GACD

——Teacher Wu

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前言

This is a GACD note-book for xupt. If there has much error in note-book,forgive me. It's just writes for me.

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目录

第一章　Linear Equations

1.1 Introduction

Bezier curves are widely used in computer graphics and related fields. They are used to model smooth curves and surfaces, and are used in computer animation, CAD, and other fields. The curves, which are related to Bernstein polynomials, are named after Pierre Bézier, who used it in the 1960s for designing curves for the bodywork of Renault cars. Generalizations of Bézier curves to higher dimensions are called Bézier surfaces, of which the Bézier triangle is a special case.

1.2 Bezier Curves

Definition 1.2.1 (*Bezier Curve*)

$$F(t) = P_0 + \sum_{i=1}^{n} a_i f_i^n(t)$$

$$f_i^t(t) = \frac{-t^i}{(i-1)!}\frac{d^{i-1}}{dt^{i-1}}\left[\frac{(1-n)^n - 1}{t}\right] = \sum_{j=1}^{n}(-1)^{i+j}\binom{n}{j}\binom{j-1}{i-1}t^j$$

Remark: *This polynomial funcitons is given by Bezier*

Now we can write the Bezier curve as

1.2.1 Bezier Curves definition

Definition 1.2.2

n power Bezier curve is given by:

$$P(t) = \sum_{i=0}^{n} b_i B_i^n(t), t \in [0,1]$$

Remark: *The $b_i \in R^3$ is the control point of the curve. The $B_i^n = \binom{n}{j}(1-t)^{n-i}t^i$ is the Bernstein polynomial.*

1.2.2 Bernstein polynomial characterization

1. unit decompostion

$$1 = [t + (1-t)]^n = \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} = \sum_{i=0}^{n} B_i^n(t)$$

2. non-negative

$$B_i^n(t) \geq 0$$

3. The endpoint $B_i^n(0) = \begin{cases} 1, & i=0 \\ 0, & i \neq 0 \end{cases}$, $B_i^n(1) = \begin{cases} 1, & i=n \\ 0, & i \neq n \end{cases}$, $\frac{dB_i^n(t)}{dt}\Big|_{t=0} = \begin{cases} -n, & i=0 \\ n, & i=n \\ 0 = i \neq 0, n \end{cases}$,

$\frac{dB_i^n(t)}{dt}\Big|_t = 1 = \begin{cases} -n, & i=0 \\ n, & i=n \\ 0 = i \neq 0, n \end{cases}$

4. Symmetry

$$B_i^n(t) = C_{n-i}^n (1-t)^{n-i} t^i = B_{n-i}^n (1-t)$$

5. Derivative

$$\frac{dB_i^n(t)}{dt} = n\left[B_{i-1}^{n-1}(t) - B_i^{n-1}(t)\right]$$

6. Recursion

$$B_i^n(t) = (1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t)$$

7. The Maxium : The max of

$$B_i^n(t) is B_{\lfloor n/2 \rfloor}^n(t)$$

8. elevation

$$B_i^n(t) = (1 - \frac{i}{n+1})B_i^{n+1}(t) + \frac{i+1}{n+1}B_{i-1}^{n+1}(t)$$

9. partion formula:

$$B_i^n(ct) = \sum_{n}^{j} B_i^j B_j^n(t)$$

10. integral formula:

$$\int_0^1 B_i^n(t)dt = \frac{1}{n+1}$$

11. conversion formula with power basis:

$$t^j = \sum_{i=j}^{n} \frac{C_{n-j}^{i-j}}{C_n^i} B_i^n(t)$$

12. Recursion formula:

$$P_n(t) = P_n(b_0, b_1, ..., b_n; t) = (1-t)P_{n-1}(b_0, b_1, ..., b_{n-1}; t) + tP_{n-1}(b_1, b_2, ..., b_n; t)$$

13. end point char:

$$P_n(0) = b_0, P_n(1) = b_n$$

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