Chapter 4 Three –Dimensional Graphics

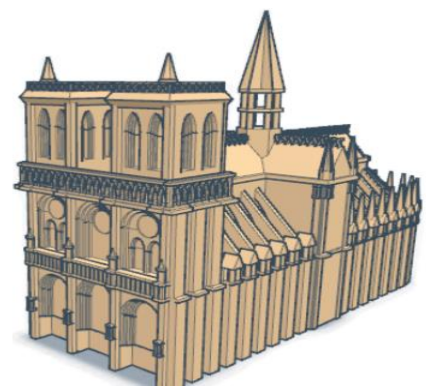
Three-Dimensional Graphics:

Three-dimensional space is a geometric 3-parameters model of the physical universe (without considering time) in which all known matter exists. These three dimensions can be labeled by a combination of length, breadth, and depth. Any three directions can be chosen, provided that they do not all lie in the same plane.

An object that has height, width and depth, like any object in the real world is a 3 dimensional object.

3D Transformations:

Just as 2D-transfromtion can be represented by 3x3 matrices using homogeneous co-ordinate can be represented by 4x4 matrices, provided we use homogenous co-ordinate representation of points in 3D space as well.

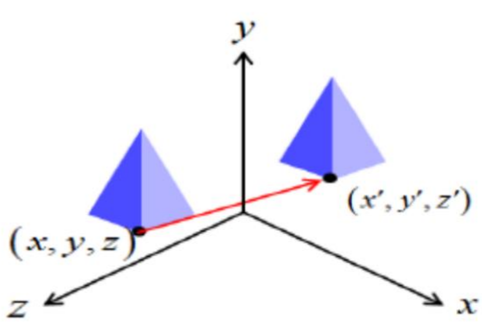


1. Translation
2. Rotation
3. Scaling
4. Reflection
5. Shear

Translation:

Translation in 3D is similar to translation in the 2D except that there is one more direction parallel to the z-axis. If, tx, ty, and tz are used to represent the translation vectors. Then the translation of the position P(x, y, z) into the point P' (x', y', z') is done by:

$$x' = x + t_x, \quad y' = y + t_y, \quad z' = z + t_z$$



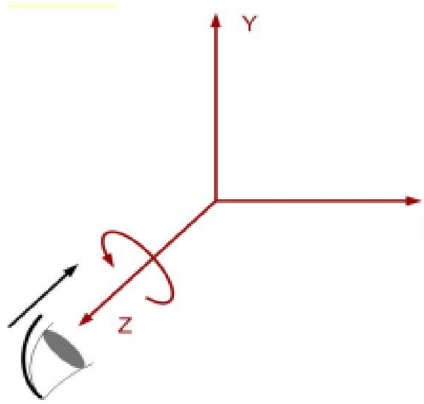
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = T.P$$

Rotation:

- Rotation About z -axis:

  Z-component does not change



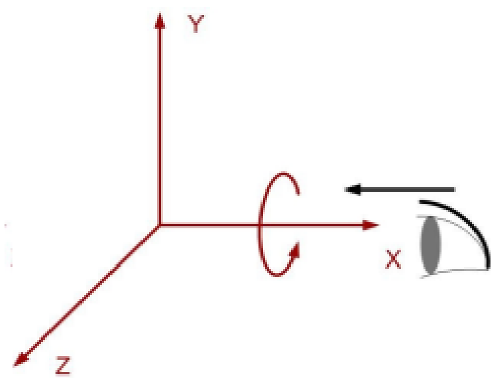
$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = R_z(\theta).P$$

- Rotation About x -axis:

  x-component does not change



$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta$$
$$x' = x$$

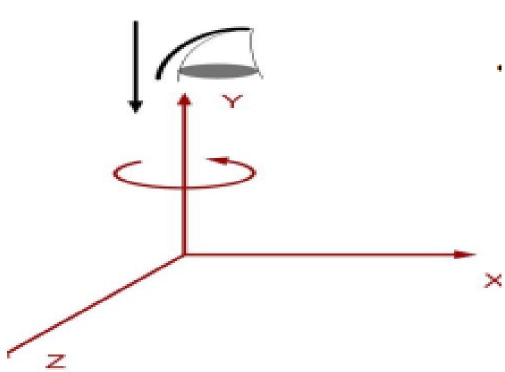
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = R_x(\theta).P$$

- Rotation About y -axis:

  Y-component does not change

Y-Axis Rotation



$$z' = z\cos\theta - x\sin\theta$$
$$x' = z\sin\theta + x\cos\theta$$
$$y' = y$$

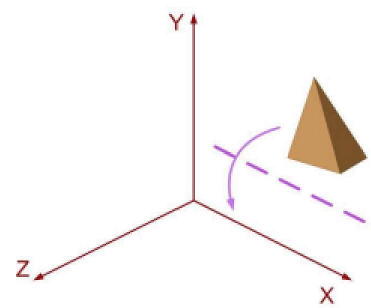
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = R_y(\theta).P$$

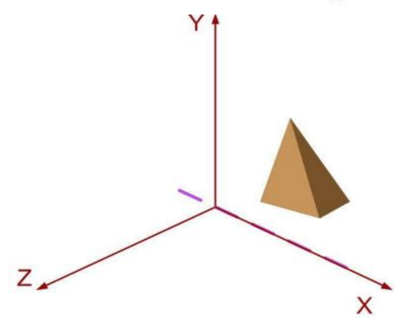
General 3D Rotations:

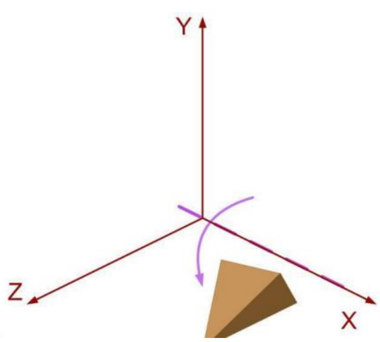
(a) ***Rotation about an axis parallel to any of the co-axis***: When an object is to be rotated about an axis that is parallel to one of the co-ordinate axis, we need to perform series of transformation.

i. Translate the object so that the rotation axis coincides with the parallel co-ordinate axis.
ii. Perform the specified rotation about the axis.
iii. Translate the object so that the rotation axis is moved to its original position.



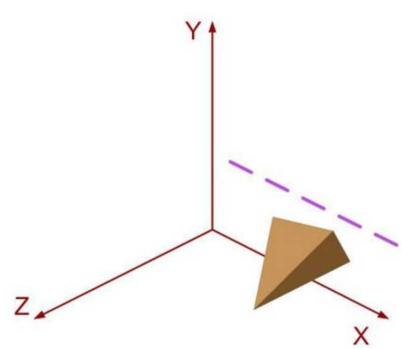
Step 1: Translate the object so that the rotation axis coincides with the parallel co-ordinate axis.





Step 2: Perform the specified rotation about the axis.

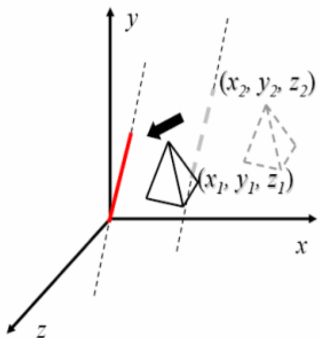
Step 3: Translate the object so that the rotation axis is moved to its original position.



b) Rotation about an axis not parallel to any of the co-axis:

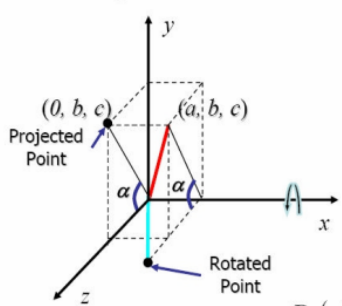
i. Translate the object such that rotation axis passes through coordinate origin
ii. Rotate the axis such that axis of rotation coincides with one of the co-ordinate axis.
iii. Perform the specific rotation about the ordinate axis.
iv. Apply inverse rotation to bring the rotation axis back to its original orientation.
v. Apply inverse translation to bring the rotation axis back to its original position.

- Step 1. Translate



$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Step 2. Rotate about x axis by α

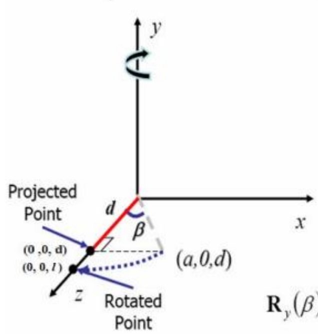


$$\sin\alpha = \frac{b}{\sqrt{b^2+c^2}} = \frac{b}{d}$$

$$\cos\alpha = \frac{c}{\sqrt{b^2+c^2}} = \frac{c}{d}$$

$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Step 3. Rotate about y axis by β (clockwise)

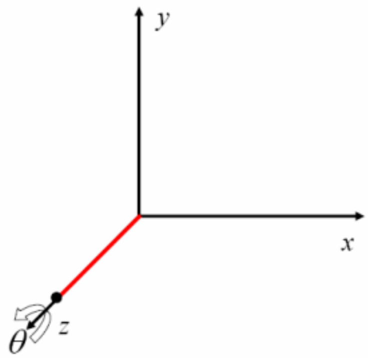


$$\sin\beta = \frac{a}{l}, \qquad \cos\beta = \frac{d}{l}$$

$$l^2 = a^2 + b^2 + c^2 = a^2 + d^2$$

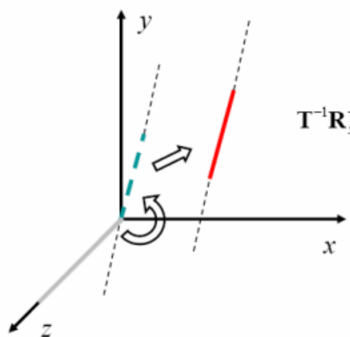
$$\mathbf{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} d/l & 0 & -a/l & 0 \\ 0 & 1 & 0 & 0 \\ a/l & 0 & d/l & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Step 4. Rotate about z axis by the angle $\boldsymbol{\theta}$



$$\mathbf{R}_z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

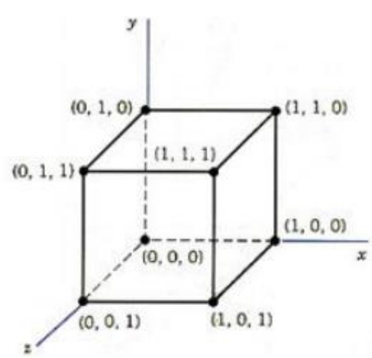
- Step 5. Reverse transformation



$$\mathbf{T}^{-1}\mathbf{R}_x^{-1}(\alpha)\mathbf{R}_y^{-1}(\beta)=\begin{bmatrix}1 & 0 & 0 & x_1\\ 0 & 1 & 0 & y_1\\ 0 & 0 & 1 & z_1\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha & 0\\ 0 & -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\beta & 0 & \sin\beta & 0\\ 0 & 1 & 0 & 0\\ -\sin\beta & 0 & \cos\beta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

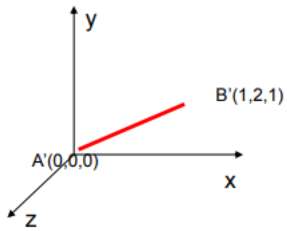
$$\mathbf{R}(\theta)=\mathbf{T}^{-1}\mathbf{R}_x^{-1}(\alpha)\mathbf{R}_y^{-1}(\beta)\mathbf{R}_z(\theta)\mathbf{R}_y(\beta)\mathbf{R}_x(\alpha)\mathbf{T}$$

Q. Find the new coordinates of a unit cube 90^0 –rotated about an axis defied by its endpoints A(2,1,0) and B(3,3,1).



A Unit Cube

Step1. Translate point A to the origin

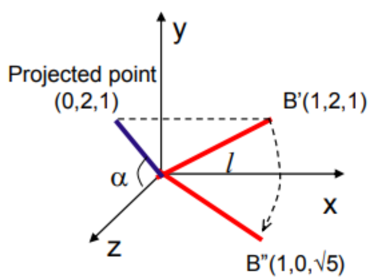


$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 2. Rotate axis *A'B'* about the x axis by and angle α, until it lies on the *xz* plane.

$$\sin\alpha = \frac{2}{\sqrt{2^2+1^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

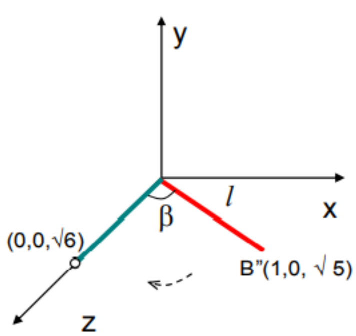
$$\cos\alpha = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$



$$l = \sqrt{1^2+2^2+1^2} = \sqrt{6}$$

$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}}{5} & -\frac{2\sqrt{5}}{5} & 0 \\ 0 & \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 3. Rotate axis *A'B"* about the y axis by and angle β until it coincides with the z axis.



$$\sin\beta = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

$$\cos\beta = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$$

$$\mathbf{R}_y(\beta) = \begin{bmatrix} \frac{\sqrt{30}}{6} & 0 & -\frac{\sqrt{6}}{6} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 4. Rotate the cube 90° about the z axis

$$\mathbf{R}_z(90°) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Finally, the concatenated rotation matrix about the arbitrary axis AB becomes,

$$\mathbf{R}(\theta) = \mathbf{T}^{-1}\mathbf{R}_x^{-1}(\alpha)\mathbf{R}_y^{-1}(\beta)\mathbf{R}_z(90°)\mathbf{R}_y(\beta)\mathbf{R}_x(\alpha)\mathbf{T}$$

$$\mathbf{R}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} & 0 \\ 0 & -\frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{30}}{6} & 0 & \frac{\sqrt{6}}{6} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{30}}{6} & 0 & -\frac{\sqrt{6}}{6} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{6}}{6} & 0 & \frac{\sqrt{30}}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{5}}{5} & -\frac{2\sqrt{5}}{5} & 0 \\ 0 & \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.166 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiplying $\mathbf{R}(\theta)$ by the point matrix of the original cube

$$[\mathbf{P}'] = \mathbf{R}(\theta) \cdot [\mathbf{P}]$$

$$[\mathbf{P}'] = \begin{bmatrix} 0.166 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.650 & 1.667 & 1.834 & 2.816 & 2.725 & 1.742 & 1.909 & 2.891 \\ -0.558 & -0.484 & 0.258 & 0.184 & -1.225 & -1.151 & -0.409 & -0.483 \\ 1.467 & 1.301 & 0.650 & 0.817 & 0.726 & 0.560 & -0.091 & 0.076 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

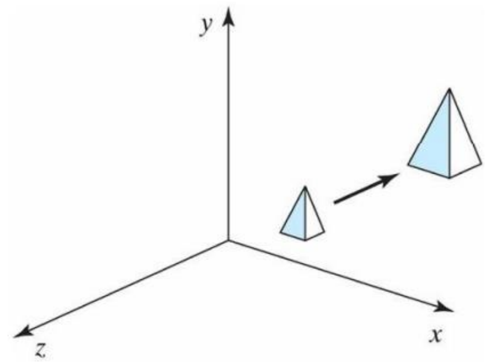
Scaling:

Scaling about origin

X' = X . Sx

Y' = Y . Sy

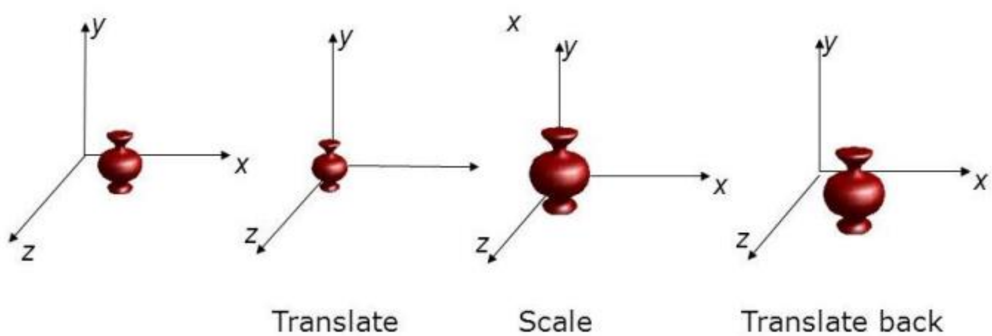
Z' = Z . Sz



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

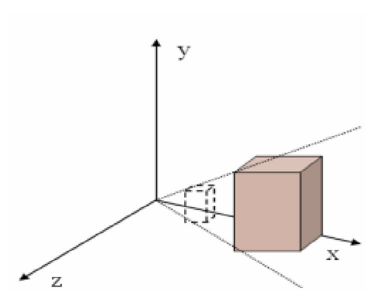
$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$

Scaling about an arbitrary point or Fixed point (xf, yf, zf)



Fixed Point Scaling

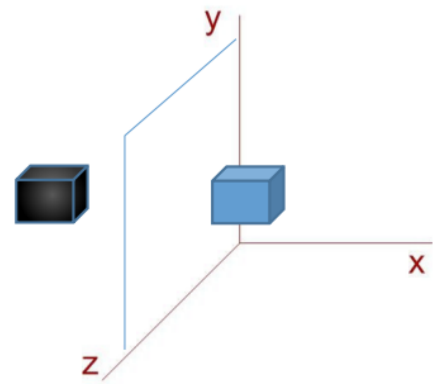
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & (1-s_x)x_f \\ 0 & s_y & 0 & (1-s_y)y_f \\ 0 & 0 & s_z & (1-s_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$S(x_f, y_f, z_f, s_x, s_y, s_z) = T(x_f, y_f, z_f).S(s_x, s_y, s_z).T(-x_f, -y_f, -z_f)$$

Reflection

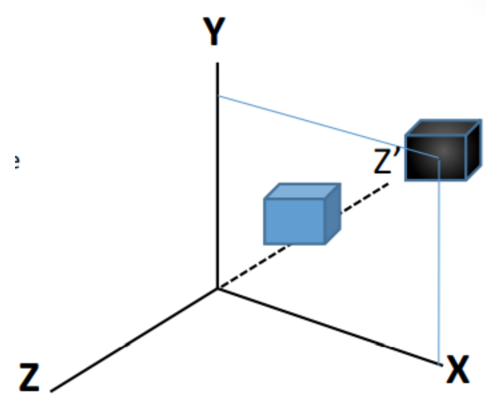
i) Reflection about yz plane



X' = -X
Y' = Y
Z' = Z

$$T_{yz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

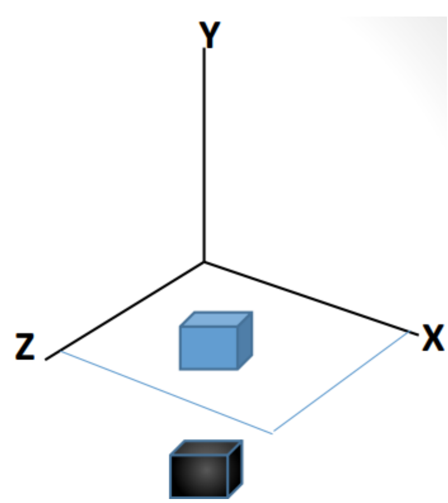
ii) Reflection about XY plane



X' = X
Y' = Y
Z' = -Z

$$T_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii) Reflection about XZ plane



X' = X
Y' = -Y
Z' = Z

$$T_{yz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Shear:

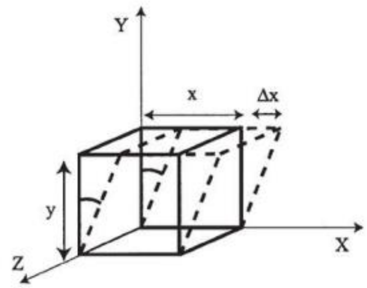
Shearing transformations can be used to modify object shapes.

Z-axis Shear

This transformation alters x- and y-coordinate values by an amount that is proportional to the z value, while leaving the z coordinate unchanged, i.e.

$$x' = x + S_{hx} .z$$
$$y' = y + S_{hy} .z$$
$$z' = z$$

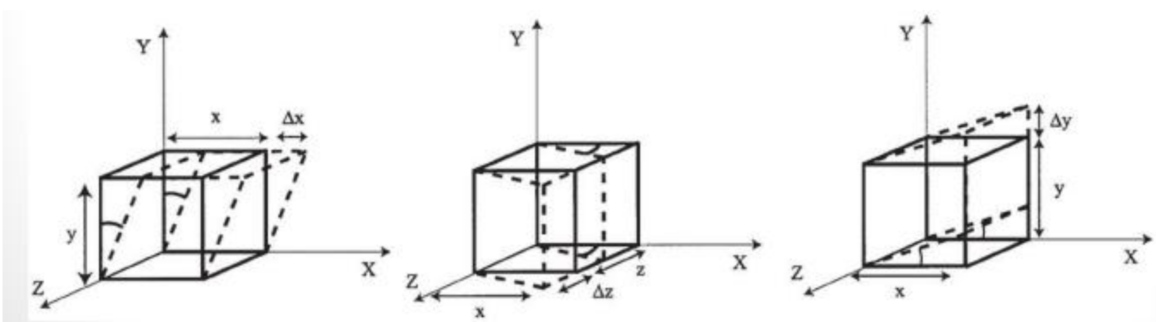
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & S_{hx} & 0 \\ 0 & 1 & S_{hy} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Similar X-axis shear and Y-axis shear is given as

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_y & 1 & 0 & 0 \\ sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Q. Given a 3D triangle with points (0, 0, 0), (1, 1, 2) and (1, 1, 3). Apply shear parameter 2 on X axis, 2 on Y axis and 3 on Z axis and find out the new coordinates of the object.

Solution:

Given-

- Old corner coordinates of the triangle = A (0, 0, 0), B(1, 1, 2), C(1, 1, 3)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2
- Shearing parameter towards Y direction (Sh_z) = 3

Shearing in X Axis-

For Coordinates A(0, 0, 0)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 0 = 0$
- $Z_{new} = Z_{old} + Sh_z \times X_{old} = 0 + 3 \times 0 = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

For Coordinates B(1, 1, 2)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$
- $Z_{new} = Z_{old} + Sh_z \times X_{old} = 2 + 3 \times 1 = 5$

Thus, New coordinates of corner B after shearing = (1, 3, 5).

For Coordinates C(1, 1, 3)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$
- $Z_{new} = Z_{old} + Sh_z \times X_{old} = 3 + 3 \times 1 = 6$

Thus, New coordinates of corner C after shearing = (1, 3, 6).

Thus, New coordinates of the triangle after shearing in X axis = A (0, 0, 0), B(1, 3, 5), C(1, 3, 6).

Shearing in Y Axis-

For Coordinates A(0, 0, 0)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} = 0$
- $Z_{new} = Z_{old} + Sh_z \times Y_{old} = 0 + 3 \times 0 = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

For Coordinates B(1, 1, 2)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$
- $Y_{new} = Y_{old} = 1$
- $Z_{new} = Z_{old} + Sh_z \times Y_{old} = 2 + 3 \times 1 = 5$

Thus, New coordinates of corner B after shearing = (3, 1, 5).

For Coordinates C(1, 1, 3)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$
- $Y_{new} = Y_{old} = 1$
- $Z_{new} = Z_{old} + Sh_z \times Y_{old} = 3 + 3 \times 1 = 6$

Thus, New coordinates of corner C after shearing = (3, 1, 6).

Thus, New coordinates of the triangle after shearing in X axis = A (0, 0, 0), B(3, 1, 5), C(3, 1, 6).

Shearing in Z Axis-

For Coordinates A(0, 0, 0)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Z_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} + Sh_y \times Z_{old} = 0 + 2 \times 0 = 0$
- $Z_{new} = Z_{old} = 0$

Thus, New coordinates of corner A after shearing = (0, 0, 0).

For Coordinates B(1, 1, 2)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Z_{old} = 1 + 2 \times 2 = 5$
- $Y_{new} = Y_{old} + Sh_y \times Z_{old} = 1 + 2 \times 2 = 5$
- $Z_{new} = Z_{old} = 2$

Thus, New coordinates of corner B after shearing = (5, 5, 2).

For Coordinates C(1, 1, 3)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}, Z_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Z_{old} = 1 + 2 \times 3 = 7$
- $Y_{new} = Y_{old} + Sh_y \times Z_{old} = 1 + 2 \times 3 = 7$
- $Z_{new} = Z_{old} = 3$

Thus, New coordinates of corner C after shearing = (7, 7, 3).

Thus, New coordinates of the triangle after shearing in X axis = A (0, 0, 0), B(5, 5, 2), C(7, 7, 3).

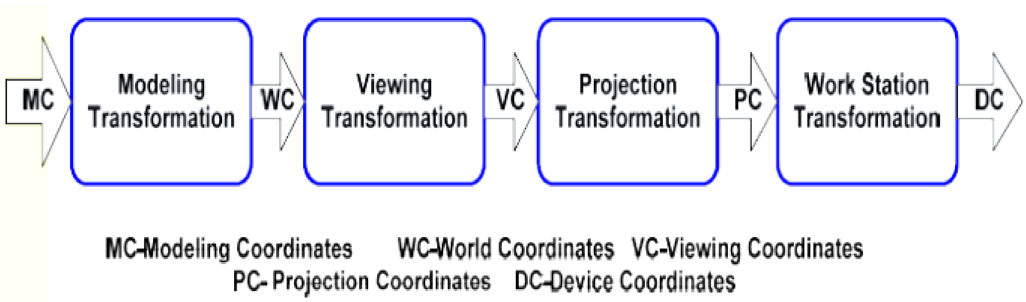
Chapter 4 Three –Dimensional Graphics

3D Viewing Pipeline:

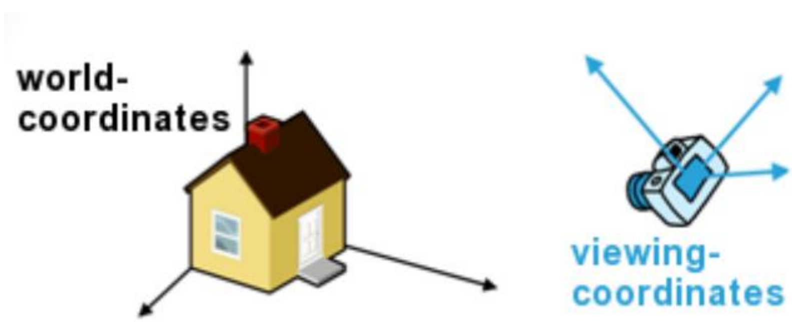
The viewing-pipeline in 3 dimensions is almost the same as the 2D viewing-pipeline. Only after the definition of the viewing direction and orientation (i.e., of the camera) an additional projection step is done, which is the reduction of 3D-data onto a projection plane:

World to screen viewing transformation:

But in 3D graphics application, we have to consider spatial position (i.e. an object can be viewed from the front, from above or form the back) or we could generate a view of what we would see if we were standing in the middle of a group of objects or inside a single object, such as buildings.



Additionally, 3D descriptions of object must be projected onto the flat viewing surface of the output device.



- The steps for computer generation of a view of a three dimensional scene are analogous to the process involved in taking a photograph.
- To take a snapshot, we first need to position the camera at a particular point in space, then need to decide on the camera orientation. (i.e. which way do we point the camera and how should we rotate it around the line of sight to set the up direction for the picture). Finally, when we snap the shutter, the scene is cropped to the size of the shutter, the scene is cropped to the size of the 'window' (aperture) of the camera and light from the visible surface is projected.

- Each model or object has its own dimension and coordinate system. It is called modeling coordinate. Modeling transformation is to take all the objects in a single scene by using transformation such translation, rotation etc. To set object is called modeling translation.
- After modeling translation, the objects come to scene or a coordinate system is called world coordinate.
- To set the camera on some position, angle or orientation is called viewing transformation. From viewing transformation we get viewing coordinate.
- Projection transformation is to adjust focus, zoom in, zoom out etc. Projection transformation creates projection coordinates.
- Workstation or viewport translation is just like to click the button to save the image in the device.
- Once the scene has been modeled, world coordinate positions are converted to viewing co-ordinate.
- The VC system is used in graphics system as a reference for specifying the observer viewing position and the position of the projection plane analogous to camera film plane.
- Projection operations are performed to convert VC description of a scene to co-ordinate positions on the projection plane.
- The projection plane is then mapped to output device.

World to screen viewing transformation:

Suppose that the viewing co-ordinates are specified in world co-ordinates. We need to transform each vertex specified in world co-ordinates to view co-ordinates.

1. Translate world co-ordinate origin to viewing co-ordinates origin.
2. Apply rotation to align u, v and n with the world x, y and z.

Transformation Matrices:

Translation:

If the view reference point is specified at world position(x0, y0, z0) this point is translated to world origin with the matrix transformation.

$$\begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

The composite rotation matrix for viewing transformation is then

$$R = \begin{bmatrix} u_1 & u_2 & u_3 & 0 \\ v_1 & v_2 & v_3 & 0 \\ n_1 & n_2 & n_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Chapter 4 Three –Dimensional Graphics

Projection:

- Transformation that changes a point in n-dimensional coordinate system into a point in a coordinate system that has dimension less than n.
- Converts 3-D viewing co-ordinates to 2-D projection coordinates
- View Plane or Projection Plane: Two dimensional plane in which 3D objects are projected is called the view plane or projection plane. Simply it is a display plane on an output device

Types of Projection

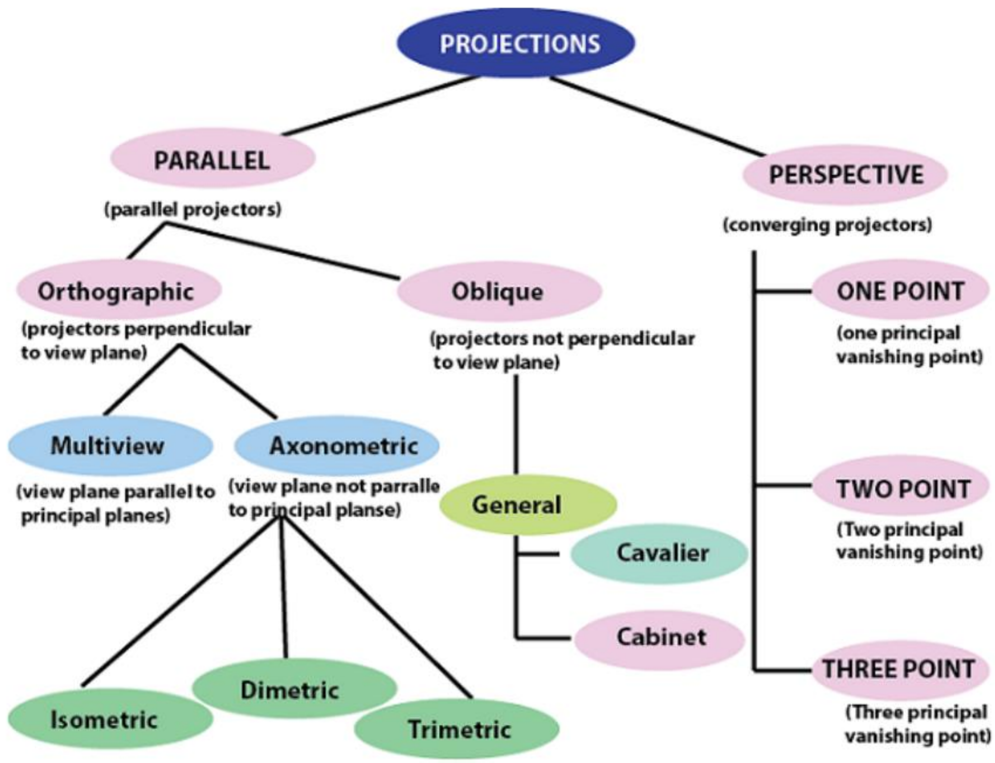
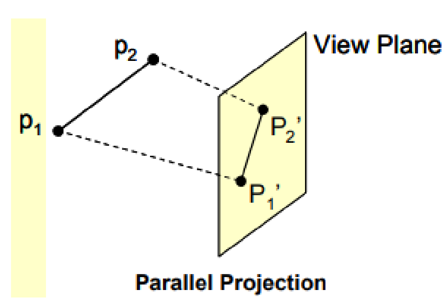


Figure. Types of projections

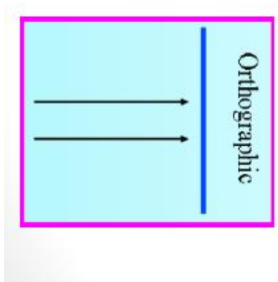
Parallel Projection:

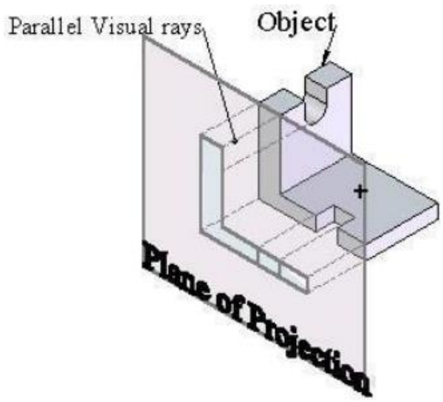
- Coordinate positions are transformed to view plane along parallel lines (projection lines)
- Preserves relative proportions of objects
- Accurate views of various sides of an object are obtained.
- Doesn't give realistic representation of the appearance of the 3-D object



Orthographic Parallel Projection:

- When projection is perpendicular to view plane then it is called orthographic parallel projection.

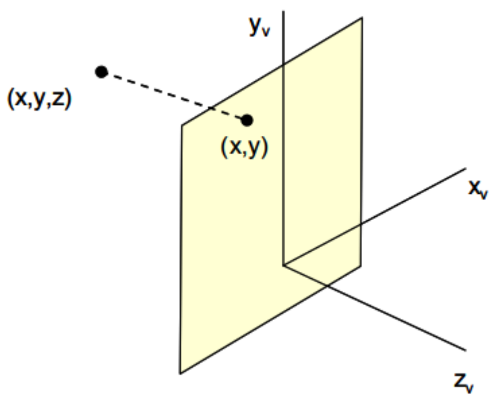




$x_p = x$

$y_p = y$

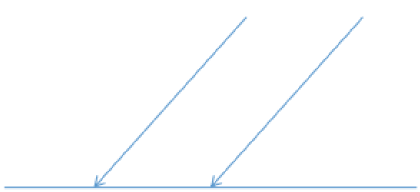
Note: **Z value is preserved for the depth information needed in depth culling and visible surface determination procedure**

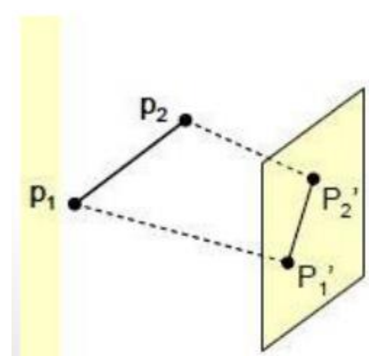


Orthographic projection of a point onto a viewing plane

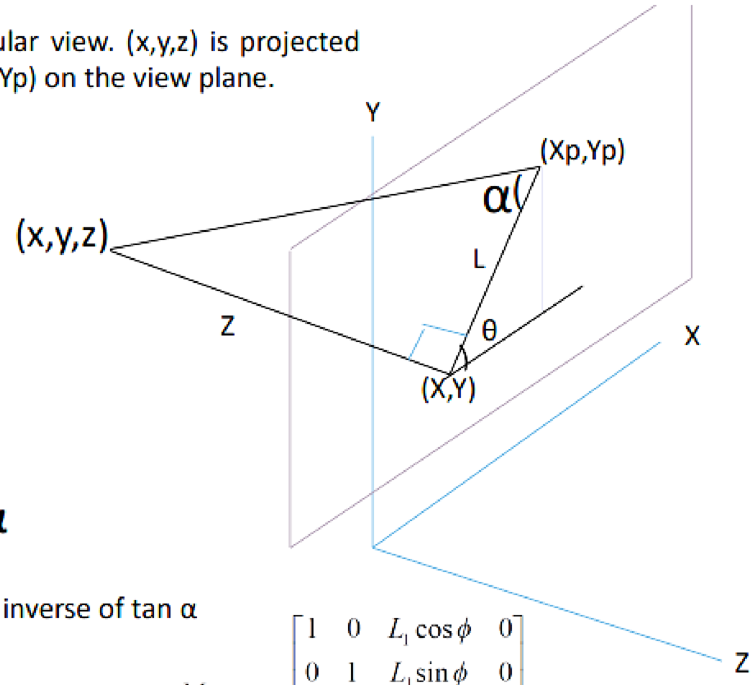
Oblique Parallel Projection:

Projectors (projection vectors) are not perpendicular to the projection plane. It preserves 3D nature of an object.





- Not perpendicular view. (x,y,z) is projected To position (Xp,Yp) on the view plane.



Cos θ = Xp / L

Xp = L Cos θ

But exact position is

Xp = X + L Cos θ

similarly

Sin θ = Yp / L

Yp= Y + L Sin θ

L depend on angle **α**

Tan α = Z / L

L= Z L_1 Where L_1 is inverse of tan α

Xp = X +Z L_1 Cos θ

Yp= Y + Z L_1 Sin θ

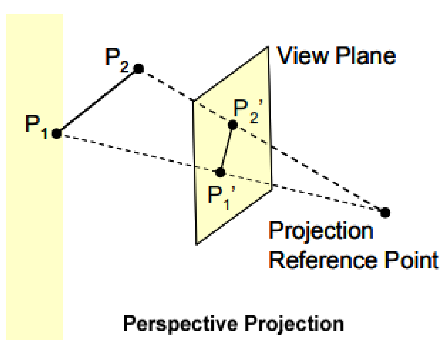
$$M_{parallel} = \begin{bmatrix} 1 & 0 & L_1\cos\phi & 0 \\ 0 & 1 & L_1\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

When α = 90, i.e. $L_1 = 0$, it is orthographic projection on z = 0 plane.

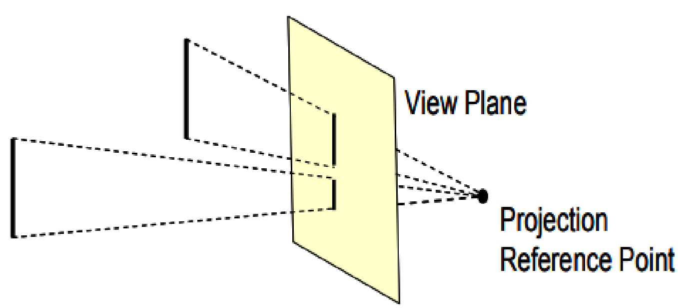
Oblique projections are generated with non-zero values for L_1.

Perspective Projection:

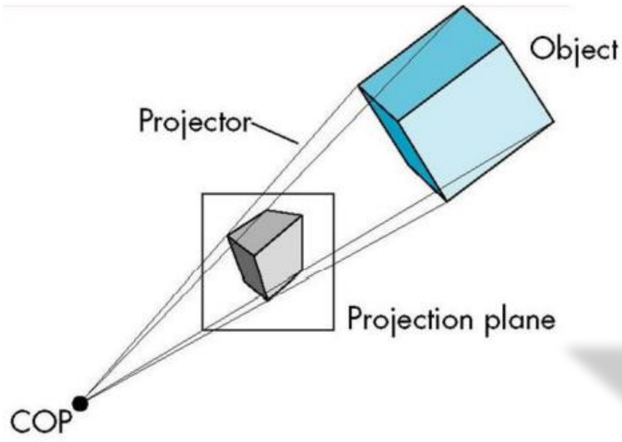
- Coordinate positions are transformed to view plane along lines (projection lines) that converges to a point called projection reference point (center of projection)
- Produce realistic view
- Does not preserve relative proportions
- Equal sized object appears in different size according as distance from view plane.

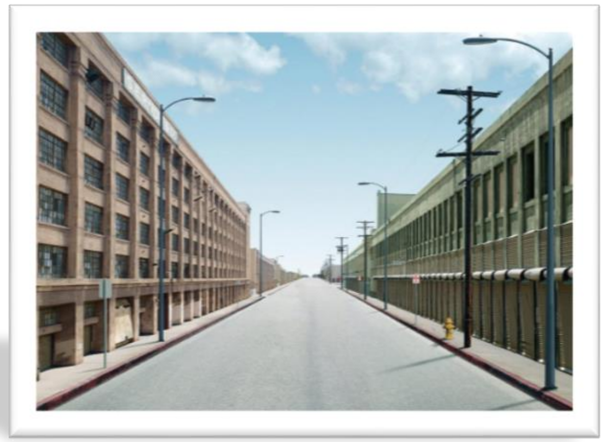


Perspective Projection



Perspective Projection of equal sized objects at different distances from the view plane





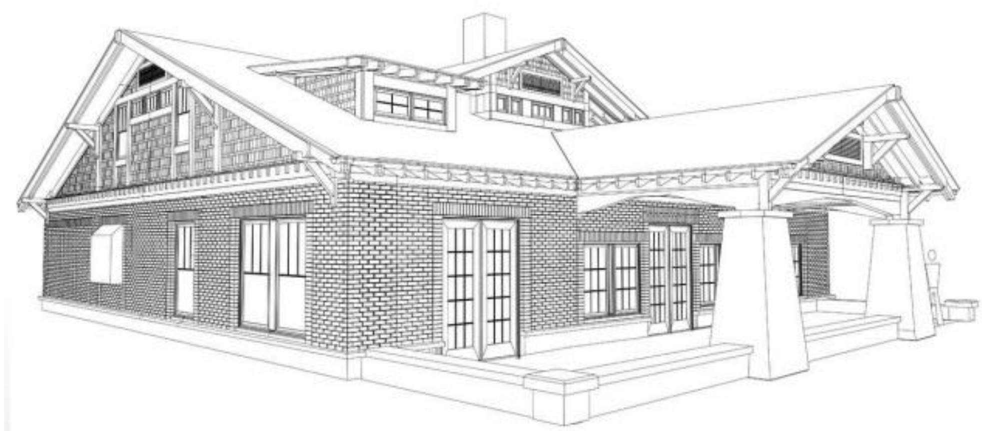
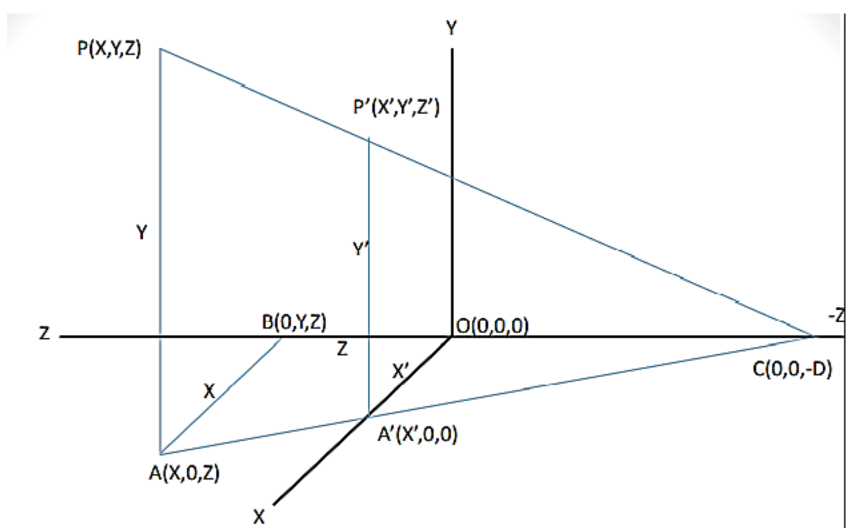
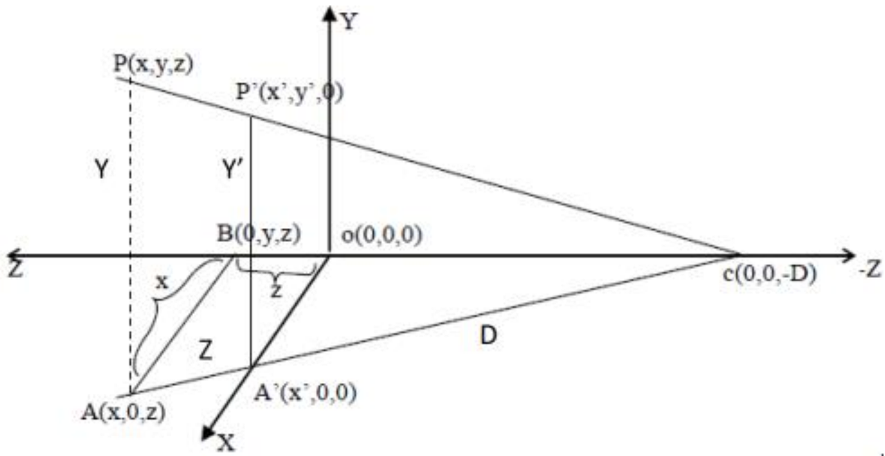


Figure. Perspective View



Here center of Projection is c (0,0,-D) along the direction of Z axis so the reference point is taken of world coordinate space Wc and the normal vector N is aligned with the y axis.

So now the view plane vp is the xy plane and center of projection is c (0,0,-D) now from similar triangles ABC and A'OC



Triangles ABC and A'OC
(x/x') = AC/A'C = (Z+D)/D
X' = (XD)/(Z+D)
And Z'=0

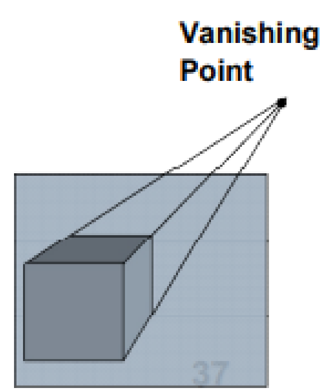
Triangles APC and A'P'C
(y/y') = (AC/AC') = (Z+D)/D
y' = (DY)/(Z+D)
And Z' =0

now in homogenous coordinates

$$\begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \frac{1}{z+D} \begin{pmatrix} Dx \\ Dy \\ 0 \\ z+D \end{pmatrix} = \frac{1}{z+D} \begin{pmatrix} D & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & D \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Vanishing Point:

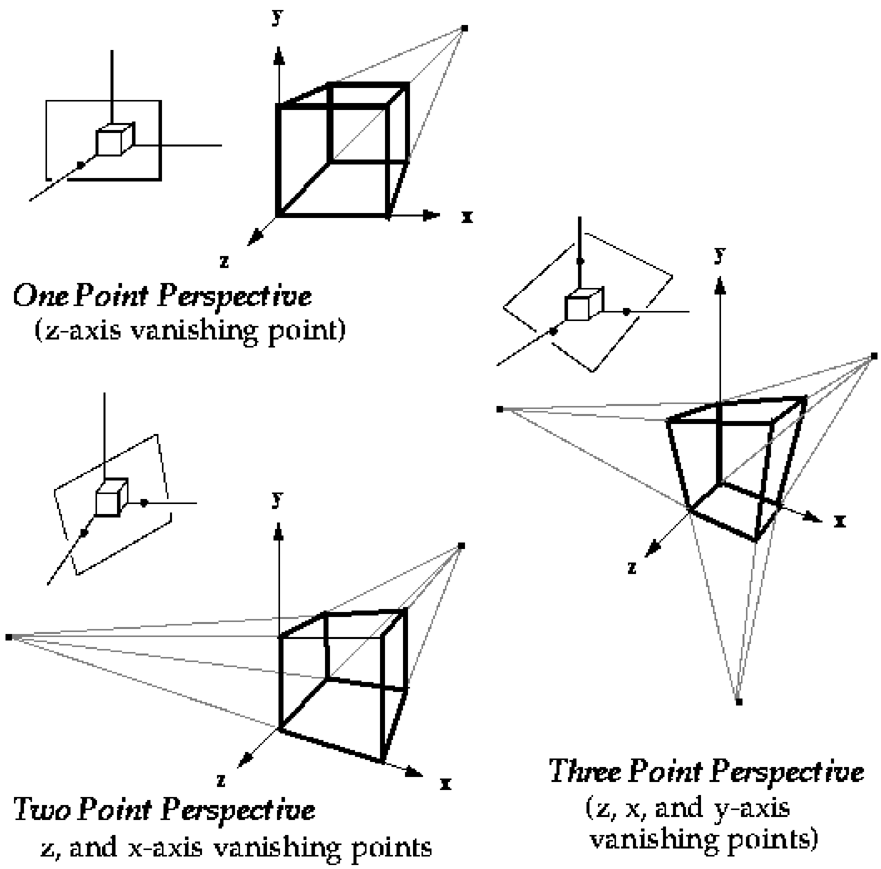
- A set of parallel lines that are not parallel to the view plane pare projected as converging lines that appear to converge at a point called vanishing point.
- A set of parallel lines that are parallel to view plane are projected as parallel lines.
- More than one set of parallel lines form more than one vanishing point in the scene.



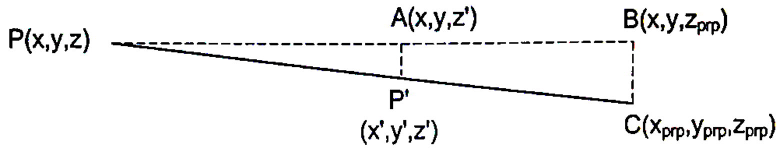
Principal vanishing point

- Principal vanishing points are formed by the apparent intersection of lines parallel to one of the three principal x, y, and z axes.
- Vanishing point for a set of parallel lines parallel to one of the principal axis of object
- We can control the number of principal vanishing point to one, two or three with the orientation of projection plane and classify as one, two or three point perspective projection.

Perspective views and z-axis vanishing point:



Perspective Projection:



$\Delta PAP'$ and ΔPBC are similar triangles.

Therefore, $\dfrac{AP'}{BC} = \dfrac{PA}{PB}$

Let $u = \dfrac{AP'}{BC} = \dfrac{PA}{PB}$ ----(1)

$AP' = x - x'$ and $BC = x - x_{prp}$

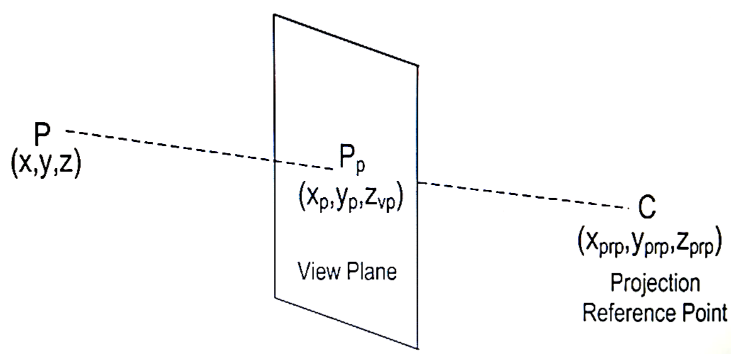
$PA = z - z'$ and $PB = z - z_{prp}$

From (1), $u = \dfrac{AP'}{BC} = \dfrac{x - x'}{x - x_{prp}}$

$\Rightarrow x - x' = (x - x_{prp})u$

$\Rightarrow x' = x - (x - x_{prp})u$ -----(2)

Similarly, $y' = y - (y - y_{prp})u$ -----(3)



From (2),(3) and(4)

When $u = 0$, $x' = x$, $y' = y$, $z' = z$

When $u = 1$, $x' = x_{prp}$, $y' = y_{prp}$, $z' = z_{prp}$

At view plane, $z' = z_{vp}$

From (4), $z_{vp} = z - (z - z_{prp})u$

$u = \dfrac{z_{vp} - z}{z_{prp} - z}$

At view plane, $x' = x_p$ and

From (2), $x_p = x - (x - x_{prp})\dfrac{z_{vp} - z}{z_{prp} - z}$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + x_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right) - - - (5)$$

Similarly, at view plane, $y' = y_p$ and

From (3), $y_p = y - (y - y_{prp})\dfrac{z_{vp} - z}{z_{prp} - z}$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + y_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right) - - - (6)$$

Let $d_p = Z_{prp} - Z_{vp}$, which is the difference between COP and view plane distances then, we can form a matrix as

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} d_p/(z_{prp} - z) & 0 & 0 & 0 \\ 0 & d_p/(z_{prp} - z) & 0 & 0 \\ 0 & 0 & 0 & z_{vp} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -z_{vp}/d_p & z_{vp}(z_{prp}/d_p) \\ 0 & 0 & -1/d_p & z_{prp}/d_p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

where $h = \dfrac{z_{prp} - z}{d_p}$

$x_p = \dfrac{x_h}{h}$, $y_p = \dfrac{y_h}{h}$

And original z co-ordinate value would be retained for visible surface and other depth processing

Special cases:

(i) Projection Reference Point on z_{view} axis:

$x_{prp} = 0$ and $y_{prp} = 0$

From (5) and (6)

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) \text{ and}$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right)$$

(ii) Projection Reference Point at origin:

$x_{prp} = 0$

$y_{prp} = 0$

$z_{prp} = 0$

From (5) and (6)

$$x_p = x\left(\frac{z_{vp}}{z}\right) \text{ and}$$

$$y_p = y\left(\frac{z_{vp}}{z}\right)$$

(iii) If the view plane is the uv plane:

$z_{vp} = 0$

From (5) and (6)

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right) - x_{prp}\left(\frac{z}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right) - y_{prp}\left(\frac{z}{z_{prp} - z}\right)$$

(iv) If the view plane is the uv plane and Projection Reference Point on z_{view} axis:

$x_{prp} = 0$, $y_{prp} = 0$ and $z_{vp} = 0$

From (5) and (6)

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right) \text{ and}$$

$$y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right)$$