Principal Component Analysis and Exploratory Factor Analysis

Module 2: PCA with Two Components

Karen Grace-Martin

Module 2 Outline

1. What is Rotation
2. Types of Rotation
3. Creating Component Scores
4. Example: Social Anxiety Scale

1. What is Rotation?

Steps for 2nd Example : Intent to Breastfeed Scale

1. Initial Extraction of the Components
2. Determine the Number of Components to Retain
3. Rotation **Skipped in Module 1**
4. Interpret the Rotated Solution
5. Create Component Scores **Skipped in Module 1**

The Variables

- HospBf Likely to breastfeed in the hospital
- HospF Likely to give formula in the hospital
- ONEmosBf Likely to breastfeed @ 1-mos
- ONEmosF Likely to give formula @ 1-mos
- FIVEmosBf Likely to breastfeed @ 5-mos
- FIVEmosF Likely to give formula @ 5-mos

Scaling

-2 extremely unlikely
-1 unlikely
0 neutral
1 likely
2 extremely likely

Initial Extraction of the Components

Total Variance Explained			
	Initial Eigenvalues		
Component	Total	% of Variance	Cumulative %
1	2.876	47.926	47.926
2	1.568	26.128	74.054
3	.803	13.375	87.430
4	.293	4.889	92.319
5	.277	4.622	96.941
6	.184	3.059	100.000

Extraction Method: Principal Component Analysis.

Determine the Number of Components to Retain

The Interpretability criteria

1. Minimum of three items with high loadings on each retained component
2. The items that load on any component make sense
3. Items loading on different components make sense
4. The **rotated** factor pattern has simple structure: Each variable has a relatively high loading (> .4) on only one component

Rotated Component Matrix[a]

	Component	
	1	2
HospBf	.012	.882
HospF	.823	.265
ONEmosBf	.105	.903
ONEmosF	.920	.017
FIVEmosBf	.301	.645
FIVEmosF	.850	.128

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.[a]

a. Rotation converged in 3 iterations.

Determine the Number of Components to Retain

Criterion	Number of Components Suggested
Kaiser	2
Total Variance Accounted for	2
Scree Plot	3
Parallel Analysis	2
MAP Test	2
Interpretability	2

Rotation

Component Matrix

	Component	
	1	2
HospBf	.570	.673
HospF	.803	-.318
ONEmosBf	.655	.631
ONEmosF	721	-.572
FIVEmosBf	.642	.306
FIVEmosF	.738	-.441

Extraction Method: Principal Component Analysis.

a. Two Components Extracted

Rotated Component Matrix

	Component	
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Extraction Method: Principal Component Analysis.

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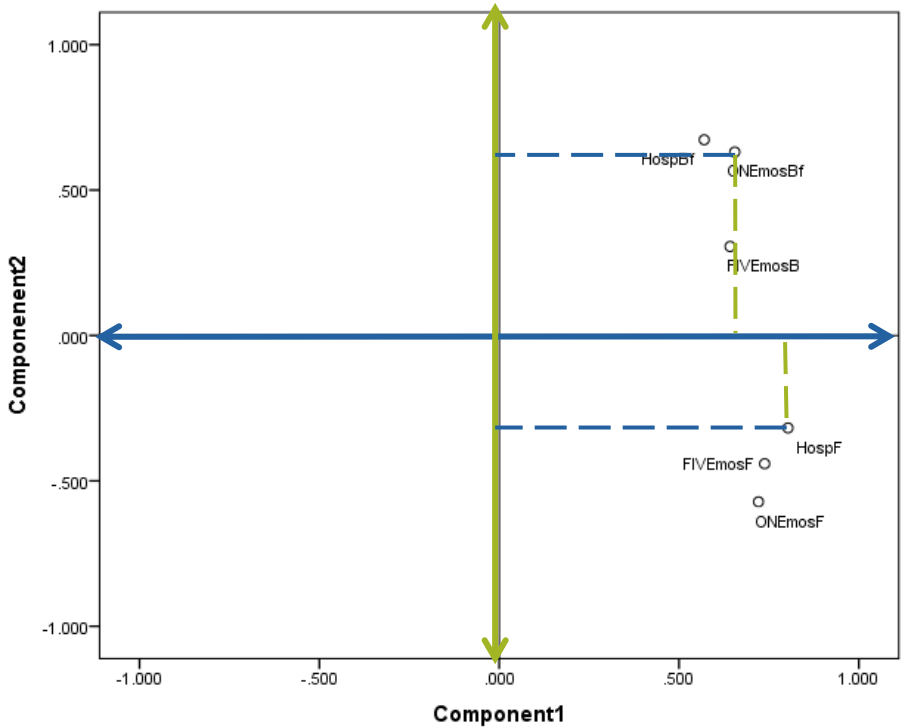
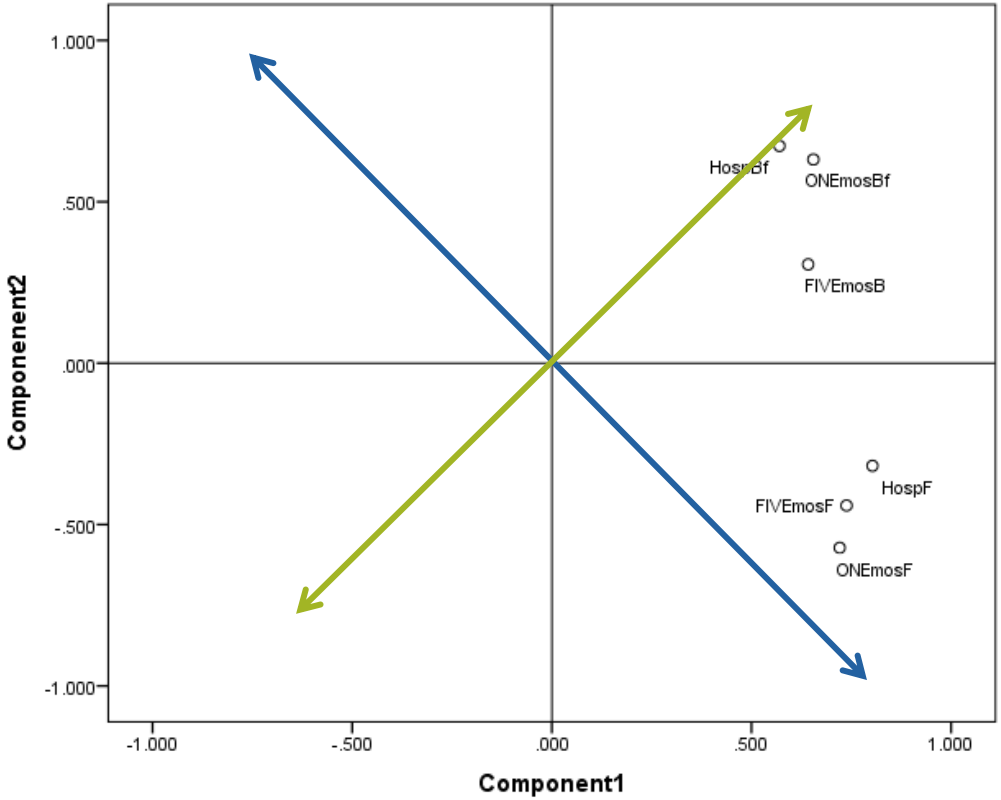
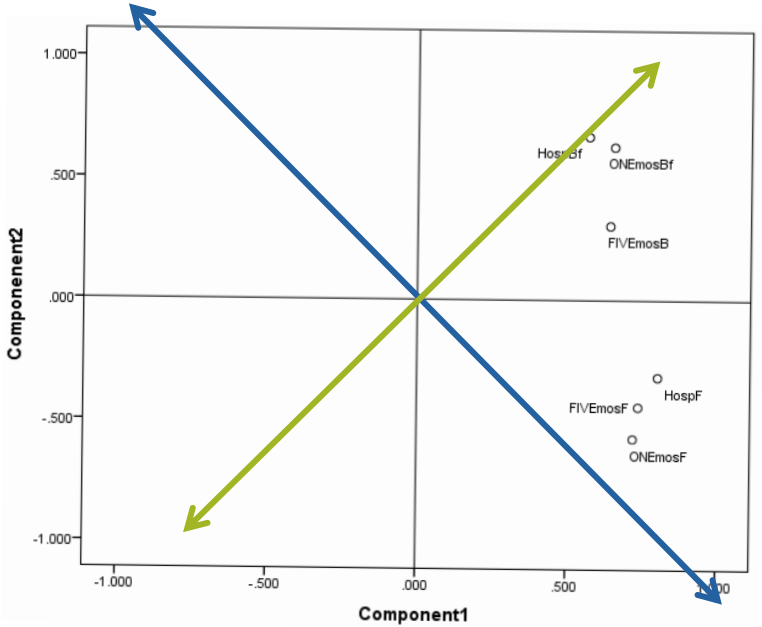


Rotation


1.000
.500
.000
-.500
-1.000
Componenent2
-1.000
-.500
.000
.500
1.000
Component1
ONEmosBf
FIVEmosB
HospF
FIVEmosF
ONEmosF

Rotation


1.000
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Componenent2
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Component1
HospBf
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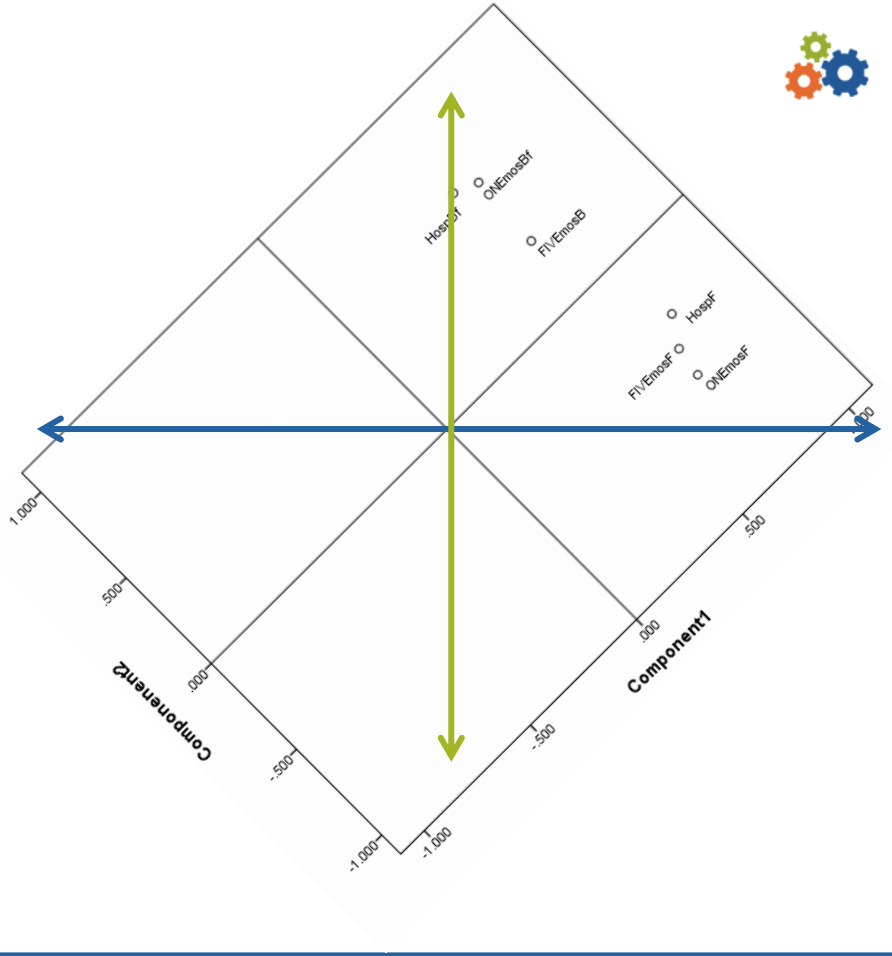
FIVEmosB
HospF
FIVEmosF
ONEmosF


HospBf
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FIVEmosB
HospF
FIVEmosF
ONEmosF
1.000
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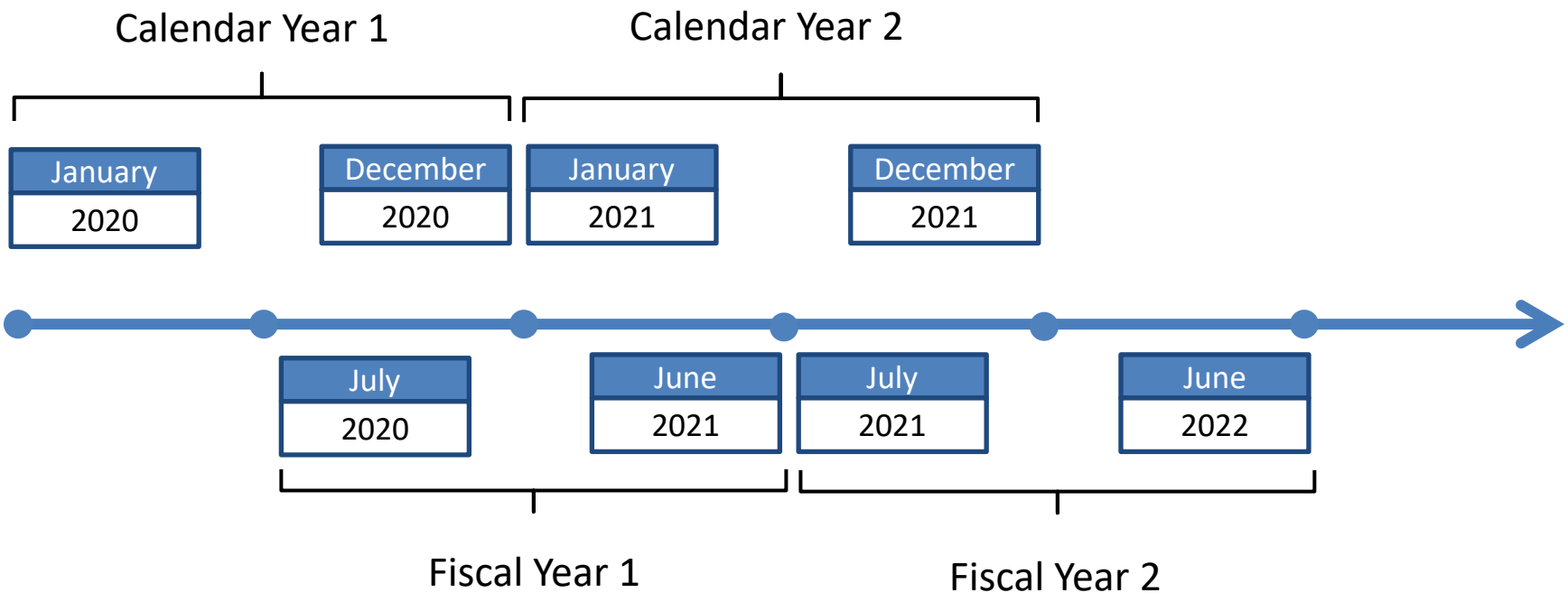
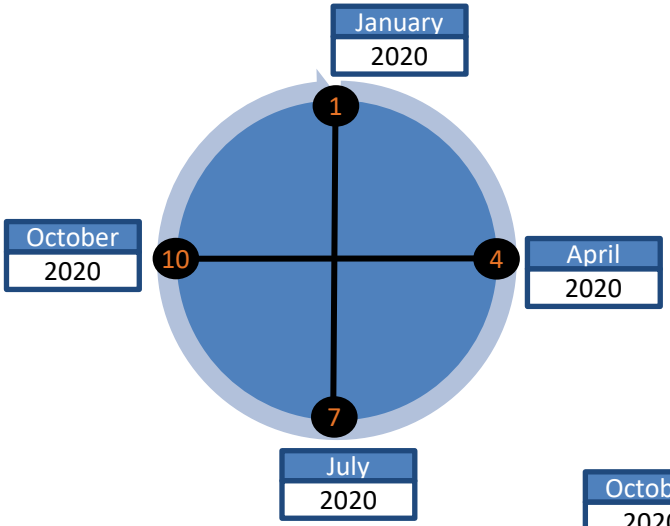
-.500
-1.000
Componenent2
-1.000
-.500
.000
.500
Component1

An Analogy: Fiscal Year

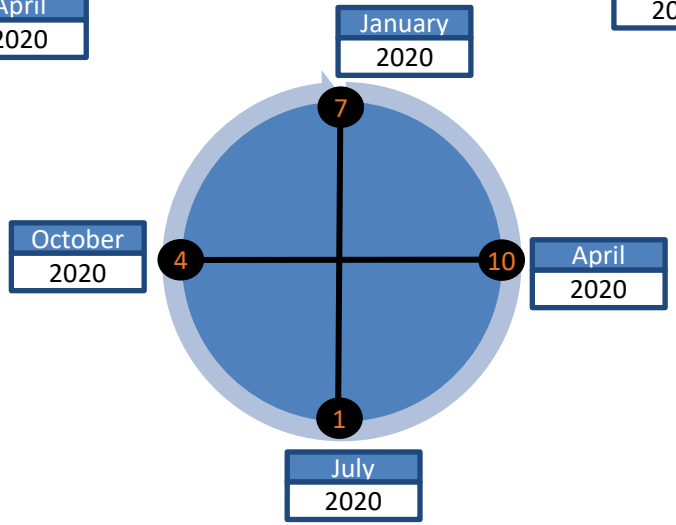


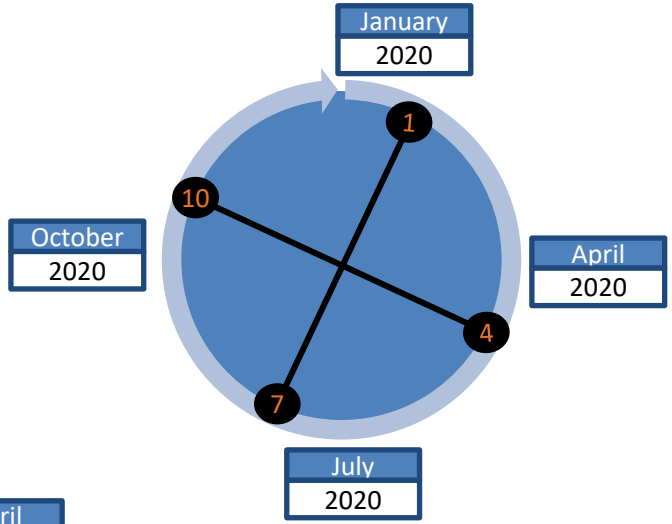
An Analogy: Fiscal Year



Fiscal Year: January = 1

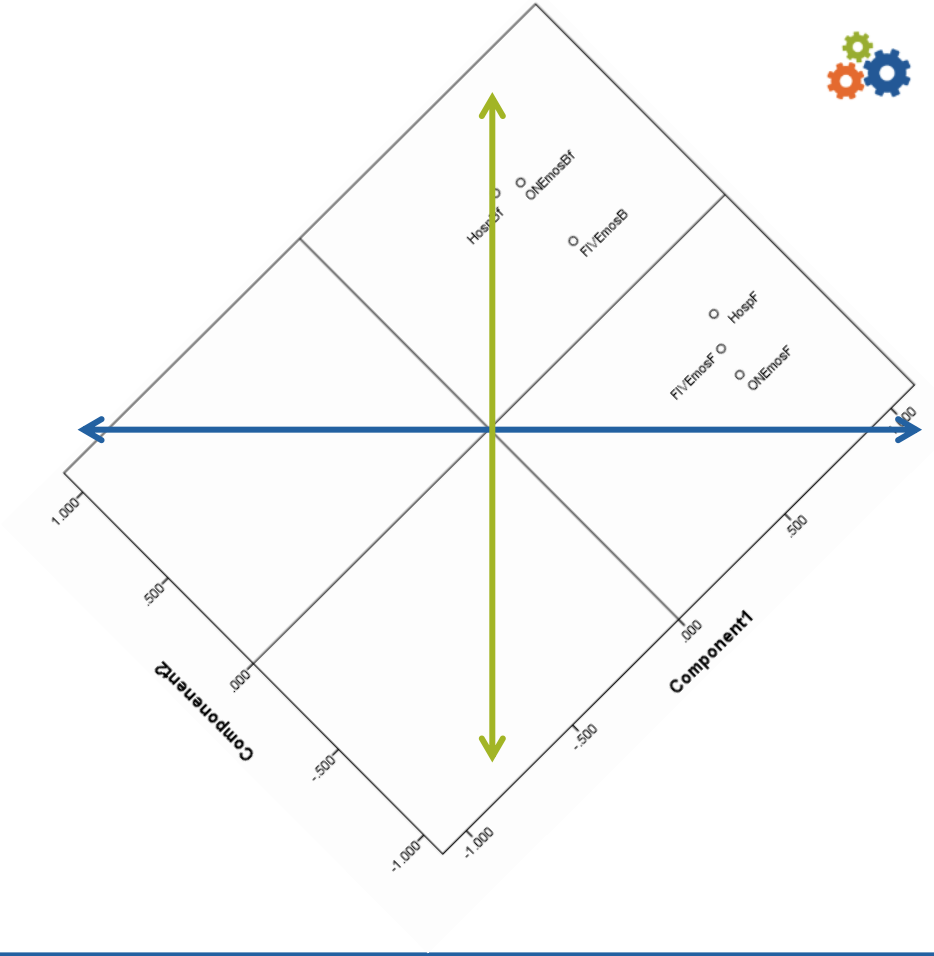
Fiscal Year: July = 1





Fiscal Year: February = 1

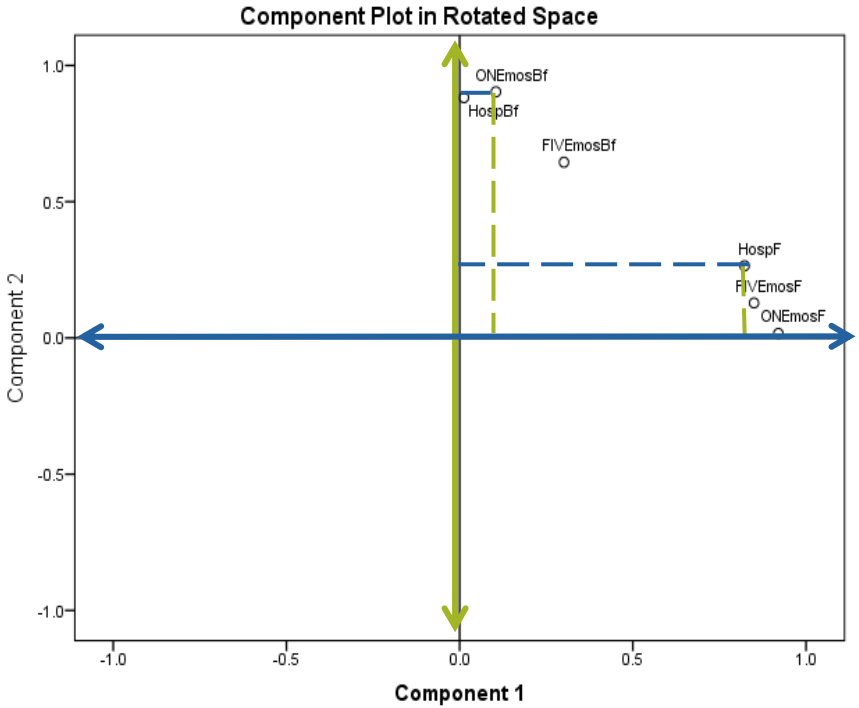
Rotation


ONEmosBf
FtvEmosB
HospF
FtvEmosF
ONEmosF
1.000
.500
.000
-.500
-1.000
Component2
Component1

Rotation

Rotated Component Matrix[a]

	Component	
	1	2
HospBf	.012	.882
HospF	.823	.265
ONEmosBf	.105	.903
ONEmosF	.920	.017
FIVEmosBf	.301	.645
FIVEmosF	.850	.128



Varimax Rotation

Original Component Matrix

	Component	
	1	2
HospBf	.570	.673
HospF	.803	-.318
ONEmosBf	.655	.631
ONEmosF	721	-.572
FIVEmosBf	.642	.306
FIVEmosF	.738	-.441

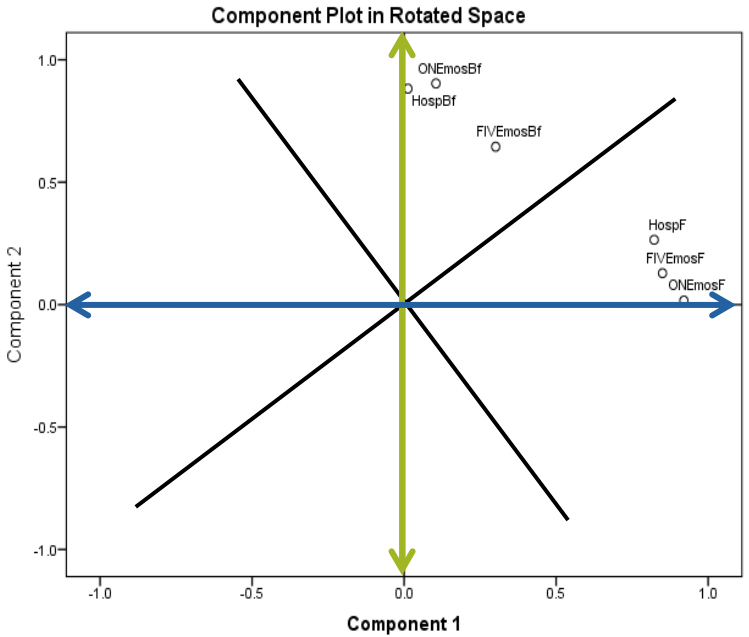
Rotated Component Matrix

	Component	
	1	2
HospBf	.012	.882
HospF	.823	.265
ONEmosBf	.105	.903
ONEmosF	.920	.017
FIVEmosBf	.301	.645
FIVEmosF	.850	.128

2. Types of Rotation

Types of Rotation

Orthogonal



Component Matrix

	Component	
	1	2
HospBf	.570	.673
HospF	.803	-.318
ONEmosBf	.655	.631
ONEmosF	721	-.572
FIVEmosBf	.642	.306
FIVEmosF	.738	-.441

Rotated Component Matrix

	Component	
	1	2
HospBf	.012	.882
HospF	.823	.265
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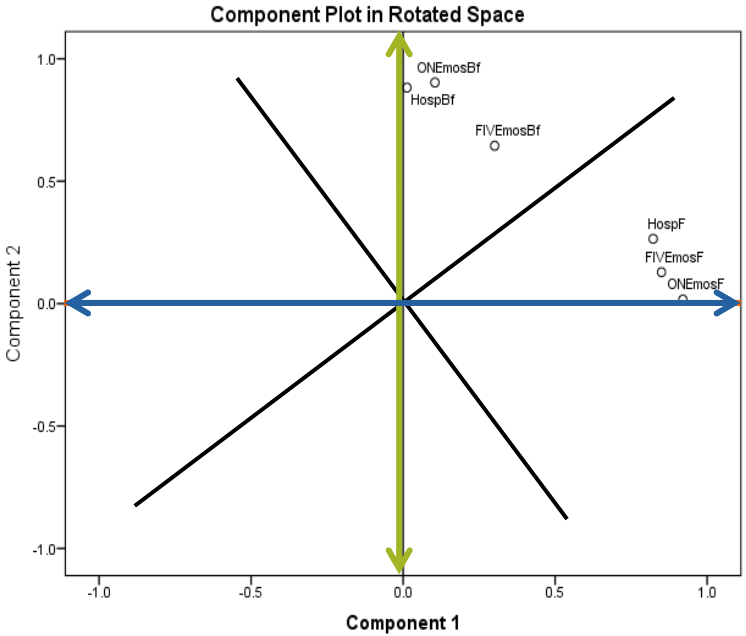
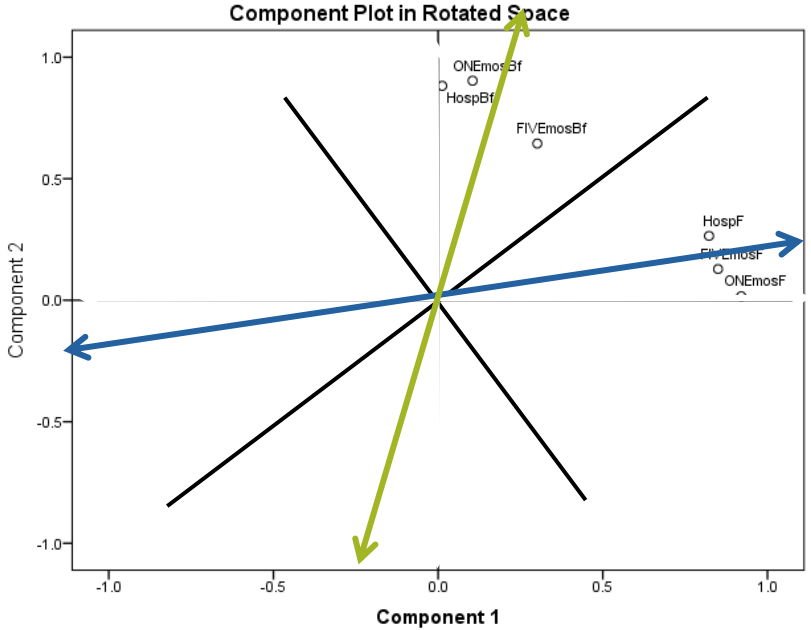
ONEmosF	.920	.017
FIVEmosBf	.301	.645
FIVEmosF	.850	.128

Types of Rotation

Orthogonal



Oblique



Common Orthogonal Types of Rotation

Varimax: maximizes the variance of loadings within a component

Quartimax: maximizes the squared item loadings so each item loads most strongly on a single component

Equamax: combines Varimax and Quartimax. Simultaneously simplifies component (columns) and items (rows)

Oblique Types of Rotation

Promax: First does varimax, then adjusts factors, allowing them to correlate. Loadings are raised to the power kappa (κ).

Higher κ → higher correlation

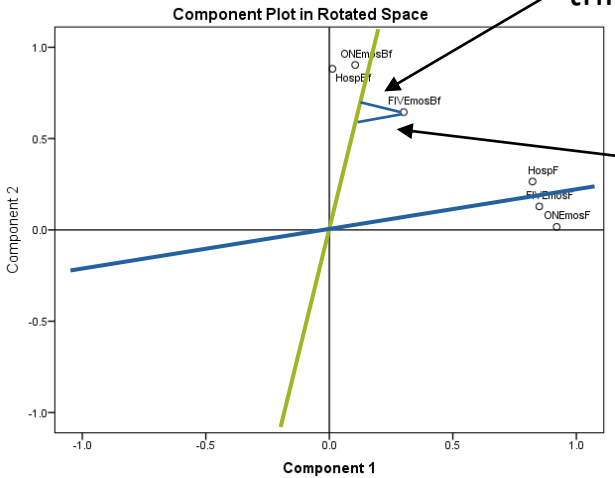
Direct Oblimin: Uses parameter delta (δ) to control correlations among factors.

Usual range: -.4 to 0

Pattern Matrix		
	Component	
	1	2
HospBf	-.113	.907
HospF	.811	.148
ONEmosBf	-.020	.915
ONEmosF	.947	-.122
FIVEmosBf	.219	.619
FIVEmosF	.859	.003

Structure Matrix		
	Component	
	1	2
HospBf	.141	.875
HospF	.852	.374
ONEmosBf	.235	.909
ONEmosF	.913	.142
FIVEmosBf	.392	.680
FIVEmosF	.859	.243

Component Correlation Matrix		
Component	1	2
1	1.000	.279
2	.279	1.000



Structure loading is perpendicular to this component

Pattern loading is parallel to the other component

Varimax

Rotated Component Matrix

	Component 1	Component 2
HospBf	.012	.882
HospF	.823	.265
ONEmosBf	.105	.903
ONEmosF	.920	.017
FIVEmosBf	.301	.645
FIVEmosF	.850	.128

Equamax

Rotated Component Matrix

	Component 1	Component 2
HospBf	.012	.882
HospF	.823	.265
ONEmosBf	.105	.903
ONEmosF	.920	.017
FIVEmosBf	.301	.645
FIVEmosF	.850	.128

Quartimax

Rotated Component Matrix

	Component 1	Component 2
HospBf	.018	.882
HospF	.824	.259
ONEmosBf	.110	.902
ONEmosF	.920	.011
FIVEmosBf	.305	.643
FIVEmosF	.851	.123

Promax

Structure Matrix

	Component 1	Component 2
HospBf	.141	.875
HospF	.852	.374
ONEmosBf	.235	.909
ONEmosF	.913	.142
FIVEmosBf	.392	.680
FIVEmosF	.859	.243

Oblimin

Structure Matrix

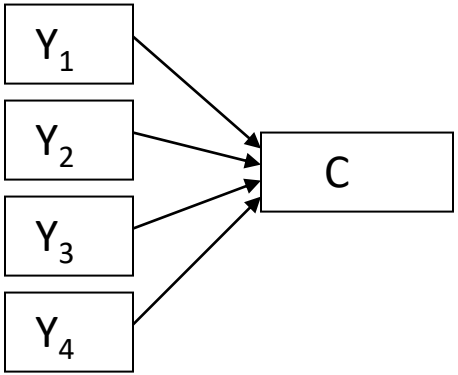
	Component 1	Component 2
HospBf	.137	.875
HospF	.852	.377
ONEmosBf	.231	.909
ONEmosF	.913	.145
FIVEmosBf	.389	.680
FIVEmosF	.859	.245

3. Creating Component Scores

Principal Component Analysis Steps

1. Initial Extraction of the Components
2. Determine the Number of Components to Retain
3. Rotation
4. Interpret the Rotated Solution
5. Create Component Scores

Create Component Scores



$C_i = w_1(Z_{i1}) + w_2(Z_{i2}) + w_3(Z_{i3}) + w_4(Z_{i4})$

w_j = component score coefficient for item j

$$\mathbf{W} = \mathbf{R}^{-1}\mathbf{S}$$

$$Z_{ij} = \frac{Y_{ij} - \bar{Y}_j}{S_{Yj}}$$ for subject i on item j

Create Component Scores

PC	FAC1_1	ZWeight	ZTotSleep	ZPredatn	ZExpos	species
2.81	2.83938	7.17917	-1.57003	.08740	1.60809	African elephant
-.20	-.17009	-.21997	-.48467	.08740	-.88445	African giant pouched rat
-.97	-.93517	-.21732	.42703	-1.26724	-.88445	Arctic Fox
-.16	-.11917	-.22006	1.29532	1.44203	-.26131	Arctic ground squirrel
1.77	1.82727	2.61157	-1.43979	.08740	1.60809	Asian elephant
.65	.72034	-.20935	-.15906	.76471	.98495	Baboon
-1.53	-1.50371	-.22106	1.98995	-1.26724	-.88445	Big brown bat
1.23	1.30240	-.04314	-.94052	.76471	1.60809	Brazilian tapir
-.87	-.82982	-.21741	.86118	-1.26724	-.26131	Cat
-.73	-.70292	-.16307	-.18077	-1.26724	-.88445	Chimpanzee

Component Score Coefficient Matrix	
	Component 1
weight	.218
totsleep	-.363
predatn	.320
expos	.404

PC = .218*Zweight - .363*Ztotsleep + .320*Zpredatn + .404*Zexpos.

Create Component Scores

Component Score Coefficient Matrix		
	Component	
	1	2
HospBf	-.052	.437
HospF	.344	.068
ONEmosBf	-.013	.440
ONEmosF	.403	-.063
FIVEmosBf	.090	.297
FIVEmosF	.365	-.002

Component Score Covariance Matrix		
Component	1	2
1	1.078	.559
2	.559	1.078

Extraction Method: Principal Component Analysis.
Rotation Method: Promax with Kaiser Normalization.
Component Scores.

PC1 = -.052*Z_HospBf + .344*Z_HospF - .013*Z_ONEmosBf + .403*Z_ONEmosF + .090*Z_FIVEmosBf + .365*Z_FIVEmosF

PC2 = .437*Z_HospBf + .068*Z_HospF + .440*Z_ONEmosBf - .063*Z_ONEmosF +.297*Z_FIVEmosBf -.002*Z_FIVEmosF

4. Example: Social Anxiety Scale

Example: Social Anxiety Scale

PrAnxAuth	I get nervous if I have to speak with someone in authority
PrAnxEye	I have difficulty making eye-contact with others
PrAnxStreet	I tense-up if I meet an acquaintance on the street
PrAnxMix	When mixing socially I feel uncomfortable
PrAnxOne	I feel tense if I am alone with just one other person
PrAnxTalk	I have difficulty talking with other people
PrAnxExp	I worry about expressing myself in case I appear awkward
PrAnxSay	I find myself worrying that I won't know what to say in social situations
PrAnxWell	I am nervous mixing with people that I don't know well
PrAnxEmbarr	I feel I'll say something embarrassing when talking
PrAnxIgn	When mixing in a group, I find myself worrying that I will be ignored
PrAnxTense	I am tense mixing in a group
PrAnxGreet	I am unsure whether to greet someone I know only slightly
PrAnxSelf	I have high self-esteem

1	Not at all
2	Slightly
3	Moderately
4	Very
5	Extremely

Steps

1. Initial Extraction of the Components
2. Determine the Number of Components to Retain
3. Rotation
4. Interpret the Rotated Solution
5. Create Component Scores

Correlation Matrix

	PrAnxAuth	PrAnxEye	PrAnxStreet	PrAnxMix	PrAnxOne	PrAnxTalk	PrAnxExp	PrAnxSay	PrAnxWell	PrAnxEmbarr	PrAnxIgn	PrAnxTense	PrAnxGreet	PrAnxSelf
PrAnxAuth	1	0.346	0.329	0.43	0.257	0.384	0.465	0.428	0.464	0.493	0.354	0.359	0.284	-0.321
PrAnxEye	0.346	1	0.48	0.51	0.403	0.387	0.346	0.345	0.491	0.321	0.311	0.487	0.364	-0.284
PrAnxStreet	0.329	0.48	1	0.555	0.511	0.499	0.395	0.415	0.456	0.399	0.415	0.485	0.475	-0.221
PrAnxMix	0.43	0.51	0.555	1	0.501	0.63	0.614	0.608	0.713	0.587	0.573	0.727	0.474	-0.387
PrAnxOne	0.257	0.403	0.511	0.501	1	0.455	0.399	0.366	0.32	0.397	0.4	0.447	0.328	-0.239
PrAnxTalk	0.384	0.387	0.499	0.63	0.455	1	0.564	0.535	0.619	0.517	0.42	0.609	0.352	-0.285
PrAnxExp	0.465	0.346	0.395	0.614	0.399	0.564	1	0.733	0.641	0.7	0.567	0.615	0.437	-0.436
PrAnxSay	0.428	0.345	0.415	0.608	0.366	0.535	0.733	1	0.695	0.69	0.621	0.694	0.465	-0.373
PrAnxWell	0.464	0.491	0.456	0.713	0.32	0.619	0.641	0.695	1	0.63	0.565	0.71	0.521	-0.405
PrAnxEmbarr	0.493	0.321	0.399	0.587	0.397	0.517	0.7	0.69	0.63	1	0.568	0.621	0.398	-0.446
PrAnxIgn	0.354	0.311	0.415	0.573	0.4	0.42	0.567	0.621	0.565	0.568	1	0.634	0.4	-0.443
PrAnxTense	0.359	0.487	0.485	0.727	0.447	0.609	0.615	0.694	0.71	0.621	0.634	1	0.488	-0.457
PrAnxGreet	0.284	0.364	0.475	0.474	0.328	0.352	0.437	0.465	0.521	0.398	0.4	0.488	1	-0.237
PrAnxSelf	-0.321	-0.284	-0.221	-0.387	-0.239	-0.285	-0.436	-0.373	-0.405	-0.446	-0.443	-0.457	-0.237	1

Run an Initial Model

Options:

1. Four Components
2. Principal Components Extraction
3. Promax Rotation
4. Scree test, Parallel Analysis, MAP test

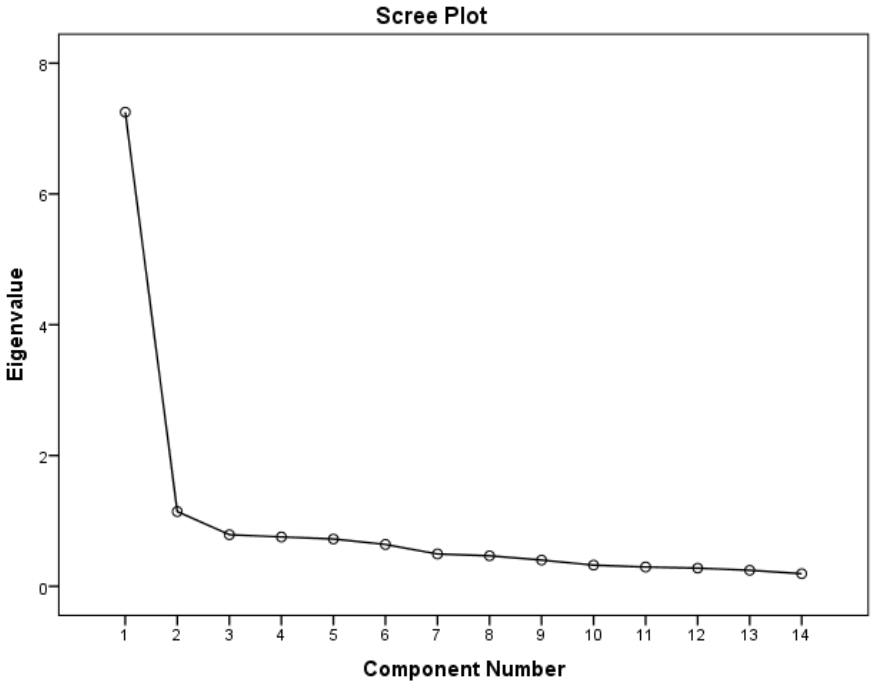
Initial Extraction of the Components

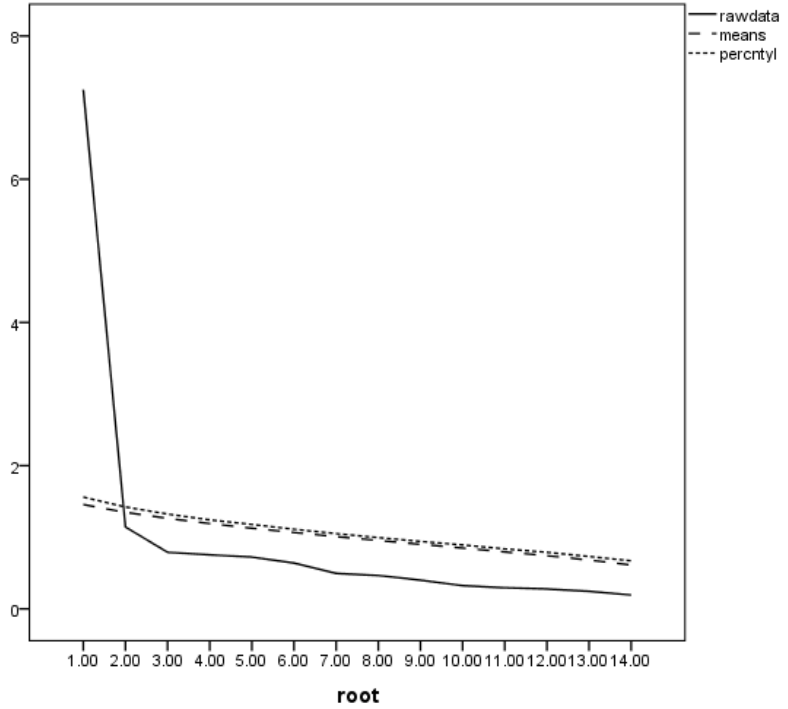
Total Variance Explained			
	Initial Eigenvalues		
Component	Total	% of Variance	Cumulative %
1	7.252	51.797	51.797
2	1.143	8.168	59.964
3	.789	5.639	65.603
4	.755	5.392	70.995
5	.723	5.168	76.162
6	.640	4.570	80.732
7	.495	3.536	84.268
8	.466	3.328	87.596
9	.401	2.867	90.463
10	.324	2.314	92.777
11	.295	2.107	94.885
12	.278	1.985	96.869
13	.245	1.752	98.621
14	.193	1.379	100.000

Extraction Method: Principal Component Analysis.

Communalities		
	Initial	Extraction
PrAnxAuth	1.000	.845
PrAnxEye	1.000	.691
PrAnxStreet	1.000	.687
PrAnxMix	1.000	.721
PrAnxOne	1.000	.668
PrAnxTalk	1.000	.575
PrAnxExp	1.000	.727
PrAnxSay	1.000	.786
PrAnxWell	1.000	.749
PrAnxEmbarr	1.000	.708
PrAnxIgn	1.000	.671
PrAnxTense	1.000	.748
PrAnxGreet	1.000	.523
PrAnxSelf	1.000	.840

Initial Extraction of the Components





Initial Extraction of the Components

Velicer's Minimum Average Partial (MAP) Test:

Average Partial Correlations

	squared	power4
.0000	.2363	.0712
1.0000	.0211	.0012
2.0000	.0219	.0014
3.0000	.0322	.0047
4.0000	.0477	.0089
5.0000	.0649	.0139
6.0000	.0795	.0223
7.0000	.1019	.0399
8.0000	.1469	.0588
9.0000	.1987	.1050
10.0000	.2715	.1509
11.0000	.3812	.2485
12.0000	.4763	.3602
13.0000	1.0000	1.0000

The smallest average squared partial correlation is: .0211

The smallest average 4rth power partial correlation is: .0012

The Number of Components According to the Original (1976) MAP Test is: 1

The Number of Components According to the Revised (2000) MAP Test is: 1

Determine the Number of Components to Retain

The Interpretability criteria

1. Minimum of three items with high loadings on each retained component

2. The items that load on any component make sense

3. Items loading on different components make sense

4. The **rotated** factor pattern has simple structure:
   Each variable has a relatively high loading (> .4) on only one component

Structure Matrix				
	Component			
	1	2	3	4
PrAnxAuth	.503	.354		.904
PrAnxEye	.815	.426	.456	.439
PrAnxStreet	.840	.447	.406	.415
PrAnxMix	.416	.759		.518
PrAnxOne	.620	.560		
PrAnxTalk	.751	.474	.542	
PrAnxExp	.793	.724	.327	.387
PrAnxSay	.460	.774	.321	
PrAnxWell	.878	.445	.330	.344
PrAnxEmbarr	-.469	-.301	-.903	
PrAnxIgn	.538	.819		
PrAnxTense	.704	.628		.379
PrAnxGreet	.837	.643	.427	
PrAnxSelf	.845	.563		.498

The Interpretability criteria

☑ 1. Minimum of three items with high loadings on each

2. Items loading on same component make sense

3. Items loading on different components make sense

☒ 4. Simple structure

Component Correlation Matrix				
Component	1	2	3	4
1	1.000	.614	.397	.408
2	.614	1.000	.217	.321
3	.397	.217	1.000	.167
4	.408	.321	.167	1.000

Structure Matrix		
	Component	
	1	2
PrAnxAuth	.598	.426
PrAnxEye	.441	.732
PrAnxStreet	.478	.821
PrAnxMix	.770	.786
PrAnxOne	.435	.732
PrAnxTalk	.664	.700
PrAnxExp	.846	.560
PrAnxSay	.852	.574
PrAnxWell	.828	.675
PrAnxEmbarr	.838	.532
PrAnxIgn	.758	.537
PrAnxTense	.818	.716
PrAnxGreet	.530	.637
PrAnxSelf	-.620	

The Interpretability criteria

☑ 1. Minimum of three items with high loadings on each

2. Items loading on same component make sense

3. Items loading on different components make sense

☒ 4. Simple structure

Component Correlation Matrix		
Component	1	2
1	1.000	.682
2	.682	1.000

Component Matrix	
	Component 1
PrAnxAuth	.579
PrAnxEye	.592
PrAnxStreet	.653
PrAnxMix	.840
PrAnxOne	.588
PrAnxTalk	.733
PrAnxExp	.803
PrAnxSay	.813
PrAnxWell	.836
PrAnxEmbarr	.787
PrAnxIgn	.734
PrAnxTense	.846
PrAnxGreet	.616
PrAnxSelf	-.540

The Interpretability criteria

1. ☑ Minimum of three items with high loadings on each
2. ☑ Items loading on same component make sense
3. Items loading on different components make sense
4. ☑ Simple structure

Determine the Number of Components to Retain

Criterion	Number of Components Suggested
Kaiser	2
Total Variance Accounted for	4
Scree Plot	1
Parallel Analysis	1
MAP Test	1
Interpretability	1

One Component Solution

Total Variance Explained			
	Initial Eigenvalues		
Component	Total	% of Variance	Cumulative %
1	7.252	51.797	51.797
2	1.143	8.168	59.964
3	.789	5.639	65.603
4	.755	5.392	70.995
5	.723	5.168	76.162
6	.640	4.570	80.732
7	.495	3.536	84.268
8	.466	3.328	87.596
9	.401	2.867	90.463
10	.324	2.314	92.777
11	.295	2.107	94.885
12	.278	1.985	96.869
13	.245	1.752	98.621
14	.193	1.379	100.000

Component Matrix	
	Component
	1
PrAnxAuth	.579
PrAnxEye	.592
PrAnxStreet	.653
PrAnxMix	.840
PrAnxOne	.588
PrAnxTalk	.733
PrAnxExp	.803
PrAnxSay	.813
PrAnxWell	.836
PrAnxEmbarr	.787
PrAnxIgn	.734
PrAnxTense	.846
PrAnxGreet	.616
PrAnxSelf	-.540

Communalities		
	Initial	Extraction
PrAnxAuth	1.000	.336
PrAnxEye	1.000	.351
PrAnxStreet	1.000	.427
PrAnxMix	1.000	.706
PrAnxOne	1.000	.346
PrAnxTalk	1.000	.537
PrAnxExp	1.000	.645
PrAnxSay	1.000	.661
PrAnxWell	1.000	.699
PrAnxEmbarr	1.000	.619
PrAnxIgn	1.000	.538
PrAnxTense	1.000	.715
PrAnxGreet	1.000	.379
PrAnxSelf	1.000	.291

One Component Solution

Total Variance Explained			
	Initial Eigenvalues		
Component	Total	% of Variance	Cumulative %
1	6.990	53.771	53.771
2	1.094	8.413	62.185
3	.772	5.937	68.122
4	.729	5.607	73.728
5	.640	4.922	78.650
6	.558	4.294	82.945
7	.466	3.585	86.529
8	.402	3.096	89.625
9	.325	2.496	92.122
10	.296	2.277	94.399
11	.282	2.166	96.565
12	.249	1.915	98.480
13	.198	1.520	100.000

Component Matrix	
	Component
	1
PrAnxAuth	.577
PrAnxEye	.594
PrAnxStreet	.663
PrAnxMix	.844
PrAnxOne	.594
PrAnxTalk	.741
PrAnxExp	.801
PrAnxSay	.816
PrAnxWell	.838
PrAnxEmbarr	.783
PrAnxIgn	.728
PrAnxTense	.844
PrAnxGreet	.623

Communalities		
	Initial	Extraction
PrAnxAuth	1.000	.333
PrAnxEye	1.000	.353
PrAnxStreet	1.000	.440
PrAnxMix	1.000	.712
PrAnxOne	1.000	.352
PrAnxTalk	1.000	.549
PrAnxExp	1.000	.641
PrAnxSay	1.000	.666
PrAnxWell	1.000	.702
PrAnxEmbarr	1.000	.613
PrAnxIgn	1.000	.529
PrAnxTense	1.000	.712
PrAnxGreet	1.000	.388

One Component Solution

All Fourteen Variables

PrAnxAuth	PrAnxEye	PrAnxStreet	PrAnxMix	PrAnxOne	PrAnxTalk	PrAnxExp	PrAnxSay	PrAnxWell	PrAnxEmbarr	PrAnxIgn	PrAnxTense	PrAnxGreet	PrAnxSelf	FAC1_1
1	1	1	1	1	1	1	2	1	1	1	1	2	5	-1.32324
2	2	2	1	1	2	2	2	2	1	1	2	4	3	-.29025
3	1	2	3	2	2	3	3	99	99	99	99	99	99	.
2	1	1	1	1	1	1	1	1	1	1	1	2	3	-1.19152
2	1	1	1	1	1	1	1	2	1	1	1	1	3	-1.16545
1	2	1	2	1	1	1	1	2	2	2	2	1	2	-.65656
2	2	3	2	1	1	1	3	2	3	3	3	3	3	.24925
3	2	2	2	1	1	3	4	4	3	4	2	2	3	.65436
1	1	1	1	1	1	1	1	2	1	1	2	2	5	-1.20560
3	2	2	4	1	3	2	3	4	1	2	2	3	1	.76042

Component Score Coefficient Matrix

	Component 1
PrAnxAuth	.080
PrAnxEye	.082
PrAnxStreet	.090
PrAnxMix	.116
PrAnxOne	.081
PrAnxTalk	.101
PrAnxExp	.111
PrAnxSay	.112
PrAnxWell	.115
PrAnxEmbarr	.109
PrAnxIgn	.101
PrAnxTense	.117
PrAnxGreet	.085
PrAnxSelf	-.074