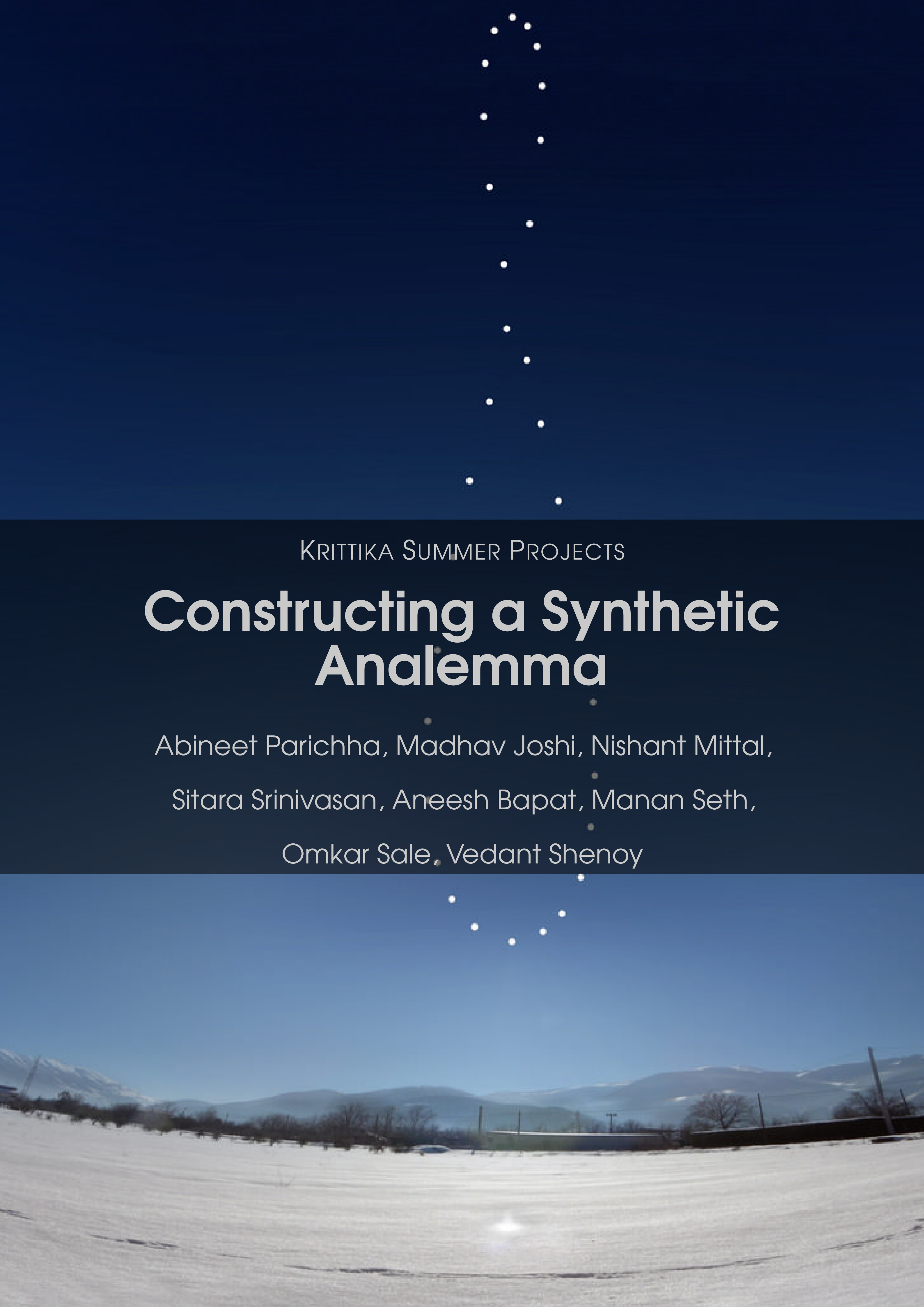

Krittika Summer Projects

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Abineet Parichha[1], Madhav Joshi[2], Nishant Mittal[2], Sitara Srinivasan[3], Aneesh Bapat[2,4], Manan Seth[2,4], Omkar Sale[2,4], and Vedant Shenoy[2,4]

[1]Indian Institute of Science Education and Research Mohali, Knowledge city, Sector 81, SAS Nagar, Manauli 140306, India

[2]Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

[3]Naxxatra Sciences and Collaborative Research, Bangalore, Karnataka, India

[4]Krittika-The Astronomy Club of IIT Bombay, Powai, Mumbai 400076, India

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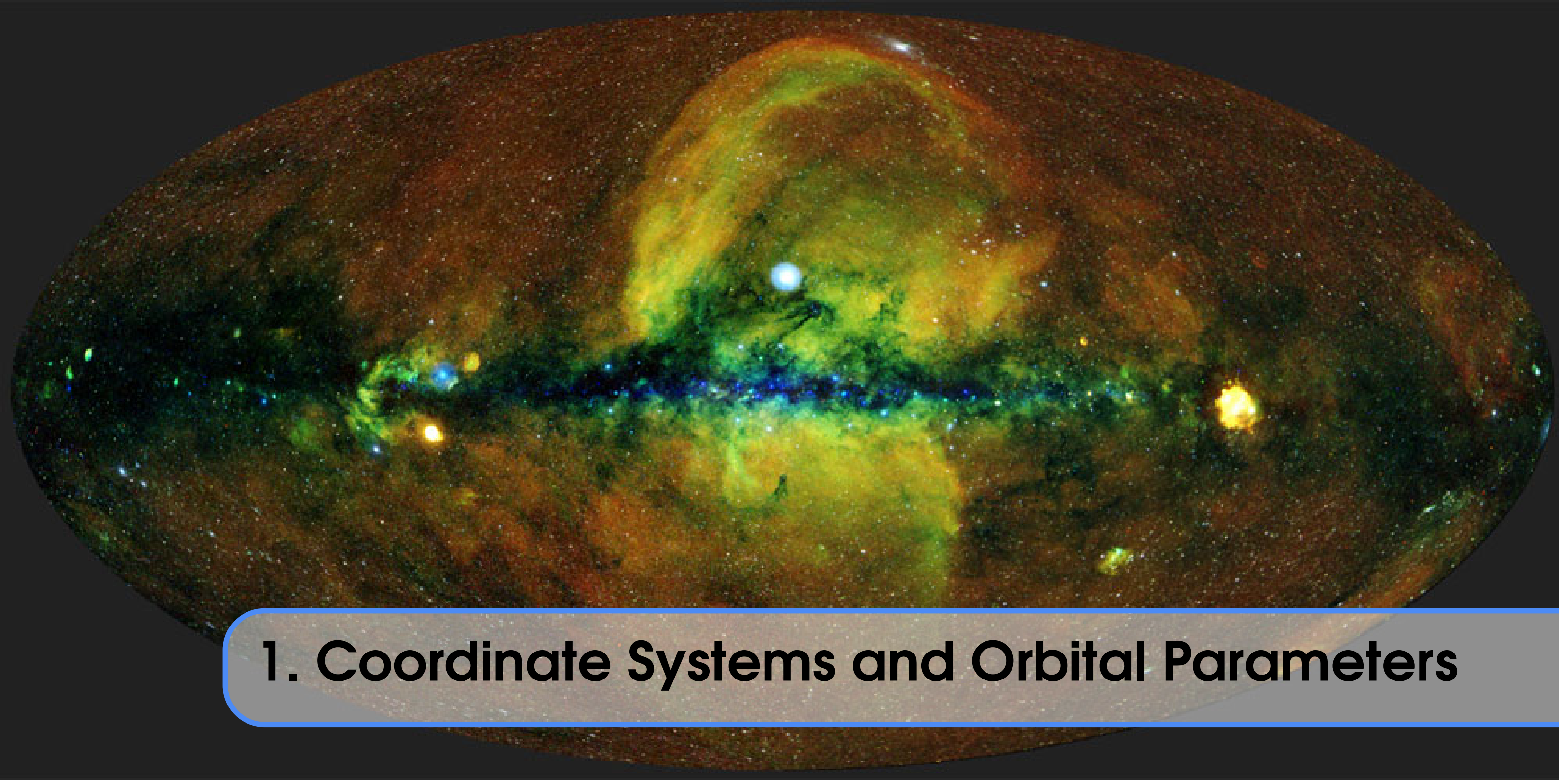


1. Coordinate Systems and Orbital Parameters

1.1 Equation of motion

In an inertial frame of reference, we have

$$\ddot{\mathbf{r}}_{earth} = \frac{Gm_{sun}}{|\mathbf{r}_{earth} - \mathbf{r}_{sun}|^3}(\mathbf{r}_{sun} - \mathbf{r}_{earth})$$

$$\ddot{\mathbf{r}}_{sun} = \frac{Gm_{earth}}{|\mathbf{r}_{earth} - \mathbf{r}_{sun}|^3}(\mathbf{r}_{earth} - \mathbf{r}_{sun})$$

Subtracting the two equations, we obtain the differential equation governing the position of earth relative to sun.

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} \tag{1.1}$$

where $\mu = G(m_{earth} + m_{sun})$ and $\mathbf{r} = \mathbf{r}_{earth} - \mathbf{r}_{sun}$.

Equation (1.1) is of 2^{nd} order and thus, can be solved provided a set of **six** arbitrary parameters, say $\mathbf{r}$ and $\dot{\mathbf{r}}$ at some time t_0. Another approach is to consider a set of parameters, called orbital parameters, which can be experimentally determined and solve equation (1.1) using them.

1.2 Some conserved quantities

1.2.1 Angular Momentum

Defined as : $\mathbf{L} = m_{earth}(\mathbf{r} \times \dot{\mathbf{r}})$. Instead, we prefer to use angular momentum per unit mass.

$$\mathbf{k} = \mathbf{r} \times \dot{\mathbf{r}} \tag{1.2}$$

Differentiating equation (1.2), we have

$$\begin{aligned}\dot{\mathbf{k}} &= (\dot{\mathbf{r}} \times \dot{\mathbf{r}}) + (\mathbf{r} \times \ddot{\mathbf{r}}) \\ &= \mathbf{r} \times (-\frac{\mu}{r^3}\mathbf{r}) && \text{substituting equation (1.1)} \\ &= \mathbf{0}\end{aligned}$$

Angular momentum of earth about the sun being conserved indicates that its trajectory is constrained to a plane, with $\mathbf{k}$ being normal to the plane. This is referred to as the orbital plane of earth.

1.2.2 Eccentricity vector

Defined as :

$$\mathbf{e} = -\frac{1}{\mu}(\mathbf{k} \times \dot{\mathbf{r}} + \mu\frac{\mathbf{r}}{r}) \tag{1.3}$$

Differentiating equation (1.3), we have

$$\begin{aligned}-\mu\dot{\mathbf{e}} &= \dot{\mathbf{k}} \times \dot{\mathbf{r}} + \mathbf{k} \times \ddot{\mathbf{r}} + \mu(\frac{\dot{\mathbf{r}}}{r} - \frac{\dot{r}\mathbf{r}}{r^2}) && \because \dot{\mathbf{k}} = \vec{0} \\ &= \mathbf{k} \times (-\frac{\mu}{r^3}\mathbf{r}) + \mu(\frac{\dot{\mathbf{r}}}{r} - \frac{\dot{r}\mathbf{r}}{r^2}) && \text{substituting equation (1.1)} \\ &= -\frac{\mu}{r^3}[(\mathbf{r} \times \dot{\mathbf{r}}) \times \mathbf{r}] + \mu(\frac{\dot{\mathbf{r}}}{r} - \frac{\dot{r}\mathbf{r}}{r^2}) && \text{substituting equation (1.2)} \\ &= \frac{\mu}{r^3}[\mathbf{r}(\mathbf{r} \cdot \dot{\mathbf{r}}) - r^2\dot{\mathbf{r}}] + \mu(\frac{\dot{\mathbf{r}}}{r} - \frac{\dot{r}\mathbf{r}}{r^2}) \\ r\dot{r} &= r\frac{d}{dt}(\mathbf{r} \cdot \mathbf{r})^{0.5} = r(\frac{1}{2r})(2\mathbf{r} \cdot \dot{\mathbf{r}}) = \mathbf{r} \cdot \dot{\mathbf{r}} \\ \implies -\mu\dot{\mathbf{e}} &= \mu(\frac{\dot{r}\mathbf{r}}{r^2} - \frac{\dot{\mathbf{r}}}{r}) + \mu(\frac{\dot{\mathbf{r}}}{r} - \frac{\dot{r}\mathbf{r}}{r^2}) \\ &= \vec{0}\end{aligned}$$

Thus, the vector $\mathbf{e}$ is also conserved. From the definitions, it follows that

$$\mathbf{k} \cdot \mathbf{e} = -\frac{1}{\mu}[\mathbf{k} \cdot (\mathbf{k} \times \dot{\mathbf{r}}) + \frac{\mu}{r}(\mathbf{k} \cdot \mathbf{r})] = \vec{0}$$

Thus, $\mathbf{e} \perp \mathbf{k}$ and lies in the orbital plane of earth. In section 1.3, we shall see that the trajectory of earth is actually an ellipse with $e = |\mathbf{e}|$ as its eccentricity and $\mathbf{e}$ points towards the perihelion.

1.2.3 Energy Integral

Integrating equation (1.1), we have

$$\begin{aligned}&\int \ddot{\mathbf{r}} \cdot d\mathbf{r} = -\int \frac{\mu}{r^3}\mathbf{r} \cdot d\mathbf{r} \\ \implies &\int_a^{\dot{r}} \dot{r}d\dot{r} = -\int_\infty^r \frac{\mu}{r^2}dr \\ \implies &\frac{\dot{r}^2 - a^2}{2} = \frac{\mu}{r} \\ \implies &\frac{d}{dt}(\frac{\dot{r}^2}{2} - \frac{\mu}{r}) = 0\end{aligned}$$

We define the energy integral h as :

$$h = \frac{\dot{r}^2}{2} - \frac{\mu}{r} \tag{1.4}$$

The energy integral h can also be obtained from k and e.
From equation (1.3),

$$\begin{aligned}
|\mathbf{e}|^2 &= \left| -\frac{1}{\mu}(\mathbf{k} \times \dot{\mathbf{r}} + \mu \frac{\mathbf{r}}{r}) \right|^2 \\
\implies \mu^2 e^2 &= |\mathbf{k} \times \dot{\mathbf{r}}|^2 + \mu^2 + \frac{2\mu}{r}[\mathbf{r} \cdot (\mathbf{k} \times \dot{\mathbf{r}})] \\
\implies \mu^2(e^2 - 1) &= k^2 \dot{r}^2 + \frac{2\mu}{r}[\mathbf{k} \cdot (\dot{\mathbf{r}} \times \mathbf{r})] && \because \mathbf{k} \perp \dot{\mathbf{r}} \implies |\mathbf{k} \times \dot{\mathbf{r}}| = k\dot{r} \\
\implies \mu^2(e^2 - 1) &= 2k^2 \left(\frac{\dot{r}^2}{2} - \frac{\mu}{r} \right) && \text{substituting equation (1.2)}
\end{aligned}$$

Substituting equation (1.4),

$$\mu^2(e^2 - 1) = 2hk^2 \tag{1.5}$$

In section 1.3, we shall see that the value of h determines whether the earth-sun system is a bound system.

1.3 Solving for orbit

To obtain the trajectory of earth around the sun, we compute

$$\begin{aligned}
\mathbf{r} \cdot \mathbf{e} &= -\frac{1}{\mu}[\mathbf{r} \cdot (\mathbf{k} \times \dot{\mathbf{r}}) + \mu r] && \text{substituting equation (1.3)} \\
\implies re\cos f &= -\frac{1}{\mu}[\mathbf{k} \cdot (\dot{\mathbf{r}} \times \mathbf{r})] - r \\
\implies r(1 + e\cos f) &= \frac{k^2}{\mu} && \text{substituting equation (1.2)} \\
\implies r &= \frac{k^2/\mu}{1 + e\cos f} && (1.6)
\end{aligned}$$

In equation (1.6), f refers to the angle measured from $\mathbf{e}$ to $\mathbf{r}$. It is called the "true anomaly". We observe that the equation for trajectory of earth is that of a conic section in terms of polar coordinates (r, f) with e as eccentricity. The perihelion is

Eccentricity	Energy Integral	System	Trajectory
$e = 0$	$h < 0$	Bound	Circle
$1 > e > 0$	$h < 0$	Bound	Ellipse
$e = 1$	$h = 0$	Unbound	Parabola
$e > 1$	$h > 0$	Unbound	Hyperbola

Table 1.1: *The earth-sun being a bound system, $e < 1$ and orbit elliptical.*

defined as the point of closest approach between earth and sun. From equation (1.6), we see that r is minimum for $f = 0$. Hence, $\mathbf{e}$ points towards the perihelion.

$$r_{min} = \frac{k^2/\mu}{1 + e} \tag{1.7}$$

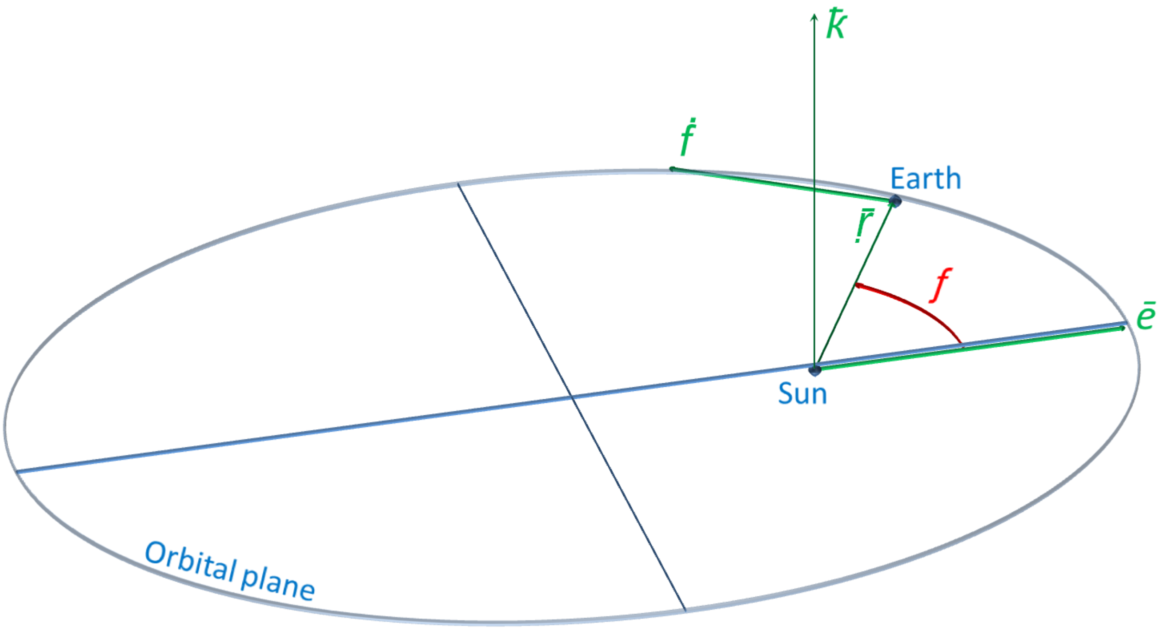


Figure 1.1: Earth's orbit around the sun

1.4 Orbital Parameters

The six orbital parameters are :

- eccentricity e
- semi-major axis a
- inclination of orbital plane i, w.r.t a reference plane
- argument of perihelion ω
- longitude of ascending node Ω, w.r.t a reference axis
- time of perihelion τ

Equation (1.6) merely provides the trajectory of the earth in terms of e and k. Semi-major axis a can be substituted for k (see section 1.5. The next three parameters(i, ω, Ω) specify the orientation of the orbital plane w.r.t a reference plane and an axis. Ascending node is defined as the vector pertaining to the line of intersection of orbital plane and reference plane. The argument of perihelion ω The angle measured from ascending node and $\mathbf{e}$ (direction of perihelion). The longitude of ascending node Ω is the angle measured from reference axis to the ascending node. These five parameters$(e, a, i, \omega, \Omega)$ describe complete geometry of the orbit. See figure 1.2

To obtain position of an object in its orbit as a function of time, we need to solve for the true anomaly $f(t)$ (see section 1.6), where time of perihelion$(f = 0)$ τ is used as a limit.

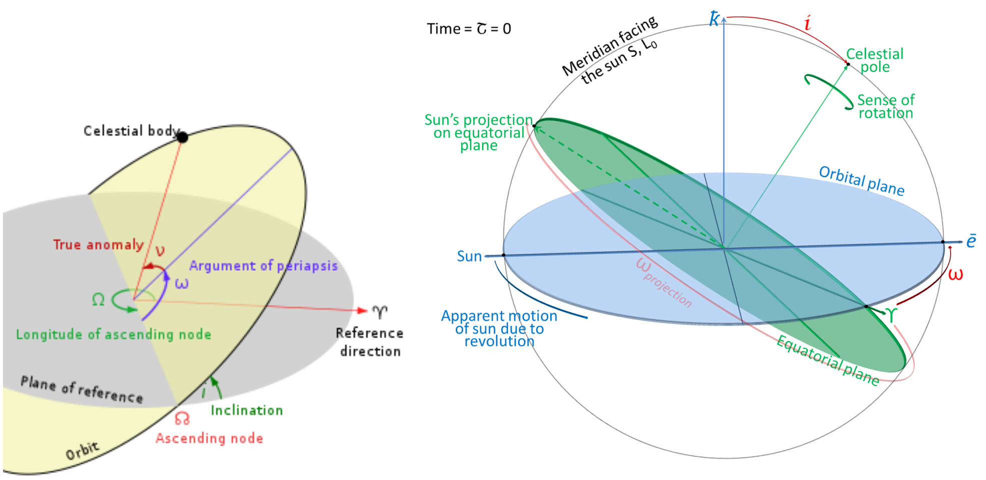


Figure 1.2: (i)Orbital elements with sun at origin (ii)Earth at the origin

For the earth-sun system,

- $e = 0.0167086$
- $a = 1.000001018$ AU $= 149.60 \times 10^6$ km
- The equatorial plane of earth is taken to be the reference plane. Thus, the inclination i is same as the *obliquity of the ecliptic* of earth, roughly $23.46°$
- Ascending node is called *Vernal Equinox* ♈(also called the first point of Aries). The reference axis is the Vernal equinox itself.
  $\omega = 85.901°$
- $\Omega = 0°$
- $\tau = 0$ sec

1.5 Semi-major axis and Period

From the figure, it is clear that

$$r_{min} = a - ae$$

$$\implies \frac{k^2/\mu}{1+e} = a(1-e) \qquad \text{substituting equation (1.7)}$$

$$\implies k = +\sqrt{\mu a(1-e^2)} \tag{1.8}$$

Equation (1.8) can, thus, be used to replace k with a in equation (1.6).

$$\text{The rate of area swept } \dot{A} = \frac{1}{2}|\mathbf{r} \times \dot{\mathbf{r}}|$$

$$= k/2 \qquad \text{substituting equation (1.2)}$$

$$= \frac{1}{2}\sqrt{\mu a(1-e^2)} \qquad \text{substituting equation (1.8)} \tag{1.9}$$

Therefore, equal areas are swept in equal intervals of time $[\because \dot{\mathbf{k}} = \vec{0}]$

$$\begin{aligned}
\text{Area of an ellipse} &= \pi ab = \pi a^2\sqrt{1-e^2} \\
\text{Period of revolution } P &= \frac{\pi a^2\sqrt{1-e^2}}{\dot{A}} \\
&= \frac{2\pi}{\sqrt{\mu}}a^{3/2} \qquad \text{substituting equation (1.9)} \qquad (1.10)
\end{aligned}$$

Hence, period of revolution P depends only on semi-major axis a (independent of e).

1.6 Solving for Anomaly

Consider the motion of earth in cylindrical coordinates with $(\hat{\mathbf{r}}, \hat{\mathbf{f}}, \hat{\mathbf{k}})$ as the basis, the orbital plane as the X-Y plane, f measured from $\mathbf{e}$, $\mathbf{k}$ as Z axis and sun as the origin. Refer figure 1.1

$$\begin{aligned}
\mathbf{r} &= r\hat{\mathbf{r}} \\
\dot{\mathbf{r}} &= \dot{r}\hat{\mathbf{r}} + r\dot{\hat{\mathbf{r}}} = \dot{r}\hat{\mathbf{r}} + r\dot{f}\hat{\mathbf{f}} \\
\mathbf{k} &= \mathbf{r}\times\dot{\mathbf{r}} = r^2\dot{f}\hat{\mathbf{k}} \\
\implies \dot{f} &= k/r^2 = \frac{\mu^2(1+e\cos f)^2}{k^3} \qquad \text{substituting equation (1.6)} \\
&= \frac{\mu^{1/2}(1+e\cos f)^2}{a^{3/2}(1-e^2)^{3/2}} \qquad \text{substituting equation (1.8)}
\end{aligned}$$

$$\implies \int_0^f \frac{(1-e^2)^{3/2}}{(1+e\cos f)^2}df = \int_\tau^t \frac{2\pi}{P}dt \qquad \text{substituting equation (1.10)} \qquad (1.11)$$

One way to obtain $f(t)$ from equation (1.11) is to integrate it numerically. Another way is to reduce this equation by substituting with mean M and eccentric E anomalies and solve.

The mean anomaly is defined as :

$$M = \frac{2\pi}{P}(t-\tau) \qquad (1.12)$$

Placing $e = 0$ in equation (1.11), we see that the mean anomaly can be thought of as the true anomaly of earth if it were in an orbit with eccentricity e equal to 0 (i.e. circular orbit centred at the sun) and same period P (which necessitates same value of semi-major axis a, see equation (1.10)). Also,

$$\langle\dot{f}\rangle = \langle\dot{M}\rangle = \frac{2\pi}{P}$$

The eccentric anomaly E is defined as :

Consider a circle centred at the centre C of the elliptical orbit of earth with radius equal to the semi-major axis a and a perpendicular PP′, to $\mathbf{e}$, passing through a point P on the orbit with true anomaly f. The angle subtended by the arc, between the intercept P′ made by the perpendicular on the circle and the perihelion, on the centre is called the eccentric anomaly E corresponding to the true anomaly f. From the figure 1.3,

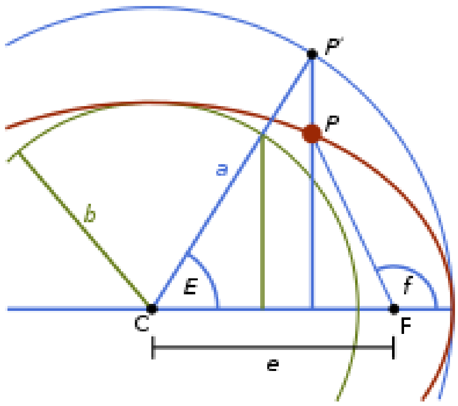


Figure 1.3: Eccentric anomaly

$$\begin{aligned}
& CP' \cos E = CF + r\cos f \\
& a\cos E = ae + r\cos f \\
\implies & \cos E = e + \frac{(1-e^2)\cos f}{1+e\cos f} && \text{substituting equations (1.6) and (1.8)} \\
\implies & \cos E = \frac{e+\cos f}{1+e\cos f} && (1.13) \\
\implies & \sin E = 1-(\cos E)^2 = \frac{\sqrt{1-e^2}\sin f}{1+e\cos f} && (1.14)
\end{aligned}$$

The inverse transformations are

$$\cos f = \frac{\cos E - e}{1-e\cos E} \qquad (1.15)$$

$$\sin f = \frac{\sqrt{1-e^2}\sin E}{1-e\cos E} \qquad (1.16)$$

The following relations are, thus, obtained

$$dE = \frac{\sqrt{1-e^2}}{1+e\cos f}df \qquad \text{diff. eq (1.13) and subst. eq (1.14)} \qquad (1.17)$$

$$\frac{1-e^2}{1+e\cos f} = 1-e\sin E \qquad \text{substituting equation (1.15)} \qquad (1.18)$$

Now, we substitute equations (1.17) and (1.18) in the LHS, and equation (1.12) in the RHS of equation (1.11) to obtain

$$\begin{aligned}
& \int_0^E (1-e\sin E)dE = M \\
\implies & M = E - e\sin E && (1.19)
\end{aligned}$$

Equation (1.19) is called Kepler's equation. Given an arbitrary time t, we can obtain the mean anomaly M from eq (1.12), find the root of Kepler's equation i.e. E for that M and finally, the true anomaly f from equations (1.15) and (1.16).

Therefore, the equations (1.12), (1.19), (1.15), (1.16), (1.6) and the orbital parameters, all together give the position of earth about the sun as a function of time.

1.7 Analytical solutions

To better study the behaviour of the variables and properties associated with the analemma, it is useful to obtain the true anomaly f in an algebraic expression involving mean anomaly M (substitute for time t) and eccentricity e. Beginning with equation (1.19), we see that $E-M$ is a periodic function with a period of 2π. It can thus be expanded into a Fourier series.

$$
\begin{aligned}
E-M &= \sum_{n=1}^{\infty} a_n \sin(n\frac{2\pi}{L}M) && L = period = 2\pi \\
&= \sum_{n=1}^{\infty} a_n \sin(nM) && (1.20) \\
\text{where } a_n &= \frac{2}{L}\int_{-\pi}^{+\pi}(E-M)\sin(nM)dM \\
&= \frac{2}{\pi}\int_{0}^{+\pi} e\sin(E)\sin(nM)dM && \text{subst. eq. (1.19)} \\
&= \frac{2}{\pi}\Big[e\sin(E)(-\frac{\cos(nM)}{n})\Big|_{0}^{+\pi} && E_{M=0}=0 \\
&\quad + \frac{1}{n}\int_{0}^{+\pi}\frac{\partial e\sin(E)}{\partial M}\cos(nM)dM\Big] && E_{M=\pi}=\pi \\
&= \frac{2}{n\pi}\int_{0}^{+\pi}\cos(nM)(dE-dM) && \text{diff. eq. (1.19)} \\
&= \frac{2}{n\pi}\Big[\int_{0}^{+\pi}\cos(nM)dE - \int_{0}^{+\pi}\cos(nM)dE\Big] \\
&= \frac{2}{n\pi}\Big[\int_{0}^{+\pi}\cos(nM)dE - \sin(nM)\Big|_{0}^{+\pi}\Big] \\
&= \frac{2}{n\pi}\int_{0}^{+\pi}\cos[nE-ne\sin(E)]dE && \text{subst. eq. (1.19)} \\
&= \frac{2}{n}J_n(ne),\ J = \text{Bessel function of 1st order} && (1.21)
\end{aligned}
$$

Substituting equation (1.21) in (1.20),

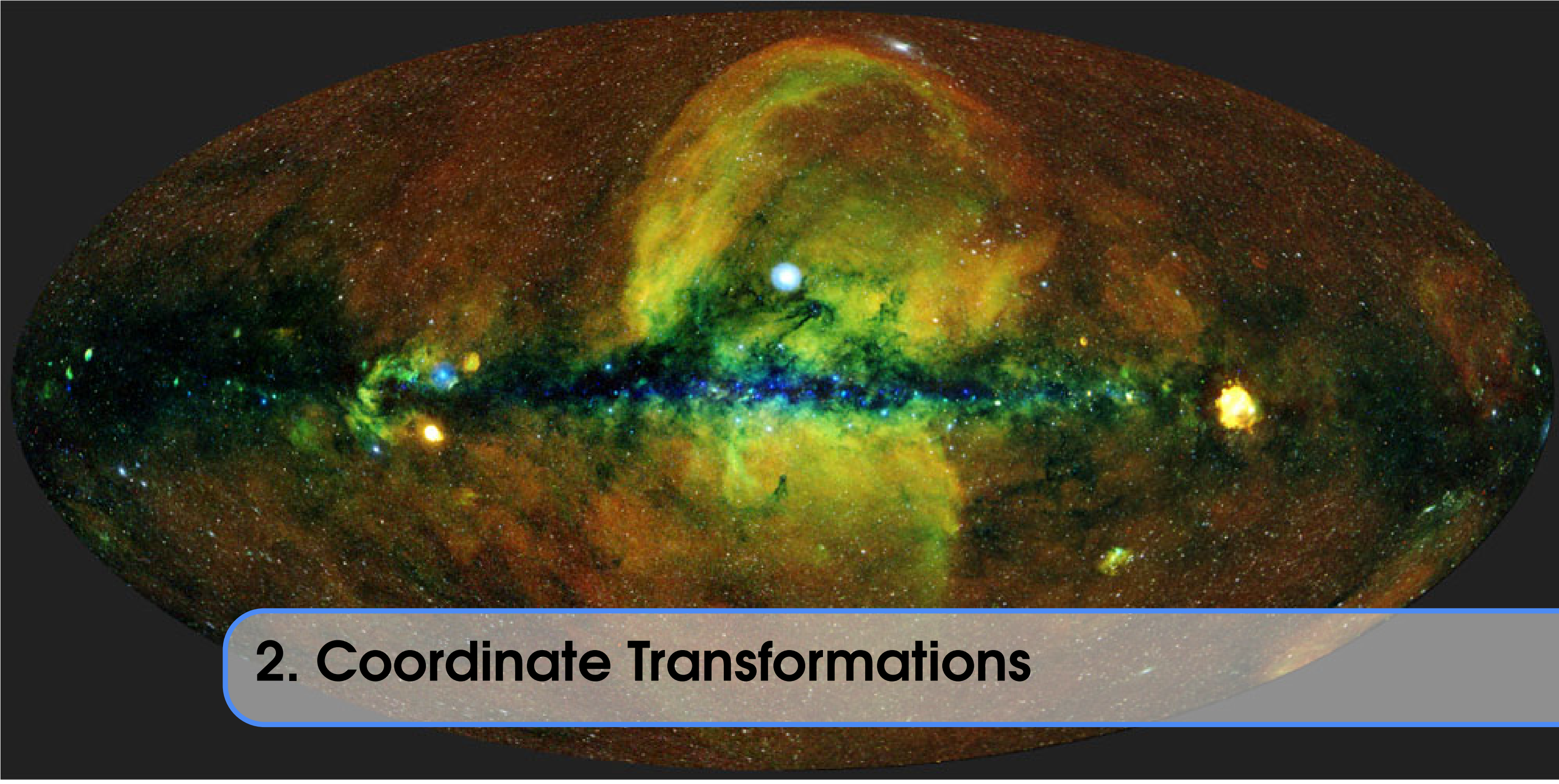
$$
\begin{aligned}
E &= M + \sum_{n=1}^{\infty}\frac{2}{n}J_n(ne)\sin(nM) \\
&= M + (e-\frac{e^3}{8})\sin M + \frac{e^2}{2}\sin 2M + \frac{3e^3}{8}\sin 3M + o(e^4) && (1.22)
\end{aligned}
$$

Expanding equation (1.15) into a Taylor series, we have

$$
f = E + (e+\frac{e^3}{4})\sin E + \frac{e^2}{4}\sin 2E + \frac{e^3}{12}\sin 3E + o(e^4) \qquad (1.23)
$$

Substituting equation (1.22) in (1.23), we have the equation of centre $f-M$

$$f-M=(2e-\frac{e^3}{4})\sin M+\frac{5e^2}{4}\sin 2M+\frac{13e^2}{12}\sin 3M+o(e^4) \tag{1.24}$$



2. Coordinate Transformations

2.1 Changing Coordinate axes

First let's see how to convert 3D Cartesian coordinate system with unit vectors along the axes $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ to some other system with with unit vectors along the axes $\hat{\imath}'$, $\hat{\jmath}'$, $\hat{k}'$.

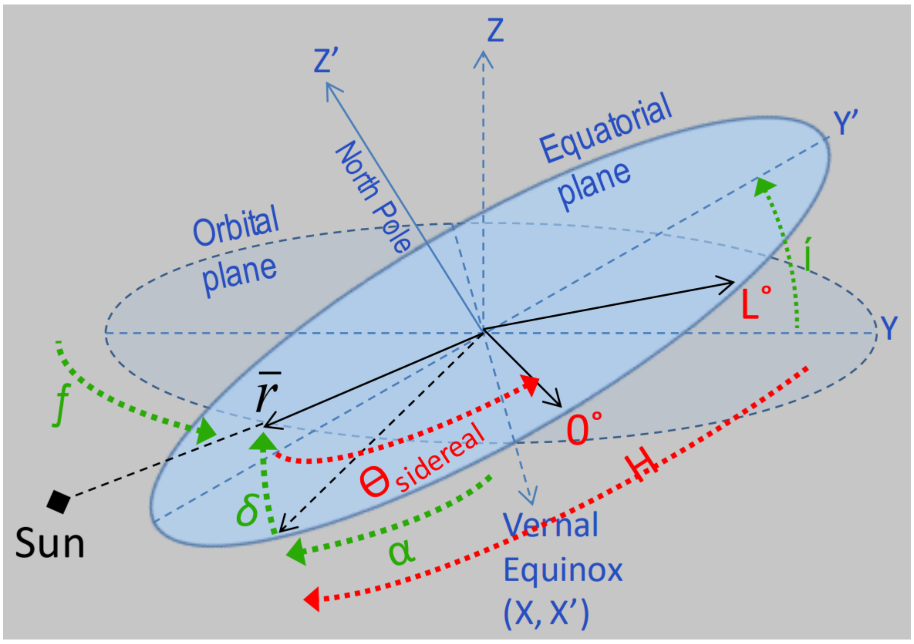
Theorem 2.1.1 — Coordinate Conversion. Say we have a vector $\vec{\mathbf{r}} = \mathbf{x}\hat{\imath} + \mathbf{y}\hat{\jmath} + \mathbf{z}\hat{k}$ in Cartesian system. For converting to a system with unit vectors $\hat{\imath}'$, $\hat{\jmath}'$ and $\hat{k}'$, we find $\mathbf{x}'$, $\mathbf{y}'$ and $\mathbf{z}'$ as:

$$\mathbf{x}' = \vec{\mathbf{r}} \cdot \hat{\imath}' \quad \mathbf{y}' = \vec{\mathbf{r}} \cdot \hat{\jmath}' \quad \mathbf{z}' = \vec{\mathbf{r}} \cdot \hat{k}' \quad \text{hence,} \tag{2.1}$$

$$\begin{pmatrix} \mathbf{x}' \\ \mathbf{y}' \\ \mathbf{z}' \end{pmatrix} = \begin{pmatrix} \hat{\imath}\cdot\hat{\imath}' & \hat{\jmath}\cdot\hat{\imath}' & \hat{k}\cdot\hat{\imath}' \\ \hat{\imath}\cdot\hat{\jmath}' & \hat{\jmath}\cdot\hat{\jmath}' & \hat{k}\cdot\hat{\jmath}' \\ \hat{\imath}\cdot\hat{k}' & \hat{\jmath}\cdot\hat{k}' & \hat{k}\cdot\hat{k}' \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{pmatrix} \tag{2.2}$$

2.2 RA, DEC and HA

Given true anomaly (f) of sun, argument of periapsis (ω = angle between Vernal equinox and eccentricity vector i.e. a vector from sun along periapsis) and inclination of the planet's rotational axis (i), we know the position of sun with respect to the orbital plane as the X-Y plane and Vernal equinox as X-axis and Z-axis facing up.
For now we focus on green values in the figure below.



$$\mathbf{x} = \sin f \quad \mathbf{y} = -\cos f \quad \mathbf{z} = 0$$

For simplicity it is assumed that $\omega = \frac{\pi}{2}$. For some other value of ω just replace f with $f - (\frac{\pi}{2} - \omega) = f + \omega - \frac{\pi}{2}$. Then $\mathbf{x} = -\cos(f+\omega), \quad \mathbf{y} = \sin(f+\omega), \quad \mathbf{z} = 0.$

Using **Theorem 3.1.1** we find the position of sun with respect to equatorial plane as X′-Y′ plane just by tilting the original X-Y plane about X-axis by an angle equal to the angle of inclination (i). Therefore:

$$\begin{pmatrix} \hat{i}\cdot\hat{i}' & \hat{j}\cdot\hat{i}' & \hat{k}\cdot\hat{i}' \\ \hat{i}\cdot\hat{j}' & \hat{j}\cdot\hat{j}' & \hat{k}\cdot\hat{j}' \\ \hat{i}\cdot\hat{k}' & \hat{j}\cdot\hat{k}' & \hat{k}\cdot\hat{k}' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix}$$

From the figure, considering the radius to be 1 unit, we can calculate the components of $\mathbf{r}$ along $\mathbf{x}', \mathbf{y}', \mathbf{z}'$ in terms of declination(δ) and right ascension(α). Also, we reiterate the values of $\mathbf{x}, \mathbf{y}, \mathbf{z}$ from above.

$$\begin{pmatrix} \mathbf{x}' \\ \mathbf{y}' \\ \mathbf{z}' \end{pmatrix} = \begin{pmatrix} \cos\delta\cos\alpha \\ -\cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \quad \textit{and} \quad \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{pmatrix} = \begin{pmatrix} \sin f \\ -\cos f \\ 0 \end{pmatrix}$$

Substituting above values in **Theorem 3.1.1**:

$$\begin{pmatrix} \cos\delta\cos\alpha \\ -\cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix} \begin{pmatrix} \sin f \\ -\cos f \\ 0 \end{pmatrix}$$

From the above equation we can determine the values of δ and α. Be careful about the range of δ and α.

$$\delta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \quad \textit{and} \quad \alpha \in [0, 2\pi)$$

Generalising the above equation by replacing f with $f+\omega-\frac{\pi}{2}$ as was explained below the figure, we have:

$$\begin{pmatrix} \cos\delta\cos\alpha \\ -\cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix} \begin{pmatrix} -\cos(f+\omega) \\ \sin(f+\omega) \\ 0 \end{pmatrix}$$

Now looking at the red values in the above figure.
H is hour angle
L° is observer's longitude
0° is that longitude on the equator which is facing the sun or closest to the sun when Earth was at periapsis for that particular year.
$\theta_{sidereal}$ is the angle between the periapsis and the reference 0° along equatorial plane.

$$\theta_{sidereal} = \frac{2\pi}{T_{sidereal}} t$$

where $T_{sidereal}$ is the period of sidereal day and t is time for which $\theta_{sidereal}$ is required. After converting longitude from degrees (L°) to radians (L), we find Hour Angle (H) by seeing in the figure as:

$$H = \left(\theta_{sidereal} + L - \left(\frac{\pi}{2} - \alpha\right)\right) \mod 2\pi, \quad where \quad \alpha = Right\,Ascension$$

The $\frac{\pi}{2}-\alpha$ in the above relation is $2\pi-\alpha-\pi-\omega_{projection} = \pi-(\alpha+\omega_{projection}$ where $\omega_{projection}$ is the ω projected on the equatorial plane. It can be calculated by using spherical trigonometry cosine rule. The result taking into account the quadrant of $\omega_{projection}$ is as follows:

$$\omega_{projection} = \begin{cases} \left(\tan^{-1}\left(\tan\omega\cos i\right)\right) + \pi, & \text{if } \frac{\pi}{2} < \omega < \frac{3\pi}{2} \\ \left(\tan^{-1}\left(\tan\omega\cos i\right)\right) \mod 2\pi, & \text{otherwise} \end{cases}$$

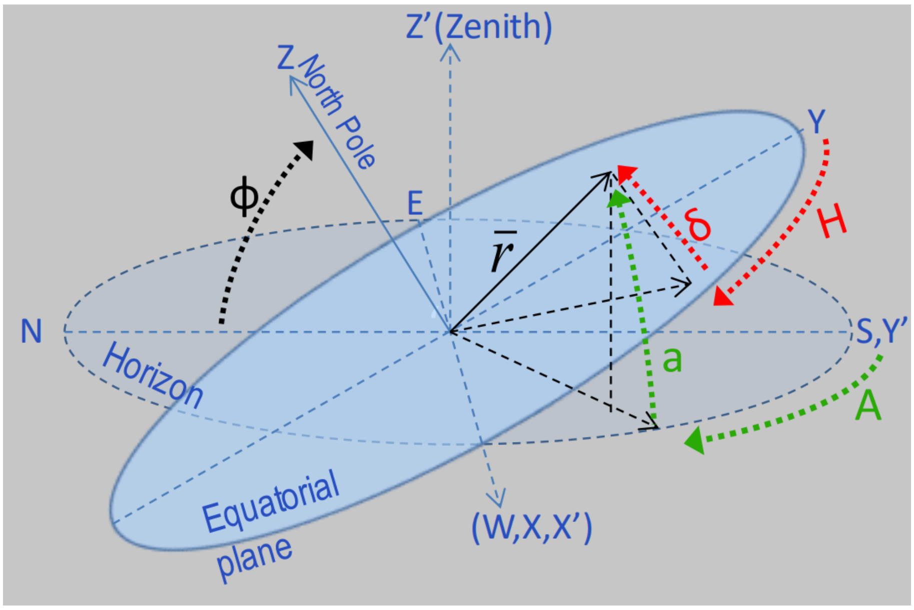
For the point with longitude L_0° on the equator closest to sun at periapsis, replace L with $(L-L_0)$ i.e. relative longitude.
Hence generalising the Hour angle formula by replacing the above mentioned becomes:

$$H = (\theta_{sidereal} + (L-L_0) + (\omega_{projection} + \alpha) - \pi) \mod 2\pi, \quad where \quad \alpha = Right\,Ascension$$

2.3 ALT and AZ

From the previous section, we know the Hour Angle (H) and Declination (δ) which are with respect to the equatorial plane. So now we take the equatorial plane as the X-Y plane with West direction as the X-axis and the North Celestial pole as the Z-axis.

We now want to calculate the location with respect to the Horizon as X'-Y' plane and West as X'-axis and Zenith as Z'-axis as shown in the figure below.



Where a is the Altitude, A is the Azimuth and ϕ is the latitude of the observer. So we have cartesian coordinates $(\mathbf{x},\mathbf{y},\mathbf{z})$ in terms of δ and H as:

$$\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\\ \mathbf{z}\end{pmatrix} = \begin{pmatrix}\cos\delta\cos H\\ \cos\delta\sin H\\ \sin\delta\end{pmatrix}$$

And the cartesian coordinates $(\mathbf{x}',\mathbf{y}',\mathbf{z}')$ in terms of Altitude (a) and Azimuth (A) as:

$$\begin{pmatrix}\mathbf{x}'\\ \mathbf{y}'\\ \mathbf{z}'\end{pmatrix} = \begin{pmatrix}\cos a\cos A\\ \cos a\sin A\\ \sin a\end{pmatrix}$$

Also by looking at the figure we have the following relation between the unit vectors:

$$\begin{pmatrix}\hat{i}\cdot\hat{i}' & \hat{j}\cdot\hat{i}' & \hat{k}\cdot\hat{i}'\\ \hat{i}\cdot\hat{j}' & \hat{j}\cdot\hat{j}' & \hat{k}\cdot\hat{j}'\\ \hat{i}\cdot\hat{k}' & \hat{j}\cdot\hat{k}' & \hat{k}\cdot\hat{k}'\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & \sin\phi & -\cos\phi\\ 0 & \cos\phi & \sin\phi\end{pmatrix}$$

Substituting the above values in **Theorem 3.1.1**, we have the relation between (a,A) and (δ,H) as:

$$\begin{pmatrix}\cos a\cos A\\ \cos a\sin A\\ \sin a\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & \sin\phi & -\cos\phi\\ 0 & \cos\phi & \sin\phi\end{pmatrix}\begin{pmatrix}\cos\delta\cos H\\ \cos\delta\sin H\\ \sin\delta\end{pmatrix}$$

From the above equation we can determine the values of a and A. Again being cautious about the range of a and A.

$$a \in \left[\frac{-\pi}{2},\frac{\pi}{2}\right] \quad \textit{and} \quad A \in [0,2\pi)$$

2.4 Summary

To find RA-DEC (α, δ) from inclination (i), true anomaly (f) and argument of periapsis (ω).

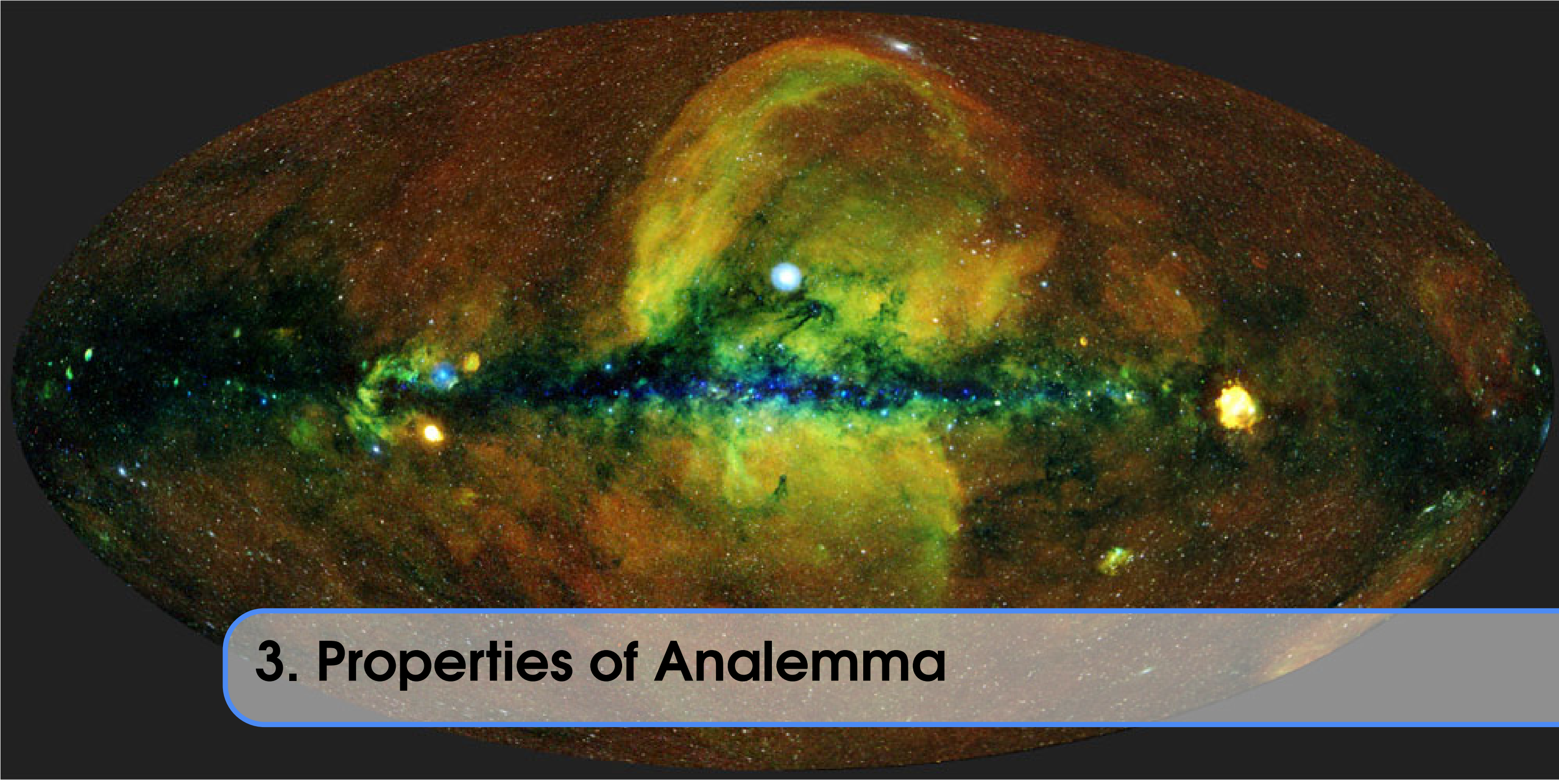
$$\begin{pmatrix} \cos\delta\cos\alpha \\ -\cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix} \begin{pmatrix} -\cos(f+\omega) \\ \sin(f+\omega) \\ 0 \end{pmatrix}$$

To calculate hour angle (H)

$$H = (\theta_{sidereal} + (L - L_0) + (\omega_{projection} + \alpha) - \pi) \mod 2\pi, \quad where \quad \alpha = Right\,Ascension$$

To calculate Alt-Az (a,A) from latitude (ϕ), declination (δ) and hour angle (H).

$$\begin{pmatrix} \cos a\cos A \\ \cos a\sin A \\ \sin a \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\phi & -\cos\phi \\ 0 & \cos\phi & \sin\phi \end{pmatrix} \begin{pmatrix} \cos\delta\cos H \\ \cos\delta\sin H \\ \sin\delta \end{pmatrix}$$



3. Properties of Analemma

3.1 Why an Analemma?

"Mean Sun" is defined as the apparent position of the sun from earth if eccentricity e and inclination i were both zero. Therefore,

$$
\begin{aligned}
f_{\text{mean sun}} &= \frac{2\pi}{P}t = M && \text{substituting } e=0 \text{ in equation} && (3.1)\\
\alpha_{\text{mean sun}} &= \omega + \pi + f_{\text{mean sun}} && \text{substituting i=0 in equation} \\
&= \omega + \pi + \frac{2\pi}{P}t && \text{substituting equation (3.1)} && (3.2)
\end{aligned}
$$

We define the *equation of time* eqt as :

$$
\begin{aligned}
eqt &= \alpha - \alpha_{\text{mean sun}} \\
&= \alpha - (\omega + \pi + \frac{2\pi}{P}t) && \text{substituting equation (3.2)} && (3.3)
\end{aligned}
$$

Now, consider the following figures :

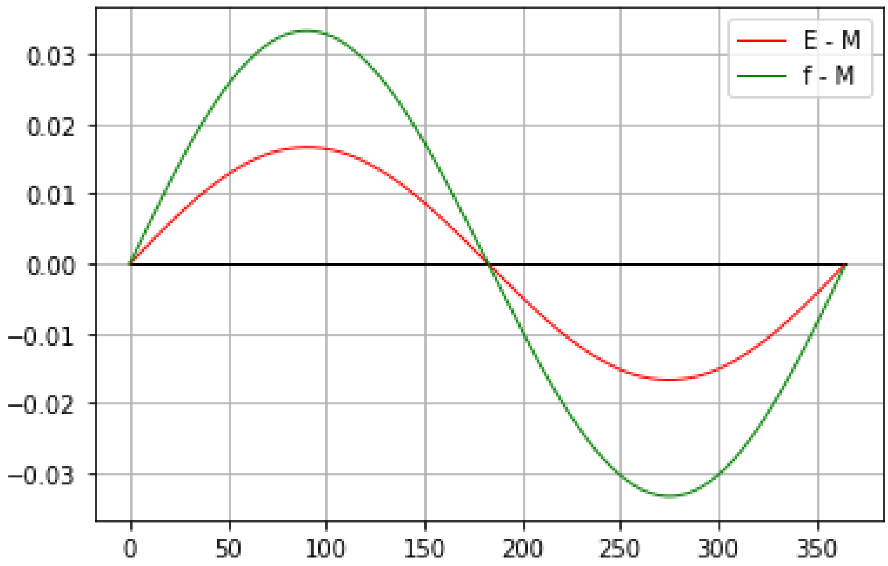


Figure 3.1: Variation of the anomalies w.r.t the mean for $e = 0.0167$. The earth's orbit being elliptical results in deviation of true anomaly from the mean. There are 3 nodes at $M = 0°, 180°, 360°$

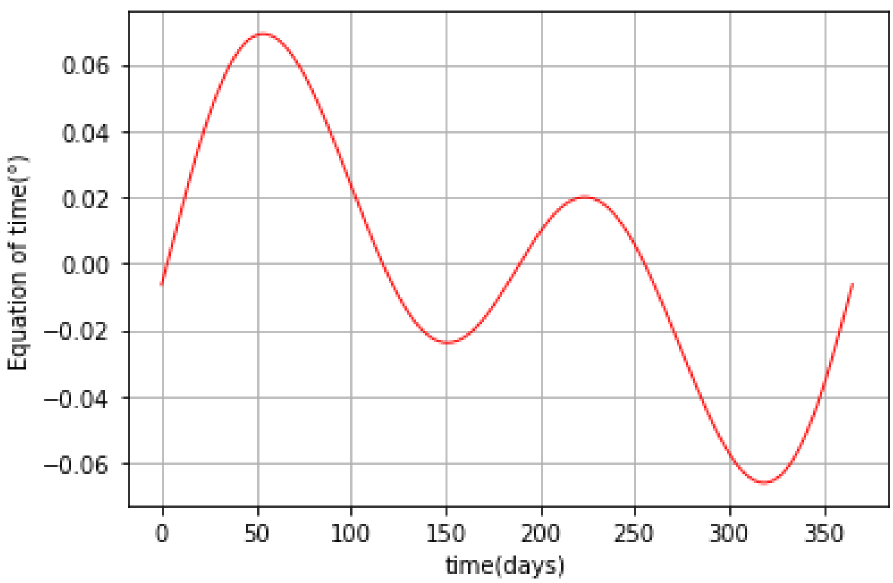


Figure 3.2: Equation of time vs time for $e = 0.0167, i = 23.5°, \omega = 85.901°$. The deviation of equation of time from zero is due to eccentricity and inclination. Inclination adds 2 more nodes about $M = 180°$.

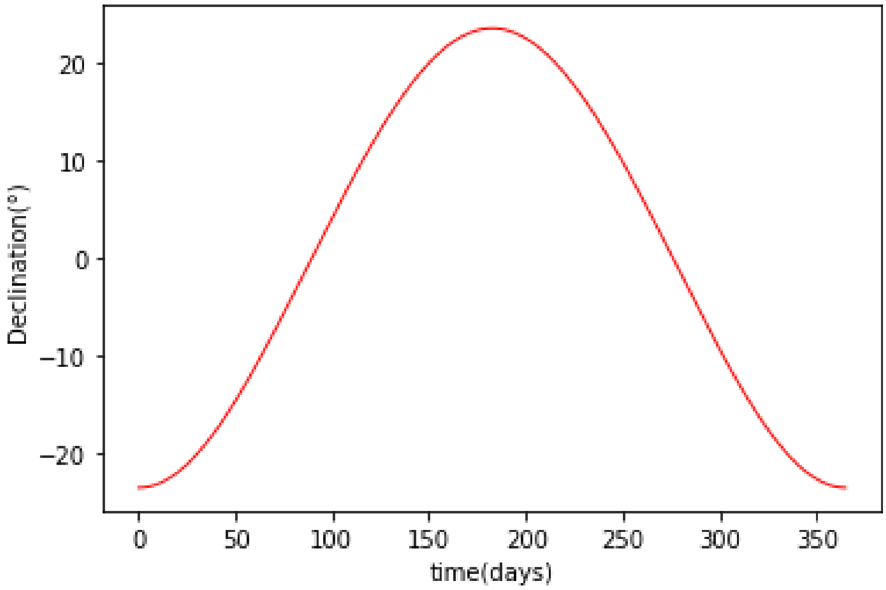


Figure 3.3: Variation of declination vs time for $e = 0, i = 23.5°, \omega = 85.901°$. The inclination of earth's axis of rotation causes the variation of declination δ.

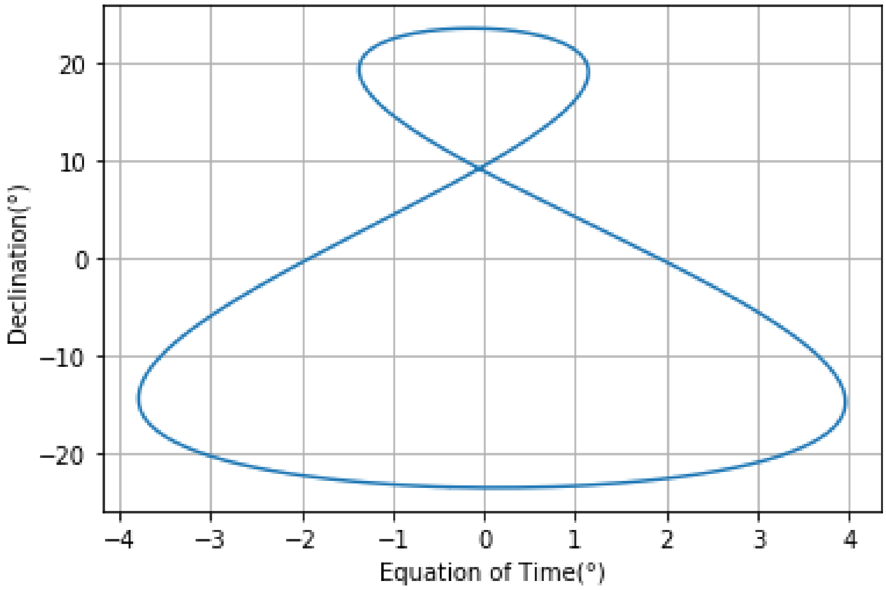


Figure 3.4: An analemma appears in the plot of eqt vs δ

3.2 The Analemma we see

Suppose :
at $t = \tau = 0$, the longitude where the sun is overhead is lon_0(radians), the observer's longitude lon(radians), and sidereal period T_{sdrl}.

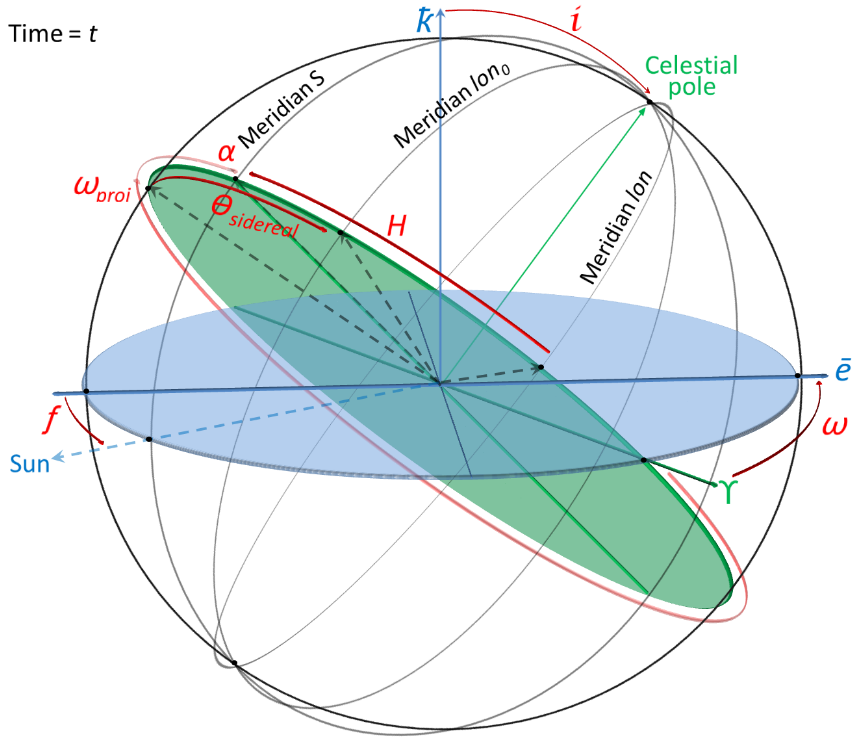


Figure 3.5: Visualisation with earth at the origin Geometric significance of orbital parameters as visualised from earth

From the figure 3.5, at some time t,

$$\theta_{\text{sdrl}} = (\frac{2\pi}{T_{sdrl}}t)\%2\pi \tag{3.4}$$

$$\begin{aligned} \alpha_{\text{obs}} &= (\omega_{\text{proj}} + \theta_{\text{sdrl}} + lon - lon_0)\%2\pi \\ &= (\omega_{\text{proj}} + \frac{2\pi}{T_{\text{sdrl}}}t + lon - lon_0)\%2\pi \qquad \text{substituting equation (3.4)} \end{aligned} \tag{3.5}$$

$$\begin{aligned} H &= \alpha_{obs} - \alpha \\ &= \Big[\omega_{\text{proj}} + \frac{2\pi}{T_{\text{sdrl}}}t + lon - lon_0 \\ &\quad - (eqt + \omega + \pi + \frac{2\pi}{P}t)\Big] \qquad \text{subst. eq. (3.3) and (3.5)} \\ &= \Big[-eqt + 2\pi(\frac{1}{T_{\text{sdrl}}} - \frac{1}{P})t \\ &\quad + (-\pi + lon - lon_0) + (\omega_{\text{proj}} - \omega)\Big]\%2\pi \end{aligned} \tag{3.6}$$

The mean solar day T_{solar} is defined as :

$$\frac{1}{T_{\text{solar}}} = \frac{1}{T_{\text{sdrl}}} - \frac{1}{P} \tag{3.7}$$

For the earth, $T_{\text{solar}} = 24$ hrs.

Suppose the local time of taking observations at observer's longitude lon is lt (same units as that of T_{solar}). At $t = \tau = 0$,

$$\text{local time at longitude } lon_0 = \frac{T_{\text{solar}}}{2}$$
$$\text{local time at longitude } lon = \frac{T_{\text{solar}}}{2} + (\frac{lon - lon_0}{2\pi})T_{\text{solar}}$$

Thus, the time of first observation t_0 is given by

$$\begin{aligned} t_0 &= lt - \text{local time at longitude } lon \\ &= \left[lt - \frac{T_{\text{solar}}}{2\pi}(\pi + lon - lon_0)\right] \% T_{\text{solar}} \\ \implies \frac{2\pi}{T_{\text{solar}}} lt &= \left[\frac{2\pi}{T_{\text{solar}}} t_0 + (\pi + lon - lon_0)\right] \% 2\pi \end{aligned} \tag{3.8}$$

Substituting equation (3.7) in equation (3.6)

$$\begin{aligned} H = \Big[&- eqt + \frac{2\pi}{T_{\text{solar}}} t + (+\pi + lon - lon_0) \qquad\qquad (-\pi)\%2\pi = +\pi \\ &+ (\omega_{\text{proj}} - \omega) \Big] \% 2\pi \end{aligned}$$

For $t = t_0 + nT_{\text{solar}}$,

$$\begin{aligned} H = \Big[&- eqt + \frac{2\pi}{T_{\text{solar}}} t_0 + (+\pi + lon - lon_0) \\ &+ (\omega_{\text{proj}} - \omega) \Big] \% 2\pi \\ = \Big[&- eqt + \frac{2\pi}{T_{\text{solar}}} lt + (\omega_{\text{proj}} - \omega) \Big] \% 2\pi \qquad \text{substituting equation (3.8)} \end{aligned} \tag{3.9}$$

It is necessary for the observations to be made in intervals of T_{solar} so that the data points in eqt are isometric to that in H and the analemma as seen in figure 3.4 is preserved in $\delta - H$ data set as well. It is only for $t = t_0 + nT_{\text{solar}}$ that the time dependence of hour angle H vanishes and it turns out as a mere translation of equation of time eqt. Also notice, equation (3.9) suggests hour angle is independent of lon and lon_0, which is to be expected as the assigning of longitude and local time values to the meridians is synonymous.

The analemma plot in alt-az is only a rotaion(an isometry) of that in $\delta - H$. Thus, all analemmas, observed at any latitude, longitude and local time, are isometric to the analemma in figure 3.4, differing only in their position and tilt.

3.3 Factors affecting the Analemma

In this section, we shall consider the parameters - (i)eccentricity e (ii)inclination i (iii)argument of perihelion ω (iv)semi-major axis a (v)latitude of observer lat (vi)longitude of observer lon, longitude of overhead sun lon_0 at $t = 0$ and local time of observation lt - and their effect on the characteristics of the analemma.

3.3.1 Eccentricity

- The plots are made for $i = 23.5°$, $\omega = 85.901°$, while varying eccentricity e

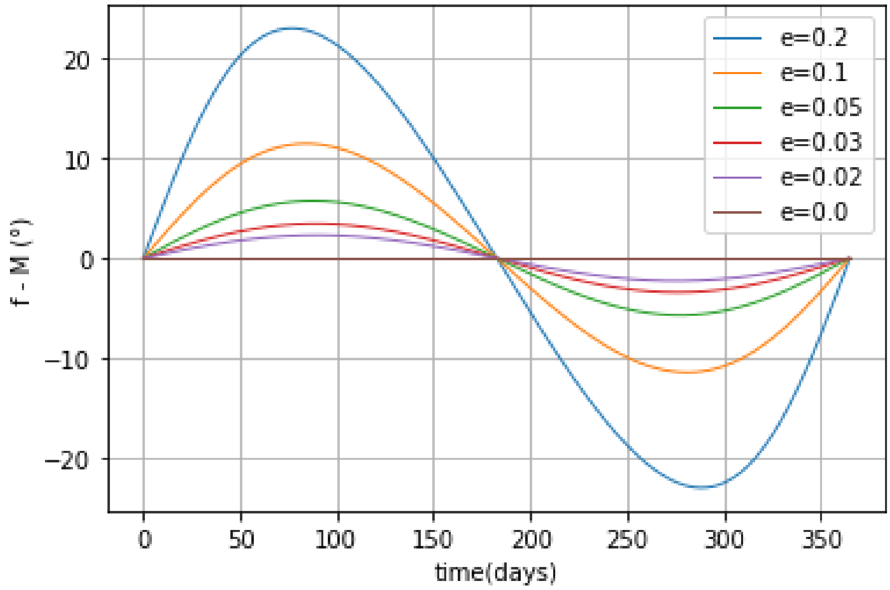


Figure 3.6: Equation of centre vs time

- As seen in figure 3.6, the amplitude of equation of centre $f - M$ increases with increase in eccentricity e.

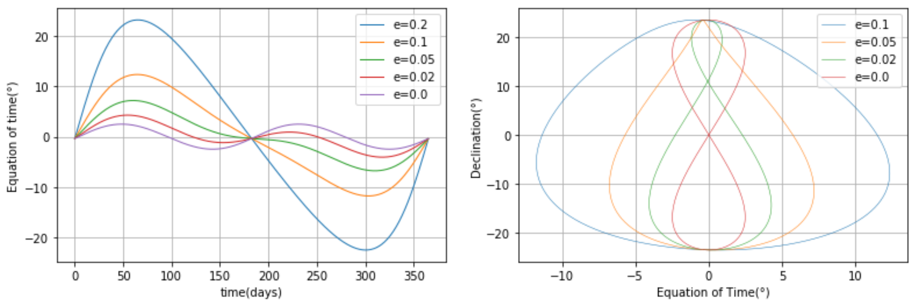


Figure 3.7: (i)Equation of time vs time (ii)analemma

- Greater the eccentricity, greater the spread in equation of time and hence, greater the width, perimeter and area of the analemma, as seen in figure 3.7.
- Increasing the eccentricity has no effect on the height of the entire analemma. However, it causes the height of one of the lobes to increase and that of the other to decrease, until there's only one lobe left.
- For a given inclination $i = 23.5°$, no. of nodes reduces from 5 to 3 after a particular value of eccentricity e, making the analemma single-lobed (for $e = 0.05, 0.1, 0.2$), instead of bi-lobed (for $e = 0.0, 0.02$).

3.3.2 Inclination

- The plots are made for $e = 0.0167$, $\omega = 85.901°$, while varying inclination i

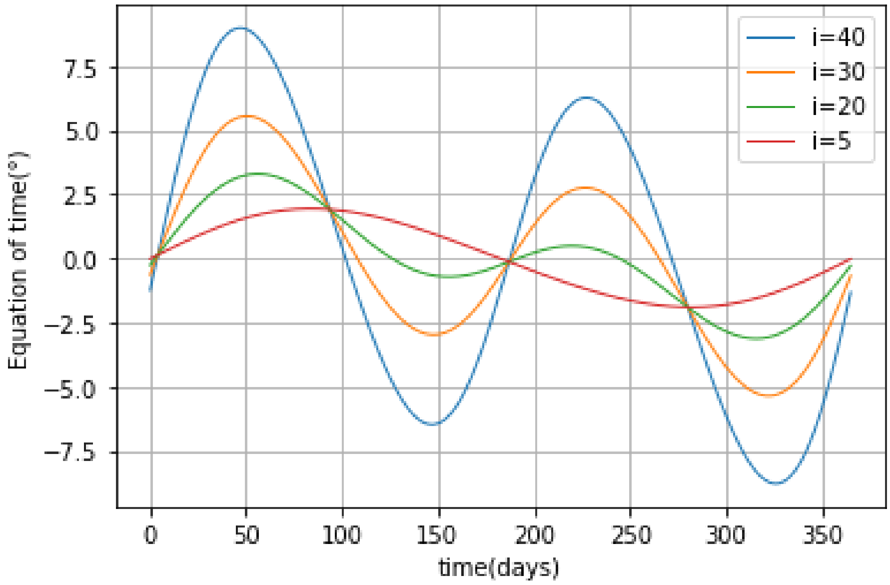


Figure 3.8: Equation of time vs time

- As seen in figure 3.8, the amplitude of equation of time increases with increase in inclination i. So does the width, perimeter and area of the analemma.
- It is interesting to note that all the curves pass through four points about $t = [0,90,270,360]$ days, which correspond to the points where $\alpha = [3\pi/2, 0, \pi/2, \pi]$. At these points, the circular measures of the sun's projection on both the orbital and the equatorial planes from the vernal equinox are equal, regardless of i. Thus the following relation : $\alpha - [f - (\pi/2 - \omega)]\%(2\pi) = 3\pi/2$, holds. Thus, these points correspond to $f = [x, x+\pi/2, x+\pi, x+3\pi/2]$, where $x = \pi/2 - \omega$, making α at these points independent of i. Since $\alpha_{\text{mean sun}}$ is independent of i as well, the equation of time at all these points turns out to be the same.

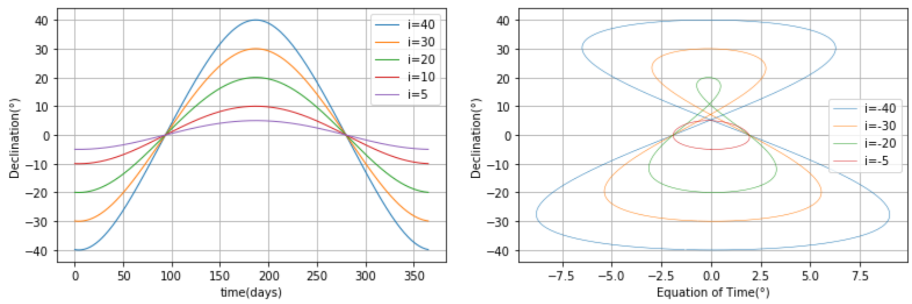


Figure 3.9: $(i)\delta$ vs time (ii)analemma

- Greater the inclination, greater the spread in declination δ and hence, greater the height of the analemma, as seen in figure 3.9.
- At the equinoxes i.e. $\alpha = [0, \pi]$ or $f = [x+\pi/2, x+3\pi/2]$, $\delta = 0°$. Minimum and maximum $\delta = [-i, +i]$ are reached for $\alpha = [3\pi/2, \pi/2]$ or $f = [x, x+\pi]$ respectively.
- For the given eccentricity $e = 0.0167$, the no. of nodes increases from 3 to 5 after a particular value of i, making the analemma bi-lobed instead of single-lobed.
- The following figures show the variation of height, width, perimeter and area

of the analemma against eccentricity e and inclination i.

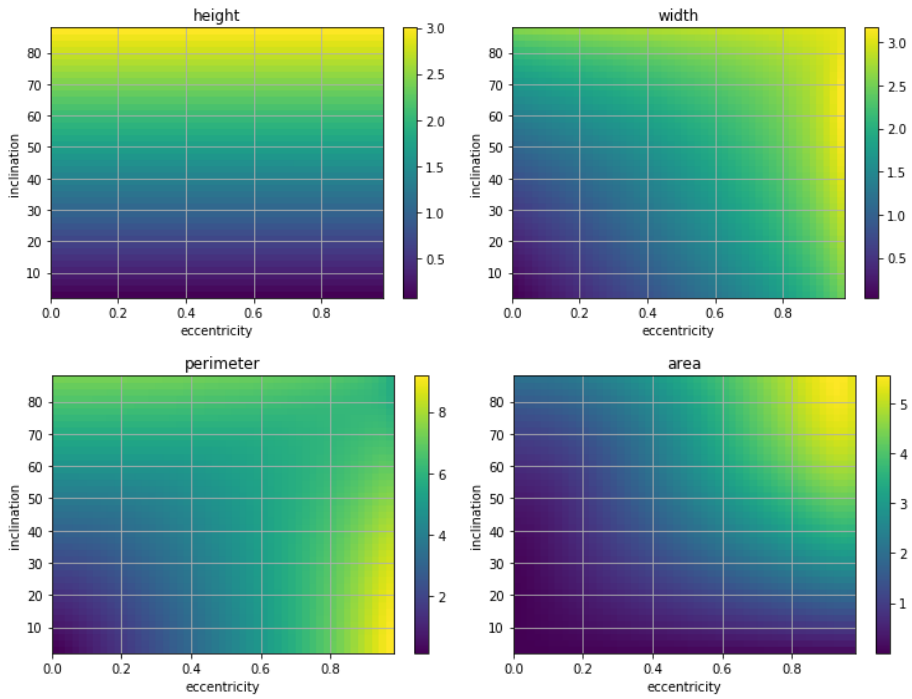


Figure 3.10: Properties of the entire analemma

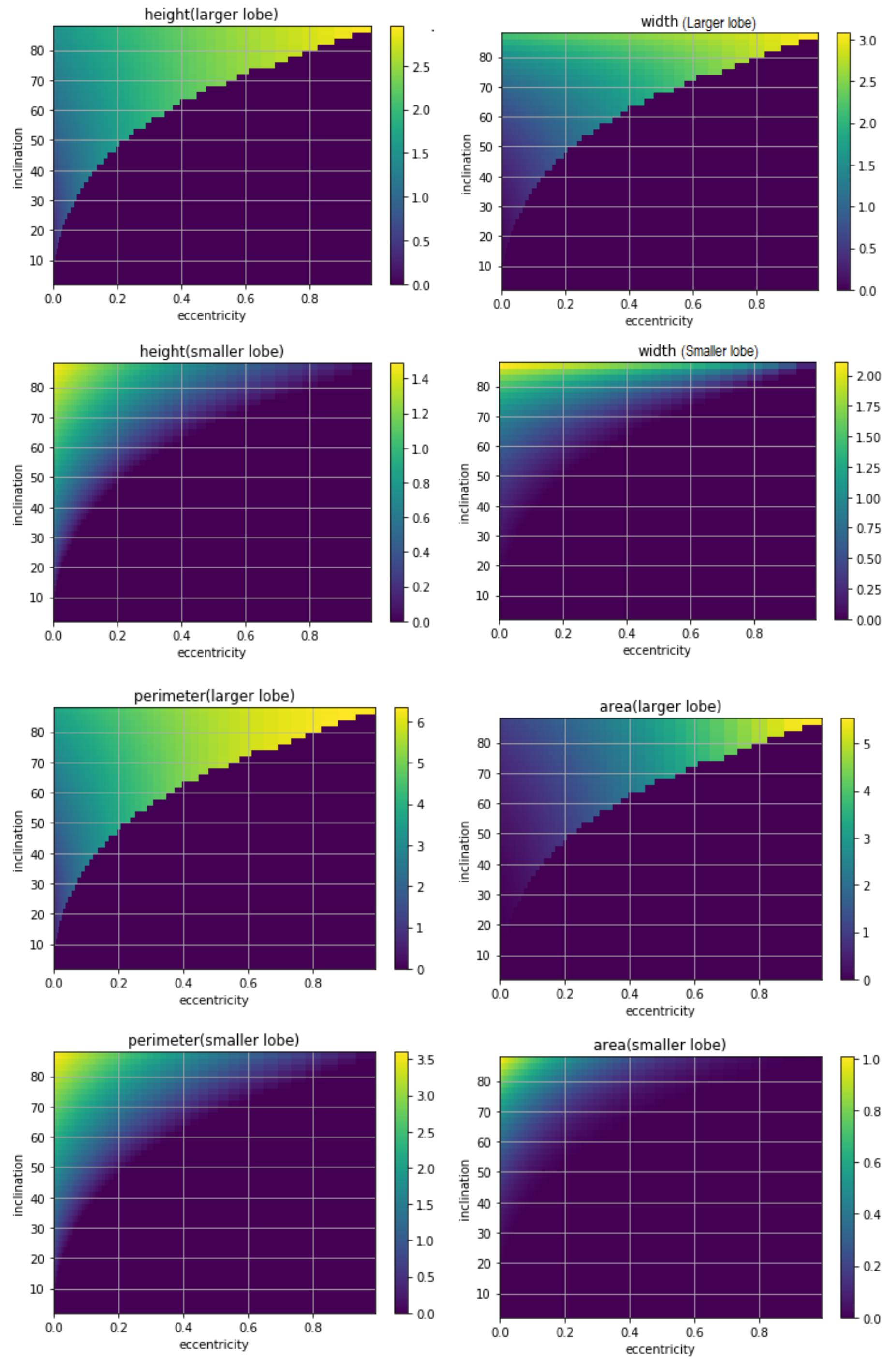


Figure 3.11: Properties of lobes of the analemma

- The values of eccentricity and inclination determine whether the analemma seen from a planet will be single- or bi-lobed. The following figures represent the region of separations of sinle- and bi-lobed analemmas, along with scatter points of the planets in our solar system.

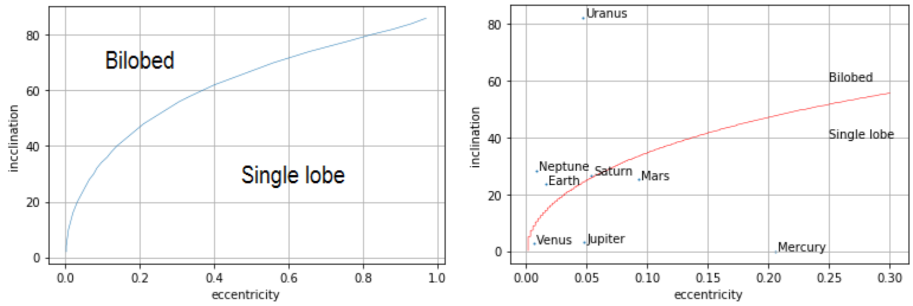


Figure 3.12: Curve separating single- and bi-lobed analemmas

3.3.3 Argument of Perihelion

- The following figures are for $e = 0.0167$ and $i = 23.5°$

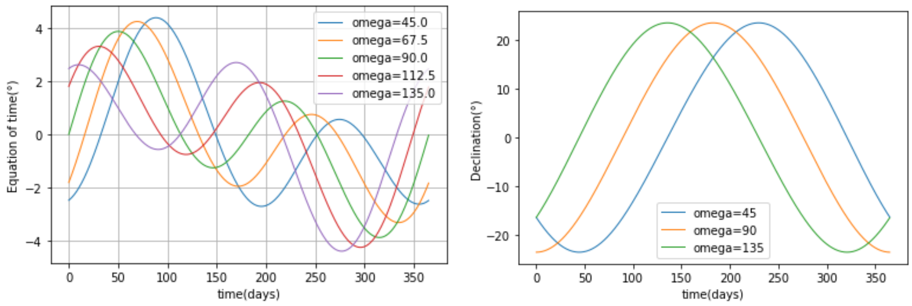


Figure 3.13: (i)Equation of time vs time $(ii)\delta$ vs time for varying ω

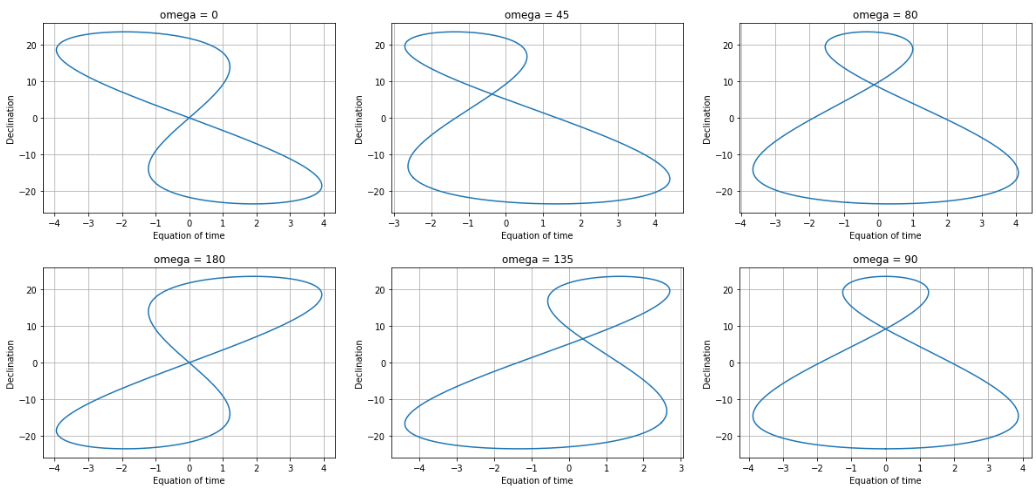


Figure 3.14: Analemmas for various ω

- Changing ω changes the amplitude as well as the phase of equation of time (see figure 3.13)and hence, skews the analemma, affecting the width, area and perimeter of the analemma.
- Varying ω merely changes the phase in declination, see figure 3.13, which affects the heights of individual lobes, but not the total height of the analemma.

3.3.4 Semi-major axis

Varying the semi-major axis a, keeping all other factors constant, changes the period of revolution P only. Thus, there will be more no. of data points tracing out the same analemma in the sky.

3.3.5 Latitude of observer

Latitude does appear in calculation of δ and equation of time. Hence, it does not affect the geometrical properties of the analemma. It is required in the conversion of $eqt - \delta$ into $alt - az$ coordinates and thus, it causes a mere rotation of the analemma in the observer's horizon sphere, along the meridian perpendicular to the equator, passing through celestial pole.

3.3.6 Local time of observation

Again, same as latitude, it does not affect any of the geometrical properties. It affects the hour angle H (see equation (3.9)) only and hence, causes rotation of the analemma along the equator in the observer's horizon sphere.

3.4 Verification against DE405 Model

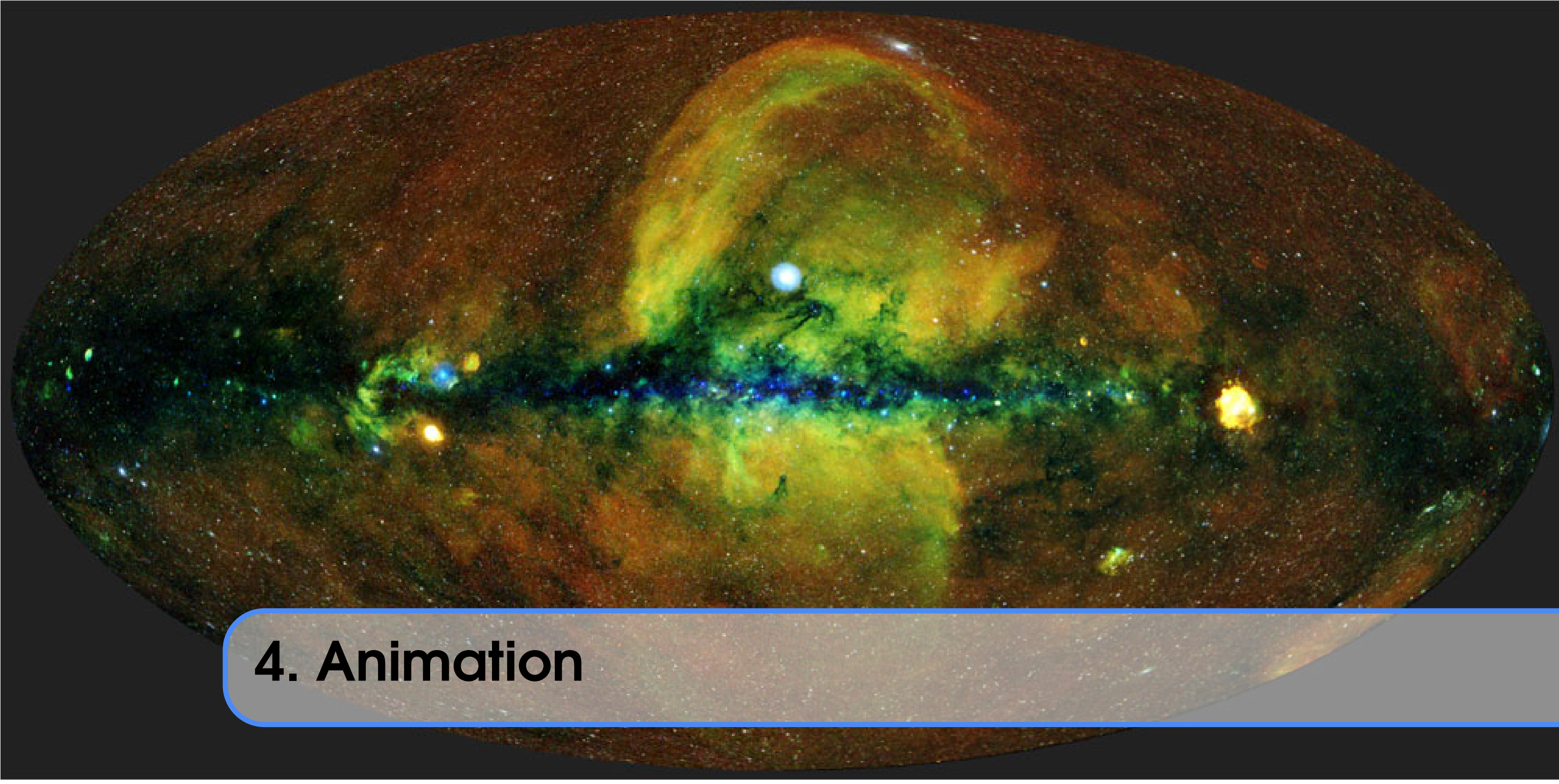
Next, we compare our generated analemma with the data from 2020 ephemeris of sun (see `http://astropixels.com/ephemeris/sun/sun2020.html`), w.r.t height, width, perimeter and area. We also test against sparse data sets, differing in frequency of observations. The algorithm to calculate these properties from the discrete data points, interpolates them to obtain a continuous plot, and hence, the categories of linear and cubic interpolation.

Height(radians)		Observed			Generated		
		Lobe 1	Lobe 2	Total	Lobe 1	Lobe 2	Total
	Linear				0.570115	0.250173	0.820288
	Cubic				0.570126	0.250173	0.820299
1 pic/2 days	Linear				0.570148	0.25014	0.820288
	Cubic				0.57019	0.250141	0.820331
1 pic/4 days	Linear	0.551879	0.257681	0.80956	0.570218	0.24986	0.820078
	Cubic	0.555035	0.257681	0.812716	0.570362	0.250071	0.820432
1 pic/7 days	Linear	0.563704	0.245392	0.809096	0.570338	0.249949	0.820288
	Cubic	0.565952	0.245887	0.81184	0.57073	0.24995	0.82068
1 pic/14 days	Linear	0.560464	0.248254	0.808718	0.570702	0.249586	0.820288
	Cubic	0.562614	0.249152	0.811766	0.572122	0.249587	0.821708
1 pic/28 days	Linear	0.556977	0.251711	0.808688	0.571944	0.237085	0.809029
	Cubic	0.564365	0.252839	0.817204	0.577379	0.248165	0.825544

Width(radians)		Observed		Generated			
		Lobe 1	Lobe 2	Lobe 1	Lobe 2	Lobe 2	Total
1 pic/day	Linear			0.131152	0.0413856	0.250173	0.820288
	Cubic			0.131156	0.0413856	0.250173	0.820299
1 pic/2 days	Linear			0.131126	0.041346	0.25014	0.820288
	Cubic			0.131156	0.0413856	0.250141	0.820331
1 pic/4 days	Linear	0.130838	0.0391477	0.131126	0.0413453	0.24986	0.820078
	Cubic	0.131478	0.0397854	0.131156	0.0413856	0.250071	0.820432
1 pic/7 days	Linear	0.128532	0.0394406	0.130866	0.0412266	0.249949	0.820288
	Cubic	0.128665	0.0395096	0.131156	0.0413861	0.24995	0.82068
1 pic/14 days	Linear	0.1356	0.046036	0.129903	0.0403694	0.249586	0.820288
	Cubic	0.137097	0.0476758	0.131218	0.0414046	0.249587	0.821708
1 pic/28 days	Linear	0.128966	0.0531224	0.129903	0.0403694	0.237085	0.809029
	Cubic	0.134251	0.0539593	0.132216	0.0408959	0.248165	0.825544

Perimeter (radians)		Observed			Generated		
		Lobe 1	Lobe 2	Total	Lobe 1	Lobe 2	Total
1 pic/day	Linear				1.2028	0.516347	1.71915
	Cubic				1.20282	0.51863	1.72145
1 pic/2 days	Linear				1.20282	0.516248	1.71907
	Cubic				1.2035	0.516296	1.71979
1 pic/4 days	Linear	1.27144	0.539675	1.81111	1.20279	0.515967	1.71876
	Cubic	1.30267	0.541678	1.84434	1.21174	0.516157	1.72789
1 pic/7 days	Linear	1.19603	0.512177	1.70821	1.20257	0.515341	1.71791
	Cubic	1.20592	0.522828	1.72875	1.22703	0.51592	1.74295
1 pic/14 days	Linear	1.1811	0.510247	1.69135	1.20126	0.513083	1.71434
	Cubic	1.19479	0.525798	1.72059	1.26594	0.5152	1.78114
1 pic/28 days	Linear	1.16073	0.512455	1.67319	1.19633	0.499733	1.69606
	Cubic	1.1811	0.526512	1.70761	1.35001	0.512949	1.86296

Area(sterad)		Observed			Generated		
		Lobe 1	Lobe 2	Total	Lobe 1	Lobe 2	Total
1 pic/day	Linear				0.048868	0.00705769	0.055925
	Cubic				0.0488718	0.00705921	0.055931
1 pic/2 days	Linear				0.0488568	0.00705338	0.055910
	Cubic				0.0488718	0.00705934	0.055931
1 pic/4 days	Linear	0.0426824	0.00645157	0.0491339	0.0488119	0.00703637	0.055848
	Cubic	0.0427158	0.00647072	0.0491866	0.0488724	0.00705953	0.055931
1 pic/7 days	Linear	0.0448453	0.00627013	0.0511155	0.0486993	0.00698085	0.055680
	Cubic	0.0450707	0.00633311	0.0514038	0.0488755	0.00705953	0.055935
1 pic/14 days	Linear	0.0451197	0.00599354	0.0511133	0.0481331	0.00674557	0.054878
	Cubic	0.045596	0.00628239	0.0518784	0.0488526	0.00705956	0.055912
1 pic/28 days	Linear	0.042083	0.00556488	0.0476479	0.0459114	0.00585697	0.051768
	Cubic	0.0442707	0.00663247	0.0509032	0.0486456	0.00699977	0.055645



4. Animation

4.1 Introduction to Blender

Blender3D is an amazingly powerful free open-source graphics software with a wide tool set suitable for almost any 3D graphical tasks. Creating animated films, visual effects, art, 3D printed models, motion graphics, interactive 3D applications, virtual reality and computer games are only a few of the applications of this software. With a relatively small learning curve, the basics of this software can be learnt in under a month with the ability to render out more impressive and satisfying work within four months. Of course, this has a near endless possibilities and even professionals occasionally stumble upon a few new tricks.

While a majority of blender users use the point,click,drag and create based on visual appeal, this project involved a certain level of accuracy and precision in the final outputs. Considering this, Blender provides a useful API and all procedural steps can be implemented via script. This also reduces repetitive effort and aids in providing finer control over the positioning of objects and timings in animations.

4.2 Basic Elements of Blender

Blender offers a wide palette of tools and objects that one can make use of for their animations. The most fundamental elements of any animation include the camera, lighting, environment settings and the objects of focus. We can consider each element in further detail.

4.2.1 Cameras in Blender

The camera is the most important object for rendering images and animations. The camera largely defines the appearance of the final resulting render. Cameras are invisible in renders, so they do not have any material or texture settings. The camera lens options allow the user to modify and control how 3D objects are projected in the 2D image. Perspective, orthographic and panoramic cameras are some of the types of cameras that you can choose from, depending on the final style and structure of the animation you wish to render. The focal length or field of view can be modified to change the extent of the zoom. This helps you determine what fraction of the scene is visible in the final animation.

4.2.2 Lighting in Blender

Proper lighting can make all the difference when it comes to the quality and feel of the rendered animation. There are numerous methods to illuminate a scene in Blender. The most common one is to add a “Lamp” , which can be of the following five types:

- Sun
- Point
- Area
- Hemi
- Spot

The above lighting options have different features, such as the direction, power, area of impact, and so on. Using a "point" light could be useful to render a sunrise or a sunset, while "area" could be more useful for uniformly lighting up a planet against a dark background. In astronomy-related animations, which is our current domain of interest, lighting becomes important to highlight subtler features or terrains of objects, although very often we will use a luminous object in place of a light, to create a star, for example. We will explore such luminous shaders under the "textures" section.

4.2.3 Setting up the Environment

The "world" defines the environment that the scene is in. The surface shader sets the background and environment lighting, either as a fixed color, sky model or HDRI texture.

5. Replicating this work

5.1 Prerequisite Knowledge

While we will attempt to provide instructions such that it is easy for anyone to follow, a base level knowledge of Python along with an idea of how to navigate around Blender would be ideal.
For a basic idea about Python and a few basic tutorials on Blender

We have split our work into two files, the first one to generate the analemma and another to show the corresponding position of the Earth's position in its orbit.

5.2 Analemma

The main analemma script is structured such that the main analemma generating functions are kept in a separate folder from which the necessary functions are imported. These are custom functions that give us the synthetic analemma data in the desired coordinate system. for our purposes, we take 52 data points in the Cartesian system. 52 spheres are then initiated at the first data point which are stored in an array for future reference. These spheres are equipped with a simple material made by connecting a RGB node to emission shader. See picture 5.1 for the whole material setup. Different strength values are used for the placeholder spheres and main sun spheres.

Then a frame array was created by separating the total frames at regular intervals using numpy's linspace function. Using this to set the frames, locations of the spheres are changed according to the data points and keyframes added. The camera location must be at the origin of blender space since that is how we have set up the data points. The orientation and focal length is then changed so as to have the entire analemma in view. For the environment, an HDRI image is added as a world texture. Other still shots rendered are minor variations of the procedure detailed above.

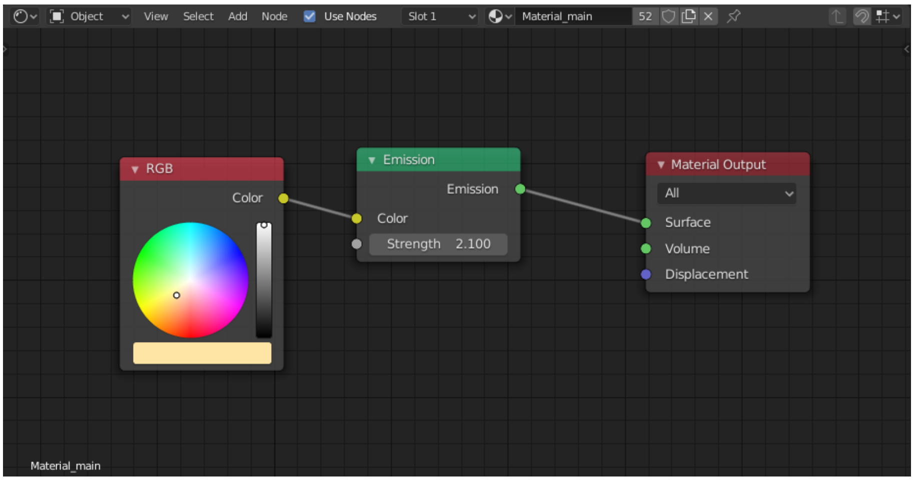


Figure 5.1: Material node structure for sun spheres

5.3 Orbit Animation

For the orbit animation, a similar workflow was followed. The materials were made using the node editor with the Sun using a lava texture and a NASA image for the Earth material. Data points for the orbit spheres were obtained by obtaining 365 points in the Ra-Dec system from above mentioned function. These data points were then rotated to lie on the XY plane. This made it easier to position the camera. The axis of the Earth was a simple thin cylinder joined to the sphere via a few lines of code. The material for the orbit spheres is similar to the material used for the spheres in the Analemma animation. The world(background) material was created by simply adding noise to a black background. Another script was added in the Functions folder to locate the points of interest from the raw data, i.e. the extremities of the orbit along with the autumn and spring equinox and summer and winter solstice. We marked these points in red by changing the material via a line in the script.

5.4 Bringing it Together

To add the orbit animation as an inset as well as the text, we used Adobe After Effects. Blender is not particularly suited for such tasks hence we used a video editor to add the final touches and complete our animation

All blend files along with complementary materials can be found in the official GitHub repository.

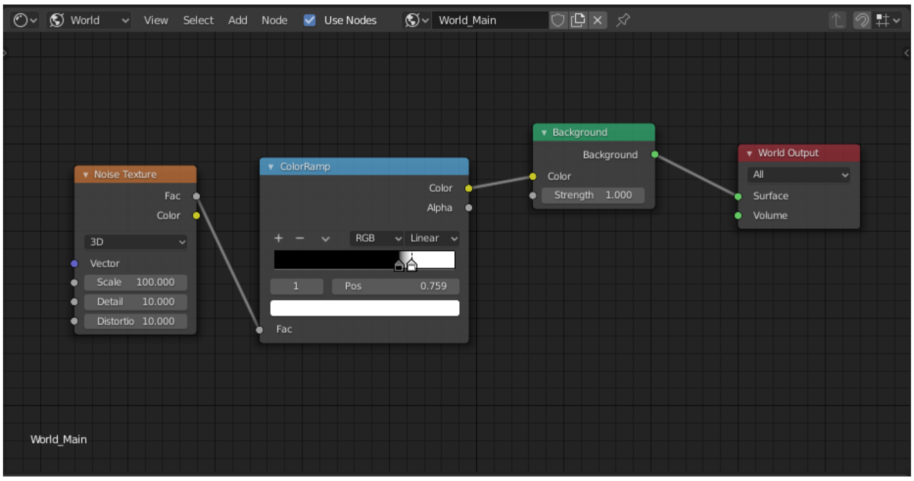


Figure 5.2: Background material used in Orbit Animation

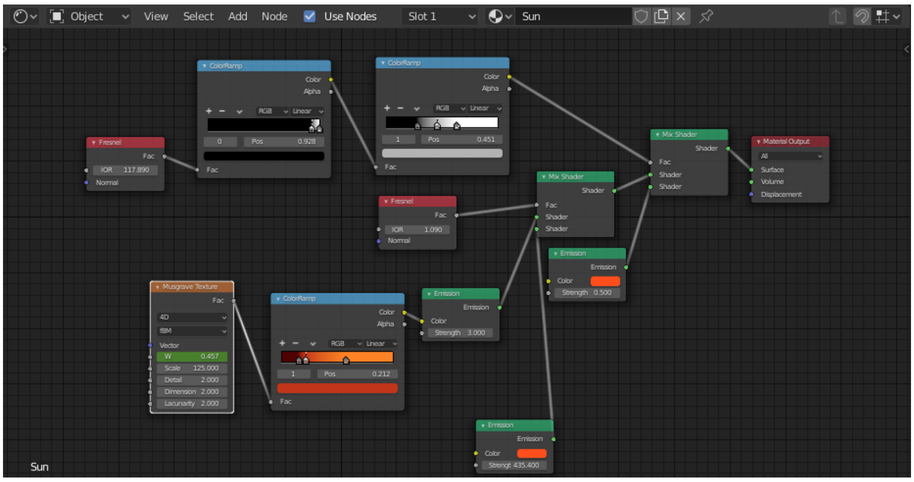


Figure 5.3: Sun Material used in Orbit animation

References