Thrust 1: Individual decision making under computational and cognitive constraints

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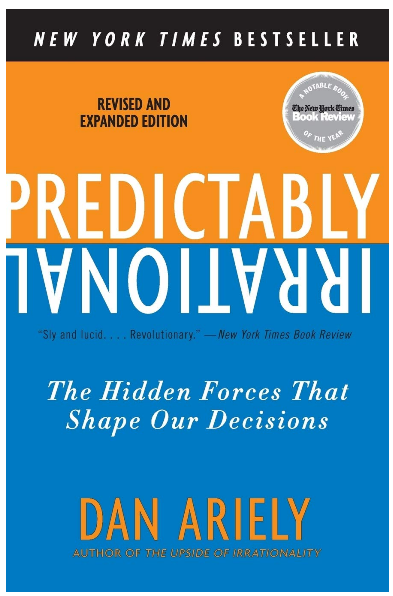
The big picture

Many studies have shown that humans are "predictably irrational"

- they do not act in a fully rational way, as assumed by standard economic theory
- but their deviations from rational behavior are quite systematic



Can we explain "predictably irrational" human behavior as the outcome of computational and cognitive constraints?

We focus on four well-studied problems:

- multi-armed bandits:
  - You want to choose the best slot machine, given m choices
  - Classic exploitation/exploration problem
  - Models managing research investments, portfolio selection, ...

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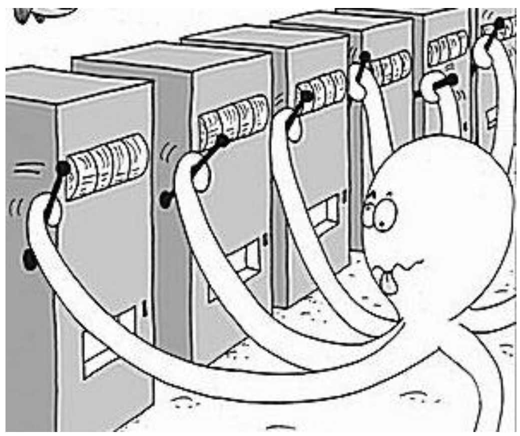
Conclusions:

- Computational limitations help explain human behavior
- Building human-like behavior into our algorithms can significantly improve performance!
- "Irrational" behavior is not always so irrational

Multi-armed bandits (MABs)

[Liu and Halpern]



Exploration vs. *Exploitation*

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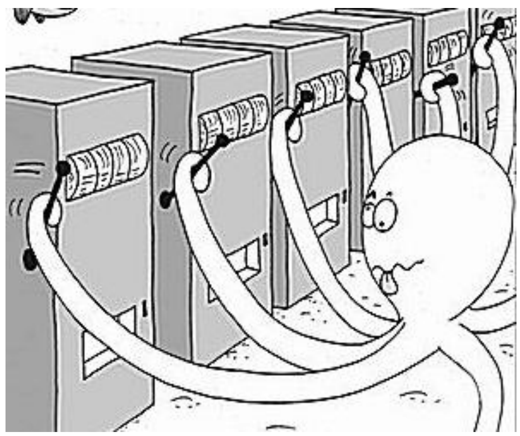

Exploration vs. *Exploitation*

A K-armed MAB can be identified with a tuple $(\mu_1, \ldots, \mu_K)$

- each component represents an arm
- μ_i is a distribution over the possible rewards of arm i
    - We take the possible rewards to be 1 ("success") or 0 ("failure").
    - We assume the distributions do not vary over time.

MAB protocols

There are many protocols for finding the best arm. Two of the best-known are:

- *Thompson sampling:* uses Bayesian methods
- *Epsilon-greedy: explores* (play a random arm) with probability ϵ and otherwise *exploits* (play the current best arm)

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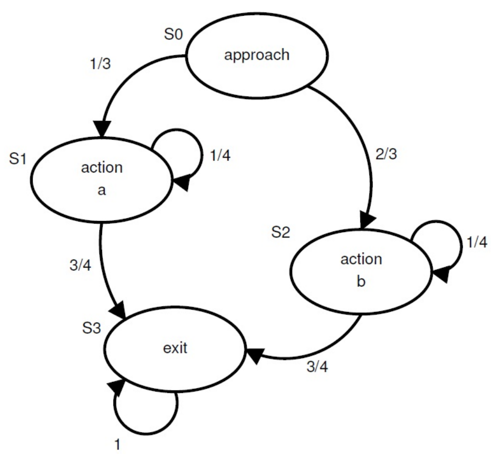
- not very "human-like"!

Our goal is not to find the best protocol, but to explain human behaviors.

Probabilistic finite automata (PFA)

To capture resource-boundedness, we model people as probabilistic finite automata (PFA).

- Just like deterministic finite automata, except that we allow probabilistic state transitions.



Key Ideas of PFA

- We play each arm sequentially against a “virtual arm”
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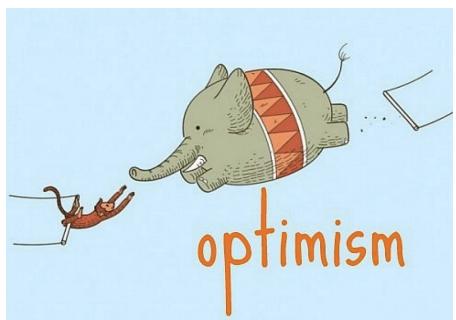

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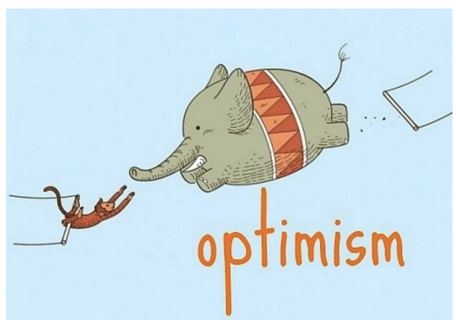

- if no arm can beat the virtual arm, we slowly lower its success probability
- We build in a *negativity bias*:
  - we quickly eliminate arms that do not beat the virtual arm, but are reluctant to declare an arm a winner

- Experiments show that our aspiration-level protocol does extremely well in practice
  - It does better that ϵ-greedy
  - It does not do as well as Thompson sampling (which is known to be optimal) but that is an inherent problem
    - For all finite-state protocols P, there exists an $\epsilon > 0$ such that after N steps, the regret is $> \epsilon N$.
    - Thompson sampling has logarithmic regret
- The protocol's performance degrades gracefully as we decrease the number of states, with human-like biases becoming more emphasized.

The Ranger-Poacher Game

[Liu and Halpern]

- There are two player, a ranger and a poacher, and a fixed number n of sites that rhinos might go to.
- The distribution of rhinos at each site is commonly known.
  - e.g., may have $(.8, .9, .3, .2)$: with probability .8, there is a rhino at site 1, with probability .9, there is a rhino at site 2, . . .
- It's a zero-sum game:
  - If the poacher catches a rhino, then he gets $+1$; the ranger gets -1
  - if the ranger catches the poacher, then she gets $+1$, the poacher gets -1.
- The game has a unique Nash equilibrium (NE).

Fictitious Play

There are many protocols that converge to NE. We consider perhaps the best studied: *fictitious play* (FP):

- At each step, players best respond to the mixed strategy where the probability that an action is played is the frequency with which that strategy has been played thus far.
  - This requires an unbounded number of states to implement, since again, we must keep track of history.
  - It is easy to approximate this with an FP
- Although the convergence time of FP is slow, it has been very well-studied in large part because it is so natural.

PFAs play FP

With only finitely many states, a PFA must approximate frequency that the opponent has gone to each site:

- Assume that the memory has the form $[q_1, \ldots, q_n]$, where $q_1 + \cdots + q_n \leq M$
  - q_i is (an approximation to) the number of times that the ranger has been to site i in the last M steps
  - If the poacher observes the ranger go to site i then
    - q_i is increased by 1
    - q_j is chosen at random according to its frequency ($\frac{q_j}{q_1+\cdots+q_n}$) and is decreased by 1
  - Experiments show that this gives quite a good approximation to the actual frequency for $M = 100$.

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  - For smaller M, the approximation fluctuates more around the actual frequency, which leads to *probability matching*
    - The smaller M is, the more the poacher goes to sites proportional to the probabilities of rhinos being there.
    - The ranger continues to play NE, because probability matching is essentially the NE strategy for the ranger
    - Key observation: Best responding + variations in estimates due to small memory lead to probability matching!

The Significance of Significance

From a human perspective, some events are more significant than others.

- Observing a potentially poisonous snake is far more significant than observing a beetle.

Significant events are typically ones with very bad outcomes:

- Kahneman and Tversky observe that we tend to overweight the negative

In the ranger-poacher game, we take the significant events to be

- Getting caught, for the poacher
- The poacher catching a rhino, for the ranger

We overweighted these events in the PFA, so as to be able to reproduce what we observed in our Amazon Turk experiments:

- Some poachers go to sites with high rhino likelihood less often that would be predicted by NE
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  - Intuitively, they are trying to avoid being caught
- Big surprise: taking significance into account not only replicated observed results, but significantly improved performance of the PFA, especially with little memory.

MTurk Experiments

We ran experiments on Amazon Turk (MTurk), using a number of different rhino distributions, with people playing the role of poacher.

People largely cluster into three groups:

- *Level-0*: nonstrategic, visit all sites with equal probability
  - We suspect that these are often people trying to finish the game as quickly as possible, just to get the base payment

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- *Level-1*: probability match with rhino distribution
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  - people could still be best responding!
- *Level-2:* Best responding to level-1 rangers, so go to sites with higher rhino distribution less often
  - As we saw, this can be explained by overweighting negative outcomes
  - Key point: this behavior is quite rational!

Moral Decision Making

[S. Levine, M. Kleiman-Weiner, L. Schulz, J. Tenenbaum, F. Cushman]

Current theories of moral decision-making use two main approaches to choosing the “moral” action:

- Rules
  - precompiled answers that apply to a wide range of cases
- Expected utility calculations
  - Perform the action that maximizes expected utility (EU)

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Perhaps the function of moral decision-making is to maximize EU

- Whose utility?
  - The agent's? The agent's community?
- Even ignoring that, doing EU calculations is hard!
- Rules may provide a way for agents to get desired results in a computationally-efficient way.

Universalization

On the other hand, rules are limited:

- Sometimes, there is no obvious rule that exists to guide us
  - How do we create novel rules for novel circumstances?
- Sometimes a simple heuristic gives the wrong answer
  - When is it is acceptable to override/modify a heuristic rule?
    - E.g., when is it acceptable to cut in line?

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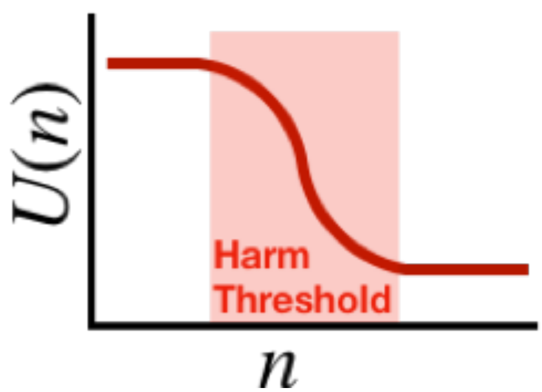
Universalization is a tool that resource-bounded agents can use to create new rules and refine their current rules

- It asks "what if everyone felt at liberty to do that?"
- Helps us figure out when and how to override rules.

Testing Universalization

- We are interested in seeing to what extent people use universalization and feel it appropriate
- We tested whether subjects use universalization in a class of collective action problems called "threshold problems", when there are no pre-agreed rules
  - There are two possible actions
  - If few people choose one of the actions, the are better off, and there are no negative consequences for the group
  - If many people choose that action things go badly for everyone
  - E.g., climate change

Universalization captures (some of) our moral judgment process.



Modeling Universalization

The critical components of the model are

- The number n_i of "interested parties"
  - the people who would act if they felt at liberty to do so
  - If everyone felt at liberty, then only the people who were interested in doing the action would do it.
- The utility consequences of those people acting

Assumption: the probability that the action would be found acceptable given that n_i people performed it has the form

$$P_{Univ}(Acceptable) = \frac{1}{1 + e^{\tau(U(0)-U(n_i))+\beta}}$$

- This is a standard "soft max"
- Note the dependence on $U(0) - U(n_i)$
  - This measures the harm done by n_i agents performing the action.

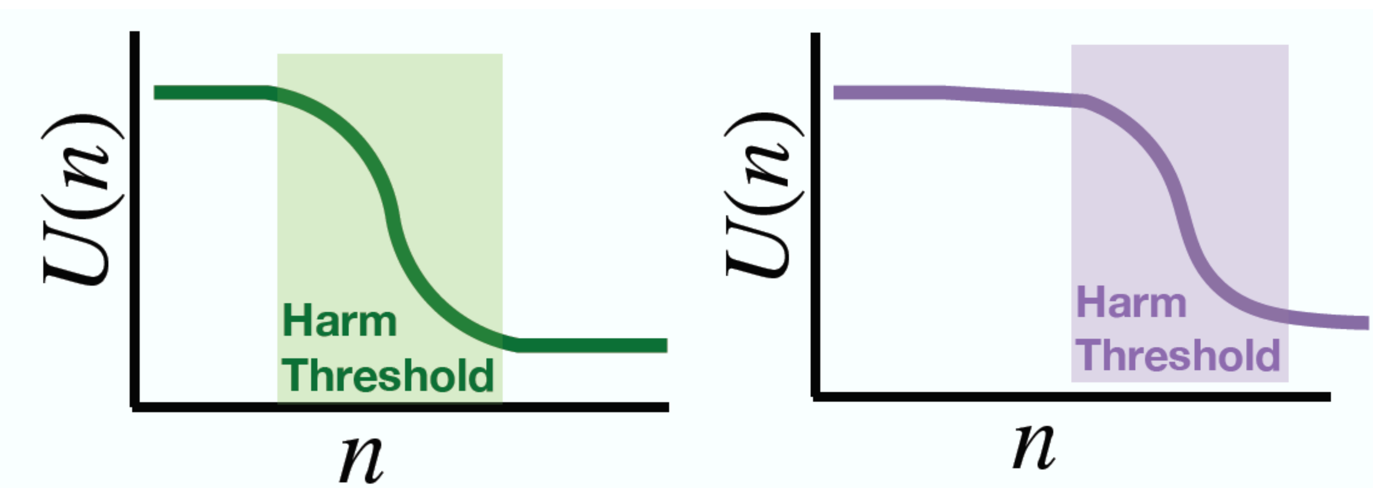
Experiment

Over-fishing scenario

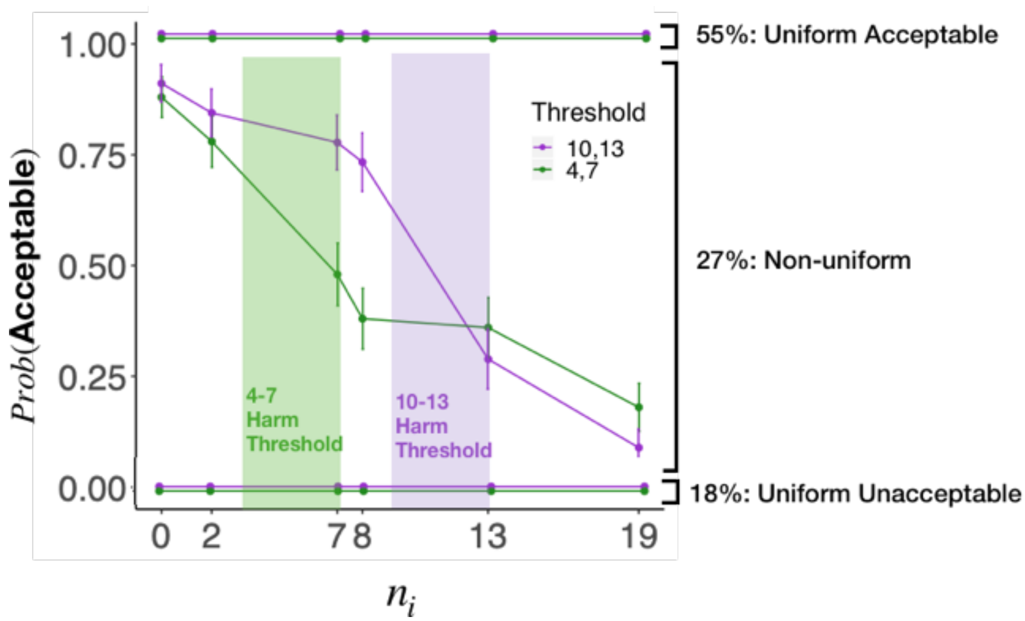
- Everyone in the village can fish sustainably with the traditional fishing method
- A new method allows them to catch more fish, but would lead to the fish going extinct if many people use the hook

Two parameters of interest:

- the number of interested parties (0-20 people)
- the utility consequences of multiple people using the new hook (which affects the harm threshold)



Moral Judgments: Experimental Results



- The model fits the data really well for the 27% whose judgments affected by n_i
- The others seem to care only about the outcome

Universalization: Creating new rules

- Given a stable number of interested parties and a stable harm threshold, a new simple rule can be created:
  - it is OK/not OK to use the new fishing hook
- The output of universalization reasoning can then be codified/amortized and reused in future cases that have the same structure

Current work

[S. Levine, J. Halpern, M. Kleiman-Weiner, J. Tenenbaum]

In communities of resource-bounded agents, some universalizable actions might not be morally permissible because of cognitive constraints.

- An agent might not be able to understand the intended universalizable action or figure out what it is

All universalizable actions should also be ...

- legible?
  - Can others figure out what you're doing
- tamperproof?
  - Not easily gameable
  - A policy like "use the hook when your relatives visit" might result in me inviting my relatives frequently
- robust?
  - Nothing disastrous happens if people deviate
- communicable?
  - Is it easy to explain what you're doing

Stereotype Formation

Cases where people evaluate each other.

- Formal settings like hiring or admissions.
- Informal evaluation in everyday interaction.

Where do negative stereotypes come from? [Greenwald-Banaji 95]

- In the presence of low information or limited available time, we are more prone to fall back on stereotypes.
- These stereotypes often work to the detriment of groups that are already disadvantaged.

Can we find a formal basis for these properties?

- Can such a model suggest useful interventions?

Screening Decisions and Disadvantage

A stylized scenario:

Applicants and feature vectors:

- Applicants are described by (Boolean) variables $x = (x^{\langle 1 \rangle}, x^{\langle 2 \rangle}, \ldots, x^{\langle k \rangle})$.
- Function f describes productivity $f(x)$ of applicant with features x.
- Plan: Sort by f-value, admit top r fraction.

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Group membership:

- Applicants can belong to *advantaged* group A or *disadvantaged* group D. Extended feature vector (x, A) or (x, D).
- Function f is independent of group: $f(x, A) = f(x, D) = f(x)$.

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Disadvantage:

- $\mu(x, \gamma) =$ fraction of population with features x and group γ.
- Likelihood-ratio condition: if $f(x) > f(x')$, then

$$\frac{\mu(x, A)}{\mu(x, D)} > \frac{\mu(x', A)}{\mu(x', D)}.$$

Simplification

- True criterion is conjunction of $x^{\langle 1\rangle}$ and $x^{\langle 2\rangle}$.
- Applicants from A have $x^{\langle i\rangle} = 1$ with prob. $2/3$.
- Applicants from D have $x^{\langle i\rangle} = 1$ with prob. $1/3$.

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	f	μ
1	1	D	1	$1/18$
1	1	A	1	$4/18$
1	0	D	0	$2/18$
1	0	A	0	$2/18$
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0	0	A	0	$1/18$

At all admission rates $r \leq 5/18$, all admitted have f-value 1, with a $1/5$ fraction from group D.

Now suppose we simplify f by using only $x^{\langle 1 \rangle}$, not both features.

- Perhaps collecting $x^{\langle 2 \rangle}$ is too expensive.
- (For larger instances) Interpretability or cognitive complexity.
- Out-of-sample generalization.
- Removing a variable that confers some of the disadvantage.

Simplification

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	avg f	μ
1	any	any	$5/9$	1/2
0	any	any	0	1/2

$\longleftarrow$

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	f	μ
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- An f-approximator: collapse rows; assign each applicant their expected f-value conditional on what we know about them; admit in this order.
- Now at all admission rates $r \leq 5/18$:
  average f-value is $5/9$ (not 1)
  fraction from group D is $1/3$ (not $1/5$).
- Relative to true f, gains in equity, losses in efficiency.

Simplifying may confer many of the aforementioned benefits.
But it also causes two potential difficulties.

First Problem: Incentived Bias

Simplification transforms disadvantage into bias:

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	avg f	μ
1	any	A	$2/3$	$1/3$
1	any	D	$1/3$	$1/6$
0	any	A	0	$1/6$
0	any	D	0	$1/3$

$\longleftarrow$

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	avg f	μ
1	any	any	$5/9$	$1/2$
0	any	any	0	$1/2$

Dropping $x^{\langle 2\rangle}$ creates an (f-maximizing) incentive to use the group membership variable γ in a way that hurts group D.

- Incentive only arises because $x^{\langle 2\rangle}$ is invisible; with true f, no incentive to consult value of γ.
- A basic mechanism for stereotype formation in everyday life [Leyens et al 1994, Greenwald-Banaji 1995].
- Connected to models of statistical discrimination [Arrow 1973, Coate-Loury 1993, Hu-Chen 2018].
- Empirical analogues: "ban the box" policies and effect discrimination [Agan-Starr 2016, Doleac-Hansen 2016, Shoag-Veuger 2016.]
- Related: drug tests [Wozniak 2015], credit history [Bartik-Nelson 2016].

Second Problem: Pareto-Improvement

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	avg f	μ
1	1	D	1	$1/18$
1	any	any	$1/2$	$8/18$
0	any	any	0	$1/2$

$\longleftarrow$

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	avg f	μ
1	any	any	$5/9$	$1/2$
0	any	any	0	$1/2$

Let g and h be two f-approximators.

- h strictly improves on g in efficiency if for every admission rate r, average f-value of admitted set under h is at least average f-value of admitted set under g; and strictly greater for at least one value of r.
- h strictly improves on g in equity if analogous condition holds for the fraction of members of group D who are admitted.

Pareto-improvement:

- Approximator on left strictly improves approximator on right: for any monotone preferences for efficiency and equity, approximator on left is an improvement.
- Empirical analogues in settings like United Steelworkers v. Weber (1974) on programs for selected members of underrepresented groups.

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	f	μ
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simplification

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	avg f	μ
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incentivized bias

Pareto-improvement

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	avg f	μ
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0	any	A	0	1/6
0	any	D	0	1/3

$x^{\langle 1\rangle}$	$x^{\langle 2\rangle}$	γ	avg f	μ
1	1	D	1	1/18
1	any	any	1/2	8/18
0	any	any	0	1/2

General Result

Informal version of a general result [Kleinberg-Mullainathan]:

For every Boolean function f with real-valued outputs satisfying the disadvantage condition and a genericity assumption, and for every simplification g of it (partitioning feature vectors into cells by fixing variables):

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	f	μ
1	1	D	1	1/18
1	1	A	1	4/18
1	0	D	0	2/18
1	0	A	0	2/18
0	1	D	0	2/18
0	1	A	0	2/18
0	0	D	0	4/18
0	0	A	0	1/18

simplification

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	avg f	μ
1	any	any	5/9	1/2
0	any	any	0	1/2

incentivized bias

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	avg f	μ
1	any	A	2/3	1/3
1	any	D	1/3	1/6
0	any	A	0	1/6
0	any	D	0	1/3

Pareto-improvement

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	avg f	μ
1	1	D	1	1/18
1	any	any	1/2	8/18
0	any	any	0	1/2

(a) There is always an f-approximator that strictly improves g in both efficiency and equity.

(b) If g does not use group membership, then adding group membership as a variable increases efficiency and reduces equity.

The Nature of the Disadvantage Condition

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	f	μ
1	1	D	.9	.06
1	1	A	.9	.04
1	0	D	.6	.02
1	0	A	.6	.06
0	1	D	.2	.07
0	1	A	.2	.06
0	0	D	.02	.35
0	0	A	.02	.34

$\longrightarrow$

$x^{\langle 1 \rangle}$	$x^{\langle 2 \rangle}$	γ	avg f	μ
1	any	D	.825	.08
1	any	A	.72	.10
0	any	D	.05	.42
0	any	A	.047	.40

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An example where the mean f-value in group A exceeds the mean f-value in group D (a weaker form of disadvantage), but:

- The f-approximator g that uses $x^{\langle 1 \rangle}$ and γ cannot be Pareto-improved.
- The f-approximator h that uses only $x^{\langle 1 \rangle}$ creates an incentive to use γ in a way that favors group D (not A).
- This is a reflection of Simpson's Paradox.
- The technical underpinning of our main theorem can be viewed as proving an "anti-Simpson" result.

Open question: What is the weakest specification of disadvantage where the result holds?

Main Combinatorial Lemma

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Assume the likelihood-ratio condition for disadvantage:
if $f(x) > f(x')$, then $\dfrac{\mu(x,A)}{\mu(x,D)} > \dfrac{\mu(x',A)}{\mu(x',D)}$.

Consider any non-trivial partition of the feature vectors for A into cells and (separately) the feature vectors for D into cells.

- Assign each feature vector (x,γ) a value $g(x,\gamma)$ equal to the (measure-weighted) average of f in its cell.
- Then there exists a feature vector x for which $g(x,A) > g(x,D)$.

Reflections

Simplifying a function f with groups A and D.

- Creates an incentive to use group membership in a way that hurts group D.
- Can always be strictly improved in both efficiency and equity.

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1	0	D	0	2/18
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Ongoing open questions:

- Consider other formulations of simplicity.
  Large alternate category: linear approximations to f.
- Consider other formulations of the disadvantage condition.
  What is the weakest condition for which these results hold?
- Studying the space of all simplifications of f w.r.t. efficiency and equity.
- Further implications for empirical analysis and interventions.