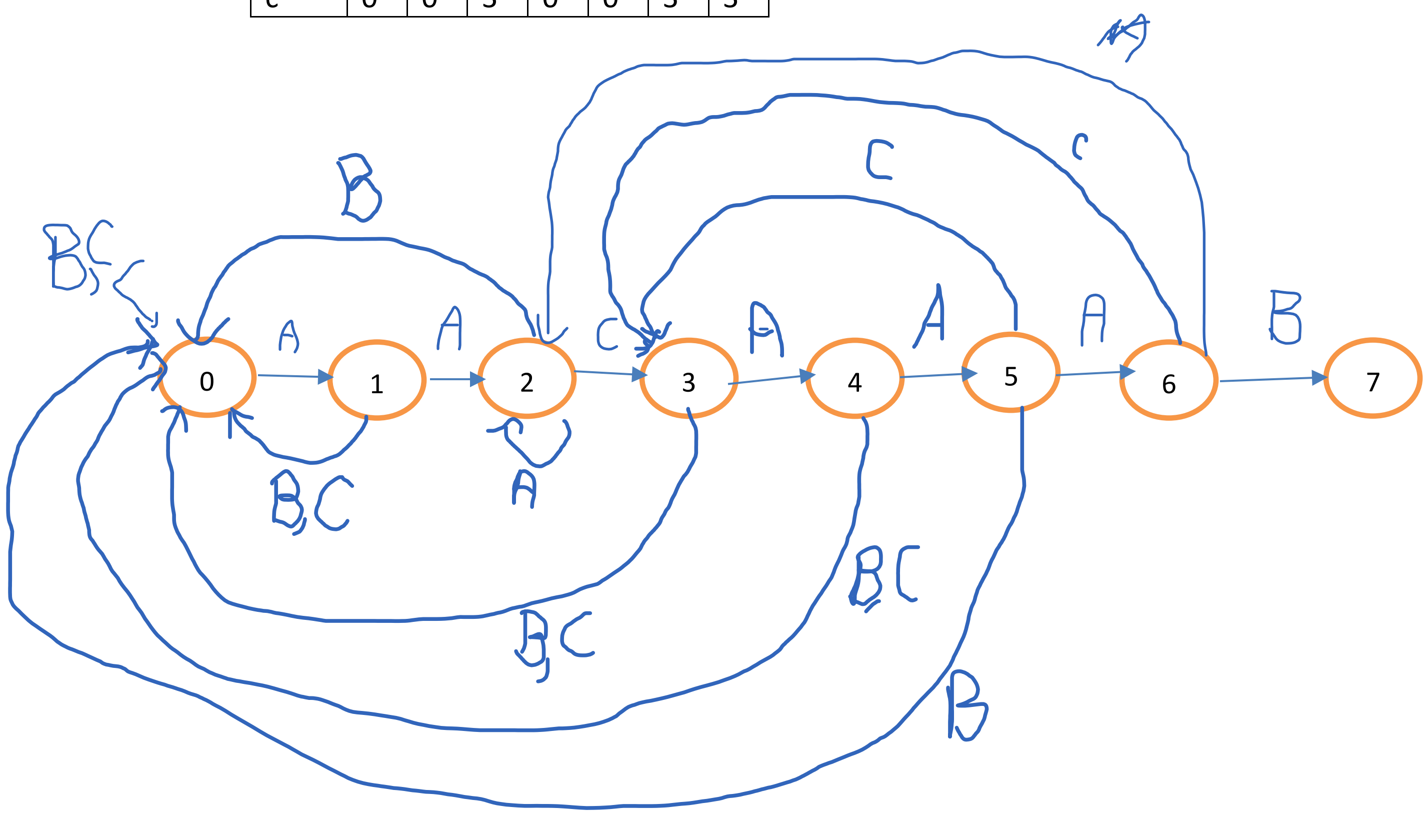
Assignment 4

1. **Draw the KMP DFA for the following pattern string: AACAAAB.**

State	0	1	2	3	4	5	6
a	1	2	2	4	5	6	2
b	0	0	0	0	0	0	7
c	0	0	3	0	0	3	3



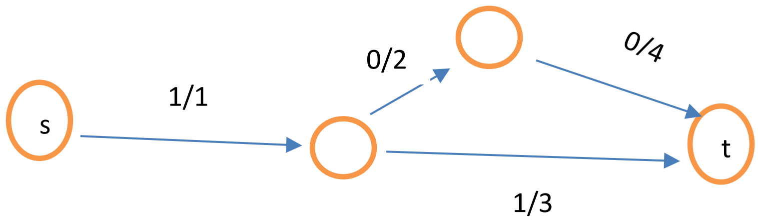
2. **Construct an example where the Boyer-Moore algorithm performs poorly**

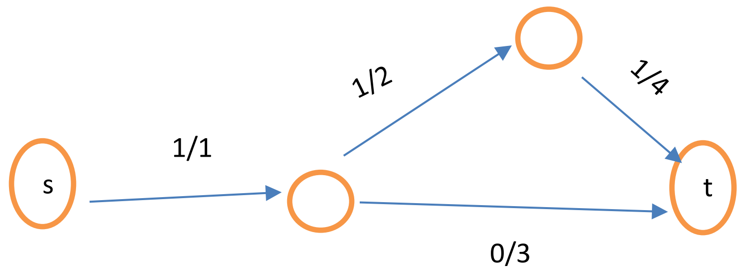
Text = ddcdcdcd ddcdcdcd ddcdcdcd ddcdcdcd . . .
Pattern = cdcdcdcd

We can see that the majority of the shifts are shift by two using the Boyer-Moore algorithm. The running time will be O(nm).

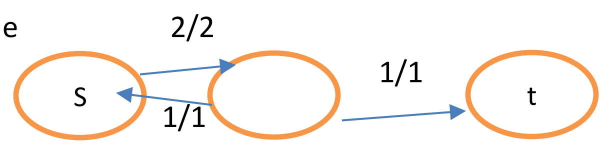
3. **True or false. If true, provide a short proof. If false, give a counterexample.**

a-False



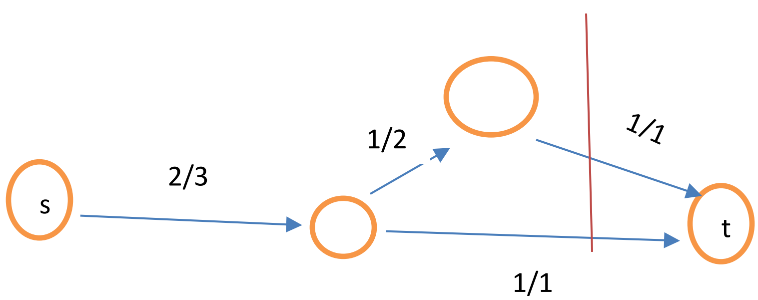


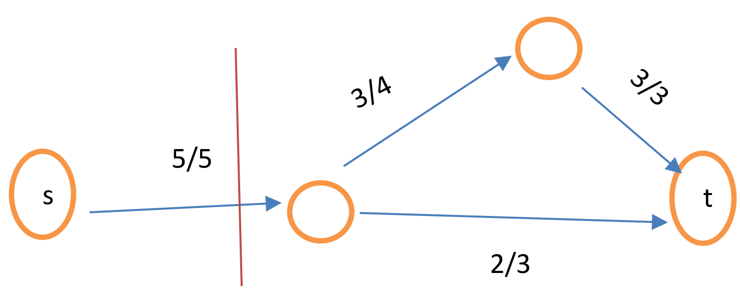
b-True Example



c-False

let k = 2





4- **Show how Ford-Fulkerson algorithm can be used to find a maximum flow in a node capacitated network.**

For every vertex v other than the source and sink, we split it into two vertices v_{in} and v_{out}, adding one edge (v_{in}, v_{out}) of capacity c(v), and then converting every edge (u, v) to an edge (u, v_{in}) and every edge (v, w) to an edge (v_{out}, w). Now if there is there is a flow of value r in this new graph, then there is a flow of value r in the original graph that respects all node capacities: we simply use the flow obtained by contracting all edges of the form(v_{in}, v_{out}).Conversely, if there is a flow of value r in the original graph, then we can use it to construct a flow of value r in the new graph. The flow using v in the original graph will now pass through the edge(v_{in}, v_{out}), and it will not exceed this edge capacity due to the node capacity condition in the original graph. Thus, we see that this problem is reduced to the standard maximum flow. Since we can use **Ford-Fulkerson algorithm** to solve the standard maximum flow, we can use it to find **a maximum flow in a node-capacitated network.**

5- Two paths in a graph are edge-disjoint if they have no edge in common.
Disjoints Path Problem: Given a digraph G = (V, E) and two nodes s and t, find the maximum number of edge-disjoint s-t paths. Show how this problem can be reduced to the max flow problem.

Let's assign unit capacity to every edge. Suppose maximum flow value is k. Suppose, there exists 0-1 flow f of value k. Consider edge (s, u) with f(s, u) = 1. By conservation, there exists an edge (u, v) with f(u, v) = 1. Continue until reach t, always choosing a new edge with Produces k (not necessarily simple) edge-disjoint paths. Thus, the maximum number of edge-disjoint s-t paths equals to the maximum flow value k.