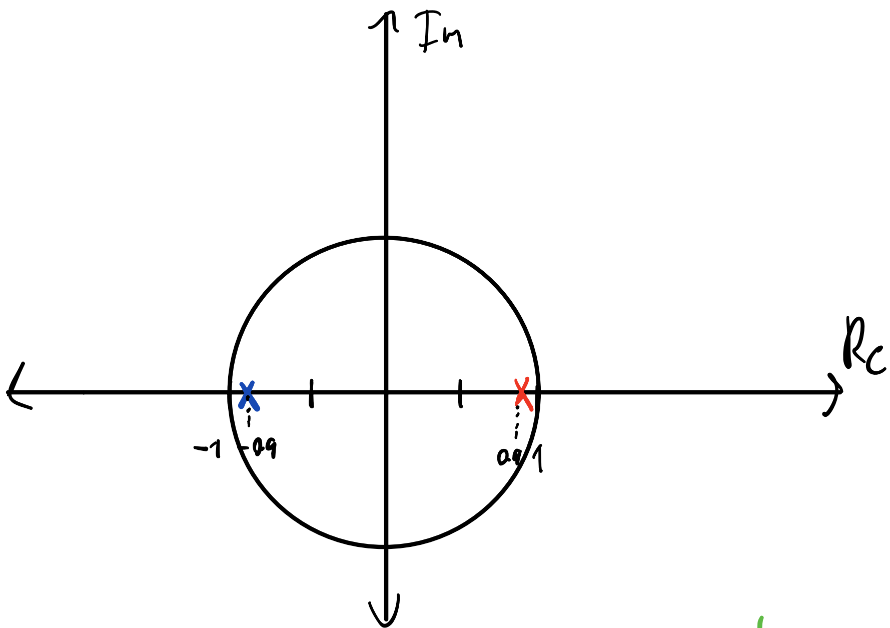
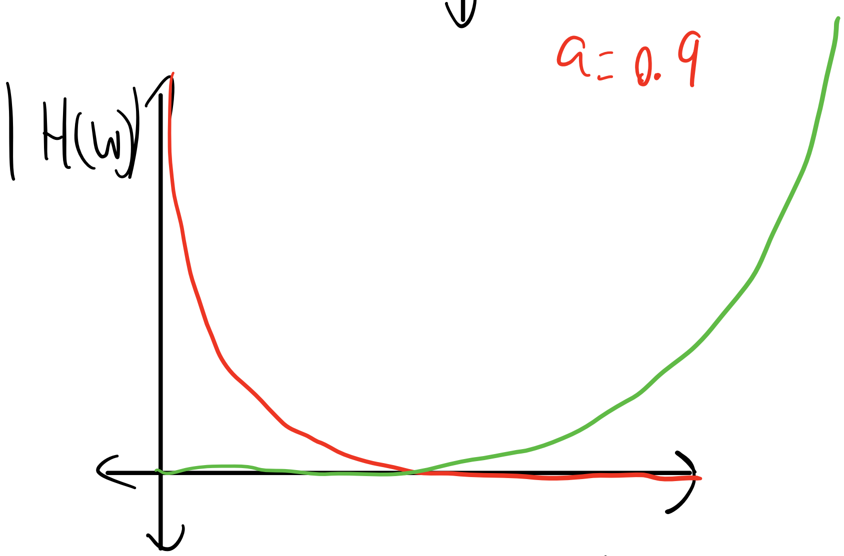
problem 1!

$$H(z) = \frac{1}{1 - az^{-1}}$$

a) $a = 0.9$, $a = -0.9$





We see that a= 0.9 lowpass, a= -0.9 highpass

b) Yes it works as expected.

Problem 2)

Causal digital filter:

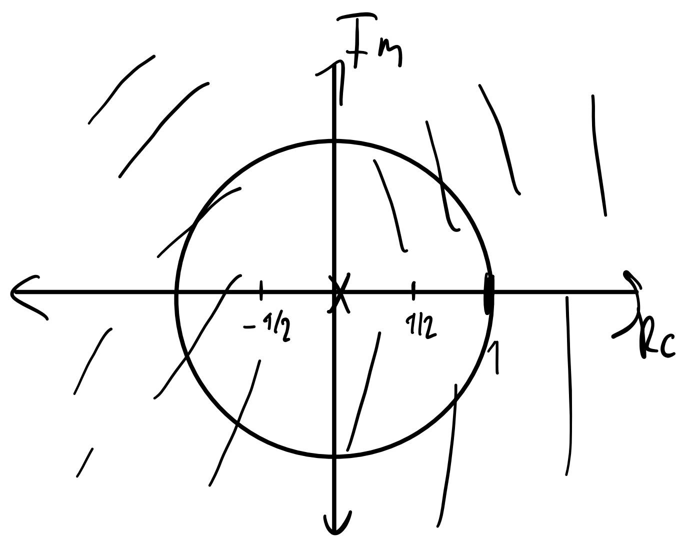
$$H(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})}, \quad ROC: |z| > \frac{1}{2}$$

a)

$$H_I(z) = \frac{1}{H(z)} = (1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1}) = \underline{\underline{1-\frac{1}{4}z^{-2}}}$$

With ROC: $|z| \neq 0$

b)



Since the unit-circle is inside of the ROC, the System is stable (it is also an FIR-filter)

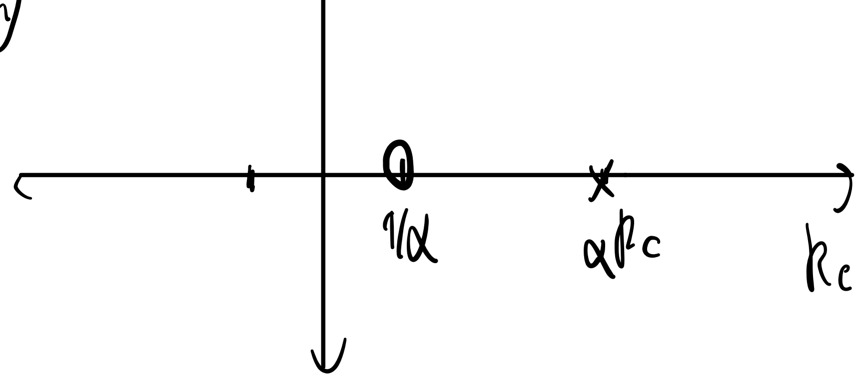
c) A system is called minimum-phase if all zeroes and poles are inside the unit circle. We can see from the figure over that this is true.

d) For linear phase filters the zeroes occur in reciprocal pairs. This means since we have zeroes in $z_1 = 1/2$ and $z_2 = -1/2$ we should also have in $z_3 = \frac{1}{z_1} = 2$, and $z_4 = \frac{1}{z_2} = -2$. Since this is not true, the filter does not have linear phase characteristics.

Problem 3)

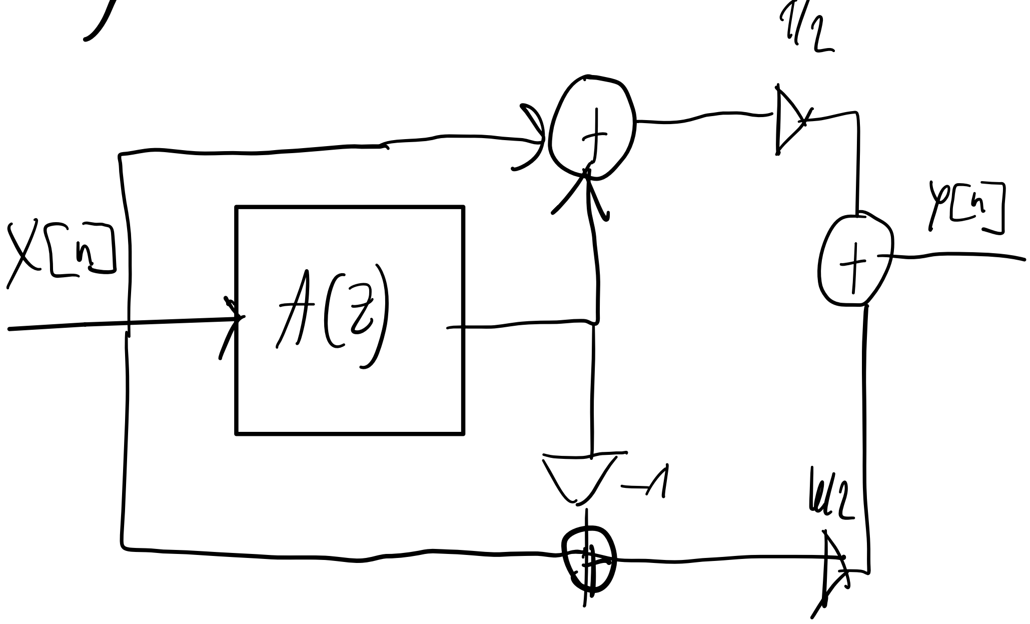
$$A(z) = \frac{\alpha - z^{-1}}{1 - \alpha z^{-1}}$$

a)



Since the filter has a zero in $\frac{1}{\alpha}$ and a pole in α the filter is an All-pass filter.

b)



$$Y(z) = X(z)H(z)$$

$$Y_{up}(z) = \left(X(z) + X(z)A(z)\right)\frac{1}{2}$$

$$Y_{up}(z) = \left(1 + A(z)X(z)\right)\frac{1}{2}$$

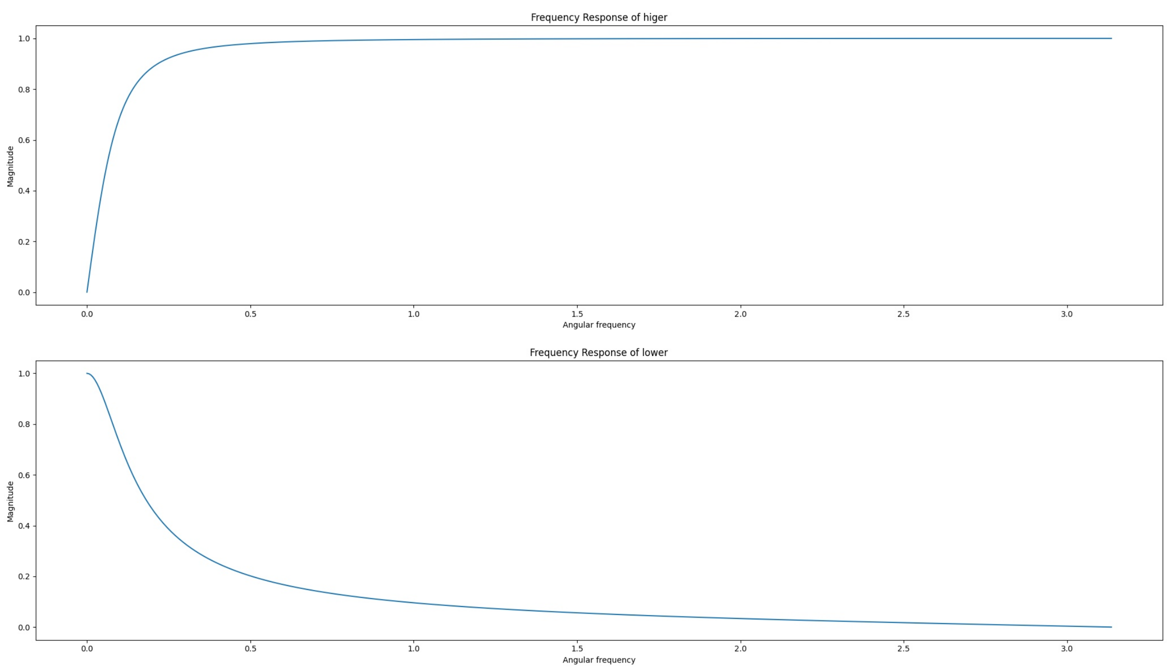
$$\Rightarrow H_{up}(z) = \frac{Y_{up}(z)}{X(z)} = \left(1 + \frac{\alpha - z^{-1}}{1-\alpha z^{-1}}\right)\frac{1}{2}$$

$$Y_{low}(z) = \left(\left(X(z) A(z) \right) (-1) + X(z) \right) \frac{k}{2}$$

$$H_{low}(z) = \left(1 - A(z) \right) \frac{k}{2}$$

$$H_{low}(z) = \left(1 - \frac{\alpha - z^{-1}}{1 - \alpha z^{-1}} \right) \frac{k}{2}$$

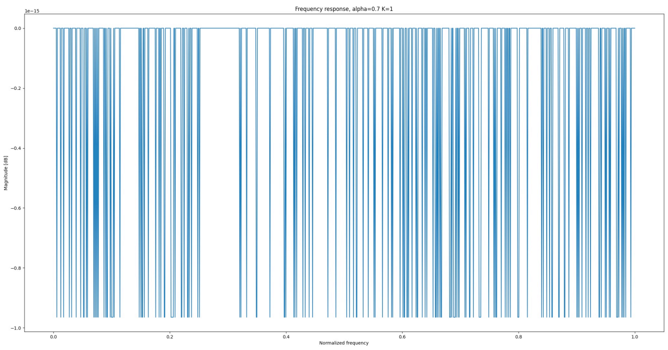
Filter:

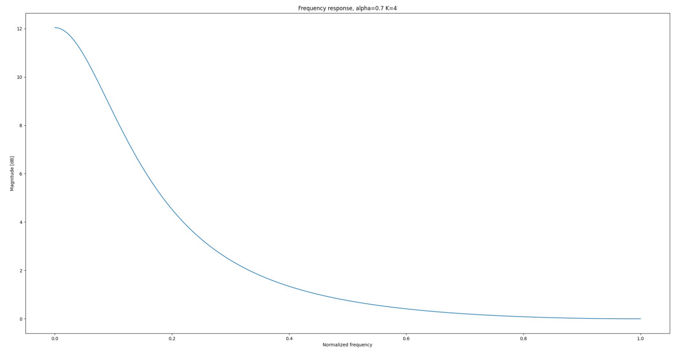


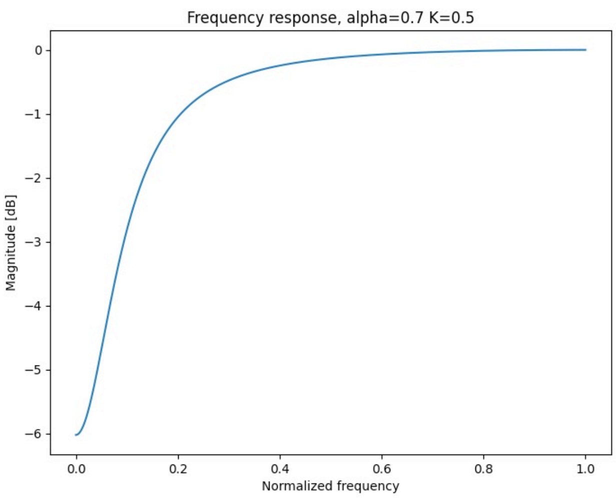
We can see that the upper is a highpass filter, while the lower is a lowpass filter.

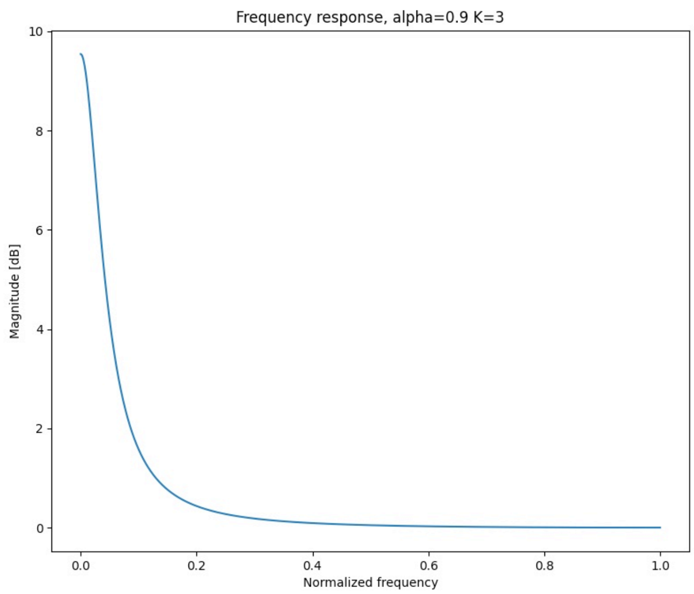
c)

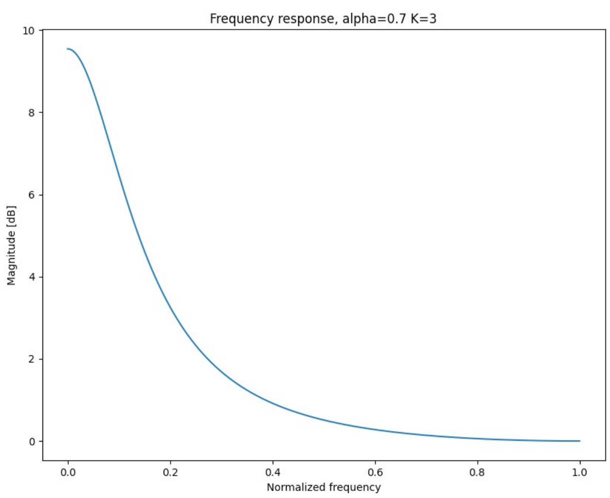
Something wrong with this plot.

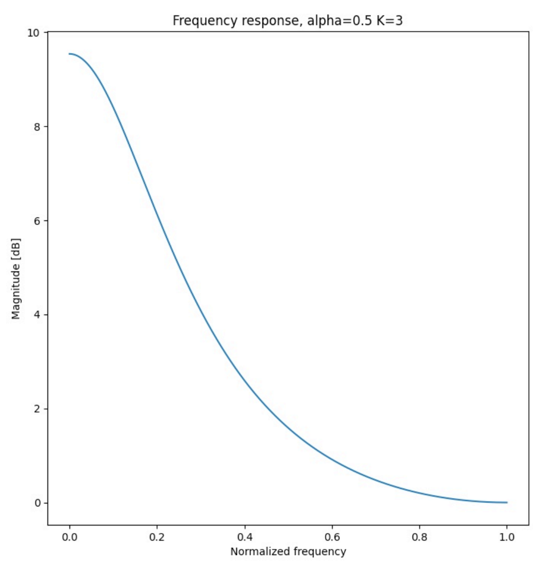


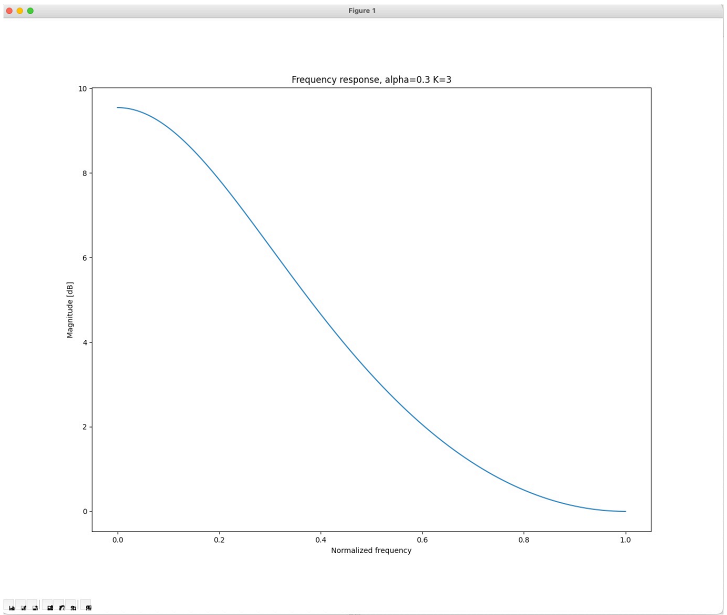












- Since K is an amplitude amplifer at the lower branch wich is a lowpass. K will adjust how much the lowpass contributes relative to the highpass, and therefore controls the boost/cut at lower frequencys.

ω_B

- α controls the bandwith of the filter

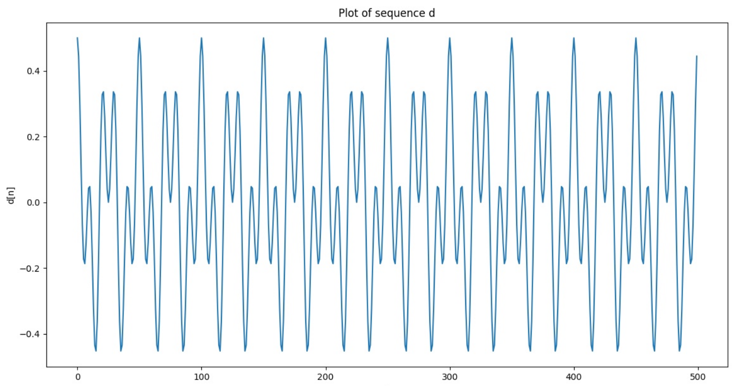
Problem 4)

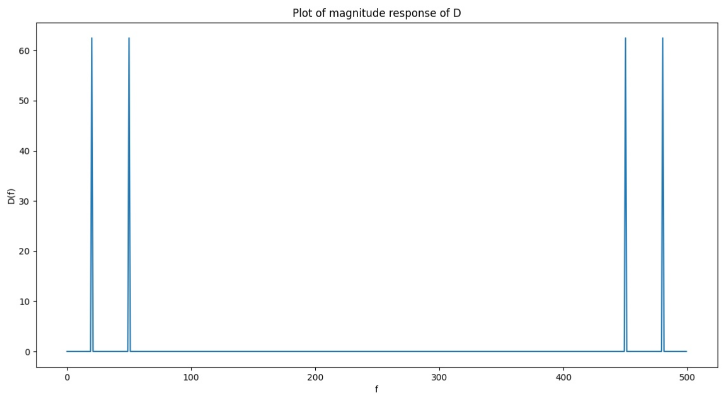
$$d(n) = A_x \cos(2\pi f_x n) + A_y \cos(2\pi f_y n), \quad 0 \leq n \leq L-1$$

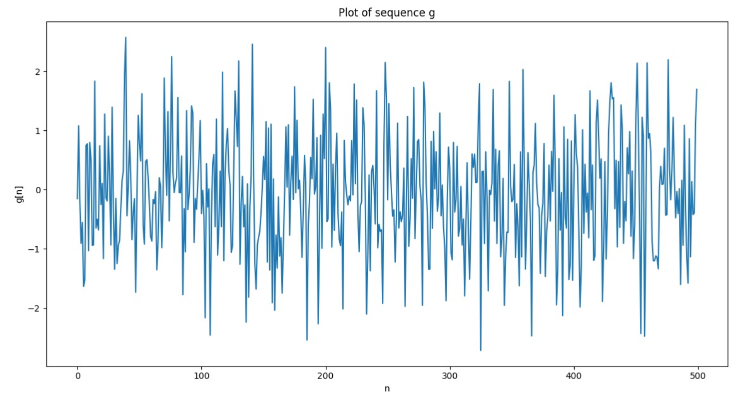
$$A_x = A_y = 0.25, \quad f_x = 0.04, \quad f_y = 0.10, \quad L = 500$$

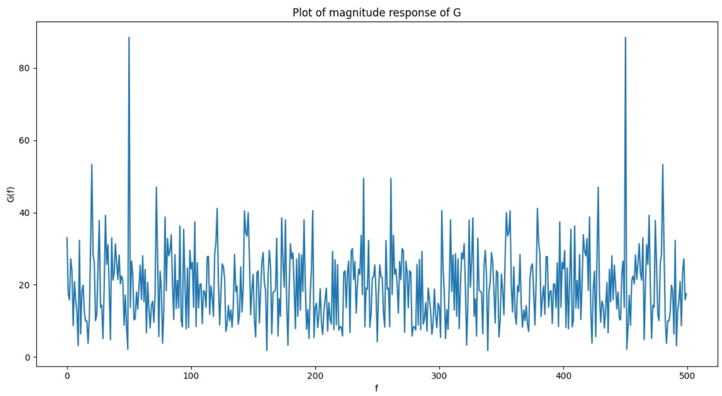
$$g(n) = d(n) + e(n)$$

a)









b) We want a digital-resonator, that filters everything out exept the 2 frequencys of the Sinusoids.
Since we want the filter to be steep, the poles should be close to the unit circle. (Almost one it)

$$P_{1,2} = re^{\pm j\omega_0}, \quad \text{Choose } r = 0.99 \text{ (to get steep filter)}$$

$$P_{1,2} = 0.99e^{\pm j\omega_0}, \quad \omega_0 = 2\pi f_{x,y}$$

$$H_x(z) = \frac{(1+z^{-1})(1-z^{-1})}{(1-0.99e^{j2\pi f_x}z^{-1})(1-0.99e^{-j2\pi f_x}z^{-1})}$$

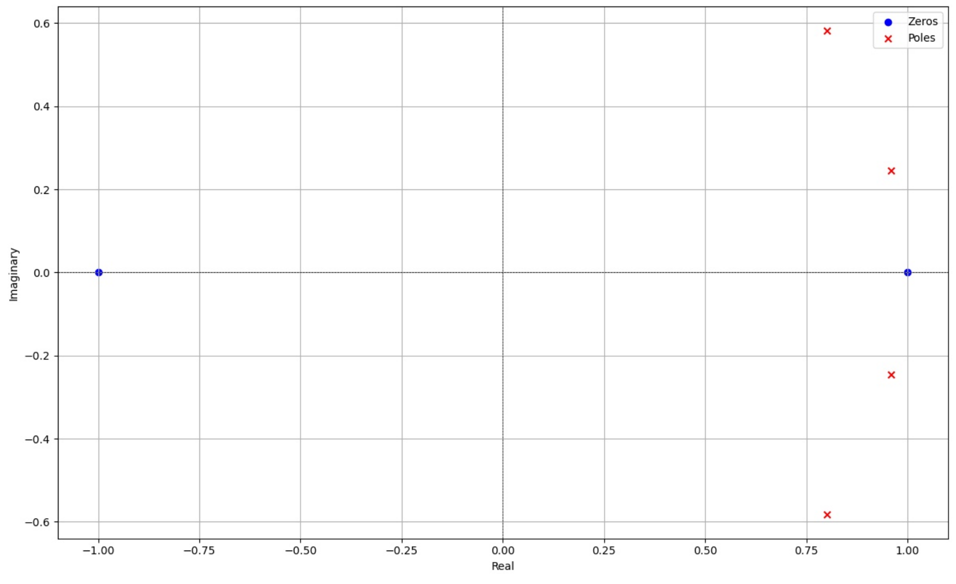
$$H_y(z) = \frac{(1+z^{-1})(1-z^{-1})}{(1-0.99e^{j2\pi f_y}z^{-1})(1-0.99e^{-j2\pi f_y}z^{-1})}$$

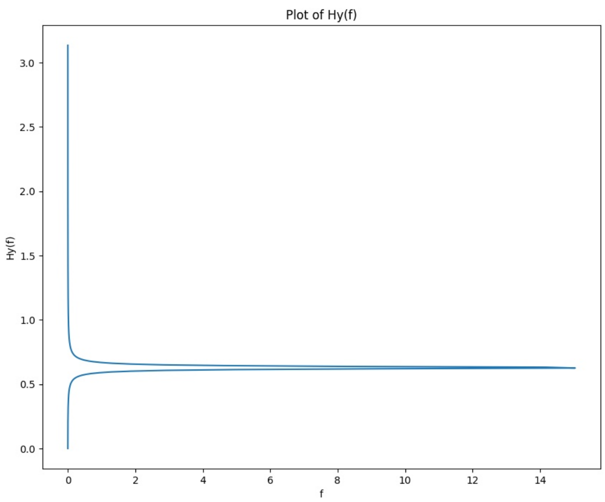
$$P_1 = 0.99e^{j2\pi f_x}, \quad 0.99e^{-j2\pi f_x}$$

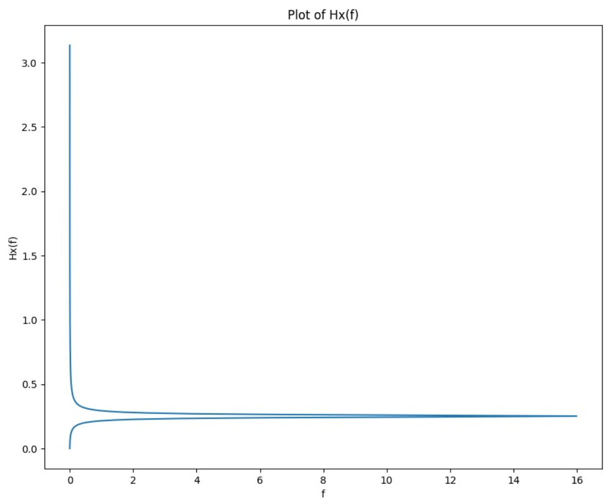
$$P_1 = 0.99\cos(2\pi f_x) + j\,0.99(2\pi f_x)$$

$P_2 =$ Same but $f_x \mapsto f_y$

Plots!

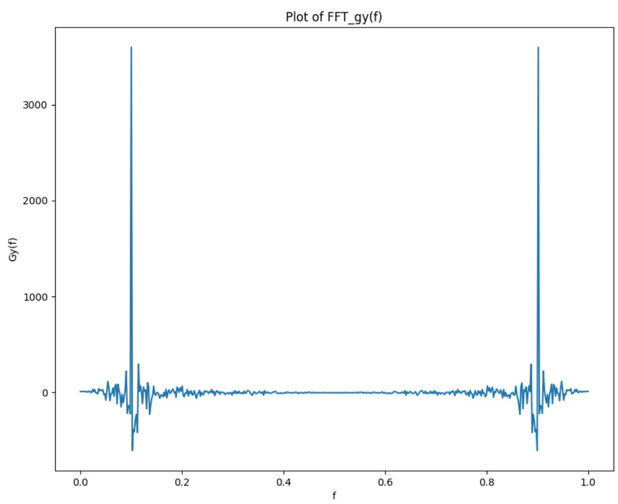


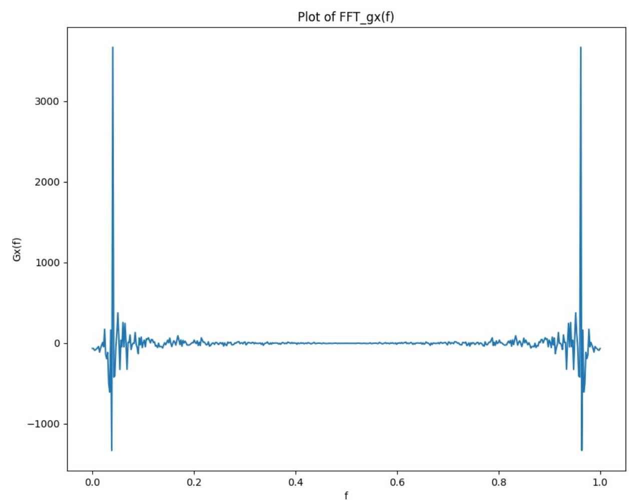


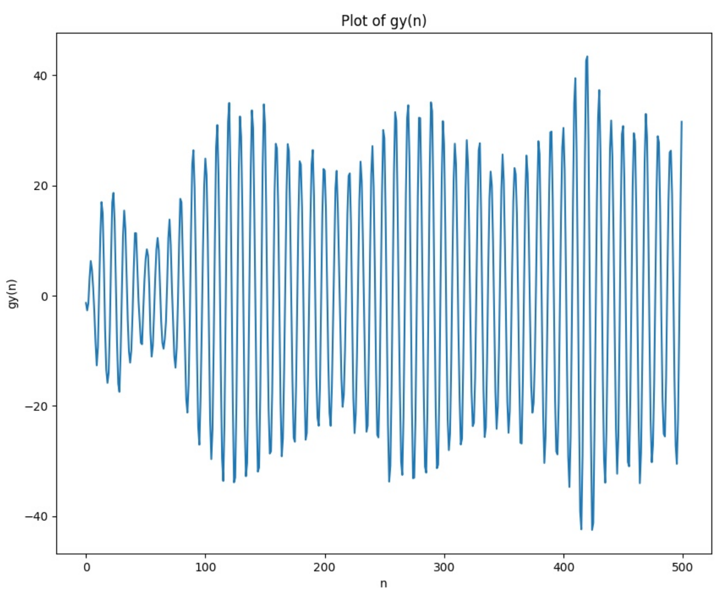


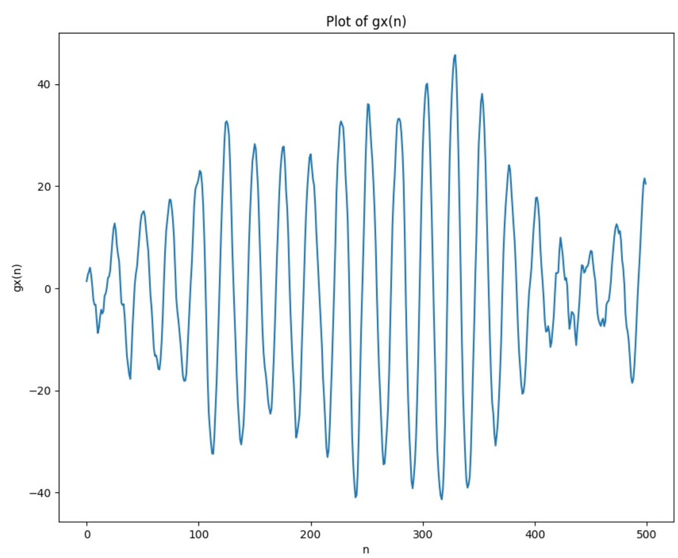
These are scaled wrong but
I cant find out why?

c)









- As we can see above from $g_x(n)$ and $G_x(f)$, the filter filters out most other frequencys and "only" lets f_x thru. The same can be said for $g_y(n)$ and $G_y(f)$ but for f_y. So it works as expected.

d) Combining the two filters gives:

$$|H(f)| = \left| \frac{(1+z^{-1})(1-z^{-1})}{(1-0.99e^{j2\pi f_x}z^{-1})(1-0.99e^{-j2\pi f_x}z^{-1})} + \frac{(1+z^{-1})(1-z^{-1})}{(1-0.99e^{j2\pi f_p}z^{-1})(1-0.99e^{-j2\pi f_p}z^{-1})} \right|$$

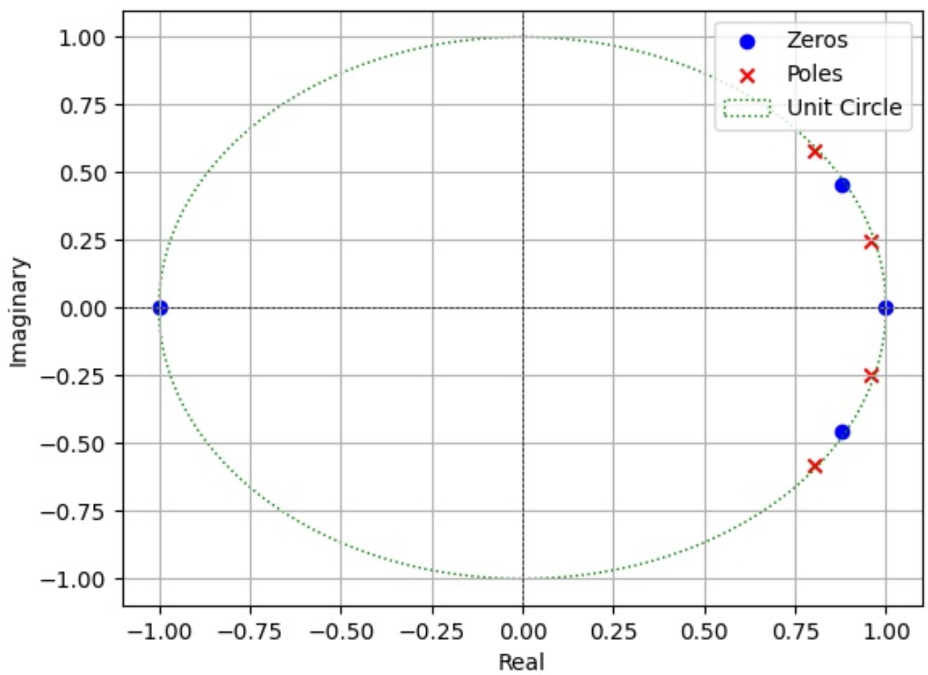
$$= \frac{(1+z^{-1})(1-z^{-1})}{(1-p_x z^{-1})(1-p_x^* z^{-1})} + \frac{(1+z^{-1})(1-z^{-1})}{(1-p_p z^{-1})(1-p_p^* z^{-1})}$$

$$= \frac{(1+z^{-1})(1-z^{-1})(1-p_y z^{-1})(1-p_y^* z^{-1}) + (1+z^{-1})(1-z^{-1})(1-p_x z^{-1})(1-p_x^* z^{-1})}{(1-p_x z^{-1})(1-p_x^* z^{-1})(1-p_y z^{-1})(1-p_p^* z^{-1})}$$

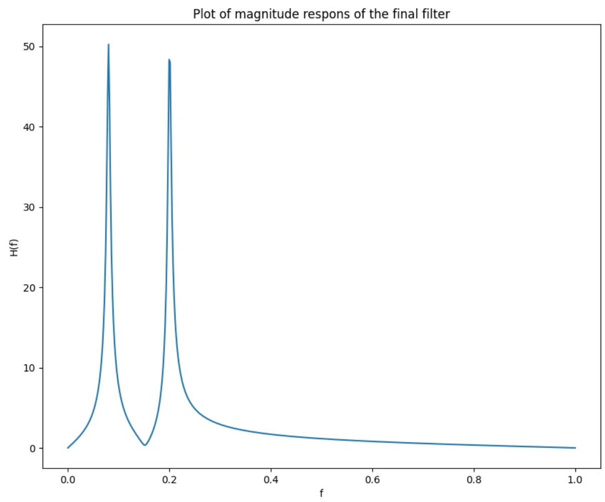
$$= \frac{(1+z^{-1})(1-z^{-1})\left((1-p_y z^{-1})(1-p_p^* z^{-1}) + (1-p_x z^{-1})(1-p_x^* z^{-1})\right)}{(1-p_x z^{-1})(1-p_x^* z^{-1})(1-p_y z^{-1})(1-p_p^* z^{-1})}$$

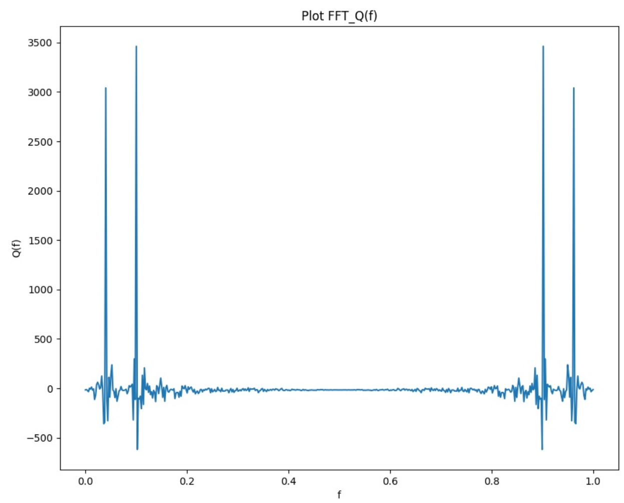
We can see we have two new Zrees and all the same poles. The two zeroes are kinda nasty to find with math so i do this with the help of computer.

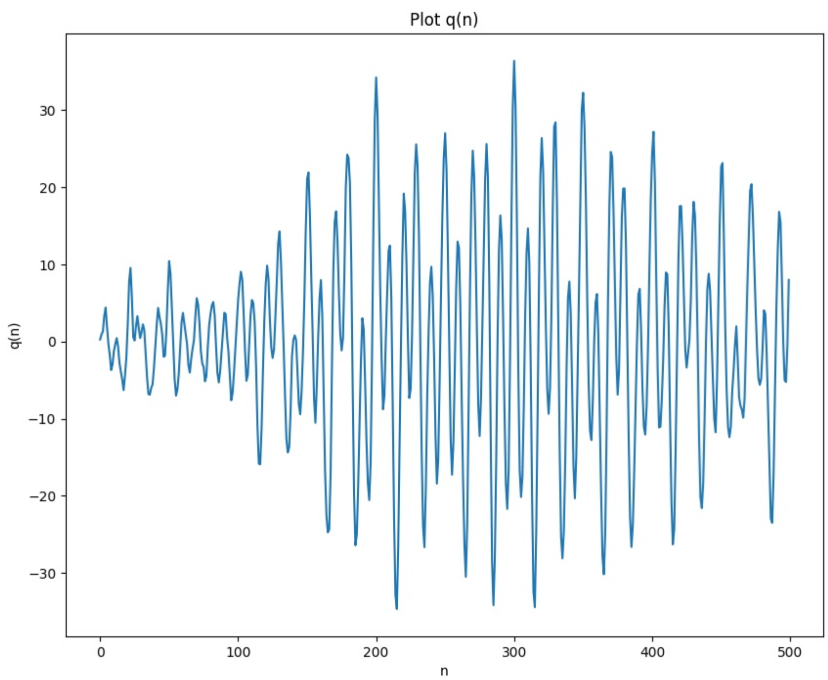
Plots:



- We can see from the plot we got 2 new Complex conjugant zeroes as expected.







As we can see the filter works as expected and we have successfully filtered out the noise of e(n). This is clear from the frequency domain Q(f).