In [ ]:

```python
import pandas as pd
```

In [ ]:

```python
df=pd.read_csv("BTC_1sec.csv")
df
```

Out[ ]:

	Unnamed: 0	system_time	midpoint	spread	buys	
0	0	2021-04-07 11:32:42.122161+00:00	56035.995	0.01	0.000000	0
1	1	2021-04-07 11:32:43.122161+00:00	56035.995	0.01	0.000000	0
2	2	2021-04-07 11:32:44.122161+00:00	56035.995	0.01	0.000000	0
3	3	2021-04-07 11:32:45.122161+00:00	56035.995	0.01	0.000000	0
4	4	2021-04-07 11:32:46.122161+00:00	56035.995	0.01	0.000000	0
...	...	...	...	...	...	
7891	7891	2021-04-07 13:44:13.122161+00:00	56779.145	0.01	433.215830	603
7892	7892	2021-04-07 13:44:14.122161+00:00	56779.145	0.01	24649.470500	0
7893	7893	2021-04-07 13:44:15.122161+00:00	56779.145	0.01	470.983617	0
7894	7894	2021-04-07 13:44:16.122161+00:00	56769.255	11.23	0.000000	36917
7895	7895	2021-04-07 13:44:17.122161+00:00	56770.515	0.01	3582.593101	361

7896 rows × 156 columns

In [ ]:

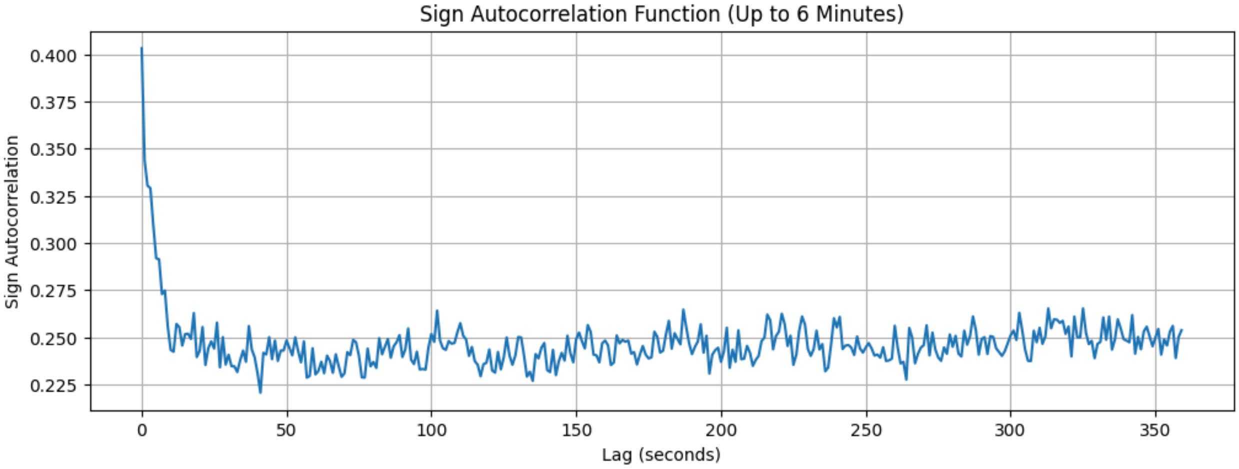
```python
import numpy as np

df["eps"] = np.sign(df["buys"] - df["sells"])
```

In [ ]:

```python
import numpy as np

def sign_autocorrelation(eps, max_lag):
    autocorr = []
    eps = eps.dropna().values
    for lag in range(1, max_lag + 1):
        corr = np.mean(eps[:-lag] * eps[lag:])
        autocorr.append(corr)
    return autocorr
acf_values=sign_autocorrelation(df['eps'], max_lag=360)
```

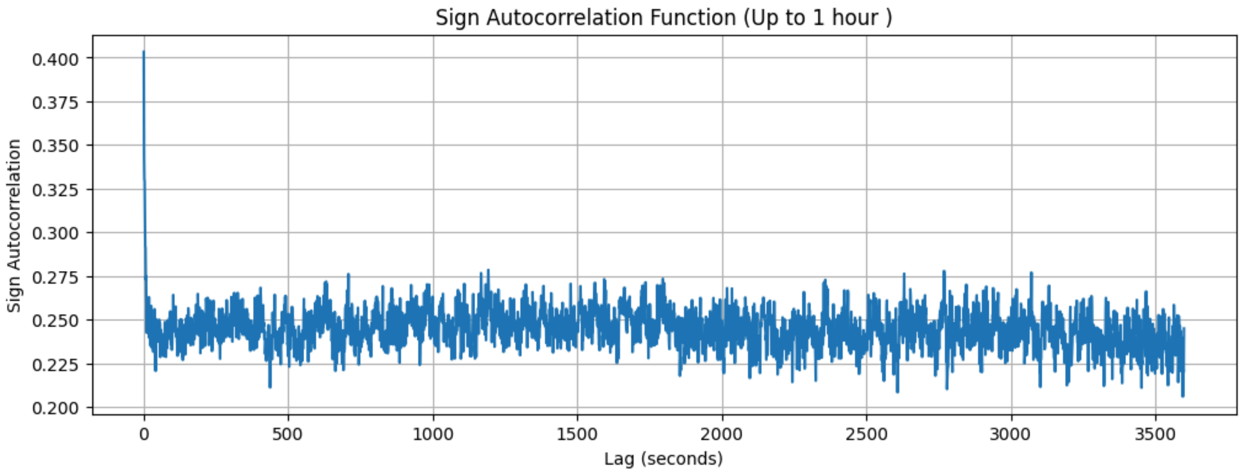
In [ ]:

```
import matplotlib.pyplot as plt

plt.figure(figsize=(12, 4))
plt.plot(acf_values)
plt.xlabel("Lag (seconds)")
plt.ylabel("Sign Autocorrelation")
plt.title("Sign Autocorrelation Function (Up to 6 Minutes)")
plt.grid(True)
plt.show()
```



In [ ]:

```
def sign_autocorrelation(eps, max_lag):
    autocorr = []
    eps = eps.dropna().values
    for lag in range(1, max_lag + 1):
        corr = np.mean(eps[:-lag] * eps[lag:])
        autocorr.append(corr)
    return autocorr
acf_values=sign_autocorrelation(df['eps'], max_lag=3600)


plt.figure(figsize=(12, 4))
plt.plot(acf_values)
plt.xlabel("Lag (seconds)")
plt.ylabel("Sign Autocorrelation")
plt.title("Sign Autocorrelation Function (Up to 1 hour )")
plt.grid(True)
plt.show()
```



In [ ]:

```
pip install nolds
```

```
Collecting nolds
  Downloading nolds-0.6.2-py2.py3-none-any.whl.metadata (7.0 kB)
Requirement already satisfied: numpy<3.0,>1.0 in /usr/local/lib/python3.11/dis
t-packages (from nolds) (2.0.2)
Requirement already satisfied: future in /usr/local/lib/python3.11/dist-package
s (from nolds) (1.0.0)
Requirement already satisfied: setuptools in /usr/local/lib/python3.11/dist-pac
kages (from nolds) (75.2.0)
Downloading nolds-0.6.2-py2.py3-none-any.whl (225 kB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 225.7/225.7 kB 5.5 MB/s eta 0:00:00
Installing collected packages: nolds
Successfully installed nolds-0.6.2
```

In [ ]:

```
import nolds
hurst = nolds.hurst_rs(df['eps'].values)
print(f"Hurst exponent: {hurst:.4f}")
```

```
Hurst exponent: 0.6295
```

In [ ]:

```
from scipy.optimize import curve_fit
import numpy as np

def power_law(k, a, alpha):
    return a * k ** (-alpha)

lags = np.arange(1, len(acf_values) + 1)
acf = np.array(acf_values)
popt, _ = curve_fit(power_law, lags[:2000], acf[:2000], p0=(1, 0.1))
a_fit, alpha_fit = popt

print(f"Fitted power-law: ρ(k) ≈ {a_fit:.4f} * k^(-{alpha_fit:.4f})")
```

```
Fitted power-law: ρ(k) ≈ 0.2494 * k^(-0.0016)
```

In [ ]:

```
df['market_order_volume'] = df['buys'] + df['sells']  # or however you estimat
df['side'] = np.where(df['buys'] > 0, 'buy', 'sell')
```

```
df['pi'] = 0
buy_aggressive = (df['side'] == 'buy') & (df['buys'] >= df['asks_notional_0'])
sell_aggressive = (df['side'] == 'sell') & (df['sells'] >= df['bids_notional_0
df.loc[buy_aggressive | sell_aggressive, 'pi'] = 1
```

In [ ]:

```
def conditional_sign_correlation(df, max_lag=100):
    eps = df['eps'].values
    pi = df['pi'].values
    results = {('MO0', 'MO0'): [], ('MO0', 'MO1'): [], ('MO1', 'MO0'): [], ('M

    for lag in range(1, max_lag + 1):
        for pi1 in [0, 1]:
            for pi2 in [0, 1]:
                valid_idx = (df.index[:-lag])[ (pi[:-lag] == pi1) & (pi[lag:]
                num = np.mean(eps[:-lag][valid_idx] * eps[lag:][valid_idx])
                p1 = np.mean(pi == pi1)
                p2 = np.mean(pi == pi2)
                denom = p1 * p2 if p1 > 0 and p2 > 0 else np.nan
                c_val = num / denom if denom else np.nan
                results[(f'MO{pi1}', f'MO{pi2}')].append(c_val)

    return results
```

In [ ]:

```
import numpy as np
import pandas as pd


df['buy_volume'] = df[[f'bids_market_notional_{i}' for i in range(15)]].sum(ax
df['sell_volume'] = df[[f'asks_market_notional_{i}' for i in range(15)]].sum(a
df['best_ask_volume'] = df['asks_notional_0']
df['best_bid_volume'] = df['bids_notional_0']
df['side'] = np.where(df['buy_volume'] > df['sell_volume'], 'buy', 'sell')
df['mo_volume'] = np.where(df['side'] == 'buy', df['buy_volume'], df['sell_vol
df['opposite_quote'] = np.where(df['side'] == 'buy', df['best_ask_volume'], df
df['pi'] = (df['mo_volume'] >= df['opposite_quote']).astype(int)

lags = np.arange(1, 3601)
results = {'C11': [], 'C10': [], 'C01': [], 'C00': []}

for lag in lags:
    for pi1, pi2, key in [(1, 1, 'C11'), (1, 0, 'C10'), (0, 1, 'C01'), (0, 0,
        mask = (df['pi'][:-lag].values == pi1) & (df['pi'][lag:].values == pi2
        if np.any(mask):
            e1 = df['eps'][:-lag].values[mask]
            e2 = df['eps'][lag:].values[mask]
            results[key].append(np.mean(e1 * e2))
        else:
            results[key].append(np.nan)

C = pd.DataFrame(results, index=lags)
```

In [ ]:

```
C
```

Out[ ]:

	C11	C10	C01	C00
1	0.468707	0.294545	0.233636	0.455386
2	0.494737	0.253033	0.231369	0.371063
3	0.414330	0.256585	0.242991	0.358178
4	0.457286	0.262899	0.257774	0.347898
5	0.404110	0.233387	0.221053	0.339810
...	...	...	...	...
3596	0.295154	0.216842	0.171617	0.201828
3597	0.231441	0.270327	0.132670	0.225794
3598	0.179487	0.284501	0.204013	0.208399
3599	0.308017	0.232162	0.146465	0.224377
3600	0.271028	0.292100	0.205835	0.234119

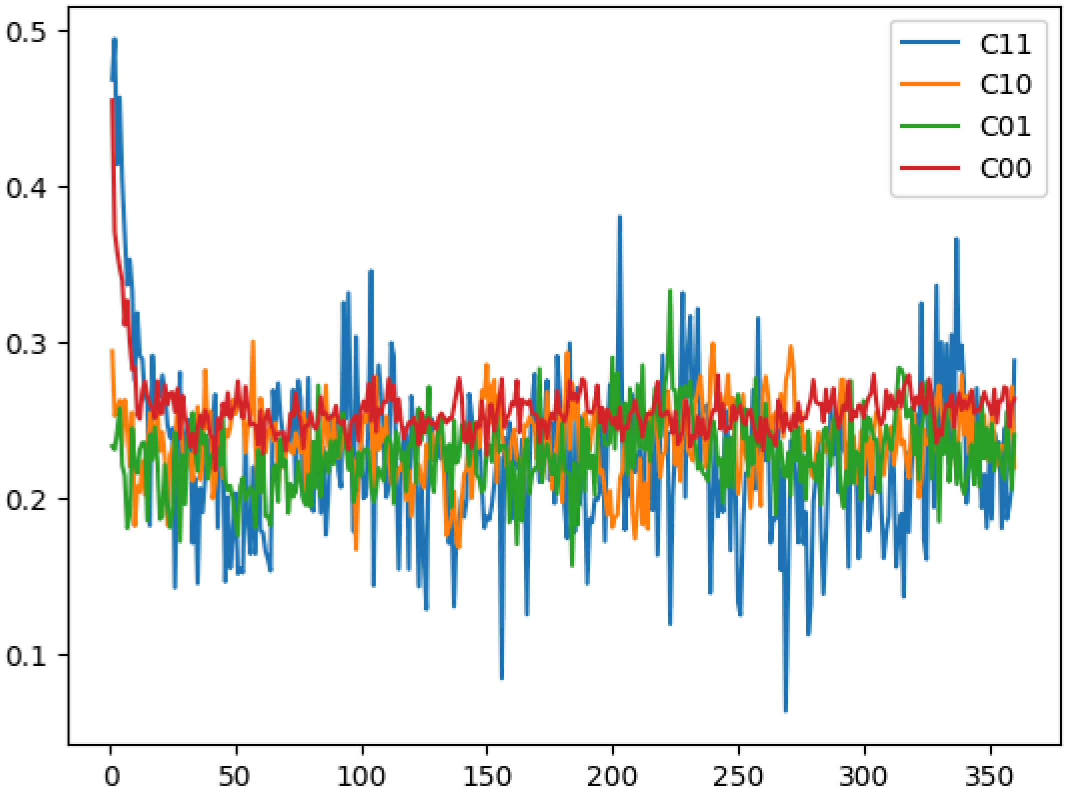
3600 rows × 4 columns

In [ ]:

```
plt.plot(C['C11'][:360], label = 'C11')
plt.plot(C['C10'][:360], label = 'C10')
plt.plot(C['C01'][:360], label = 'C01')
plt.plot(C['C00'][:360], label = 'C00')
plt.legend()
```

Out[ ]:

```
<matplotlib.legend.Legend at 0x7e22be6f9ad0>
```



In [ ]:

```
eps = df['eps'].values
cumulative_sum = np.cumsum(eps)
indices = np.arange(1, len(eps)+1)

df['osi'] = cumulative_sum / indices
```

In [ ]:

```
df
```

Out[ ]:

	Unnamed: 0	system_time	midpoint	spread	buys	
0	0	2021-04-07 11:32:42.122161+00:00	56035.995	0.01	0.000000	0
1	1	2021-04-07 11:32:43.122161+00:00	56035.995	0.01	0.000000	0
2	2	2021-04-07 11:32:44.122161+00:00	56035.995	0.01	0.000000	0
3	3	2021-04-07 11:32:45.122161+00:00	56035.995	0.01	0.000000	0
4	4	2021-04-07 11:32:46.122161+00:00	56035.995	0.01	0.000000	0
...	...	...	...	...	...	
7891	7891	2021-04-07 13:44:13.122161+00:00	56779.145	0.01	433.215830	603
7892	7892	2021-04-07 13:44:14.122161+00:00	56779.145	0.01	24649.470500	0
7893	7893	2021-04-07 13:44:15.122161+00:00	56779.145	0.01	470.983617	0
7894	7894	2021-04-07 13:44:16.122161+00:00	56769.255	11.23	0.000000	36917
7895	7895	2021-04-07 13:44:17.122161+00:00	56770.515	0.01	3582.593101	361

7896 rows × 167 columns

In [ ]:

```python
df['sign'] = df['side'].map({'buy': 1, 'sell': -1})
sign_series = df['sign'].dropna().values
```

In [ ]:

```python
def generate_dar_signs(T, K, p, seed=None):

    np.random.seed(seed)
    max_lag = len(K)
    signs = np.random.choice([-1, 1], size=max_lag).tolist()  # Initialize his

    for t in range(T):
        # Sample lag ℓ
        lag = np.random.choice(np.arange(1, max_lag+1), p=K)
        past_sign = signs[-lag]
        new_sign = past_sign if np.random.rand() < p else -past_sign
        signs.append(new_sign)

    return np.array(signs[max_lag:])
```

In [ ]:

```python
sign_series
```

```
Out[ ]: array([-1, -1, -1, ..., -1,  1,  1])
```

In [ ]:

```
from statsmodels.tsa.stattools import acf


max_lag = 3600
empirical_corr = acf(sign_series, nlags=max_lag, fft=True)
```

In [ ]:

```
def power_law_K(gamma, max_lag):
    lags = np.arange(1, max_lag+1)
    weights = lags ** (-gamma)
    return weights / weights.sum()
```

In [ ]:

```
def generate_dar_signs(T, K, p, seed=None):
    np.random.seed(seed)
    max_lag = len(K)
    signs = np.random.choice([-1, 1], size=max_lag).tolist()  # initial histor

    for _ in range(T):
        lag = np.random.choice(np.arange(1, max_lag+1), p=K)
        past_sign = signs[-lag]
        new_sign = past_sign if np.random.rand() < p else -past_sign
        signs.append(new_sign)

    return np.array(signs[max_lag:])
```

In [ ]:

```
from sklearn.metrics import mean_squared_error
import matplotlib.pyplot as plt

def fit_dar_model(empirical_corr, max_lag, gamma_range, p_range):
    best_score = np.inf
    best_params = None
    best_sim_corr = None

    for gamma in gamma_range:
        K = power_law_K(gamma, max_lag)
        for p in p_range:
            synthetic = generate_dar_signs(10000, K, p, seed=42)
            sim_corr = acf(synthetic, nlags=max_lag, fft=True)
            score = mean_squared_error(empirical_corr[1:], sim_corr[1:])  # sk
            if score < best_score:
                best_score = score
                best_params = (gamma, p)
                best_sim_corr = sim_corr

    return best_params, best_sim_corr
```

In [ ]:

```
gamma_range = np.linspace(1.2, 2.0, 10)   # power-law tail exponents
p_range = np.linspace(0.5, 0.99, 10)      # herding strength

best_params, best_sim_corr = fit_dar_model(empirical_corr, max_lag, gamma_rang
gamma_opt, p_opt = best_params
```

```
print(f"Best gamma: {gamma_opt:.2f}, Best p: {p_opt:.2f}")
```

```
Best gamma: 1.20, Best p: 0.77
```

In [ ]:

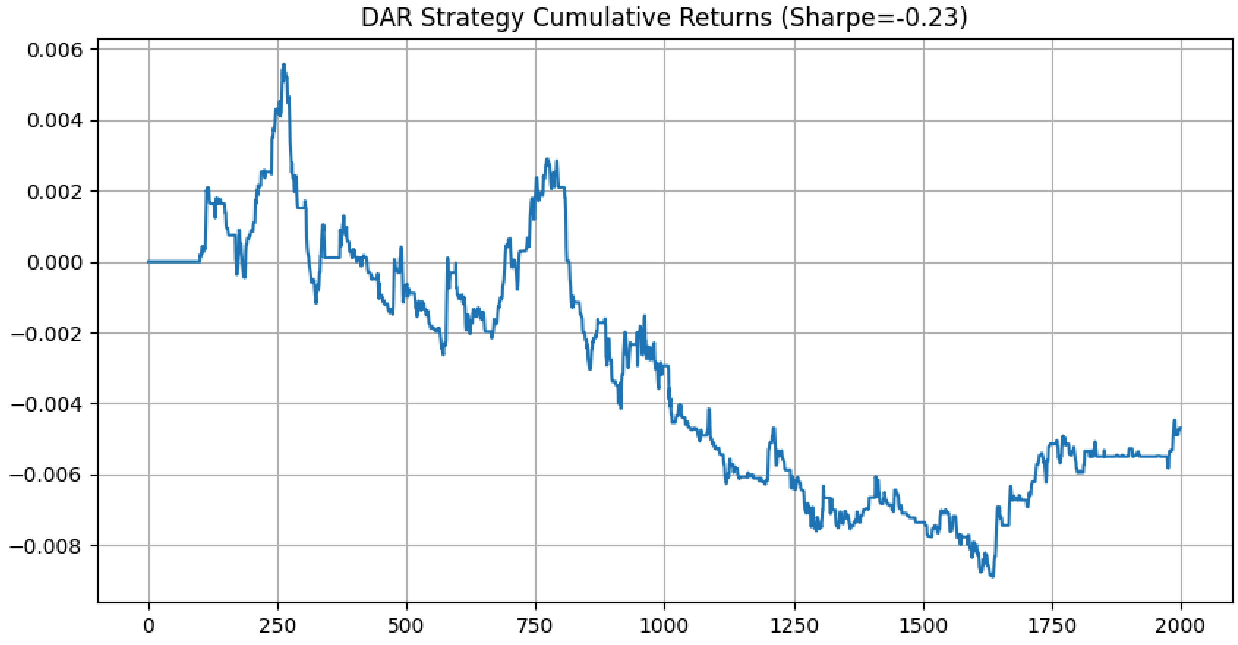
```
df = df[:2000]
```

In [ ]:

```
import numpy as np
import pandas as pd
from statsmodels.tsa.stattools import acf
from sklearn.metrics import mean_squared_error
import matplotlib.pyplot as plt


df['sign'] = df['side'].map({'buy': 1, 'sell': -1})
sign_series = df['sign'].dropna().values
max_lag = 100
empirical_corr = acf(sign_series, nlags=max_lag, fft=True)

def power_law_K(gamma, max_lag):
    lags = np.arange(1, max_lag+1)
    weights = lags ** (-gamma)
    return weights / weights.sum()

def generate_dar_signs(T, K, p, seed=None):
    np.random.seed(seed)
    max_lag = len(K)
    signs = np.random.choice([-1, 1], size=max_lag).tolist()
    for _ in range(T):
        lag = np.random.choice(np.arange(1, max_lag+1), p=K)
        past_sign = signs[-lag]
        new_sign = past_sign if np.random.rand() < p else -past_sign
        signs.append(new_sign)
    return np.array(signs[max_lag:])

def fit_dar_model(empirical_corr, max_lag, gamma_range, p_range):
    best_score = np.inf
    best_params = None
    best_sim_corr = None
    for gamma in gamma_range:
        K = power_law_K(gamma, max_lag)
        for p in p_range:
            synthetic = generate_dar_signs(10000, K, p, seed=42)
            sim_corr = acf(synthetic, nlags=max_lag, fft=True)
            score = mean_squared_error(empirical_corr[1:], sim_corr[1:])
            if score < best_score:
                best_score = score
                best_params = (gamma, p)
                best_sim_corr = sim_corr
    return best_params, best_sim_corr

gamma_range = np.linspace(1.2, 2.0, 10)
p_range = np.linspace(0.5, 0.99, 10)
best_params, best_sim_corr = fit_dar_model(empirical_corr, max_lag, gamma_rang
gamma_opt, p_opt = best_params
```

```
K_opt = power_law_K(gamma_opt, max_lag)

df['log_return'] = np.log(df['midpoint']).diff()
df['predicted_sign'] = 0

for t in range(max_lag, len(df)):
    past_signs = df['sign'].iloc[t - np.arange(1, max_lag + 1)].values
    pred = (2 * p_opt - 1) * np.dot(K_opt, past_signs)
    df.at[df.index[t], 'predicted_sign'] = np.sign(pred)

df['strategy_return'] = df['predicted_sign'].shift() * df['log_return']
df.dropna(subset=['strategy_return'], inplace=True)

sharpe = df['strategy_return'].mean() / df['strategy_return'].std() * np.sqrt(
cumulative = df['strategy_return'].cumsum()

plt.figure(figsize=(10, 5))
plt.plot(cumulative)
plt.title(f'DAR Strategy Cumulative Returns (Sharpe={sharpe:.2f})')
plt.grid()
plt.show()
```

```
/tmp/ipython-input-30-2738277585.py:8: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/sta
ble/user_guide/indexing.html#returning-a-view-versus-a-copy
  df['sign'] = df['side'].map({'buy': 1, 'sell': -1})
/tmp/ipython-input-30-2738277585.py:51: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/sta
ble/user_guide/indexing.html#returning-a-view-versus-a-copy
  df['log_return'] = np.log(df['midpoint']).diff()
/tmp/ipython-input-30-2738277585.py:52: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/sta
ble/user_guide/indexing.html#returning-a-view-versus-a-copy
  df['predicted_sign'] = 0
/tmp/ipython-input-30-2738277585.py:59: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/sta
ble/user_guide/indexing.html#returning-a-view-versus-a-copy
  df['strategy_return'] = df['predicted_sign'].shift() * df['log_return']
/tmp/ipython-input-30-2738277585.py:60: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/sta
ble/user_guide/indexing.html#returning-a-view-versus-a-copy
  df.dropna(subset=['strategy_return'], inplace=True)
```



DAR Strategy Cumulative Returns (Sharpe=-0.23)