

Infinitesimi

$$\lim_{x \rightarrow \pm\infty} \frac{C}{x} =$$

$$\text{caso 1) } C > 0 \begin{cases} \lim_{x \rightarrow +\infty} \frac{C}{x} = 0^+ \\ \lim_{x \rightarrow -\infty} \frac{C}{x} = 0^- \end{cases}$$

$$\text{caso 2) } C = 0 = \lim_{x \rightarrow \pm\infty} \frac{C}{x} = 0$$

$$\text{caso 3) } C < 0 \begin{cases} \lim_{x \rightarrow +\infty} \frac{C}{x} = 0^- \\ \lim_{x \rightarrow -\infty} \frac{C}{x} = 0^+ \end{cases}$$

Inoltre

$$\lim_{x \rightarrow +\infty} \frac{0^+}{x} = 0^+$$

$$\lim_{x \rightarrow -\infty} \frac{0^-}{x} = 0^-$$

$$\lim_{x \rightarrow +\infty} \frac{0^-}{x} = 0^-$$

$$\lim_{x \rightarrow -\infty} \frac{0^+}{x} = 0^+$$

Con i logaritmi $f(x) = \log(x)$

$$\lim_{x \rightarrow +\infty} \log(x) = +\infty$$

$$\lim_{x \rightarrow 0^+} \log(x) = -\infty$$

Tabelle

$$\frac{+\infty}{0^+} = +\infty$$

$$\frac{-\infty}{0^+} = -\infty$$

$$(+\infty)^n = +\infty \quad \forall n > 0$$

$$(+\infty)^n = 0 \quad \forall n < 0$$

$$\sqrt[n]{(+\infty)} = +\infty \quad \text{th intero}$$

$$\sqrt[n]{(-\infty)} = -\infty \quad \text{th int. disp}$$

$$\frac{+\infty}{0^-} = -\infty$$

$$\frac{-\infty}{0^-} = +\infty$$

$$(+\infty)^{(+\infty)} = +\infty$$

$$(+\infty)^{(-\infty)} = 0^+$$

$$\frac{C > 0}{0^+} = +\infty$$

$$\frac{C > 0}{0^-} = -\infty$$

$$\frac{C < 0}{0^+} = -\infty$$

$$\frac{C < 0}{0^-} = +\infty$$

Metodo di integrazione generale #1

$y = \frac{N(x)}{D(x)}$ ① Divisione fra $N \in D$ se $\text{gr}(N) \geq \text{gr}(D)$

② fattorizzazione di D

③ decomporre la frazione in fratti semplici

④ integrazione dei pezzettini

Esempio 1

$$\int \frac{x^3 - 3x - 1}{x^2 - x - 2} dx$$

$$\frac{x^3 - 3x - 1}{x^2 - x - 2} = x + 1 + \frac{1}{x^2 - x - 2}$$

$$\begin{array}{r|l} x^3 - 3x - 1 & x^2 - x - 2 \\ -x^3 + x^2 + 2x & \\ \hline & +x^2 - x - 1 \\ & -x^2 + x + 2 \\ \hline & 1 \end{array}$$

② $x^2 - x - 2 = (x-2)(x+1)$

③ $\frac{1}{x^2 - x - 2} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{Ax + A + Bx - 2B}{(x-2)(x+1)} =$

$$\frac{x(A+B) + A - 2B}{(x-2)(x+1)} \quad \begin{cases} A+B=0 \\ A-2B=1 \end{cases} \quad \begin{cases} A=-B \\ -B-2B=1 \\ \downarrow \\ -3B=1 \\ B=-\frac{1}{3} \end{cases} \quad \begin{cases} A=\frac{1}{3} \\ B=-\frac{1}{3} \end{cases}$$

$$\frac{1}{(x-2)(x+1)} = \frac{1}{3} \cdot \frac{1}{x-2} - \frac{1}{3} \cdot \frac{1}{x+1}$$

④ $\int \frac{x^3 - 3x - 1}{x^2 - x - 2} dx = \int \frac{1}{3(x-2)} - \frac{1}{3(x+1)} dx$

$$\int x dx + \int dx + \frac{1}{3} \int \frac{1}{(x-2)} dx - \frac{1}{3} \int \frac{1}{(x+1)} dx$$

$$\begin{array}{l}
 \downarrow \quad \downarrow \\
 \frac{x^2}{2} + x + \frac{1}{3} \ln(x-2) - \frac{1}{3} \ln(x+1) + c \\
 \frac{x^2}{2} + x + \frac{1}{3} \ln \frac{(x-2)}{(x+1)} + c
 \end{array}
 \quad
 \begin{array}{l}
 y = x-2 \\
 dy = 1 dx \\
 z = x+1 \\
 dz = 1 dx
 \end{array}$$

Esempio 2

$$\int \frac{1}{x^2-1} dx \quad \textcircled{1} \deg(D) > \deg(N)$$

$$\textcircled{2} x^2-1 = (x-1)(x+1)$$

$$\textcircled{3} \frac{1}{(x-1)(x+1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1) + B(x+1)}{(x+1)(x-1)} =$$

$$\frac{Ax - A + Bx + B}{(x+1)(x-1)} = \frac{x(B+A) - A + B}{(x+1)(x-1)} = \begin{cases} A+B=0 \\ B-A=1 \end{cases} \quad \begin{cases} A=-B \\ B+B=1 \end{cases}$$

$$\begin{cases} A = -\frac{1}{2} \\ B = \frac{1}{2} \end{cases}$$

$$\textcircled{4} \int \frac{1}{x^2-1} dx = \int -\frac{1}{2} \cdot \frac{1}{x+1} + \frac{1}{2} \frac{1}{x-1} dx$$

$$-\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx = -\frac{1}{2} \ln(x+1) + \frac{1}{2} \ln(x-1) + c$$

$$= \frac{1}{2} \ln \left(\left| \frac{x+1}{x-1} \right| \right) + c$$

Metodo Generale #2

- Denom 2° g non scomponibile
 - Potenza di n maggiore
- } problema!

Esempio 1

$$\int \frac{1}{x^3+x} dx$$

① non serve $\deg(D) < \deg(N)$

② $x^3+x = x(x^2+1)$

③ $\frac{1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)} =$

$$\frac{x^2(A+B)+Cx+A}{x(x^2+1)}$$

$$\begin{cases} A+B=0 \\ C=0 \\ A=1 \end{cases}$$

$$\begin{cases} A=1 \\ B=-1 \\ C=0 \end{cases}$$

④ $\int \frac{1}{x^3+x} dx = \int \frac{1}{x} dx - \frac{1}{2} \int \frac{2x}{x^2+1} dx = \ln(|x|) - \frac{1}{2} \ln|x^2+1| + C$

$$= \ln \left| \frac{x}{\sqrt{x^2+1}} \right| + C$$

passavo dove come risultato:
ln e arctan

Esempio 2

$$\int \frac{x+1}{x^3-2x^2+x} dx$$

① non serve

② $x^3-2x^2+x = x(x^2-2x+1) = x(x-1)^2$

③ $\frac{x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} =$

$$\frac{A(x-1)^2 + Bx(x-1) + Cx}{x(x-1)^2} = \frac{x^2(A+B) + x(C-2A-B) + A}{x(x-1)^2}$$

$$\begin{cases} A+B=0 \\ C-2A-B=1 \\ A=1 \end{cases}$$

$$\begin{cases} A=1 \\ B=-1 \\ C=2 \end{cases}$$

④ $\int \frac{x+1}{x(x-1)^2} dx = \int \frac{1}{x} dx - \int \frac{1}{x-1} dx + 2 \int \frac{1}{(x-1)^2} dx$

$$\ln |x| - \ln |x-1| + 2 \frac{(x-1)^{-1}}{-1} + C$$

$$\ln \left| \frac{x}{x-1} \right| - \frac{2}{x-1} + C$$

Se per caso $\frac{1}{x(x-3)^3} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \frac{D}{(x-3)^3}$