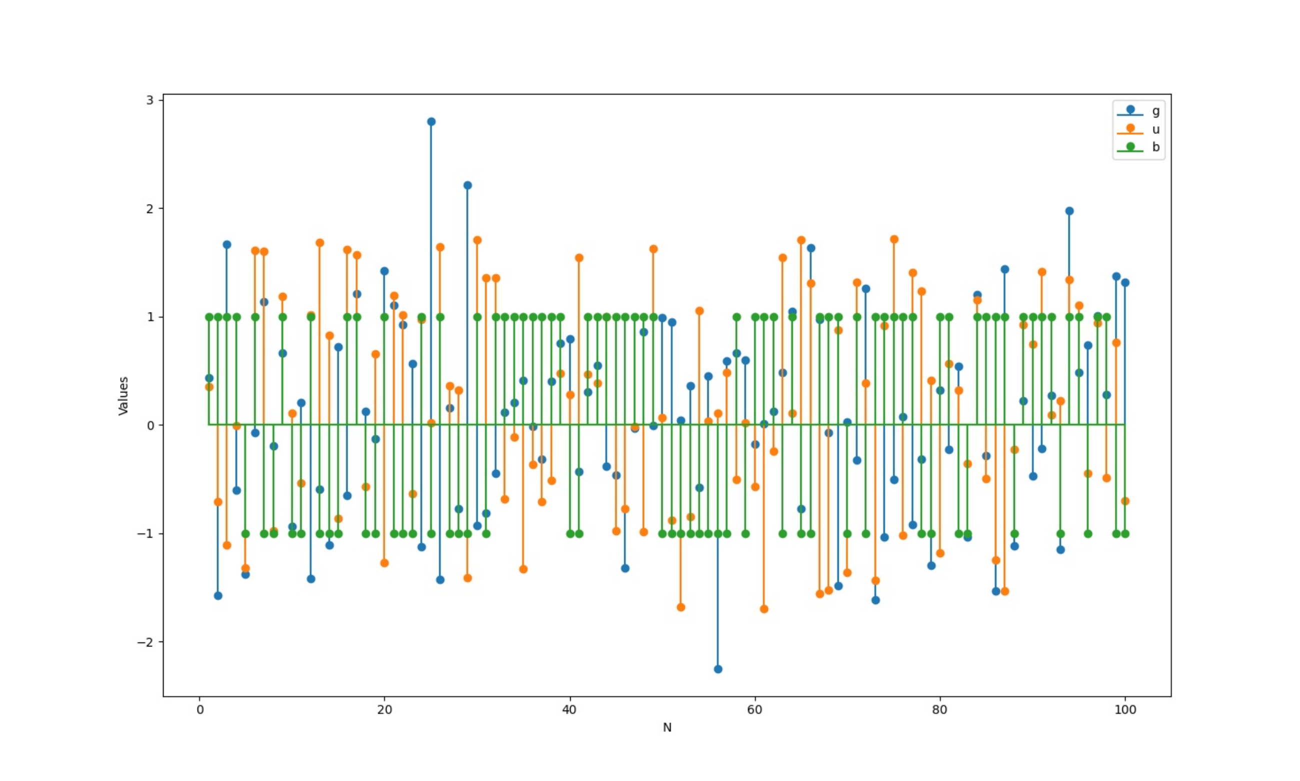
problem 1)

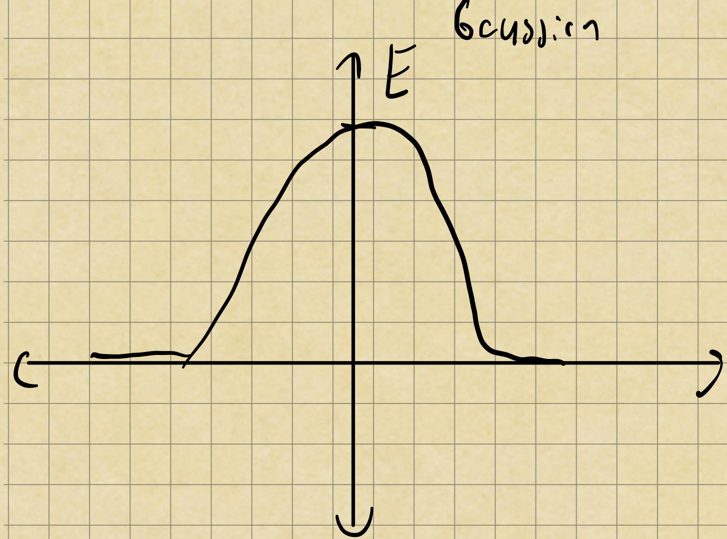
a) Unit - Variance:

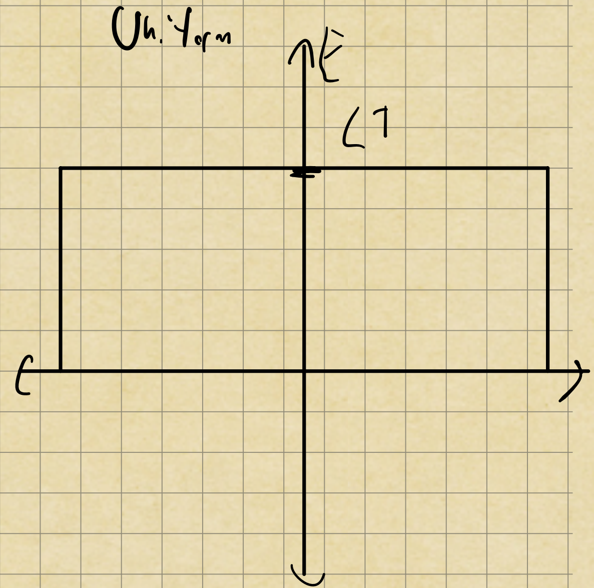


- All noises samples are uncorrelated.
  In other words, if you were given one sample their is no information about the value of the next one.

- Binary is limited to 2- values 1 and −1, where as the uniform and gaussian has a interval.

- Uniform is equally likely to take any value within a specified range but in contrast the gaussian follows a normal distrubution.





b)

For binary

$$p_X = \begin{cases} 0.5, & x = \pm 1 \\ 0, & \text{else} \end{cases}$$

Gaussian:

$$p_X = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Uniform:

$$p_X = \begin{cases} \frac{1}{2\sqrt{3}}, & x \in (-\sqrt{3}, \sqrt{3}) \\ 0, & \text{else} \end{cases}$$

Compute mean - value:

$$\mu_x = E\{X[n]\} = \int_{-\infty}^{\infty} x\, f_X(x)\, dx = 0$$

Gaussian:

$$\mu_x = \int_{-\infty}^{\infty} \overbrace{\underbrace{x}_{\text{odd}} \cdot \frac{1}{\sqrt{2}}\, \overbrace{e^{-\frac{x^2}{2}}}^{\text{even}}}^{\text{odd}}\, dx$$

Since it is an odd function and symmetric interval

$$\Rightarrow \underline{\underline{\mu_x = 0}}$$

Uniform:

$$\mu_x = \int_{-\infty}^{\infty} \frac{1}{2\sqrt{3}} \cdot x \, dx = \left[\frac{x^2}{4\sqrt{3}}\right]_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \frac{3}{4\sqrt{3}} - \left(\frac{3}{4\sqrt{3}}\right)$$

$$= \underline{\underline{0}}$$

For binary

$$\mu_x = p(1)\cdot 1 + p(-1)\cdot(-1)$$

$$= 0.5 - 0.5 = \underline{\underline{0}}$$

Autocorrelation function:

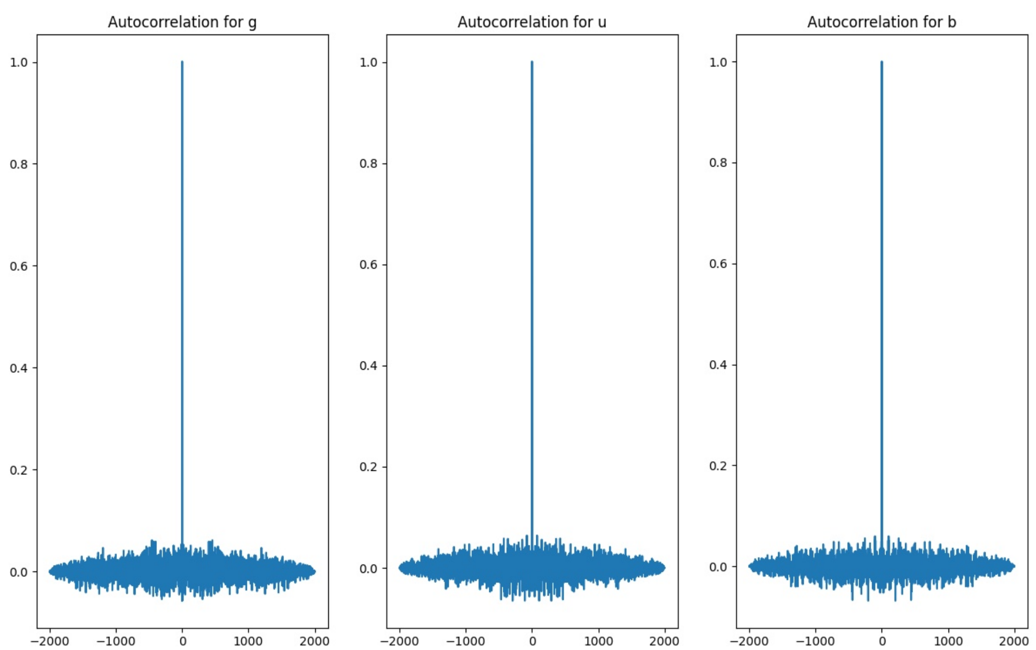
$$R_X(k) = E\,X(n)\,X(n-k) = \sigma^2\delta(k)$$

Power density correlation:

$$S_X = \mathrm{DTFT}\left\{R_X(k)\right\} = \underline{\underline{\sigma^2}}$$

c)

```
Eier@dhcp-10-24-21-119 DIGSIG Øving 7 % /usr/bi
Sample Mean for g: -0.006095568348404223
Theoretical Mean for g: 0
Sample Mean for u: 0.004009038886484537
Theoretical Mean for u: 0
Sample Mean for b: 0.001
Theoretical Mean for b: 0
```



- As we can see from the plots and values the theoretical values are pretty accurate.

Problem 2)

Random signal $X[n]$ is generated by filtering white Gaussian noise $W[n]$ with variance $\sigma_w^2 = \frac{3}{4}$ by a Causal filter with transfer function

$$H(z) = \frac{1}{1 + \frac{1}{2}z^{-1}}$$

a)

- The mean value of $m[n] = 0$
- The auto correlation of $r_{ww}[k] = \sigma_w^2 \, \delta(k) = \frac{3}{4}\delta(k)$
- $\Gamma_{ww}(f) = \frac{3}{4}$, of w_n

This gives values for $X[n]$:

$$m_x = E\left[X[n]\right] = m_w \sum_{k=\infty}^{\infty} h(k) = 0$$

$$r_{xx}(m) = r_{ww}(m) \cdot r_{hh}(m)$$

Where

$$r_{hh}(m) = \sum_{-\infty}^{\infty} h(n)\,h(n+m)$$

and $h(n) = Z^{-1}\left\{ H(z) \right\} = \left(-\frac{1}{2}\right)^n u(n)$

This gives

$$r_{hh}(m) = \sum_{-\infty}^{\infty} \left(-\frac{1}{2}\right)^n u(n) \left(-\frac{1}{2}\right)^{n+m} u(n+m)$$

We first find for $m \geq 0$

$$r_{hh}(m) = \sum_{0}^{\infty} \left(-\frac{1}{2}\right)^n u(n) \left(-\frac{1}{2}\right)^{n+m} u(n+m)$$

$u(n) = 1$ $\Rightarrow$ $r_{hh}(m) = \sum_{0}^{\infty} \left(-\frac{1}{2}\right)^{2n+m}$

$u(n+m) = 1$

$$= \left(-\frac{1}{2}\right)^m \sum_{0}^{\infty} \left(-\frac{1}{4}\right)^n$$

$$= \underline{\frac{4}{3}\left(-\frac{1}{2}\right)^m}$$

for $m < 0$

we know: $r_{hh}(m) = r_{hh}(-m)$

$$\Rightarrow \quad r_{hh} = \frac{4}{3}\left(-\frac{1}{2}\right)^{|m|} \quad , \forall m$$

$$\Rightarrow r_{xx}(m) = r_{ww}(m) * r_{hh}(m) = \frac{3}{4}\delta(m) * \left(\frac{4}{3}\right)\left(-\frac{1}{2}\right)^{m}$$

$$= \underline{\underline{\left(-\frac{1}{2}\right)^{|m|}}}$$

Power density spectrum:

$$\Gamma_{xx}(\omega) = \Gamma_{ww}(\omega)\left|H(\omega)\right|^2$$

where $H(\omega) = H(z)\Big|_{z=e^{j\omega}}$

$$\Rightarrow \left|H(\omega)\right|^2 = H(\omega)H(\omega)^* = \frac{4}{\underline{5+4\cos\omega}}$$

We know that:

$$\Gamma_{ww}(w) = \sigma_w^2(w) = \frac{3}{4}$$

$$\Rightarrow \quad \Gamma_{xx}(w) = \frac{3}{4} \cdot \frac{4}{5 + 4\cos w} = \frac{3}{5 + 4\cos w}$$

power of the signal:

$$P_x = E\left[x^2(n)\right] = r_{xx}(0) = 1$$

b)

Estimators:

$$\hat{m}_x = \frac{1}{N} \sum_{0}^{N-1} x(n)$$

power:

$$\hat{P}_x = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n)$$

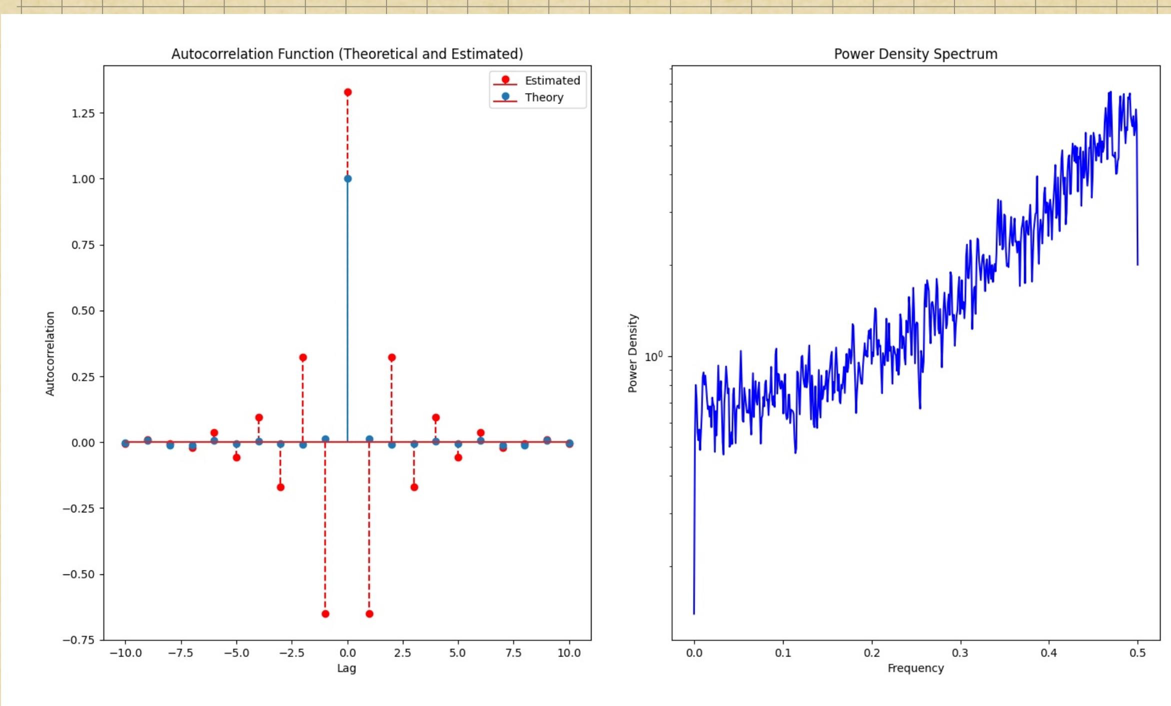
Auto Correlation:

$$\hat{r}_{xx}(l) = \begin{cases} \frac{1}{N}\sum_{0}^{N-1} x(n)x(n+l), & 0 \le |l| \le N-1 \\ 0, & N \le |l| \end{cases}$$

Power spectral density:

$$\hat{\Gamma}_{xx} = \mathrm{DFT}(\hat{r}_{xx}(l))$$

c)



d) Couldnt figure this one out.

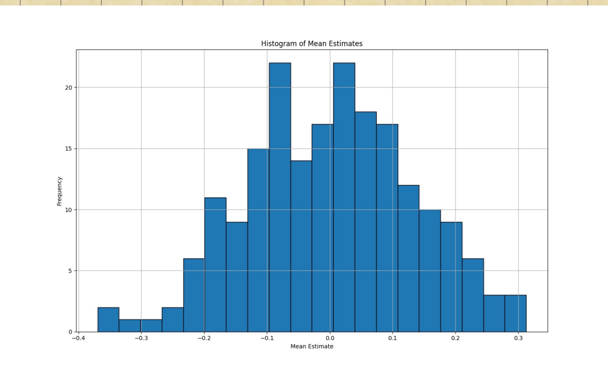
e) Have done this

problem 3)

a)

```python
#Task3a)
K = 20

# Initialize an array to store the mean estimates
mean_estimates = []

# Repeat the mean calculation for 200 segments
for i in range(200):
    # Extract a segment of length K from the signal
    segment = x[i * K: (i + 1) * K]

    # Calculate the mean of the segment
    segment_mean = np.mean(segment)

    # Append the mean estimate to the list
    mean_estimates.append(segment_mean)
```

b)



c)

Result from Code:

```
Estimated Mean: 0.0017699369465278897
Estimated Variance: 0.020333427929561641
```

d)

```
For K = 20:
Estimated Mean: 0.009194283868357246
Estimated Variance: 0.015414014133601595

For K = 40:
Estimated Mean: 0.00949630515671533
Estimated Variance: 0.014223619659367224

For K = 100:
Estimated Mean: 0.025640978212525256
Estimated Variance: 0.01276697397870169
```

e) Increasing the segment length K aligns with theoretical expectations by reducing the bias and variance, this results in a more accurate and less variable mean estimates for $\hat{\mu}_X$.