** From what I know such operation is incorrect.

However if we are dealing with 3 x 3 Orientation matrix this would project the vector onto frame ¡a¿., This page from mcm notes provides nice explanations about it:

Question: In the distance zeroing problem, if one considers the derivative of the norm of the distance, then ...
Correct answer: The end-effector reaches the goal position reducing the norm to zero exponentially. The end-effector velocity might include components orthogonal to the current distance vector

Why? - Alternative solution(2nd) to the distance zeroing problem is to make the norm of the distance vector r to go to zero. The 2nd control law shows

Question 7

Correct

Mark 1.00 out of 1.00

The i-th column of the generalized jacobian matrix $\dot{x} = J(q)\dot{q}$ with $\dot{x} = [\omega, v]^\top$ is

- a. $\begin{bmatrix} k_i \\ k_i \end{bmatrix}$ if the joint is rotational, and $\begin{bmatrix} 0 \\ k_i \end{bmatrix}$ is the joint is a prismatic one.
- b. $\begin{bmatrix} k_i \times r_{e/i} \\ k_i \end{bmatrix}$ if the joint is rotational, and $\begin{bmatrix} 0 \\ k_i \end{bmatrix}$ is the joint is a prismatic one.
- c. $\begin{bmatrix} k_i \\ k_i \times r_{e/i} \end{bmatrix}$ if the joint is rotational, and $\begin{bmatrix} 0 \\ k_i \end{bmatrix}$ is the joint is a prismatic one. ✔
- d. $\begin{bmatrix} k_i \\ k_i \times r_{e/i} \end{bmatrix}$ if the joint is rotational, and $\begin{bmatrix} k_i \\ 0 \end{bmatrix}$ is the joint is a prismatic one.
- e. $\begin{bmatrix} 0 \\ k_i \end{bmatrix}$ if the joint is rotational, and $\begin{bmatrix} k_i \\ 0 \end{bmatrix}$ is the joint is a prismatic one.

Your answer is correct.

The correct answer is:
$\begin{bmatrix} k_i \\ k_i \times r_{e/i} \end{bmatrix}$ if the joint is rotational, and $\begin{bmatrix} 0 \\ k_i \end{bmatrix}$ is the joint is a prismatic one.

Figure 19: Enter Caption

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Question 8

Partially correct

Mark 0.25 out of 1.00

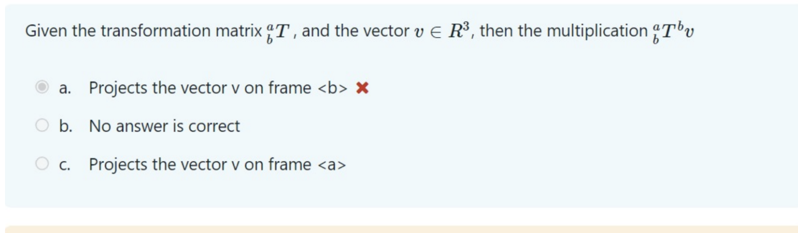
In the distance zeroing problem, if one considers the derivative of the norm of the distance, then

- a. The end-effector always reaches the goal position by following a straight path.
- b. The end-effector reaches the goal position reducing the norm to zero exponentially. The end-effector velocity might include components orthogonal to the current distance vector.
- c. The end-effector always reaches the goal position by following a curvilinear path.
- d. The end-effector reaches the goal position reducing the norm to zero exponentially. The end-effector velocity necessarily includes components orthogonal to the current distance vector. ☑

Your answer is partially correct.

The correct answer is:
The end-effector reaches the goal position reducing the norm to zero exponentially. The end-effector velocity might include components orthogonal to the current distance vector.

Figure 20: Enter Caption


Given the transformation matrix ${}^a_bT$, and the vector $v \in R^3$, then the multiplication ${}^a_bT{}^bv$

a. Projects the vector v on frame <b>

b. No answer is correct

c. Projects the vector v on frame <a>

Figure 21: Enter Caption

that the distance vector r(with purple on the figure) is rotating and therefore it has components orthogonal to the distance which will not change the vector in length but will make it rotate. The first solution is a 3D computation while the second one is a one dimensional computation.

2.2.1 Orientation Matrices

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$$ {}^a\boldsymbol{v} = {}^a\boldsymbol{i}_b x_b + {}^a\boldsymbol{j}_b y_b + {}^a\boldsymbol{k}_b z_b, \tag{2.15}$$

which rearranged in matrix form becomes

$$ {}^a\boldsymbol{v} = \begin{bmatrix} {}^a\boldsymbol{i}_b & {}^a\boldsymbol{j}_b & {}^a\boldsymbol{k}_b \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = {}^a_b\boldsymbol{R}\,{}^b\boldsymbol{v}, \tag{2.16}$$

which represents the formula for the change of coordinates of a projected vector.

Now, by inverting the above we get

$$ {}^b\boldsymbol{v} = {}^a_b\boldsymbol{R}^{-1}\,{}^a\boldsymbol{v} = {}^a_b\boldsymbol{R}^\top\,{}^a\boldsymbol{v}. \tag{2.17}$$

However, if the change of coordinate problem had been casted with the roles of frames $\langle a\rangle$ and $\langle b\rangle$ swapped, we would have obtained

$$ {}^b\boldsymbol{v} = {}^b_a\boldsymbol{R}\,{}^a\boldsymbol{v}, \tag{2.18}$$

then, due to the arbitrariness of $\boldsymbol{v}$, it follows that we must necessarily have that

$$ {}^a_b\boldsymbol{R}^\top = {}^b_a\boldsymbol{R}, \tag{2.19}$$

which represents the simple formula for knowing the orientation of a first frame with respect to a second one, once the orientation of the second one is known with respect to the first. In particular, the above equation implies the following inner structure of ${}^a_b\boldsymbol{R}$

$$ {}^a_b\boldsymbol{R} = \begin{bmatrix} {}^a\boldsymbol{i}_b & {}^a\boldsymbol{j}_b & {}^a\boldsymbol{k}_b \end{bmatrix} = \begin{bmatrix} {}^b\boldsymbol{i}_a^\top \\ {}^b\boldsymbol{j}_a^\top \\ {}^b\boldsymbol{k}_a^\top \end{bmatrix}, \tag{2.20}$$

which provides a very easy mean for immediately looking at the mutual orientations between two given frames.

Figure 22: Enter Caption

In the distance zeroing problem, if one considers the derivative of the norm of the distance, then

- a. The end-effector always reaches the goal position by following a curvilinear path.
- b. The end-effector always reaches the goal position by following a straight path.
- c. The end-effector reaches the goal position reducing the norm to zero exponentially. The end-effector velocity necessarily includes components orthogonal to the current distance vector.
- d. The end-effector reaches the goal position reducing the norm to zero exponentially. The end-effector velocity might include components orthogonal to the current distance vector.

Your answer is incorrect.

The correct answer is:
The end-effector reaches the goal position reducing the norm to zero exponentially. The end-effector velocity might include components orthogonal to the current distance vector.

Figure 23: Zeroing problem

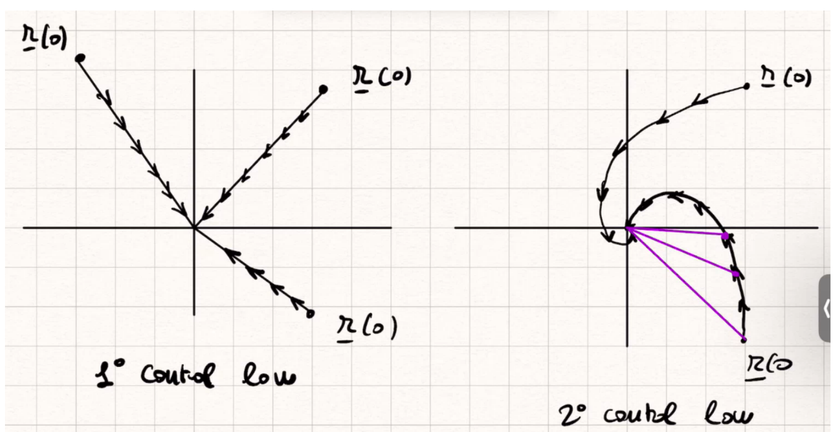

Figure 24: Two possible solutions for distance zeroing problem