GACD

——Teacher Wu

ManJack

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前言

This is a GACD note-book for xupt. If there has much error in note-book,forgive me. It's just writes for me.

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目录

第一章　Bezier Curves

1.1　Introduction

Bezier curves are widely used in computer graphics and related fields. They are used to model smooth curves and surfaces, and are used in computer animation, CAD, and other fields. The curves, which are related to Bernstein polynomials, are named after Pierre Bézier, who used it in the 1960s for designing curves for the bodywork of Renault cars. Generalizations of Bézier curves to higher dimensions are called Bézier surfaces, of which the Bézier triangle is a special case.

1.2　Bezier Curves

Definition 1.2.1 (*Bezier Curve*)

$$F(t) = P_0 + \sum_{i=1}^{n} a_i f_i^n(t)$$

$$f_i^t(t) = \frac{-t^i}{(i-1)!}\frac{d^{i-1}}{dt^{i-1}}\left[\frac{(1-n)^n - 1}{t}\right] = \sum_{j=1}^{n}(-1)^{i+j}\binom{n}{j}\binom{j-1}{i-1}t^j$$

Remark: *This polynomial funcitons is given by Bezier*

Now we can write the Bezier curve as

1.2.1　Bezier Curves definition

Definition 1.2.2

n power Bezier curve is given by:

$$P(t) = \sum_{i=0}^{n} b_i B_i^n(t), t \in [0, 1]$$

Remark: *The $b_i \in R^3$ is the control point of the curve. The $B_i^n = \binom{n}{j}(1-t)^{n-i}t^i$ is the Bernstein polynomial.*

Property 1.2.1

1. *unit decompostion*

$$1 = [t + (1-t)]^n = \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} = \sum_{i=0}^{n} B_i^n(t)$$

2. *non-negative*

$$B_i^n(t) \geq 0$$

3. *The endpoint* $B_i^n(0) = \begin{cases} 1, & i = 0 \\ 0, & i \neq 0 \end{cases}$, $B_i^n(1) = \begin{cases} 1, & i = n \\ 0, & i \neq n \end{cases}$, $\frac{dB_i^n(t)}{dt}\Big|_{t=0} = \begin{cases} -n, & i = 0 \\ n, & i = n \\ 0 = i \neq 0, n \end{cases}$,

$\frac{dB_i^n(t)}{dt}\Big|_{t} = 1 = \begin{cases} -n, & i = 0 \\ n, & i = n \\ 0 = i \neq 0, n \end{cases}$

4. *Symmetry*

$$B_i^n(t) = C_{n-i}^n (1-t)^{n-i} t^i = B_{n-i}^n (1-t)$$

5. *Derivative*

$$\frac{dB_i^n(t)}{dt} = n\left[B_{i-1}^{n-1}(t) - B_i^{n-1}(t)\right]$$

6. *Recursion*

$$B_i^n(t) = (1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t)$$

7. *The Maxium : The max of*

$$B_i^n(t) is B_{\lfloor n/2 \rfloor}^n(t)$$

8. *elevation*

$$B_i^n(t) = (1 - \frac{i}{n+1})B_i^{n+1}(t) + \frac{i+1}{n+1}B_{i-1}^{n+1}(t)$$

9. *partion formula:*

$$B_i^n(ct) = \sum_{n}^{j} B_i^j B_j^n(t)$$

10. *integral formula:*

$$\int_0^1 B_i^n(t)dt = \frac{1}{n+1}$$

11. *conversion formula with power basis:*

$$t^j = \sum_{i=j}^{n} \frac{C_{n-j}^{i-j}}{C_n^i} B_i^n(t)$$

12. *Recursion formula:*

$$P_n(t) = P_n(b_0, b_1, ..., b_n; t) = (1-t)P_{n-1}(b_0, b_1, ..., b_{n-1}; t) + tP_{n-1}(b_1, b_2, ..., b_n; t)$$

13. *end point char:*

$$P_n(0) = b_0, P_n(1) = b_n$$

14. *the Bezier curve will not extend outside the boundary of the convex polygon formed by its control points.*

15. *Geometric Invaiance: The Bezier curve is invariant under affine transformation.*

Definition 1.2.3 (*de Casteljau's algorithm*)

$$\begin{cases} P_t^0(t_0) = b_i, i = 0, 1, ..., n \\ P_i^j(t) = (1-t)P_i^{j-1}(t) + tP_{i+1}^{j-1}(t) \end{cases}$$

Remark: *In the end,we can get $P_0^n(t)$ and that $P_0^n(t)$ is $P_n(t_0)$. And The original curves is divided into two curves. The two curves are the same as the original curve.$P_n(P_0^0(t), P_0^1(t), \cdots, P_0^n(t_0); t)$ and $P_n(P_0^n(t_0), P_1^{n-1}(t_0), \cdots, P_n^0(t_0); t)$*

1.2.2 Bezier Curves other forms

Representing Bezier curve using edge vectors:

Definition 1.2.4

We know The

$$P_n(t) = \sum_{i=0}^{n} B_i^n(t) b_i, 0 \le t \le 1$$

let

$$a_0 = b_0, a_i = b_i - b_{i-1}, i = 1, 2, ..., n$$

we have

$$P_n = \sum_{i=0}^{n} f_{i,n}(t) a_i, 0 \le t \le 1.$$

where

$$\begin{cases} f_{0,n}(t) = 1 \\ f_{i,n}(t) = 1 - \sum_{j=0}^{i-1} B_j^n(t) \text{ or } f_{i,n}(t) = \sum_{j=i}^{n} B_j^n(t) \end{cases}$$

The same as we can conclude that Bezier curves's Derivative.

Assume that

$$P_n(t) = \sum_{i=0}^{n} B_i^n(t) b_i$$

we can detive

1. $\frac{dP_n(t)}{dt} = nP_{n-1}(\Delta b_0, \Delta b_1, ..., \Delta b_{n-1}; t)$
2. $\frac{d^2P_n(t)}{dt^2} = n(n-1)P_{n-2}(\Delta^2 b_0, \Delta^2 b_1, ..., \Delta^2 b_{n-2}; t)$
3. $\frac{d^kP_n(t)}{dt^k} = \frac{n!}{(n-k)!}P_{n-k}(\Delta^k b_0, \Delta^k b_1, \ldots, \Delta^k b_{n-2}; t)$

Remark:

1. $\dot{P}_n(0) = n\Delta b_0, \dot{P}_n(1) = n\Delta b_{n-1}$

证明： *Proof of* $\dot{P}_n(0) = n\Delta b_0$*:*

$$\begin{aligned} \dot{P}_n(0) &= n\sum_{i=0}^{n-1} B_i^{n-1}(0)\Delta b_i = \sum_{i=0}^{n-1} \binom{n-1}{i} t^i(1-t)^{n-i-1}\bigg|_{t=0} \Delta b_i \\ &= n\Delta b_0 \end{aligned}$$

Proof of $\dot{P}_n(1) = n\Delta b_{n-1}$*:*

$$\begin{aligned} \dot{P}_n(1) &= n\sum_{i=0}^{n-1} B_i^{n-1}(1)\Delta b_i = \sum_{i=0}^{n-1} \binom{n-1}{i} t^i(1-t)^{n-i-1}\bigg|_{t=1} \Delta b_i \\ &= n\Delta b_{n-1} \end{aligned}$$

■

2. In which that :$\Delta b_i = b_{i+1} - b_i$*,* $\Delta^k b_i = \Delta^{k-1} b_{i+1} - \Delta^{k-1} b_i$

bezier curve and bernstein polynomials

Definition 1.2.5

$$B_i^n(t) = \binom{n}{i}(1-t)^{n-i}t^i$$

so we can get the bezier curve:

第二章　B spline curse

2.1　B Spline History

2.2　B Spline curve' definition and its properties

2.3　de Boor alorgithm

2.4　The deviation of B spline curve

2.5　B spline curse and it's deviation

The power k to B spline curve is very important. It's decide the deviation of B spline curve. And the deviation of B spline curve is also decide by the knot vector. So we need to know the deviation of B spline curve.

2.6　B spline and Bezier curve

Using k power to B spline $s(u) = \sum_{i=0}^{n} N_i^k(u)$ and Bezier curve $s(u) = \sum_{i=0}^{n} B_i^n(u)$, we can get the relationship between B spline and Bezier curve. The curve is equaled between B spline and Bezier

Definition 2.6.1 ()

The control poind of terminate B spline curve is $s(u) = \sum_{i=0}^{k} d_i N_i^k(u)$ and the control poind of terminate Bezier curve is $P(t) = \sum_{i=0}^{k} b_i B_i^k(t)$.Then we can get the relationship between B spline and Bezier curve.

$$[b_0, b_1, ..., b_k]^T = M[d_0, d_1, ..., d_k]^T$$

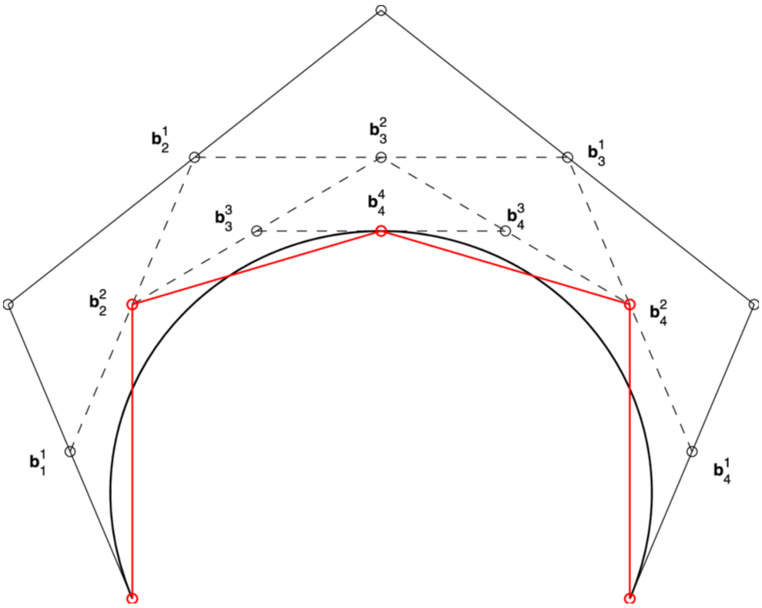

图 2.1:

2.6.1 de Rham alorgithm

de Rham alorgithm can deviate a series of S_k ‘limitations is that $C(S_0, T) = \lim_{k \to \inf} S_k$ subsubsectionde Rham Curce Continuity

Property 2.6.1 (*Continuity*)

when $0 < \lambda_{i,k}, \mu_{i,k}, \lambda_{i,k} + \mu_{i,k} < 1$,*de Rham curves* S_k *is continuted*。*Assum that* $< p_{i-1}^k p_i^k, p_i^k p_{i=1}^k >$

Property 2.6.2 (*Fractional dimension*)

1. *F has unnumberable many structure*
2. *F is not formular, but it's can be described by computer*
3. *Different F has the same as topological dimension*
4. *F's fractional dimension is larger than it's topological dimension*

第三章 Bernsterin-Bezier 三角形曲面片

设 T 是参数 uv-平面上的标准三角形，其顶点为:

$$V_1 = [1, 0]^T, V_2 = [0, 1]^T, V_3 = [0, 0]^T$$

uv-平面上任意 u 关于 T 的重心坐标为:(u,v,w), 其中:

$$U = uV_1 + vV_2 + wV_3, u + v + w = 1$$

Remark: *We can know that :*

1. *The number of control point is* $\frac{(n+1)(n+2)}{2}$
2. *The degree of the curve is* n^2

二元的 Bernstein 多项式的定义:

Definition 3.0.1

Bernstein 多项式定义为:

$$B^n_{i,j,k}(u, v, w) = \frac{n!}{i!j!k!}u^i v^j w^k, i + j + k = n, u + v + w = 1$$

二元的 Bernstein 多项式的性质:

1. 权性: $B^n_{i,j,k}(u, v, w) = 1$
2. 正性: $B^n_{i,j,k}(u, v, w) \geq 0$
3. 递推性: $B^n_{i,j,k}(u, v, w) = uB^{n-1}_{i-1,j,k}(u, v, w) + vB^{n-1}_{i,j-1,k}(u, v, w) + wB^{n-1}_{i,j,k-1}(u, v, w)$

Definition 3.0.2

The n power of Triangular Bézier Surface Patcha's definition:

$$\mathcal{P}(u, v, w) = \sum_{i+j+k=n} B^n_{i,j,k}(u, v, w)P_{i,j,k}$$

Remark:

1. *The endpoint interpolation property of the triangular Bézier surface patch:*

$$P(1,0,0) = P_{0,0,n}, P(0,1,0) = P_{0,n,0}, P(0,0,1) = P_{n,0,0}$$

2. *The edge of the triangular Bézier surface patch is a Bézier curve:*

3. *The triangular Bézier surface patch is a convex hull of the control point*