Online Math Challenge

IISc Pravega 2014

pravega.org

1 Rules

1. Each of the questions carry equal weightage.
2. The solutions should be neatly written and then scanned or typed and should be emailed to gonitsora@pravega.org before January 20th 2014.
3. Extra observations or comments will be awarded with extra points, but be brief and logical.
4. There can be more than one solution to a problem, and extra points would be awarded if someone gives more than one solution.
5. No team entries allowed.
6. All students from any educational institutions are permitted to participate.
7. Prizes will be awarded after verification of the participant's identity.
8. Winner will be declared based on number of correct answers and time taken to solve.

2 Problems

1. How many times in a day is it impossible to tell the time by a clock with identical hour and minute hands, provided we can always tell whether it is a.m. or p.m.? What happens if we cannot distinguish between a.m. and p.m.?
2. What is the maximum number of unit squares that can touch each other?
3. A fully booked theatre is about to screen *A Beautiful Mind* to a group of 100 mathematicians. The 1st mathematican was thinking about the *Riemann Hypothesis* and somehow misplaced his ticket. As a consequence, he could not find the seat alloted to him and decided to sit at a random seat. From then on, any mathematician if he finds his seat occupied, politely goes and seats at a random seat. What is the probability that the 100th mathematician seats in his own seat?
4. Imagine you are playing a new version of the game of chess, called *Pravega chess*. The rules are same, with just one exception. You have to play $2k$ moves at a time, where k is any natural number. Prove that, black who plays second always has a non-losing strategy.