Exam 2 ECE5410

1. (exam1.nb, exam1_h.m)

   (a) The transfer function is simplified in Mathematica

$$H(z) = Y(z) / U(z) = \frac{-0.00806 + 0.3321\, z - 0.832\, z^2 + 0.4\, z^3}{0.376 - 0.93\, z - 0.25\, z^2 + z^3}$$

   (b) (c) Poles and zeros are
   Poles = {{z -> -1.02}, {z -> 0.45}, {z -> 0.82}}
   Zeros = {{z -> 0.02}, {z -> 0.50}, {z -> 1.55}}

(d) Controlability matrix is

$$\begin{pmatrix} 0.475 & -0.74 & 0.4 \\ -1.10825 & 0.995 & -1.1 \\ 2.433 & -2.54 & 2.3 \end{pmatrix}$$

Eigenvalue of Ctrl is {3.9562, -0.168574, -0.0176275}. The magnitude of the first one is more than 200 times as the third one.

(e) Observability matrix is

$$\begin{pmatrix} 0.3 & 0.9 & 0.06 \\ 0.096 & 0.477 & 0.438 \\ 0.4347 & 0.32145 & -0.2301 \end{pmatrix}$$

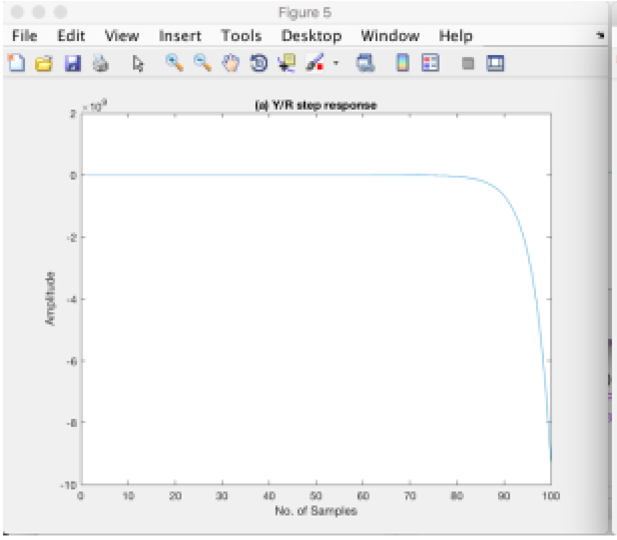
The magnitude of eigenvalue is {0.961523, 0.331218, 0.331218}, all smaller than 1.

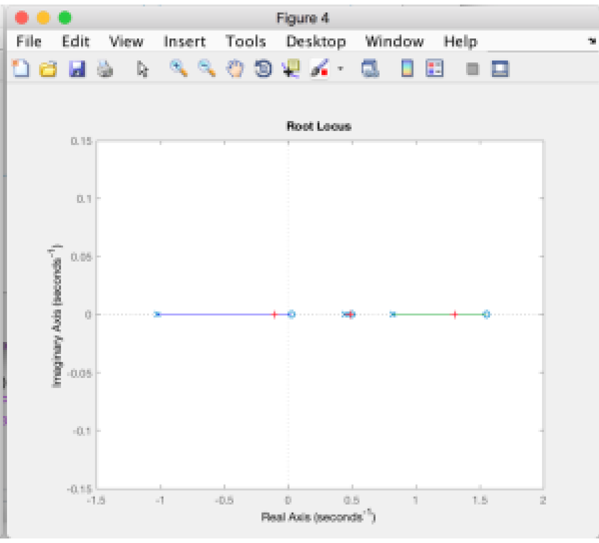
(f) It is minimal. The orders of denominator and numerator of transfer function are all 3, which matches the size of the matrix A. Also, the Det[ctrl] = 0.011756, Det[obsr] = 0.105484, the system is controllable and observable.

(g) It is not strict causal, as the order of denominator equals to the order of numerator.

(h) Matlab rootlucus() is used to find the g. (exam1_h.m)
We can select
Ö~áå=Z
===NPKMQVM
éçäÉë=Z
====NKPMMV
====MKQUUO
===J MKNMOP

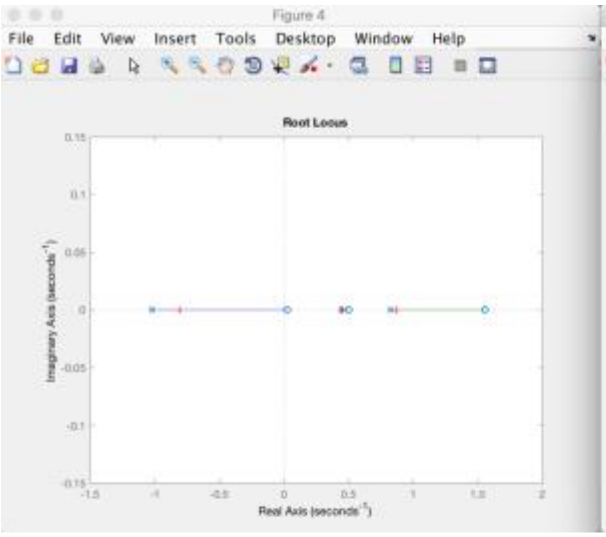
to make the transfer function unstable. Below are the rootlocus and step response of Y/R, showing that it is unstable.

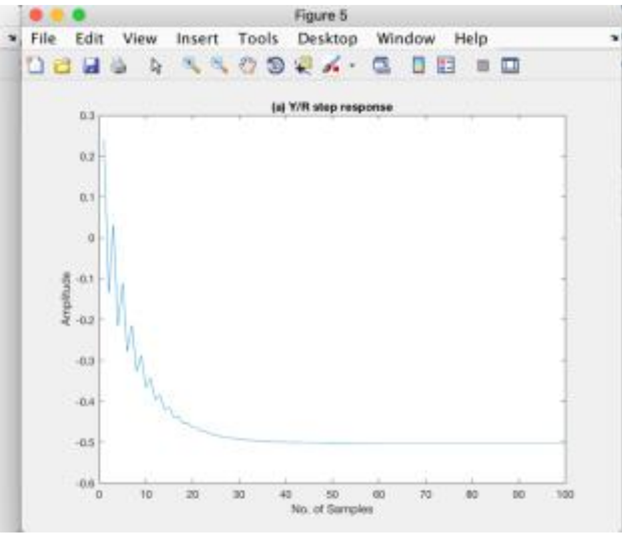




(i) Select
Ö~áå=Z

====MKSMQO
éçäÉë=Z
===J MKUMUP
====MKUTMM
====MKQRPO
The rootlocus and step response below show that the system is stable.





(j)Making close-loops poles complex-valued is impossible as the root locus is on the real axis.

2. (a)

0.3760 < r < 0.7218

Using place() in Matlab, place the poles on

dp=[0.6 + 0.15 * ii, 0.6 - 0.15 * ii, 0.2, 0.3];

Then we can find k

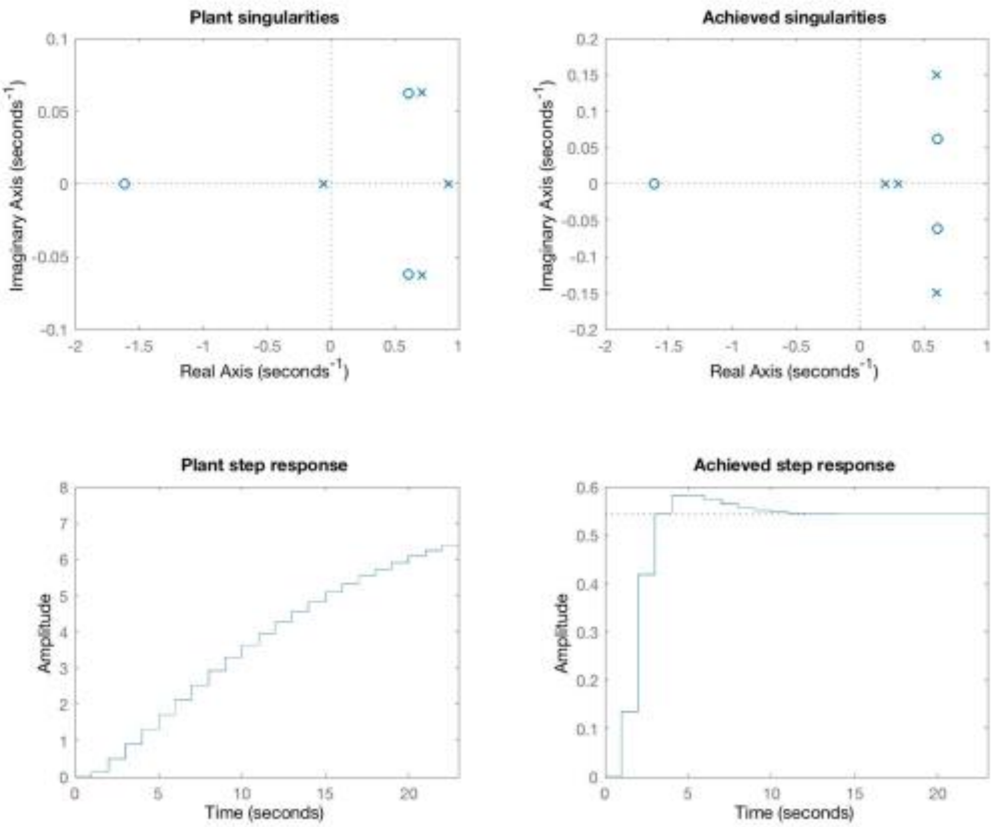
â=Z

====OKNUMM===JPKOQOV====OKUUNN====RKRSQM

The final value of y is 0.5439 and the max of y is 0.5823, meaning that the present overshoot is 7% .

In y, the 9^{th} is the last one that exceeds 2% range of the final value. Therefore the settling time is 9 samples. The step response is shown below.

6	0.5823
7	0.5737
8	0.5643
9	0.5567
10	0.5514
11	0.5480
12	0.5460



(b) exam2_b.m

Select the eigenvalues of (A – lc) as [0.5 0.3 0.1 0.05], use place to compute the l as

ä=Z

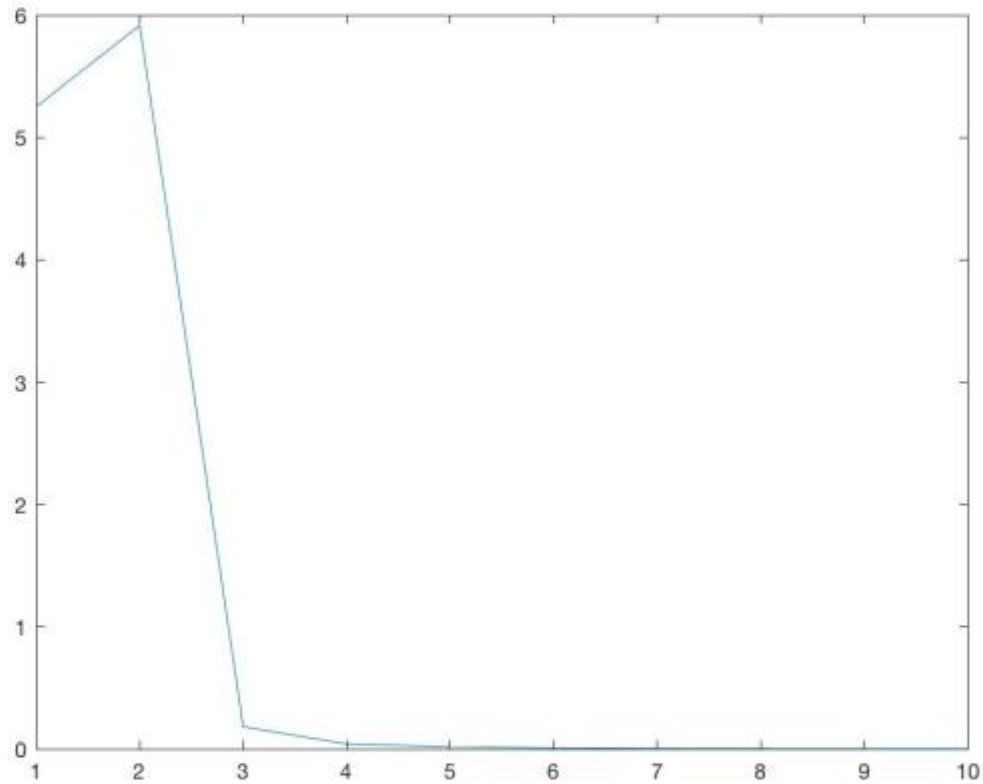
====NKONPP

====MKMONP

===J MKNQUO

====   MKOSOM

Then plot the xerrorT.xerror. The max of the error is below 6 and it decays to below 0.01 at t = 6.

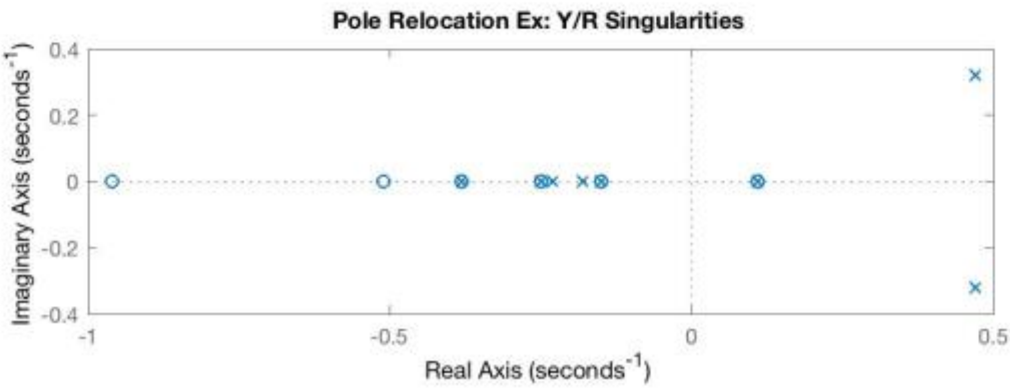


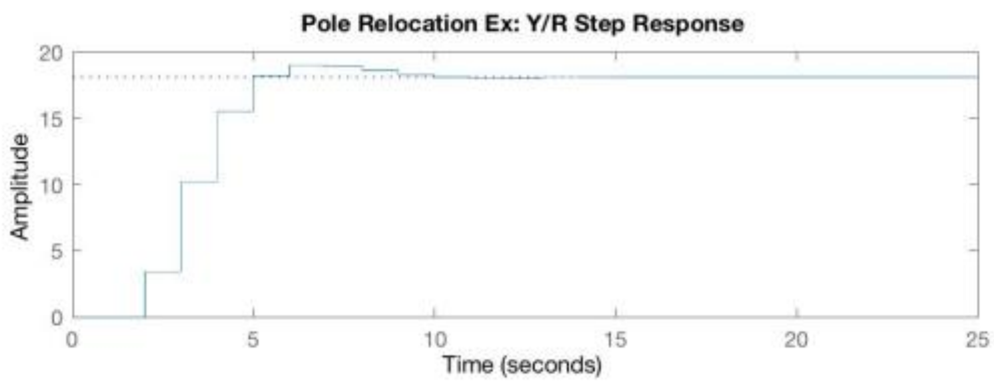
3. (a) Use Matlab dfsp to simulate. (exam3_a.m)

PN(z) = F(z) = MD(z) = N(z) = Poly(cancelled poles)
= 1z^4 + 0.67z^3+0.1037z^2 - 0.006595 z - 0.0015675

PD(z) = G(z) = 1.0000z^4 + 3.1000z^3 + 5.6873z^2 + 4.7993z+ 1.3123 (vector g)
MN(z) = M(z) = 1.0733z^3 -1.8021z^2 + 1.0819z - 0.2347 (vector m)

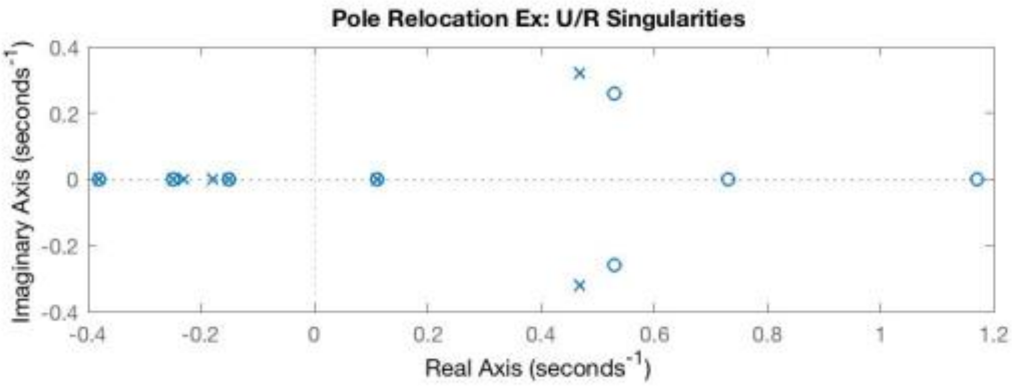
(b) Use dstep() to plot the step response
The maximum of step response and stable value of response is
óã~ñ=Z
===NVKMORM
ñ=Z=
===T
Ñáå~ä=Z
===NUKMUUT
Therefore, the percent overshoot is 5.17%. The time of the first peak is t = 7

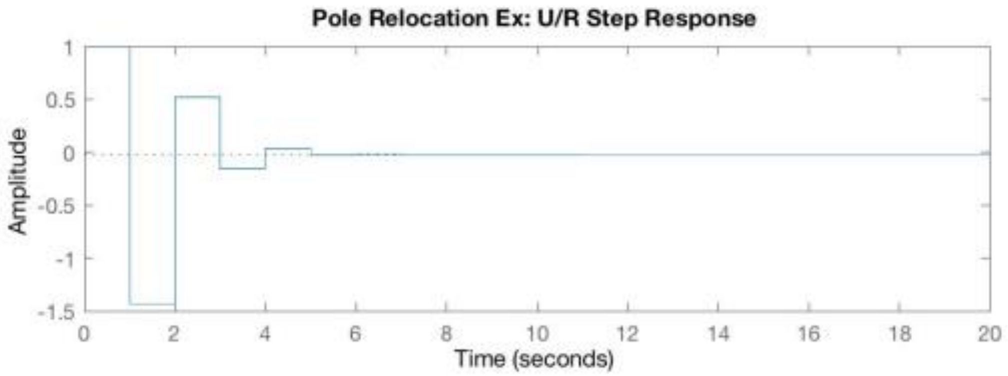




(c) Plot the step response of U/R. The maximum value of u is 1.43.
ã~ñì =Z

=====NKQPMM





4. (a)

Use place command to compute the k and l

k = place(A,b,dcp)

l = place(A',c',dop)'

We can get

â=Z

===J MKNRTS====MKOMOU===J MKOOQQ====MKORTT

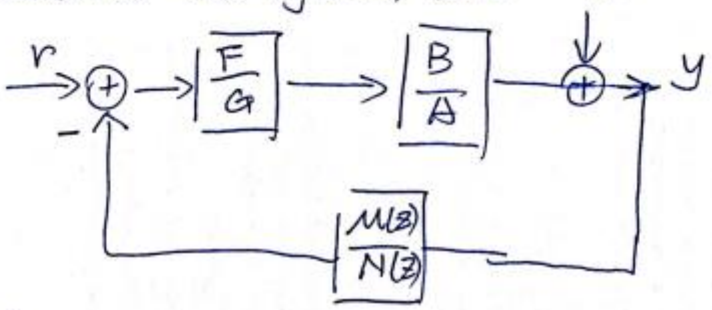
ä=Z

===J QKPNON
====MKQSPS
===NQKSSSP
===J SKMQT M

(b)

Exam 4 (b).

Convert the system into



$$\frac{B(z)}{A(z)} = c(zI - A^{-1})b + d = ss2tf(A, B, c, d).$$

$$\frac{F(z)}{G(z)} = 1 - k(zI - A + lc + bk)^{-1} \cdot b$$
$$= ss2tf(A - lc - bk, b, -k, 1)$$

$$\frac{M(z)}{N(z)} = k(zI - A + lc)^{-1} \cdot l$$
$$= ss2tf(A - lc, l, k, 0)$$

$$\frac{Y(z)}{W(z)} = 1/\left(1 + \frac{M(z)F(z)B(z)}{AGN \cdot}\right) = \frac{AGN}{AGN + BFM}.$$

Use ss2tf() in Matlab to compute B, A, F, G, M and N. Then we can use convolution to compute Y/W,

vò=Z

==` çäì ãåë=N=í Üêçì ÖÜ=V
===JNKMMMM====MKNMMM====QKPTRM===JNKVQTN===JPKSPST====NKOQOP====MKVTMO==J
MKNVQN===J MKMVUS
==` çäì ãåë=NM=í Üêçì ÖÜ=NP
====MKMMRU====MKMMOR===J MKMMMN===J MKMMMM

tò=Z
==` çäì ãåë=N=í Üêçì ÖÜ=V
===NKMMMM===J MKNMMM===J MKTMTR====MKNMSM====MKPORO====MKMSNS===J MKMPQN===J

MKMNQQ===JMKMMNN
==`çäì ãåë=NM=í Üêçì ÖÜ=NP
====MKMMMP====MKMMMN====MKMMMM====MKMMMM

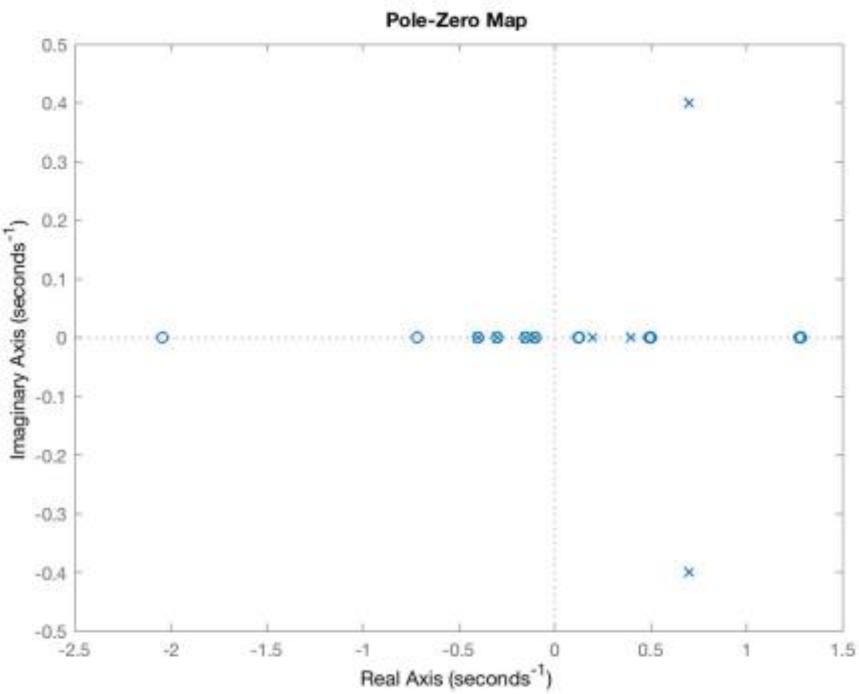
Y/W =

$$\frac{0. - 0.0001 z + 0.0025 z^2 + 0.0058 z^3 - 0.0986 z^4 - 0.1941 z^5 + 0.9702 z^6 + 1.2423 z^7 - 3.6367 z^8 - 1.9471 z^9 + 4.375 z^{10} + 0.1 z^{11} - 1. z^{12}}{0. + 0.0001 z^2 + 0.0003 z^3 - 0.0011 z^4 - 0.0144 z^5 - 0.0341 z^6 + 0.0616 z^7 + 0.3252 z^8 + 0.106 z^9 - 0.7075 z^{10} - 0.1 z^{11} + 1. z^{12}}$$

Poles and zeros are

Poles	Zeros
===MKTMMM=H=MKQMMMá	===JOKMQNM
===MKTMMM=J=MKQMMMá	====NKOUOM
===MKQMMM=H=MKMMMMá	====NKOTST
===MKOMMM=H=MKMMMMá	===JMKTNRV
==JMKQMMM=H=MKMMMMá	====MKRMMM
==JMKQMMM=H=MKMMMMá	====MKQVPM
==JMKPMMM=H=MKMMMMá	===JMKQMMM
==JMKPMMM=H=MKMMMMá	===JMKPMMM
==JMKNRMM=H=MKMMMMá	===JMKNRMM
==JMKNRMM=H=MKMMMMá	====MKNPMP
==JMKNMMM=H=MKMMMMá	====MKNOQV
==JMKNMMM=H=MKMMMMá	===JMKNMMM

Pzmap is shown as



5. (a) exam5_a.m

$$\begin{pmatrix} x(z+1) \\ \hat{x}(z+1) \\ v(z+1) \end{pmatrix} = \begin{pmatrix} A & -bk_1 & bk_2 \\ lc & A-lc-bk_1 & bk_2 \\ -c & 0 & 1 \end{pmatrix} \begin{pmatrix} x(z) \\ \hat{x}(z) \\ v(z) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r(z).$$

$$\det \begin{bmatrix} zI-A & bk_1 & -bk_2 \\ -lc & zI-A-lc-bk_1 & -bk_2 \\ c & 0 & z-1 \end{bmatrix} = 0.$$

$$\Rightarrow \det\left[zI - \left(\begin{pmatrix} A & 0 \\ -c & 1 \end{pmatrix} - \begin{pmatrix} b \\ 0 \end{pmatrix}(k_1 \ \ -k_2)\right)\right] \det(zI - A + lc).$$

$\therefore$ Use place $\left(\begin{pmatrix} A & 0 \\ -c & 1 \end{pmatrix}, \begin{pmatrix} b \\ 0 \end{pmatrix}, \text{desire poles}\right)$

to solve $(k_1, -k_2)$

Use place $(A^T, C^T, \text{desire zeros})$

to solve l.

Then we can get

âN=Z

====NKNQUP===J OKVQMO===J OKOSSP====MK MUQR

âO=Z=

====OKNTUS

ä=Z

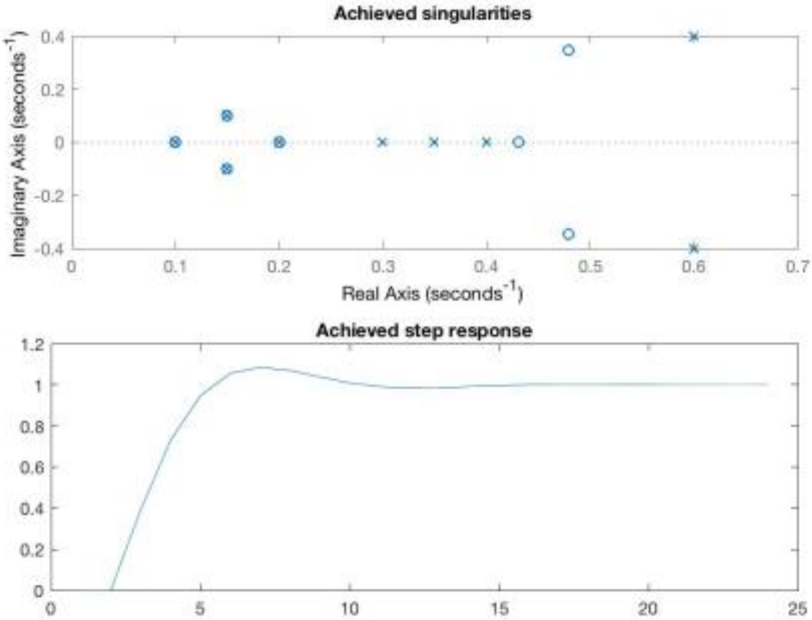
===ORKOQSR

===J NKQQPV

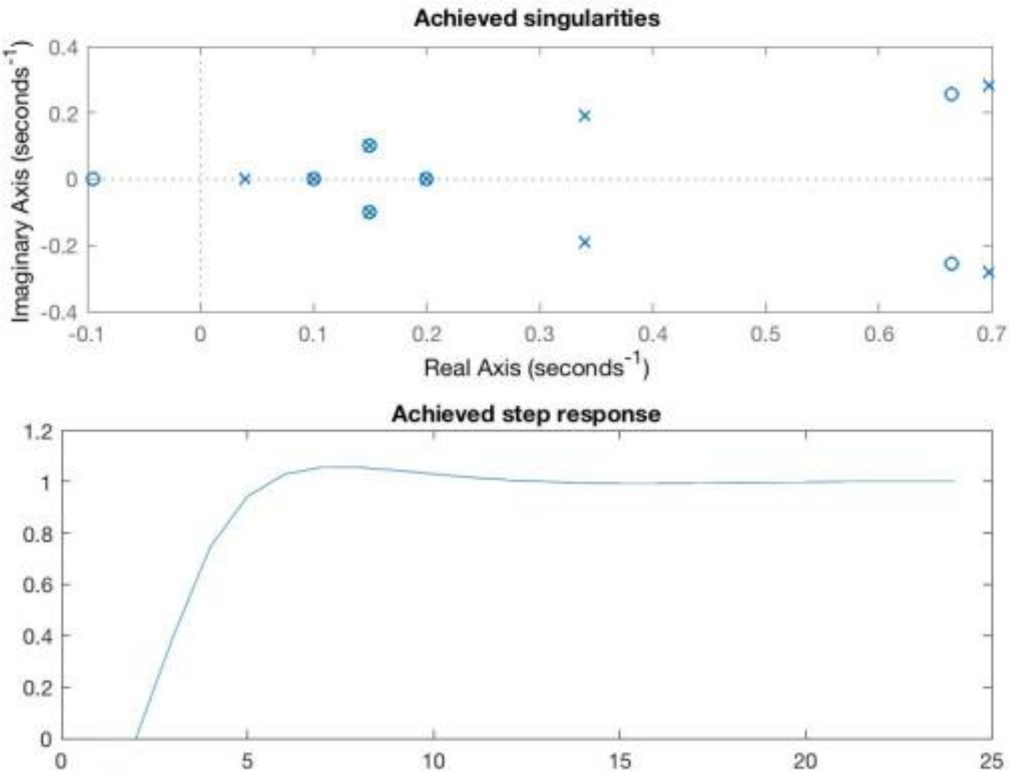
====MKRNRS

===J OKMVSV

Then we can plot the step response. We can see it has unity DC gain. Poles and zeros desired are shown in pzmap.



(b) Use the same k1,k2 and l to compute the step response of the system with bbar.



The maximum of output y is 1.0567, and the final value of y is 1, meaning that the present overshoot is 5.67%, DC gain is 1.

maxy =

1.0567

y(10) is the last one exceeds 2% of the final value, so the settling time is 11 samples.

6	1.0297
7	1.0567
8	1.0561
9	1.0446
10	1.0303
11	1.0171
12	1.0069
13	1.0000
14	0.9961