Propulsion Bonus Point Report

Mach & Cheese Group

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1 Introduction

The goal of this report is to develop a ballistic model for a BATES (Ballistic Test and Evaluation System) motor and reproduce the time evolution of the pressure as a function of the burning time, starting from a set of experimental data by using a statistical method, the Monte Carlo analysis [5]. The set of experimental data included 27 pressure traces, each representing a static firing, from 9 batches of nominally identical solid propellants, each characterized by 3 pressure curves, obtained by changing the throat area of the nozzle. The sampling frequency of each trace is 1 kHz. Thanks to the Bayern-Chemie (BC) method [4], the action and the burning times can be defined, which in turn let us obtain the parameters of the Vielle's law [1], a and n, and the characteristic velocity $c*$. Finally, the Monte Carlo analysis has also been used to compute the relative uncertainties of t_b, a, n and $c*$.
The batches of propellant all have the same configuration, shown in table 1:

Name	Percentage	Density (g/cm^3)
Aluminum	68%	1.95
Ammonium perchlorate	18%	2.7
HTPB	14%	0.92

Table 1: Chemical composition of propellant

The nozzle configurations and the scheme of the BATES motor are shown in table 2 and figure 1:

Pressure level	Throat diameter, mm
High	21.81
Medium	25.26
Low	28.80

Table 2: Nozzle configurations of BARIA motors

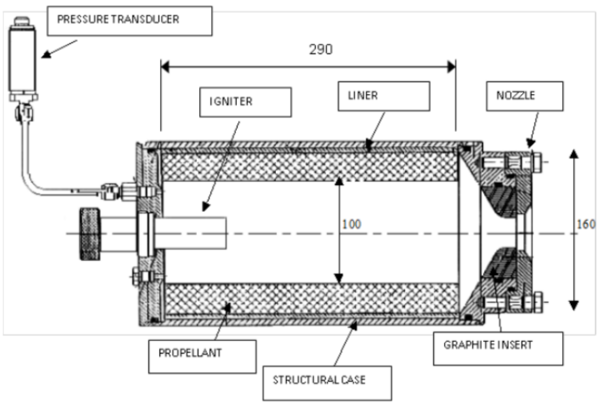
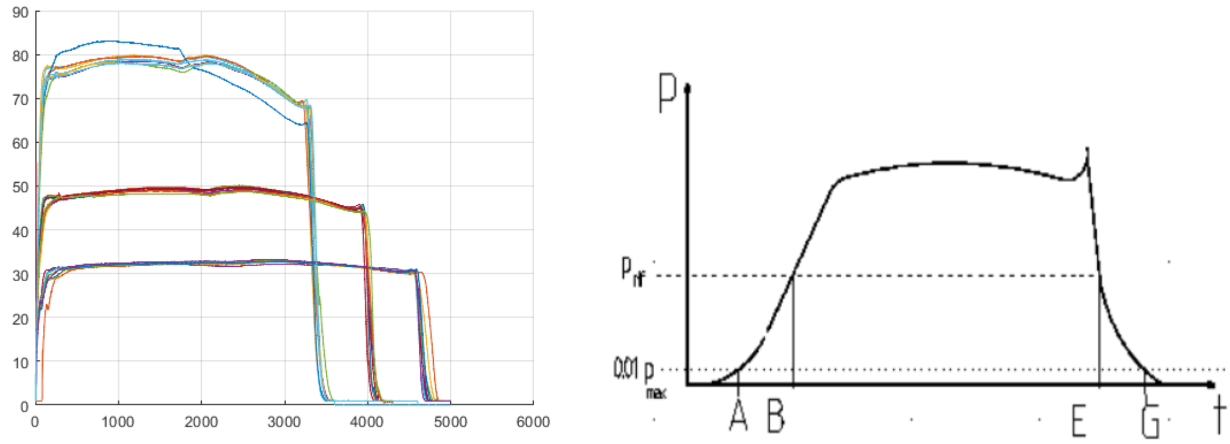


Figure 1: BATES-kind rocket motor

2 Approach and Model used

2.1 BC method

As stated previously, in order to compute the burning rate the BC method was used. In figure 2 the pressure traces along with the BC markers are shown:



(a) Pressure traces (high, medium, low) of this experiment

(b) BC pressure and time markers

Figure 2: Graphical representation of pressure traces with BC markers [4]

This method consisted of evaluating the burning rate through the web thickness and the burning time, which is defined as $t_{burn} = t_E - t_B$, where t_E and t_B are the time instant when the pressure trace crosses the reference pressure. The latter is defined as: [2] [3]

$$p_{ref} = \frac{I_1}{t_G - t_A} \qquad with \quad I_1 = \frac{\int_{t_A}^{t_G} pdt}{2} \tag{1}$$

With t_A and t_G being the time instants when the pressure traces reaches 1% of the maximum pressure. The burning rate, the effective pressure and the characteristic velocity were then computed:

$$r_b = \frac{web}{t_{burn}} \qquad p_{eff} = \frac{\int_{t_E}^{t_B} pdt}{t_{burn}} \tag{2}$$

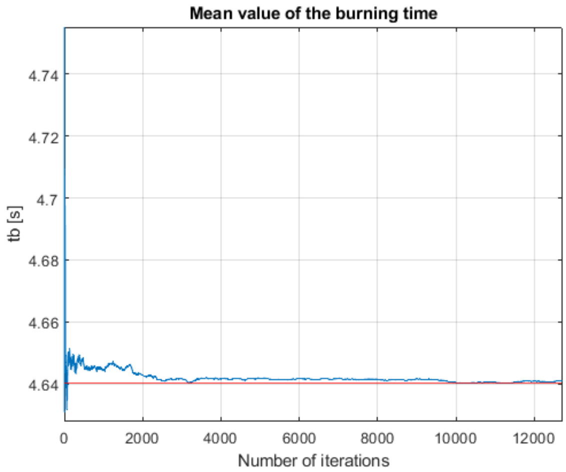
While for $c*$ its definition was used in order to compute it, using the trapezoidal integration method and then calculating the mean value. The results of these computations are shown in table 3:

Parameter	Nominal value	Standard Deviation	Unit
a	1.7271	0.01821	$mm/s \cdot bar^n$
n	0.3821	0.00271	–
$c*$	1517.23	23.6301	m/s

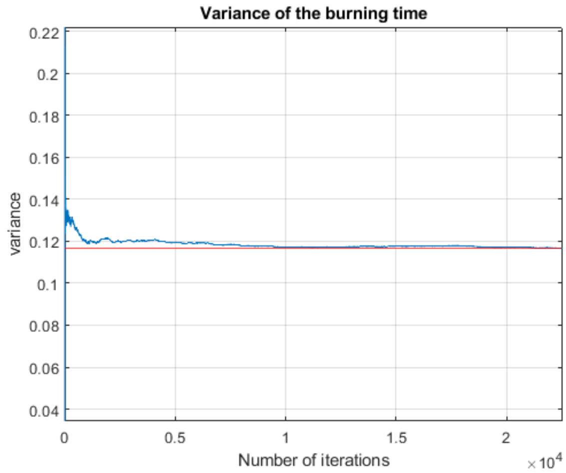
Table 3: Values computed

2.2 Monte Carlo analysis

Once the theoretical model is defined, it's crucial now to let the parameters vary, picking them randomly from a Gaussian distribution, in order to obtain different pressure traces and, consequently, different burning times. The first step is the choice of the number of simulations N; in order to have a swift convergence with a decent computation time but still remaining into a Gaussian distribution, a population of N=30 was chosen. After this, we let every parameter vary N times in the ballistic equation. For every variation of the triplets, the pressure trace is computed and the corresponding burning time is also calculated. At the end of the process a vector of N^3 burning times is obtained. Every burning time is associated with an unique triplet (a_i, n_i, c_i^*), which is extracted from the corresponding shuffled sample vector. Finally, we plot the mean value and the variance of the burning time:



(a) Mean value of burning time convergence



(b) Variance of burning time convergence

Figure 3: Convergence of burning time (N=30)

3 Conclusions

The tables with the uncertainties are shown below. Looking at the plot, the convergence of the mean value outpaces that of the standard deviation, and most importantly, the standard deviations on input parameters $(a, n, c*)$ are attenuated on the uncertainty of the burning time, even after performing more simulations. However, the relatively high uncertainty value on a varies slightly in each simulation, because its standard deviation depends on the random values of a chosen from the Gaussian distribution.

Parameter	**% Uncertainty**
u_a	1.0544 %
u_n	0.7092 %
u_{c*}	1.557452 %
u_{t_b}	0.014722 %

Table 4: Uncertainties on input parameters and burning time

In general, the uncertainty of the parameters (Table 4) obtained from the Monte Carlo analysis confirms that the output of our model is robust to input noise. In addition, this uncertainty confirms that by using the BC method to derive burn characteristics from the experimental data and integrating these into the ballistic model, we obtained simulated pressure curves that closely correlate with the experimental results.

References

[1] Jasmin Terzic Berko Zecevic Mario Baskarad Alan Catovic and Sabina Serdarevic-Kadic. “Prediction of Internal Ballistic Parameters of Solid Propellant Rocket Motors”. In: (2011).

[2] R.S. Fry etc. *Solid propellant burning rate measurement methods used within the NATO propulsion community*.

[3] T.E. Kallmeyer and L.H. Sayer. *Differences Between Actual and Predicted Pressure-Time Histories of Solid Rocket Motors*.

[4] PoliMi. *The Bayern Chemie method-Flipped class on SRM internal ballistic*.

[5] Unkown. *Monte Carlo method for propagation of uncertainties*.