AMATH 251 Final Project: SIRD Financial Contagion Model for Systemic Corporate Defaults

Man Nok Ho

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Abstract

In this project, we develop and use an SIRD differential equation system to study the impact of three major economic crises (2008 Global Financial Crisis, 2000 Dotcom Bubble Crash, and 1997 Asian Financial Crisis) on firms in the affected country/region. The goal is to determine whether the SIRD model, traditionally used for epidemiology, effectively models the contagion of financial default during sudden shocks. Parameters for the system were calibrated based on historical economic data and the methodology outlined in Martinez [2021], then a numerical simulation using the Fourth-order Runge-Kutta (RK4) method was used to plot and analyze the results. We find that the SIRD framework effectively captures the main features of the three cases, and provides a way to compare the severity and structure of different economic crises.

1 Introduction

In the modern financial system, firms are connected through a system of loans, investments, and contracts. This highly interconnected nature means that should one firm experience financial difficulty or *default* (be unable to pay back their obligations, i.e. bankrupt), their insolvency is likely to impact to other firms in its industry. This phenomenon is known as *financial contagion,* since this financial distress spreads across firms through these contractual networks, much like how a disease spreads through a population [Allen and Gale, 2000].

There is ample historical support for the idea of financial contagion. A study by Giesecke et al. [2011] analyzing 150 years of bond default rate data showed that on a macro scale, defaults tend to occur in clustered waves rather than as independent events – examples include the Great Depression and the 2008 Financial Crisis. Intuitively, times of economic recession will affect most companies' abilities to generate revenue and pay off its debt, leading to widespread default. Another major reason is *counterparty risk:* because firms have interdependent balance sheets through loans and investments, the default of one firm can trigger a cascade of defaults in its sector [Jarrow and Yu, 2001]. Allen and Gale [2000] proved this for the banking system using a model of four banks with interconnected deposits. Davis and Lo [2001] described this default interdependence by modelling it as an infection-like transmission mechanism, with the idea that a firm can either default directly or be 'infected' by the default of a correlated firm. While Davis and Lo focus on a probabilistic approach, we can adopt this 'contagion' framework to analyze the proliferation of defaults during economic downturns using differential equations.

2 Methods

The model adapts the classic SIR framework, first proposed in *'A contribution to the mathematical theory of epidemics'* by Kermack and McKendrick [1927], with an additional 'deceased' group (aka SIRD framework), to model financial credit contagion over time. The model partitions the population of firms into four subgroups that change with respect to time:

- $S(t)$: proportion of firms **susceptible** to the financial crisis.
- $I(t)$: proportion of firms **infected** by the crisis (i.e. facing financial stress).

- $R(t)$: proportion of firms **recovered** from the crisis, **or naturally immune** (due to isolation from the crisis or preventive policies).
- $D(t)$ proportion of firms **defaulted** (gone bankrupt) due to the crisis.

To model the change of these proportions over time, we will adopt the following system of equations proposed by Khan et al. [2024] for epidemics modelling:

$$\begin{cases} \frac{dS}{dt} = \mu(1-S) - \beta SI \\ \frac{dI}{dt} = \beta SI - (\mu + \alpha + \gamma)I \\ \frac{dR}{dt} = \alpha I - \mu R \\ \frac{dD}{dt} = \gamma I \end{cases}$$

where $S + I + R + D = 1$ for all $t \geq 0$, each function is non-negative, and:

- β =the *contagion rate*, i.e. the rate at which the financial distress spreads across firms. This rate concerns the transition from S to I.
- α =the *recovery rate*, i.e. the rate at which distressed firms resolve their financial issues and become immune to the crisis. This rate concerns the transition from I to R.
- δ =the *default rate*, i.e. the rate at which an infected firm will default due to the crisis. This rate concerns the transition from I to D. To keep this model simple, we make no distinction as to the reason for default, and assume that all defaults are caused by the crisis.
- μ =the *natural birth/death rate*, i.e. the rate at which firms enter and exit the market naturally (assumed equal). All firms are 'born' in S and 'die' naturally at the same rate in S, I, and R.

To simulate the system of DEs presented above, we will use the Runge-Kutta Fourth Order (RK4) Method. The method will be applied with varying time-steps to assess and compare stability, convergence, and error order. RK4 is adopted due to its high accuracy relative to its computational complexity, especially when compared to lower-order methods like the Euler or Improved Euler method. Using historically-based parameters, we will solve for the system of equations using RK4 and compare the results against the actual default data, in order to determine the accuracy of the model.

3 Simulation

We will simulate the following historical financial crises: the 2008 Global Financial Crisis, the 2000 Dotcom bubble crash, and the 1997-1998 Asian Financial Crisis. The parameter values for each scenario are as follows:

Parameter	2008 GFC	2000 Dotcom	1997 Asia
μ	0.00028	0.00030	0.00033
α	0.00472	0.00494	0.00461
δ	0.00083	0.00061	0.00094
β	0.01137	0.01569	0.00897

Table 1: Parameter values (per day) for each of the three studied crises.

US Library of Congress corporate entry data was used for the natural growth and death rate (μ); from this data we see that the difference between entry and exit rates are very small, supporting our assumption of an equal growth and death rate in the short term [Levin, 2024]. The annual rate was scaled by 1/365 to attain a daily rate. Next, for the recovery (α) and default (δ) rates, we estimate using the formulae outlined in Martinez [2021]:

$$\text{expected recovery time} \approx \frac{1}{\alpha + \delta}, \qquad \frac{\delta}{\alpha} = \text{ratio of deaths to recoveries.}$$

For all cases, we will assume a firm takes on average 180 days to recover. To find the death-to-recovery ratio, we use the peak bond default rates among speculative-grade (i.e. financially distressed) issuers in the respective crisis periods. The adopted default rates (d) are 15% for 2008 [Emery et al., 2010], 11% for 2000 [Hamilton et al., 2007], and 17% for 1997 [Tennant, 2007]. From this, we get:

$$\frac{\delta}{\alpha} = \frac{d}{1-d} \implies \delta = \frac{\alpha d}{1-d}, \quad 180 = \frac{1}{\alpha + \frac{\alpha d}{1-d}} \implies \alpha = \frac{1-d}{180}$$

Lastly, the contagion (β) rate was estimated using the the method in Martinez [2021]:

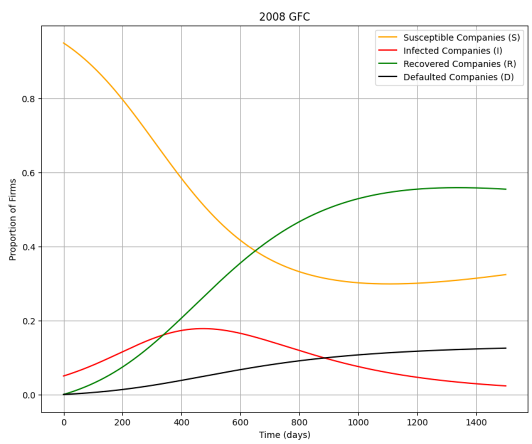
$$\beta = (\alpha + \delta)\frac{\ln(S_0/S_\infty)}{I_0 + S_0 + S_\infty} \quad (S_0 = S(0), \ \ I_0 = I(0), \ \ S_\infty = \lim_{t\to\infty} S(t))$$

The assumptions for the initial and final proportions in each scenario are as follows:

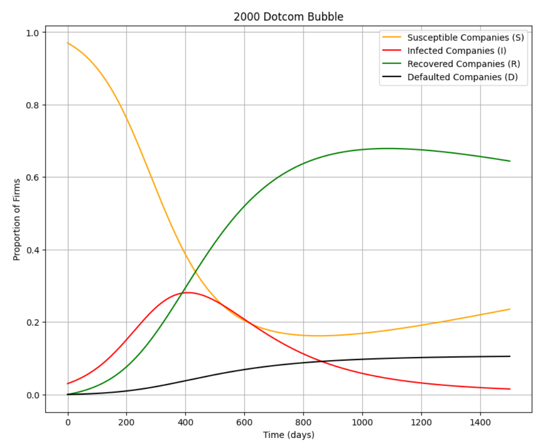
Parameter	2008 GFC	2000 Dotcom	1997 Asia
S_0	0.95	0.97	0.96
I_0	0.05	0.03	0.04
S_∞	0.10	0.05	0.15
Implied β	0.01137	0.01569	0.00897

Table 2: Population assumptions for each of the 3 studied crises.

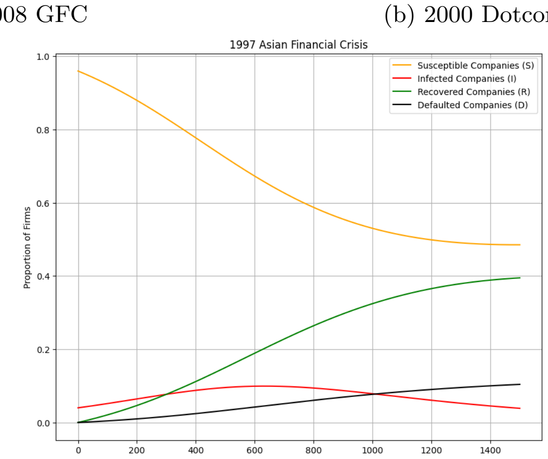
From this, an RK4 numerical simulation of 1500 days at $h = 0.1$ gives the following plots:



(a) 2008 GFC



(b) 2000 Dotcom Bubble Crash



(c) 1997 Asian Financial Crisis

Figure 1: RK4 Simulation Plots, with each crisis simulated for 1500 days at time-step $h = 0.1$.

4 Analysis

Crisis	Final S	Final I	Final R	Final D	Peak I value (time)
2008 GFC	0.3240	0.0232	0.5548	0.1253	0.1780 ($t = 472.6$)
2000 Dotcom	0.2417	0.0156	0.6610	0.1080	0.2815 ($t = 409.3$)
1997 Asia	0.4854	0.0387	0.3950	0.1041	0.0993 ($t = 633.5$)

Table 3: Summary Table of RK4 Simulation Results.

The simulated SIRD results align reasonably well with the actual documented conditions during the three crises. In the 2008 Global Financial Crisis shown in 1a, the model produces a broad and persistent wave of distress, with infection peaking at 17.80% and over 60% of firms eventually leaving the susceptible state. This is similar to real conditions, where global credit markets froze and widespread firm-level stress was experienced across all sectors, not just the financial sector [International Monetary Fund, 2009]. The timing of the crisis peak was also generally accurate, as infections peaked at day 472, just over 1 year after the crisis began; Moody's monthly data also shows that defaults peaked in early 2009 [Emery et al., 2010]. The final default share of 12.53% closely aligned with the actual speculative-grade default rate of roughly 12.97% in 2009, but exceeds the total default rate of 5.35% [Emery et al., 2010]. When considering all firms rather than just speculative-grade ones, the higher default rate in the model can be interpreted as also capturing permanent exits, distressed mergers, and company delistings as opposed to just formal defaults.

The Dotcom Bubble simulation shown in 1b produced the highest infection peak of 28.15% and the lowest final susceptible share of 24.17%, reflecting the severe collapse of the technology-startup ecosystem in 2000–2002. During the crisis, the Nasdaq index fell 77% in two years [Goldman Sachs, 2019], and estimates suggest that only 48% of Dotcom firms managed to survive through 2004 after the collapse [Goldfarb et al., 2006]. Again, the peak infection occurring just over 1 year after the crisis began, and subsiding by year 3, matches well with the historical timeline. The default rate (10.80%) aligns closely with the 2002 tech-industry default rate of 8.57% [Emery et al., 2010], but much lower than the over 50% failure rate estimate as it does not capture the other corporate failures in the industry that did not necessarily end in default. Many Dotcom startups did not enter formal bankruptcy but instead shut down, ran out of funding, or were absorbed by more resilient competitors.

The Asian Financial Crisis simulation shown in 1c shows the weakest infection peak (9.93%) and the largest remaining susceptible population (48.54%), reflecting the fact that this crisis was severe on a regional level but relatively contained globally. The crisis affected Southeast Asian countries and South Korea most severely, while places like Singapore, mainland China, and Hong Kong remained relatively insulated from the crisis and did not suffer massive default/currency crises [Pang, 2000]. Since our model used default data for all of Asia-Pacific, and accounted for the regional diversity by reducing the initial 'infected' proportion, the model did not capture these variations in crisis severity. This explains why the final default rate (10.41%) was the lowest despite having the highest speculative default rate. The timeframe for this model was also not very accurate, as default data from Tennant [2007] suggests that the crisis had largely subsided by late 1998-1999, but the model suggests an infection peak at day 633 (nearly 2 years into the crisis).

5 Conclusion

Due to the lack of concrete examples of SIRD modelling for these economic scenarios, it is not possible to compare our parameter estimations with other economic precedents. However, we found that our adopted methodology of parameter selection simulates the historical crisis conditions reasonably well. The relative ordering of infection peaks and default magnitudes, with 2008 causing the most defaults and the Dotcom crisis having the most intense peak, is generally consistent with the historical data, as shown above. In all three simulations, the recovered populations grow significantly, which reflects the fact that most firms in these crises experienced significant stress but did not end up defaulting.

One shortcoming of our model, most evident in the Dotcom and AFC examples, is the lack of crisis-specific data to more accurately calculate the parameters. For example, we used 'default rates' as a proxy for 'death rate' for the Dotcom crisis, but many firms 'died' without defaulting, which led to the model understating the number of tech startups that collapsed in this period. Likewise, the AFC model could be improved by using country-specific default rates for heavily impacted countries (e.g. Thailand, Indonesia). Narrowing the 'population' to one country in this way would have better captured the severity of the crisis.

A potential improvement would be to separate default risk into *systematic* (unrelated to crisis) and *non-systematic* (crisis-caused) risk, since our model currently assumes all defaults during the crisis are non-systematic. This was done in a proposed SIRS model by An [2022], which split the infected group into I_1 (systematic) and I_2 (non-systematic), with different contagion rates for each. This would more accurately capture the dynamics of the crisis and allow us to better compare the results with non-crisis times.

In conclusion, the contagion model provides an useful framework to modelling the spread of financial stress during times of crisis. While it fails to capture policy responses and the specific 'network effect' between individual firms, it still provides insight onto the intensity and timeframe of each economic shock.

6 Final Code

The final code can be accessed on Github through this link.

References

Franklin Allen and Douglas Gale. Financial contagion. *Journal of Political Economy*, 108(1), 2000.

Haorui An. Using sirs model to study the risk of cross- contagion in financial markets. *Proceedings of the 2022 International Conference on Mathematical Statistics and Economic Analysis*, 2022.

Mark Davis and Violet Lo. Infectious defaults. *Quantitative Finance*, 1(4):382–387, 2001.

Kenneth Emery, Sharon Li, Ou, and Jennifer Tennant. Corporate default and recovery rates, 1920-2009. *Moody's Investors Service*, 2010.

Kay Giesecke, Francis A. Longstaff, Stephen Schaefer, and Ilya Strebulaev. Corporate bond default risk: A 150-year perspective. *Journal of Financial Economics*, (102):233–250, 2011.

Brent Goldfarb, David Kirsch, and David A. Miller. Was there too little entry during the dot com era? *Journal of Financial Economics*, 86(1), 2006.

Goldman Sachs. The late 1990s dot-com bubble implodes in 2000. 2019.

David T. Hamilton, Sharon Ou, Frank Kim, and Richard Cantor. Corporate default and recovery rates, 1920-2006. *Moody's Investors Service*, 2007.

International Monetary Fund. Global financial stability report: Responding to the financial crisis and measuring systemic risks. *World Economic and Finamoncial Surveys*, 2009.

Robert A. Jarrow and Fan Yu. Counterparty risk and the pricing of defaultable securities. *The Journal of Finance*, 56(5):1765–1800, 2001.

William O. Kermack and Anderson G. McKendrick. A contribution to the mathematical theory of epidemics. *Proceedings of the Royal Society*, 115(772):700–721, 1927.

Harun-Or-Rashid Khan, Mostak Ahmed, and Alam M. M. Sarker. Optimal control of an sird model with data-driven parameter estimation. *Results in Control and Optimization*, 2024.

Adam G. Levin. Is u.s. entrepreneurship declining? *Library of Congress*, 2024.

Vincente Martinez. A modified sird model to study the evolution of the covid-19 pandemic in spain. *Symmetry*, 13(4), 2021.

Eul-Soo Pang. The financial crisis of 1997–98 and the end of the asian developmental state. *Contemporary Southeast Asia*, 22(3), 2000.

Jennifer Tennant. Default and recovery rates of asia-pacific corporate bond and loan issuers, excluding japan, 1990 -1h2007. *Moody's Corporate Finance*, 2007.