CS140B Mechanics

American University of Armenia
Course Project
Springs

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1 `spring::move()`

From the textbook, Section 3.7, we have

$$x(t) = A\sin\omega t + B\cos\omega t \tag{1}$$

From this, we form a system

$$\begin{cases} x(0) = x_0 = A\sin(\omega t_0) + B\cos(\omega t_0) \\ v(0) = \frac{\partial x}{\partial t} = v_0 = A\omega\cos(\omega t_0) - B\omega\sin(\omega t_0) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} B = \frac{x_0 - A\sin(\omega t_0)}{\cos(\omega t_0)} \Rightarrow \\ \frac{v_0}{\omega} = A\cos(\omega t_0) - B\sin(\omega t_0) = A\cos(\omega t_0) - \frac{\sin(\omega t_0)(x_0 - A\sin(\omega t_0))}{\cos(\omega t_0)} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} B\cos(\omega t_0) = x_0 - A\sin(\omega t_0) \\ \frac{v_0\cos(\omega t_0)}{\omega} = A\cos(\omega t_0)^2 - \sin(\omega t_0)(x_0 - A\sin(\omega t_0)) = \\ \qquad = A\cos(\omega t_0)^2 - x_0\sin(\omega t_0) + A\sin(\omega t_0)^2 = \\ \qquad = A(sin(\omega t_0)^2 + \cos(\omega t_0)^2) - x_0\sin(\omega t_0) = \\ \qquad = A - x_0\sin(\omega t_0) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A = \frac{v_0\cos(\omega t_0)}{\omega} + x_0\sin(\omega t_0) \\ B\cos(\omega t_0) = x_0 - \left(\frac{v_0\cos(\omega t_0)}{\omega} + x_0\sin(\omega t_0)\right)\sin(\omega t_0) = \\ \qquad = x_0 - x_0\sin(\omega t_0)^2 - \frac{v_0\sin(\omega t_0)\cos(\omega t_0)}{\omega} = \\ \qquad = x_0(1 - \sin(\omega t_0)^2) - \frac{v_0\sin(\omega t_0)\cos(\omega t_0)}{\omega} = \\ \qquad = x_0\cos(\omega t_0)^2 - \frac{v_0\sin(\omega t_0)\cos(\omega t_0)}{\omega} \end{cases} \Rightarrow$$

$$\begin{cases} A = x_0\sin(\omega t_0) + \frac{v_0\cos(\omega t_0)}{\omega} \\ B = x_0\cos(\omega t_0) - \frac{v_0\sin(\omega t_0)}{\omega} \end{cases} \tag{2}$$

Therefore, by plugging (2) into (1), we derive

$$x(t) = \left(x_0\sin(\omega t_0) + \frac{v_0\cos(\omega t_0)}{\omega}\right)\sin(\omega t) + \left(x_0\cos(\omega t_0) - \frac{v_0\sin(\omega t_0)}{\omega}\right)\cos(\omega t) \tag{3}$$

where $\omega = \omega(k, m) = \sqrt{\frac{k}{m}}$ (4) for a spring with stiffness k and mass m.

Based on these formulas, we create the following member functions for the `spring` class:

- `spring::move()` based on (1) and (3)
- `spring::A()` based on (2)
- `spring::B()` based on (2)
- `spring::omega()` based on (4)

2 `spring::in_series()`, `spring::in_parallel()`

We make use of formulas derived during lectures and obtained as a solution of Problem 3.19:

$$k_{eff}^{\text{parallel}} = k_1 + k_2$$

$$k_{eff}^{\text{series}} = \frac{k_1 k_2}{k_1 + k_2}$$