Heuristics in A* Algorithm for solving 8-Puzzle Problem

Manal Anetallah Alsahafi

Introduction

The 8-puzzle is one of the standard problems in artificial intelligence since invented by Sam Loyd in 1870s [1]. It is a 3 x 3 grid structured frame area containing tiles in irregular order with one tile missing and each tile labeled 1 through 8. The goal is to reach the final state by sliding the tiles into different positions (left, right, above, and below) using the available space to arrange the numbers on the tiles such that they match the final arrangement see figure 1.

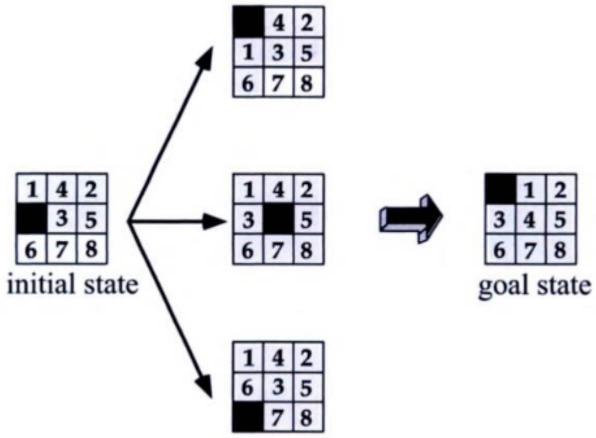

Figure 1: The 8-Puzzle with initial state (left) goal state (right)

Various algorithms were designed to increase the efficiency of the system in providing an optimal solution using lesser space possible and time. Solving the 8-puzzle using informed search methods make the computer order the moves intelligently to find the shortest solution. These methods gain information about the cost of moves that took to get the goal from any given state by using a heuristic function at the beginning of the search. The basic heuristic function h(n) that all informed search methods rely on is the value of computing the difference between a given state to the goal state. The higher h(n) value, the more estimated path length to the goal the method less ideal. The search algorithm known as A* (Hart et al., 1968) is a heuristics-based informed search algorithm that has become popular in the

Artificial Intelligence literature to solve path-finding problems. A* search algorithm combines Greedy with breadth-first search algorithm. At each step of expansion, the A* select the node that has the lowest cost estimate f(n). The estimate f(n) has the following relation:

$$f(n) = g(n) + h(n)$$

where g(n) is the minimal cost from root node to current node **n**, and h(n) is an estimation of the cost required from current node to goal node.

Motivation

In order for the methods to be able to solve optimally, differentiation between a good move and a bad one in the heuristic function should be considered with a tradeoff between the accuracy of a heuristic and how expensive it is to compute it. Therefore, in this work, we focus on these heuristic functions that were applied in previous studies and compare them in order to discover the best performance with A* method.

Literature Review

Over the years, a lot of studies focused on finding its optimal solutions using the 8-puzzle as a prominent platform to analyze heuristic search algorithms in the AI context.

In 1983, the most popular heuristic functions used for the N-puzzle problem are the number of misplaced tiles and the sum of the distances of the mismatched tiles from their proper destinations where the missing tile is not counted in this case Manhattan distance was the best to measure these distances. (Pearl, J.) described a new method to solve a problem of 8-puzzle by generating heuristics mechanically which means solving a relaxed problem by deleting constraints from the original problem.

In 2001, (Kunkle, D. R.) implemented the A* algorithm with Manhattan Distance as a heuristic function besides one penalty move added for every directly reversed tile. This combination made the heuristic more powerful and provided an approximately 16% gain in informedness over Manhattan Distance alone and 18%in the most difficult problem where the number of states examined to find an optimal solution was reduced from 10804 to 9176.

In 2012, (Tuccar, M.) analyzed the A* algorithm with two admissible heuristics which are Manhattan distance and misplaced tiles. The results showed the Manhattan heuristic dominates misplaced tiles heuristics in many aspects. In the number of maximum node expanded, the misplaced tiles required 1,433,773 whereas in Manhattan it required only 15,157 to solve the problem. In time consuming Manhattan being very fast compared to misplaced tiles. In the bigger grids than 3x3 like 2x5, 2x6, 3x4, 3x5, 4x4, which are called complex 8-puzzle problems, misplaced tiles did not give a solution while Manhattan did.

In 2013, (Kuruvilla, el al.) compared the 5 popular search methods in solving N-puzzle problem which are Breadth First Search, Depth First Search, A* Search, Greedy Best First Search and the Hill Climbing Search based on the number of nodes Generated, Expanded, Dropped and the Solution Length. The size of N-Puzzle was 8 and the basic heuristics function was used in this study for the A* algorithm. The researchers found the A* method is the best choice for complex and short solutions in the 8-puzzle problems.

In 2016, (Iordan, A. E.) introduced the double of the Chebyshev distance as a new heuristic function and compared its result with the most recorded so far in solving 8-puzzle problems which are the number of misplaced tiles (Hamming heuristic) and the Manhattan heuristic. All heuristics are implemented using the A* algorithm and the comparison is done based on the space complexity that is measured by the number of generated nodes, the expanded nodes, and by the effective branching factor besides running time to measure the time complexity. The new function was the best performance since the number of the generated and expanded nodes were strictly less than the number of the two traditional heuristics. The author also introduced new criteria to compare the heuristic searches by finding the optimal branching factor b* for the three heuristics using Newton's method. The A* algorithm using Chebyshev heuristic drives to an optimal solution in a way that appears to be linear. So, he concluded that the Chebyshev heuristic dominates the Manhattan and Hamming heuristics from the space complexity point of view. Since the time complexity was compared depending on the effective branching factor, the Chebyshev function had O((b*)m) which is better than the other heuristics.

In 2018, (Gayatri, et al.) analyzed the performance of 5 different search algorithms on the 8-puzzle problem to find the shortest solution based on space complexity and the number of moves taken in the average time which are Breadth First Search, Depth First Search, A*, Hill Climbing and Steepest Ascent Hill Climbing. The authors introduced a new approach to reduce the cost without raising the time that is by combining the values for Hamming distance and Manhattan distance as a new heuristic function. The result showed that A* with the new heuristic had the best for almost all the cases and was better compared to using one heuristic.

Summary Result

A summary of the comparison of the 5 studies based on the author, in which years, the heuristic functions that are used, and number of the maximum node expanded as observed in table 1.

Table 1: Summary Comparison of the 5 Studies

Study done by	in Years	using Heuristic	Maximum node expanded
Kunkle, D. R.	2001	Combining Manhattan and one penalty move (MP)	MP = 9176
Tuccar, M.	2012	Comparing Manhattan (M) and Hamming (H) heuristic	M = 15,157 H = 1,433,773
Kuruvilla, el al.	2013	Comparing Uninformed and Informed Methods that use Manhattan heuristic (M)	From 9 generated nodes, the maximum expanded M = 4
Iordan, A. E.	2016	Comparing 3 heuristic: Chebyshev distance (C) Manhattan (M) Hamming (H)	C = 490 M = 700 H = 3,800
Gayatri, el al.	2018	Comparing Uninformed and Informed Methods that use combining of Manhattan and Hamming heuristic (MH)	MH = 127

Conclusion and Future Work

All these studies from 2001 to 2018 concluded that the A* search is the best choice to find the optimal solution to the 8-puzzle problem. In addition, using the A* search method with the combined two different heuristics is better than simply one heuristic function. From our comparison, we can see the best heuristic function with the A* method combining Manhattan and Hamming heuristics which has the minimum node expanded from one aspect of the space complexity. For future work, we need to implement these 5 heuristic functions for more realistic results.

Reference

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