Problem 1)

$$x(n) = \begin{cases} 0.9^n & , n = 0, \ldots, N_x - 1 \\ 0 & , \text{else} \end{cases}$$

$N_x = 28$

a)

Using DTFT:

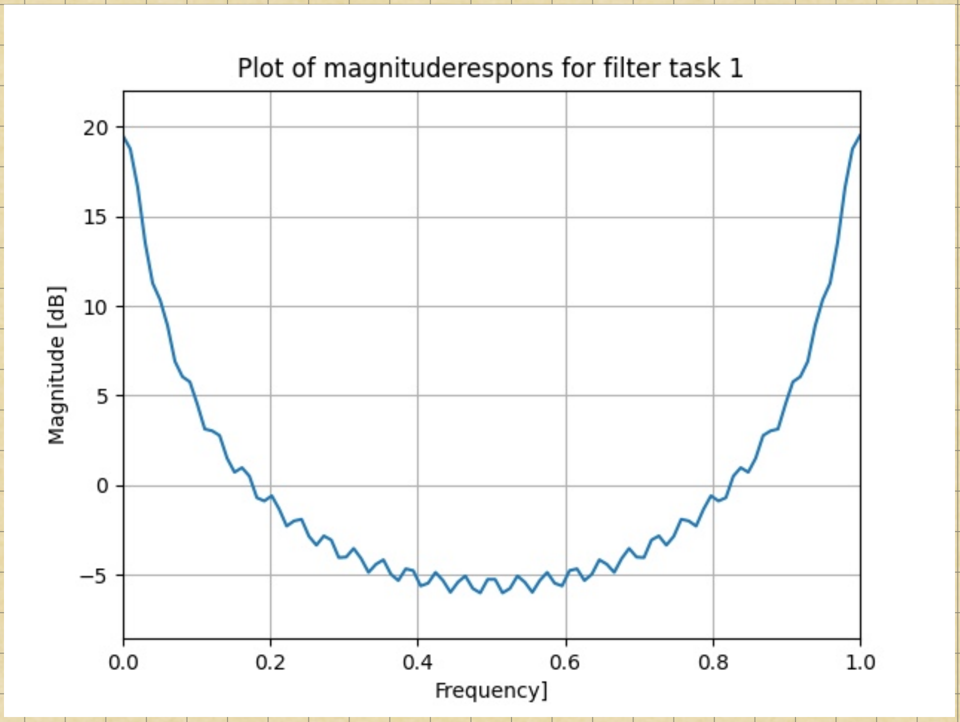
$$X(f) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j2\pi f n}$$

$$= \sum_{n=0}^{N_x - 1} 0.9^n \cdot e^{-j2\pi f n}$$

$$= \sum_{n=0}^{N_x - 1} \left(0.9 e^{-j2\pi f}\right)^n$$

$$X(f) = \frac{1 - \left(0.9 e^{-j2\pi f}\right)^{27}}{1 - 0.9 e^{-j2\pi f}}$$

plot for $f \in [0,1)$



b) Use fft to compute $X(k) = DFT\{x(n)\}$, (Discret fourier transform)

$$DFT: \; X(k) = \sum_{n=0}^{N_x-1} x(n) e^{-\frac{j2\pi k}{N_x} n}, \; k = 0, \ldots, N_x - 1$$

$$= \sum_{n=0}^{27} \left(0.9 e^{-\frac{j2\pi k}{28}}\right)^n$$

$$X(k) = \frac{1 - \left(0.9 e^{-\frac{j2\pi k}{28}}\right)^{27}}{1 - 0.9 e^{-\frac{j2\pi k}{28}}}, \; k = 0, \ldots, 27$$

Code that I used

Øving6Opp1.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.fft import fft, ifft

#Frequency where I want to plot
f = np.linspace(0,1,100)
#Here is the filter
H_f = (1-(0.9*np.e**(-1j*2*np.pi*f))**27)/(1-0.9*np.e**(-1j*2*np.pi*f))
#Task 1 b)
Nx = 28
k = np.linspace(0,Nx-1,Nx)
n = np.linspace(0,Nx-1,Nx)
x_n = 0.9**n
#Takse 1b) result
X_k1 = fft(x_n,int(Nx/4))
X_k2 = fft(x_n,int(Nx/2))
X_k3 = fft(x_n,int(Nx))
X_k4 = fft(x_n,int(2*Nx))
# Plot the magnitude response
plt.figure()
plt.title("Plot of magnituderespons for filter task 1")
plt.plot(f, 20 * np.log10(np.abs(H_f)))
plt.xlabel("Frequency]")
plt.ylabel("Magnitude [dB]")
plt.margins(0, 0.1)
plt.grid(True)
plt.show()
```

c) Find f that corresponds to the $k=1$, for the four DFT's computed in b.

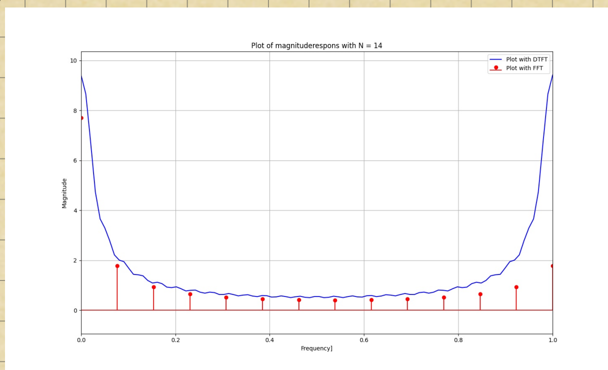
$$X(k=1) = X(f)$$, what is f?

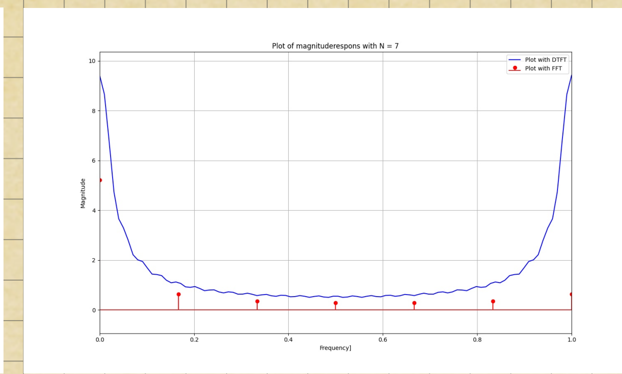
We know that $f = \frac{k}{N}$, $k = 0, \dots, N$, where N varies from $N \in [N_x/4, N_x/2, N_x, 2N_x]$

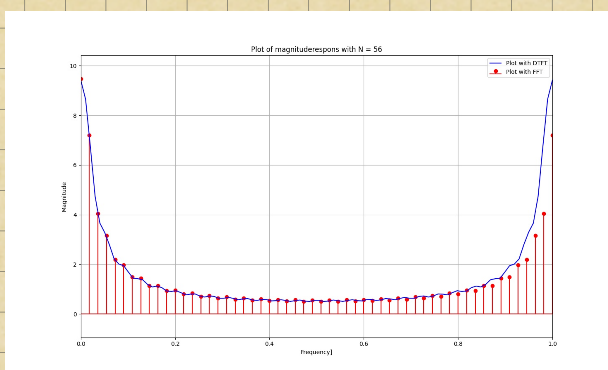
So for the four DFT's:

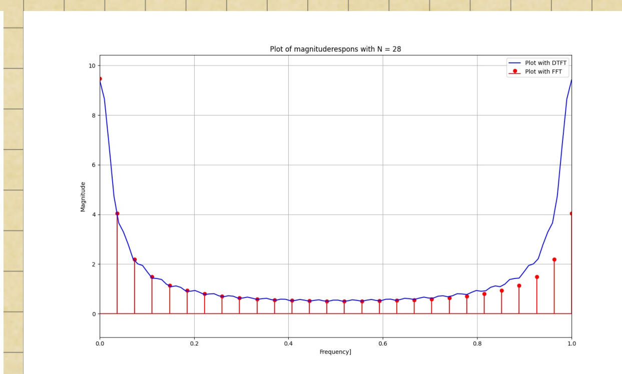
1) $f = \frac{1}{\frac{N_x}{4}} = \underline{\underline{\frac{4}{N_x}}}$, 2) $\underline{\underline{f = \frac{2}{N_x}}}$, 3) $\underline{\underline{f = \frac{1}{N_x}}}$, 4) $\underline{\underline{f = \frac{1}{2N_x}}}$

d)









We can clearly see that the bigger the DFT length the more similar the DFT and DTFT is. I think that for $N \geq N_K$, the FFT value should be exact the DTFT-values. IDK why this is not the case in the plots

e) Because of the Symmetry and Periodicity of DFT and DTFT.

problem 2)

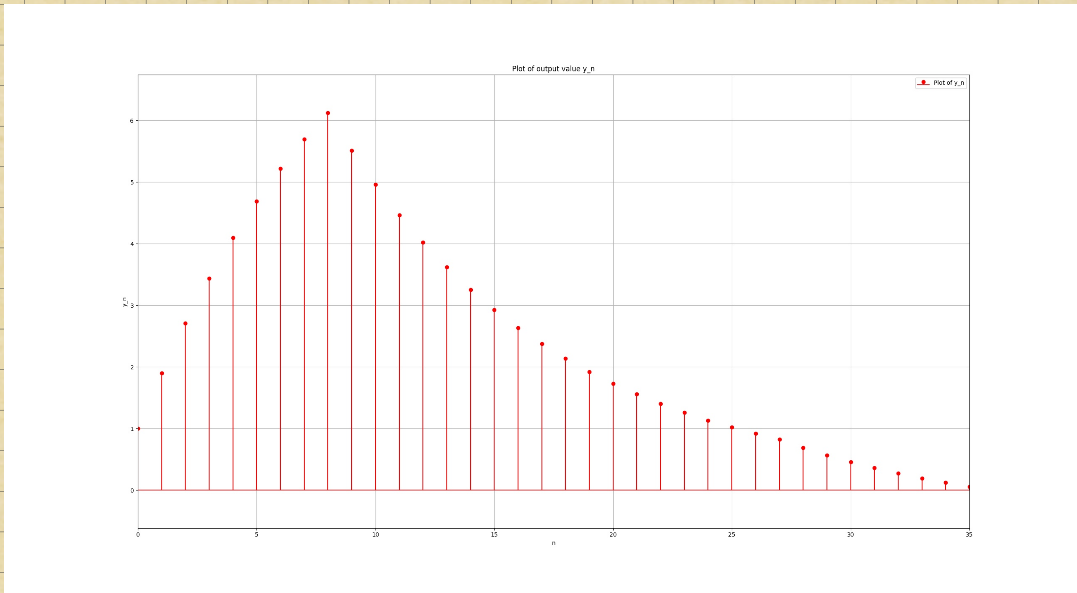
a) Sequence of $X(n)$ filtered thru FIR-filter:

$$h(n) = \begin{cases} 1, & n = 0, 1, \ldots, N_h - 1 \\ 0, & \text{else} \end{cases}$$

$N_h = 9$

Theory says N_y should be

$$N_y = N_x + N_h - 1 = \underline{36}$$

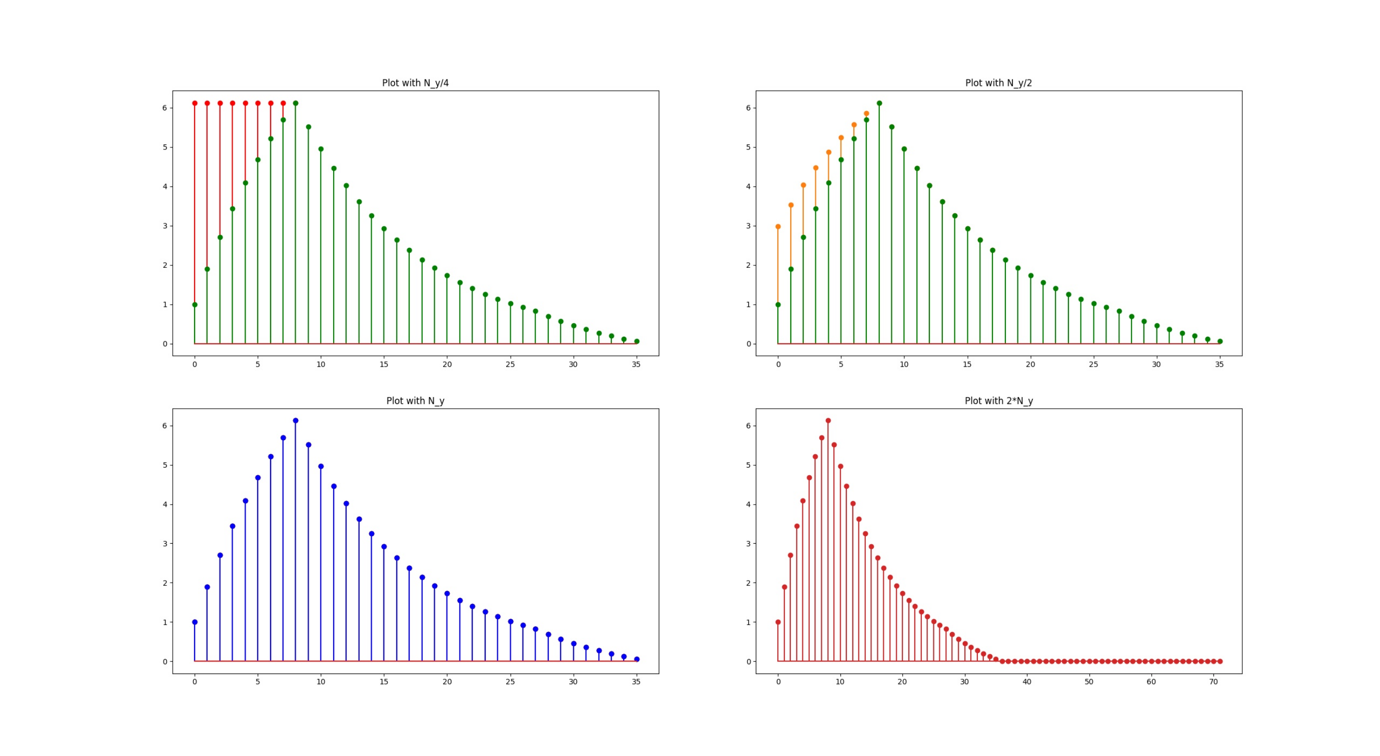


We can see that the value of $p(n)$ N_y is 36, as theory would have it.

b) plot frequency domain using DFT/IDFT:

As we saw from the previous task the DFT should be run with lengths $\geq N_p$.

Plots from code!



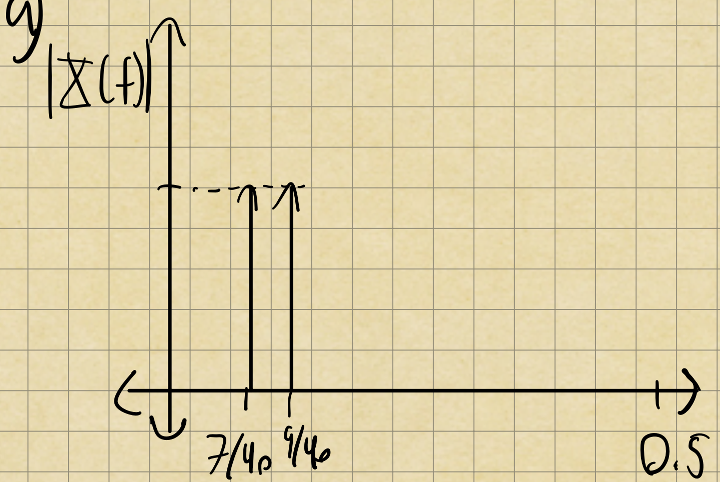
- As we can see the length of the DFT has to atleast be equal to N_y, so we dodge time domain aliasing.

This is verified by the plots

problem 3:

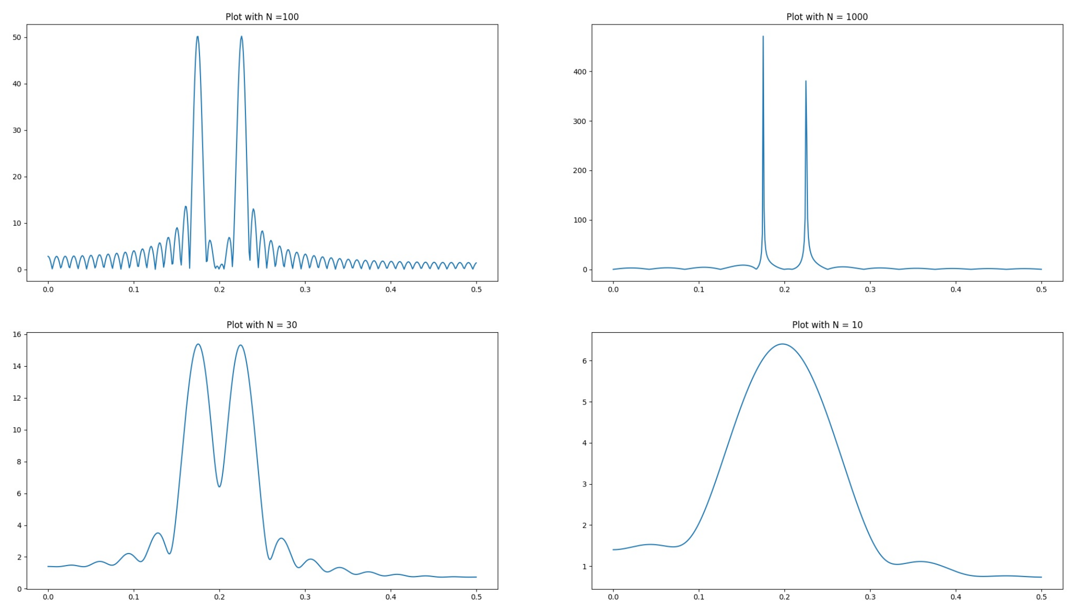
$$x(n) = \sin(2\pi f_1 n) + \sin(2\pi f_2 n), \quad f_1 = 7/40$$
$$f_2 = 9/40$$

a)



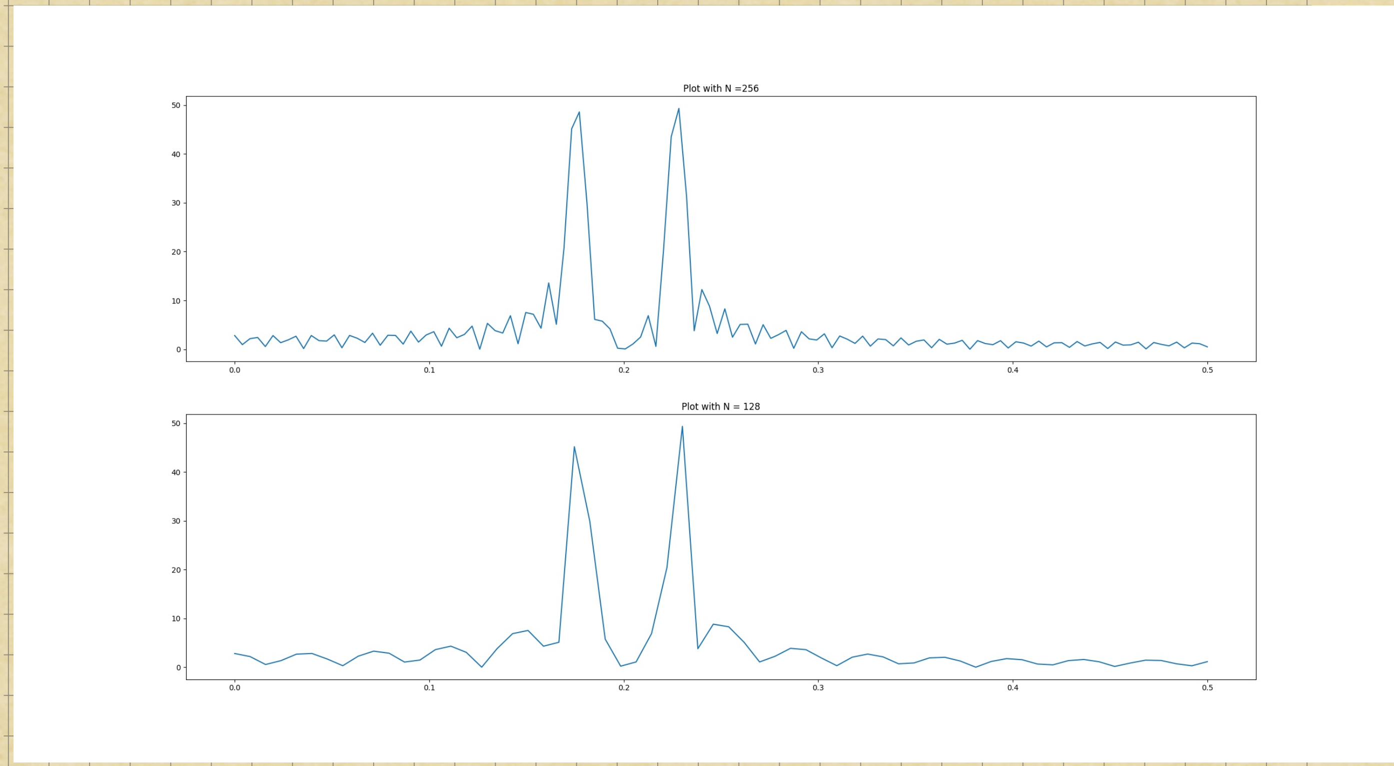
It would repeat itself at $1-f_1$, and $1-f_2$.

b)



- We can clearly see that the shorter the segment n is the less accurate the plot is. At N=10 we cant find the 2 frequencys anymore.

c)



- The higher the DFT the better the value
  (Less spacing between samples)

Task 4)

a) The FFT is an effective way to Calculate the DFT. By splitting it up in many small DFT's Using Symmetry and periodicity of W_N^{kn}

b) Radiax-2 FFT is an way to Compute DFT when $N = 2^n$. It is done by deviding the sequence of $x(n)$ into two Sequences, $f1(n)$, $f2(n)$, of length $N/2$.
By doing this we can shorten the number of operation by nearly half. Nice.

c) All N values:

N^2 Complex multiplications

d)

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, \quad k = 0, \dots, N-1$$

$$= \sum_{n_{odd}} x(n) W_N^{kn} + \sum_{n_{even}} x(n) W_n^{kn}$$

$$= \sum_{m=0}^{(N/2)-1} x(2m) W_N^{2mk} + \sum_{m=0}^{(N/2)-1} x(2m+1) W_N^{k(2m+1)}$$

$$W_N^2 = W_{N/2}$$

$$X(k) = \sum_{m=0}^{(N/2)-1} f_1(m) W_{N/2}^{mk} + W_N^k \sum_{m=0}^{(N/2)-1} f_2(m) W_{N/2}^{km}$$

$$= F_1(k) + W_N^k F_2(k)$$

perodicity:

$$X(k) = F_1(k) + W_N^k F_2(k)$$

$$X(k+N/2) = F_1(k) - W_N^k F_2(k)$$

e)

Number of Complex multiplications is

$$2(N/2)^2 + N/2 = \frac{N^2}{2} + \frac{N}{2} = \underline{\underline{\frac{N}{2}\left(N+1\right)}}$$