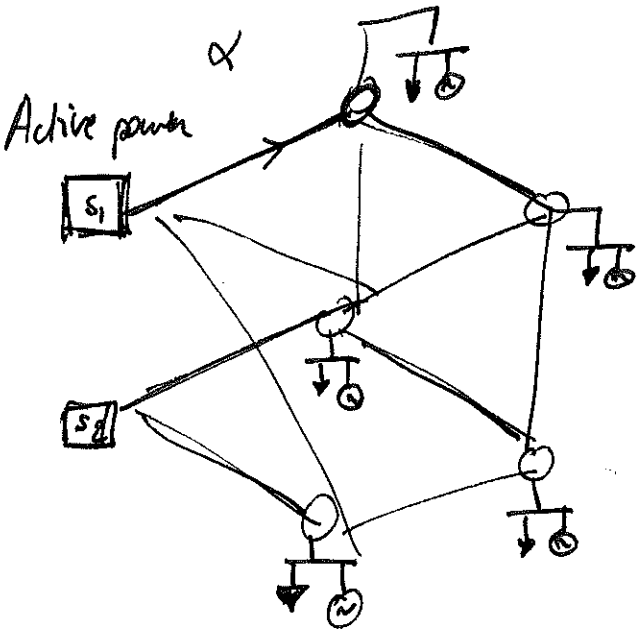
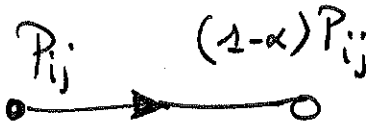


Network flow.

~~Kirchhoff~~

P limits.



Data : Set of time steps $\mathcal{T}$.

Set of lines candidates $\mathcal{L}$

Set of substation nodes. $\mathcal{N}_s$

Set of nodes. $\mathcal{N} \supset \mathcal{N}_s$.

cost of building line $l \in \mathcal{L}$: c_l^{line}

" " " substation $i \in \mathcal{N}_s$: $c_i^{substation}$.
per kw installed.

Set of grid users $\mathcal{N}_u$

$$\mathcal{N} = \mathcal{N}_s \cup \mathcal{N}_u \quad , \mathcal{N}_s \cap \mathcal{N}_u = \emptyset.$$

Power limit on line l $\overline{P}_{ij}$.
Max substation power $\overline{\delta}_i$.
(Losses in line l)

Cost of genset c_i^{genset}.
cost of fuel. c_i^{fuel} €/kwh.
cost of electricity $c_i^{imp.}$ / $c_i^{imp,grid}$
injection fee $c_i^{exp.}$ / $c_i^{exp,grid}$
connection cost $c_i^{connection}$

Data

CO_2 costs:

$CO_2^{imp,substation}$	$\frac{kg\,CO_2}{kWh}$	from transmission grid. (1')
CO_2^{fuel}	$\frac{kg\,CO_2}{kWh}$	from genset
CO_2^{genset}	$\frac{kg\,CO_2}{kW}$	of installed genset.
CO_2^{line}	$kg\,CO_2$	per line.
$CO_2^{substation}$	$\frac{kg\,CO_2}{kW}$	per substation capacity.

γ: interest rate for DSO.

Δt: time step duration.

(Max CO_2 Budget)

~~demand~~ consumption

P_{i,t_n}

Variables for the DSO (upper level)

$P_{ij,t}$ → active power from $i \to j$.

α_{ij} : build line ij (1) or not (0).

β_i : build substation (1) or not (0).

S_i : Max power drawn from substation i.

$P^{substation,imp}_{i,t}$: power from transmission grid.

$P^{substation,exp}_{i,t}$: to

CO_2 budget.

Variables for grid user i

$P^{imp}_{i,t}$ power imported at time t.

$P^{exp}_{i,t}$ — exported —

$P^{genset}_{i,t}$ — generated —

P^{genset}_i size of genset

P^{grid}_i grid imp./exp capacity.

DSO Costs: $\min \sum_{l \in \mathcal{L}} c_{ij}^{line} \alpha_{ij} + \sum_{i \in \mathcal{N}_S} c_i^{substation} S_i$.

G.U.i costs: $\min \sum_{t} p_{i,t}^{imp} \left(c_i^{imp} + c_i^{(imp,grid)} \right) - \sum_{t} p_{i,t}^{exp} \left(c_i^{exp} - c_i^{(exp,grid)} \right)$

$+ \sum_{t} p_{i,t}^{genset} c_i^{genset\ fuel}$ } Variable

$+ p_i^{genset} c_i^{genset} + p_i^{grid} c_i^{connection}$.

fixed

DSO constraints :

Budget balance : (1+f) DSO costs $\leq$

(4)

$$\sum_i P_i^{grid} \cdot c_i^{connection} + \sum_{i,t} \left(P_{i,t}^{imp} \, c_i^{imp,grid} + P_{i,t}^{exp} \, c_i^{exp,grid} \right) \Delta t \quad (d1)$$

$$CO_2 \text{ budget} = \sum_{\substack{i \in \mathcal{N}_S \\ t \in \mathcal{T}}} P_{i,t}^{substation,imp} \, CO_2^{imp,substation} + \sum_{\substack{i \in \mathcal{N}_m \\ t \in \mathcal{T}}} P_{i,t}^{genset} \, CO_2^{fuel} + \sum_{i \in \mathcal{N}_m} P_i^{genset} \, CO_2^{genset} + \sum_{i \in \mathcal{N}_S} S_i \, CO_2^{substation} + \sum_l \alpha_{ij} \, CO_2^{lin} \quad (d2)$$

(CO_2 budget $\leq$ Max CO_2 budget).

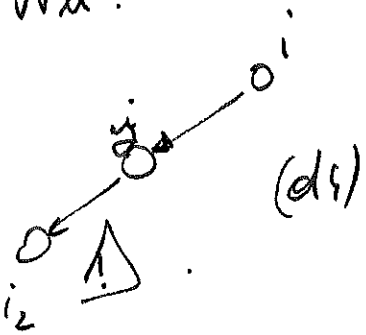
DSO constraints (cont'd)

Power balance at substation : $\forall i \in \mathcal{N}_S$

$$P_{i,t}^{imp,\,substation} - P_{i,t}^{exp,\,substation} = \sum_{j \in Neighbours(i)} P_{i,j,t} \quad (d3)$$

Power balance at grid user bus : $\forall j \in \mathcal{N}_u$:

$$P_{j,t}^{imp} - P_{j,t}^{exp} = \sum_{i \in \mathcal{N}(j)} P_{ij,t} \quad (d4)$$



Max power in a line :

$$P_{i,j,t} \leq \alpha_{ij} \overline{P_{ij}} \qquad \forall i,j \in \mathcal{L}. \quad (d5)$$

Max power at substation :

$$S_i \leq \beta_i \overline{S_i} \quad (d6)$$

$$P_{i,t}^{substation,\,k} \leq S_i \qquad \forall k \in \{imp, exp\} \quad (d7)$$

Radiality

$$\sum_{l \in \mathcal{L}} \alpha_{ij} = |\mathcal{N}_u| \quad (d8).$$

User constraints.

Power balance :

$$P_{i,t}^{imp} - P_{i,t}^{exp} = P_{i,t}^{consumption} - P_{i,t}^{genset} \quad \forall i,t. \quad (\mu_1)$$

Genset : $P_{i,t}^{genset} \leq \overline{P}_i^{genset}$ (μ_2)

Import limit : $P_{i,t}^{imp} \leq \overline{P}_i^{grid}$ (μ_3)

Export limit : $P_{i,t}^{exp} \leq \overline{P}_i^{grid}$. (μ_4)

~~$\left(P_{i,t}^{imp} - P_{i,t}^{exp} \leq \overline{P}_i^{grid} \right.$~~

~~$\left. P_{i,t}^{exp} - P_{i,t}^{imp} \leq \overline{P}_i^{grid} \right)$~~ .