



1) Hvis vi kommer til \mathcal{T} , d , c kan vi $\cos(\theta)$

$\cos(\theta) = \frac{\text{højde}}{\text{hypotenus}}$ / Tidsforsinkeligen mellem de to signaler er givet som $\mathcal{T} = \frac{X}{c} = \frac{d \cos(\theta)}{c}$

$$\Rightarrow \frac{\mathcal{T} c}{d} = \cos(\theta)$$

2)

a) $f = \frac{c \mathcal{T}}{d}$ /

Relativ usikkerhed:

$$\left| \frac{\Delta \mathcal{T}}{\mathcal{T}} \right|_{\text{mækl}} = 0,01 \quad / \quad \left| \frac{\Delta d}{d} \right|_{\text{mækl}} = 0,02$$

$$\Delta f = \left| \frac{\partial f}{\partial \gamma} \cdot \Delta \gamma \right|_{\text{max}} + \left| \frac{\partial f}{\partial d} \cdot \Delta d \right|_{\text{max}}$$

$$= \left| \frac{c}{d} \cdot \Delta \gamma \right|_{\text{max}} + \left| \frac{-c\gamma}{d^2} \cdot \Delta d \right|_{\text{max}}$$

$$= \left| \frac{c\gamma}{d} \cdot \frac{\Delta \gamma}{\gamma} \right|_{\text{max}} + \left| -\frac{c\gamma}{d} \cdot \frac{\Delta d}{d} \right|_{\text{max}}$$

$$\Delta f = \left| f \cdot \frac{\Delta \gamma}{\gamma} \right|_{\text{max}} + \left| f \cdot \frac{\Delta d}{d} \right|_{\text{max}}$$

$$= f \left(\left| \frac{\Delta \gamma}{\gamma} \right| + \left| \frac{\Delta d}{d} \right| \right)$$

$$\left| \frac{\Delta f}{f} \right|_{\text{max}} = 0,01 + 0,02 = 0,03 = 3\%$$

b) Siden den relative usikkerheten beregnet i forrige oppgave er konstant så er den uavhengig av d, c, σ osv.

c)

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/usr/bin/python3 "/Users/Eier/Documents/NTNU/6
Eier@dhcp-10-24-39-85 TTT4280 % /usr/bin/pytho
ppgave2.py"
The max value is: 0.030007698464872057
The min value is: 0.02802826274418313
The relative uncertainty is: 0.03
Eier@dhcp-10-24-39-85 TTT4280 %
```

Stemmer bra med beregnet teoretisk verd., men stemmer ikke alltid.

Oppgave 3)

$$\theta = \cos^{-1} t = \cos^{-1} \left(\frac{c\sigma}{d} \right)$$

$$\begin{aligned} a) \quad \Delta\theta &= \left| \frac{\partial\theta}{\partial\sigma} \cdot \Delta\sigma \right| + \left| \frac{\partial\theta}{\partial d} \cdot \Delta d \right| \\ &= \left| \frac{\sigma \cdot c/d \cdot \Delta\sigma}{\sqrt{1 - \left(\frac{c\sigma}{d}\right)^2} \cdot \sigma} \right| + \left| -\frac{1}{\sqrt{1 - \left(\frac{c\sigma}{d}\right)^2}} \cdot -\frac{c\sigma}{d^2} \cdot \Delta d \right| \end{aligned}$$

$$= \left| \frac{\frac{\gamma c}{d} \cdot \frac{\Delta \gamma}{\gamma}}{-\sqrt{1 - \left(\frac{c\gamma}{d}\right)^2}} + \frac{\frac{c\gamma}{d} \cdot \frac{\Delta d}{d}}{\sqrt{1 - \left(\frac{c\gamma}{d}\right)^2}} \right|$$

$$\cos(\theta) = \frac{c\gamma}{d}$$

$$= \left| \frac{\cos(\theta)}{-\sqrt{1 - \cos^2(\theta)}} \cdot \frac{\Delta \gamma}{\gamma} + \frac{\cos(\theta)}{\sqrt{1 - \cos^2(\theta)}} \cdot \frac{\Delta d}{d} \right|$$

$$= \left| \frac{\cos(\theta)}{\sqrt{\sin^2(\theta)}} \cdot q_{01} + \frac{\cos(\theta)}{\sqrt{\sin^2(\theta)}} \cdot q_{02} \right|$$

$$\Delta \theta = \left| \frac{\cos(\theta)}{\sin(\theta)} \right| \left(|q_{01} + q_{02}| \right)$$

$$\left| \frac{\Delta \theta}{\theta} \right|_{\max} = \frac{1}{\tan(\theta) \cdot \theta} (0,03) = \frac{0,03}{\tan(\theta) \theta}$$

b) D_{eff} er uavhengig av $\theta = \cos^{-1}\left(\frac{c}{d}\right)$. D_{eff} er altså uavhengig av J, d, θ .
 D_{eff} relativ feil er ∞ når $\theta \rightarrow 0$.

c)

by implementing nonpreconditioning
 The max value is: 2.499740894933999

Task 9)

$$\frac{\sigma_j}{j} = 0,01, \quad \frac{\sigma_d}{d} = 0,02, \quad \sigma_j = 0,01j, \quad \sigma_d = 0,02d$$

$$\sigma_\theta^2 = \sigma_j^2 \cdot \left(\frac{\partial \theta}{\partial j} \right)^2 + \sigma_d^2 \left(\frac{\partial \theta}{\partial d} \right)^2$$

$$= \sigma_j^2 \left(\frac{\frac{c/d}{\sqrt{1 - \left(\frac{cj}{d}\right)^2}}}{\left(\frac{cj}{d}\right)^2} \right)^2 + \sigma_d^2 \left(\frac{\frac{cj/d^2}{\sqrt{1 - \left(\frac{cj}{d}\right)^2}}}{\left(\frac{cj}{d}\right)^2} \right)^2$$

$$= (0,01j)^2 \left(\frac{c^2/d^2}{1 - \left(\frac{cj}{d}\right)^2} \right) + (0,02d)^2 \left(\frac{c^2j^2/d^4}{1 - \left(\frac{cj}{d}\right)^2} \right)$$

$$= (0,01^2 + 0,02^2) \left(\frac{c^2j^2/d^2}{1 - \left(\frac{c^2j^2}{d^2}\right)} \right), \quad \cos^2(\theta) = \frac{c^2j^2}{d^2}$$

$$= (0,01^2 + 0,02^2) \frac{\cos^2(\theta)}{1 - \cos^2(\theta)}$$

$$\sigma_\theta^2 = (0,0005) \frac{\cos^2(\theta)}{\sin^2(\theta)}$$

$$\sqrt{\sigma^2} = \sqrt{\frac{0,0005}{\tan^2(\theta)}}$$

$$\frac{\sigma}{\theta} = \sqrt{\frac{0,0005}{\tan^2(\theta)}} \cdot \frac{1}{\theta} = \frac{0,022}{\theta \cdot \tan(\theta)}$$
