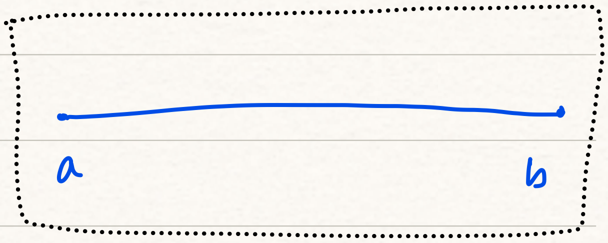
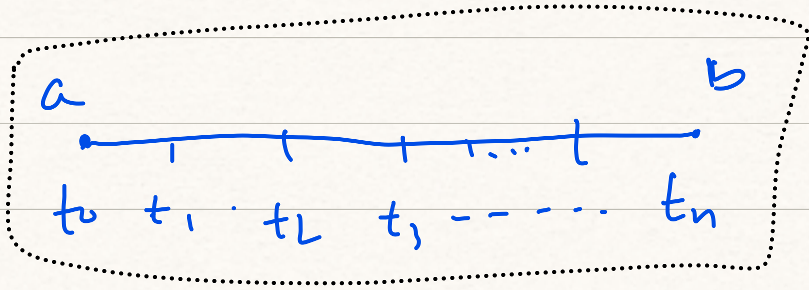
Integration

Consider the interval $[a,b]$



If we subdivide it into n subintervals we get (for example)

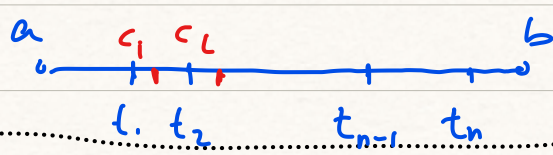


We call the increasing sequence $P = \{t_0, t_1 \ldots t_n\}$ a partition of the interval $[a,b]$

The length of the i^{th} subinterval is given by

$$\Delta t_i = t_i - t_{(i-1)}$$

Next let $c_i \in [t_i - t_{i-1}$



We then define:

Defn Reimann sum

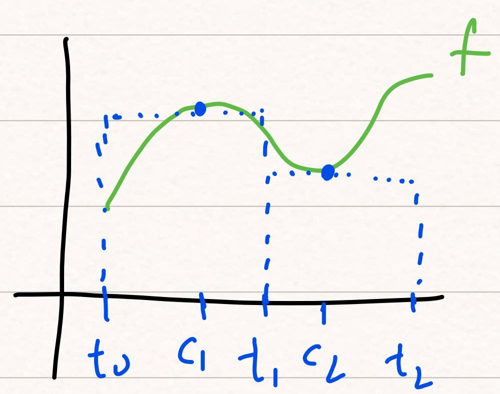
Given a bounded function f and a partition over the interval $[a,b]$ with $c_i \in [t_{i-1}, t_i]$ a Riemann sum of f w.r.t. P is

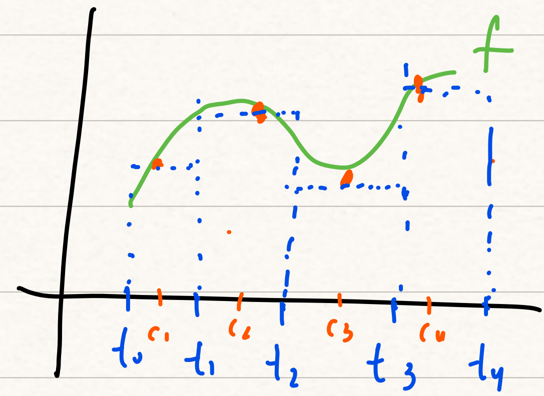
$$\boxed{S = S(f, P) = \sum_{i=1}^{n} f(c_i) \cdot \Delta t_i}$$

Notes:

- different $\boxed{\text{partition } P}$ or different choices for the $\boxed{c_i}$ will yield different values of $\boxed{S}$
- the value $\boxed{n}$ can change from one $\boxed{\text{Riemman Sum}}$ to the next

e.g.





$S = f(c_1)(t_1 - t_0) + f(c_2)(t_2 - t_1)$

$S = f(c_i) . \Delta t_i$

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Key idea

- Shrink all $\boxed{\Delta t_i}$ down to zero thus increasing the $\boxed{\text{"resolution"}}$ of the sum. We will end up with an $\boxed{\infty, 0}$ situation which will hopefully balance out to give a finite value, call it $\boxed{I}$

- if it turns out that the value of $\boxed{I}$ is independent of the $\boxed{\text{particition } P}$ and values $\boxed{c_i}$ then we say f is $\boxed{\text{integrable}}$

- we denote $\boxed{\|P\| = \max(\Delta t_1, \Delta t_2, \ldots \Delta t_n)}$ so the previous condition can be written as $\boxed{\|P\| \to 0 \text{ as } n \to \infty}$

- if our bounded function f is $\boxed{\text{integrable}}$ with value $\boxed{I}$ then we write

$$\boxed{I = \int_a^b \underbrace{f(t)}_{\text{integrand}} . \overbrace{dt}^{\text{variable of Integration}}}$$

dimits of Integration

The notation $\boxed{\int_a^b f(t).dt}$ is called the

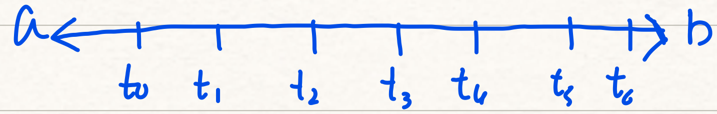
$\boxed{\text{definite integral of } f \text{ from a to b}}$

Note that $\int_a^b f(t).dt = \int_a^b f(c).dc = \int_a^b f(a).da$ etc.

$$\left[\text{think of how } \sum_{i=1}^{n} (f_i) = \sum_{K=1}^{n} f(K) . \sum_{P=1}^{n} f(P)\right.$$

Def^n The regular partition of an interval $[a,b]$ is where $\Delta t_i = \frac{b-a}{n}$ for each i.

That is, we divide $[a,b]$ into n intervals of equal width

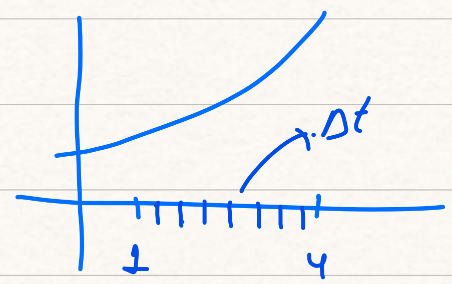


Ex. If we knew that $f(x) = e^x$ was integrable then one way to calculate its integral over $[1,4]$ would be:

$$\int_1^4 e^x . dx = \lim_{n \to \infty} \sum_{i=1}^{n} e^{c_i} \Delta t_i \quad \text{where} \quad c_i \in [t_{i-1}, t_i]$$

Assuming the regular partition we get

$$\Delta t_i = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n} = \Delta t$$



We can choose $\square$ any way we like. One common way is to build a right hand Riemman Sum (R) by letting

$$c_i = a + i\Delta t = 1 + \frac{3i}{n}$$

We could also make a

by choosing $c_i = a + (i-1)\Delta t$

$= 1 + \frac{3(i-1)}{n}$

Using , we would get

But is $f(x) = e^x$???

Thm Integrability Condition

If f is on $[a,b]$ then f is integrable on $[a,b]$.

Note: if f is bounded with finitely many

then it is also integrable

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