Chapter 8 Illumination and Surface Rendering method

Illumination and surface rendering model:

- Once visible surface has been identified by hidden surface algorithm, a shading model is used to compute the intensities and color to display for the surface. For realistic displaying of 3d scene, it is necessary to calculate appropriate color or intensity for that scene.
- *An **illumination model** is also called **lighting model** and sometimes called as a **shading model** which is used to calculate the intensity of light that we should see at a given point on the surface of an object.*
- *A **surface-rendering algorithm** uses the intensity calculations from an illumination model.*
- A **surface-rendering method** determine the pixel colors for all projected positions in a scene

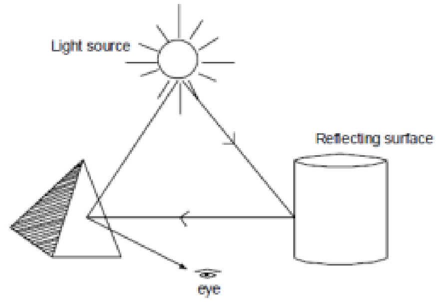
Illumination models involve number of factors like:

- Optical properties of the surfaces (transparency, reflectivity, surface texture)
- Relative positions of the surfaces in a scene.
- Color and position of the light sources and
- Position and orientation of the viewing plane.

Light Source:

- Object that radiates energy are called light sources, such as sun, lamp, bulb, fluorescent tube etc.
- Sometimes light sources are referred as light emitting object and light reflectors. Generally light source is used to mean an object that is emitting radiant energy e.g. Sun.

Total Reflected Light = Contribution from light sources + contribution from reflecting surfaces



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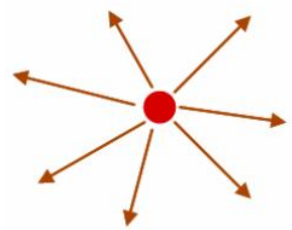
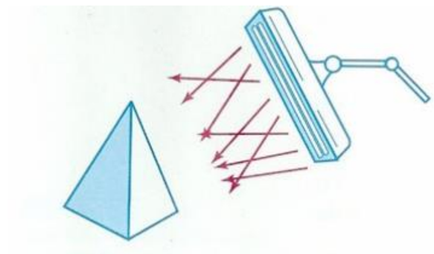
1. **Point Source:** when light source model is a reasonable approximation for sources whose dimensions are small compared to the size of objects in a scene. e.g. Sun
2. **Distributed Light Source:** Area of source is not small compared to the surfaces in the scene e.g. Fluorescent lamp


Point Light Source


Distributed Light Source

Light Reflection:

When light is incident on opaque surface part of it is reflected and part of it is absorbed.

$$\therefore \mathbf{I = A+R}$$

The amount of incident light reflected by a surface depends on the type of material. Shining material reflects more incident light and dull surface absorbs more of the incident light. For transparent surfaces, some of the incident light will be reflected and some will be transmitted through the material.

1. **Diffuse Reflection:** It is the reflection of light from a surface such that a ray incident on the surface is scattered at many angles rather than at just one angle. It exhibit Lambertian reflection, meaning that there is equal luminance when viewed from all directions lying in the half-space adjacent to the surface.
2. **Specular Reflection:** It is the mirror-like reflection of waves such as light, from a surface. In this process, each incident ray is reflected at the same angle to the surface normal as the incident ray, but on the opposing side of the surface normal is the plane formed by incident and reflected rays. The law of reflection states that for each incident ray the angle of incidence equals the angle of reflection, and the incident, normal, and reflected directions are coplanar.

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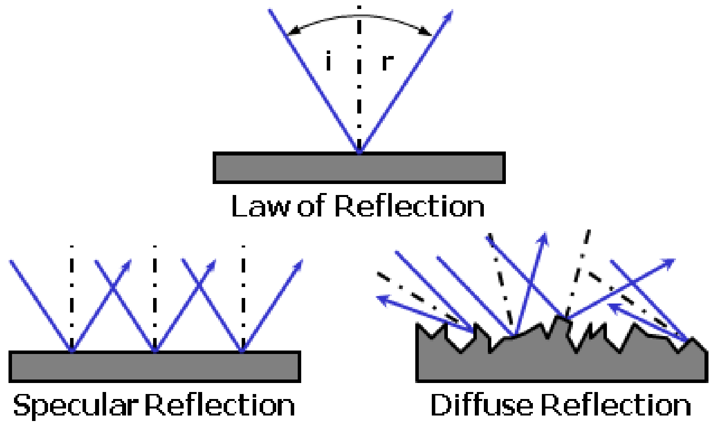


Reflection

Illumination models:

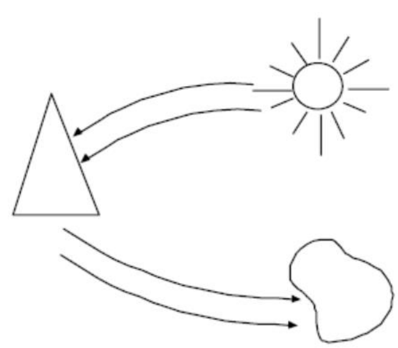
Illumination models are used to calculate light intensities that we should see at a given point on the surface of an object. Lighting calculations are based on the optical properties of surfaces, the background lighting conditions and the light source specifications. All light sources are considered to be point sources, specified with a co-ordinate position and an intensity value (color).

Some illumination models are:

1. **Ambient Light**
2. **Diffuse Reflection:**
3. **Specular Reflection and Phong Model**

1. Ambient Light:

- A surface that is not exposed directly to light source still will be visible if nearby objects are illuminated.
- The combination of light reflections form various surfaces to produce a uniform illumination is called ambient light or ***background light***.
- Ambient light means the light that is already present in a scene, before any additional lighting is added. It usually refers to natural light, either outdoors or coming through windows etc. It can also mean artificial lights such as normal room lights.



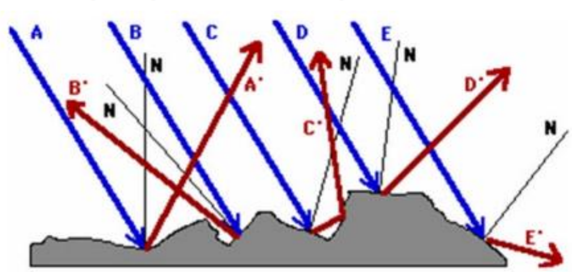
- Ambient light has no spatial or directional characteristics and amount on each object is a constant for all surfaces and all directions. But the intensity of the reflected light for each surface depends on the optical properties of the surface.
- ***If I_a is the amount of ambient light incident on any surface, the ambient light reflection is given by ambient illumination equation,***

$$I_{amb} = k_a I_a$$

- Where **K_a** is **ambient reflectivity** or **ambient reflection coefficient**. The amount of light reflected from an object's surface is determined by **K_a**, which ranges from 0 to 1. It is a material property.

2. **Diffuse Reflection:**
   - Surfaces are rough
   - Incident light is scattered with equal intensity in all directions
   - Surfaces appear equally bright from all direction
   - Such surfaces are called ideal diffuse reflectors (also referred to as Lambertian reflectors)
   - Color of an object is determined by the color of the diffuse reflection of the incident light.
   - If any object surface is red then there is a diffuse reflection for red component of light and all other components are absorbed by the surface.
   - The ***diffuse-reflection coefficient***, or ***diffuse reflectivity***, **K_d** (varying from 0 to 1) define the fractional amount of the incident light that is diffusely reflected.
   - ***The parameter K_d (actually function of surface color) depends on the reflecting properties of material so for highly reflective surfaces, the K_d nearly equals to 1 and for a very dull surface it is nearly equals to 0.***
   - If a surface is exposed only to ambient light, we can express the intensity of the diffuse reflection at any point on the surface as:

$$\mathbf{I_{amb,Diff}} = \mathbf{k_d} \cdot \mathbf{I_a}, \text{ (where } I_a = \text{intensity of ambient light)}$$



Diffuse Reflection From a Surface

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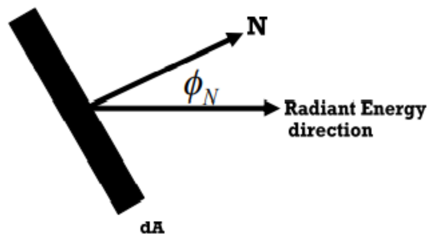
Ideal Diffuse Reflector:

The radiant energy from any small surface dA in any direction relative to surface normal is proportional to **cos Φ**. That is, brightness depends only on the angle **Φ** between the light direction L and the surface normal N.

Light intensity α cos Φ.

$$I = dA \cos\phi_N$$



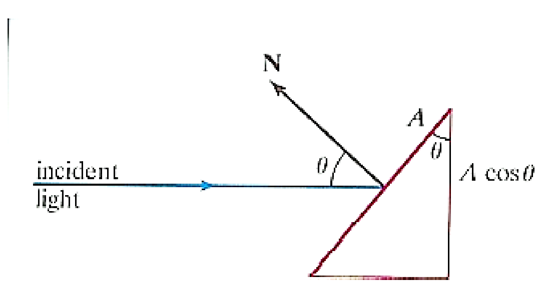
Thus, for Lambertian reflection the intensity of light is the same over all viewing direction. Even though there is equal light scattering in all directions from a perfect diffuse reflector, the brightness of the surface does depend on the orientation of the surface relative to the light source.

Surface perpendicular to the direction of incident light appears brighter than the one with some oblique angle to the direction of the incoming light.

- Light intensity is independent of angle of reflection.
- Light intensity depends on angle of incidence

If θ is angle of incidence between the incoming light direction and the surface normal then the projected area of a surface path perpendicular to light direction is proportional to **cosθ.**

When cosΘ is
+ve surface is illuminated
-ve light source is behind the surface



If I_l is the intensity of the point light source, then the diffuse reflection equation for a point on the surface is,

$$I_{l,diff} = K_d I_l \cos\theta.$$

A surface is illuminated by a point source only if the angle of incidence is in the range 0^0 to 90^0 for which $\cos\theta$ is in the range 0 to 1. When $\cos\theta$ is negative, the light source is behind the surface.

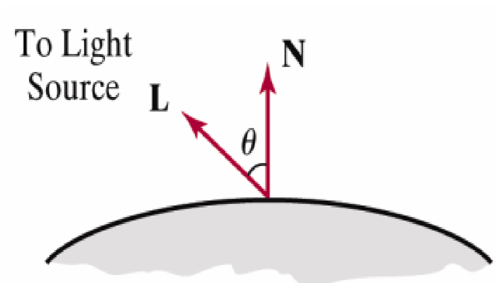
If $\vec{N}$ is the unit normal vector to a surface and $\vec{L}$ is the direction vector to the point source from a position on the surface then,

$$\vec{N}.\vec{L} = \cos\theta$$

$$\cos\theta = \frac{\vec{N}.\vec{L}}{/\vec{N}/./\vec{L}/}$$

and

$$I_{l,diff} = K_d I_l (\vec{N}.\vec{L})$$



I_l : the intensity of the light source

k_d : diffuse reflection coefficient,

N : the surface normal (unit vector)

L : the direction of light source, (unit vector)

We can combine the ambient and point source in intensity calculations to obtain an expression for the total diffuse reflection

$$I_{diff} = K_a I_a + K_d I_l (\vec{N}.\vec{L})$$

Where both I_a and K_d depends upon surface material properties and are assigned values in the range from 0 to 1.

$$I = \begin{cases} k_a I_a + k_d I_l (N \cdot L) & \text{if } N \cdot L > 0 \\ k_a I_a & \text{if } N \cdot L \le 0 \end{cases}$$

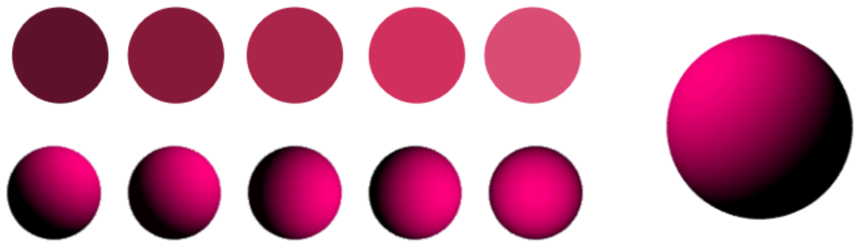
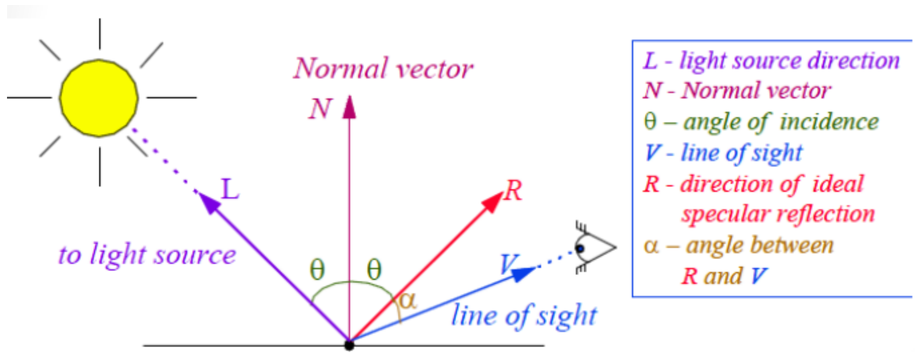


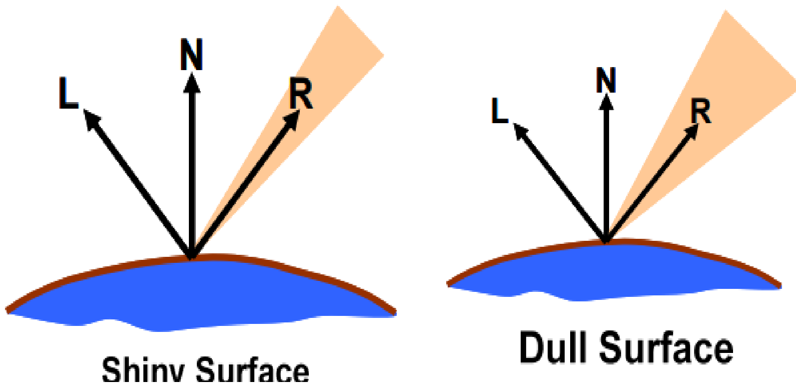
Fig: sphere illuminated with different intensity ambient light
Illuminated with varying direction light source

Specular reflection and the Phong model:

- When we look at an illuminated shiny surface, such as polished metal, an apple, we see a highlight or bright spot, at certain viewing direction. Such phenomenon is called specular reflection,
- It is the result of total or near total reflection of the incident light in a concentrated region around the " **specular reflection angle = angle of incidence**".



- Perfect reflector (mirror) reflects all lights to the direction where angle of reflection is identical to the angle of incidence.
- It accounts for the **highlight.**
- Shiny surface have a narrow specular-reflection range, and dull surfaces have a wider reflection range.

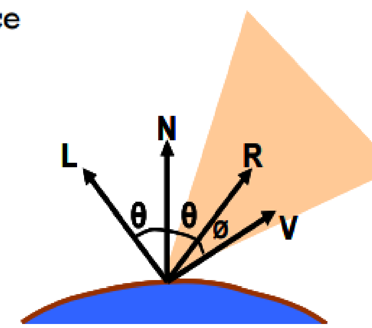


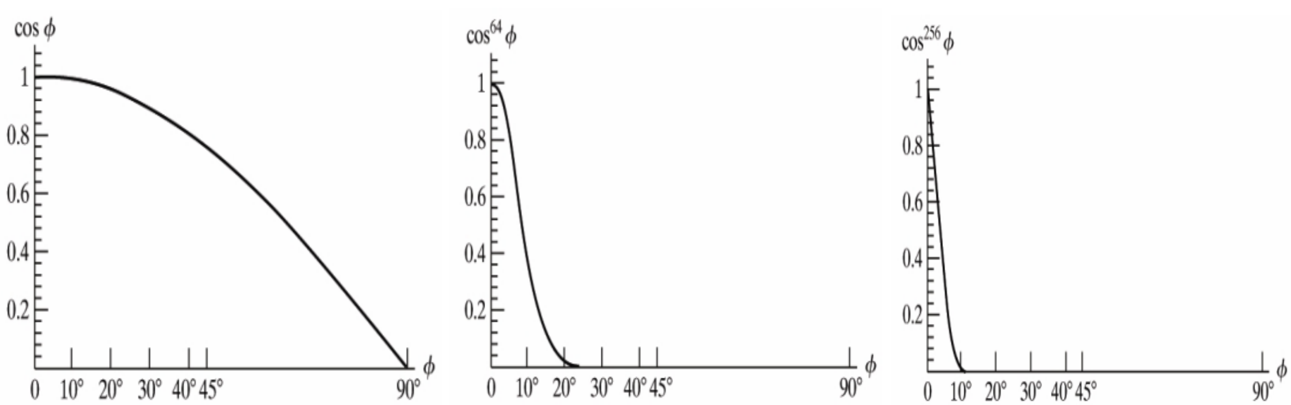
- Phong model, developed by phong Bui-Tuong, is used to calculate the specular reflected range, which sets the **intensity of specular reflection proportional to** $\cos^{n_s}\phi$
- Ø can be assigned values in the range 0^0 to 90^0 (cos Ø = 0 to 1)
- Shiny surface have narrow Ø and dull surface wider Ø.
- **Specular reflection parameter** n_s is determined by type of surface that we want to display.
- A very shiny surface is modeled with a large value for n_s (say 100), and duller surface is assigned smaller values (say 1) and for a perfect reflector, n_s is infinite.

Monochromatic specular intensity variations can be approximated using **specular-reflection coefficient**, $W(\boldsymbol{\theta})$ for each surface

$$I_{spec} = w(\theta) I_l \cos^{n_s}\phi$$

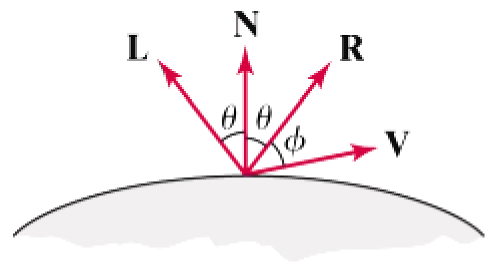
At $\boldsymbol{\theta} = 90^0$, $w(\boldsymbol{\theta}) = 1$ → all incident light is reflected





$$I_{spec} = k_s I_l \cos^{n_s} \phi = k_s I_l (R \cdot V)^{n_s}$$

N = unit normal Vector
L = unit vector towards light source
R = unit vector to specular reflection direction
V = unit vector towards viewer
ϕ = angle between R and V



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Specular reflection angle equals angle of incidence θ.

I_l : intensity of the incident light
k_s : color-independent specular coefficient
n_s : specular reflection parameter (depends upon surface)

- Specular reflection coefficient $\mathbf{K_s}$ is a material property. For some material, $\mathbf{K_s}$ varies depending on θ. $\mathbf{K_s = 1}$ if θ = 90.
- ***Calculating the reflection vector R***

$$\cos\phi = \frac{\vec{V}.\vec{R}}{|\vec{V}||\vec{R}|} = \vec{V}.\vec{R}$$

$$\text{so, } I_{spec} = K_s I_l (\vec{V}.\vec{R})^{n_s}$$

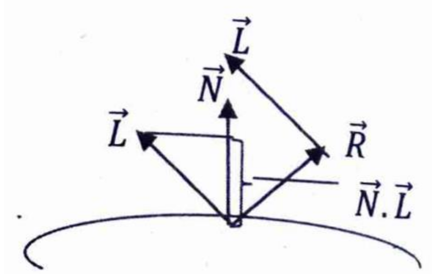


Figure Calculation of vector R by considering projections onto them direction of the normal vector N

The projection of $\vec{L}$ onto the direction of the normal vector is obtained with the dot product $\vec{N}.\vec{L}$.

(The vector projection of $\vec{L}$ onto $\vec{N}$ is $\frac{\vec{N}.\vec{L}}{|\vec{N}|^2}\vec{N}$

The scalar projection $\vec{L}$ onto $\vec{N}$ is $\frac{\vec{N}.\vec{L}}{|\vec{N}|}$)

So,

$$\vec{R} + \vec{L} = (2\vec{N}.\vec{L})\,\vec{N}$$

$$\vec{R} = (2\vec{N}.\vec{L})\,\vec{N} - \vec{L}$$

Combined diffuse and specular reflection,

Point Light Source:

$$\begin{aligned} I &= I_{diff} + I_{spec} \\ &= K_a I_a + k_d I_l (\vec{N}.\vec{L}) + k_s I_l (\vec{V}.\vec{R})^{n_s} \end{aligned}$$

Multiple light sources (n light sources)

$$I = K_a I_a + \sum_{i=1}^{n} I_{1i}[K_d(\vec{N}.\vec{L_i}) + K_s(\vec{V}.\vec{R_i})^{n_s}]$$

Intensity Attenuation:

- As radiant energy from a point source travels through space, its amplitude is attenuated by the factor $\mathbf{1/d^2}$, where d is the distance that the light has traveled.
- Which means a surface close to the light source (small d) receives a higher surface incident intensity from the source than a distant surface (large d).
- So, for realistic lighting effects, we should take into account the intensity attenuation, otherwise, it produces unrealistic effect as we will be illuminating all surfaces with the same intensity, no matter how far they might be from the light source.
- But simple point source illumination model does not always produce realistic picture, if we use the factor $\mathbf{1/d^s}$ to attenuate intensities, as it produces very little variation when d is large.
- Graphics package have compensated this problem by using universal quadratic attenuation function as the attenuation factor.

$$f(d) = \frac{1}{a_0 + a_1 d + a_2 d^2}$$

- Where d is the distance to the light source. And $\mathbf{a_0}$**,** $\mathbf{a_1}$ and $\mathbf{a_2}$ are properties of the light which are called “Constant attenuation”, “Linear attenuation”, and “quadratic attenuation” of the light source. By default $\mathbf{a_0}$ is one and $\mathbf{a_1}$**,** $\mathbf{a_2}$ are Zero, which means that there is no attenuation.

Using the attenuation function

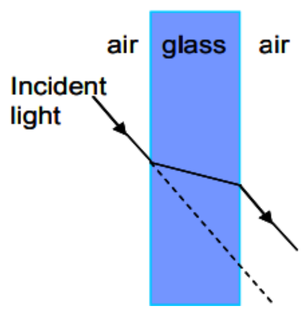
$$I = K_a I_a + \sum_{i=1}^{n} f(d_i) I_{1i}[K_d(\vec{N}.\vec{L_i}) + K_s(\vec{V}.\vec{R_i})^{n_s}]$$

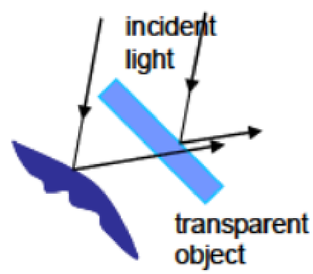
Chapter 8 Illumination and Surface Rendering method

Transparency:

- Transparent surface produces both reflected and transmitted light.
- Light intensity depends on relative transparency and position of light source or illuminated object behind or in front of the transparent surface.
- To model transparent surface, intensity contribution of light from various sources (illuminated objects) that are transmitted from the surface must be considered in the intensity equation.
- Both diffuse and specular reflection take place on transparent surface.
- Diffuse effects are important for partially transparent surfaces such as frosted glass
- The Snell's law is used to calculate the refracted ray direction

$$\sin\theta_r = \frac{\eta_i}{\eta_r}\sin\theta_i$$





Transmitted intensity I_{trans} through a transparent surface from a background object and Reflected intensity I_{refl} from the transparent surface with ***transparency coefficient*** k_t is given by

$$I = (1-k_t)I_{refl} + k_t I_{trans}$$

where $(1-k_t)$ is ***opacity factor***

Transparency effects Implementation:

1. *Process opaque objects first to determine depths for visible opaque surfaces.*
2. *Depth positions of the transparent objects are compared to the values previously stored in the depth buffer.*
3. *If any transparent surface is visible, its reflected intensity is calculated and combined with the opaque-surface intensity previously stored in the frame buffer.*
4. *Visible transparent surfaces are then rendered by combining their surface intensities with those of the visible and opaque surfaces behind them.*

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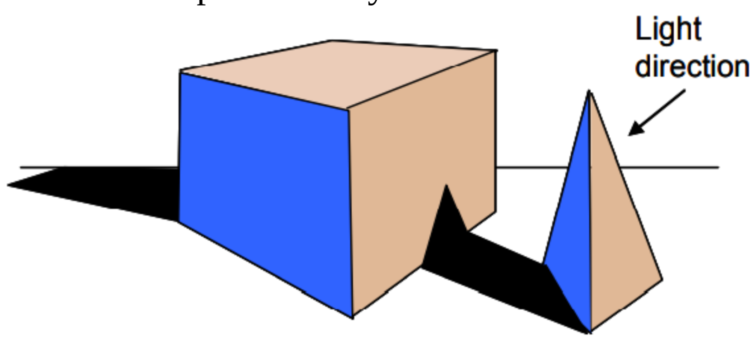
Shadow:

- Hidden surface method with light source at the view position can be used
- The shadow area for all light sources are determined and these shadows could be treated as a surface pattern arrays.



- Shadow can help to create realism. Without it, a cup, e.g., on a table may look as if the cup is floating in the air above the table.
- By applying hidden-surface methods with pretending that the position of a light source is the viewing position, we can find which surface sections cannot be "seen" from the light source => shadow areas.

Assigning Intensity Level:

- The intensity calculated by illumination model must be converted to the allowable intensity of a particular graphics system.
- The difference between intensities 0.20 and 0.22 should be perceived same as that of 0.80 and 0.88.
- The intensity level in a monitor should be spaced so that the ratio of successive intensities is constant.

for n+1 successive intensity levels

$$I_1/I_0 = I_2/I_1 = \ldots\ldots\ldots I_n/I_{n-1} = r$$

$$I_k = r^k I_0$$

also $I_n = 1 \rightarrow r = (1/I_0)^{1/n}$

so, $I_k = I_0^{(n-k)/n}$

- Lowest intensity value $\mathbf{I_0}$ depends on the characteristics of the monitor (I0 ranges from 0.005 to around 0.025).
- The highest intensity value is 1.
- For color system (blue color for example)

$$I_{Bk} = r_B^{\,k} I_{B0}$$

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Polygon Rendering Methods:

- Objects usually follows polygon-mesh approximation.
- Illumination model is applied to fill the interior of polygons.
- Two ways of polygon surface rendering:
    1. Single intensity for all points in a polygon
    2. Interpolation of intensities for each point in a polygon

Methods:

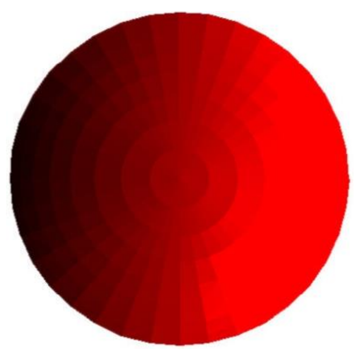
1. **Constant/Flat Shading**
2. **Gouraud Shading**
3. **Phong Shading**

1. Constant Shading:

- It is also called flat shading.
- It is fast and simple.
- A single intensity is calculated for each polygon and all points over the surface of the polygon are then displayed with the same intensity value.
- Useful for quickly displaying the general appearance of a curved surfaces.

PROCEDURE

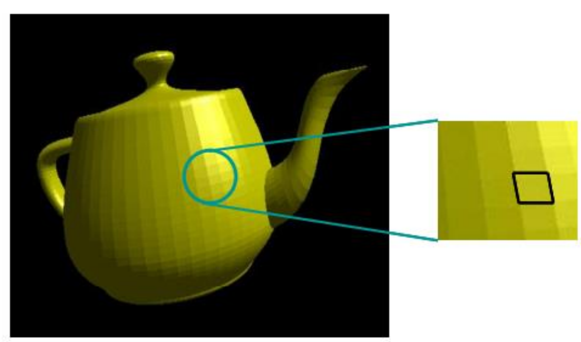
1. Take a point on the object surface and calculate the intensity
2. Render the surface with same intensity throughout the surface
3. Repeat above procedure for each polygon surface



ASSUMPTIONS

1. Object is a polyhedron
2. light sources should be sufficiently (i.e. **N.L** and attenuation function are constant)
3. Viewing position is sufficiently far (i.e. **V.R** is constant over the surface)

- The sharp intensity discontinuation is seen in the border between two polygons.



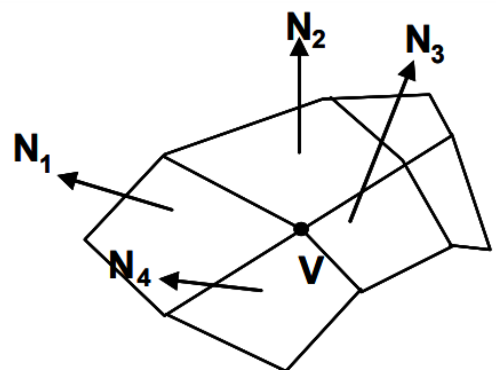
2. Gouraud Shading:

- It uses the intensity interpolation method.
- Renders a polygon surface by linearly interpolating intensity values across the surface.
- Intensity discontinuity at the edges of polygons is eliminated by matching intensity values of each polygon with adjacent polygons thus eliminating the intensity discontinuation that can occur in flat shading.
- Produces more realistic results, but requires consider more calculations.

Each polygon surface is rendered with Gouraud shading by performing the following calculations.

1. **Determine the average unit normal vector at each polygon vertex:**
   It is obtained by averaging the surface normal of all polygons sharing the vertex.

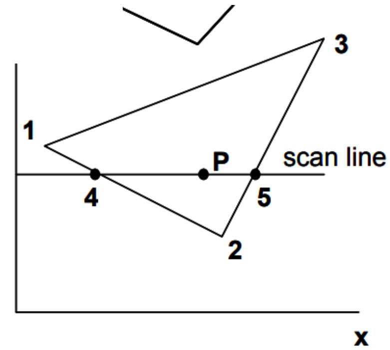
$$N_v = \frac{\sum_{k=1}^{n} N_k}{\left|\sum_{k=1}^{n} N_k\right|}$$



2. **Calculate each of the vertex intensities by applying an illumination model:**

$$\begin{aligned} I &= I_{diff} + I_{spec} \\ &= K_a I_a + k_d I_l (\vec{N}.\vec{L}) + k_s I_l (\vec{V}.\vec{R})^{n_s} \end{aligned}$$

3. **Linearly interpolate the vertex intensities over the polygon surface:**
   For each scan line, the intensity at the intersection of the scan line with a polygon edge is linearly interpolated from the intensities at the edge is endpoints.



- The intensity at point 4 is linearly interpolated from the intensities at vertices 1 and 2.
- The intensity at point 5 is linearly interpolated from intensities at vertices 2 and 3.
- An interior point P is then assigned an intensity value that is linearly interpolated from intensities at positions 4 and 5.

$$I_4 = \frac{y_4 - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y_4}{y_1 - y_2} I_2$$

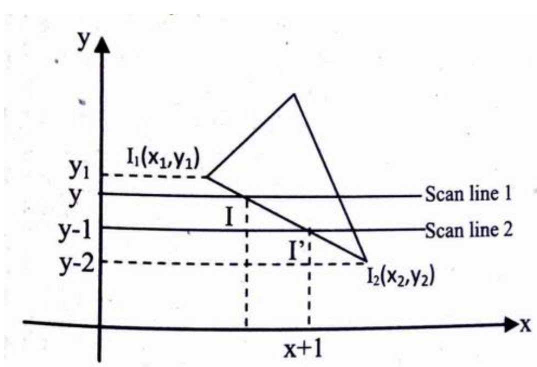
Similarly,

$$I5 = \frac{y5-y3}{y2-y3}*I2 + \frac{y2-y5}{y2-y3}*I3$$

Then, along the scan line between the polygon edges are obtained by interpolating intensities at the intersection of scan line and polygon edges

$$I_p = \frac{x_5 - x_p}{x_5 - x_4} I_4 + \frac{x_p - x_4}{x_5 - x_4} I_5$$

And, along the edge we make incremental calculations for intensity values



$$I = \frac{y-y2}{y1-y2}*I_1 + \frac{y1-y}{y1-y2}*I_2$$

$$
\begin{aligned}
I' &= \frac{(y-1)-y_2}{y_1-y_2}\times I_1+\frac{y_1-(y-1)}{y_1-y_2}\times I_2\\
&= \frac{y-y_2-1}{y_1-y_2}\times I_1+\frac{y_1-y+1}{y_1-y_2}\times I_2\\
&= \frac{y-y_2}{y_1-y_2}\times I_1-\frac{I_1}{y-y_2}+\frac{y_1-y}{y_1-y_2}\times I_2+\frac{I_2}{y_1-y_2}\\
&= I-\frac{I_1}{y_1-y_2}+\frac{I_2}{y_1-y_2}\\
&= I+\frac{I_2-I_1}{y_1-y_2}
\end{aligned}
$$

$$\text{then, } I' = I + \frac{I2-I2}{y1-y2}$$

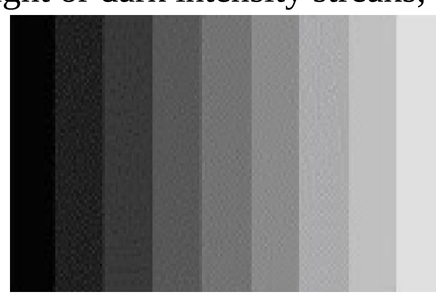
We make, similar calculations to obtain successive intensity values along horizontal line.

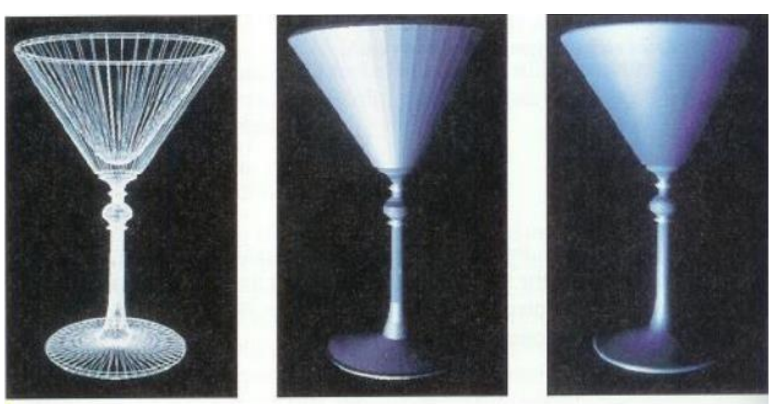
Advantages:

- Removes the intensity discontinuities associated with the constant shading model.

Disadvantages:

- Linear intensity interpolation can cause bright or dark intensity streaks, called **Mach Bands** to appear on the surface. This effect can be reduced by increasing the number of polygon while representing the object or Phong shading.
- Mach band effect is Optical illusion





Polygon Approximation Flat Shading Gouraud Shading

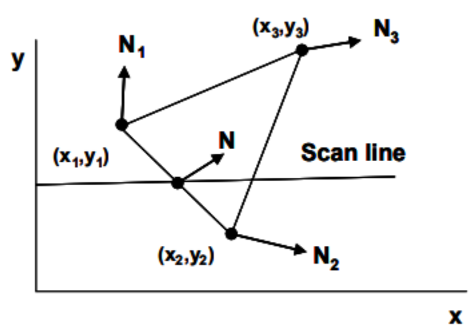
3. Phong Shading (Normal Vector Interpolation shading):

- A more accurate method for rendering a polygon surface is Phong shading, or normal vector interpolation shading which first interpolate normal vectors, and then apply the illumination model to each surface point. It displays more realistic highlights on a surface and greatly reduces the Mach band effect.
- A polygon surface is rendered using Phong shading by carrying out the following steps:
  1. Determine average unit normal vectors at each polygon vertex

$$N_v = \frac{\sum_{k=1}^{n} N_k}{\left|\sum_{k=1}^{n} N_k\right|}$$

  2. Linearly interpolate the vertex normals over the surface of the polygon

$$N = \frac{y - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y}{y_1 - y_2} N_2$$



  3. Apply an illumination model along each scan line to calculate projected pixel intensities for the surface points

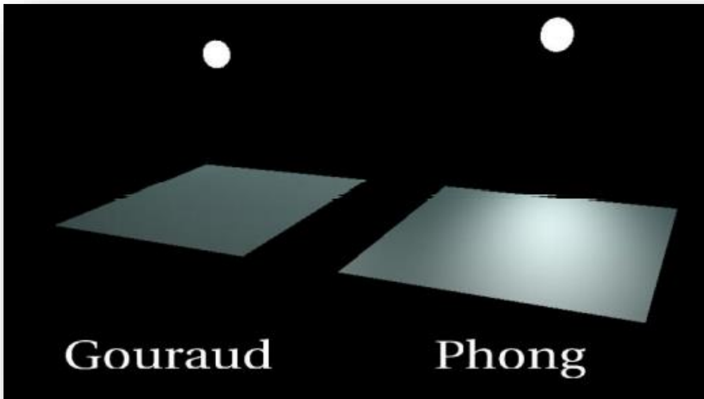
Advantages:

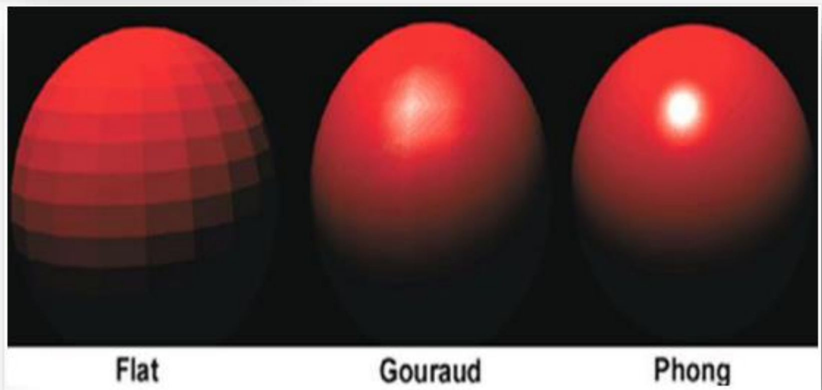
- Displays more realistic highlights in a surface
- Phong shading is more accurate
- Reduce mach band effect.

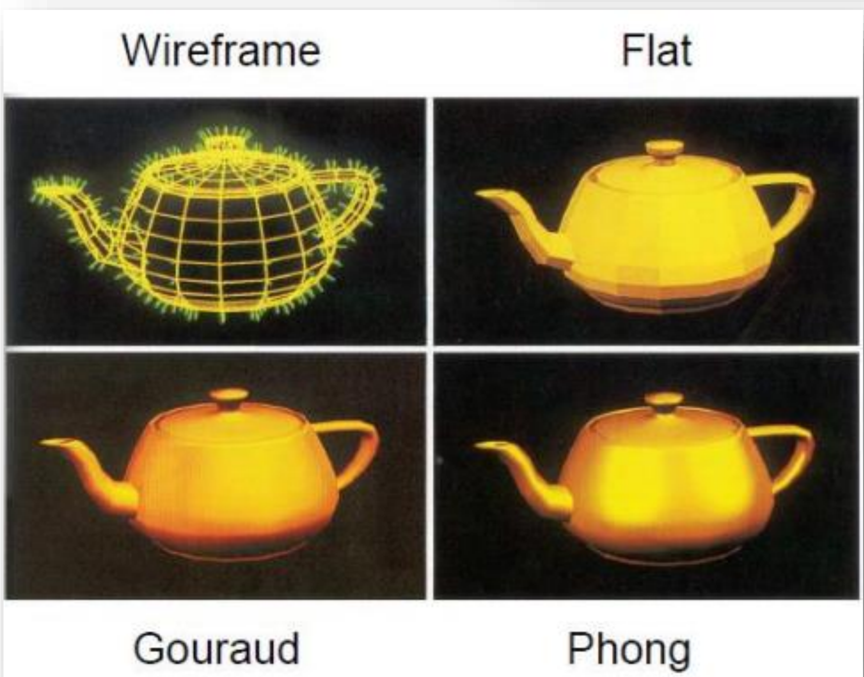
Disadvantages:

- requires considerably more calculations

Chapter 8 Illumination and Surface Rendering method

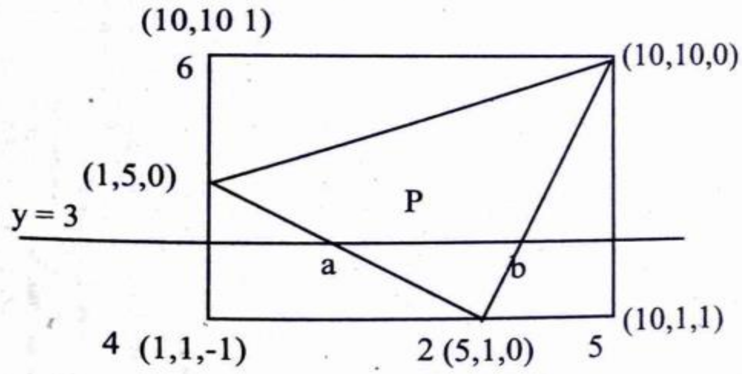






[Compiled By: Er. Shankar Bhandari]

Q. ***Find out the intensity of light reflected from the midpoint P on scan line y = 3 in the above given figure using Gouraud shading model. Consider a single point light source located at positive intensity on z-axis and assume vector to eye as (1,1,1). Given d = 0, k = 1, la = 1, l_l = 10, K_s = 2, K_a = K_d = 0.8 for use in a simple illumination model.***



Solution

Step 1 Calculate unit normal vectors $\hat{N}_1$, $\hat{N}_2$, $\hat{N}_3$ at vertices 1, 2 and 3 respectively.

The normal vectors at the vertices can be approximated by averaging the cross product of all the edges that terminate at the vertices. It is important that the order of vectors should be so chosen that the cross product yields outward normal vectors only.

The normal vectors at 1,

$$\vec{N}_1' = V_1V_2 \times V_1V_3 + V_1V_3 \times V_1V_6 + V_1V_4 \times V_1V_2$$

$$= (4\hat{i}-4\hat{j})\times(9\hat{i}+5\hat{j}) + (9\hat{i}+5\hat{j}) \times (5\hat{j}+\hat{k}) + (-4\hat{j}-\hat{k}) \times (4\hat{i}-4\hat{j})$$

$$= \hat{i}-13\hat{j}+117\hat{k}$$

The unit normal at 1,

$$\hat{N}_1 = \frac{\vec{N}_1'}{|\vec{N}_1'|}$$

$$= \frac{\hat{i}-13\hat{j}-117\hat{k}}{\sqrt{1^2+(-13)^2+117^2}}$$

$$= 0.01\hat{i}+0.11\hat{j}+0.99\hat{k}$$

Similar at 2,

$$\vec{N}_2' = V_2V_3 \times V_2V_1 + V_2V_1 \times V_2V_4 + V_2V_5 \times V_2V_3$$
$$= 13\hat{i}+\hat{j}+117\hat{k}$$

$$\hat{N}_2 = \frac{\vec{N}_2'}{|\vec{N}_2'|}$$
$$= -0.11\hat{i}+0.001\hat{j}+0.99\hat{k}$$

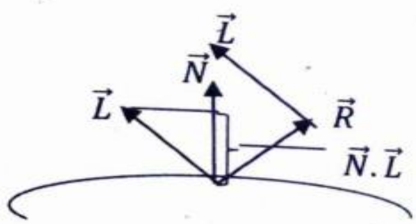
Similarly at 3,

$$\vec{N}_3' = -4\hat{i}-4\hat{j}+146\hat{k}$$

$$\hat{N}_3 = \frac{\vec{N}_3'}{|\vec{N}_3'|}$$
$$= -0.03\hat{i}-0.03\hat{j}-0.99\hat{k}$$

Step 2

Calculate unit reflection vectors $\vec{R}_1$, $\vec{R}_2$, $\vec{R}_3$ at vertices 1, 2, and 3 respectively



The projection of $\vec{L}$ onto the direction of the normal vector is obtained with the dot product $\vec{N}.\vec{L}$.

So,

$$\vec{R} + \vec{L} = (2\vec{N}.\vec{L})\vec{N}$$
$$\vec{R} = (2\vec{N}.\vec{L})\vec{N} - \vec{L}$$
$$\vec{R}_1 = 2(\vec{N}_1.\vec{L})\,\vec{N}_1 - \vec{L}$$
$$= 2((0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}).\hat{k})(0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}) - \hat{k}$$

$$= (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k})$$

$$\vec{R}_2 = 2(\vec{N}_2 . \vec{L})\vec{N}_2 - \vec{L}$$

$$= (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k})$$

$$\vec{R}_3 = 2(\vec{N}_3 . \vec{L})\,\vec{N}_3 - \vec{L}$$

$$= (-0.06\hat{i} - 0.06\hat{j} + 0.96\hat{k})$$

Step 3

Calculate intensities I_1, I_2, I_3 at vertices 1, 2, and 3 respectively

$$I_1 = I_a K_a + \frac{Il(K_d(\vec{N}_1.\vec{L}) + K_s)\vec{R}_1.V)^{ns})}{K+d}$$

$$\vec{N}_1 . \vec{L} = (0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}) . \hat{k}$$

$$= 0.99$$

$$\vec{R}_1 . \vec{V} = (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k}).(0.58\hat{i} + 0.58\hat{j} + 0.58\hat{k})$$

$$= 0.44$$

$$I_1 = (1)(0.10) + (10.1)((0.10)(0.99) + (0.80)(0.44)^2)$$

$$= 2.64$$

$$I_2 = I_a K_a + \frac{Il(Kd(N2L) + Ks)R2.V)^{ns})}{K+d}$$

$$\vec{N}_2 . \vec{L} = (0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}) . \hat{k}$$

$$= 0.99$$

$$\vec{R}_2 . \vec{V} = (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k}).(0.58\hat{i} + 0.58\hat{j} + 0.58\hat{k})$$

$$= 0.44$$

$$I_2 = (1)(0.10) + (10.1)((0.10)(0.99) + (0.80)(0.44)^2)$$

$$= 2.64$$

Similarly, calculate $\vec{N}_3 . \vec{L}$, $\vec{R}_3 . \vec{V}$ and we get

$$I_3 = 3.09$$

Step 4

Interpolate intensities I_a, I_b and I_p at a, b, p respectively

Referring the figure,

The scan line y = 3 containing point p intersects the edges 1-2 and 3-2 respectively at a and b.

$$\frac{x_1 - x_a}{x_1 - x_2} = \frac{y_1 - y}{y_1 - y_2}$$

$$\frac{x_2 - x_b}{x_2 - x_3} = \frac{y_2 - y}{y_2 - y_3}$$

Using the slope of the edges the co-ordinates of a and b are found to be (3,3,0) and (6.11,3,0) respectively .

The coordinates of p, the midpoint of a b is found (4.56, 3,0). Now we have apply 3 stage interpolation technique to determine I_p

$$\frac{I_1 - I_a}{I_1 - I_2} = \frac{y_1 - y}{y_1 - y_2}$$

$I_a = 2.64$ $\qquad$ $I_b = 2.74$

$x_a = 3$ $\qquad$ $x_b = 6.11$

$x_p = 4.555$

$$\frac{I_a - I_p}{I_a - I_b} = \frac{x_a - x_p}{x_a - x_b}$$

$I_p = 2.69$