$\theta_{24} = (\theta_2 + \theta_4)$

Let $R = a_3 C_{24} - d_3 S_2$ | $R \neq 0$ for nondegenerate cases.

$$\begin{cases} P_x = -C_1 R \;;\; P_y = -S_1 R \\ P_z = d_1 - a_3 S_{24} - d_3 C_2 \end{cases}$$

Also; $T_d(3,1) = -S_{24}$
$T_d(3,2) - S_{24}$

$S_{24} = -T_d(3,1)$
$C_{24} = -T_d(3,2)$
$\theta_{24} = \text{atan2}[-T_d(3,1), -T_d(3,2)]$

I can solve for unknowns in the order $\theta_1 \rightarrow \theta_{24} \rightarrow (\theta_2, d_3) \rightarrow \theta_4$

(1) θ_1 since $P_x = -R\cos\theta_1$ & $P_y = -R\sin\theta_1$

$\theta_1 = \text{atan2}(-P_y, -P_x)$

equivalent $\theta_1 = \text{atan2}(P_y, P_x) + \pi$

② Finding θ_{24} from third row of T_0^4.

No need $\theta_{24} = \text{atan2}\left[-T_4(3,1), -T_4(3,2)\right]$ if I specify a fixed number.

③ $R = a_3\cos(\theta_2+\theta_4) - d_3\sin\theta_2$

$= -(P_x\cos\theta_1 + P_y\sin\theta_1)$??

Now that I know θ_1, I can solve for R.

④ Finding θ_2 and d_3.

$R = a_3 C_{24} - d_3 S_2$

$P_z = d_1 - a_3 S_{24} - d_3 C_2$

$\Rightarrow d_3 S_2 = a_3 C_{24} - R = N$ — ①

$d_3 C_2 = d_1 - a_3 S_{24} - P_z = C$ — ②

$①^2 + ②^2$

$d_3^2(S_2^2 + C_2^2) = N^2 + C^2$

$d_3 = \pm\sqrt{N^2 + C^2}$ (if $d_3 \geq 0$, the prismatic extends positively)

$\theta_2 = \text{atan2}(N, C)$

⑤ Finding θ_4.

$\theta_4 = \theta_{24} - \theta_2$

Degenerate cases.

1. d_3 yield 2 signs ($\pm$)
So we have to choose a physically meaningful d_3:
$d_3 \in [d_{min}, d_{max}]$

2. If $N^2 + C^2 < 0$ numerically, d_3 will be a complex number and hence the pose will be unreachable

I will pick a simple fixed orientation for the wrist center
say $\theta_{24} = -\pi/2$

Using kinematics model only
position-only IK