Infinitesimi

$\lim_{x\to\pm\infty} \frac{c}{x} =$

caso 1) $c>0$ → $\lim_{x\to+\infty} \frac{c}{x} = 0^+$; $\lim_{x\to-\infty} \frac{c}{x} = 0^-$

caso 2) $c=0 = \lim_{x\to\pm\infty} \frac{c}{x} = 0$

caso 3) $c<0$ → $\lim_{x\to+\infty} \frac{c}{x} = 0^-$; $\lim_{x\to-\infty} \frac{c}{x} = 0^+$

Inoltre

$\lim_{x\to+\infty} \frac{0^+}{x} = 0^+$ $\quad$ $\lim_{x\to-\infty} \frac{0^-}{x} = 0^-$

$\lim_{x\to+\infty} \frac{0^-}{x} = 0^-$ $\quad$ $\lim_{x\to-\infty} \frac{0^-}{x} = 0^+$

Con i logaritmi

$f(x) = \log(x)$

$\lim_{x\to+\infty} \log(x) = +\infty$ $\quad$ $\lim_{x\to0^+} \log(x) = -\infty$

Tabella

$\frac{\pm\infty}{0^+} = +\infty$ $\quad$ $\frac{-\infty}{0^+} = -\infty$

$\frac{+\infty}{0^-} = -\infty$ $\quad$ $\frac{-\infty}{0^-} = +\infty$

$(+\infty)^{(+\infty)} = +\infty$ $\quad$ $(+\infty)^{(-\infty)} = 0^+$

$\frac{c>0}{0^+} = +\infty$ $\quad$ $\frac{c>0}{0^-} = -\infty$

$\frac{c<0}{0^+} = -\infty$ $\quad$ $\frac{c<0}{0^-} = +\infty$

$(+\infty)^n = +\infty \quad \forall n>0$

$(+\infty)^n = 0 \quad \forall n<0$

$\sqrt[n]{(+\infty)} = +\infty$ $\forall n$ intero

$\sqrt[n]{(-\infty)} = -\infty$ $\forall n$ int. dispari

Metodo di integrazione generale #1

$y = \frac{N(x)}{D(x)}$

1. Divisione fra N e D se $gr(N) \geq gr(D)$
2. Fattorizzazione il den
3. decomporre la frazione in fratti semplici
4. Integrazione dei pezzettini

Esempio 1

$$\int \frac{x^3 - 3x - 1}{x^2 - x - 2}\, dx$$

1)

$$\begin{array}{r|l} x^3 \;0x^2 - 3x - 1 & x^2 - x - 2 \\ -x^3 + x^2 + 2x & x + 1 \\ \hline \quad + x^2 - x - 1 & \\ \quad - x^2 + x + 2 & \\ \hline \quad\quad / \;\; / \;\; 1 & \end{array}$$

$$\frac{x^3 - 3x - 1}{x^2 - x - 2} = x + 1 + \frac{1}{x^2 - x - 2}$$

2) $x^2 - x - 2 = (x-2)(x+1)$

3) $$\frac{1}{x^2 - x - 2} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{Ax + A + Bx - 2B}{(x-2)(x+1)} =$$

$$\frac{x(A+B) + A - 2B}{(x-2)(x+1)} \qquad \begin{cases} A + B = 0 \\ A - 2B = 1 \end{cases} \qquad \begin{cases} A = -B \\ -B - 2B = 1 \end{cases} \qquad \begin{cases} A = \frac{1}{3} \\ B = -\frac{1}{3} \end{cases}$$

$$-3B = 1$$

$$B = -\frac{1}{3}$$

$$\frac{1}{(x-2)(x+1)} = \frac{1}{3} \cdot \frac{1}{x-2} - \frac{1}{3} \cdot \frac{1}{x+1}$$

4) $$\int \frac{x^3 - 3x - 1}{x^2 - x - 2}\, dx = \int \frac{1}{3(x-2)} - \frac{1}{3(x+1)}\, dx$$

$$\int x\, dx + \int dx + \frac{1}{3}\int \frac{1}{(x-2)}\, dx - \frac{1}{3}\int \frac{1}{(x+1)}\, dx$$

$y = x-2$ $\quad dy = 1\,dx$ $\qquad z = x+1$ $\quad dz = 1\,dx$

$$\frac{x^2}{2} + x + \frac{1}{3}\ln(x-2) - \frac{1}{3}\ln(x+1) + c$$

$$\frac{x^2}{2} + x + \frac{1}{3}\ln\frac{(x-2)}{(x+1)} + c$$

Esempio 2

$$\int \frac{1}{x^2-1}\,dx$$

① $\deg(D) > \deg(N)$

② $x^2 - 1 = (x-1)(x+1)$

③ $$\frac{1}{(x-1)(x+1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)} =$$

$$\frac{Ax - A + Bx + B}{(x+1)(x-1)} = \frac{x(B+A) - A + B}{(x+1)(x-1)} = \begin{cases} A+B=0 \\ B-A=1 \end{cases} \quad \begin{cases} A=-B \\ B+B=1 \end{cases}$$

$$\begin{cases} A = -\frac{1}{2} \\ B = \frac{1}{2} \end{cases}$$

④ $$\int \frac{1}{x^2-1}\,dx = \int -\frac{1}{2}\cdot\frac{1}{x+1} + \frac{1}{2}\frac{1}{x-1}\,dx$$

$$-\frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx = -\frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(x-1) + c$$

$$= \frac{1}{2}\ln\left(\left|\frac{x+1}{x-1}\right|\right) + c$$

Metodo Generale #2

- Denom 2° g non scomponibile
- Potenza di un monomio

} problema!

Esempio 1

$$\int \frac{1}{x^3+x}\,dx$$

① non serve $\deg(D) < \deg(N)$

② $x^3+x = x(x^2+1)$

③ $\frac{1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)} =$

$\frac{x^2(A+B)+Cx+A}{x(x^2+1)}$ $\quad \begin{cases} A+B=0 \\ C=0 \\ A=1 \end{cases} \quad \begin{cases} A=1 \\ B=-1 \\ C=0 \end{cases}$

④ $\int \frac{1}{x^3+x}\,dx = \int \frac{1}{x}\,dx - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \ln(|x|) - \frac{1}{2}\ln|x^2+1| + c$

$= \ln\left|\frac{x}{\sqrt{x^2+1}}\right| + c$

possono dare come risultati ln e arctan

Esempio 2

$$\int \frac{x+1}{x^3-2x^2+x}\,dx$$

① non serve

② $x^3-2x^2+x = x(x^2-2x+1) = x(x-1)^2$

③ $\frac{x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} =$

$\frac{A(x-1)^2+Bx(x-1)+Cx}{x(x-1)^2} = \frac{x^2(A+B)+x(C-2A-B)+A}{x(x-1)^2}$

$\begin{cases} A+B=0 \\ C-2A-B=1 \\ A=1 \end{cases} \quad \begin{cases} A=1 \\ B=-1 \\ C=2 \end{cases}$

④ $\int \frac{x+1}{x(x-1)^2}\,dx = \int \frac{1}{x}\,dx - \int \frac{1}{x-1}\,dx + 2\int \frac{1}{(x-1)^2}\,dx$

$$\ln|x| - \ln|x-1| + 2\frac{(x-1)^{-1}}{-1} + c$$

$$\ln\left|\frac{x}{x-1}\right| - \frac{2}{x-1} + c$$

Se per caso $\frac{1}{x(x-3)^3} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \frac{D}{(x-3)^3}$