TEST SIMULATION 1

Propositional Logic

$$T\left\{\begin{array}{l} 1.\ \neg(p \to q) \\ 2.\ \neg(\neg(p \wedge q) \to \neg r) \\ 3.\ ((q \wedge r) \to (p \vee \neg q)) \end{array}\right.$$

$T \overset{?}{\models} p \vee r$

L.C. $p \vee r$ is logical consequence of T iff $T \cup \{\neg(p \vee r)\}$ is inconsistent iff

(*) $(\neg(p \to q)) \wedge (\neg(\neg(p \wedge q) \to \neg r)) \wedge ((q \wedge r) \to (p \vee \neg q)) \wedge (\neg(p \vee r))$

is unsatisfiable

SAT

We start by converting the formula (*) in CNF

To perform the conversion we convert each simple conjunct (The formula (*) is "almost" a CNF)

$\boxed{\neg(p \to q)} \Rightarrow \neg(\neg p \vee q) \Rightarrow (\neg\neg p \wedge \neg q) \Rightarrow$

$(\alpha \to \beta) \equiv (\neg\alpha \vee \beta)$ $\quad \neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$ $\quad \neg\neg\alpha \equiv \alpha$

$\Rightarrow \boxed{p \wedge \neg q}$ CNF

$\boxed{\neg(\neg(p \wedge q) \to \neg r)} \Rightarrow \neg(\neg \underbrace{\boxed{\neg(p \wedge q)}}_{\alpha} \vee \underbrace{\boxed{\neg r}}_{\beta}) \Rightarrow$

$(\alpha \to \beta) \equiv (\neg\alpha \vee \beta)$

$\Rightarrow \neg((p \wedge q) \vee \neg r) \Rightarrow \neg(p \wedge q) \wedge \neg\neg r \Rightarrow$

$\Rightarrow \boxed{(\neg p \vee \neg q) \wedge r}$ CNF

$((q \wedge r) \to (p \vee q)) \Rightarrow \neg(q \wedge r) \vee (p \vee q) \Rightarrow$

$(\neg q \vee \neg r) \vee (p \vee q) \Rightarrow \neg q \vee \neg r \vee p \vee q \Rightarrow T$

Tautology $\quad \underline{\alpha \vee \neg\alpha}$

- p
- $\neg q$
- $\neg p \vee \neg q$
- r

CNF Corresponding To The Theory Γ

$\neg(p \vee r) \Rightarrow \neg p \wedge \neg r$

Check whether The CNF φ is satisfiable.

$$\varphi \left[\begin{array}{l} p \\ \neg q \\ \neg p \vee \neg q \\ r \\ \neg p \\ \neg r \end{array} \right.$$

The formula is not satisfiable as seen by performing unit clause propagation (V.E. And/or DPLL)

φ is not satisfiable, Therefore $\Gamma \cup \{\neg(p \vee r)\}$ is inconsistent, And thus $p \vee r$ is A logical consequence of Γ.

First Order Logic

1. $\forall x . \neg A(x, x)$
2. $\forall x . \forall y . (P(x, y) \to A(x, y))$
3. $\forall x . \forall y . (\exists z \; P(z, y) \wedge A(x, z) \to A(x, y))$

1. Nobody is his own sucestor
2. If x is y's parent then x is also y's sucestor
3. If there exists a z who is y's parent and x is z's ancestor, then x is also y's ancestor

Are the three sentences consistent?

For a set of sentences to be consistent, there should be at least one interpretation that satisfies all of them.

$I = \langle D, \delta \rangle$ D Domain
δ interpretation function

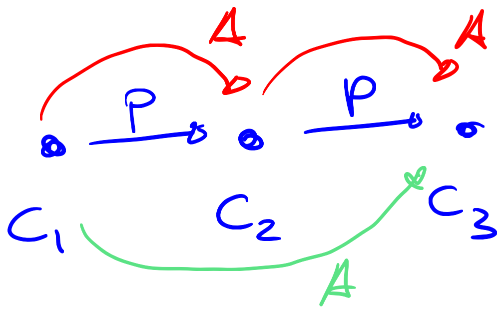
$D = \{c\}$ $\delta(P) = \emptyset$ $\delta(A) = \emptyset$

$\models_I \forall x. \neg A(x,x)$ True because $A(c,c)$ is false

$\models_I \forall x. \forall y (P(x,y) \to A(x,y))$ True because $P(c,c)$ is false

$\models_I \forall x. \forall y (\exists z. P(z,y) \wedge A(x,z) \to A(x,y))$ True $P(c,c)$ is false

Also a more complex interpretation would do ...



$P(c_1, c_2)$ $P(c_2, c_3)$

$A(c_1, c_2)$ $A(c_2, c_3)$

$A(c_1, c_3)$

For (2) to be satisfied by this interpretation we need also $\rightarrow$

$\forall x \forall y \; P(x,y) \rightarrow A(x,y)$

For (3) to be satisfied we need also $\rightarrow$

... but you need to be careful to satisfy all the formulas at the same time

$$I \begin{cases} D = \{m, s, j\} \\ \rho(P) = \{(m,s), (s,j)\} \quad \rho(A) = \emptyset \end{cases}$$

$m :=$ mary $\quad j :=$ juliet

$s :=$ susan

$$m \xrightarrow{P} s \xrightarrow{P} j$$

$\models_I \forall x. \neg A(x,x)$ is satisfied (trivially $\rho(A) = \emptyset$)

$\models_I \forall x \forall y (P(x,y) \to A(x,y))$ not satisfied because, e.g.

$P(m,s)$ but not $A(m,s)$ since $\rho(A) = \emptyset$

The interpretation DOES NOT satisfy all sentences (we found at least one for which it does not)

$\exists x . P(x,x)$ is a logical consequence?

We believe $\exists x . P(x,x)$ is NOT logical consequence of ①, ②, ③. It is sufficient to show an interpretation that satisfies ①, ②, ③ and not $\exists x . P(x,x)$. We have them already:

$I: \; D = \{c\} \quad \rho(P) = \rho(A) = \emptyset \qquad P(c,c)$ is false

$I: \; D = \{c_1, c_2, c_3\} \quad \rho(P) = \{(c_1, c_2), (c_2, c_3)\}$

$\rho(A) = \{(c_1, c_2), (c_2, c_3), (c_1, c_3)\}$

$P(c_i, c_i)$ is false for all $i \in \{1, 2, 3\}$