Approximating C_{free} space topology by constructing Vietoris-Rips complex

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***Abstract*— In this work, we present a memory efficient representation of the roadmap that approximates and measures these approximations of the underlying topology of the C_{free} space. First, we perform sampling in the configuration space while meeting preconditions in the workspace and connect samples such that the resulting graph becomes a *skeleton* graph that can be used to construct a Vietoris-Rips (VR) complex. Second, we show that a series of topological collapses can be performed to remove vertices from a graph so that the resulting graph reconstructs homotopy equivalent shape to the η-offset of the C_{free} space. In our experiments, we use a range of sample points in three different test environments, construct VR-complex from densely connected graphs, perform topology collapses and plan path trajectories from preserved samples after the collapse for different robot scenarios. Our result shows that on satisfying certain preconditions, the VR-complex gives homotopy equivalent shape to the η-offset of the C_{free} space.**

I. Introduction

Understanding and describing the structure of the configuration space ($\mathcal{C}_{space}$) is one of the fundamental challenges in robotic motion planning. If we can fully describe the geometry and the topology of the configuration space, the motion planning will become trivial as the robot can be viewed as a point in the configuration space. However, as the configuration space can be arbitrarily complex compared to the workspace, it is difficult to answer even the seemingly simple questions about the connectivity of the space.

In recent decades, one of the most often applied methods in motion planning is the sampling-based methods which attempt to explore the connectivity of some portion of the space through sampling. These methods are very effective in problems relative open configuration spaces. Even though over the years, many asymptotically optimal motion-planning algorithms including PRM* and RRT* [26] have been proposed, the generated samples still only covers an immeasurable subset of the configuration, unless the number of samples reaches infinity.

In this work, we attempt to answer the question about the connectivity of the configuration space through the construction of a set of Vietoris-Rips (VR) complex. The set of VR-complexes constructed using the described properties will be able to approximate the homotopy of the underlying space from which the samples are generated. The details of the topological equivalence will be introduced in Section III-D. In addition, we will show that the constructed VR complexes can perform topological collapses by removing non-common vertices among complexes, which will result in a relatively sparse representation.

Extracting topological properties of the configuration space is not a new idea and many good results have been presented. One of the most often extracted information is "persistent homology", which describes holes present i.e., obstacles, in the configuration space. This extracted information can isolate and describe obstacles, therefore can be useful to guide the motion planning. The aim of our work however, is not identifying objects and obstacles present in the configuration space, but provide a tool to efficiently describe properties of the C_{free} space, so that the resulting description can be directly used for finding paths in the configuration space.

Our approach can be briefly described as follows, 1), we first generate samples and connect them locally, so that they satisfy a set of properties describing the *convexity* of the subspace occupied by the samples. These satisfied properties guarantee that the constructed VR-complexes homotopically equivalent to the underlying space. 2), once the equivalence is established, we will remove samples from the representation that does not affect the topological properties of the complexes, by performing topological collapses. In addition, as all the remaining samples belong to one or more VR-complexes, i.e. local cliques, we can also remove the edges from the representation, recording only which VR-complex a sample belongs to. 3), the resulting set of samples can be used to build a roadmap for the C_{free} space. If a path connecting a pair of start and goal configurations exists within the space described by the VR-complexes, the proposed methods can find such a solution. Figure 1 shows the process flow of our approach.

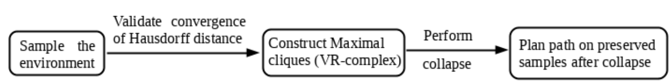


Fig. 1: Our approach

We admit that this is still a preliminary work on using topological tools for motion planning. There are many potential extensions that can be built upon the current results. For example, even though our approach is not currently incremental, the capability of preserving topological information of the sampling space while being memory efficient can be beneficial for planning in complex spaces. Also, the presented approach currently only finds a path but makes no guarantee about the optimality of the path, which is part of the future work we are currently working on. Interestingly, we can change/refine the sampling parameters used to generate samples for VR-complex construction if a path that we know exists is not returned by the current roadmap. We, therefore, will expand the spaces approximated by the VR-complexes to include the path. We perform experiments in three different environments and compare results with an optimal path planning algorithm and show improvement in time needed to generate path trajectories for different robot scenarios.

II. Related Work

Topological features are defined as the basic representation of mathematical or geometrical space and refer to a feature that supports continuity, connectivity, and convergence that is established and maintained based on geometric coincidence. These topological features can be extracted using various mathematical concepts such as sheaf theory, persistent homology, Vietoris-Rips (VR) complexes, and landmarking approach. Past results have shown the beneficial use of the topological feature for improved behaviors or actions of machines in areas like signal processing, cohomology, topological motion planning, etc.

To understand the application of homotopy classes for 2D and 3D space objects, Bhattacharya et al. in [6], [8], [9], proposed the use of homology classes for 2D objects, as the application of homotopy classes cannot be practically applied to robots path planning problems. They proposed an application of complex analysis and electromagnetism for the path planning through the 3D objects with K genus (holes in the obstacles) by using the concept of homotopy classes. Later in his work, a more practical application approach was researched, Bhattacharya et al. in [7], used the concept of persistent homology to find the homology class of trajectories that is most persistent for a given probability map. The work proposed the use of persistent homology concept to solve the fundamental problem of goal-directed path planning in an uncertain environment represented by probability map.

Research by Pokorny et. al. in [30], studied homotopy classes of trajectories in general configuration space using Delaunay-Cech Complexes filtration method and abstract the global information of trajectories using persistent homology. Pokorny et. al. further showed in [31], the application of a sampling-based approach to topological motion planning that is fully data-driven in nature The work also used the Delaunay-Cech filtration method to filter the data from the point-cloud dataset and improvises Dijkstras algorithm to generate distance vector trajectories for a source vertex.

The above-cited research has shown improvements more inclined towards extracting topological information of the space and then performing approximate sampling with performance guarantees. These methods, however, do not provide a measure of the approximation that has been performed.

A. Sampling Based Motion Planning

Sampling-based methods [14] are a state-of-the-art approach to solving motion planning problems. These methods are known to be probabilistic complete because the probability of finding a solution if it exists tends towards 1 as the number of samples generated also increases. Sampling-based methods are broadly classified into two main classes: graph-based methods such as the Probabilistic Roadmap Method (PRM) [27] and tree-based methods such as Expansive-Space tree planner (ESTs) [24] and Rapidly-exploring Random Tree (RRT) [28]. PRM variants consider different topology which include uniformly generating samples in the environment [27], sampling near obstacles [2], [4], [12], [23], [35], sampling with constraints placed on the robots [29] and planning with uncertainty in the environment [25]. Other methods exist that investigate the heterogeneous nature of the planning environment using reinforcement learning [15]–[18], [33].

III. Preliminaries

A. VR-complex and Čech-complex

There has been various research done to better understand the VR and Čech complexes and how they provide topologically correct approximations. More formally, the VR and Čech complex can be defined as follows:

Given a set X of points in Euclidean space E, the VR complex R(X) is the abstract simplicial complex whose k-simplices are determined by subsets of k + 1 points in X with a diameter that is at most ε, whereas the Čech-complex C(X) is the abstract simplicial complex where a subset of k + 1 points in X determines a k-simplex if and only if they lie in a ball of radius $\varepsilon/2$.

In work by Attali et. al., [5] they presented mathematical proofs to show VR complexes can provide topologically correct approximations of shapes with the notion of distances between points in the metric space. This previous research provides conditions under which the VR complex of the point set at some scale reflects the homotopy type of the shape for a finite point set that samples a shape.

To perform better topological analysis, Chambers et. al. in [13] presented a concept called shadow complexes – a projection map from the VR complex to Euclidean n-space that has as its image an n-dimensional approximation to the homotopy type of sampled space. However, the projection map did not preserve higher-order topological data for planar sets, nor did it preserve fundamental group data for point sets in dimension larger than three.

Previous work has shown that VR Complex have been used to extract the topological feature of space using various methods. In [36], Zomorodian proposes three different algorithms to faster compute VR Complex from a generated neighborhood graph of topological space. He further discussed the application of topology data analysis methods in recovering the topology of the sampled space in [37].

In this work, we use the concept of VR Complex to get topological approximations of the configuration space to enhance path planning by providing a measure of the approximation to better guide sampling in difficult regions.

B. Simplicial collapses

A simplicial complex K, i.e., a collection of sets closed under the subset operation, it is a generalization of a graph and is useful in representing higher-than-pairwise connectivity relationships. The elements of any set are called vertices and the set itself is called a simplex. Topological thinning (simplicial collapse) [10] is an important preprocessing operation that aims to shrink simplicial complexes to a smaller, simpler simplex which retains a lot of the significant information of the space. For example, A thinned simplicial complex is a subcomplex of K with the condition that all the faces of the maximal simplices are shared. Thus, it can no longer be possible to collapse them.

To establish the conceptual understanding of topology collapse, the work in [21] gave a theoretical proof of simplicial collapse of simplicial complexes with the implementation shown on folded manifolds. Presenting the application of simplicial collapses, research in [22], proposed a method for computing the cohomology ring of three-dimensional (3D) digital binary-valued pictures via a simplicial complex and algebraic thinning using topological representations. Research in [34], presented a novel algorithm for simplifying homology and hole location computations on a complex by reducing it to it's core using a strong collapse (a concept of simplicial collapse).

In this work, the concept of simplicial collapse will be investigated. In particular, the simplicial collapse will be used to reduce the complexity of maximal simplices through vertex deletion down to a core simplex on maintaining the topological structure of the configuration space.

C. Hausdorff Distance

The Hausdorff distance measures how far two subsets of a metric space are from each other [1]. In this work, we measure Hausdorff distance (ϵ) between set P (point cloud) and set X (boundary points representing and covering the entire $\mathcal{C}_{free}$). The algorithm uses a convex hull to find the boundary points of point cloud set P. The value of ϵ is calculated as the difference of boundary points of two sets P and X as shown in Figure 2. As the sample points get denser in the $\mathcal{C}_{space}$, the value of ϵ decreases and becomes constant after the certain number of sample points in the $\mathcal{C}_{space}$. Since the points sampled in the $\mathcal{C}_{space}$ are not point object but takes the constraint of the robot into consideration, the value of ϵ reaches constant value within a finite range of points.

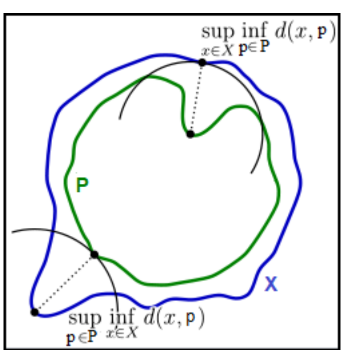


Fig. 2: Hausdorff distance for set P and X

D. From VR complex to sampled-space topology

Generally, a VR-complex does not preserve the topology of the underlying sampled space. However, in [5], the authors showed that a VR complex can be retracted to a Čech complex to approximate the topology of the underlying sampled space. Let us define the *flag complex* of a graph G, denoted Flag G as the maximal simplicial complex whose 1-skeleton is G. More precisely, this is the largest simplicial complex sharing with the Čech complex the same 1-skeleton. In addition, let us denote the VR-complex $R(P, t)$ the abstract simplicial complex whose k-simplices correspond to subsets of $k+1$ points in P with a diameter that is at most $2t$. The Čech complex $C(P, t)$ as the abstract simplicial complex whose k-simplices correspond to subsets of $k+1$ points that can be enclosed in a ball of radius t. Define α as an *inert value* of P if $\mathrm{Rad}(\delta) \neq \alpha$ for all non-empty subsets $\delta \subset P$.

Then, given any point set $P \in \mathbb{R}^n$ and any real numbers $\alpha, \beta \geq 0$ with $\alpha \leq \beta$, define the flag complex of any graph G satisfying $R(P, \alpha) \subset \text{Flag } G \subset R(P, \beta)$ an (α, β)-almost Rips complex of P. Also, let $v_n = \sqrt{\frac{2n}{n+1}}$. We can then have the following theorem, which is Theorem 7 from [5].

Theorem 1. *Let $P \subset \mathbb{R}^n$ be a finite set of points. For any real numbers $\beta \geq \alpha \geq 0$ such that α is an inert value of P and $c_P(v_n\beta) < 2\alpha - v_n\beta$, there exists a sequence of collapses from any (α, β)-almost Rips complex of P to the Cech complex $C(P, \alpha)$.*

Further, the graph can be shown to be homotopy equivalent to η-offset of the sampling space X, from Theorem 10 in [5].

Theorem 2. *Let ϵ, α and β be three non-negative real numbers such that $\alpha \leq \beta$ and $\eta = 2\alpha - v_n\beta - 2\epsilon > 0$. Let P be a finite set of points whose Hausdorff distance to a compact subset X is ϵ or less. Then, any (α, β)-almost Rips complex of P is homotopy equivalent to the η-offset of X whenever α is an inert value of P and $h_X(v_n\beta + \epsilon) < 2\alpha - v_n\beta - 2\epsilon$.*

where $\mathrm{Hull}(X)$ denotes the convex hull of X, and

$$h_X(t) = d_H(\mathrm{Hull}(X, t)|X) \tag{1}$$

$$Hull(X, t) = \bigcup_{\substack{\emptyset \neq \delta \subset X \\ \mathrm{Rad}(\delta) < t}} \mathrm{Hull}(\delta) \tag{2}$$

From the theorem, we can derive that in order to use a graph-like structure to approximate the underlying homotopy of the sampling space, we need to first have sufficiently dense samples, so that P is no more than ϵ away from the set X based on Hausdorff distance. Here, X is the set we would like to approximate using samples in P. Recall, Hausdorff distance $d_H(X, Y)$ is

$$\begin{aligned} d_H(X, Y) &= \max\{\sup_{x\in X} \inf_{y\in Y} d(x, y), \sup_{y\in Y} \inf_{x\in X} d(x, y)\} \\ d(y, X) &= \inf_{x\in X} d(y, x) \\ d_H(Y|X) &= \sup_{y\in Y} d(y, X) \end{aligned}$$

Therefore, if the samples P satisfy the above properties, we can construct a graph based on P and use the relations to approximate the underlying homotopy of X, even when the number of samples is small. Compared to the sampling-based motion planning approaches, where the connectivity is *guaranteed* when the number of samples reaches infinity, the proposed method yields a bound on the number of samples. On the other hand, given a set of samples P, we can also compute the relevant parameters to derive how much of the sample space X has the samples covered, where X can be $\mathrm{C}_{\mathrm{free}}$ in the case of motion planning.

IV. Methodology

A. Sampling the configuration space

We generate samples in the $\mathcal{C}_{free}$ that satisfies sampling conditions as indicated in Theorem 1 and 2 previously described in Section III-D. These samples are used to compute Hausdorff distances (ϵ) with the boundary points of the workspace, set X. The Hausdorff distance between two compact sets X and P is analyzed on generating different range of sample points at a time in the workspace. The convex hull points of set P is calculated to compute the two-sided Hausdorff distance with set X, i.e. the supremum distance between sets X and P. In [5], the authors state that if the Hausdorff distance is smaller than some notion of topological feature size of the shape, then the output is topologically correct. Taking this into consideration from theorem 2 we validate the expression $2\epsilon < 2\alpha - v_n\beta$, where $\beta = \alpha$ in our experiments. On verifying the sampling condition in the workspace, the output is a densely sampled $\mathcal{C}_{space}$ graph G.

B. Collapsing a VR-complex

The convex hull of any nonempty subset of the $n+1$ points that define an n-simplex is called a face of the simplex (complex). A maximal face (facet) is any simplex in a complex that is *not* a face of any larger simplex. Given $\tau, \delta \in K$, if $\tau \subset \delta$, in particular $\dim \tau < \dim \delta$, and δ is a maximal face of K and no other maximal face of K contains τ, then τ is called a *free face*. A *simplicial collapse* of K is the removal of all simplices γ such that $\tau \subseteq \gamma \subset \delta$.

Given a simplicial complex K of dimension $n \geq d$, a d-skeleton of K is the subcomplex of K consisting of all the faces of K that have dimension at most d. Then, a graph can be used to represent the 1-skeleton of K, and let us refer to the graph as the *underlying graph* and denote the graph as G_K. For simplicity, in this work we will refer to the 0-skeleton of K as vertices of G_K, and faces of K that have dimension 1 as edges of G_K. Then, we can derive the following results.

Lemma 1. *Given a complex K and its underlying graph G_K, let δ be a maximal face of K, if a vertex v of G_K is a subset of δ ($v \subset \delta$) and no other maximal face of K contains v, then there exist a sequence of simplicial collapses on K that can remove vertex v.*

Proof: Let there exist a sequence of free faces $s_0, s_1, s_2, \dots, s_m$, so that $s_0 \subset s_1 \subset s_2 \subset \dots \subset s_m \subset \delta$ and $s_0 = v$. Let s_1 be one of the edges on G_K with v being one endpoint of the edge, let s_2 be the tetrahedron containing s_1, etc. Because each s_i is a free face, a simplicial collapse can remove it. Then, let the sequence of collapse start from s_m, and move towards s_0. Each collapse of s_i will not change the fact that s_i still is a free face of δ. Therefore, v can be removed. □

Then, we can extend the results to get the following theorem.

Theorem 3. *Given a complex K and its underlying graph G_K, let δ be a maximal face of K, and let V_s be the set of all the vertices v where v is a subset of δ and no other maximal face of K contains v. Then, after removing all vertices in V_s, there are no free faces on δ.*

Proof: Let us assume that after removing all vertices in V_s, there still exists at least one free face $\tau \subset \delta$. If τ is of dimension 0, then it is a vertex that only belongs to δ, so it must have been part of V_s, so τ can only be of dimension 1 or above. If τ is of dimension 1, i.e. an edge on G_K, then at least one vertex of the edge will belong only to δ otherwise the edge cannot be a free face. Therefore, removing all vertices of V_s will remove this edge. Inductively, we can extend this to higher dimensions. Therefore, there cannot be any free face left after removing all vertices in V_s. □

C. From roadmaps to collapsed VR-complex

Algorithm 1 constructs vietoris-rips complex using Quick-cliques library from [20] on a sampled graph. The maximal clique technique uses a hybrid algorithm to compute quick cliques. These cliques are binary represented to perform $\oplus$ operation on it to simplicially collapse by pruning vertices from the graph.

The algorithm returns a sampled graph with vertices of non-colliding regions of $\mathcal{C}_{space}$ after completing topological collapse on the graph. This densely sampled graph gives an approximate topological shape representation of the objects and available free region in the $\mathcal{C}_{space}$.

Algorithm 1 Graph-Collapse(G)

Input: Let G be the sampled graph, M be maximal clique, B be set of binary representation for each clique and T be set of vertices on topological collapse.

for all nodes in graph G **do**
  compute maximal clique M.
while M is not empty **do**
  for each clique in M **do**
    if node in clique **then**
      Set binary value '1' for node in B
    else
      Set binary value '0' for node in B
if B is not empty **then**
  T = B $\oplus$ B
  for each node in T **do**
    project node in graph G_{new}.
return G_{new}

Each node in the graph was represented in binary form based on the clique in which it belongs. For the graph with n nodes, the binary representation of a clique (or a sub-graph) is the binary string of length n in which the i^{th} character is "1" if the clique (sub-graph) contains the i^{th} node and "0" if the clique (sub-graph) does not contain the i^{th} node. During $\oplus$ operation, the algorithm checks for i^{th} node with '0' to perform the pruning process. The algorithm collapses all the edges between the two nodes and removes nodes that are not part of the cliques from the graph. The resulting topological reconstruction of the environment gives a sampled facet graph G_{new}.

V. Experiments and Results

A. Experimental Setup

All experiments were executed on a Dell Optiplex 7040 desktop machine running OpenSUSE operating system and were implemented in C++.

We perform experiments in three different environments as shown in Figure 3 and generate samples ranging from 100 to 10,000. The environments are taken from the Parasol Lab benchmarks at Texas A & M University [3].

- **ZigZag environment**: This is a 2D environment with structured obstacles placed randomly as shown in Figure 3a and 3b. We test with 2 robots ranging of 2DOF and 4DOF respectively.
- **Heterogeneous 3D**: This is a 3D maze environment with walls and narrow passages between the walls. The robot with a toroidal shape has to pass through maze-like tunnels to reach the end as shown in Figure 3c.
- **Helico**: This is a city representation environment with tall buildings and cable wires between buildings as shown in Figure 3d. The robot is a rigid body representation designed as a helicopter and has the ability to change it vertical position based on the goal position.

B. VR Complex Libraries

We perform preliminary experiments with two libraries that apply VR-complexes to produce simplices. We compare results and determine what library is most suited for our approach.

GUDHI library [11] computes persistent homology (PH) of a sequence of simplicial complexes using a fast and memory efficient approach. These simplicial complexes can be constructed using different methods available within the library. We used the VR-complex package available in this library to construct simplicial complexes. The time complexity of the algorithm is $O(v^2d+m^2d)$, here d is the dimension of the complex, v is the number of vertices, m is the number of maximal simplices in the graph.

Quick-cliques library [19], [20] generates faster maximal cliques on modifying Bron-Kerbosch algorithm by Tomita et. al. [32]. We used a hybrid algorithm that applies VR-complex approach to construct simplices. The time complexity of the algorithm is $O(3^{d/3}nd)$ with n vertices and degeneracy d. Since VR-complexes are also known as clique complexes, the algorithm tries to generate maximal cliques as a result.

Table I and II provides comparison results for GUDHI and Quick-cliques library in 2 DOF ZigZag environment with and without obstacles present. Investigating the results, we can see that Quick-cliques library computes maximal cliques faster than the GUDHI library as the number of nodes increases hence our choice for using the Quick-clique library.

Library	Number of Nodes	Cliques	Time taken (sec)
GUDHI	100	210	**0.01**
Quick-Cliques	100	51	0.042274
GUDHI	10000	33552695	289.31
Quick-Cliques	10000	892190	**0.014901**

TABLE I: Constructing Rips complex in a 2 DOF ZigZag environment without obstacles

Library	Number of Nodes	Cliques	Time taken (sec)
GUDHI	100	268	**0.02**
Quick-Cliques	100	1378	0.042939
GUDHI	10000	81463172	675.6
Quick-Cliques	10000	1443062	**50.072735**

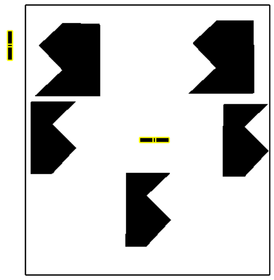
TABLE II: Constructing Rips complex in 2 DOF ZigZag environment with obstacles

C. Sampling Improvement Results

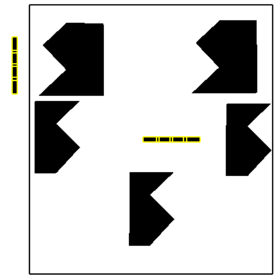
We perform two sets of experiments on our three testbeds, as described in Section V-A, both with obstacles and no obstacles in the environment. We first perform experiments for the sampling conditions of P based on the $2\epsilon < 2\alpha - v_n\beta$, where $\beta = \alpha$ preconditions as previously discussed in Section IV-A and another condition as defined in [5], states that as the sampled space becomes denser, the Hausdorff distance (ϵ) reduces or approaches a constant value. Secondly, we utilize the dense sampled space graph G to a construct VR-complex and perform the topology collapse. Our results show that after a topology collapse, the coverage of $\mathcal{C}_{space}$ is not compromised.

1) Sampling conditions: In Figure 3 and 4, the Hausdorff distance (ϵ) decreases in an empty environment as well as in environment with obstacles. The trend as shown in Figure 5b clearly satisfy the conditions stated in [5] which states "the value of ϵ will become constant above radius of the circle covering the $\mathcal{C}_{space}$". The purple and blue bars (2ϵ)(E) and the green and yellow bars ($2\alpha - v_n\beta$) (A) in the histogram represented in Figure 4a to 5b show that in all cases the condition is satisfied.

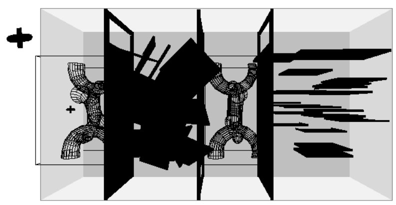
A particular case in the Helico environment as seen in Figure 5b, here the ϵ value reads a low value at the initial stage and subsequently increases as the graph gets denser before leveling off and then becoming constant. The position of a robot in this environment is at the corner of the $\mathcal{C}_{space}$, so when samples are generated earlier on, they are generated only near the boundary of the $\mathcal{C}_{space}$ and hence ϵ value is low as the number of samples increases in the environment to produce better coverage. The values of ϵ converges to constant as it reaches 10000 sampled nodes in all the environments as shown in Figure 6.



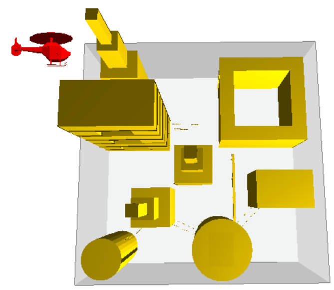
(a) **2 DOF ZigZag (snake-like)**



(b) **4 DOF ZigZag (snake-like)**

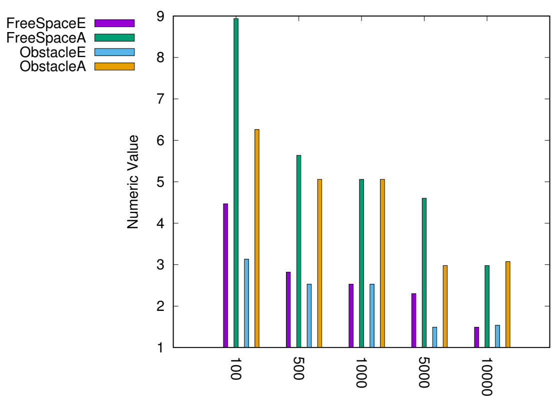


(c) **Heterogeneous 3D (toroidal plus)**

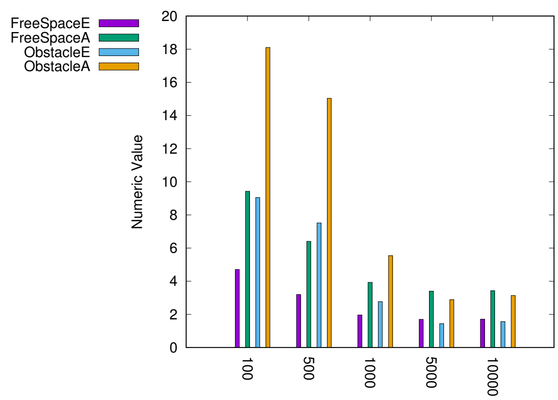


(d) **Helico (helicopter)**

Fig. 3: Environments Studied

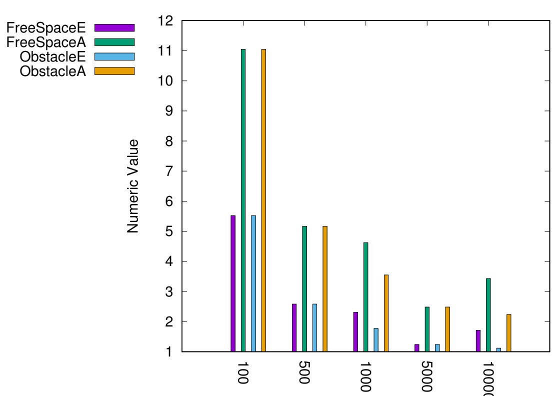


(a) **2 DOF ZigZag**

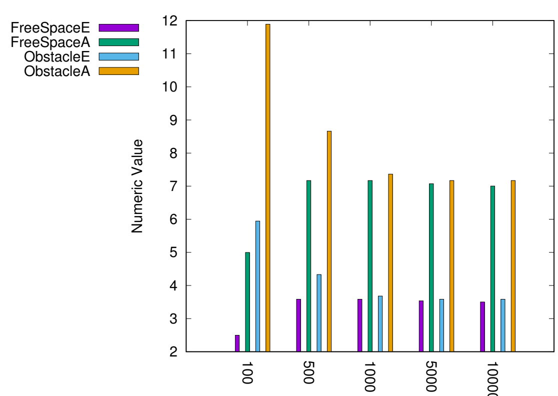


(b) **4 DOF ZigZag**

Fig. 4: ϵ and α trends in obstacle and free environments

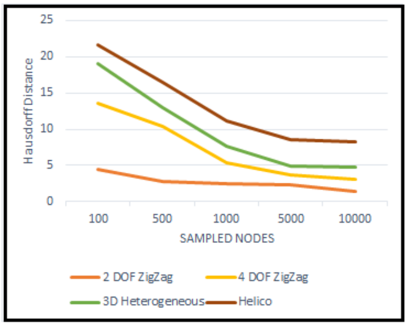


(a) **Heterogeneous 3D**

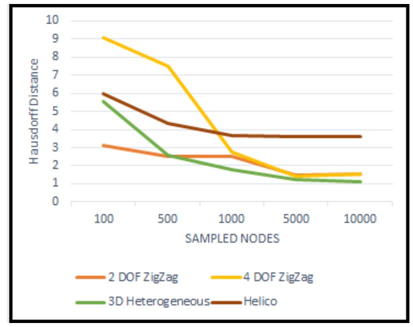


(b) **Helico**

Fig. 5: ϵ and α trends in obstacle and free environments



(a) $\mathcal{C}_{free}$ **environment**



(b) $\mathcal{C}_{obst}$ **environment**

Fig. 6: Convergence of Hausdorff distance in obstacle and free environments

2) Topology Collapse: Table III contains results for topology collapse experiments that utilize theorems and algorithms presented in Section IV-B and IV-C. Results substantiate the ability to delete vertices thus substantiating Lemma 1. We show a 40 to 90% reduction across all the environments which is promising towards deleting vertices while still maintaining the topological information of the space with the produced maps.

Figure 7 gives a pictorial representation for two environments produced after the graph topology collapse.

In addition, because the samples we generated are used to construct VR-complex, which are locally complete subgraphs (cliques), we skip the storage of all the edges and store only the information of which cliques a vertex belongs to. Therefore, the storage needed to store the entire resulting roadmap is a linear scale of the number of samples left after the collapse, which is comparable to k-nearest neighbor PRM but provides much richer topological information.

D. Planning with homology equivalent samples

Table IV and V shows the comparison results of path generated by PRM* [26] and the path generated by our approach after the collapse in different environments in terms of total path cost and time needed to build a path. We report time to connect and query the environment alone to allow for fairness in our comparisons. The results show an order of magnitude improvement in all environments studied.

VI. Discussion and Future Work

The work presented has shown that the Vietoris-Rips Complex and the Čech Complex have homotopy equivalence beneficial to improving approximate sampling algorithms and gives a much-needed measure of this approximation. The reconstructed $\mathcal{C}_{space}$ has proven to be more helpful in path planning while reducing the computation time and memory.

The approach can have application in a dynamic real-world environment where the path planning of robot can be performed with minimum computation time on each smaller portion of the environment a robot can view at the time of traversal and combine all together to get a better understanding of the actual environment.

In future work, we will further enhance the approach to identify critical points in $\mathcal{C}_{space}$, i.e. sample points closest to the $\mathcal{C}_{space}$ curvature, and using the properties of Vietoris-Rips, perform path planning on smaller sized graphs.

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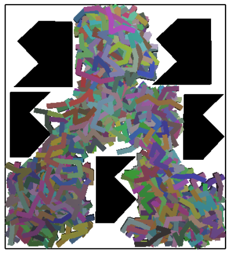
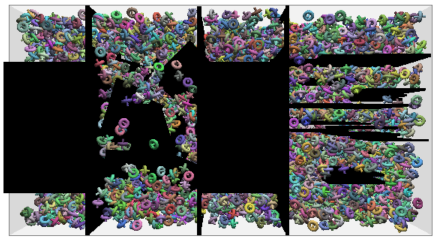
Environment	Nodes Before	Nodes After- Free	% Reduction	Nodes After- Obstacle	% Reduction
2DOF Zig Zag	10,000	5081	49.2	4826	51.7
4DOf Zig Zag	10,000	637	93.6	896	91.1
Heterogeneous 3D	10,000	4968	50.3	5061	49.3
Helico	10,000	5041	49.6	5023	49.8

TABLE III: **Results after the Topology Collapse in the Free and Obstacle Environment**



(a) **4 DOF ZigZag**



(b) **Heterogeneous 3D**

Fig. 7: Environments after the Topological collapse

$\mathcal{C}_{free}$ Environment	Our Approach	PRM*	$\mathcal{C}_{obst}$ Environment	Our Approach	PRM*
2DOF ZigZag	**62.1342**	229.922	2DOF ZigZag	**53.0861**	129.904
4DOf ZigZag	**64.7357**	11146.4	4DOf ZigZag	**2.52541**	24089.9
Heterogeneous 3D	**62.6602**	DNF	Heterogeneous 3D	DNF	DNF
Helico	**55.8919**	DNF	Helico	**58.4688**	82967

TABLE IV: **Path planning time (in seconds) in the Free and Obstacle Environments**

$\mathcal{C}_{free}$ Environment	Our Approach	PRM*	$\mathcal{C}_{obst}$ Environment	Our Approach	PRM*
2DOF ZigZag	**1003**	1438	2DOF ZigZag	**827**	1553
4DOf ZigZag	**916**	1324	4DOf ZigZag	**893**	1258
Heterogeneous 3D	**3714**	DNF	Heterogeneous 3D	DNF	DNF
Helico	**1806**	DNF	Helico	**1338**	2698

TABLE V: **Path planning cost in the Free and Obstacle Environments**

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