

# Practice Problems on Random Variables and Probability Distributions

## Problem 1

A coin is tossed three times. Let  $X$  be the number of heads. Find the probability function of  $X$ .

**Solution:**

Sample space:  $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Values of  $X$ : 3 (HHH), 2 (HHT, HTH, THH), 1 (HTT, THT, TTH), 0 (TTT)

$$f(x) = \begin{cases} \frac{1}{8} & x = 0, 3 \\ \frac{3}{8} & x = 1, 2 \end{cases}$$

## Problem 2

Roll a fair die. Let  $X$  be the square of the number shown. Find the probability function of  $X$ .

**Solution:**

Outcomes:  $1-6 \Rightarrow X \in \{1, 4, 9, 16, 25, 36\}$

Each outcome has probability  $\frac{1}{6}$ , so:

$$f(x) = \begin{cases} \frac{1}{6} & x \in \{1, 4, 9, 16, 25, 36\} \end{cases}$$

## Problem 3

A die is rolled. Let  $X = 1$  if an odd number occurs, and 0 otherwise. Find the distribution of  $X$ .

**Solution:**

Odd numbers:  $1, 3, 5 \Rightarrow P(X = 1) = \frac{3}{6} = \frac{1}{2}$

Even numbers:  $2, 4, 6 \Rightarrow P(X = 0) = \frac{1}{2}$

$$f(x) = \begin{cases} \frac{1}{2} & x = 0, 1 \end{cases}$$

## Problem 4

Let  $X$  be a continuous random variable with PDF:  $f(x) = 2x$ , for  $0 \leq x \leq 1$ . Find the cumulative distribution function (CDF)  $F(x)$ .

**Solution:**

$$F(x) = \int_0^x 2t \, dt = x^2, \quad \text{for } 0 \leq x \leq 1$$

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

## Problem 5

Let  $f(x) = 3x^2$  for  $0 \leq x \leq 1$ . Find the probability  $P(0.2 < X < 0.4)$ .

**Solution:**

$$P(0.2 < X < 0.4) = \int_{0.2}^{0.4} 3x^2 \, dx = [x^3]_{0.2}^{0.4} = 0.4^3 - 0.2^3 = 0.064 - 0.008 = 0.056$$

## Problem 6

Let  $X$  have PDF  $f(x) = cx^2$ ,  $0 \leq x \leq 2$ . Find the value of  $c$ .

**Solution:**

$$\int_0^2 cx^2 \, dx = 1 \Rightarrow c \cdot \frac{8}{3} = 1 \Rightarrow c = \frac{3}{8}$$

## Problem 7

A random variable has PDF  $f(x) = c(4x - x^2)$  for  $0 \leq x \leq 2$ . Find the constant  $c$ .

**Solution:**

$$\int_0^2 c(4x - x^2) \, dx = c \left[ 2x^2 - \frac{x^3}{3} \right]_0^2 = c \left( 8 - \frac{8}{3} \right) = c \cdot \frac{16}{3} = 1 \Rightarrow c = \frac{3}{16}$$

## Problem 8

Let  $X$  have PDF  $f(x) = 2(1 - x)$ ,  $0 \leq x \leq 1$ . Find the CDF  $F(x)$ .

**Solution:**

$$F(x) = \int_0^x 2(1 - t)dt = 2 \left( x - \frac{x^2}{2} \right) = 2x - x^2$$

## Problem 9

Let  $f(x) = c(1 - x^2)$ ,  $-1 \leq x \leq 1$ . Find the probability  $P(0 < X < 0.5)$ .

**Solution:**

$$\int_{-1}^1 c(1 - x^2)dx = c \cdot \frac{4}{3} = 1 \Rightarrow c = \frac{3}{4}$$

$$P(0 < X < 0.5) = \int_0^{0.5} \frac{3}{4}(1 - x^2)dx = \frac{3}{4} \left[ x - \frac{x^3}{3} \right]_0^{0.5} = \frac{3}{4}(0.5 - 0.0417) = 0.34375$$

## Problem 10

Let  $X$  have PDF  $f(x) = cx^2$ ,  $0 \leq x \leq 1$ . Find the CDF and evaluate  $F(0.6)$ .

**Solution:**

$$\int_0^1 cx^2dx = 1 \Rightarrow c = 3$$

$$F(x) = \int_0^x 3t^2dt = x^3 \Rightarrow F(0.6) = (0.6)^3 = 0.216$$